On the Use of the Bass Model for Forecasting Pecuniary Damages: a Reappraisal

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Abstract

In a 2006 paper in the Journal of Forensic Economics, Tomlin & Wazzin purport to show the inapplicability of the Bass Model for routine, mundane estimation of pecuniary damages (Tomlin & Wazzan, 2006). *We agree. A Bass model is better suited for appraising but-for estimates of lost sales when the environment constitutes a homogeneous product viewed as innovative or novel by its prospective customers and sales and marketing efforts benefit from diffusion via social networks.*

We argue that when confronted with an underlying diffusion data generating process of a but-for sales effort, the task at hand is twofold: (i) to determine the rate of sales increase, and (ii) to identify the apex of the but-for sales path. Given these tasks we show that a linear model is unsuited for purposes of illustrating counterfactuals. The Bass model, on the other hand, reproduces the underlying data-generating process

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more adequately. We re-examine the Bass model using a more conventional simulation study to compare the accuracy of the Bass Model to a competing linear model. Our results uphold the generality of the Bass model – especially when modeling counterfactual performance of products perceived as novel and innovative by its prospective customers.

JEL Codes: C52, C53, O31

“Only a moron would bring a sword to a gunfight."

Derek Landy

Introduction

Accurate and timely forecasts are central to the calculation of damages across any number of litigation scenarios including business interruption, tortious interference, contract disputes, patent infringement. But evaluating but-for sales forecast in a litigation context is not simple, because there is no rigorous answer to the fundamental counterfactual question underscoring the tort: but-for this event, what would sales have been? Moreover, there is a competing tension underlying an expert’s modeling effort. On the one hand methodological soundness compels an expert to accurately represent the underlying data-generating-process (DGP). On the other hand, the courts value parsimony.

A DGP has observable features or “tells.” The modeling effort must necessarily address the DGP’s features, the underlying context; the expert can finesse this task in one of two ways. The expert may find it expedient to overlook any DGP tells and set forth defensible abridgments and assumptions to explain and substantiate a preferred model. Or the DGP’s features may compel a, specific, model or class of models. Thus, a sound, coherent narrative based on the extant record and domain knowledge underscoring the modeling effort, preferably embedded within a cohesive economic or financial framework or theory may enhance the robustness of the expert’s contribution and its forensic appeal.

The *Journal of Forensic Economics* published a paper in 2006 – by Tomlin and Wazzan arguing that the S-curve approach to estimated damages runs afoul of Daubert

(Tomlin & Wazzan, 2006).[1] Tomlin & Wazzan ("the authors") conclude that S-curve models "do not provide proper foundation for calculating damages in litigation."[2] Their argument relies on two assumptions that drive the core of their analysis.

First, Tomlin & Wazzan emphasize that an appropriate model should consider what they call "*the parameters underlying those sales patterns*," referring to the variables and context characterizing the underlying DGP. Relatedly, the authors spend considerable efforts noting how the Bass model should be applied solely to situations that derived from diffusion processes. We cannot disagree. Unfortunately, the authors promptly ignore their own advice and proceed to illustrate their criticism of the Bass model by conducting an empirical study in markets not necessarily characterized as having diffusion networks underscoring firm sales efforts. To be sure, to disregard whether an industry's sales efforts can be characterized as a social diffusion process is unsurprising. For the most part, there existed scant awareness of the commercial advantages of internet-based social and commercial networks at the time their data was drawn.

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[1] Daubert refers to the standard used by a triers of fact to assess whether expert witness testimony is based on scientifically valid reasoning and therefore appraise its admissibility; the standard was first articulated in *Daubert v. Merrell Dow Pharmaceuticals Inc., 509 U.S. 579 (1993)*, and further refined in *Kumho Tire Co. v. Carmichael* (1999). Under the Daubert standard, the factors that may be considered in determining whether the proffered methodology is valid are: (1) whether the theory or technique in question can be and has been tested; (2) whether it has been subjected to peer review and publication; (3) its known or potential error rate; (4)the existence and maintenance of standards controlling its operation; and (5) whether it has attracted widespread acceptance within a relevant scientific community.

[2] S-curve models are an instance of what are more generally known as diffusion models; within the family of diffusion models, the Bass model is the most popular and is widely used (Ofek, 2005) (Watts & Gilbert, 2014).

Second, confronting the S-shaped future realization of but-for sales, the authors "fit" their model by *assuming* the point where the apex, or peak, occurs and disregard the fact that one of the advantages of models such as the Bass model is that they allow the expert to *estimate* the apex.

Our objective in this paper is to demonstrate the reasonableness of the Bass model for estimating but-for damages for underlying business environments that have at least two key features. First, in instances where the product under consideration is perceived as innovative by its prospective customers; and, second, where it can be shown that social, consumer, or online media networks contributed to the commercial diffusion of the product (Ofek, 2005) (Watts & Gilbert, 2014).

We present our reasoning and empirical work here in the following steps: in the next section we dive a little deeper into Tomlin & Wazzan's assumptions. We then explain the modeling characteristics of the Bass Model. The fourth section provides a simulation study where we contrast the performance of a linear model to a Bass model when the underlying context can be framed as a diffusion process. The last section concludes.

The Model is Not the System

Tomlin & Wazzan reach back to a 1986 book by Mahajan and Wind to list assumptions underlying the effective use of a diffusion model (Mahajan & Wind, 1986). According to the authors, a failure to fit every one of Mahajan and Wind's assumptions impugns the diffusion effort. Yet the literature on diffusion modeling and Bass models has accommodated any number of variants over the years; advances incorporating features into the original model ranging from product own-prices to the impact of competitors and more. The extant literature on the applications of diffusion models *for sales and marketing* *alone* is simply massive. Starting with the landmark Iowa empirical corn studies of Ryan and Gross followed by the theoretical armature laid out by Rogers we lately find comprehensive studies of the Bass model as the analytical framework underscoring the study of memes,[3] tweets, and videos and other artifacts of our social media era (Ryan & Gross, 1943) (Rogers, 2003) (Ofek, 2005) (Watts & Gilbert, 2014) (Kumar, Nagpal, & Venkatesan, 2002).[4] Use of the bass model is found across durables, non-durables, pneumonia, covid cases and fatalities (Eryarsoy, Delen, Davazdahemami, & Topuz, 2021), politics and social sciences, marketing, medicine, information systems, finance, and economics. The model is used in its elementary form, modified, complemented, enhanced, combined. The model parameters are estimated in multiple ways: maximum likelihood, online social media scraping (Fan, Che, & Chen, 2017), drawn from experience, simulated, dynamic programming, and feedback mechanisms. And even more to the point, the Bass model is prevalent in forecasts of products with scant or even no historical data (Ganjeizadeh, Lei, Goraya, & Olivar, 2017) (Albers, 2004).

Yet, to paraphrase Box, the model is not the system – so any representation of the underlying DGP will necessarily be incomplete. More perplexing is Tomlin & Wazzan's

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[3] Memes are the simplest cultural units that spread between different individuals and may gain collective attention within a community or culture The term is attributed to Dawkins who set forth the concept that memes constitute a cultural analogy to genes. Equating memes and genes explains how innovations, ideas, catchphrases, melodies, rumors, or fashion trends disseminate through a population.

[4] Everett Rogers first produced the idea that the percentage of a market adopting a product, cumulative sales per capita of a product, or even sales per capita often follow an S-shaped curve. The graph shows a cumulative percentage of adopters over time slow at the start, more rapid as adoption increases, then levelling off until only a small percentage of laggards have not adopted.

insistence on criticizing the applicability of diffusion models in instances where sales paths have been realized. The sales path is known: there is no but-for.

The authors cite Rogers:

The S-curve, it must be remembered, is innovation-specific and system specific, describing the diffusion of a particular new idea among the member-units of a particular system. [5]

With this statement the authors seem to understand when and where conditions recommending a Bass model exist. Yet Tomlin & Wazzan present an empirical study that deliberately disregards their apprehensions, the Mahajan and Wind characteristics, the insight from citing Rogers. In other words, they disregard their admonition that diffusion models be relegated to modeling sales of products perceived as novel, or innovative and marketed and sold via social media networks.

We obtained annual net sales data for the period 1995-2004 for all US firms available on Standard and Poor's Compustat North America database.

They proceed to fit a Bass model to the sales data – assuming various "peaks" or sales apex. Unsurprisingly, they find the Bass models fall short.

we applied a simple implementation of the "S" curve approach to a large sample of U.S. firms under various assumptions of the ultimate level of revenues (the asymptote or peak) expected to be achieved (Tomlin & Wazzan, 2006).

This is unfortunate because the apex or peak of the but-for sales naturally defines predicted sales. Thus, in

[5] Tomlin & Wazzan, 2003, citing Rogers, 1995.

assuming the apex amount you necessarily set forth your conclusion.

The Bass Model

Why the popularity? The Bass model is aimed at modeling a diffusion process. To illustrate: the growth in sales of a new product represents a diffusion process. The process entails a situation where someone "introduces" a new idea, process, or product – a product that may not be "objectively" new as measured by the lapse of time since its first use or launch. The perceived novel product is then communicated through certain channels over time among participants in a social or commercial networked system (Rogers, 2003). And since many commercial situations fit this general description – the Bass model can be set forth to fit the diffusion pattern which typically consists of time series describing total adoptions and the adoption rate.

A careful premeditation of the context in which the model is deployed is necessary to ensure its relevance. We do not just need to have a partial or fragmentary prediction model, but we need to understand the entire system we are modeling including the extent of product heterogeneity including its complexity, the nature of competitors, and the impact of exogenous events.

Context matters. If the context in which the product is launched and marketed can be represented by a diffusion process, then the evolution of sales will traverse states that can be modeled. These are the "tells." The presence of patents effectively enhances the perception of "newness." Last, and relatedly the rise of and preeminence of social networks have permeated modern-day marketing and sales. Internet-based communications have reduced the costs of communications and enhanced the importance of networks in the promulgation of innovative novel products, ideas, and services.

Societies and institutions provide means for sellers to diffuse information through various channels. For instance, firms share knowledge of innovations with their immediate network (customers), through social media, or broadcast it via public media (TV, newspapers, specialized media outlets, etc.) throughout the network.

The Bass model principle underscoring the diffusion of an innovation is simple.

"The probability of adopting by those who have not yet adopted is a linear function of those who had previously adopted" (Bass's Basement Research Institute, 2010).

The Bass model is parsimonious; it sets forth three parameters: *p*, *q* and *m* to model and forecast the diffusion trajectory of a single-purchase innovation. Bass imputed conceptual meaning to these parameters to render them useful marketing and sales constructs. This theory stipulates those adopters of an innovation can be classified into two mutually exclusive groups (Mahajan et al., 1990).

Mathematically, the probability that a prospective adopter would adopt an innovation at time t, given that he has not yet adopted till now, is given by:

$$f(t)/(1- F(t)) = p + qF(t)$$

Where:

- f(t) is the density function in time to adoption.
- F(t) is the cumulative fraction of adopters in time t.
- *p* is the coefficient of innovation.
- *q* is the coefficient of imitation and.
- *m* is the market potential.

The coefficient of innovation, *p*, is considered the probability of adoption at a certain point in time because of external influences like mass media promotional activities.

The coefficient of imitation, *q*, is considered the probability of adoption at a certain point in time because of internal influences including inter alia various social media platforms, and including word of mouth (Bass, 1969; Mahajan et al., 1990).

Equation (1) yields the following differential equation (Mahajan, Muller, & Bass, 1990). The cumulative sales N(t) at time t is given by:

$$n(t) = dN(t)/dt = p[m - N(t)] + (q/m)N(t)[m - N(t)$$

Estimation of Bass parameters p, q and m are necessary to forecast diffusion. To draw timely forecasts, the challenge to forecasters arises in the form of estimation of Bass parameters with limited data.

The resulting development of cumulative sales over time (expressed as penetration of market potential) is shown in Figure 1.

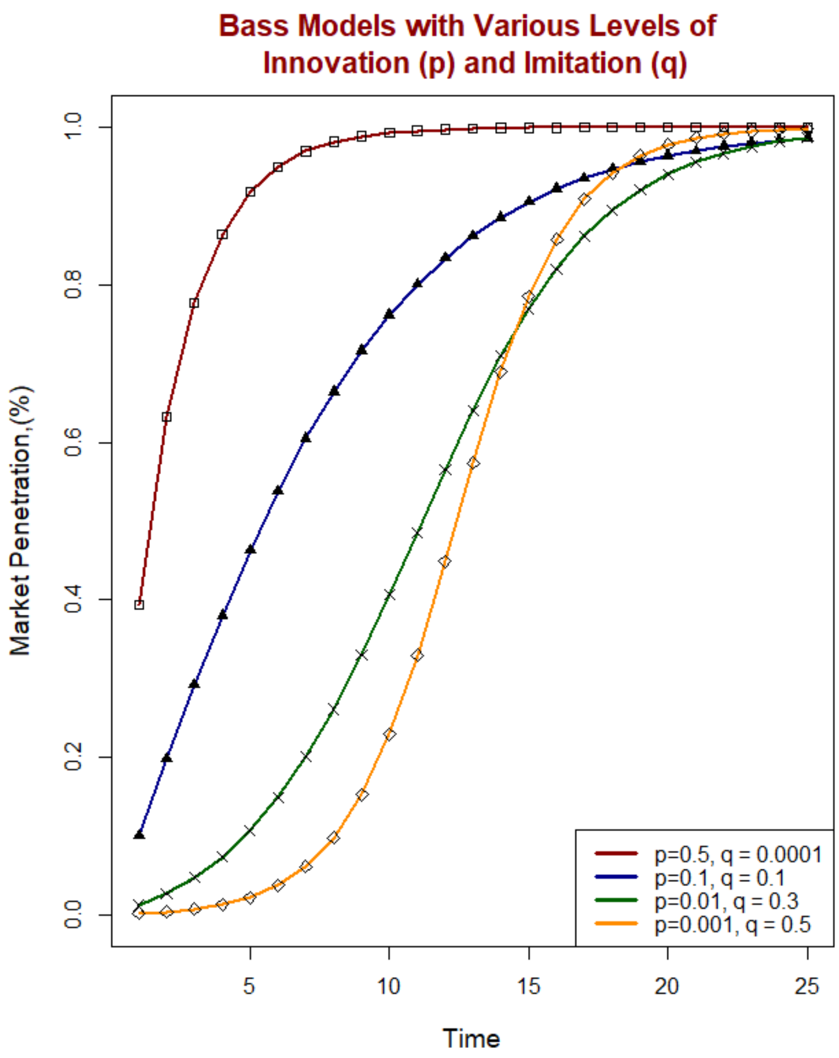


Figure 1

Knowing these parameter values one can reconstruct a variety of diffusion curves which is depicted in Figure 1 for different values.

Simulation Study

The most common principle for choosing among models is to contrast their forecasting accuracy. Accordingly, we conducted simple simulation study with three objectives: (i) to illustrate the modeling flexibility and parsimony of the Bass Model; (ii) to contrast the performance of the Bass model and a linear OLS model in approximating but-for sales in a hypothetical pecuniary damages scenario where the underlying DGP is drawn from a formal network process; and, (iii) to appraise the accuracy of the two competing models.

We accomplish this in two steps. First, we show how to design a synthetic data set from a hypothetical social and/or commercial network to demonstrate how it constitutes the DGP underscoring the S-shaped sales adoption curve.

We then segment the resulting simulated product adoption/sales profile into two sections. One segment is used to fit the competing models and the other segment – considered the "damages period" - is used to gauge the accuracy of the model fit. The objective is for the models under consideration to accurately predict the post event values, based on accuracy measures such as Root Mean Square Error (RMSE).

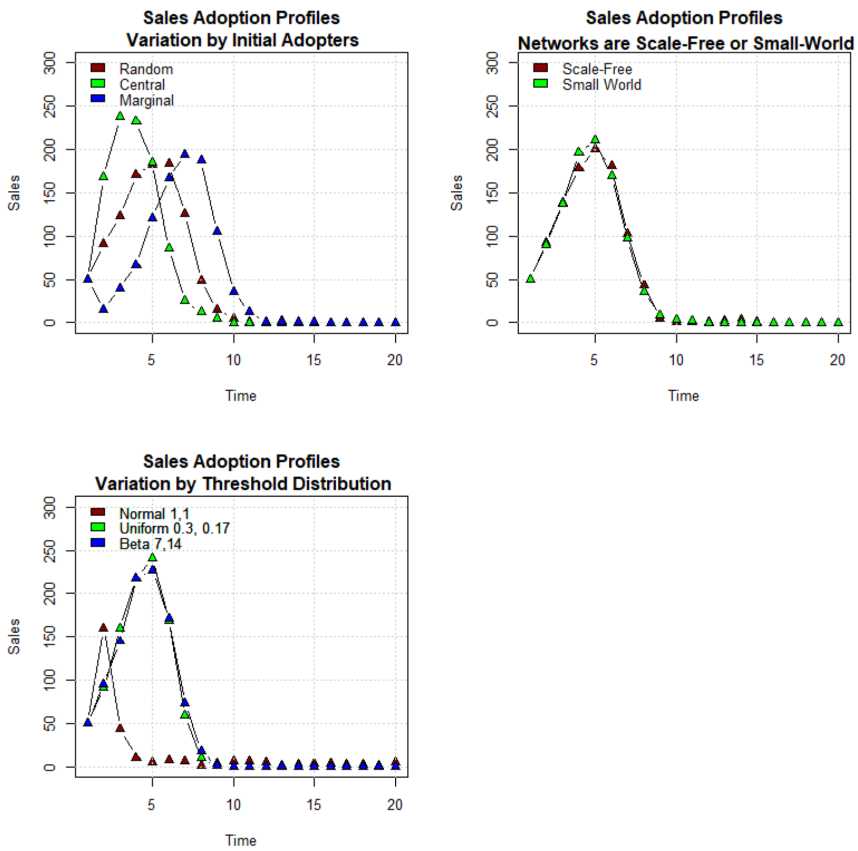
A Network Model as a Lost Sales Data Generating Process

Studies of simulation models of diffusion aim to illustrate how key network features affect the rate at which an innovative product spreads in a commercial sales effort; features such as: (i) initial adopters or seeds; (ii) network structure; (iii) the threshold distribution; (iv) the influence mechanism. (Valente & Vega Yon, 2020) (Trusov, Rand, & Joshi, 2013).

Based on the methodology set forth in Valente and Vega Yon, we construct our synthetic data by showing how variation in initial seeds and in network structure affect commercial sales diffusion processes (Valente & Vega Yon, 2020). These two parameters interact in ways that either slow or accelerate diffusion, and their values can also affect the final cumulative magnitudes of a diffusion simulation – such as cumulative Sales.

Network Parameters:
Size: 100 nodes
Density 6%
Initial Adopters: 5%
Time: 20 time periods
Seeding: Random
Network Structure: Small World
Threshold Distribution: Beta (7,14)
Mechanism: Structural Equivalence

Figure 2



Diffusion Curves by Simulation Conditions are shown in Figure 2. Results indicate that diffusion was fastest when (a) marginal members seeded the diffusion process, (b) either scale-free or small-world, and (c) threshold distribution was a beta (7,14) or a uniform (0.03, 0.17) distribution.

Discussion

All the sales adoption profiles generated above display s-shaped curves. We extract one draw from the multiple synthetic data set to illustrate our claim that the Bass model displays better accuracy in fitting a hypothetical but-for scenario given the observed "tells."

Figure 1 shows the DGP we use for our analysis and the associated Bass model parameters. The underlying DGP is based on a specific process but could represent any of the sales adoption processes deployed over the multiple networks modeled in our study.

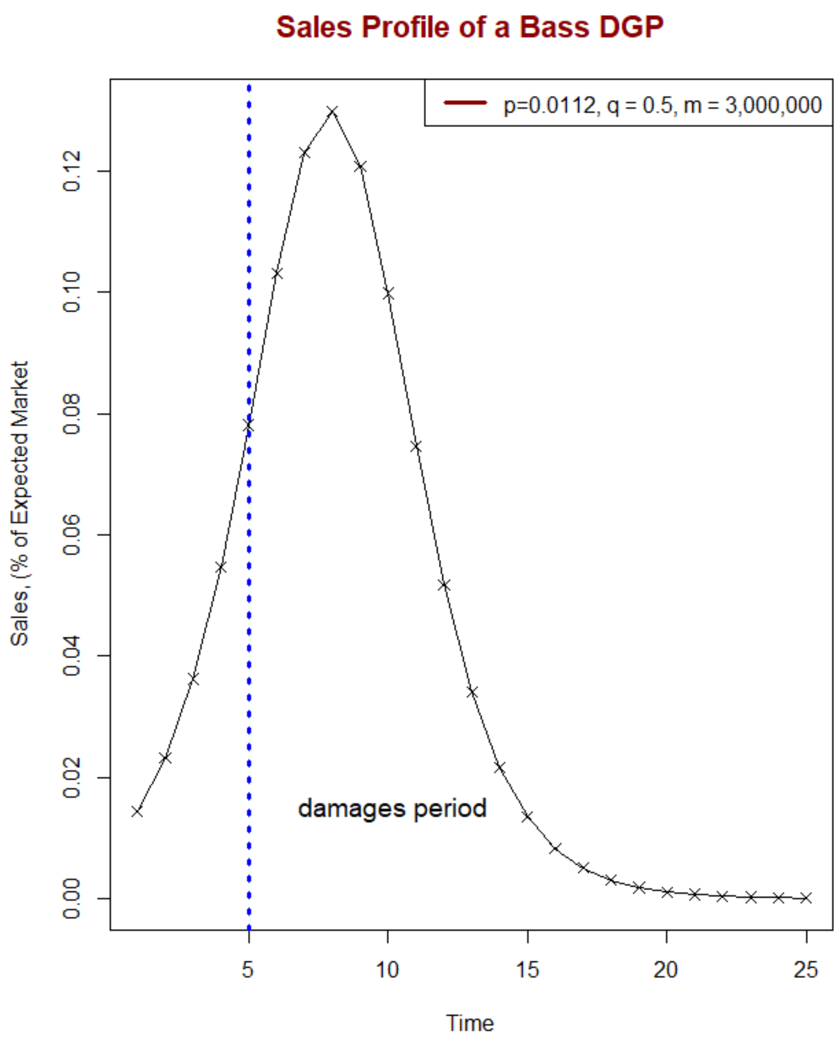


Figure 3

Estimating the Demand Forecast

We truncate the Sales data to construct an existing set of five data points drawn from the DGP in Figure 1. This toy data represents the only data available to the forensic analyst.

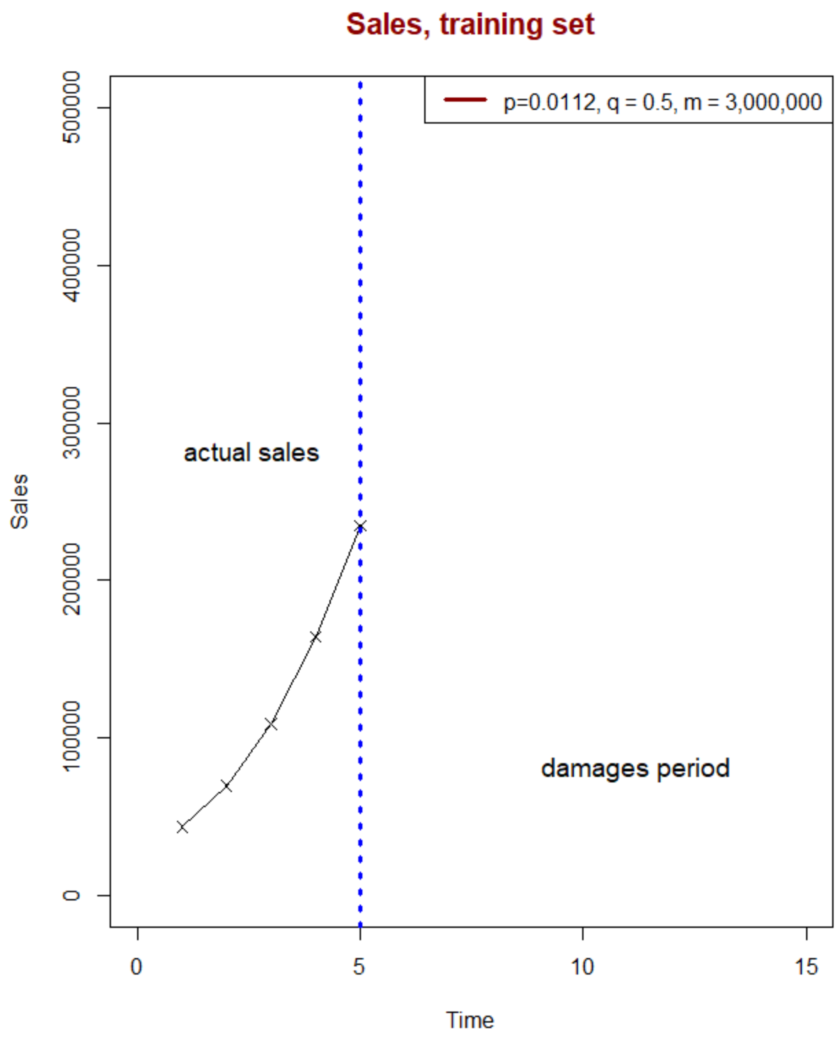


Figure 4

We fit a Bass model and the same linear model in Tomlin & Wazzan to the data.[6]

$$Sales = \alpha + \beta^{*}Time + \varepsilon$$

We then forecast the next twenty data points and compare the forecast accuracy with the actual (simulated) Sales. The results can be seen in Figure 4. The linear model is grossly inaccurate in the damages period – for it fails to recognize the apex of the Sales function.

Table 1
Sales Forecast Accuracy

Model	**RMSE**
Linear	650207
Bass	95583

The Root Mean Square Error reflects model fit; table 1 shows that the Bass model outperforms the linear model by a factor of seven.

[6] We use the R-packages *netdiffuseR* and *diffusion* to construct the synthetic data and fit all models. We also reproduce the modeling effort in Excel. Code (and spreadsheet) is available upon request.

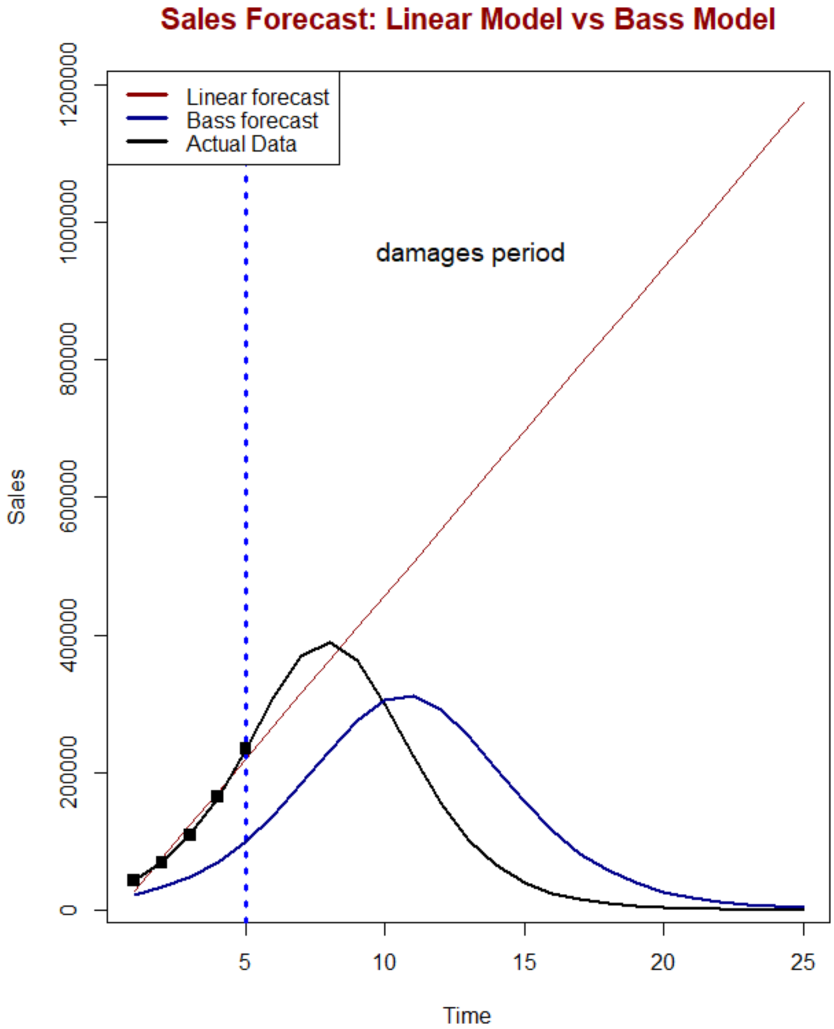


Figure 5

The graph displays the two resulting forecasts from the model fit. The measure of accuracy indicates a clearly better fit for the Bass model.

Concluding Comments

We had a twofold objective in this paper. First, we wanted to unpack the Bass model for forensic economists. The Bass model promises to be an increasing common method to estimate damages in litigation occurring in markets for innovative products in this era of viral marketing across multiple social and commercial media platforms.

Second, we empirically demonstrated how a Bass damages forecast fared when paired against a linear regression model. The Bass model clearly outperformed the linear model.

In doing so, we think the Bass model necessarily shares the robustness of linear models to Daubert challenges when used for modeling lost sales in innovation markets. Relatedly, our results vigorously rebut the critique set forth by Tomlin & Wazzan (Tomlin & Wazzan, 2006).

Our key message is not that the Bass model is always adequate to any modeling effort, nor that the linear model is always better, as suggested in earlier research. Rather, we show that there are identifiable conditions in which relying on a bespoke data-generating process is rational.

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