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Identifying behaviour in a multiproduct oligopoly: Incumbents’ reaction to tariffs dismantling∗

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Preliminary

Abstract

The Spanish automobile market of the nineties experienced a perfectly foreseeable tariff dismantling and a strong demand downturn, with the observed result of an apparently sharpened producer competition in products and perhaps in prices. This paper is aimed at testing whether or not there really was a change in pricing behavior, using a structural model of competition. To answer that question, we specify, estimate and test semiparametric pricing equations with panel data for 164 models belonging to the 31 firms which competed in the market. The specification includes several equilibriums as alternative estimating models, considering prominently tacit coalitions by which a group of firms sets prices, taking into account the cross effects on their demands. The statistical test selects as the best model given the data an unbroken coalition of domestic and European producers. Comparative results using tight demand side specifications show that an inadequate specification of the demand side may induce wrong inferences.

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1. Introduction

At the beginning of the nineties, the Spanish automobile market completed a tariff dismantling planned since the Adhesion to the EEC. This fact, perfectly foreseeable since years before, complicated with a strong demand downturn (see Figure 1), lead to an apparently sharpened producer competition, clearly in products and perhaps in prices. Domestic producers (installed multinational firms) and foreign producers (European and non-European) introduced new models and increased model turnover, engaged in network investment and high advertising expenditures, while some signs of price competition seemed to appear. This paper is aimed at testing whether or not there really was a change in pricing behaviour, using a structural model of oligopolistic multiproduct firms which compete in a product differentiated market.

We understand by behaviour the particular strategies, in a set of well defined market-specific equilibrium concepts, which are sustained at a given moment. Clearly all producers, multiproduct firms with a rough average of more than three car models on the market at any given moment, must be assumed internalizing optimally the cross effects of their model pricing. Moreover, it is natural to assume that firms continuously adjusted model prices to their environment (demand evolution, entry and changes in characteristics of rival models), independently of the type of pricing equilibrium. The addressed question is whether, in addition, the environmental changes induced a change in firms’ pricing strategies, modifying their degree of rivalry.

To try an answer to this question, we develop the pricing equation implications of a series of equilibriums in the form of alternative estimating models. Among these equilibriums we consider prominently tacit coalitions, by which a group of firms sets prices taking into account the cross effects on their demands, and the change of these coalitions. We then relatively assess the models by testing which one best fits the data.

We specify and estimate the pricing equation with (monthly) panel data on quantity, prices and characteristics for 164 car models belonging to the 31 firms which competed in the Spanish market during the period 1990-96. We derive a semiparametric method
to simultaneously test for behaviour and estimate the own and cross elasticities from the specification, estimation and test of these pricing equations. A key question is how demand side information is treated. We specify price equations free of any restriction on the price effects coming from functional form demand side constraints, and this turns out to be important for the conclusion. Despite all appearances, our statistical test reveals that pricing is consistent with an unbroken tacit coordination of domestic and European car producers and a competitive behaviour on the part of the coming Asian brands. This conclusion is shown to be perfectly compatible with a broader dynamic game in which all producers may be competing in product investments (entry, advertising...)

The type of exercise that we perform can have some general interest. We try to uncover whether a policy change combined with a demand downturn triggered a behavioral change and which change. A similar issue becomes relevant when any exogenous market event may trigger a change in behavior: e.g. demand changes, approval of a merger or regulatory change, irruption of an innovation...As they now stand, quantitative methods of analysis of market competition have largely avoided the question of identifying behaviour and changes in behavior, with and without government intervention. It seems useful a further development of techniques to assess the impact of these changes.

Let us briefly comment on the relevant literature. Some of the pioneering works in the “new empirical industrial organization” were motivated by and focussed on the analysis of behavior changes (Porter, 1983; Bresnahan, 1987¹). More generally, this set out the question of the precise identification of firms’ behavior². A detailed specification of a set of market equilibrium behavioral alternative (static) outcomes in a product differentiated market, and the test among them given the data, was carried by Gasmi, Laffont and Vuong (1992). Only a few works have focussed on this type of testing (see for example for a recent application Jaumandreu and Lorences, 2002), but many discuss the potential effects of different behaviors or use, at some point, alternative behavioral assumptions.

¹See also Bresnahan 1981, which develops the model and assess the impact of imports.
²As something different from the use and empirical measurement of "conjectural variations" (on this use see Bresnahan’s 1989 survey).
Exercises of market modelling, concerned with assessing market power and describing its sources, e.g. product differentiation versus price coordination, discuss the likelihood of different behaviors. Nevo 2001, for example, compares the markups implied by his estimated elasticities, under the alternative behavioral assumptions of Bertrand-Nash competition and collusion, with the real industry markups, to conclude that pricing is non-collusive and markups come from product differentiation. Pinkse and Slade 2003 use the estimated elasticities to evaluate the effects of real and potential mergers, using the Bertrand equilibrium after concluding that is not rejected by the data. Both examples have in common relying exclusively on the estimation of demand systems.

On the other hand, two examples of the use of pricing relationships under alternative behavioral specifications just come from examples on the automobile market. Berry, Levinsohn and Pakes 1999 estimate their oligopoly model for the American automobile industry under the three alternative assumptions (to Bertrand equilibrium) that firms play Cournot, that there is a "mixed" equilibrium in which Japanese firms set quantities, and that Japanese firms play Bertrand as do the rest, but colluding among them. Goldberg and Verboven, 2001\(^3\), in their automobile model for five European countries, also estimate the model under the alternative assumption that firms in the UK collude. The conclusion of these exercises seems at first glance somewhat disappointing. The first paper concludes that estimated parameters are quite similar and that differences seem really do not matter for policy conclusions. The second finds the models indistinguishable in terms of fit. However, all estimates of the pricing relationships are carried out constraining markups to have the value determined by the demand estimated elasticities, either simultaneously (first paper) or even sequentially (second paper) to the demand parameters estimation. As Berry, Levinsohn and Pakes 1999 point out, using "the estimated elasticities to investigate the cost side of the model...would be more flexible and impose less structure."

More generally, a rich methodology for the specification and estimation of demand in industries with product differentiation has been developed since the papers by Berry, 1994, and by Berry, Levinsohn and Pakes, 1995. Berry, Levinsohn and Pakes 2004 show in partic-

\(^3\)See also Goldberg 1995.
ular how this methodology may be precise in estimating the patterns of substitution. But the rich modelling of the demand side has often come at the cost of constraining behavior. In fact, price relationships have been exploited more as auxiliary equations for demand identification than as genuine sources of information. Here, we explore the extension of ideas and techniques of the recent advances to a framework which addresses the identification of firms’ behavior through pricing equations.

The methodology consists of specifying and estimating pricing equations which nest the unobservable marginal cost and the margins established by firms. Margins can be shown to be in general a function of the firm expected demand price effects, firms’ market shares and behavior. By specifying alternative behaviors, one ends with a series of models which predict different margins which depend on different ways on observed shares. We derive a semiparametric specification to simultaneously test for behavior and estimate own and cross elasticities from the price equations, free of constraints imposed by functional form assumptions on the form of demand. We find that estimation is possible, easy, and gives sensible results. The comparative results using tight demand side specifications show that an inadequate specification of the demand side may induce wrong inferences. By the same reason, the extended practice of doing inferences about market competition using exclusively demand models can be highly misleading.

The rest of this paper is organized as follows. Section 2 explains in detail the competition changes that took place in the Spanish market and descriptively explores the price data. Section 3 discusses the way to specify and test for behavior. Section 4 is devoted to detail the semiparametric specification and estimation techniques that we apply to the pricing equations, and Section 5 to explain the empirical results. Section 6 concludes. An Appendix develops a series of technical details which we use at different points of our exercise.
2. Competition changes

At the start of the nineties, the Spanish automobile market\textsuperscript{4} was served by three types of car producers: domestic producers, European foreign producers and non-European foreign producers, just then beginning to enter the market. The domestic producers were the multinationals with plants installed in Spain during the seventies and the eighties, aimed at exporting an important part of production, manufacturing in them some of the car models they sold\textsuperscript{5}. The European foreign producers were the multinational European producers without manufacturing in Spanish territory, and the non-European foreign producers were firstly exclusively Asian producers, sometimes possessing an incipient production in European territory. Tables 1 and 2 report some basic facts about the structure and evolution of the market.

Domestic producers accounted for seven brands belonging to five groups (Citroen-Peugeot, Ford, Opel, Renault and Seat-VW), which coincided with the most important non-Japanese world producers with the absence of Fiat and Chrysler (recall that Opel is a GM subsidiary). They had dominated the Spanish market during the eighties, and they started the nineties with a joint market share of 82\% (see Table 1). At this time the European foreign producers’ supply consisted of 14 brands\textsuperscript{6}, with a joint share of only 16\%, but with an important presence in the upper segments (e.g., more than half of the cars of the highest segment). And non-European producers accounted initially for 5 Asian brands, representing all together just a market share of 2\%. This number grew up to 9 brands in the following years\textsuperscript{7}, and the American Chrysler entered the market in 1992.

\textsuperscript{4}The Spanish market was at the time about 1 million cars sold a year, a non-negligible size from the European perspective.
\textsuperscript{5}In 1990, they sold in the domestic market 39\% of the domestic production. Production capacity grew faster than the market in the following years and, by 1996, the proportion of production going to domestic sales was only 25\%. Notice that Spain was at the time the 3rd European and the 5th World car producer.
\textsuperscript{6}Audi, Alfa-Romeo, BMW, Fiat, Jaguar, Lada, Lancia, Mercedes, Porshe, Rover, Saab, Skoda, Volvo and Yugo.
\textsuperscript{7}Honda, Hyundai, Mazda, Nissan, and Toyota were in the market at the start of the 90’s, Mitsubishi and Suzuki entered in 1990, Subaru in 1991 and Daewo in 1995.
Tariff and non-tariff protection made it unprofitable to import cars from abroad during the early eighties, dampening even the import of the models from domestic producers not produced in Spain. All imported cars in 1985 amounted to only 13% of sales. But this year the Spanish Adhesion Treaty to the EEC, setting the transition framework to full integration in the single market of 1992, firmly established a different perspective. Tariffs on cars imported from the EEC had to be decreased as stipulated from the then-current value of 36.7% to zero by the beginning of 1993. And tariffs on cars imported from third countries had to be reduced from the then-current value of 48.9% to the common EEC tariff of 10%.

This perspective immediately started a new competition preparing the coming open market, stimulated by a very dynamic demand (see Figure 1). Domestic producers enlarged the range of models distributed in the market with models imported from their production in plants abroad, while foreign producers entered new models. Imports had risen to 32% of sales by 1990 (recall that only 18% are imports by foreign producers) and product variety was already quite high (79 marketed models, see Table 2). But, the beginning of the nineties, when tariffs reached the minimum and at a moment in which demand transitorily experienced a stagnation and then a sharp downturn (see Table 1 and Figure 1), seems to trigger a new competition intensity.

Competition during the nineties adopted several dimensions: product behavior resulted in a high rate of model introduction and turnover, producers heavily invested in construction and enlargement of sales networks, engaged in a sharp increase of advertising and seem to start some price competition which consumers perceived through promotional advertising.

The entry of car models, both replacing old models and introducing in the Spanish market models absent until this time, was particularly important. In the years following 1990, 104 models entered the market and 59 exited, which implies 123 marketed models by the end of 1996 (see Table 2). Entry was important from the beginning, but notice that exit increases after the first years (see Table 1), a sign of more acute product competition. Asian cars

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*We take as an exit the fact that monthly sales persistently go under some minimum threshold. This can be obviously determined either because consumers stop buying this particular model or the brand decides...*
accounted for a disproportionate share of this entry, but entry by the European foreign
producers and even domestic producers is also important. The role of replacement can be
seen by noting that 90% of exits are separated from a model entry by the same brand by less
than 48 months. Advertising expenditures suddenly jump in 1993, with the expenditure by
unit sold during the period 1993-96 being 170% of the amount during 1990-92.

Among all these competition changes, the focus of this paper is on pricing. In particular,
did the dismantling of tariffs, perhaps complicated with the demand downturn, change
firms’ price behavior? Foreign firms found themselves able to sell at significantly lower
prices for the same received prices. Domestic producers experienced the same change for the
models which were introduced from abroad and, at the same time, they expected increased
competition for all their models, including enlarged substitutes and lower rivals’ prices.
All producers are multiproduct firms, with several car models on the market at a given
moment, which implies that they must be assumed to optimally internalize the cross effects
of their models pricing. Moreover, firms are continuously adjusting each model price to the
changing environment (sales level, entry and changes in characteristics), independently of
the game they play. The central question is whether, in addition to all this, the environment
induced any change in firms’ pricing strategies, modifying their degree of rivalry, in the sense
explained in the next section.

To acquire an impression of possible pricing behavior changes in our sample period, the
cost changes induced by quality changes must be disentangled. With this aim, we will use
the hedonic coefficients resulting from regressing prices on car characteristics. Let us define
the price corrected by quality changes as

\[ \tilde{p}_{jt} = p_{jt} - (x_{jt} - x_{j0})\hat{\beta} \]  

(1)

where \( x_{jt} \) is the vector of characteristics of model \( j \) at moment \( t \), \( x_{j0} \) stands for this vector
when the model enters the sample, and \( \hat{\beta} \) represents the cost per unit of characteristic
retire it from sales or some mix of both aspects. Increased exits can be taken in both cases as a sign of
increased product competition.
estimated in the hedonic regression\(^9\). Averages of these quality-corrected prices will change with the entry and exit of models, which embody idiosyncratic qualities that shift the mean. To correct for these effects, let us define quality change and entry-corrected prices as

\[
\bar{p}_t = \frac{1}{N} \sum_j (\bar{q}_{jt} - (x_{j0} - \bar{x})\hat{\beta})
\]

where \(\bar{x}\) is the sample mean of attributes and \(N\) is the number of models at date \(t\). Entry and quality change-corrected prices, depicted as indices, give the change in prices which may be attributed to reasons other than quality-induced cost variations. Of course, they can show cost changes attributable to other reasons, but they are likely to clearly reflect possible changes in pricing.

Figures 2, 3 and 4 summarize the results of descriptively exploring price changes. Figure 2 represents simple average monthly prices for the three producer types, deflated by the consumer price index, and the average received prices; that is, the price received by producers after deducting the relevant tariffs\(^{10}\). The figure highlights an apparent parallel evolution of European and domestic received prices during the period, at a different level determined by the diverse sales composition, and a sharp decrease of the Asian received prices. Figure 3 represents the evolution of received prices differencing out the quality-induced cost variations (normalized to unity the first year), and Figure 4 represents the evolution differencing out the quality composition effects of entry and exit. The hedonic corrections work very well, and in particular denote that quality increments of marketed cars are introduced at a similar pace for all producers, particularly after 1992, and that Asian entry mainly consists of models directed to compete in the lowest segments as time goes by. Notice how the sequential corrections notably reduce the range of variation of what remains of prices variation.

\(^9\)We employ the coefficients corresponding to one of the estimated models (see section 5), but the exercise produces very similar results using alternative estimates.

\(^{10}\)Tariffs during the years 1990, 1991 and 1992 are estimated as 12.37, 8.25 and 4.125% for European cars, and 23.6, 18.7, 13.8% for non-European cars. Since the beginning of 1993, only remains the 10% tariff for the non-European.
Figure 4 highlights several points. Firstly, all prices tend to show a fall during the first three years (1990-92) and some recovery at some point of the following subperiod. This suggests partly procyclical pricing, matching the demand evolution reported above, which does not contradict the possible change in pricing. Secondly, Asian car prices show a sharp new decrease by the year 1993. Asian producers seem to price more aggressively when the transitory tariff period reaches its end. These sensible changes in relative pricing (notice that the biggest difference is on average less than 15%) suggests that, since 1993 onwards, the market may be working on a different equilibrium.

3. Testing pricing behaviour with price equations.

This section presents the framework to specify and test for pricing behavior by means of price equations. Firms are assumed to be multiproduct, competing in a product-differentiated industry given products and their characteristics. Behaviour consists of the particular strategies, among the set of well-defined equilibrium concepts for price games, sustained by firms. A wide range of behaviours may be covered and the framework is consistent in particular with a broad class of dynamic oligopoly games. The equations stem from market equilibrium relationships between prices and output represented by shares. Firms’ shares are hence endogenous variables, given by the relevant (in principle unspecified) demand system. Testing behavior consists of assessing which equilibrium best fits the data.

3.1 Basic setting.

Let us assume a product-differentiated industry consisting of $F$ multiproduct firms, indexed $f = 1, \ldots, F$. Each firm produces $J_f$ products and there are in total $J = \sum_f J_f$ products. When we generically refer to a product $j$, it is implicitly assumed to be one of the products of the set $j = 1, \ldots, J_f$ produced by firm $f$. Demand for each product $j$ is a function of the $J \times 1$ vector of prices $p$ and all the vectors of products characteristics $x_1 \ldots x_J$. Write demand, for the sake of simplicity, as a function of prices only in the shares form $q_j = s_j(p)M$, where $M$ is market size (usually the number of potential consumers).
Share $s_0(p)$ stands for the fraction of consumers buying nothing. Let $s$ be the $J \times 1$ vector of shares and define the $J \times J$ price effects matrix $D = \{\frac{\partial s_k}{\partial p_j}\}$, where row $j$ collects the own and cross-demand effects of price $j$. Product $j$ constant marginal cost, assumed here to be known to simplify notation, is $c_j$\(^{11}\).

3.2 Behaviour.

Assume that prices are strategic complements (reaction curves are upward-sloping)\(^{12}\). Therefore, any price increase on a product generates a positive externality on the other product profits, including rival firms’ product profits. It seems simply natural to assume that in any case firms care about internalizing the cross-price effects of their own profits (they are not “myopic”). That is, firm $f$ sets prices by maximizing $\sum_{k \in J_f} (p_k - c_k)q_k$. But firms can also set prices which internalize the positive cross-price effects among a group of rivals. That is, they can form price coalitions (Deneckere and Davidson, 1985), by maximizing $\sum_{k \in J_h} (p_k - c_k)q_k$, where summation is extended to the $J_h = \sum_{f \in h} J_f$ products of the firms which take part in the coalition\(^{13}\). Let $H$ be the number of coalitions. Belonging to a coalition implies setting prices to maximize over a set of products which contain the own products as a subset. Hence, from now on, we will speak exclusively about the grouping of products at the level of coalition without loss of generality (a situation in which real coalitions are irrelevant can be thought of as consisting of so many one-member coalitions as firms in the market).

Prices maximize profits of the relevant set of products given the other prices, and we will write this as $p_j = \arg\max \{\sum_k \delta_{jk}(p_k - c_k)q_k|\delta_j\}$, where $\delta_j$ is a $1 \times J$ vector of ones and zeroes, with element $\delta_{jk}$ being the indicator of inclusion of product $k$ in the relevant profits sum (i.e., $\delta_{jk} = 1$ if $k$ belongs to $j$’s coalition and $\delta_{jk} = 0$ otherwise).

\(^{11}\)The model can be extended to allow for non-constant marginal costs by specifying relevant marginal cost as $c_j(1 + k)$, where $k$ is the elasticity of cost with respect to output.

\(^{12}\)With goods being economic substitutes this is the usual case, although it is not the unique possibility. The model, however, can be extended to any other situation.

\(^{13}\)Here we assume that firms enter price coalitions with all their products. Other equilibria can be considered, but notably complicate the analysis and notation.
Let us stack all vectors $\delta_j$ in a $J \times J$ matrix $\delta$. Notice that we are simply specifying the “one-period” price interactions, not the full conditions for each equilibrium. Pricing behaviour will be identified without addressing the details of the relevant whole equilibrium. This allows for a great generality in the equilibriums encompassed. For example, the particular set of interactions may be the result of a Nash perfect equilibrium under strategies corresponding to repeated games. In fact, price coalitions are usually understood as sustainable under repeated games (see, for example, Tirole 1989). The price game can be also the result of the per-period adjustment of prices in any dynamic oligopoly game of investment (e.g. on advertising or capacity; see, for example, Bajari, Benkard and Levin, 2005). One limitation are dynamic price games, or games in which the prices of one period impact the demand of future periods. If this situation is thought as the relevant the setting should be adapted.

3.3 The determination of prices.

With uniproduct Bertrand players or "myopic" multiproduct Bertrand players $\delta = I$. With multiproduct Bertrand players, each row $j$ has ones in the entries corresponding to the rest of products produced by the firm that owns $j$. With price coalitions, the ones of a row expand to all the products of firms in the coalition. The set of FOC conditions which define the price equilibrium corresponding to the relevant behaviour can be written in matrix form as

$$s + (\delta \circ D)(p - c) = 0$$

where $\circ$ represents Hadamard product (element by element product). Equilibrium prices are easily obtained as

$$p = c - (\delta \circ D)^{-1}s$$

System (3) consists of $J$ equations in the form $p_j = c_j + m_j(D, \delta, s)$, and shows that margins corresponding to a particular equilibrium are an equilibrium-specific function of demand price effects and firm shares. These equations are structural equilibrium relation-
ships, relating the endogenous variables $p$ and $s$, with $s$ determined additionally through an arbitrary system of demands. A useful property of equation (3) may be summarized in the following

Property 1. Product $j$ margin can be written as a linear combination of the shares of the products included in the coalition, with weights which are a function of the coalition submatrix of demand price effects.

Proof. See Appendix.

A useful implication of this property is that is that $\delta \circ (\delta \circ D)^{-1} = (\delta \circ D)^{-1}$.

### 3.4 Expected price effects.

System (3) could already be the base for testing behaviour\textsuperscript{14}, but it seems reasonable and useful to be more specific about the matrix $D$ of price effects. Suppose a market with many differentiated products and consumers heterogeneous in income and tastes who decide among varieties (discrete choice). With $M$ consumers, firm which sets $p_j$ will take into account price effects across the expected quantities $E(q_k) = P(k)M$ and hence computes average partial price effects $\frac{\partial P(k)}{\partial p_j}$. A useful property is the following

Property 2. If each consumer endowed with income $y$ and tastes heterogeneity $v$ is characterized by a set of conditional probabilities $P(j|y,v)$ of buying the different goods (included the outside alternative), and a change in price $p_j$ changes her probabilities of buying $j$ by a fraction $\alpha(y,p_j)$\textsuperscript{15} and varies the rest of probabilities consistently in proportion to their

\textsuperscript{14}Equation (3) suggests that behavior can be tested by comparing the fit of alternative pricing equations, with behavior imposed on each equation through the constraints on the price derivatives that each equilibrium implies. In addition, an important implication of the above proposition is that, if price effects can be considered stable, and hence estimable, econometric specifications of behavior may be obtained by simply including the relevant rivals’ shares among the right-hand side variables.

\textsuperscript{15}The price effect may be simplified to $\alpha(y)$, but $\alpha(y,p_j)$ is theoretically sounder if we think of the price effects consistent with a general enough indirect utility function.
relative weights, average market price effects of good \( j \) have the form

\[
E \left[ \frac{\partial P(j)}{\partial p_j} \right] = \alpha_j P(j)(1 - P(j))
\]

(3)

\[
E \left[ \frac{\partial P(k)}{\partial p_j} \right] = \alpha_j P(j)P(k)\theta_{jk}
\]

where \( P(j) \) stand for the aggregate probability that \( j \) is bought and \( \alpha_j = \alpha_j^*(1 + \omega_j) \) and \( \theta_{jk} = (1 + \omega_{jk})/(1 + \omega_j) \), with \( \alpha_j^*, \omega_j \) and \( \omega_{jk} \) representing moments computed over the distribution of consumers heterogeneity (marginal utilities of income and buying probability covariances respectively). The \( \theta_{jk} \) values are positive and can range from zero to values well above 1.

Proof. See Appendix.

These market price effects under consumer heterogeneity vaguely remember logit elasticities but they differ in two important aspects. Firstly, the \( \alpha \) coefficients are product specific. Moment \( \alpha_j \) is marginal utility of income averaged over the distribution of income conditioned in choosing \( j \), or “average marginal utility of income for buyers of good \( j \)”.

Secondly, cross-price effects are scaled by factors which depend on the \( \omega \) covariance terms. These terms, which measure “proximity” or degree of substitutability of the products, can be thought of as reflecting substitution over the distribution of consumers’ heterogeneity.

These aggregate price effects follow by aggregation under arbitrary consumer heterogeneity. Notice that no parametric assumption is necessary, although parametric demand models general enough, such as BLP random coefficients model, generate aggregated price effects of this type. BLP techniques estimate these price effects by simulating the unobservable consumer heterogeneity. The interesting thing about price equations is that maximizing firms can be assumed as computing these average effects from their own knowledge about heterogeneity.

3.5 Price equations.

Property 3.

Plugging expressions (4) into matrix \( D \) of (3) and inverting the matrix it can be shown
that firms pricing equations can be written as

\[ p_j \simeq c_j + \frac{1}{\alpha_j} [1 + P(j)] + \sum_{k \neq j} \delta_{jk} \frac{\theta_{jk}}{\alpha_k} P(k) \]

Proof. See Appendix.

Under the assumption that aggregated shares converge in probability to aggregated probabilities, we get an equation in terms of observed shares and parameters which are functions of product-specific average marginal utilities and moments of probabilities over the distribution of consumers’ heterogeneity. Modeling cost in terms of observed and unobserved attributes, adding the time dimension and an error term we then have the price equation

\[ p_{jt} = \beta_0 + x_{jt}\beta + \frac{1}{\alpha_j} (1 + s_{jt}) + \sum_{k \neq j} \delta_{jk} \frac{\theta_{jkt}}{\alpha_k} s_{jt} + \xi_j + u_{jt} \]

(4)

The first two terms of the right hand side model marginal cost in "hedonic" terms (assume \(x\) includes characteristics and squares of characteristics in deviation from their sample means) while \(\xi\) stands for unobserved product-specific costs. The third and fourth terms stand for the margin that the firm would set for product \(j\) -given the other prices- in the absence of even multiproduct pricing (myopic behavior). The non-zero terms of the following sum account for the relevant terms as the result of multiproduct pricing (\(\delta_{jk} = 1\) when \(k\) is produced by the firm which produces \(j\)) and strategic behavior (\(\delta_{jk} = 1\) when \(k\) is produced by a firm which prices coordinately with the firm which produces \(j\)).

Equations like (5), representing equilibrium pricing by the firm, are attractive because:

a) they do not impose any functional-form structure on the price effects, and b) different behaviors raise nested models. Unknown coefficients \(\alpha\) and \(\theta\) may be estimated as coefficients of the shares. Consistent estimation imply the use of IV, because shares are endogenous, but tests between equilibria may be easily carried out as tests of exclusion restrictions. The main problem is that we will probably need to estimate a number of parameters which can be very high, and that the number of parameters increases with the degree of collusion. With \(H\) price coalitions (think of multiproduct firms that price independently as one-firm coalitions, without loss of generality), this number is \(\sum_h J_h^2 \leq J^2\) which may easily be far
from the already likely high lower bound $J$. Hence, equations like (4) can only be estimated for a very small number of products and enough repeated observations for each product $j$, either over time or across markets.

The situation partially improves when similar price effects can be assumed for groups of analogous products or “nests.” But any reduction in the dimensionality based on constraining the price effects tends to cast doubts on the results of the testing of behavior, which relies on evaluating the (own and cross) structure of these effects.

4. Semiparametric specification and estimation.

To overcome the problem of parameter dimensionality we are going to consider a semiparametric alternative for equation (5), valid for $J$ large, which avoids imposing strong previous constraints on the form of the price effects.

Notice that $\frac{1}{\alpha_j}(1 + s_{jt})$ can be written as $(\frac{1}{s_{jt}} + 1)\frac{s_{jt}}{\alpha_j}$ and recall that (alternative) $\delta_j's$ are a-priori specifications representing the behavior we want to test for. We substitute two unknown functions for the two varying-parameter expressions that remain: $\frac{s_{kt}}{\alpha_k} = g(s_{kt})$ for all $k$ (i.e. including $j = k$), and $\theta_{jkt} = \theta(d_{jkt})$, where $d_{jk}$ is a measure of the distance between products $k$ and $j$, $k \neq j$, in the characteristics space. As the easiest alternative we are going to use the Euclidean distance.

Equation (5) can now be written as

$$p_{jt} = \beta_0 + x_{jt}^j + \left(\frac{1}{s_{jt}} + 1\right)g(s_{jt}) + \sum_{k \neq j} \delta_{jk} \theta(d_{jkt})g(s_{kt}) + \xi_j + u_{jt}. \tag{5}$$

or, to simplify notation, defining conveniently $w(d_{jkt}) = \theta(d_{jkt})$ if $j \neq k$ and $w(d_{jjt}) = (1/s_{jt} + 1)$ we can use the more compact form.

$$p_{jt} = \beta_0 + x_{jt}^j + \sum_k \delta_{jk} w(d_{jkt})g(s_{kt}) + \xi_j + u_{jt}. \tag{6}$$

Model (6-7) is a semiparametric equation which includes two interacted unknown functions, each one with its own economic interpretation. To estimate them we are going to use series estimators, so the sum in (6) will be the tensor product usually employed to specify
multivariate unknown functions\textsuperscript{16} We are going to specify $g(s_k) = \rho_0 + \sum_i \rho_i s_k^i$ and impose positivity on the $\theta$ effects by specifying $\theta(d_{jk}) = \exp(\lambda_0 + \sum_i \lambda_i d_{jk}^i)$. Notice that the fact that the $g(.)$ function appears in (6) multiplied by a known factor and that both functions are interacted allows in principle for the identification of the constants separately to $\beta_0$.

With both series estimators having the same number of terms $I$, the number of parameters to be estimated by reason of the unknown functions are $2I$. Identification can be intuitively though of in the following way. Approximate $\theta(.)$ by the sum of a constant and terms in powers of $d$. The product of this approximation of $\theta(.)$ by $g(.)$ is, for a given term $k$ of the sum, a polynomial including the tensor product of the powers of $d$ and $s$. The sum of these polynomials across $k$, gives a new polynomial in terms of sums of products of powers of $d$ and $s$ across goods. This polynomial would constitute an identifiable linear model on whose coefficients we are in fact imposing nonlinear constraints.

Estimation can be carried out by means of a nonparametric two stage least squares method (Newey and Powell, 2003). We use the version of Ai and Chen (2003) or "sieve minimum distance" estimator. We proceed as follows. Dummies for each one of the models to account for the fixed effects and the other parameters which enter linearly (time dummies, characteristics and the squares of characteristics) are "concentrated out". This leaves us with a search for the parameters of the two unknown functions. We set the problem as a nonlinear GMM problem where we use as instruments all the variables which enter linearly and all the sums of the products of powers of the variables on which we want to condition $d$ and $s$ (i.e. the terms of the "aggregate" polynomial). We consider the distance between products as exogenous and hence we use it as instrument. And we use as instrument $s$ lagged six months to avoid the possible correlation of more contemporaneous values with the error term of the equation. The sums are computed across all competitors, independently of the particular competition model represented by $\delta$, to keep instruments the same increasing the comparability of the estimates.

Both functions are specified as cubic polynomials and, while $g(.)$ is kept totally unrestricted, we impose (decreasing) monotonicity on the $\theta(.)$ function. Trials have included

\textsuperscript{16}This provides another possible perspective to see the estimator.
the estimation without "fixed effects," the use of higher order polynomials, the employ of different lags of $s$ and the use more instruments. It seems to be important to estimate including fixed effects and assuming the endogeneity of $s$, but the change in other details seem to change little.

The model is estimated for different pre-specified behaviors embodied through the specification. The semiparametric specification has transformed the problem however in a series of non-nested estimates which use different regressors. The testing of the models against each other is then carried out by means of the Lavergne-Vuong (1996) test for selection of regressors in nonparametric estimation when models are non-nested. The Lavergne-Vuong test compares the MSE of each two models taking into account the variance of the difference between the MSE’s of the two estimates.

5. Empirical results.

We estimate price equations using data on the car models sold on the Spanish market from 1990 to 1996 by the 31 firms with a presence in the marketplace. The data consist of unbalanced panel observations for a rather standard number of individuals (164 models\textsuperscript{17}) but with the more unusual characteristic of monthly data frequency (which gives a maximum of 84 observations per individual). Using the price equilibrium relationships established in Section 3, and the semiparametric specification and techniques described in Section 4, the final objective of this empirical exercise is to obtain estimates under the assumption of different behaviours and to test their relative likelihood given the market data. This section begins giving some details on the employed variables and equilibrium concepts, goes on some detail in explaining the results of estimating the semiparametric equations and finally compares the estimates with the results obtained with other specifications.

\textsuperscript{17}The total number of models are 182, but we must drop 18 in estimation due to the lack of enough lagged observations: the 16 entrant models of the last year and 2 models which stayed in the market less than 12 months.
5.1 Variables and equilibriums.

Our dependent variable $p_{jt}$ is producer received prices or observed prices once deducted the tariff, i.e. $p^{ob}_{jt}/(1 + \text{tariff}_{jt})$. We must carry out the simultaneous estimation of a nested marginal cost function and the firms’ markups. To estimate marginal costs, we adopt the “hedonic” approach\(^\text{18}\): we take cost as a function of a set of product attributes. Specifically, we approximate marginal cost around its mean using a quadratic polynomial with attributes entering in the form of deviations with respect to the sample mean and the squares of these deviations. We specify marginal cost as independent of output (in fact we do not observe the relevant output for most of the involved producers), we include an estimate of the relevant average unit labour costs for each producer and we allow for unobserved components of marginal cost by adding the unobservable model-specific effect.

The employed attributes are the power measure \textit{ratio cubic centimeters to weight} (CC/Weight), the fuel efficiency ratio \textit{km to liter} (Km/l), used in the particular form of the relative efficiency in city driving with respect to 90 Km/h driving, the measure of size and safety \textit{length times width} (Size), the \textit{maximum speed in km/h} (Maxspeed) and the materials use indicator \textit{weight} (Weight)\(^\text{19}\). The use of other characteristics or a more complete list does not change the main results.

We estimate the measure $d_{jk}$ of “distance” or degree of substitutability between each two products using the four first attributes. Let $x_j$ and $x_k$ represent the relevant vector of characteristics, distance is computed as $d_{jk} = \left[ (x_j - x_k)\Sigma^{-1}(x_j - x_k) \right]^{1/2}$, where $\Sigma$ represents the matrix of empirical sample covariances.

Equations are estimated for five pre-specified behaviors: myopic; Betrand multiproduct; a price coalition sustained by the domestic and European firms; the break up of this coalition in 1993 reverting to Bertrand; and full collusion. The first behaviour is taken into account

\(^{18}\)Cost estimates starting from regressions on product characteristics can be called “hedonic” because they use the methodology of the traditional hedonic price regressions (see Griliches, 1961; Rosen, 1974, and the recent discussion in Pakes, 2003). We follow the approach by Berry, Levinsohn and Pakes (1995).

\(^{19}\)We try to be deliberately close to Berry, Levinsohn and Pakes’ (1995) specification for the sake of comparisons.
as an unlikely lower bound. The second implies that behaviour was Bertrand-Nash all the
time and for all players, a common assumption in many models and estimates of this type.
The third makes the sensible assumption that domestic and European producers set prices
internalizing the cross effects of their prices; i.e., they constitute a price coalition while Asian
producers are assumed to play Bertrand. The fourth assumes that this coalition broke up
at the end of 1992, with the Domestic and European producers switching to play Bertrand.
The fifth makes the unrealistic assumption that behavior was collusion of all players all the
time.

5.2 Results from semiparametric equations.

Table 3 shows the estimates of the pricing equations under the different behavior spec-
ifications. Table 4 shows the results of the Lavergne-Vuong test for model selection. The
model representing an unbroken price coalition is clearly the closest to the data. Next sub-
section shows the degree of coincidence of this conclusion with the estimates obtained by
other methods, what turns out to be a robustness check which reinforces the interest of the
estimates. Let first explore with some detail the results of the estimate.

The most natural product of the selected model is the estimation of margins. Let us study
the implied margins and the corresponding elasticities. Despite many efforts to avoid the
problem, the \( g(.) \) function estimate remains negative for the preferred model at somewhat
less than 12% of the data, mainly in the right tail (big \( s \) values). As some negative values
for the \( g(.) \) function can be perfectly consistent with our "fixed effects" specification (both
the constant and the effects can be picking up some specific margin intercepts) we take into
account part of them. Specifically, we define a trimmed estimator \( \hat{g} \) by setting the 5% lower
values (all in the right tail) to the (still negative) value of the function at this percentile.
Then we compute margins as \((\frac{1}{s_j}+1)\hat{g}(s_j)+\sum_{k\neq j}\delta_{j\neq j}\hat{\theta}(d_{jk})\hat{g}(s_k)\), and set to zero the small
proportion of margins which result negative (2.4%). Panel A of Table 3 shows the estimated
margins and their evolution for the whole sample and for competitors grouped in domestic,
European and Asian. Panels B, C and D of Table 5 show a rough decomposition of average
margins into a part explained by Bertrand pricing, a part added by the firms’ multiproduct optimal pricing, and the part explained by price coordination. It must be taken into account that this decomposition is done given the other prices.

The selected model allows also us to estimate the own and cross elasticities consistent with these margins. The own elasticities can be estimated as \( \hat{\eta}_j = -\hat{\alpha}_j(1 - s_j)p_j \simeq -p_j/\hat{m}_j \) and the cross-elasticities as \( \hat{\eta}_{jk} = \hat{\alpha}_js_jp_j\hat{\theta}_{jk} \simeq (p_j/\hat{m}_j)\frac{s_j}{s_j}\hat{\theta}_{jk} \), where \( \hat{m} \) is an estimate of the myopic margins. We start with the estimator \( \hat{m} = \frac{1}{s_j} + 1 \). As we have, however, some margins estimated very close to zero and others set to zero part of the elasticities take high (absolute) values and (minus) infinity respectively. To avoid this we use a trimmed version (Newey, 1994) defined as follows

\[
\hat{m}(\bar{m}) = \begin{cases} 
0 & \text{if } \bar{m} = 0 \\
1 + \left( \frac{\bar{m}}{2b} \right)^2 & \text{if } 0 < \bar{m} \leq 2b \\
\bar{m} > 2b 
\end{cases}
\]

As \( b \) approaches to zero some elasticities grow arbitrarily large. As the important insight is the structure of elasticities, which remains almost invariant to changes in their absolute value, we choose a value to obtain rather conventional elasticity values (\( b = 0.01 \)).

Table 6 shows averages of the estimated elasticities grouped by standard automobile segments. Notice the important variation of the cross elasticities (next section shows that this variation is more important by far than with the estimates obtained with any other estimator). Recall that the value of cross-elasticities is governed by the estimated thetas, representing covariances of buying probabilities over the heterogeneity of consumers, which measure the intensity of cross price effects between two given car models. Table 7 shows than an average model gets a relatively intense competition of 30% of the competitor car models (\( \theta > 1 \)) and relatively weak competition from the other 70%. It would be interesting to develop some examples for particular car model cases. Table 8 give average values for the thetas, which can be taken as measures of the intensity of competition inside of the employed groupings in doing the table (values in the diagonal) and between cross segments (of-diagonal values). The values are illuminating and reveal than the grouping (of commercial origin)
5.3 Results from other specifications.

The same problem with the same sample was subjected to three other alternative estimation procedures. Firstly, we used completely parametrized price equations consistent with a nested logit specification of demand. Nests were defined according to the conventional automobile segments mentioned above, the model was flexibilized by allowing the price parameters to vary by segments, and the degree of correlation inside nests was given a standard value. Secondly, we estimated a nonlinear system consisting of the nested logit demand equation corresponding to the previous model and the corresponding price equations, imposing the cross equation constraints. Thirdly, we estimated the nonlinear system consisting of a random coefficients utility model with its corresponding price equations (BLP estimation). Shares and the share derivatives are here obtained by simulating the consumers’ income distribution and the random shocks interacted with characteristics. These estimates can be seen as an ordering of demand specifications ranked in ascending order of flexibility.

All three models are parametric and equilibriums are non-nested, so we used the Rivers and Vuong (2002) test for selection among (possibly misspecified) nonnested models, in the version in which the selection criteria is based on the value of the objective function which has been minimized (the GMM objective); i.e. the value $T = \sqrt{n}\sigma(Q_1 - Q_2)$ to be compared with critical values of a $N(0,1)$, with $\hat{\sigma}$ an estimate of the sampling variance of the difference between objectives. Values of the "first-step" objective function are used, which employ the same consistent estimator of the weighting matrix $A$, based on the same set of instruments for the series of models to be compared. Variance is taken as

$$\sigma = 4 \left[ G_1'AE_{11}AG_1 + G_2'AE_{22}AG_2 - 2G_1'AE_{12}AG_2 \right]$$

and estimated using $\hat{G}_i = \frac{1}{n} \sum m_1(\hat{\gamma})$ and $\hat{E}_{12} = \frac{1}{n} \sum m_1(\hat{\gamma})m_2(\hat{\gamma})'$.

Table 9 reports the result of testing behaviour with these estimation procedures. In the first case the change in behaviour is accepted as the model which best fits the data.
The second estimation procedure gives Bertrand for all players and time periods as the best equilibrium. The third procedure (BLP estimation) gives the same result the our semiparametric estimation, pointing out the price coalition as the model that best fits the observe data. This result seems however not to be robust to all kind of instruments and tends to assign also a too good score to collusion. Table 10 gives the estimated averages of elasticities in the second and third estimation procedures, to be compared with the averages in Table 6. Acceptation or rejection of equilibriums can be in fact traced back to the estimation of the level and structure of these elasticities.

6. Conclusion.

This paper has addressed the question of whether the Spanish car market underwent a change in pricing behavior that coincided with the tariffs dismantling attained by 1992. The answer is no, despite that the the simple observation of the data can induce to believe a change in behaviour. Careful specification of pricing equations, and their estimation by semiparametric methods, has allowed us to point out that tacit coordination in pricing maintained up to this moment by domestic and European producers is likely to have continued, given the data, versus the alternative hypothesis that this coordination broken up by this time. The specification and estimation of a random coefficients utility model and its corresponding pricing equations gives the same result. One caveat, however, is in order: this can be only one part of the whole picture. One advantage of our framework is that is consistent with the likely pricing of more complex dynamic competition models. The corresponding disadvantage is that we cannot say anything about the rest of the setting. For example, coordination in prices is likely to have been kept at the same time that was an increased competition in advertising and the building of sales networks. But the answer provides, at least, a first step for focussing on broader hypotheses and more complex structural models.

More generally, a lesson of this study is that to specify semiparametric price equations, including a flexible modelization of the own and cross price effects given some pre-specified
behavior, is possible, easy and useful. The model is identified and gives sensible results. The estimation amounts to solve a nonlinear GMM problem. The model provides estimates of margins and elasticities free of parametric demand side assumptions and permits to decide which behavior is closest to the market data as well as to base the estimates on this behavior specification. The estimated margins can be used to do welfare analysis and for policy recommendations. The comparative results using tight demand side specifications show that an inadequate specification of the demand side may induce wrong inferences. In fact, all comparisons show that demand specifications seem to constitute a potential source of bias to be taken seriously into account. By the same reason, the extended practice of doing inferences about market competition using exclusively demand models can be highly misleading. One interesting avenue for future research is how to integrate more demand side information and estimates with the flexible specification of pricing equations.
Appendix

Proof of Property 1.

To see this property, let $P_H$ be the permutation matrix which induces a re-ordering of firms (and hence products) according to the coalition they belong to. $P_H s + P_H(\delta \circ D)P_H'(p-c) = 0$ is a system equivalent to (3), and hence $P_H p = P_H c - (P_H(\delta \circ D)P_H')^{-1}P_H s$ gives the same prices. But, by definition of $\delta$, $(P_H(\delta \circ D)P_H')^{-1}$ is a block diagonal matrix.

Proof of Property 2.

Consumers are endowed with income $y$ and unobservable heterogeneity $v$, and $y$ and $v$ are distributed independent with densities $f(y)$ and $g(v)$. Marginal utility of income is modelled as depending only on $y$ for simplicity, generalization to $\alpha(y,p_j)$ is straightforward and important; heterogeneity $v$ is associated to a unique characteristic $r$ again for simplicity, and the independence assumption could be also relaxed.

Let $P(j)$ be aggregate probability of buying good $j$, and $P(j|y,v)$ probability conditional on $(y,v)$. For simplicity of notation we are systematically omitting $x$ and $p$ from the conditioning set. Note also that probabilities are conditional expectations. A more complete notation would be to write $P(j|y,v)$ as $E[\zeta_j = 1|x,p,y,v]$.

$$P(j) = \int P(j|y,v)f(y)g(v)dydv$$

A natural object of interest are average partial effects, namely average price effects $\frac{\partial P(k)}{\partial p_j}$. Assuming that integral and derivative can be interchanged,

$$\frac{\partial P(k)}{\partial p_j} = E\left[\frac{\partial P(k|y,v)}{\partial p_j}\right] = \int \frac{\partial P(k|y,v)}{\partial p_j}f(y)g(v)dydv$$

$$= \int \alpha(y)P(j|y,v)P(k|y,v)f(y)g(v)dydv$$

$$= \int \alpha(y)\left[\int P(j|y,v)P(k|y,v)g(v)dv\right] f(y)dy$$

where the third equality comes from the assumption on consumer probability changes. The term between brackets can be written as

$$P(j|y)P(k|y) + \int [P(j|y,v) - P(j|y)] [P(k|y,v) - P(k|y)] g(v)dv$$
where \( P(j|y) = \int P(j|y, v)g(v)dv \) is probability of buying \( j \) for given \( y \) averaged over the distribution of \( v \), and the second term is \( \text{Cov}_v [P(j|y, v), P(k|y, v)] \) for a given \( y \) over the distribution of \( v \). We are unable to write analytically \( P(j|y) \) but we know that it is the right probability by the Law of Iterated Expectations. Using this and then writing the first term in a similar way we have

\[
\frac{\partial P(k)}{\partial p_j} = \int \alpha(y)P(j|y)P(k|y)f(y)dy + \int \alpha(y)\text{Cov}_v[P(j|y, v), P(k|y, v)]f(y)dy
\]

\[
= \alpha_j P(j)P(k) + \text{Cov}_y[\alpha(y)P(j|y), P(k|y)]
\]

\[
+ \int \alpha(y)\text{Cov}_v[P(j|y, v)P(k|y, v)]f(y)dy
\]

where \( \alpha_j = \int \alpha(y)f(y|j)dy \).

Bayes rule is used to write \( \int \alpha(y)P(j|y)f(y)dy \) as \( [\int \alpha(y)f(y|j)dy]P(j) = \alpha_j P(j) \). Moment \( \alpha_j \) is marginal utility of income averaged over the distribution of income conditioned in choosing \( j \), or “average marginal utility of income for buyers of good \( j \)”.

Average price effects turn therefore out to be decomposable into one part which depends on average marginal utility and average probabilities and another which depends on covariance moments over the distribution of heterogeneity. As a convenient way to express the total effects, dividing and multiplying by \( \alpha_j P(j)P(k) \) we can write

\[
\frac{\partial P(k)}{\partial p_j} = \alpha_j P(j)P(k)(1 + \omega_{jk})
\]

where \( \omega_{jk} \) depends on covariance moments of the probabilities of buying \( j \) and \( k \) over the distribution of consumers’ heterogeneity. Similarly, we can write \( \frac{\partial P(j)}{\partial p_j} = -\alpha_j P(j)[1 - P(j)](1 + \omega_j) \). It is easy to check that \( \omega_j = \sum_{k \neq j} \frac{P(k)}{1 - P(j)} \omega_{jk} \), which ensures the additive properties of price effects.

Proof of Property 3.

Let \( \alpha \) and \( P \) be \( J \)-dimensional diagonal matrices collecting the \( \alpha_j \) and the \( P(j) \)’s respectively; define the matrices \( W = [1 + \omega_{jk}] \) and \( I_W = \text{diag}(W) \); and let \( p \) and \( c \) be the \( J \times 1 \) vectors of prices and costs and \( e \) a \( J \times 1 \) vector of ones. The set of \( J \) FOC conditions may be written as

\[
26
\]
\[ P e - [\delta \circ (\alpha P I_W - \alpha P W P)] (p - c) = 0 \]

and taking into account that element by element product of \( \delta \) by any diagonal matrix leaves the matrix unchanged, it is easy to arrive at the expression

\[ p = c + [I - \delta \circ (I_W^{-1} WP)]^{-1} I_W^{-1} \alpha^{-1} e \]

Elements of row \( j \) of matrix \((I_W^{-1} WP)\) add up \( P(j) + \frac{1}{1+\omega_j} \sum_{k \neq j} (1+\omega_{jk})P(k)\) and taking into account the restriction on the \( \omega_j \)'s it can be shown that this sum is \( \frac{\sum_k P(k)+\omega_j}{1+\omega_j} < 1 \). Maximum row sum matrix norm \([||\delta \circ I_W^{-1} WP||]\) \( \text{row} < 1 \) and hence \([I - \delta \circ (I_W^{-1} WP)]^{-1} = (I + \delta \circ I_W^{-1} WP + (\delta \circ I_W^{-1} WP)^2 + ...\), (Horn and Johnson 1985, Theorem 5.6.15). Neglecting the squared and following terms, which depend on the square of probabilities, and substituting into the previous equation we get

\[ p \simeq c + (I_W^{-1} + \delta \circ I_W^{-1} WP I_W^{-1}) \alpha^{-1} e \]

or, developing the expression for price \( p_j \)

\[ p_j \simeq c_j + \frac{1}{\alpha_j(1+\omega_j)} [1 + P(j)] + \sum_{k \neq j} \frac{1+\omega_{jk}}{\alpha_k(1+\omega_j)(1+\omega_k)} P(k) \]
References


Table 1
The Spanish car market in the 90s: basic statistics

<table>
<thead>
<tr>
<th>Year</th>
<th>Sales (index)</th>
<th>Sales index</th>
<th>Models entry</th>
<th>Models exit</th>
<th>No. of models¹</th>
<th>Share of domestic producers²</th>
<th>[Of which imported cars]</th>
<th>Share of European producers²</th>
<th>Share of Asian producers²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>971,466</td>
<td>100.0</td>
<td>19</td>
<td>2</td>
<td>96</td>
<td>82.0</td>
<td>[14.3]</td>
<td>16.0</td>
<td>2.0</td>
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<tr>
<td>1991</td>
<td>878,594</td>
<td>90.4</td>
<td>10</td>
<td>3</td>
<td>103</td>
<td>80.0</td>
<td>[13.7]</td>
<td>16.9</td>
<td>3.1</td>
</tr>
<tr>
<td>1992</td>
<td>973,414</td>
<td>100.2</td>
<td>16</td>
<td>7</td>
<td>112</td>
<td>81.3</td>
<td>[14.3]</td>
<td>14.6</td>
<td>3.9</td>
</tr>
<tr>
<td>1993</td>
<td>735,993</td>
<td>75.8</td>
<td>12</td>
<td>8</td>
<td>116</td>
<td>80.7</td>
<td>[19.1]</td>
<td>13.9</td>
<td>5.2</td>
</tr>
<tr>
<td>1994</td>
<td>897,492</td>
<td>92.4</td>
<td>13</td>
<td>13</td>
<td>116</td>
<td>78.6</td>
<td>[16.2]</td>
<td>15.8</td>
<td>5.3</td>
</tr>
<tr>
<td>1995</td>
<td>822,593</td>
<td>84.7</td>
<td>17</td>
<td>12</td>
<td>121</td>
<td>77.0</td>
<td>[15.2]</td>
<td>15.7</td>
<td>6.8</td>
</tr>
<tr>
<td>1996</td>
<td>897,906</td>
<td>92.4</td>
<td>16</td>
<td>14</td>
<td>123</td>
<td>75.0</td>
<td>[15.2]</td>
<td>15.8</td>
<td>8.4</td>
</tr>
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</table>

¹ At the end of the year.
² See notes to Table 2.
<table>
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<tr>
<th>Producer type</th>
<th>No. of brands</th>
<th>No. of car models</th>
<th>Brand entry</th>
<th>Models entry</th>
<th>Models exit</th>
<th>Models net entry</th>
<th>No. of car models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic</td>
<td>7</td>
<td>33</td>
<td>-</td>
<td>26</td>
<td>16</td>
<td>10</td>
<td>43</td>
</tr>
<tr>
<td>European</td>
<td>14</td>
<td>38</td>
<td>.</td>
<td>45</td>
<td>30</td>
<td>15</td>
<td>53</td>
</tr>
<tr>
<td>Asian</td>
<td>5</td>
<td>8</td>
<td>4</td>
<td>28</td>
<td>12</td>
<td>16</td>
<td>24</td>
</tr>
<tr>
<td>American</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>26</strong></td>
<td><strong>79</strong></td>
<td><strong>5</strong></td>
<td><strong>103</strong></td>
<td><strong>59</strong></td>
<td><strong>44</strong></td>
<td><strong>123</strong></td>
</tr>
</tbody>
</table>

1. Citroen-Peugeot, Ford, Opel(GM), Renault and Seat-VW
2. Audi, Alfa-Romeo, BMW, Fiat, Jaguar, Lada, Lancia, Mercedes, Porsche, Rover, Saab, Skoda, Volvo and Yugo.
3. Honda, Hyundai, Mazda, Nissan and Toyota before 1990; Mitsubishi, Suzuki, Subaru and Daewo since 1990 and after.
4. Chrysler
Table 3
Preliminary results from the estimation of semiparametric pricing equations

Dependent variable: \( \frac{p_{jt}}{1 + \tau_{jt}} \)
No. of car models: 164; Sample period\(^1\): 1991-96; No. of observations\(^1\): 7,122
Estimation method: Nonparametric two stage least squares

<table>
<thead>
<tr>
<th>Behavior</th>
<th>Myopic pricing</th>
<th>Betrand multiproduct</th>
<th>Price coalition</th>
<th>Coalition breaks up 1993</th>
<th>Collusion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Parameter</td>
<td>SE</td>
<td>Parameter</td>
<td>SE</td>
<td>Parameter</td>
</tr>
<tr>
<td>Constant</td>
<td>( -^2 )</td>
<td>-</td>
<td>0.24</td>
<td>1.207</td>
<td>1.907</td>
</tr>
<tr>
<td>Unit labor costs</td>
<td>0.126</td>
<td>0.169</td>
<td>-0.132</td>
<td>0.333</td>
<td>0.138</td>
</tr>
<tr>
<td>CC/Weight</td>
<td>-0.191</td>
<td>0.346</td>
<td>1.299</td>
<td>0.647</td>
<td>-0.029</td>
</tr>
<tr>
<td>Maxspeed</td>
<td>2.569</td>
<td>0.404</td>
<td>2.133</td>
<td>0.519</td>
<td>1.834</td>
</tr>
<tr>
<td>Km/l</td>
<td>0.549</td>
<td>0.099</td>
<td>-0.040</td>
<td>0.159</td>
<td>0.204</td>
</tr>
<tr>
<td>Size</td>
<td>1.555</td>
<td>2.728</td>
<td>-6.496</td>
<td>2.786</td>
<td>0.017</td>
</tr>
<tr>
<td>Weight</td>
<td>-1.254</td>
<td>0.394</td>
<td>0.472</td>
<td>0.800</td>
<td>-0.729</td>
</tr>
<tr>
<td>(CC/Weight)(^2)</td>
<td>0.239</td>
<td>0.098</td>
<td>0.381</td>
<td>0.120</td>
<td>0.410</td>
</tr>
<tr>
<td>(Maxspeed)(^2)</td>
<td>3.580</td>
<td>0.552</td>
<td>5.725</td>
<td>1.149</td>
<td>5.046</td>
</tr>
<tr>
<td>(Km/l)(^2)</td>
<td>-2.278</td>
<td>0.349</td>
<td>-0.573</td>
<td>0.391</td>
<td>0.102</td>
</tr>
<tr>
<td>Yearly dummies</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Model dummies</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>( g(s) ) function</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>( \theta(d) ) function</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

MSE                      1.6437 | 0.9075 | 0.3849 | 0.7459 | 0.4662

\(^1\)Total sample period and observations are 1990-96 and 9,090 respectively.
\(^2\)In the myopic pricing model the constant cannot be estimated separately from the \( g(.) \) function.
Table 4

Testing behavior with the semiparametric price equations\textsuperscript{1,2}

<table>
<thead>
<tr>
<th>Bertrand multiproduct</th>
<th>Price coalition</th>
<th>Coalition breaks up 1993</th>
<th>Collusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Myopic</td>
<td>2.43</td>
<td>3.85</td>
<td>2.73</td>
</tr>
<tr>
<td>Bertrand multiproduct</td>
<td>9.62</td>
<td>2.14</td>
<td></td>
</tr>
<tr>
<td>Price coalition</td>
<td>-6.93</td>
<td></td>
<td>-2.64</td>
</tr>
<tr>
<td>Coalition breaks up 1993</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{1} Lavergne-Vuong test for nonparametric selection of nonnested regressors: \( T = \frac{n}{2} (MSE_{\text{row}} - MSE_{\text{col}}) \) to be compared with a \( N(0, 1) \).

\textsuperscript{2} Row model versus column model. A value above (below) the critical value of 1.96 (-1.96) means that the row model is worse (better) than the column model.
Table 5
Average margin evolution 1990-1997 by producers, as predicted by selected model, and decomposition

### A. Margins (=B+C+D).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic</td>
<td>0.37</td>
<td>0.51</td>
<td>0.51</td>
<td>0.50</td>
<td>0.44</td>
<td>0.44</td>
<td>0.43</td>
</tr>
<tr>
<td>European</td>
<td>0.31</td>
<td>0.40</td>
<td>0.39</td>
<td>0.41</td>
<td>0.40</td>
<td>0.36</td>
<td>0.33</td>
</tr>
<tr>
<td>Asian</td>
<td>0.13</td>
<td>0.12</td>
<td>0.14</td>
<td>0.15</td>
<td>0.15</td>
<td>0.16</td>
<td>0.17</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>0.31</td>
<td>0.40</td>
<td>0.37</td>
<td>0.39</td>
<td>0.36</td>
<td>0.34</td>
<td>0.33</td>
</tr>
</tbody>
</table>

### B. Myopic margins.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic</td>
<td>0.07</td>
<td>0.09</td>
<td>0.09</td>
<td>0.08</td>
<td>0.09</td>
<td>0.10</td>
<td>0.09</td>
</tr>
<tr>
<td>European</td>
<td>0.13</td>
<td>0.13</td>
<td>0.11</td>
<td>0.13</td>
<td>0.13</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Asian</td>
<td>0.12</td>
<td>0.11</td>
<td>0.12</td>
<td>0.14</td>
<td>0.13</td>
<td>0.14</td>
<td>0.15</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.10</td>
<td>0.11</td>
</tr>
</tbody>
</table>

### C. Contribution of multiproduct pricing.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic</td>
<td>0.02</td>
<td>0.03</td>
<td>0.02</td>
<td>0.04</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>European</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Asian</td>
<td>0.01</td>
<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.01</td>
</tr>
</tbody>
</table>

### D. Contribution of price coalition.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic</td>
<td>0.27</td>
<td>0.39</td>
<td>0.36</td>
<td>0.36</td>
<td>0.32</td>
<td>0.31</td>
<td>0.32</td>
</tr>
<tr>
<td>European</td>
<td>0.16</td>
<td>0.25</td>
<td>0.24</td>
<td>0.26</td>
<td>0.25</td>
<td>0.24</td>
<td>0.21</td>
</tr>
<tr>
<td>Asian</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>0.18</td>
<td>0.26</td>
<td>0.24</td>
<td>0.25</td>
<td>0.22</td>
<td>0.22</td>
<td>0.20</td>
</tr>
</tbody>
</table>
### Table 6
Average own/cross-price elasticities (by segments) with semiparametric price equation

<table>
<thead>
<tr>
<th>Segment</th>
<th>Own-price elasticity</th>
<th>Cross-price elasticities (×100)</th>
<th>Small</th>
<th>Compact</th>
<th>Intermediate</th>
<th>Luxe</th>
<th>Minivan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>-10.12</td>
<td>7.47</td>
<td>4.86</td>
<td>2.65</td>
<td>1.20</td>
<td>1.69</td>
<td></td>
</tr>
<tr>
<td>Compact</td>
<td>-11.01</td>
<td>3.54</td>
<td>6.86</td>
<td>5.46</td>
<td>2.52</td>
<td>2.00</td>
<td></td>
</tr>
<tr>
<td>Intermediate</td>
<td>-4.22</td>
<td>0.44</td>
<td>1.04</td>
<td>1.16</td>
<td>0.64</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>Luxe</td>
<td>-2.04</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.04</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>Minivan</td>
<td>-1.6</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td>-5.78</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 7
The distribution of theta values

<table>
<thead>
<tr>
<th>Theta value</th>
<th>Percentage of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta \leq 0.50$</td>
<td>45.4</td>
</tr>
<tr>
<td>$0.50 &lt; \theta \leq 1$</td>
<td>24.6</td>
</tr>
<tr>
<td>$1 &lt; \theta \leq 2$</td>
<td>19.5</td>
</tr>
<tr>
<td>$2 &lt; \theta$</td>
<td>10.5</td>
</tr>
</tbody>
</table>

### Table 8
Average estimated theta values (by segments)

<table>
<thead>
<tr>
<th>Segment</th>
<th>Theta values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Small</td>
</tr>
<tr>
<td>Small</td>
<td>1.68</td>
</tr>
<tr>
<td>Compact</td>
<td>0.89</td>
</tr>
<tr>
<td>Intermediate</td>
<td>0.50</td>
</tr>
<tr>
<td>Luxe</td>
<td>0.25</td>
</tr>
<tr>
<td>Minivan</td>
<td>0.41</td>
</tr>
</tbody>
</table>
Table 9

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parametrized (nested logit) price equations</td>
<td>Coalition breaks up in 1993</td>
</tr>
<tr>
<td>Nonlinear parametric (nested logit) demand-price system</td>
<td>Bertrand multiproduct</td>
</tr>
<tr>
<td>Nonlinear system with simulation (random coefficients or BLP estimation)</td>
<td>Price coalition</td>
</tr>
</tbody>
</table>

*Rivers-Vuong (2002) test for selection among nonnested models: $T = \frac{\sqrt{n}}{\sigma}(Q_{row} - Q_{col})$ to be compared with a $N(0, 1)$.***
Table 10
Average own/cross-price elasticities and margins by segments with other estimators

Nonlinear system with simulation (random coefficients or BLP estimation), Price coalition:

<table>
<thead>
<tr>
<th>Segment</th>
<th>Own-price elasticity</th>
<th>Cross-price elasticities (×100)</th>
<th>Price-cost margins</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Small  Compact  Intermediate  Luxe  Minivan</td>
<td>1990-92 1993-96</td>
</tr>
<tr>
<td>Small</td>
<td>-2.72</td>
<td>2.67    2.02     1.46      0.91  1.22</td>
<td>0.64  0.61</td>
</tr>
<tr>
<td>Compact</td>
<td>-3.17</td>
<td>2.46    2.32     2.04      1.64  1.74</td>
<td>0.50  0.48</td>
</tr>
<tr>
<td>Intermediate</td>
<td>-3.39</td>
<td>1.01    1.19     1.25      1.27  1.14</td>
<td>0.45  0.44</td>
</tr>
<tr>
<td>Luxe</td>
<td>-3.84</td>
<td>0.19    0.33     0.48      0.71  0.52</td>
<td>0.38  0.38</td>
</tr>
<tr>
<td>Minivan</td>
<td>-3.16</td>
<td>0.19    0.27     0.34      0.46  0.39</td>
<td>0.42  0.41</td>
</tr>
<tr>
<td>Total</td>
<td>-3.35</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Nonlinear parametric demand-price system, Bertrand multproduct:

<table>
<thead>
<tr>
<th>Segment</th>
<th>Own-price elasticity</th>
<th>Cross-price elasticities (×100)</th>
<th>Price-cost margins</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Small  Compact  Intermediate  Luxe  Minivan</td>
<td>1990-92 1993-96</td>
</tr>
<tr>
<td>Small</td>
<td>-6.30</td>
<td>28.26   0.20     0.20       0.20  0.20</td>
<td>0.17  0.17</td>
</tr>
<tr>
<td>Compact</td>
<td>-7.39</td>
<td>0.18    30.09    0.18       0.18  0.17</td>
<td>0.14  0.14</td>
</tr>
<tr>
<td>Intermediate</td>
<td>-3.04</td>
<td>0.03    0.03     6.53      0.03  0.03</td>
<td>0.36  0.37</td>
</tr>
<tr>
<td>Luxe</td>
<td>-4.17</td>
<td>0.01    0.01     0.01      9.68  0.01</td>
<td>0.28  0.30</td>
</tr>
<tr>
<td>Minivan</td>
<td>-7.03</td>
<td>0.02    0.02     0.02      0.02  139.25</td>
<td>0.31  0.15</td>
</tr>
<tr>
<td>Total</td>
<td>-4.87</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 1
Sales evolution in the Spanish car market

Informe de la industria española, MCYT
Sample sales
Figure 2: Prices and received prices
Figure 3: Quality adjusted price index

- Domestic
- European
- Asian
Figure 4: Entry and quality change adjusted price index