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CARTEL STABILITY WITH QUALITY-ANCHORED BUYERS

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ABSTRACT. This note examines cartel stability in a vertically differentiated duopoly with quality-anchored buyers. It is shown that such buyers are a facilitating factor for collusion.

Keywords: *Captive Consumers*, *Cartel Stability*, *Collusion, Quality-Anchored Buyers, Vertical Product Differentiation*.

JEL Classification: D43, L13, L41.

1. Introduction

Consumers play a central role in the (not) working of markets. Perhaps one of their most important tasks is to continuously be on the lookout for good or better deals and act upon it. Recent literature has reminded us that this is everything but easy.[1] Wilson and Waddams Price (2010), for example, report that approximately one-fifth of the consumers in the UK electricity market reduced their surplus by switching suppliers. Also, it may be rational not to switch when there are significant opportunity or search costs. This task is further complicated by the fact that firms employ increasingly sophisticated tactics to obfuscate consumers, examples of which include (intertemporal) price dispersion, price discrimination, and drip pricing. Such pricing practices not only tend to put an upward pressure on competitive prices, but have also been argued to serve as a facilitating factor for collusion.[2]

Consumers look at more than just price, however. They typically also take into account the quality of a product or service, and do so to varying degrees. Some are *quality-anchored*, meaning they only consider purchasing products with a particular level of (perceived) quality. For example, customers with a high level of brand trust may choose name-brand products

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[1]See, *e.g.*, Heidhues and Kőszegi (2018) and Vickers (2021).

[2]See, *e.g.*, de Roos and Smirnov (2021) and Wenzel (2024). There is recent evidence suggesting that consumers may be reluctant to respond to collusive pricing in the short term. See Rösner (2024) as well as some of the references therein.

only and ignore private label products.[3] Others are *not quality-anchored* and base their purchasing decisions on the available price-quality combinations. For instance, buyers may prefer a higher-quality 'green product' to a lower-quality 'brown product', but still opt for the second when it is offered at a substantially lower price.

In this note, we aim to shed some light on how the share of (non-)quality-anchored buyers affects the ability of firms to collude.[4] Toward that end, we study a vertical differentiation duopoly model in the spirit of Mussa and Rosen (1978), Gabszewicz and Thisse (1979) and Shaked and Sutton (1982). Specifically, there is a standard (low-quality) producer and a premium (high-quality) producer, which interact over an infinite time horizon. Both sellers have a captive customer base consisting of quality-anchored buyers and compete for the remaining share of consumers that consider both products.

We show that a growing share of quality-anchored buyers leads to higher prices for two reasons. On the one hand, it induces firms to set higher prices in competition and, on the other hand, it increases the likelihood of collusion. As to the latter, we find that the critical discount factors to sustain the cartel depend negatively on both captive customer bases. We thus conclude that quality-anchored buyers are a facilitating factor for collusion.

The next section introduces the model. Section 3 presents the main finding. Section 4 concludes.

2. Model

We analyze an infinitely repeated price-setting duopoly game with quality differentiation. Each of the two suppliers sells a single variant of a product with quality v_i, $i = l, h$. Specifically, $v_h > v_l > 0$ so that firm h is the (relatively) high-quality producer and firm l is the (relatively) low-quality producer. Unit production costs are constant and given by $c_h \geq c_l \geq 0$. Hence, it is (weakly) more costly to produce the premium product. Firms face a common discount factor $\delta \in (0, 1)$ and, in every period $t \in \mathbb{N}$, simultaneously choose prices to maximize the discounted present value of their profits. All prices set up until $t - 1$ are common knowledge.

Demand comes from a unit mass of consumers, all of whom want to purchase at most one unit of the product. These customers have a quality valuation θ that is uniformly distributed on $[0, 1]$. A higher θ implies a higher gross utility when consuming variant v_i. Let $\overline{u} \in \mathbb{R}_{++}$ denote the buyer's 'baseline utility' when making a purchase. A consumer with valuation θ then obtains the following utility:

$$U(\theta) = \begin{cases} \overline{u} + \theta v_i - p_i & \text{when buying from firm } i = l, h \\ 0 & \text{when not buying,} \end{cases} \tag{1}$$

where p_l, $p_h \in [0, \infty)$ indicate the prices of the low- and high-quality producer, respectively.[5]

Every consumer is one of two types: *quality-anchored* or *non-quality-anchored*. Both firms serve a share of captive, quality-anchored, consumers denoted by $\alpha_i > 0$, $i = l, h$.

[3]Note that there may be other reasons for why consumers may not consider all (types of) products available, including bounded rationality, limited awareness of alternatives, search and travelling costs, *et cetera*.

[4]This note is naturally related to literature on collusion among producers of vertically differentiated goods. See, *e.g.*, Häckner (1994), Ecchia and Lambertini (1997), Symeonidis (1999), Bos and Marini (2019), Bos, Marini and Saulle (2020). None of this work considers quality-anchored consumers, however.

[5]For a two-dimensional variation of this utility specification, see Vandenbosch and Weinberg (1995).

Additionally, there is a contestable market segment where firms compete for the remaining share of $1-\alpha_l-\alpha_h$ non-captive, non-quality-anchored, consumers. Using (1), a non-quality-anchored consumer at $\theta \in [0,1]$ is indifferent between purchasing the low- and high-quality product when:

$$\theta(p_l, p_h) = \frac{p_h - p_l}{v_h - v_l}.$$

In what follows, we assume that the market is and remains covered (*i.e.*, all buyers buy).[6] Furthermore, our focus is on situations where both the standard and the premium supplier are active in the contestable segment in which case their profits are, respectively, given by:

$$\Pi_l(p_l, p_h) = (p_l - c_l) \cdot \left[\alpha_l + (1-\alpha_l-\alpha_h) \cdot \frac{p_h - p_l}{v_h - v_l}\right], \tag{2}$$

and

$$\Pi_h(p_l, p_h) = (p_h - c_h) \cdot \left[\alpha_h + (1-\alpha_l-\alpha_h) \cdot \left(1 - \frac{p_h - p_l}{v_h - v_l}\right)\right]. \tag{3}$$

Both these profit functions are strictly concave in the own price.[7] Let (p_l^*, p_h^*) denote the candidate static Nash equilibrium. We write $\Pi_i^* = \Pi_i(p_l^*, p_h^*)$ to indicate the corresponding profit of firm i, $i = l, h$.

To ensure that (p_l^*, p_h^*) is indeed an 'interior' static Nash equilibrium and there is scope for collusion, we impose the following two conditions:

C1 $p_l^* + v_h - v_l > p_h^* > p_l^*$ and $\overline{u} > p_i^* \geq c_i$ for $i = l, h$;

C2 $\Pi_i^m < \Pi_i^*$ for $i = l, h$,

where $\Pi_i^m = (\overline{u} - c_i) \cdot \alpha_i$ is firm i's maximum profit when it exclusively serves its captive customer base.

By C1, both firms serve a share of the non-captive market segment at (p_l^*, p_h^*), *i.e.*, $1 > \frac{p_h^* - p_l^*}{v_h - v_l} > 0$. Loosely speaking, this condition is met when the quality advantage of the premium producer is not too big and the cost advantage of the low-quality producer is not too big either. Moreover, costs should not be too high so that it pays to be productive at these prices. Taken together, this means that both firms can make competitive value propositions in the contestable segment. C2 ensures that the contestable segment is sufficiently important relative to the non-contestable segments so that firms do not find it in their interest to exclusively focus on the latter. This effectively requires the size of the captive segments to be within boundaries.[8]

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[6] This is a frequently-made assumption in this type of vertically differentiation models. See, for example, Tirole (1988, pp. 296-298), Ecchia and Lambertini (1997), and Bos and Marini (2019).

[7] That is,

$$\frac{\partial^2 \pi_l(p_l, p_h)}{\partial p_l \partial p_l} = \frac{\partial^2 \pi_h(p_l, p_h)}{\partial p_h \partial p_h} = -\frac{2\cdot(1-\alpha_l-\alpha_h)}{v_h - v_l} < 0.$$

[8] In the Appendix, we show that both conditions are satisfied for a broad range of parameter values. For a detailed analysis of the existence of an interior Nash equilibrium in a vertical differentiation setting with captive consumers, see Gabzsewicz, Marini and Zanaj (2023).

Under these conditions, best-responses are given by:

$$p_l(p_h) = \frac{\alpha_l \cdot (v_h - v_l)}{2(1-\alpha_l-\alpha_h)} + \frac{p_h + c_l}{2}, \tag{4}$$

$$p_h(p_l) = \frac{\alpha_h \cdot (v_h - v_l)}{2(1-\alpha_l-\alpha_h)} + \frac{p_l + c_h + v_h - v_l}{2}. \tag{5}$$

Combining yields the following static Nash equilibrium prices:

$$p_l^* = \frac{(2\alpha_l + \alpha_h) \cdot (v_h - v_l)}{3(1-\alpha_l-\alpha_h)} + \frac{v_h - v_l + 2c_l + c_h}{3}, \tag{6}$$

$$p_h^* = \frac{(2\alpha_h + \alpha_l) \cdot (v_h - v_l)}{3(1-\alpha_l-\alpha_h)} + \frac{2(v_h - v_l) + 2c_h + c_l}{3}. \tag{7}$$

Note that these prices depend positively on the quality-anchored customer bases (α_l and α_h) and negatively on the share of non-quality-anchored buyers ($1-\alpha_l-\alpha_h$).

3. Sustainability of Collusion

Now suppose that the producers make a price-fixing agreement. As in Bos and Marini (2019), we assume that cartel prices are set in a way that market shares remain at pre-collusive levels. There are at least three reasons for why such an allocation rule is appealing. First, it has a competitive appearance, which *ceteris paribus* makes it easier to keep the cartel 'under the radar' of competition law enforcers. Second, pre-cartel market shares serve as a natural focal point on how to divide the collusive profits absent side-payments. And third, there is evidence from antitrust practice showing that several discovered cartels did use this type of division rule.[9]

Given both captive customer bases, this market-sharing scheme has the implication that non-captive buyers who were indifferent between the high- and low-quality product in the pre-cartel phase should also be indifferent under the collusive regime. It, therefore, holds that:

$$\left(\frac{p_h^* - p_l^*}{v_h - v_l}\right) v_l - p_l^c = \left(\frac{p_h^* - p_l^*}{v_h - v_l}\right) v_h - p_h^c,$$

so that the cartel prices p_l^c and p_h^c satisfy the following equality:

$$p_h^c - p_l^c = p_h^* - p_l^*.$$

As market size is assumed fixed, the profit-maximizing cartel contract stipulates $p_i^c = \overline{u}$ for $i = l, h$ which is sustainable when the discount factor is sufficiently high. If not, however, firms may have to settle for a less profitable agreement with at least one binding incentive compatibility constraint. In what follows, we assume that the price-fixing agreement is sustained by means of a grim punishment so that the critical discount factors are given by:

$$\delta_i^* = \frac{\Pi_i^d - \Pi_i^c}{\Pi_i^d - \Pi_i^*}, \quad i = l, h,$$

where Π_i^d and Π_i^c are deviation and cartel profits, respectively. Note that, using the best-responses as specified in (4) and (5) above, one-period deviation profits can be written as:

[9]See, for example, Harrington (2006).

$$\Pi_l^d = \frac{1-\alpha_l-\alpha_h}{v_h-v_l} \cdot (p_l^d - c_l)^2,$$

and

$$\Pi_h^d = \frac{1-\alpha_l-\alpha_h}{v_h-v_l} \cdot (p_h^d - c_h)^2,$$

where $p_l^d = (1/2)\,(p_l^* + p_l^c)$ and $p_h^d = (1/2)\,(p_h^* + p_h^c)$ are the respective deviation prices.

We now have all ingredients available to present our main finding. The next result shows that quality-anchored buyers are a facilitating factor for collusion.

Proposition 1. *The critical discount factors for sustaining collusion (δ_l^* and δ_h^*) depend negatively on the quality-anchored customer bases (α_l and α_h).*

Proof. We prove the statement in three steps.

(i) First, note that the critical discount factors δ_l^* and δ_h^* can be written as:

$$\begin{aligned}
\delta_l^* &= \frac{\Pi_l^d - \Pi_l^c}{\Pi_l^d - \Pi_l^*} = \frac{\left(\frac{1-\alpha_l-\alpha_h}{v_h-v_l}\right)\cdot\left(p_l^d - c_l\right)^2 - (p_l^c - c_l)\cdot\left(\frac{1-\alpha_l-\alpha_h}{v_h-v_l}\right)\cdot(p_l^* - c_l)}{\left(\frac{1-\alpha_l-\alpha_h}{v_h-v_l}\right)\cdot\left(p_l^d - c_l\right)^2 - \left(\frac{1-\alpha_l-\alpha_h}{v_h-v_l}\right)\cdot(p_l^* - c_l)^2} \\
&= \frac{\frac{1}{4}\,(p_l^c - p_l^*)}{\frac{1}{4}\,(p_l^c - p_l^*) + (p_l^* - c_l)},
\end{aligned}$$

and

$$\begin{aligned}
\delta_h^* &= \frac{\Pi_h^d - \Pi_h^c}{\Pi_h^d - \Pi_h^*} = \frac{\left(\frac{1-\alpha_l-\alpha_h}{v_h-v_l}\right)\cdot\left(p_h^d - c_h\right)^2 - (p_h^c - c_h)\cdot\left(\frac{1-\alpha_l-\alpha_h}{v_h-v_l}\right)\cdot(p_h^* - c_h)}{\left(\frac{1-\alpha_l-\alpha_h}{v_h-v_l}\right)\cdot\left(p_h^d - c_h\right)^2 - \left(\frac{1-\alpha_l-\alpha_h}{v_h-v_l}\right)\cdot(p_h^* - c_h)^2} \\
&= \frac{\frac{1}{4}\,(p_h^c - p_h^*)}{\frac{1}{4}\,(p_h^c - p_h^*) + (p_h^* - c_h)}.
\end{aligned}$$

(ii) Next, note that for a given cartel contract, the critical discount factors are decreasing in the own Nash equilibrium prices as given by (6) and (7) and that:

$$\begin{aligned}
\frac{\partial p_l^*}{\partial \alpha_l} &= \frac{(v_h - v_l)\cdot(2-\alpha_h)}{3(1-\alpha_l-\alpha_h)^2} > 0, \\
\frac{\partial p_l^*}{\partial \alpha_h} &= \frac{(v_h - v_l)\cdot(1+\alpha_l)}{3(1-\alpha_l-\alpha_h)^2} > 0, \\
\frac{\partial p_h^*}{\partial \alpha_h} &= \frac{(v_h - v_l)\cdot(2-\alpha_l)}{3(1-\alpha_l-\alpha_h)^2} > 0, \\
\frac{\partial p_h^*}{\partial \alpha_l} &= \frac{(v_h - v_l)\cdot(1+\alpha_h)}{3(1-\alpha_l-\alpha_h)^2} > 0.
\end{aligned}$$

(iii) Finally, following the profit-allocation rule, the equality $p_h^c - p_h^* = p_l^c - p_l^*$ should hold both before and after the change in the captive customer share(s). Suppose, therefore, that cartel prices are raised by precisely the same amount as the Nash prices. To see that such a rise in cartel prices is sustainable, note that this would leave the numerators (*i.e.*, $1/4\,(p_i^c - p_i^*)$) and the first term of the denominators (*i.e.*, $1/4\,(p_i^c - p_i^*)$) unaltered. Yet, the second term of the denominators (*i.e.*, $(p_i^* - c_i)$) has increased, which implies a decrease in the critical discount factors. This concludes the proof.

□

To provide some intuition for this result, note that the numerator of the critical discount factors can be written as:

$$\Pi_i^d - \Pi_i^c = \frac{1 - \alpha_l - \alpha_h}{v_h - v_l} \cdot (p_i^c - p_i^d)^2, i = l, h.$$

For any given cartel price p_i^c, the optimal deviating price p_i^d is increasing in both captive customer bases (α_l and α_h). Loosely speaking, the larger the share of quality-anchored buyers, the more costly it becomes to cut price below the cartel level. By deviating from the cartel, a firm incurs a loss in its captive segment and a gain in its non-captive segment. Therefore, when the latter becomes comparatively less important, the difference between deviation and collusive profits decreases. Though this effect works in favor of collusion, it is possibly mitigated by a less effective punishment strategy. Indeed, an increase in the share of quality-anchored buyers positively affects the price-cost margins absent collusion. This may induce a decrease in $\Pi_i^c - \Pi_i^*$, which works against collusion. As Proposition 1 reveals, however, the 'punishment effect' is not strong enough to make up for the 'deviation effect'. Taken together, we therefore conclude that an increase in the share of quality-anchored consumers facilitates collusion.

It is worth highlighting that the firm with the lowest non-collusive price-cost margin is the one with the highest critical discount factor. That is,

$$\delta_l^* \gtreqless \delta_h^* \Leftrightarrow p_h^* - c_h \gtreqless p_l^* - c_l \Leftrightarrow \frac{v_h - v_l}{c_h - c_l} \gtreqless 2\frac{1 - \alpha_h - \alpha_l}{1 - 2\alpha_l}.$$

Thus, which firm has the tightest incentive constraint critically depends on the relation between quality and cost as well as on the shares of quality-anchored customers. Note further that each critical discount factor depends negatively on the number of both types of quality-anchored buyers, but that the own effect dominates the cross effect. The strongest pro-collusive effect, therefore, comes from a growth in the quality-anchored customer base of the least profitable producer.

4. Conclusion

There is now quite a rich literature identifying facilitating factors for collusion. In this note, we have argued that quality-anchored buyers should be added to that list. Specifically, we have shown that an increase in the share of quality-anchored consumers relaxes firms' incentive constraints, which *ceteris paribus* enables them to sustain (weakly) higher prices.

This facilitating effect may materialize in rather subtle ways. For example, firms may launch advertising campaigns to enhance the perceived quality of their brand(s), which is common practice in competitive markets. It can also result from changes in the market that are (partly) exogenous. For instance, a shift in income distribution might induce a larger share of consumers to exclusively consider low-quality products (*e.g.*, since they cannot afford more) or premium products instead (*e.g.*, for reasons of prestige). As we have seen, this not only leads to higher competitive prices, but also makes it more likely that the price to be paid is a coordinated one.

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Appendix

The purpose of this Appendix is to shed some light on conditions C1 and C2 by showing sufficient conditions under which they are satisfied. To that end, consider the demand specification of, respectively, the standard and premium producer when all buyers make a purchase:

$$D_l(p_l, p_h) = \begin{cases} 1-\alpha_h & \text{if} \quad \overline{u} \geq p_h \geq p_l + v_h - v_l \\ \alpha_l + (1-\alpha_l-\alpha_h)\cdot\left(\dfrac{p_h-p_l}{v_h-v_l}\right) & \text{if} \quad \overline{u} \geq p_h \text{ and } p_l + v_h - v_l > p_h > p_l \\ \alpha_l & \text{if} \quad \overline{u} \geq p_l \geq p_h \end{cases}$$

$$D_h(p_l, p_h) = \begin{cases} 1-\alpha_l & \text{if} \quad \overline{u} \geq p_l \geq p_h \\ \alpha_h + (1-\alpha_l-\alpha_h)\cdot\left(1-\dfrac{p_h-p_l}{v_h-v_l}\right) & \text{if} \quad \overline{u} \geq p_h \text{ and } p_l + v_h - v_l > p_h > p_l \\ \alpha_h & \text{if} \quad \overline{u} \geq p_h \geq p_l + v_h - v_l \end{cases}$$

Thus, for (p_l^*, p_h^*), as specified in (6) and (7), to constitute an interior static Nash equilibrium in which both firms are active in the contestable segment, it is required that: $\overline{u} \geq p_h^*$ and $p_l^* + v_h - v_l > p_h^* > p_l^*$. This is guaranteed by C1. To see when $v_h - v_l > p_h^* - p_l^*$ and $p_h^* > p_l^*$, note that:

$$v_h - v_l > p_h^* - p_l^* \equiv \frac{1}{3}(v_h - v_l)\frac{1-2\alpha_l}{1-\alpha_h-\alpha_l} + \frac{1}{3}(c_h - c_l). \tag{8}$$

Observe that the RHS of (8) is positive when $\alpha_i < 1/2$, $i = i, h$, which implies $p_h^* > p_l^*$. Moreover, for $v_h - v_l > p_h^* - p_l^*$ to hold as well it is sufficient that the premium producer creates more value (*i.e.*, $v_h - c_h > v_l - c_l$).

In addition, firms should not have an incentive to sell exclusively to their quality-anchored consumers. This is guaranteed by C2. Using (6) and (7), the condition $\Pi_l^* > \Pi_l^m$ is then satisfied when:

$$\frac{\left((v_h - v_l)(1+\alpha_l) + (c_h - c_l)(1-\alpha_h-\alpha_l)\right)^2}{9(v_h - v_l)(1-\alpha_h-\alpha_l)} > (\overline{u} - c_l)\cdot\alpha_l. \tag{9}$$

Similarly, $\Pi_h^* > \Pi_h^m$ requires that:

$$\frac{\left((2-\alpha_l)(v_h - v_l) - (c_h - c_l)(1-\alpha_h-\alpha_l)\right)^2}{9(v_h - v_l)(1-\alpha_h-\alpha_l)} > (\overline{u} - c_h)\cdot\alpha_h. \tag{10}$$

If, for example, $c_h = c_l = 0$ and $\alpha_l = \alpha_h = \alpha < 0.5$, then C1 holds and C2 is satisfied when:

$$\frac{v_h - v_l}{9\overline{u}} > \frac{\alpha\cdot(1-2\alpha)}{(1+\alpha)^2}.$$

More generally, the quality-anchored customer bases should not be too large. This is not to say that they cannot be of substantial size. Conditions C1 and C2 may, for example, hold when no less than half the market is quality-anchored. To illustrate, C1 and C2 are met if, *e.g.*, $\alpha_l = 0.25$, $\alpha_h = 0.25$, $\overline{u} = 3.5$, $c_h = 1$, $c_l = 0$, $v_h = 3$, and $v_l = 1$ (which gives $p_l^* = 2$ and $p_h^* = 3$). The same holds when the quality-anchored buyers are unevenly distributed. For example, assuming the same parameter values, the conditions are also met when the low-quality firm has hardly any anchored buyers (*i.e.*, $\alpha_l \to 0$) and half the market only thinks about the high quality product when considering a purchase (*i.e.*, $\alpha_h = 0.5$). One can construct plenty of other examples illustrating that there is a broad range of parameter values that meet the conditions imposed.