Escaping the Middle-income Trap: The Role of Strategic Public Investment and Industrial Structure Change

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Abstract

This article examines how the middle-income trap occurs in developing economies, and how fiscal policies and development strategies can help them avoid or escape this development challenge. Using a dual-sector model with public investment in infrastructure and education, we analyze how different policy choices affect growth trajectories and development outcomes. The findings suggest that appropriate fiscal policies and investment strategies are crucial for avoiding development traps and achieving sustainable economic growth. However, insufficient infrastructure investments and a high tax rate can lead to the stagnation of industrialization and economic development, potentially leading to a middle-income trap. The model also reveals an important relationship between poverty and middle-income traps, showing that even after escaping initial poverty through external assistance, countries need appropriate fiscal policies to achieve sustainable development.

Key Words middle income trap; industrial transition; public investment; fiscal policy

JEL-Classification E22; E62; H54; O11; O41

1 Introduction

In the post-World War II period, many developing countries experienced remarkable economic growth and industrialization, transforming from low-income agricultural

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economies to more industrialized ones. This transformation enabled many nations to achieve middle-income status through manufacturing and export growth. It was particularly evident in East Asian economies, where countries like Japan, South Korea, and Taiwan achieved sustained high growth rates (The World Bank, 1993). However, many other developing countries were unable to progress beyond middle-income levels and transition to high-income status, leading to what economists termed the "middle-income trap." This concept was introduced by Gill and Kharas (2007, 2015) to describe the economic phenomenon wherein countries successfully grow from low to middle-income levels but struggle to advance further into high-income status.

According to the World Bank (2024), middle-income countries are defined as nations with a gross national income per capita between $1,146 and $14,005. For example, countries like Indonesia, Thailand, Iran, and Mexico have remained in the middle-income category for several decades despite their initial rapid growth and industrialization (Robertson and Ye, 2013). As such, understanding why these countries remain stuck in middle-income status, what prevents them from achieving sustained economic development, and how they can overcome these challenges to achieve high-income status are important issues.

An extensive literature has examined the factors that contribute to the middle-income trap and potential strategies for overcoming this development challenge (Glawe and Wagner, 2016; Agénor, 2017). For example, Agénor and Canuto (2015) and Lee and Park (2024) explore how a decrease in productivity growth and human capital accumulation can lead to economic stagnation and prevent countries from escaping the middle-income trap. Agénor and Canuto (2015) develop a theoretical framework for understanding how advanced economies successfully escaped the middle-income trap through advanced infrastructure improvements, such as high-speed internet connectivity, digital networks, and advanced telecommunications infrastructure. From another perspective, Kharas and Kohil (2011) and Dabús et al. (2016) analyze how export diversification and technological upgrading can help middle-income countries escape economic stagnation and transition to high-income status. Despite this literature, the theoretical analysis of the middle-income trap remains limited compared to empirical studies that focus on understanding the underlying mechanisms and dynamics of economic growth and development in middle-income countries.

Meanwhile, research on East Asian economies like South Korea and Taiwan has provided valuable lessons about successfully transitioning from middle-income to high-income status. For example, Ohno (2009) and Aoki (2012) highlight how these economies invested heavily in education, technology adoption, and industrial upgrading to avoid

stagnation at middle-income levels. Ohno (2009) examines how to avoid the middle-income trap in Vietnam and other Southeast Asian economies through industrial upgrading policies, technology transfer mechanisms, and human capital development strategies that promote innovation and productivity growth. The author shows how successful industrial policies and strategic economic planning can drive sustainable growth and technological advancement in developing economies. Aoki (2012) regards the middle-income trap as a phenomenon where a shift from agricultural to industrial employment stagnated, and led to a slowdown in productivity growth and technological advancement in many middle-income economies. The author reveals that a key factor that caused economic growth stagnation was the lack of effective industrial transition policies, leading to inefficient resource allocation and missed opportunities for economic advancement.

Again, despite the extensive empirical studies and research on the middle-income trap, few theoretical frameworks explain the complex economic mechanism of the middle-income trap. Glawe and Wagner (2016) highlight that there are limited theoretical frameworks and mathematical models for analyzing the middle-income trap with the concept of employment transition and industrial upgrading. This research gap presents opportunities for developing more rigorous theoretical models to analyze the dynamics of structural transformation and economic development in middle-income economies. Next, Villamil et al. (2020) and Fernandes and du Oliveira (2024) use a theoretical framework which can analyze the relationship between employment transitions, industrial change, and economic growth in developing countries. This theoretical framework adopts a dual sector model, which is an extension of a Lewis development framework (Lewis, 1954), to analyze how employment transitions between traditional and modern sectors affect economic growth and development. Villamil et al. (2020) suggest that the dual-sector model can be used to analyze how employment transitions and structural change affect economic development outcomes in developing countries, whose labor market has structural heterogeneity in terms of productivity, wages, and working conditions between traditional and modern sectors. Fernandes and du Oliveira (2024) provide valuable insights into how employment transitions and structural transformation can shape economic development trajectories through fiscal policy adjustments and structural reforms. However, these theoretical frameworks still need further development to fully capture the complex dynamics of employment transitions and structural transformation in middle-income economies.

Unlike the analysis of the middle-income trap, the poverty trap concept has been more extensively studied and modeled through rigorous mathematical frameworks that

analyze economic dynamics and equilibrium conditions. The poverty trap is characterized by self-reinforcing mechanisms that keep poor countries and limited economic development. Fernandes and du Oliveira (2024) also refer to the importance of public investment in promoting economic growth for developing countries which face development trap. The poverty trap is also a significant challenge closely related to the middle-income trap, as both phenomena involve structural barriers to economic development and growth in developing nations. However, few studies have examined the relationship between poverty traps and middle-income traps from a theoretical perspective.

Here, we develop a theoretical framework to analyze how employment transitions and structural transformation affect economic growth in middle-income countries, and examine the relationship between poverty and middle-income traps. Specifically, we develop a dual-sector model that incorporates both employment transitions and poverty dynamics to analyze structural transformation patterns in middle-income economies. We find that appropriate fiscal policies and investment strategies are crucial for avoiding development traps and achieving sustainable economic growth. However, insufficient infrastructure investment and high tax rate can lead to stagnation of industrialization and economic development, potentially leading to a middle-income trap. We also reveal an important relationship between poverty traps and middle-income traps, showing that even after escaping the initial poverty through external assistance, countries need appropriate fiscal policies to achieve sustainable development.

The remainder of this work proceeds as follows. The model structure is presented in the second section. Then, the model behavior in the long run is analyzed in the third section. Cases of successful development and the poverty trap are simulated and illustrated in the fourth section. In the fifth section, the cases of middle-income trap are analyzed and illustrated. Finally, the study closes with a summary of the main conclusions.

2 Model

2.1 Model structure

We consider the middle-income trap as a situation where a country struggles to progress from middle- to high-income status due to structural challenges in industrial upgrading and structural transformation. This definition aligns with Aoki (2012). Moreover, we distinguish between the middle-income and poverty traps. A poverty trap occurs when

countries remain stuck in low-income status due to self-reinforcing mechanisms that prevent economic development and growth.

We use a dual-sector model to analyze the economic growth patterns and structural changes in developing countries, focusing on how fiscal policies impact industrial transformation to illustrate the causes of and strategies to avoid the middle-income trap. The dual-sector model is based on the Lewis (1954) development framework which analyzes economic development through the lens of labor transitions between traditional agricultural and modern industrial sectors. The traditional sector is modeled with low productivity and wages, characterized by surplus labor and traditional production methods. Meanwhile, the modern sector features higher productivity, advanced technology, and better wages. This model allows us to examine how labor allocation, productivity differentials, and technological progress influence the economic growth in the developing countries. However, the Lewis model lacks a micro foundation, which makes it difficult to analyze further development and be used rigorously in empirical research (Wang and Piesse, 2013; Villamil et al, 2020). Therefore, Villamil et al. (2020) and Zhang et al. (2022) develop a dual-sector model with micro-foundations to analyze structural transformation and economic development. Thus, we adopt a dual-sector model with micro-foundations to analyze how fiscal policies and structural transformation affect economic growth and development in developing economies.

This model is based on Felice (2016), Goes and Teixeira (2022), and Fernandes and de Oliveira (2024). While the model structure is similar to Fernandes and de Oliveira (2024), we extend their model by incorporating infrastructure investment and education spending as key determinants of productivity growth and structural transformation in developing economies. The model's main hypotheses are as follows:

1. The economy is a closed system with a labor surplus and two sectors, the traditional or subsistence sector, S, and modern sector, M.

2. The traditional sector uses labors with labor-intensive production technique, while the modern sector uses capital stocks and labors with capital-intensive production technique.

3. The S good is used only for consumption, while the M good is available for both consumption and investment in the modern sector.

4. The traditional sector's surplus labor provides a continuous supply of workers to the modern sector at a constant real wage rate.

5. The economy comprises two classes: labors and capitalists. Labors receive a wage from their work in the traditional and modern sectors, while capitalists earn profits from their investments in the modern sector.

6. The labors do not save money but spend their entire income on consumption of both S and M goods. The capitalists spend their income on consumption of S and M goods, and save a portion of their profits for investment in the modern sector. The rate of consumption between S and M goods is constant and same between labors and capitalists.

7. The government imposes a tax on the modern sector's profits to invest in two types of public goods: infrastructure and education.

8. The price is measured in real terms, where the M good is the numeraire.

This model framework enables a detailed analysis of how government policies and structural transformations affect development paths.

2.2 The household

2.2.1 The capitalist's behavior

The capitalist's problem is to maximize their utility subject to their budget constraint, considering their income from profits and their savings decisions for future investment; that is, to solve the following dynamic optimization problem:

$$\text{Max } U^c = \int_0^\infty \ln[(C_m^c)^\lambda (C_s^c)^{1-\lambda}] \exp(-\rho t)\ dt \tag{1}$$

$$\text{s.t. } \dot{K} = (1-\tau) rK - C_m^c - pC_s^c - \delta K \tag{2}$$

This equation represents the capitalist's intertemporal optimization problem, where C_s^c and C_m^c denote the consumption of traditional and modern goods, respectively. r is the profit rate, δ is the depreciation rate of capital stock, τ is the tax rate, and ρ is the time discount rate. The capitalist uses λ for consumption goods in the modern sector and $1-\lambda$ for consumption goods in the traditional sector.

Here, the state variable is the capital stock, and control variables are the consumption levels of traditional and modern goods.

The Hamiltonian of constant value is:

$H \equiv \lambda \ln C_m^c + (1-\lambda) \ln C_s^c + \mu^c \left[(1-\tau) rK - C_m^c - pC_s^c - \delta K\right]$

Applying the Principle of the Pontryagin Maximum, the equation of the consumption ratio between S and M good is:

$$\frac{C_s^c}{C_m^c} = \frac{1}{p} \cdot \frac{1-\lambda}{\lambda}, \tag{3}$$

and the Euler equation of consumption of good S and M are:

$$\frac{\dot{C}_m^c}{C_m^c} = (1-\tau)r - \rho - \delta \tag{4}$$

$$\frac{\dot{C}_s^c}{C_s^c} = (1-\tau)r - \rho - \delta - \frac{\dot{p}}{p}. \tag{5}$$

The equation of the trajectory of the capitalist's capital stock is given by (2) and (3):

$$\frac{\dot{K}}{K} = (1-\tau)r - \frac{1}{\lambda} \cdot \frac{C_m^c}{K} - \delta. \tag{6}$$

In addition, we need the transversality condition (TVC):

$$\lim_{t\to\infty} \mu^c K e^{-\rho t} = 0. \tag{7}$$

Now, let $\chi \equiv C_c^M/K$. Then, the differential equation of χ is:

$$\dot{\chi} = \left(\frac{1}{\lambda}\chi - \rho\right)\chi. \tag{8}$$

A candidate for the steady state value of χ is $\chi = 0$ or $\chi = \lambda\rho$. The steady state $\chi = 0$ is stable but $\chi = \lambda\rho$ is unstable. C_c^M is a non-predetermined jump variable, and hence, C_c^M is also a jump variable. Therefore, the capitalist chooses $\chi = \lambda\rho$ at the initial point in time. Hence, there are no transitional dynamics of χ (Taveni, 2013). From this, we obtain:

$$\frac{\dot{K}}{K} = (1-\tau)r - \rho - \delta. \tag{9}$$

In this case, the capitalist's saving rate is:

$$\begin{aligned} s^c &= \frac{rK - C_m^c - pC_s^c}{rK}. \\ &= 1 - \frac{\rho}{r} \end{aligned} \tag{10}$$

2.2.2 Labor's behavior

Labor's problem is to maximize their utility subject to their wage constraint by choosing optimal consumption levels of traditional and modern goods; that is, to solve the following utility optimization problem:

$$\text{Max } U^w = \ln[(C_m^w)^\lambda (C_s^w)^{1-\lambda}] \tag{11}$$

$$\text{s.t. } C_m^w + pC_s^w = wL \tag{12}$$

This equation represents the labor's utility maximization problem, where C_s^w and C_m^w denote the consumption of traditional and modern goods, respectively. w is the wage. Besides the capitalist, the labor uses λ for consumption goods in the modern sector and $1-\lambda$ for consumption goods in the traditional sector.

The Lagrangian is: $L \equiv \lambda \ln C_m^w + (1-\lambda) \ln C_s^w + \mu^w \left[wL - C_m^w + pC_s^w\right]$

From the first order conditions, we obtain:

$$\frac{C_s^w}{C_m^w} = \frac{1}{p} \cdot \frac{1-\lambda}{\lambda}. \tag{13}$$

2.3 The government

The government expenditure is allocated between infrastructure and education. The infrastructure, k_g, is the stock of public capital that accumulates over time through government investment and depreciates at a constant rate. We model infrastructure as a public capital that is nonexcludable, free of charge, and rival, meaning that each user competes with others for its use. Infrastructure enhances productivity in the modern sector through positive externalities and spillover effects. The education system, represented as a flow of public spending, directly impacts labor productivity in both traditional and modern sectors through skill development. The tax revenue collected from the modern sector's profits is used to finance public investments in infrastructure and education according to a fixed proportion determined by government policy.

Empirical evidence emphasizes the importance of infrastructure and education investments in driving economic development in developing countries, demonstrating their positive effects on productivity growth and industrial transformation. Unnikirishnan and Kattookaran (2020) suggest that public infrastructure investment has a significant positive impact on capital investment, which increases productivity and economic growth. Moreover, some empirical research shows the different impacts of infrastructure and education investments on productivity growth across different sectors of the economy. Arshed et al. (2022) show that infrastructure, such as roads, railways and

airports, has significant positive effects on productivity growth in modern sectors like industry but relatively limited effects in traditional sectors like agriculture. Meanwhile, education investments have positive effects on productivity growth across both agricultural and industrial sectors. These findings support the model's assumption that infrastructure and education investments have differential impacts on productivity growth across sectors, with infrastructure primarily benefiting the modern sector while education enhances productivity in both sectors.

We assume that all tax revenues are spent and the government runs a balanced budget:

$$G = G_I + G_F = \tau r K. \tag{14}$$

The government's expenditure policy can be represented by a fixed proportion between infrastructure and education investments, denoted as ϕ, where $0 < \phi < 1$ determines the proportion of tax revenue allocated to infrastructure investment. The infrastructure depreciates at a constant rate δ, as well as the capital stock's depreciation rate. As in Carboni and Medda (2011) and Fernandes and de Oliveira (2024), the public capital stock evolves according to the following differential equation:

$$\dot{K}_g = G_I - \delta K_g = \phi \tau r K - \delta K_g \tag{15}$$

Education, the flow of public capital, is

$$G_F = (1 - \phi)\tau r K. \tag{16}$$

2.4 The firms

We assume a dual-sector model with a labor surplus economy and elastic labor market conditions, where workers can move between the sectors based on wage differentials. "Labor surplus" refers to a situation with an excess labor supply in the traditional sector that can be transferred to the modern sector without reducing agricultural output. "Elastic labor supply" means that workers are willing to move between sectors in response to wage differentials, with labor supply responding flexibly to changes in relative wages between the traditional and modern sectors. The wage in the traditional sector is determined by the subsistence level of income. Meanwhile, the wage in the modern sector is determined by market forces and productivity levels, reflecting the marginal product of labor in the modern sector. This dual-sector framework allows for the analysis of labor mobility between sectors in response to economic incentives and structural changes in the economy.

2.4.1 The modern sector

The modern sector produces output using capital and labor through the Cobb-Douglas production function. The modern sector's technology is capital-intensive and exhibits increasing returns to scale. The production function in modern sector is:

$$Y_m = AK^{\alpha}(aL_m)^{1-\alpha},\ 0 < \alpha < 1, \tag{17}$$

where A is the total factor productivity, K is the capital stock, L_m is the labor employed in the modern sector and a is the labor productivity. The lowercase letters, such as k, k_g, represent per unit of effective labor (defined as $k \equiv K/aL$, etc.).

The total factor productivity, A, reflects the external effect of infrastructure and is given by:

$$A = k_g^{\beta},\ 0 < \beta < 1, \tag{18}$$

where $\alpha > \beta$ and $1-\alpha-\beta > 0$, similar to Fernandes and de Oliveira (2024). Following Sasaki (2021), we assume a Marshallian externality and profit maximizing firms regard A as exogenously given.

The labor productivity growth rate is determined by the investment in education, which is represented by:

$$\frac{\dot{a}}{a} = \hat{a}(g_F) = \varphi g_F,\ \varphi > 0. \tag{19}$$

2.4.2 The traditional sector

The traditional sector produces output using labor. Labor productivity is enhanced by education through a simple linear production function. The traditional sector's technology is labor-intensive and exhibits constant returns to scale. The production function in traditional sector is represented by:

$$Y_s = aL_s, \tag{20}$$

where L_s is the labor employed in the traditional sector.

The industrial structure transition is characterized by the movement of labor from the traditional to the modern sector, driven by productivity differentials and wage incentives. The traditional and modern sectors coexist until the surplus labor from the traditional sector is absorbed into the modern sector, leading to structural transformation and economic development. As long as surplus labor exists in the traditional sector, labor continues to migrate between the sectors. We define this situation as a

"labor surplus phase". When the surplus labor in the traditional sector is fully absorbed into the modern sector, the economy transitions from a labor surplus phase to a "mature phase". In the mature phase, the traditional sector no longer exists.

2.5 Wages and profit rate

In the traditional sector, as in Ros (2001) and Sasaki (2008), the wages are equal to the average production in effective labor units:

$$\tilde{w}_s = 1. \tag{21}$$

The tilde above the wage, w, indicates the per unit of effective labor.

The wages remain at this level until the surplus labor in the economy is absorbed into the modern sector. Meanwhile, in the modern sector, the wage is determined by the wage in the traditional sector plus a wage premium, $(f-1)$, to attract workers from the traditional sector: $\tilde{w}_m = f\tilde{w}_s$. As in Lewis (1954), this wage premium to attract workers from the traditional sector creates a dual labor market where wages in the modern sector exceed those in the traditional sector. The wage premium $(f-1)$ remains constant while both sectors simultaneously operate, with the modern sector maintaining a fixed real wage rate. For simplicity and tractability, as in Ros (2001) and Sasaki (2008), we suppose that the wage premium is equal to zero. Then, we have:

$$\tilde{w}_m = \tilde{w}_s \tag{22}$$

Moreover, when the economy achieves maturity and reaches full employment in the modern sector, wages in both sectors are determined by the marginal product of labor in the modern sector.

$$\tilde{w}_m = k_g^{\beta}(1-\alpha)\left(\frac{K}{aL_m}\right)^{\alpha} \tag{23}$$

In the mature phase, the wage per effective labor is positively correlated with the capital stock and public infrastructure, reflecting how capital accumulation and public infrastructure development influence the labor demand and raise wages in the modern sector. Meanwhile, when the number of workers in the modern sector increases, the wage tends to decrease.

The profit rate in the modern sector is determined by the marginal product of capital.

$$r = \frac{k_g^{\frac{\beta}{\alpha}}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{w^{\frac{1-\alpha}{\alpha}}} \tag{24}$$

The profit rate is positively correlated with public infrastructure, reflecting how public infrastructure investment enhances productivity in the modern sector, yielding higher returns on capital investment. Furthermore, when the wage decreases and profit for the capitalist increases, this leads to a higher investment rate and demand for the capital stock, leading to a higher profit rate (Fernandes and de Oliveira, 2024).

2.6 Short-run equilibrium

The short-run equilibrium conditions require balancing supply and demand in both sectors while accounting for labor allocation, wage rates, and capital accumulation dynamics.

2.6.1 Labor surplus phase

In the labor surplus phase ($L_s > 0$), the equilibrium conditions require balancing the labor supply and demand between the traditional and modern sectors while maintaining the fixed real wage rate. From (21) and (24), the wage and profit rate can be respectively written as:

$$\tilde{w} = 1 \tag{25}$$

$$r = \Omega k_g^{\frac{\beta}{\alpha}} \tag{26}$$

where $\Omega = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}$. The wage per natural unit of labor is equal to a.

Now, we consider the employment rate in the modern sector. From (23) and (25), we obtain:

$$\frac{L_m}{L} = (1-\alpha)^{\frac{1}{\alpha}} k k_g^{\frac{\beta}{\alpha}} < 1. \tag{27}$$

The fraction of the labor force employed in the modern sector depends on the accumulation of capital stock and public infrastructure. The fraction increases as capital accumulation and infrastructure development enhance productivity and create more employment opportunities. The remaining labor force continues to work in the traditional sector until economic development and structural transformation lead to the complete absorption of the surplus labor.

2.6.2 Mature phase

In the mature phase ($L_s = 0$), the equilibrium conditions require wages to be determined by the marginal product of labor in the modern sector, while maintaining full

employment and efficient resource allocation. From (23) and (24), the wages and profit rate can be respectively written as:

$$w = (1-\alpha)k^{\alpha}k_g^{\beta} \tag{28}$$

$$r = \alpha k_g^{\beta}k^{\alpha-1} \tag{29}$$

Unlike the labor surplus phase, the wage per unit of effective labor and profit rate are dependent on capital accumulation.

2.7 The accumulation of capital stock and public infrastructure

In the long-run equilibrium, the capital and public capital stocks evolve according to the accumulation equations derived from the optimization problems of the capitalists and the government's investment decisions.

From (9), the dynamics of capital stock per effective labor units can be defined as follows:

$$\frac{\dot{k}}{k} = (1-\tau)r - \rho - [n+\delta+\varphi(1-\phi)\tau rk] \tag{30}$$

From (15), the dynamics of public infrastructure per effective labor unit can be expressed as follows:

$$\frac{\dot{k}_g}{k_g} = \phi\tau rkk_g^{-1} - [n+\delta+\varphi(1-\phi)\tau rk] \tag{31}$$

3 Long-run equilibrium

In the labor surplus phase ($L_s > 0$), the two-dimensional nonlinear differential equations system describes the dynamics of capital accumulation and public infrastructure development. Using Equations (25), (26), (30) and (31), we get:

$$\dot{k} = \{(1-\tau)\Omega k_g^{\frac{\beta}{\alpha}} - \rho - [n+\delta+\varphi(1-\phi)\tau\Omega k k_g^{\frac{\beta}{\alpha}}]\}k \tag{32}$$

$$\dot{k}_g = \{\phi\tau\Omega k k_g^{\frac{\beta-\alpha}{\alpha}} - [n+\delta+\varphi(1-\phi)\tau\Omega k k_g^{\frac{\beta}{\alpha}}]\}k_g. \tag{33}$$

In the mature phase ($L_s = 0$), using Equations (28), (29), (30), and (31), the two-dimensional nonlinear differential equations system can be written as follow:

$$\dot{k} = \{(1-\tau)\alpha k^{\alpha-1}k_g^{\beta} - \rho - [n+\delta+\varphi(1-\phi)\tau\alpha k^{\alpha}k_g^{\beta}]\}k \tag{34}$$

$$\dot{k}_g = \{\phi\tau\alpha k^{\alpha}k_g^{\beta-1} - [n+\delta+\varphi(1-\phi)\tau\alpha k^{\alpha}k_g^{\beta}]\}k_g \tag{35}$$

The steady state equilibrium in both phases can be determined by letting the time derivatives of capital stock and public infrastructure equal to zero, and solving the resulting system of equations. The steady state of value of capital stock and public infrastructure can be denoted as (k^*, k_g^*), respectively. Then, the following two equations hold:

$$\frac{(1-\tau)}{\varphi(1-\phi)\tau} > k^* \tag{36}$$

$$\frac{\phi}{\varphi(1-\phi)} > {k_g}^* \tag{37}$$

3.1 Labor surplus phase

The system's properties in the labor surplus phase can be analyzed using a Taylor's expansion around the steady state equilibrium. This system can be analyzed by examining the eigenvalues of the Jacobian matrix to determine stability conditions and growth dynamics.

$$\begin{bmatrix} \dot{k} - k^* \\ \dot{k}_g - k_g^* \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \dot{k} - k^* \\ \dot{k}_g - k_g^* \end{bmatrix}$$

$$J_{11} = -\varphi(1-\phi)\tau\Omega {k_g^*}^{\frac{\beta}{\alpha}} < 0 \tag{38}$$

$$J_{12} = [(1-\tau) - \varphi(1-\phi)\tau k^*]\frac{\beta}{\alpha}\Omega {k_g^*}^{\frac{\beta-\alpha}{\alpha}} > 0 \tag{39}$$

$$J_{21} = [\phi - \varphi(1-\phi)k_g^*]\tau\Omega {k_g^*}^{\frac{\beta-\alpha}{\alpha}} > 0 \tag{40}$$

$$J_{22} = -\frac{\alpha-\beta}{\alpha}\phi\tau\Omega k^* {k_g^*}^{\frac{\beta-2\alpha}{\alpha}} - \frac{\beta}{\alpha}\varphi(1-\phi)\tau\Omega k^* {k_g^*}^{\frac{\beta-\alpha}{\alpha}} < 0 \tag{41}$$

In the Jacobian matrix, J_{11} is negative. An increase in capital stock per unit of efficient labor reduces the rate of change of capital stock per unit of efficient labor. In turn, an increase in public infrastructure per unit of efficient labor increases the rate of change of public infrastructure per unit of efficient labor, $J_{22} < 0$. Moreover, a rise in the capital stock per unit of efficient labor increases the profit rate, which increases the income of capitalist, and thus, tax revenue that increases the public infrastructure investment; this justifies $J_{12} > 0$. In turn, a rise in public infrastructure around the steady state equilibrium increases the rate of change of capital stock per unit of efficient labor through increased productivity and investment incentives in the modern sector, which justifies $J_{21} > 0$.

As observed, given that (k^*, k_g^*) are positive, the trace of the Jacobian matrix is negative. This is a necessary condition for the steady state equilibrium's stability in the labor surplus phase.

To achieve stability, the determinant of the Jacobian matrix must be positive, which requires certain conditions on the model parameters, and steady-state values of capital and public infrastructure. However, this determinant depends on the government's fiscal policy choices that establish the tax rate and tax revenue allocation between infrastructure and education investments.

$$\begin{aligned}\det\mathbf{J} &= J_{11}J_{22} - J_{12}J_{21} \\ &= \left[\varphi(1-\phi)\phi\tau^2 k^* + \frac{\beta}{\alpha}\varphi(1-\phi)(1-\tau)\tau k^* - \frac{\beta}{\alpha}\phi(1-\tau)\tau\right]\left(\Omega k_g^{*\frac{\beta-\alpha}{\alpha}}\right)^2\end{aligned}$$

When the determinant is positive,

$$k_g^* > -\frac{\alpha}{\beta}\cdot\frac{\phi\tau}{1-\tau}k^* + \frac{\phi\tau}{\varphi(1-\phi)} \tag{42}$$

indicating that the steady state equilibrium is locally stable in the labor surplus phase. Meanwhile, when the determinant is negative,

$$k_g^* < -\frac{\alpha}{\beta}\cdot\frac{\phi\tau}{1-\tau}k^* + \frac{\phi\tau}{\varphi(1-\phi)} \tag{43}$$

indicating that the steady state equilibrium is unstable in the labor surplus phase, which can lead to divergent economic development paths.

3.2 Mature phase

The system's properties in the mature phase can also be analyzed using a Taylor's expansion around the steady state equilibrium.

$$\begin{bmatrix} \dot{k}-k^* \\ \dot{k}_g-k_g^* \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}\begin{bmatrix} \dot{k}-k^* \\ \dot{k}_g-k_g^* \end{bmatrix}$$

$$J_{11} = -\alpha(1-\alpha)(1-\tau)k^{*\alpha-2}k_g^{*\beta} - \alpha^2\varphi(1-\phi)\tau k^{*\alpha-1}k_g^{*\beta} < 0 \tag{44}$$

$$J_{12} = [(1-\tau) - \varphi(1-\phi)\tau k^*]\alpha\beta k^{*\alpha-1}k_g^{*\beta-1} > 0 \tag{45}$$

$$J_{21} = [\phi - \varphi(1-\phi)k_g^*]\alpha^2\tau k^{*\alpha-1}k^{*\beta-1} > 0 \tag{46}$$

$$J_{22} = -\alpha(1-\beta)\phi\tau k^{*\alpha}k_g^{*\beta-2} - \alpha\beta\varphi(1-\phi)\tau k^{*\alpha}k_g^{*\beta-1} < 0 \tag{47}$$

In the Jacobian matrix, J_{11} is negative. An increase in capital stock per unit of efficient labor decreases the rate of change of capital stock per unit of efficiency labor. An increase in public infrastructure per unit of efficiency labor decreases the rate of change of public infrastructure per unit of efficient labor, $J_{22} < 0$. Moreover, a rise in the

capital stock per unit of efficient labor increases the profit rate; this increases the income of capitalist, and thus, the tax revenue that increases the public infrastructure investment, thus justifying $J_{12} > 0$. In turn, a rise in public infrastructure around the steady state equilibrium increases the rate of change of capital stock per unit of efficient labor through increased productivity and investment incentives in the modern sector, which justifies $J_{21} > 0$.

As observed, given that (k^*, k_g^*) are positive, the trace of the Jacobian matrix is negative. This is a necessary condition for the stability of the steady state equilibrium in the labor surplus phase. The determinant of the Jacobian matrix is positive, confirming the stability of the equilibrium in both phases of economic development. Therefore, the long-run equilibrium is characterized by stable growth paths in both phases.

$$\begin{aligned}\det\mathbf{J} &= J_{11}J_{22} - J_{12}J_{21} \\ &= [(1-\alpha-\beta)(1-\tau)\phi\tau + \alpha\varphi(1-\phi)\tau^2\phi k^* + \beta(1-\tau)\tau\varphi(1-\phi)k_g^*](\alpha k^{*\alpha-1}k_g^{*\beta-1})^2 \\ &> 0\end{aligned}$$

Therefore, in the long-run equilibrium in the mature phase, the steady state equilibrium is characterized by stable capital accumulation and balanced growth paths.

In the steady state, the economic growth rate is determined by the rate of technological progress and population growth.

$$g = \frac{\dot{Y}}{Y} = n + \varphi(1-\phi)\tau\alpha k^{*\alpha}k_g^{*\beta} \tag{48}$$

Therefore, the economic growth rate is determined by the effectiveness of public investments in infrastructure and education.

4 Cases of successful development and the poverty trap

In the previous section, we analyzed the stability conditions and dynamics of the economic system in both the labor surplus and mature phases, showing how different equilibrium states can emerge based on policy choices and initial conditions. In the labor surplus phase, the stability conditions and economic outcomes depend on the interaction between fiscal policies, infrastructure investment, and education spending. These collectively determine whether workers in the traditional sector can successfully transition to the modern sector and achieve sustained economic growth. In the mature

phase, the economic growth rate is also the interaction between fiscal policies, infrastructure investment, and education spending. Analyzing these economic outcomes can help explain why some countries successfully escape the middle-income trap while others remain stagnant in their development trajectory.

We use numerical simulations to analyze different scenarios of development and identify conditions for sustained economic growth and identify policy choices that can effectively promote structural transformation and help countries avoid development traps.. The simulation examines key parameters such as tax rates and infrastructure investment ratios to determine their impact on development outcomes. The parameters are as follows[1]:

Table 1: Parameter values

α	0.44
β	0.15
φ	0.2
ρ	0.05
n	0.015
δ	0.015
τ	0.2, 0.4
ϕ	0.22, 0.8

Since four possible combinations of tax rates and infrastructure investment ratios exist, we analyze how these combinations affect economic development outcomes, and the potential for escaping the middle-income trap. We use a global analysis to demonstrate how different combinations of fiscal policy parameters influence economic growth trajectories and development outcomes.

4.1 Labor surplus phase

We can analyze the parameters that determine the dynamics of the economic system in the labor surplus phase, including tax rates and infrastructure investment ratios. These parameters determine whether the economy can successfully transition from a labor surplus to mature phase, or remain trapped in underdevelopment. The simulation can help identify specific parameter thresholds and policy combinations that promote

[1]We set the parameters following Fernandes and de Oliveira (2024). In addition, we set φ in Equation (19) following Laboure and Taugourdeau (2018) and Fedotenkov and Gupta (2021).

successful economic development versus those that lead to stagnation and poverty traps.

We consider the case with relatively high tax and infrastructure investment rates: $(\tau, \phi) = (0.4, 0.8)$. These high rates can lead to rapid capital accumulation and successful economic development through increased public investment and productivity growth.

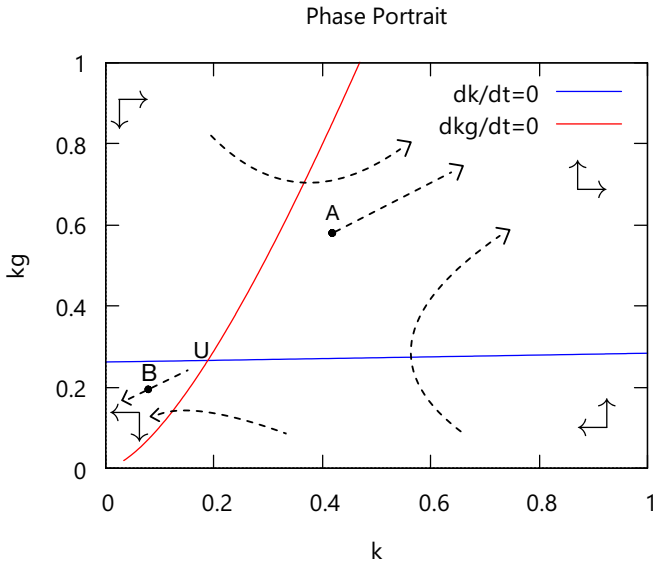

Figure 1: Labor surplus phase $\tau = 0.4$ and $\phi = 0.8$

In Figure 1, the dynamic system of (k, k_g) has a unique equilibrium, U. The point U is the unstable equilibrium point where the economy can either progress toward successful development or fall into a poverty trap depending on initial conditions (see Appendix for detailed proofs).

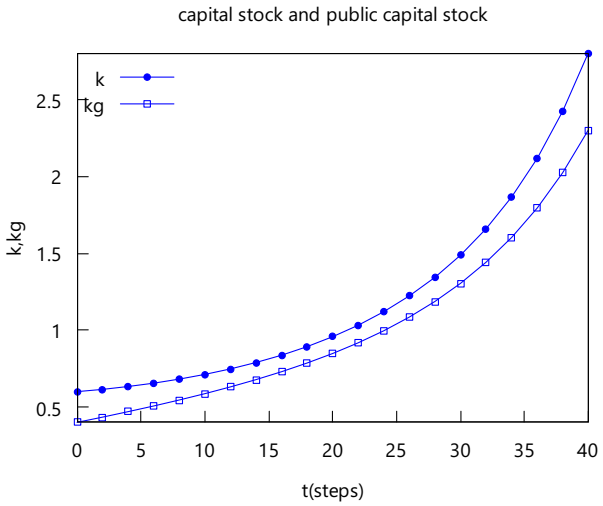

Figure 2: Initial condition A

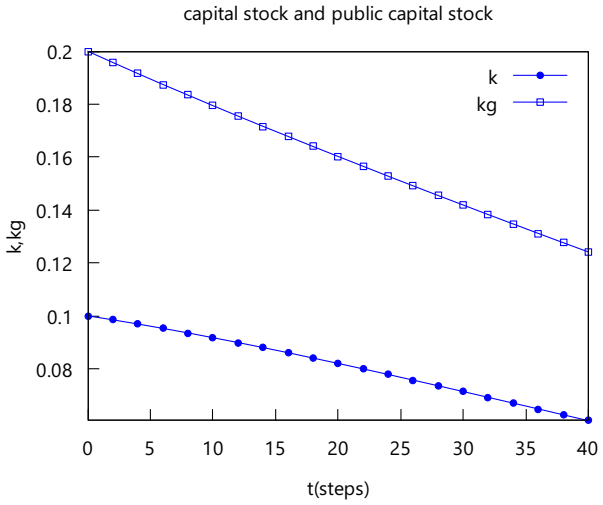

Figure 3: Initial condition B

In Figure 2, the simulation demonstrates the system's dynamic trajectories when initialized near point A in Figure 1, characterized by relatively higher levels of both capital stock and public capital per unit effective labor. In this case, the capital stock and infrastructure per effective labor increase over time, yielding successful economic development and transition to the mature phase. Meanwhile, in Figure 3, when the initial conditions are settled around point B, with relatively lower levels of capital stock

and public infrastructure, the economy declines capital and infrastructure per effective labor.

Now, we consider the employment dynamics in the modern sector. From (27), the dynamics of employment in the modern sector depend on capital stock and public capital accumulation rates, determining the absorption of surplus labor from the traditional sector. The employment rate in the modern sector increases as capital accumulation progresses, gradually reducing surplus labor from the traditional sector. By analyzing the dynamics of the employment rate in the modern sector, we can understand how the transition from labor surplus to full employment occurs, and how different policy interventions can accelerate or impede this process.

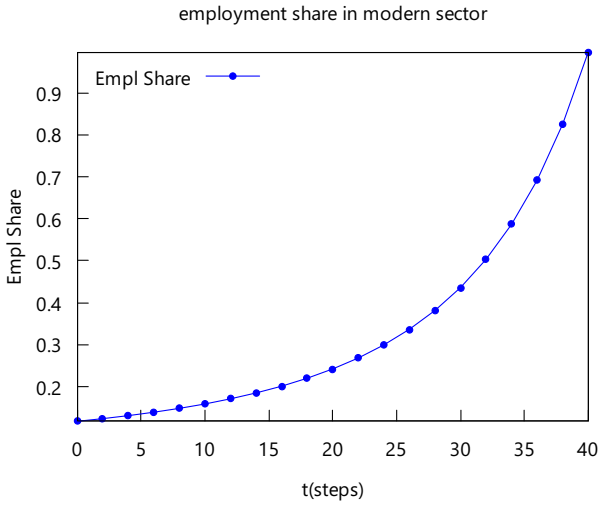

Figure 4: Initial condition A

In Figure 4, when the initial condition is settled around point A in Figure 1, the accumulation of capital stock and public capital per effective labor leads to successful development. Consequently, the labor demand in the modern sector increases, causing increased employment rates and a gradual shift of workers from the traditional to the modern sector. This leads to industrialization and successful economic development.

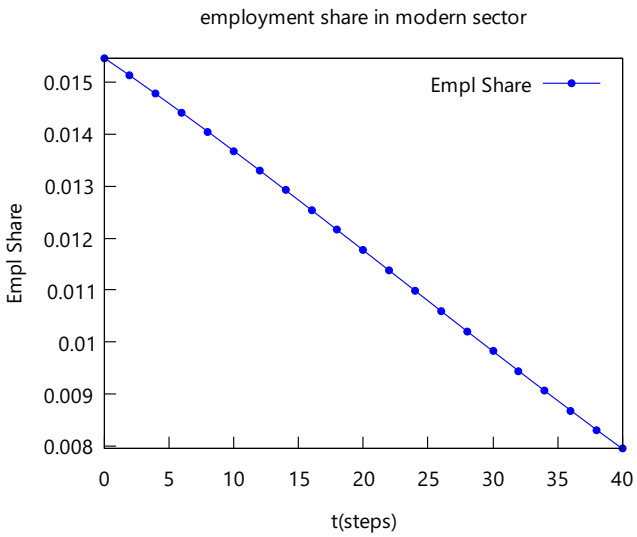

Figure 5: Initial condition B

Meanwhile, in Figure 5, when the initial condition is settled around point B in Figure 1, characterized by relatively lower levels of capital stock and public capital per effective labor, the economy stagnates and fails to achieve successful development. This situation leads to insufficient capital accumulation and infrastructure development. This causes limited employment opportunities in the modern sector and a persistent labor surplus in the traditional sector, leading to persistent poverty and underdevelopment in the economy.

This simulation demonstrates how initial conditions and policy parameters determine whether an economy successfully transitions to modern sector dominance or remains trapped in traditional sector dependence.

Besides this case, other cases with different combinations of tax rates and infrastructure investment ratios, $(\tau, \phi) = (0.2, 0.8)$, $(0.2, 0.22)$, have similar patterns and implications for economic development outcomes. These can be analyzed through similar numerical simulations and phase diagrams to understand their implications for development outcomes (see Appendix for detailed proofs). However, the other case with different combinations of tax rates and infrastructure investment ratios, $(\tau, \phi) = (0.4, 0.22)$, show different patterns and development outcomes, with varying implications for economic growth and structural transformation. The detailed analysis is provided in the later section.

4.1.1 Case of poverty trap

When the initial conditions are settled around point B with relatively lower levels of capital stock and public capital per labor, the economy may fall into a poverty trap characterized by low capital accumulation, limited infrastructure development, and persistent surplus labor in the traditional sector.

As profits in the modern sector decline, investment in the modern sector decreases, slowing capital accumulation and economic growth. In other words, the economy remains trapped in a low-level equilibrium with minimal growth and development prospects. Labor remain trapped in the traditional sector with low wages and limited opportunities for economic advancement.

Moreover, industrial structure development will not progress beyond a certain point because the labor in the traditional sector will not be absorbed by the modern sector. This can cause persistent underemployment and economic stagnation, which can be called a poverty trap.

This is consistent with Rosenstein-Rodan (1943, 1961), who emphasize that external financial assistance and investment can help break the cycle of poverty by providing the necessary capital and resources for economic development and modernization. That is, once caught in a poverty trap, this surplus labor economy can potentially be released through a substantial public investment and infrastructure development push to stimulate economic growth and modernization.

In Figure A, the initial point B can be moved to point A through external financial assistance and investment, enabling the economy to achieve successful development and modernization.

4.2 Mature phase

In the mature phase, the equilibrium conditions and dynamics follow similar patterns to the labor surplus phase but with wages determined by market forces and full employment in the modern sector.

We consider the case where the tax and infrastructure investment rates are relatively high, $(\tau, \phi) = (0.4, 0.8)$. Such high rates can foster rapid capital accumulation and successful economic development through increased public investment and productivity growth.

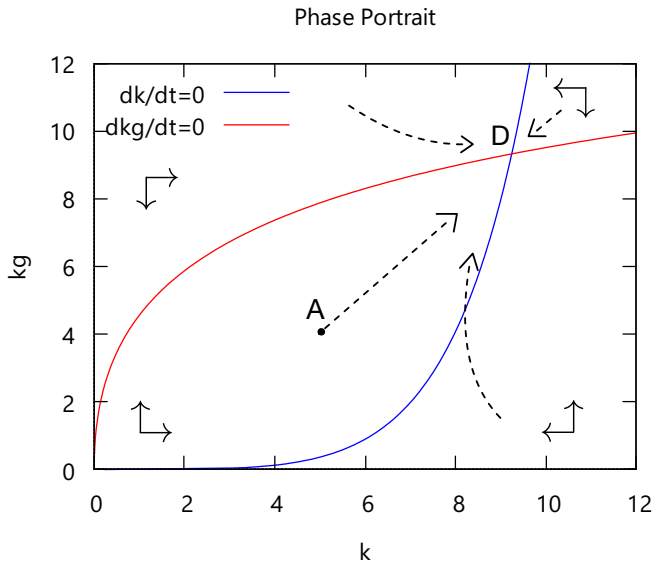

Figure 6: Mature phase $\tau = 0.4$ and $\phi = 0.8$

In Figure 6, the point D, which represents the steady-state equilibrium in the mature phase, is locally stable and characterized by balanced growth with constant capital- and infrastructure-to-labor ratios.

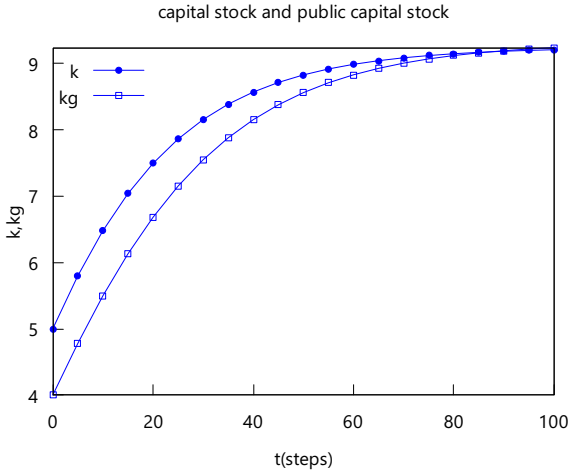

Figure 7: Initial condition A

In Figure 7, the simulation demonstrates the system's dynamic trajectories when initialized near point A in Figure 6, characterized by relatively higher capital stock and

public capital per effective labor. These two factors continue to increase and eventually converge to their steady-state values, point D.

At point A, capital stock and public capital are at sufficient levels for development. The increase in public capital enhances productivity in the modern sector, yielding increased profits, productivity, and overall economic growth in the mature phase. Thus, capitalists' profits and the government's tax revenues increase, engendering further capital accumulation and public investment. This can create a virtuous cycle of economic growth and development in the mature phase. Eventually, the capital stock and public capital converge to their steady-state values, point D, where the economy achieves stable and sustained economic growth.

In addition to this case, other cases with different combinations of tax rates and infrastructure investment ratios, $(\tau, \phi) = (0.2, 0.8)$, $(0.2, 0.22)$, $(0.4, 0.22)$, have similar patterns and implications for economic development outcomes. These can be analyzed through similar numerical simulations and phase diagrams to understand their implications for development outcomes (see Appendix for detailed proofs).

4.3 Cases of successful development: The virtuous cycle of economic growth

This section provides a further qualitative and illustrative analysis of the dynamics of economic growth by combining the unique economical equilibrium in each phase of development using the case of $(\tau, \phi) = (0.4, 0.8)$.

In Figure 8, the blue line indicates capital stock accumulation and red line indicates public capital accumulation in the labor surplus phase. The green and pink lines indicate the accumulation of the capital stock and public capital in the mature phase, respectively. In addition to these lines, from (27), the black line indicates the boundary between the labor surplus and mature phases, representing the transition point where surplus labor is fully absorbed into the modern sector. The diagram illustrates how the economy can achieve successful development through the interaction between capital accumulation and public investment across the different economic growth phases.

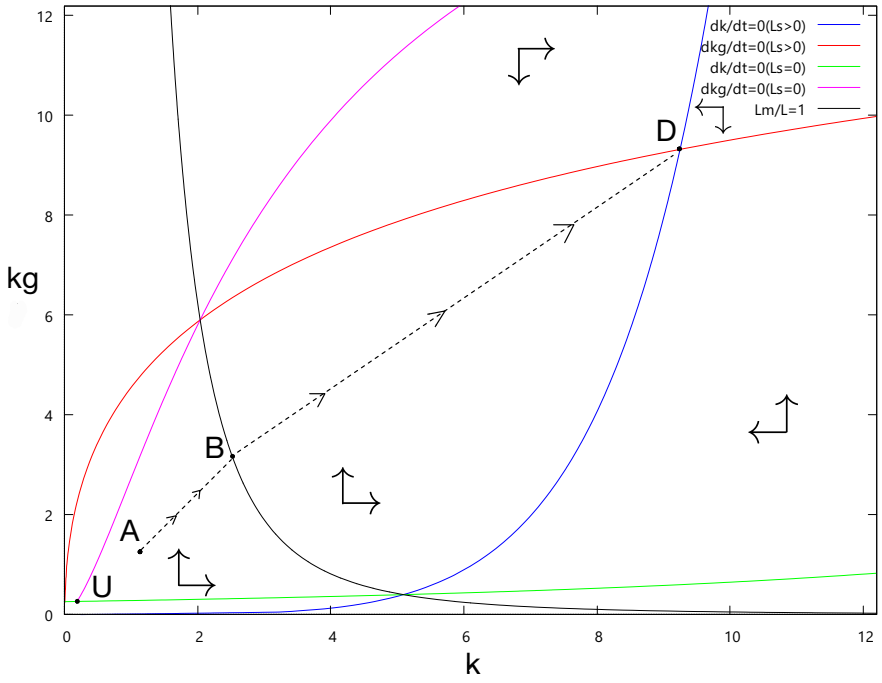

Figure 8: Successful development

When the initial condition is settled around point A, infrastructure accumulation increases the productivity in the modern sector due to spillovers and profit increases. In this case, both investment and government tax revenues increase. Then, further capital stock and public capital accumulation can proceed. Consequently, the labor demand in the modern sector increases, absorbing the surplus labor in the traditional sector and the labor transition from the traditional to the modern sector occurs, further advancing industrialization. Finally, the economy, starting from point A, approaches the black line, which implies that the surplus labor in the traditional sector has been fully absorbed.

When the economy reaches the curve, the transformation of the industrial structure is completed and the economy moves to a mature phase. When the transition is completed at point B, the accumulation of capital stock and infrastructure continues and finally converges to equilibrium point D.

This virtuous cycle of economic growth and development is characterized by increasing returns to scale, positive spillover effects, and sustained capital stock and public capital accumulation. The virtuous cycle leads to sustained economic growth, technological advancement, and improved living standards across the economy.

4.4 The economic growth rate in the mature phase

Next, we analyze the economic growth rate after the steady state in the mature phase, examining how different fiscal policies affect long-term growth outcomes depending on tax rates and infrastructure investment allocation.

First, given an infrastructure investment ratio of 0.2 and 0.8, we use Equation (48) and examine how tax rate affects economic growth rates in the mature phase (see Appendix for detailed economic growth figures).

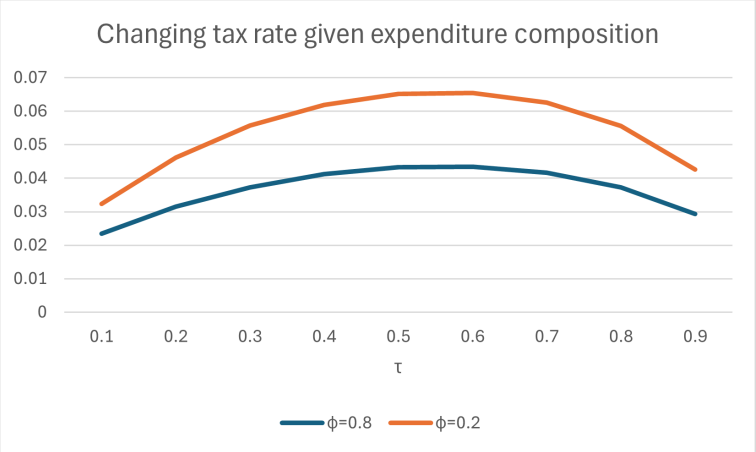

Figure 9: Economic growth rate in the mature phase

Figure 9 shows that an appropriate tax rate exists, which varies depending on the rate of allocation of public investment in infrastructure. The tax rate should not be extremely high or low, but should be set at an optimal level to public investment in infrastructure development. When comparing infrastructure investment allocation rates at the same tax level, the growth rate may be higher when the allocation to infrastructure is low and allocation to education is high.

In turn, given a tax rate of 0.2 and 0.4, we examine how infrastructure investment ratio affects economic growth rates in the mature phase (see Appendix for detailed economic growth figures).

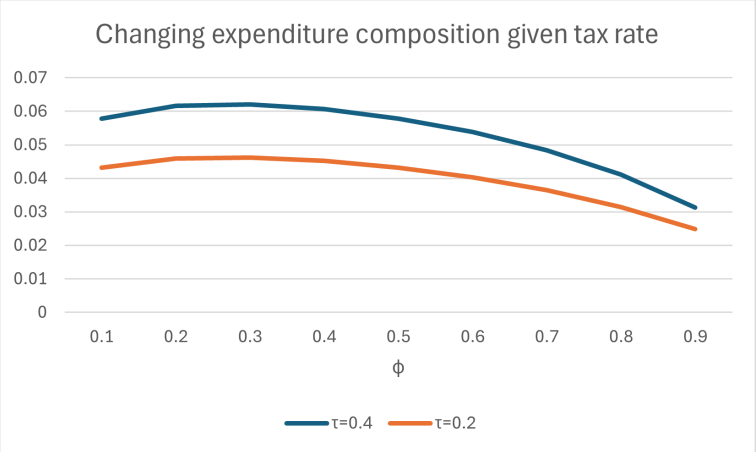

Figure 10: Economic growth rate in the mature phase

Figure 10 shows that an optimal allocation rate between infrastructure and education investment also exists, which depends on the tax rate. This optimal allocation rate should neither be extremely high nor low but should be balanced according to the tax rate level. Unlike the optimal tax rate, although extremely low allocation levels should be avoided, higher allocation to education than to public infrastructure investment may be more effective for economic growth, suggesting that education investment plays a crucial role in promoting economic development and growth in the mature phase. Therefore, the economic growth rate in the steady state is higher, when the allocation to education exceeds infrastructure investment, leading to enhanced productivity and sustained economic growth in the long run.

5 The cases of the middle-income trap

Here, we analyze how middle-income economies can become trapped in economic stagnation despite achieving initial development success and industrialization.

5.1 The cases of the middle-income trap

In the labor surplus phase, when the allocation of infrastructure investment is low and tax rate is high, $(\tau, \phi) = (0.4, 0.22)$, we get a different conclusion.

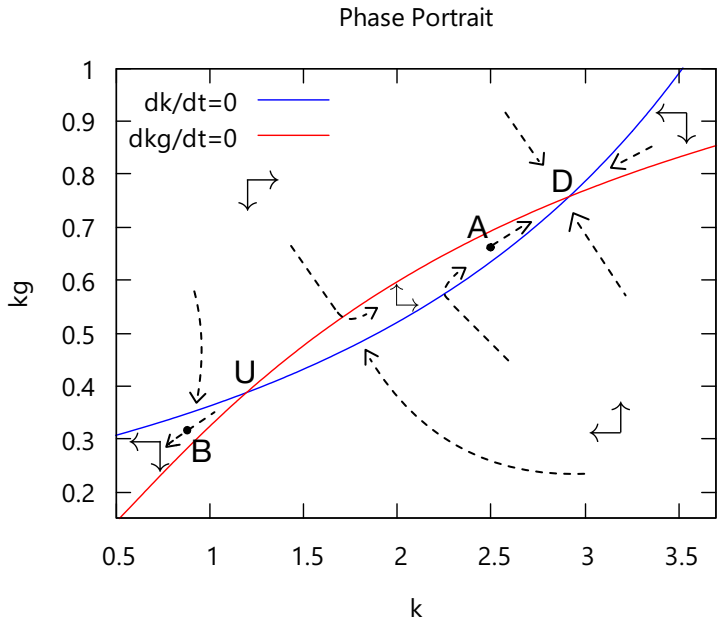

Figure 11: Labor surplus phase $\tau = 0.4$ and $\phi = 0.22$

The two demarcation curves intersect at points U and D in Figure 11, representing unstable and stable equilibrium points, respectively (see the Appendix for detailed proofs). Point U represents an unstable equilibrium where the economy can either progress toward successful development or falls into the poverty trap depending on the initial conditions. Point D represents the stable equilibrium point where the economy achieves sustained growth and development through balanced capital accumulation and infrastructure investment.

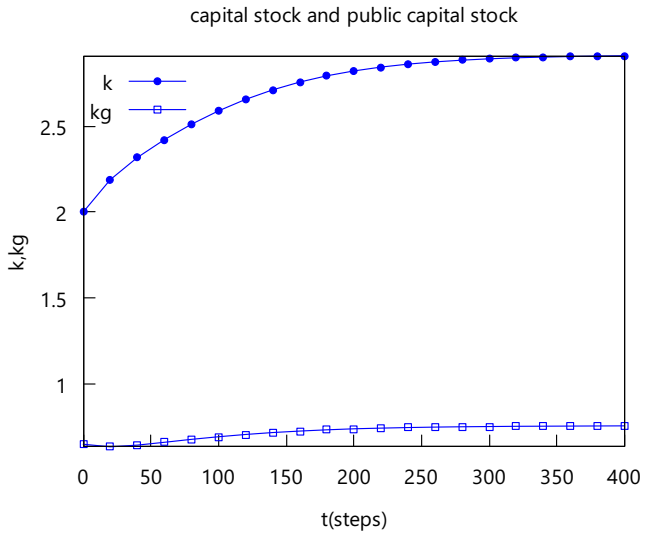

Figure 12: Initial condition A

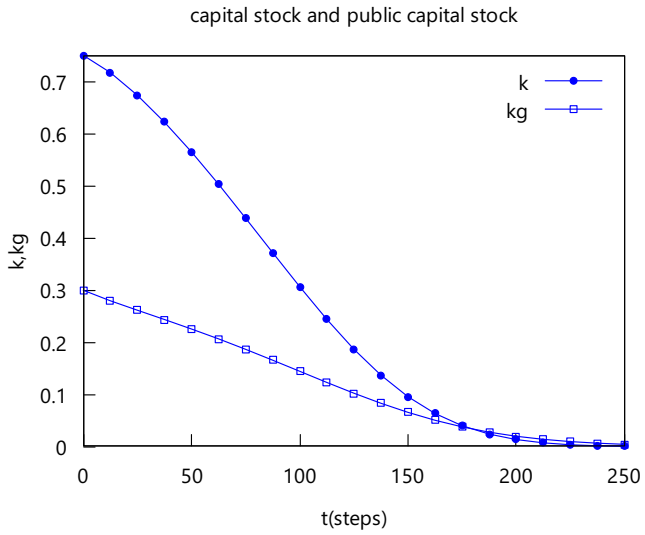

Figure 13: Initial condition B

In Figure 12, the simulation demonstrates the system's dynamic trajectories when initialized near point A in Figure 11, characterized by relatively higher levels of both capital stock and public capital per effective labor. Here, the capital stock and public capital per unit of effective labor increase initially but eventually converge to point D. Meanwhile, in Figure 13, when the initial conditions are settled around point B

with relatively lower levels of capital stock and public infrastructure, the economy falls into a poverty trap characterized by low capital accumulation, limited infrastructure development, and persistent surplus labor in the traditional sector.

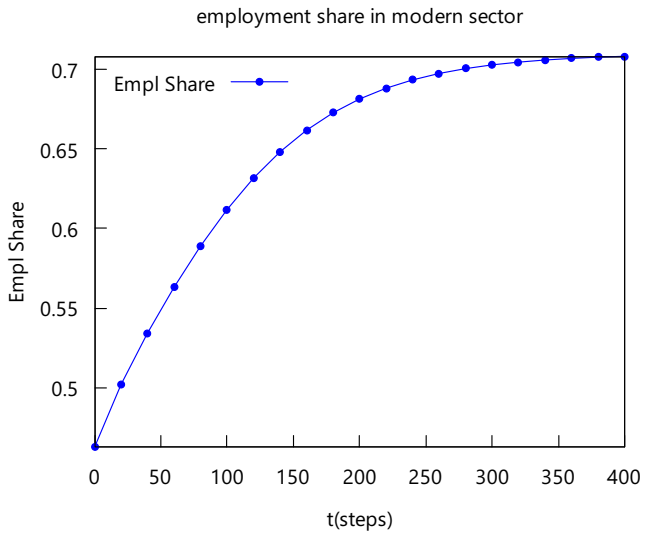

Figure 14: Initial condition A

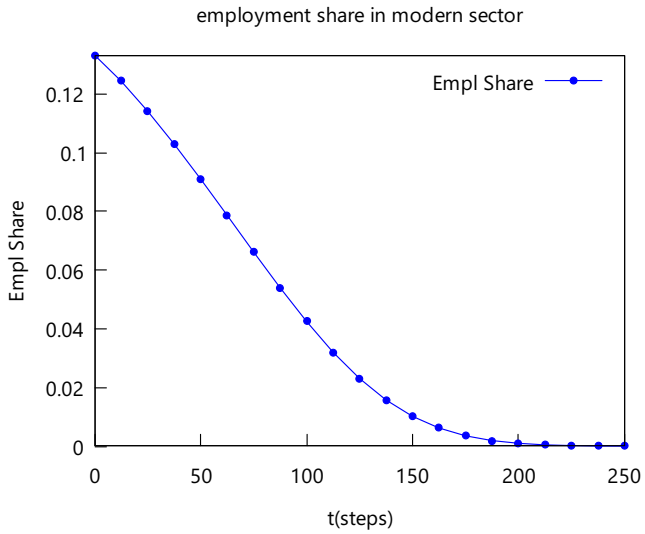

Figure 15: Initial condition B

In turn, we consider the dynamics of employment in the modern sector. When

the initial conditions are settled around point A with relatively high levels of capital stock and public infrastructure, employment in the modern sector increases as capital accumulation progresses. However, the employment rate in the modern sector eventually stabilizes at a level below full employment, indicating a middle-income trap where surplus labor persists despite initial development progress.

In the early development stages in the labor surplus phase, as in the previous section, public capital accumulation has a significant external effect on productivity in the modern sector through spillovers, increasing profits and capital accumulation. Labor demand in the modern sector increases accordingly, causing a gradual absorption of surplus labor from the traditional sector and progress in industrialization. However, although the accumulation and employment rates rise initially, the external effects of the spillover are small due to insufficient infrastructure investments, limiting the potential for sustained economic growth and development. The productivity growth rate in the modern sector eventually slows, labor demand in the modern sector also decreases, and labor absorption into the modern sector slows down or stagnates. This creates ongoing job shortages and limits economic growth, which limits both industrial progress and overall economic development. This economy remains trapped at middle-income levels, unable to achieve full industrialization and transition to high-income status despite initial progress in development.

Therefore, countries currently trapped in middle-income levels may have a traditional sector and government public investment is considered inefficient. Tran (2013) shows that middle-income countries with inefficient public investment and a persistent traditional sector struggle to achieve higher development levels, like Indonesia, the Philippines, and Vietnam. These middle-income countries need targeted policies to improve public investment efficiency and modernize their traditional sectors to overcome development barriers.

The solution to the middle-income trap requires addressing inefficient public investment and modernizing traditional sectors through targeted policy reforms and improved resource allocation. The government must set appropriate policy priorities and investment strategies to achieve a virtuous circle where the external effects of infrastructure accumulation increase the productivity and employment rate in the modern sector, and promote further capital stock and infrastructure accumulation. This solution enhances the external effects of spillovers and productivity growth, helping countries escape the middle-income trap and achieve higher development levels.

5.2 Relationship between the middle-income and poverty traps

The middle-income and poverty traps share common characteristics but differ in their underlying causes and manifestations. The poverty trap occurs at lower income levels where economies struggle to generate basic capital accumulation and infrastructure development. Meanwhile, the middle-income trap manifests in more developed economies that have progressed beyond initial industrialization but struggle to transition from middle- to high-income status due to structural challenges in industrial upgrading and structural transition.

The poverty trap has been solved by the Big Push, such as a fiscal policy and external financial assistance, which can shift the initial point condition to relatively sufficient capital and infrastructure levels. However, the risk of falling into the middle-income trap still exists. Even after successfully escaping from the poverty trap through initial investments and big push policies, countries must carefully design and implement appropriate fiscal and development policies to avoid falling into the middle-income trap. For example, if tax rates are high, but infrastructure investment is insufficient, then government policies must be designed to maintain appropriate tax rates and allocate sufficient investment to infrastructure to promote sustained economic growth and development.

Thus, to avoid the middle-income trap and achieve sustainable economic growth, government policies in the initial phase of development are important. Here, the public capital allocation ratio to such as infrastructure needs to be increased. The external effects of infrastructure spillover increase the productivity in the modern sector. Thus, the profits and capital accumulation increase, engendering successful economic development and sustainable growth in the long run. Such a virtuous circle of infrastructure investment and economic development can help countries avoid middle-income traps while achieving sustainable long-term growth.

Even if a country seems to have succeeded in development once through foreign investment and foreign aid, it may still fall into the middle-income trap if appropriate fiscal policies and development strategies are not carefully designed. In other words, appropriate fiscal policies and development strategies are crucial for avoiding the middle-income trap, even after initial development success based on external investment.

6 Conclusion

To analyze the causes of and strategies to avoid the middle-income trap, we developed an economic development model with two sectors and public investment in infrastructure and education to examine how different fiscal policies and development strategies affect growth trajectories and development outcomes. We use a dual-sector model with public investment, and analyze how fiscal policies and development strategies affect economic outcomes. The model effectively illustrates how policy choices, investment allocation, and initial economic conditions determine whether countries achieve sustainable development or become trapped. Interestingly, even after successfully escaping from poverty traps through external investment and aid, countries need appropriate fiscal policies and development strategies to achieve sustainable economic growth and avoid falling back into development traps.

Our conclusions are summarized as follows: First, the model demonstrates that appropriate fiscal policies and infrastructure investment are essential for avoiding the middle-income trap and achieving sustainable economic development. In the labor surplus phase, sufficient infrastructure investment and appropriate tax rates are essential for promoting economic growth and successful development through enhanced productivity and capital accumulation in the modern sector. Meanwhile, insufficient infrastructure investment and high tax rates can lead to the middle-income trap, preventing economies from achieving full industrialization and higher development stages. The economy may remain trapped in the traditional sector with low productivity and wages, leading to persistent development challenges and limited growth potential, and stagnate in industrialization and economic development, resulting in a middle-income trap. Thus, avoiding the middle-income trap may be possible without relying on foreign aids.

Second, the model showing that balanced investment in infrastructure and education alongside appropriate tax rates can create virtuous development cycles. To achieve sustainable economic growth and avoid development traps, governments must implement appropriate fiscal policies, which can achieve a virtuous circle in which infrastructure investment and capital accumulation lead to increased productivity and employment in the modern sector, promoting further development and sustainable economic growth through enhanced spillover effects and technological advancement.

Third, in the steady state of the mature phase, investment allocation between infrastructure and education plays a crucial role in determining economic growth rates. Higher education investment generally leading to better outcomes in terms of productivity and development. This result is consistent with Ag´enor and Canuto (2012).

However, we should empirically investigate how education investment affects economic growth and development outcomes through an empirical analysis of different countries' experiences.

Finally, even after successfully escaping from poverty through external assistance and large policies, appropriate fiscal policies and development strategies remain critical for avoiding development traps and achieving sustainable growth. Few studies have analyzed the relationship between poverty and middle-income traps in theoretical research. This relationship directly affects how countries develop their economic policies and growth strategies.

Future work can explore two main possible extensions: First, extending the model to include international trade and foreign investment can help analyze how external factors affect development trajectories and trap dynamics. Matsuyama (1992) has shown how these two factors can significantly influence economic development patterns and outcomes. In today's globalized economy, considering how international trade and foreign investment affect development patterns and trap dynamics can provide valuable insights into modern economic development challenges.

Second, recent studies focusing on the middle-income trap have focused on premature deindustrialization, wherein middle-income countries experience manufacturing employment and output declines before reaching high-income status. In a premature deindustrialization context, many middle-income countries face declining manufacturing sectors and slower productivity growth, making it harder to achieve high-income status through traditional industrialization paths. Therefore, future research should analyze how premature deindustrialization affects development outcomes and policy responses in middle-income countries through theoretical and empirical research. This expanded theoretical framework can provide a more comprehensive understanding of how countries can effectively navigate development challenges and avoid stagnation at middle-income levels.

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A Stability Conditions

The relationship between the equilibrium point and the conditional equation of stability is shown in the following figure.

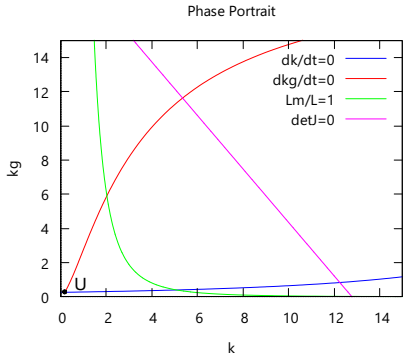

Figure 16: Labor surplus phase
$\tau = 0.4$, $\phi = 0.8$

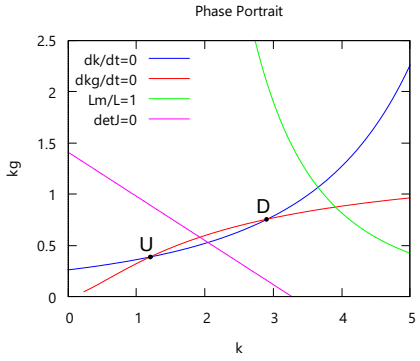

Figure 17: Labor surplus phas
$\tau = 0.4$, $\phi = 0.22$

In the figure 16, the point U is located below the curve, det$\mathbf{J} = 0$, and thus satifies (43). In the figure 17, the point U also satifies (43). Therefore, the point U is the unstable equilibrium point.

Meanwhile, the point D in the figure 17 is located above the below the curve, det$\mathbf{J} = 0$, and thus satisifies (42). Therefore, the point D is the stable equilibrium point.

B Other phase diagrams and detailed economic growth figures

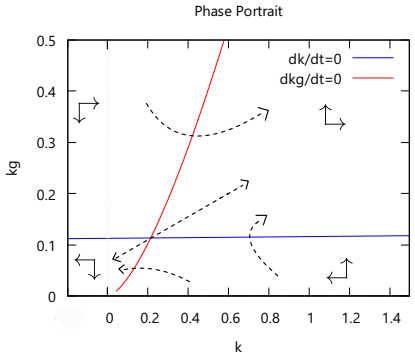

Figure 18: Labor surplus phase $\tau = 0.2$, $\phi = 0.8$

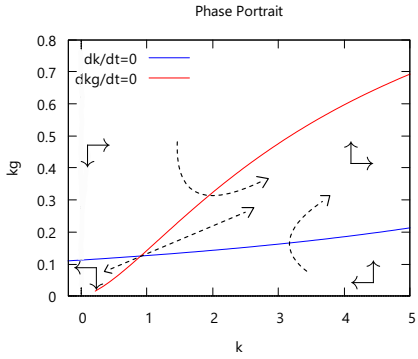

Figure 19: Labor surplus phase $\tau = 0.2$, $\phi = 0.22$

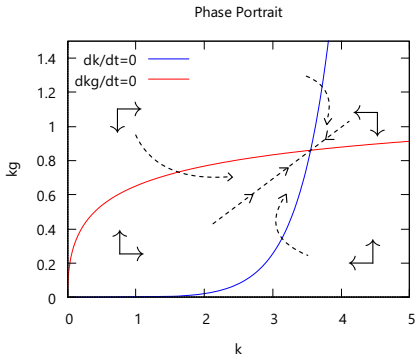

Figure 20: Mature phase$\tau = 0.4$, $\phi = 0.22$

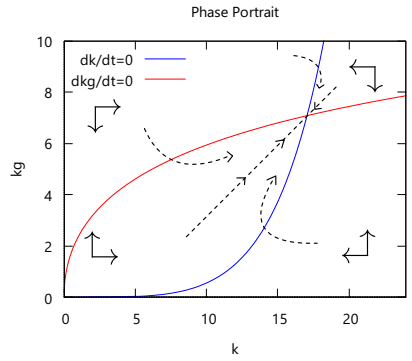

Figure 21: Mature phase$\tau = 0.2$, $\phi = 0.8$

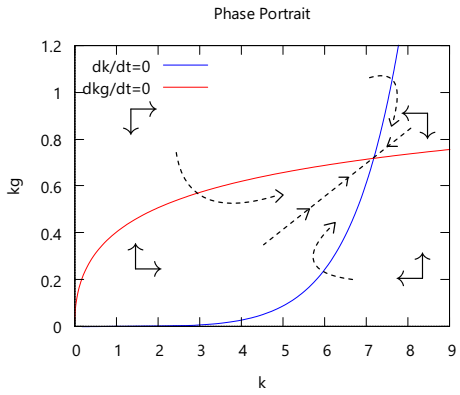

Figure 22: Mature phase$\tau = 0.2$, $\phi = 0.22$

Table 2 shows the detailed growth rates in Figure 9 and table 3 shows the detailed growth rate in the figure 10.

	g	
	$\phi = 0.8$	$\phi = 0.2$
$\tau = 0.1$	0.02350	0.03232
$\tau = 0.2$	0.03144	0.04609
$\tau = 0.3$	0.03729	0.05566
$\tau = 0.4$	0.04118	0.06189
$\tau = 0.5$	0.04324	0.06514
$\tau = 0.6$	0.04342	0.06543
$\tau = 0.7$	0.04156	0.06249
$\tau = 0.8$	0.03721	0.05554
$\tau = 0.9$	0.02934	0.04255

Table 2: Economic growth rate :Changing tax rate given expenditure composition

	g	
	$\tau = 0.4$	$\tau = 0.2$
$\phi = 0.1$	0.05783	0.04316
$\phi = 0.2$	0.06162	0.04589
$\phi = 0.3$	0.06204	0.04620
$\phi = 0.4$	0.06064	0.04518
$\phi = 0.5$	0.05786	0.04318
$\phi = 0.6$	0.05380	0.04028
$\phi = 0.7$	0.04837	0.03644
$\phi = 0.8$	0.04118	0.03144
$\phi = 0.9$	0.03129	0.02481

Table 3: Economic growth rate :Changing expenditure composition given tax rate