Common Ownership with Unlisted Suppliers of Perfectly Complementary Inputs

Tsuritani, Ryosuke

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Ryosuke Tsuritani†

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Abstract

Since unlisted firms' shares are not publicly traded, common ownership only affects listed firms and has no direct impact on unlisted ones. We investigate the welfare implications of this asymmetry between listed and unlisted upstream suppliers of perfectly complementary inputs. This study considers a vertically related market with S perfectly complementary inputs, in which L sole listed upstream suppliers and $S - L$ sole unlisted upstream suppliers sell each input through linear wholesale prices to the two listed downstream manufacturers that compete à la Cournot. We find that the input price of each listed supplier is higher than that of each unlisted supplier only when the number of listed suppliers is small. The key factor contributing to this result is the price sensitivity of listed suppliers. We also find that an optimal rate of common ownership may exist for consumers and society, depending on the proportion of listed suppliers in the supply chain.

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†Faculty of Economics, Nagasaki University, 4-2-1 Katafuchi, Nagasaki-city, Nagasaki, 850-8506, Japan. e-mail: *tsuritanir@gmail.com*

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1 Introduction

Common ownership is a widely observed phenomenon worldwide. Institutional investors own a variety of shares in major listed firms within the same industries, such as the U.S. airline and banking industries (Azar et al., 2018). Consider the common ownership in the Japanese automotive supply chain. In April 2025, The Master Trust Bank of Japan and the Custody Bank of Japan had more than $5\% - 16\%$ of the stocks of automobile manufacturers, Toyota, Honda, Nissan, Mazda, Suzuki, and automotive parts suppliers, such as Denso, Aisin, Asahi Kasei, Mitsubishi Electric, Furukawa Electric, Yokohama Rubber, Sumitomo Rubber, Nippon Steel, Kobe Steel, Nidec.

Common ownership appears to be effective in the Japanese automotive supply chain. However, major suppliers such as Yazaki, Astemo, and Multimatic are not listed. Furthermore, this supply chain has a limited number of listed suppliers: only around 900 firms are listed among Nissan's counterparties and their respective counterparties, while approximately $12,000$ firms remain unlisted.[1] Since institutional investors could not purchase the stocks of the unlisted firms, common ownership may have only a slightly impact on this supply chain. How does the asymmetry of the listed and unlisted firms affect the welfare in a vertical market with common ownership? Is there a welfare-maximizing rate of common ownership?

In a vertical market with downstream listed manufacturers and upstream listed suppliers, common ownership alleviates downstream competition and reduces the multiple margin distortion. Due to this trade-off, when downstream competition is weak, common ownership enhances consumer surplus and social welfare; conversely, when competition is strong, common ownership may have the opposite effect. However, the impact of common ownership on welfare is unclear when perfectly complementary inputs, such as auto parts, are procured by listed and unlisted sole suppliers. Since common ownership does not affect unlisted suppliers, reducing multiple margin distortion might become ineffective. This can lead to higher input

[1] For details, see "Nissan Motor Group Domestic Supplier Survey" by Teikoku Databank (Japanese): https://www.tsr-net.co.jp/data/detail/1200759_1527.html.

prices and reduce welfare.

We consider a vertical market where manufacturers 1 and 2 produce final goods using S kinds of perfect complementary inputs, which S sole suppliers procure through linear input prices. The two manufacturers and L suppliers are listed, whereas $S - L$ suppliers are unlisted. Due to common ownership, listed firms maximize the sum of their own profits and a portion of the other listed firms' profits. For simplicity, we assume a linear inverse demand for the final goods and zero marginal costs, except for the input prices.

We find that the input prices of listed suppliers may be higher than those of unlisted suppliers. The key mechanism is in the price sensitivity of the listed suppliers. When the number of listed suppliers is one, common ownership makes the downstream firms less sensitive to the input price of the listed supplier than that of unlisted suppliers. As a result, the listed suppliers could set a higher input price than unlisted suppliers. When the number of listed suppliers exceeds one, however, this ranking of input prices might be reversed. Due to the common ownership link, a listed supplier is concerned about the profits of the listed suppliers and manufacturers. In other words, while the input prices of the unlisted suppliers are sensitive to the input prices for the same manufacturer, those of the listed suppliers are sensitive to all the input prices for manufacturers. Since the input prices are strategic substitutes, the high input prices of listed suppliers cannibalize, thereby becoming less than those of unlisted suppliers.

We find that a welfare-maximizing rate of common ownership exists when the proportion of listed suppliers in the supply chain is medium. This result is in sharp contrast with Chen et al. (2024) and Matsumura et al. (2025), who analyze vertically related markets with common ownership that works for all firms, thereby showing that the optimal rate of common ownership for the welfare is none ($= 0$) or full ($= 1/2$). The intuition is as follows. When most suppliers are listed, common ownership effectively mitigates the multiple marginalization and reduces input prices. Thus, the optimal rate of common ownership for welfare is full. Conversely, when few suppliers are listed, due to the existence of the upstream nonparticipants of common ownership, common ownership does not effectively mitigate the multiple marginalization. Since common ownership alleviate downstream competition, the optimal rate of common ownership for welfare is zero. When the proportion of listed suppliers is intermediate, the effects of mitigating multiple marginalization and alleviating downstream competition cancel each other out. Therefore, the optimal rate of common ownership for welfare exists between none and full.

Many scholars have recently paid attention to the welfare effects of common ownership. Referring to the financial data in Azar et al. (2018), Elhauge (2015) claimed that common ownership might have anti-competitive effects. Azar et al. (2018) state that common ownership in the US airline industry has larger anti-competitive effects than the US competitive authorities expected. López and Vives (2019) analyze the relationship between common ownership and firms' R&D investment with spillovers, showing that common ownership may promote R&D and improve welfare.

Only a few studies analyze common ownership in vertically related markets. Lømo (2024) analyzes a vertically related market with common ownership that works for only the downstream firms. The author finds that overlapping ownership may raise, reduce, or have no effect on input prices, depending on the demand curvature of total output. Chen et al. (2024) analyze the vertically related markets in which common ownership works for the downstream firms and a common upstream supplier. They show that common ownership is more likely to improve welfare monotonously when there are fewer downstream firms and a greater degree of product differentiation. Matsumura et al. (2025) analyze the successive oligopoly model where common ownership works for all of the upstream and downstream firms. They show that common ownership might increase welfare monotonously, depending on the competitiveness in the upstream and downstream markets. However, none of the above studies considers perfectly complementary inputs, which are the main focus of our study. We demonstrate that a welfare-maximizing rate of common ownership exists, depending on the proportion of listed suppliers.

We also contribute to the extensive literature on perfectly complementary inputs in vertically related markets. This literature have analyzed various topics, such as vertical integration (Laussel, 2008; Reisinger and Tarantino, 2019), vertical separation (Matsushima and Mizuno, 2013), conglomerate mergers (Etro, 2019; Kadner-Graziano, 2023; Spulber, 2017), entry (Nariu et al., 2021), exclusive contracts (Kitamura et al., 2018), non-discriminatory commitment (Li and Shuai, 2019; Tsuritani, 2025), sequential bargaining with labor unions (Chongvilaivan et al., 2013), make-or-buy decisions (Sim and Kim, 2021), and mutual outsourcing (Arai and Matsushima, 2023; Milliou and Serfes, 2025). However, these studies do not take common ownership into account, leaving this aspect unexplored. Our study is the first to pay attention to the price sensitivity of listed suppliers, which is a key mechanism underlying our main results.

The remainder of this paper is as follows. Section 2 describes our model. Deriving equilibrium outcomes in Section 3, we compare these outcomes in Section 4. In section 5, we conclude.

2 Baseline Model

We consider a vertically related market with $S \geq 2$ monopolistic upstream suppliers and duopolistic downstream manufacturers. Each symmetric listed supplier $l \in \{1, \cdots, L\}$ and unlisted supplier $s \in \{L+1, \cdots, S\}$ produces a perfectly complementary input and sells it to manufacturers i and j, $i \in \{1, 2\}$ and $i \neq j$. Manufacturers produce one unit of the homogeneous final product by one unit of each input (i.e., Leontief production technology). We denote the inverse demand function $p = 1 - q_1 - q_2$, where p is the price of the final goods, and q_i is the output of manufacturer i. We assume that all firms' marginal costs are zero, except for the input price to each manufacturer from listed supplier l (w_{li}) and unlisted supplier s (w_{si}).

Accordingly, supplier s's profits are $\pi_s = w_{s1}q_1 + w_{s2}q_2$, and manufacture i's profits are $\pi_i = (p - \sum_{s=1}^{S} w_{si})q_i$. We denote consumer surplus and social welfare by $CS = (q_1 + q_2)^2/2$ and $SW = CS + \pi_1 + \pi_2 + \sum_{s=1}^{S} \pi_s$, respectively.

We consider the common ownership for two manufacturers and L listed suppliers. This ownership structure corresponds to the Japanese automotive supply chain. Automakers and some large suppliers are listed, whereas other large suppliers, such as Yazaki, Astemo, Mitsubishi Electric, and others, are unlisted. Following López and Vives (2019) and Chen et al. (2024), manufacturer i and supplier $l \in \{1, \cdots, L\}$ have the following objective functions:

$$\psi_i = \pi_i + \lambda \left(\pi_j + \sum_{l=1}^{L} \pi_l \right), \tag{1}$$

$$\psi_l = \pi_l + \lambda \left(\pi_1 + \pi_2 + \sum_{k=1}^{L} \pi_k - \pi_l \right), \tag{2}$$

where $\lambda \in [0, 1)$ is the degree of common ownership. As of Chen et al. (2024), we focus on the realistic case where λ is less than 1/2. An overly large λ may lead to government intervention via anti-monopoly legislation.

The game's timing is as follows: In stage 1, listed supplier s and unlisted supplier l set the input price w_{si} and w_{li}, respectively. In stage 2, given input prices, manufacturers face Cournot competition. We solve the game using backward induction.

3 Calculating Equilibrium

In this section, we derive the equilibrium result. From the first-order conditions of (1), i's reaction function is as follows:

$$q_i = \frac{1}{2} \left(1 - (1+\lambda) q_j - (1-\lambda) \sum_{l=1}^{L} w_{li} - \sum_{s=L+1}^{S} w_{si} \right). \tag{3}$$

We observe that as the rate of common ownership λ increases, downstream firm i becomes less sensitive to the rival's quantities, $-\lambda q_j$, and discount listed supplier's input prices, $\lambda \sum_{l=1}^{L} w_{li}$.

From (3), we obtain the equilibrium outcomes in stage 2 as follows:

$$q_i = \frac{1-\lambda-2(1-\lambda)\sum_{l=1}^{L} w_{li} + (1-\lambda)(1+\lambda)\sum_{l=1}^{L} w_{lj} - 2\sum_{s=L+1}^{S} w_{si} + (1+\lambda)\sum_{s=L+1}^{S} w_{sj}}{(1-\lambda)(3+\lambda)}, \tag{4}$$

This equation suggests that as the rate of common ownership λ increases, M_i's quantity becomes less elastic to the input prices of unlisted suppliers than those of listed suppliers.

In stage 1, given the equilibrium quantities in stage 2 of equation (4), listed supplier $l \in \{1, \cdots, L\}$ and unlisted supplier $s \in \{L+1, \cdots, S\}$ simultaneously and independently choose the input price to optimize ψ_l and π_k. From the first-order conditions, the reaction functions of l and k for manufacturer i are as follows:

$$w_{li}^R = \frac{\begin{bmatrix} 3+2\lambda-\lambda^2-\left(3+\lambda-4\lambda^2\right) w_{-li} - \lambda\left(1-\lambda\right) w_{-lj} \\ -\left(3+\lambda-2\lambda^2\right) w_{si} - \lambda\left(1+\lambda\right) w_{sj} - \left(3+\lambda-2\lambda^2\right) w_{-si} - \lambda\left(1+\lambda\right) w_{-sj} \end{bmatrix}}{6\left(1-\lambda\right)\left(1+\lambda\right)}, \tag{5}$$

$$w_{si}^R = \frac{1-w_{-si}-(1-\lambda)w_{li}-(1-\lambda)w_{-li}}{2},$$

where the superscript R expresses the reaction function, the subscript $-l$ denotes the listed suppliers other than l, and the subscript $-s$ denotes the unlisted suppliers other s, respectively. These reaction functions suggest the following three observations. First, the input prices are strategic substitutes for each other. Second, due to common ownership, only the listed suppliers' input prices for a manufacturer depend on those for the other manufacturer. Third, the greater the common ownership rate, the more (less) sensitive the listed (unlisted) supplier is to the other input prices, respectively.

Solving S system of FOCs, we obtain the following equilibrium input prices:

$$w_l^* = \frac{(3-\lambda)(\lambda+1)}{(1-\lambda)\left(\lambda+4\lambda L+(\lambda+3)S(\lambda L+1)-\lambda(\lambda+3)L^2+3\right)},$$
$$w_s^* = \frac{(3+\lambda)(\lambda L+1)}{\lambda(4L-(\lambda+3)L^2+1)+(\lambda+3)(\lambda L+1)S+3},$$

where the superscript $*$ represents equilibrium outcomes. Table 1 summarizes the other subgame outcomes.

Table 1 Equilibrium outcomes of the model

	Equilibrium outcomes
p_i^*	$\frac{\lambda-\lambda(\lambda+3)L^2+2\lambda L+(\lambda+3)S(\lambda L+1)+1}{\lambda-\lambda(\lambda+3)L^2+4\lambda L+(\lambda+3)S(\lambda L+1)+3}$
$q_1^*+q_2^*$	$-\frac{2(\lambda L+1)}{\lambda(L((\lambda+3)L-4)-1)+(\lambda+3)(-S)(\lambda L+1)-3}$
π_i^*	$\frac{(\lambda+1)(\lambda L+1)(\lambda+2\lambda L-1)}{(\lambda-1)(\lambda-\lambda(\lambda+3)L^2+4\lambda L+(\lambda+3)S(\lambda L+1)+3)^2}$
π_l^*	$\frac{2(\lambda-3)(\lambda+1)(\lambda L+1)}{(\lambda-1)(\lambda-\lambda(\lambda+3)L^2+4\lambda L+(\lambda+3)S(\lambda L+1)+3)^2}$
π_s^*	$\frac{2(\lambda+3)(\lambda L+1)^2}{(\lambda-\lambda(\lambda+3)L^2+4\lambda L+(\lambda+3)S(\lambda L+1)+3)^2}$
CS^*	$\frac{2(\lambda L+1)^2}{(\lambda-\lambda(\lambda+3)L^2+4\lambda L+(\lambda+3)S(\lambda L+1)+3)^2}$
SW^*	$\frac{2(\lambda L+1)\left(\lambda-\lambda(\lambda+3)L^2+3\lambda L+(\lambda+3)S(\lambda L+1)+2\right)}{(\lambda-\lambda(\lambda+3)L^2+4\lambda L+(\lambda+3)S(\lambda L+1)+3)^2}$

4 Static Comparison

In this section, we compare equilibrium outcomes and derive welfare implications. First, we compare the input prices of the listed and unlisted suppliers. The following proposition summarizes this comparison:

Proposition 1. *The listed supplier's input price* w_l^* *is higher than the unlisted supplier's input price* w_s^* *if the number of listed suppliers is small. Formally,* $L < \dfrac{4}{3 - 2\lambda - \lambda^2}$. [2]

Proof. See Appendix *A*.

This result suggests that the listed suppliers may set a higher input price than the unlisted suppliers if the number of listed suppliers is low. Intuitively, common ownership allows listed firms to coordinate to increase their profits. This coordination effect is evident in the equilibrium quantity in stage 2 in equation (4), which shows that as the common ownership rate increases, downstream listed manufacturers become less (more) sensitive to the input price of listed (unlisted) suppliers. Therefore, when the number of listed suppliers is small (i.e., $L < \dfrac{4}{3 - 2\lambda - \lambda^2}$), they could set a higher input price than unlisted suppliers.

On the other hand, why is the equilibrium input price ranking in Proposition 1 reversed when the number of listed suppliers is large? To explain this, we focus on the reaction functions of suppliers w_{li}^R and w_{si}^R in equations (5). As we explained, when the number of listed suppliers is one, the coordination effect of common ownership allows the listed supplier to set higher input prices than unlisted suppliers. However, when the number of listed suppliers is more than one, their high input prices can lead to cannibalization. w_{li}^R implies that, if the coordination effect led the input prices of the other listed suppliers (i.e., w_{-li} and w_{-lj}) to be high, then listed supplier l would set a low input price. In contrast, w_{si}^R implies that due to common ownership, unlisted supplier s sets its input price with little regard to the input prices of listed suppliers. Therefore, the high sensitivity of listed suppliers' input prices causes them to set lower prices than unlisted suppliers. This is why the equilibrium input price ranking reverses when there are many listed suppliers.

[2] Since $\lambda \in (0, 1/2)$, the necessary condition is $L \leq 2$.

Finally, we consider consumer surplus and social welfare. The following proposition summarizes the welfare implications of common ownership in the vertically related markets with unlisted suppliers of perfectly complementary inputs:

Proposition 2. *Consumer surplus and social welfare have the optimal* $\lambda^* \in (0, 1/2)$ *when the proportion of listed suppliers in the supply chain is medium. Formally,* $\dfrac{L^3 + 16L^2 - 4L - 4}{(2+L)^2} < S < 3L^2 - L - 1$.

Proof. See Appendix *A*.

These results suggest that a welfare-maximizing rate of common ownership may exist in the vertically related markets, depending on the number of input types and listed suppliers. This result contrasts with Chen et al. (2024) and Matsumura et al. (2025), which analyzes vertically related markets and shows that the rate of common ownership monotonically increases or decreases the welfare. Note that the optimal rate of common ownership exists within a broad range: if $L = 2$, this condition is $15/4 < S < 9$; if $L = 3$, this condition is $31/5 < S < 23$; if $L = 4$, this condition is $25/3 < S < 43$. Note also that if the number of downstream manufacturers is one, an increase in common ownership rate monotonically increases the welfare.

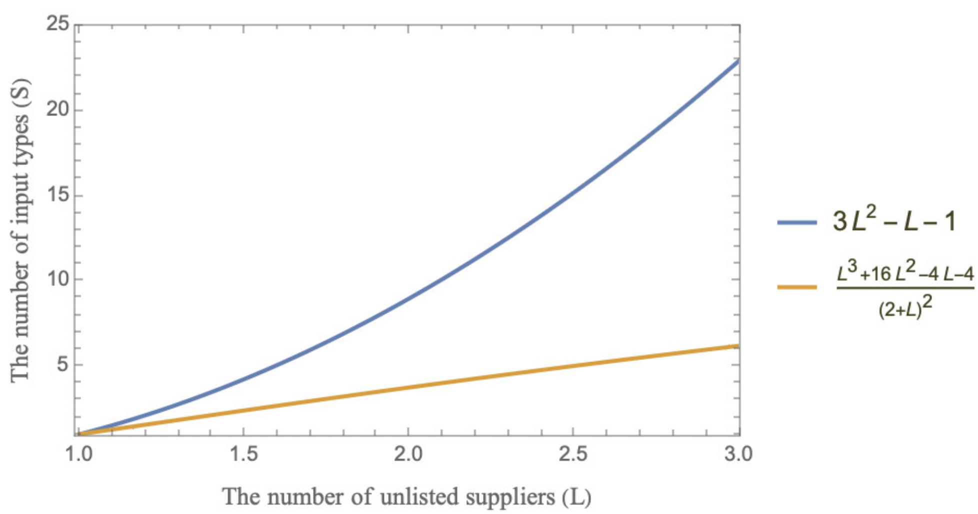

Fig. 1 The Threshold Values of Proposition 2

Figure 1 describes the condition that the optimal rate of common ownership for the welfare is in $(0, 1/2)$. The horizontal axis denotes L, and the vertical axis denotes S. Figure 1 suggests the following two observations. First, the region between the blue and orange lines expresses the formal inequality in Proposition 2. Thus, we can confirm that when the proportion of listed suppliers in the supply chain is medium, a welfare-maximizing rate of common ownership exists in the interval $(0, 1/2)$. Second, if the number of listed suppliers is one, the blue and orange lines intersect at that point. Therefore, to obtain the result in Proposition 2, the analysis should include a realistic scenario involving more than two listed suppliers and more than four input types (i.e., $L \geq 2$ and $S \geq 4$).

The intuition is as follows. When all suppliers are listed (i.e., $S = L$), common ownership mitigates the multiple margin distortion and lowers input prices. Consequently, the optimal common ownership rate for the welfare λ^* is $1/2$. Since the welfare is continuous in L, this result also holds if the proportion of listed suppliers is large (i.e., $S < \dfrac{L^3 + 16L^2 - 4L - 4}{(2 + L)^2}$). When few suppliers are listed, due to the existence of the upstream nonparticipants of common ownership, it does not effectively mitigate the multiple margin distortion. As a result, common ownership greatly alleviates downstream competition, and the optimal rate of common ownership for welfare is zero. Since the welfare is continuous in L, this result also holds if the proportion of listed suppliers is small (i.e., $3L^2 - L - 1 < S$). When the proportion of listed suppliers is intermediate, the effects of mitigating multiple marginalization and alleviating downstream competition cancel each other out. Therefore, the optimal rate of common ownership for welfare exists between none and full.

5 Conclusion

We analyze a supply chain with common ownership and perfectly complementary inputs provided by listed and unlisted sole suppliers. This analysis suggests that common ownership might result in higher input prices of listed suppliers compared to those of unlisted ones. The key of this result is the price sensitivity of the listed supplier. Furthermore, we find that, due to the existence of the upstream nonparticipants of common ownership, there might exist an optimal rate of common ownership for consumers and society. These results demonstrate that the asymmetry between listed and unlisted is important to welfare analysis in vertically related markets with common ownership.

Appendix

A Proof

Proof of Proposition 1

Comparing the equilibrium input prices of l and s, we obtain

$$w_{li}^{*}-w_{si}^{*}=\frac{\lambda(4-(1-\lambda)(3+\lambda)L)}{(1-\lambda)\left(3+\lambda+4\lambda L+(\lambda+3)(S+\lambda LS-\lambda L^{2})\right)} \tag{A1}$$

Since $S>L$ and $\lambda\in(0,1/2)$, we confirm that $w_{li}^{*}-w_{si}^{*}>0$ if $4/(3-2\lambda-\lambda^{2})$. □

Proof of Proposition 2

The equilibrium consumer surplus CS^{*} is

$$CS^{*}=\frac{2\left(1+L\lambda\right)^{2}}{\left(3+\lambda+4L\lambda-L^{2}\lambda\left(3+\lambda\right)+S\left(3+\lambda\right)\left(1+L\lambda\right)\right)^{2}} \tag{A2}$$

and the derivative of CS^{*} by λ is

$$\frac{\partial CS^{*}}{\partial\lambda}=\frac{4\left(1+L\lambda\right)Z}{T^{3}} \tag{A3}$$

where $Z\equiv-1-L+3m^{2}-S+2L^{2}\lambda-2LS\lambda+L^{3}\lambda^{2}-L^{2}S\lambda^{2}$ and $T\equiv 3+3S+\lambda-4L\lambda+3L^{2}\lambda+S\lambda+3mS\lambda-L^{2}\lambda^{2}+LS\lambda^{2}$. We start by analyzing the sign of T. If $L=0$, then $T=(1+S)^{2}(3+\lambda)^{3}>0$. If $L=S$, then $T=3+3S+\lambda+5S\lambda>0$. If $L\neq S$ and

$L > 0$, then solving $T = 0$ for λ yields

$$\lambda_{T1} = \frac{-\left(1 + 4L + S + 3L(S-L)\right) + \sqrt{-12L(S-L)(1+S) + \left(1 + S + L(4 - 3L + 3S)\right)^2}}{2L(S-L)},$$
$$\lambda_{T2} = \frac{-\left(1 + 4L + S + 3L(S-L)\right) - \sqrt{-12L(S-L)(1+S) + \left(1 + S + L(4 - 3L + 3S)\right)^2}}{2L(S-L)}.$$

Since $S > L > 0$, λ_{T1} and λ_{T2} are real solutions, and we obtain $\lambda_{T2} < 0$ and $\lambda_{T1}\lambda_{T2} = \frac{3(1+S)}{L(S-L)} > 0$. Hence, $\lambda_{T1} < 0$. Since T's coefficient of λ^2 is $L(S-L) > 0$, we confirm $T > 0$. Thus, we observe that the sign of $\frac{\partial CS^*}{\partial \lambda}$ depends only on Z. Next, we analyze the sign of Z. If $L = 0$, then $Z = -1 - S < 0$. If $L = S$, then $Z = -1 - 2S + 3S^2 > 0$. If $L \neq S$ and $L > 0$, then solving $Z = 0$ for λ yields

$$\lambda_{Z1} = \frac{-(S-L) + \sqrt{(L-1)(3L+1)(S-L)}}{L(S-L)},$$
$$\lambda_{Z2} = \frac{-(S-L) - \sqrt{(L-1)(3L+1)(S-L)}}{L(S-L)}.$$

Since $S > L > 0$, λ_{Z1} and λ_{Z2} are real solutions and we obtain $\lambda_{Z2} < 0$. Since $\lambda \in (0, 1/2)$, we will check when $0 < \lambda_{Z1} < 1/2$ is satisfied. Solving $\lambda_{Z1} = 0$ and $\lambda_{Z1} = 1/2$, we observe that $0 < \lambda_{Z1} < 1/2$ is satisfied if $(-4 - 4L + 16L^2 + L^3)/(2+L)^2 < S < -1 - L + 3L^2$. Since Z's coefficient of λ^2 is $-L(S-L) < 0$, Z attains a local maximum in $(0, 1/2)$ if $(-4 - 4L + 16L^2 + L^3)/(2+L)^2 < S < -1 - L + 3L^2$. Therefore, CS^* has the optimal value of λ in $(0, 1/2)$ if $(-4 - 4L + 16L^2 + L^3)/(2+L)^2 < S < -1 - L + 3L^2$.

The equilibrium social welfare SW^* is

$$SW^* = \frac{2(1 + L\lambda)\left(2 + \lambda + 3L\lambda - L^2\lambda(3+\lambda) + S(3+\lambda)(1+L\lambda)\right)}{\left(3 + \lambda + 4L\lambda - L^2\lambda(3+\lambda) + S(3+\lambda)(1+L\lambda)\right)^2} \tag{A4}$$

and the derivative of SW^* by λ is

$$\frac{\partial SW^*}{\partial \lambda} = \frac{RZ}{T^3} \tag{A5}$$

where Z and T have already defined at ($A3$), and $R = 2 + 6S + 2\lambda + 4L\lambda - 6L^2\lambda + 2S\lambda + 6LS\lambda - 2L^2\lambda^2 + 2LS\lambda^2$. As before, we analyze the sign of R. If $L = 0$, then $R = 2 + 6S + 2\lambda + 2S\lambda > 0$. If $L = S$, then $R = 2 + 6S + 2\lambda + 6S\lambda > 0$. If $L \neq 0$ and $L \neq S$, then solving $R = 0$ for λ yields

$$\lambda_{R1} = \frac{-(1 + 2L + S + 3L(S - L)) + \sqrt{(1 + 3L)^2(1 - L + S)^2 + 4L(S - L)(1 + 3S)}}{2L(S - L)},$$
$$\lambda_{R2} = \frac{-(1 + 2L + S + 3L(S - L)) - \sqrt{(1 + 3L)^2(1 - L + S)^2 + 4L(S - L)(1 + 3S)}}{2L(S - L)}.$$

Since $S > L > 0$, λ_{R1} and λ_{R2} are real solutions, and we obtain $\lambda_{R2} < 0$ and $\lambda_{R1}\lambda_{R2} = \frac{1+3S}{L(S-L)} > 0$. Hence, $\lambda_{R1} < 0$. Since R's coefficient of λ^2 is $2L(S - L) > 0$, we confirm $R > 0$. Thus, we observe that the sign of $\frac{\partial SW^*}{\partial \lambda}$ depends only on Z as $\frac{\partial CS^*}{\partial \lambda}$. Therefore, SW^* has the optimal value of λ in $(0, 1/2)$ if $(-4 - 4L + 16L^2 + L^3)/(2 + L)^2 < S < -1 - L + 3L^2$. $\square$

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