

# The Equality of the Natural Rates of Interest, Inflation and Economic growth, and Its Implications for Monetary Policy

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### The Equality of the Natural Rates of Interest, Inflation and Economic growth, and Its Implications for Monetary Policy

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#### Abstract

This paper considers the role of savings as that of managing the tradeoff between the amount consumed of present varieties and the investment into improving the quality of varieties in the future. Under this framework, it is shown that the rates of interest, inflation and economic growth are equal and derived from the same phenomenon: innovation. Consequently, this leads to a monetary policy recommendation, namely to attempt to bind the rates of inflation and interest to the directly observable real rate of economic growth.

## 1 The Equality of the Natural Rates of Interest, Inflation and Economic Growth

If the economy were static, the unknown would either not exist at all, or not be accessible, or there would be no notion of its existence, and it would follow a repetitive pattern of exchanges, where the same goods would always be consumed every year, and the total value of the economy, representing total utility, would always be the same.

However, the human being reasons and is capable of imagining what does not yet exist, but which can come to exist if they themselves create it, transforming the surrounding world into preferred physical forms for the satisfaction of their needs. Thus, the initial economy does not perpetuate itself, but rather changes continuously.

As such, like there is a market where present final goods are exchanged, there is also a capital market where quantities of present goods are traded for tools that enable the production of innovative goods to be consumed in the future.

This is the role of savings in a growth theory context and the decision that economic agents face: to balance the quantity of present-quality goods to consume and the pace at which the quality of that quantity of goods increases in each year through innovation, i.e., the rate at which the economy transforms. The present paper introduces a model which represents this dynamic and is then led to the conclusion that the natural rates of interest, inflation and economic growth are one, which is a relevant result in guiding monetary policy.

We begin with the maximization problem

$$max_{s_t} \sum_{t=0}^{infinity} \beta^t U_t = \sum_{t=0}^{infinity} \beta^t q_t(s_{t-1}) u(c_t)$$

Where total utility at t depends on the quantity consumed  $c_t$  and the quality of the present good  $q_t$ . We assume a constant potential quantity consumed of each quality-level good across all periods, thus setting  $c_t = 1 - s_t$ .

The objective function is subject to the constraint  $\frac{q_{t+1}}{q_t} = g(s_t)$ , meaning that the capacity to innovate for t+1 relative to the quality level at t is constant and given by a function  $g(s_t)$ .

The maximization problem simplifies to

$$max_{s_t} \sum_{t=0}^{infinity} \beta^t q_t(s_{t-1})u(1-s_t)$$

Solving for  $s_t$  leads to the FOC

$$q_t u'(1 - s_t) = \beta q'_{t+1}(s_t) u(1 - s_{t+1})$$

The LHS is the marginal utility of consumption today. Note that marginal utility is given by  $U' = q_t u'(1-s_t) + q'_t u(1-s_t)$  and since  $q_t$  is a state variable at t, the second term equals zero.

The RHS represents the marginal utility of savings or capital. While the physical process of cause and effect starts from capital and ends in the new final good, the chain of value flows the other way around, *i.e.*, the value of capital is derived solely from the value of what it produces, or it would not be valued at all. Thus, the marginal utility of savings in the current period is given by the discounted value of the marginal increase in utility that it generates through a quality improvement in the next period across all consumed units.

As such, the optimality condition equates the marginal utilities of consumption and savings in each period. When savings are low, the marginal utility of consumption is low and the marginal utility of savings is high, leading the economic agent to save more until they are equalized.

The fact that the ability to innovate in quality for the next period relative to the current period's level of quality stays the same leads to a constancy in the savings rate. Mathematically,

$$q_{t+1}' = g'(s_t)q_t$$

which leads to a simplied form of the FOC as

$$u'(1-s^*) = \beta g'(s^*)u(1-s^*)$$

Where all terms are time-independent.

Lastly, we are led to the introduction of prices into the model. Relative prices are given by relative marginal utilities; thus, the price of the present final good equals the current price of capital. Following the definitions of the rate of interest and inflation, we arrive at an equality of both.

$$\pi = \frac{P_{t+1}}{P_t} = \frac{U'_{t+1}}{U'_t} = \frac{q_{t+1}u'(1-s^*)}{q_tu'(1-s^*)} = g(s^*)$$

And

$$r = \frac{P_{t+1}}{P_{k,t}} = \frac{U'_{t+1}}{U'_{k,t}} = g(s^*)$$

We see that inflation and interest are representative of the same phenomenon. While r encapsulates the expected utility benefit from sacrificing a unit of consumption in the present, inflation demonstrates that this utility benefit has been achieved.

Finally, looking at economic growth as the change in total value in the economy:

$$g = \frac{U_{t+1}}{U_t} = \frac{q_{t+1}u(1-s^*)}{q_tu(1-s^*)} = g(s^*)$$

So, the three rates are equal to the rate of innovation, or the rate of improvement of the economy.

### 2 Implications for Monetary Policy

Although with some exceptions, such as Hayek (2015), it is almost consensual that the State should hold a monopoly over money and that only one currency should exist in a nation, and for good reason. In a market where money is a good like any other and firms compete to have their own currency being exchanged, a downfall in the value of one currency could bring to ruin the lifetime savings of the agents who were clients of that currency, and without stability assured, the economy's growth through savings and innovation could not happen to the same degree, due to issues of trust in any given currency.

With this in mind, for a long time it has been discussed how monetary policy should be managed by the central banks, and, as Wicksell (1936) put it: "it is, however, widely believed that what is most desirable of all is a state of affairs in which prices are rising slowly but steadily". Friedman and Schwartz (2008), in a seminal work, reached a similar conclusion, and now the European Central Bank and others have set goals for inflation of around 2%. These statements have been the product of empirical studies and observational experience throughout modern economic history. They are also the consequence of asking what inflation does, *i.e.*, the consequences of different levels of inflation for economic growth and stability, and provide a generally consensual answer. However, this paper aims to answer what does inflation, i.e., what are its fundamental causes and what its natural rate is. By describing the mechanism through the dynamics of growth bring natural inflation about, I believe this paper achieves just that.

Any rates of inflation and interest that deviate from the previously cited low, but positive, rates, cause a disruption in the stability of the economy, as has been argued by various schools of thought from the Austrians to the Monetarists. It is not the aim of this paper to participate in that discussion, but rather to say that it is avoidable altogether by choosing rates of interest and inflation that match the directly observable real rate of economic growth.

#### References

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