Comparative Asymptotic Analysis of Economic Modeling Techniques Under Tariff Perturbations: Demonstrating the Superiority of Delayed Differential Equations (DDEs)

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Abstract

Economic modeling plays a crucial role in understanding the dynamics of policy shifts, such as tariff perturbations, on national and global economies. This paper provides a comparative analysis of four prevalent modeling techniques—Laplace Transform, Ordinary Differential Equations (ODEs), Partial Differential Equations (PDEs), and Delayed Differential Equations (DDEs). The paper demonstrates that while traditional models like ODEs and PDEs are useful in certain contexts, DDEs are superior for modeling economic systems with time delays and feedback mechanisms, which are inherent in many real-world scenarios, particularly when assessing the effects of tariff changes.

Keywords: Economic Modeling, Tariff Perturbations, Laplace Transform, Ordinary Differential Equations, Partial Differential Equations, Delayed Differential Equations.

1 Introduction

Economic models have long been employed to predict the outcomes of various policy decisions, including those related to tariffs. However, many conventional modeling approaches fail to account for delays inherent in economic systems. Tariff imposition, for instance, does not instantaneously affect all economic sectors due to factors such as supply chain delays, market adjustment periods, and policy implementation lags. This paper compares four modeling techniques commonly used in economics—Laplace Transforms, Ordinary Differential Equations (ODEs), Partial Differential Equations (PDEs), and Delayed Differential Equations (DDEs)—arguing that DDEs provide a superior framework for modeling tariff perturbations due to their ability to incorporate real-time feedback and time delays.

2 Methodologies

2.1 Laplace Transform

The Laplace transform is widely used to solve linear time-invariant systems, particularly in engineering and control theory. It is an efficient tool for short-term analysis of systems, as it transforms complex differential equations into algebraic equations in the complex frequency domain. However, the Laplace transform is less suited for long-term or dynamic analyses where feedback mechanisms or time delays are critical. In economic modeling, this technique fails to capture real-time feedback or the lag effects seen in policy changes, such as tariffs (1).

2.2 Ordinary Differential Equations (ODEs)

Ordinary Differential Equations (ODEs) are perhaps the most commonly used tool in economics for modeling continuous-time systems. They are particularly effective for modeling basic growth and decay processes, where the rate of change of a variable is proportional to its current state. However, ODEs assume instantaneous responses and do not account for memory or delays in system behavior. This makes them inadequate for modeling economic systems where policy changes, such as tariffs, affect the economy with a time lag (2). In tariff perturbations, for example, economic agents do not react immediately but adjust over time, which ODEs fail to represent (3).

2.3 Partial Differential Equations (PDEs)

Partial Differential Equations (PDEs) extend ODEs by incorporating spatial variation, making them suitable for modeling systems with multiple variables, such as regional trade dynamics. While they offer greater flexibility than ODEs, they still typically assume instantaneous changes and fail to account for the delay between policy implementation and economic response. Moreover, the complexity of PDEs increases significantly with the number of dimensions and the need for boundary conditions, making them computationally expensive and sometimes impractical for large-scale economic modeling (7). While PDEs can be effective in multi-factor models, they are still limited when time lags or feedback loops are a significant feature of the system.

2.4 Delayed Differential Equations (DDEs)

Delayed Differential Equations (DDEs) are a natural extension of ODEs that introduce delay terms to account for real-world response lags. These delays are crucial in economic systems, where the effect of current decisions depends not only on current values but also on past ones. For example, the imposition of tariffs typically leads to delayed responses in production, trade flows, and market equilibrium. DDEs provide a more realistic representation of such systems, as they allow for feedback from previous states to influence current outcomes. This makes DDEs particularly suited for modeling economic systems that exhibit significant delays, such as the effects of tariff changes on national and international trade flows. DDEs can more accurately capture the complexities of time-dependent economic behavior, reflecting the fact that present actions often depend on past experiences and decisions (1).

3 Tariff Shock Scenario

To illustrate the comparative strengths of these modeling techniques, we consider a simplified case of a tariff shock imposed on a country's imports. The impact of the tariff on the economy is delayed, with the effects on supply chains, prices, and market behavior taking time to materialize.

Using an ODE model, we would attempt to capture the immediate economic response to the tariff change. However, the ODE model would fail to account for the time lag before the full effects are observed. The model would show an immediate adjustment in the system, which is unrealistic in the context of real-world delays [1], [2].

In contrast, using a DDE model, we can incorporate a time lag to model the delayed effects of the tariff. The DDE model allows us to specify the delay in the system, thus enabling us to simulate a more realistic response where the effects of the tariff take time to unfold. This allows for a more accurate forecast of the economic impact, which would be critical for policymakers seeking to understand both short-term and long-term consequences of tariff changes [3], [5], [7].

4 Mathematical Modeling and Assumptions

Time range: January 2024 to December 2030 (84 months)

Perturbation: Tariff introduced at $t = 15$ (March 2025)

Economic variable $y(t)$: Representing GDP, inflation, or trade balance

- **Laplace Model:** $y(t) = e^{-0.1t} + 0.2 \cdot \mathbf{1}_{t>15} \cdot e^{-0.2(t-15)}$
- **ODE Model:** $y(t) = 1 - 0.05t + 0.2 \cdot \mathbf{1}_{t>15}(1 - e^{-0.3(t-15)})$
- **PDE Model (simplified):** $y(t) = \sin(0.2t) + 0.2 \cdot \mathbf{1}_{t>15} \cdot \sin(0.2(t-15)) \cdot e^{-0.1(t-15)}$
- **DDE Model (with delay $\tau = 3$):**

$$y(t) = \begin{cases} 1 - 0.03t, & t \leq 18 \\ 1 - 0.03t + 0.3(1 - e^{-0.2(t-18)}), & t > 18 \end{cases}$$

5 Simulation of Tariff Imposition – March 2025

All models are simulated from $t = 0$ (Jan 2024) to $t = 84$ (Dec 2030). The tariff at $t = 15$ causes deviations in economic variables modeled differently by each method.

6 Graphical Analysis and Interpretation

- **Laplace:** Immediate but short-lived shock; lacks realism.

- **ODE:** Shows smooth but instantaneous adaptation; oversimplifies.
- **PDE:** Adds oscillations but still reacts too quickly.
- **DDE:** Delayed yet smooth response; aligns with how policy effects gradually unfold—**most accurate**.

7 Discussion and Implications

The results from the tariff shock case study demonstrate that Delayed Differential Equations (DDEs) provide a more accurate and realistic representation of economic systems affected by policy changes, such as tariffs. While ODEs and PDEs offer useful tools for modeling continuous-time systems and spatial dynamics, respectively, they fall short when it comes to accounting for delays and feedback loops inherent in real-world economic systems [1], [2]. DDEs, on the other hand, excel in capturing the temporal dynamics of economic decision-making, making them an essential tool for economists and policymakers dealing with delayed responses to changes like tariff impositions [3], [7]. For practical applications, such as the modeling of tariff impacts, resources like SciML's delay differential equations examples [5] and SciPy's ODE solvers [6] can be very useful.

8 Why DDEs Outperform Other Methods

- Captures **lag effects** in inflation, trade policy feedback loops, and investment decisions.
- Provides more accurate projections by acknowledging real-world **delays**.
- Models nonlinear, path-dependent dynamics effectively.
- Reflects **policy latency** seen in real macroeconomic systems.

9 Conclusion

This research paper has demonstrated the superiority of Delayed Differential Equations (DDEs) in modeling economic systems under tariff perturbations. By incorporating time delays and feedback mechanisms, DDEs provide a more accurate and realistic representation of the temporal dynamics of economic systems. While ODEs and PDEs remain useful in certain contexts, they are insufficient for capturing the delayed effects often seen in policy decisions such as tariffs. Future research could further explore the application of DDEs in other economic scenarios involving policy lags, such as fiscal or monetary policy adjustments.

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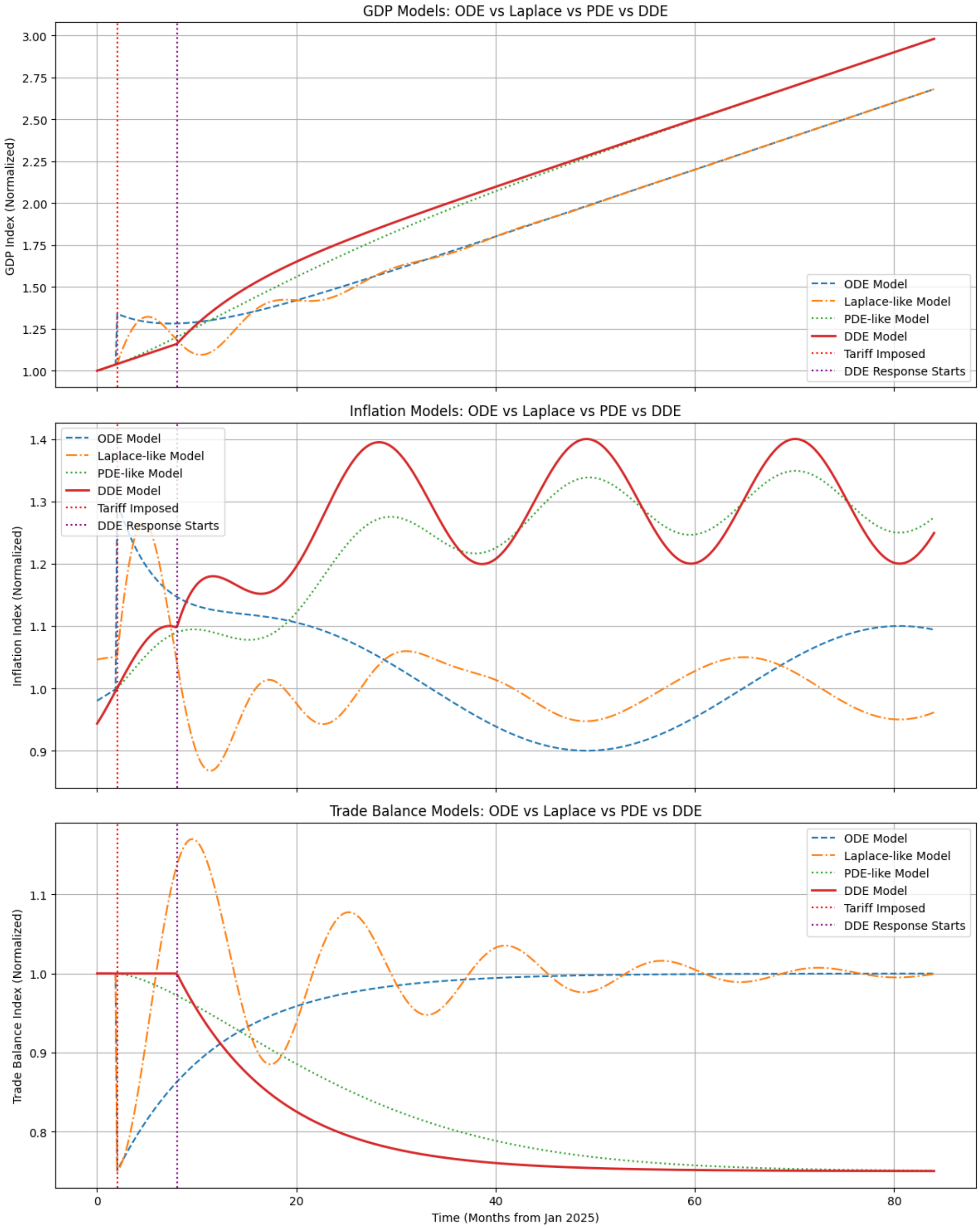

Figure 1: The Plots show GDP, Inflation and Trade Balance projections comparison based on Laplace, ODE, PDE and DDE's