Dynamics and Optimal Monetary-Fiscal Policy in Fiscally Dominant Economies with Occasionally Inflexible Monetary Authorities

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1 Introduction

1.1 Topic and Motivation

There is a growing, theoretically and empirically motivated interest in dynamics and optimal macroeconomic policy under *Fiscal Dominance*. This is particularly the case in the context of development economies, in which constraints placed by institutional organisation in developing economies on the monetary and fiscal policy-making stand out as frictional aspects of import to the understanding macro transmission and optimal stabilisation policy design, and how this might differ from the insights from the (generally NK) modelling of advanced economies. Addressing such growing interest in Fiscally Dominant economies, in this paper we generalise a sticky-price DSGE model to admit both Monetary Dominance (MD), with active monetary policy and passive fiscal policy (i.e. a mainstream NK closure), and Fiscal Dominance (FD), in which the reverse applies, as alternative institutional designs or ”second-order” devices for addressing inflation path indeterminacy, and study the implications of occasional inflexibility on the part of a generally passive monetary authority. In particular, building on the Fiscal Dominance closure of the model as a benchmark, we extend the model to account for frictions introduced by the theoretically-relevant and empirically plausible possibility *occasionally occurring* inflexibility on the passive monetary policy side: chiefly, in the form of ceilings on the quantity of nominal debt or bond issuance at which the economy is willing to operate at any point in time.

As we see them, such ceilings might be isomorphic to either existing or debated legislative or constitutional ”fiscally prudent” commitments to keep in check debt-

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financing in contexts in which fiscal rules and institutions are not able to offer the non-explositivity guarantees associated to the canonically ”passive” treasury in the New Keynesian model, or to exogenous political pressures, or to the reluctance[1] of a monetary authority concerned with public finances to allow bond sales beyond some quantity. In this latter case, occasional policy inflexibility on debt side restricting further debt financing emerges not only as a friction introduced by empirical institutional organisation, but also as a ”last-resort” or unconventional policy option exercisable by a Central Bank. Such occasional policy inflexibility option might be conceivably exercised when the Bank has knowledge of (a) the severely inflationary implications of allowing for unchecked growth in nominal debt in a fiscally dominant economy even under optimal inflation-targeting monetary policy[2], and (b) as Kumhof et al. have initially pointed out in their exploration of a benchmark fiscal dominance economy (2010), that they are the only other actor who can act on a concern for the stabilisation of nominal debt path below some exogenously arising target.

Essentially, this can be thought of as a Fiscal Dominance regime in which the monetary authority might refuse (or be forced to refuse) to potentially print *as much more money* or allow for as much unchecked money growth – in order to redeem a growing extant bond balance – between periods as would be required by a passive monetary policy always adjusting flexibly to accommodate fiscal activity (or inactivity): they refuse to potentially have to monetise debt past some level. Contrary to the pure or unconstrained Fiscal Dominance models recent contributions (cf. Kumhof et al., 2010; Cochrane, 2023), and with an eye to examining dynamics and optimal policy under more robust models of Fiscal Dominance, we view this exercise as essentially relaxing a heroic assumption of an always-flexible passive policy side to a case in which passivity of the monetary authority holds *conditionally*: specifically, as far as debt growth under fiscal behaviour does not ”cross a line” relative to initial, steady state conditions but no further. From a theoretical-methodological point of view, because this inflexibility involves an occasionally binding constraint with endogenous timing, the paper essentially proposes and analyses an extension of the benchmark FD model to allow for non-linearities in the form of *endogenous* regime switches between a ”normal times” fiscally dominant economies in which occasionally inflexible policy is not binding and one in which it is.

The modelling exercise we propose is motivated on the back of and enables us to raise and discuss two questions key to furthering our understanding macroeconomic transmission and optimal policy-design in Fiscally Dominant economies. *First*, a positive question on how the presence of such occasional, endogenously biting inflexibility and resulting non-linearity impacts the transmission of typi-

[1] Ruled out by construction in the benchmark case proposed here and in the FD literature, and motivating our ”inflexibility” label for the case

[2] i.e. where a Fiscal Theory of the Price Level, or inflation-financing of debt, applies as we show. cf. Woodford

cal demand-side shocks to transitional dynamics relative to the friction-less FD benchmark: are transitional dynamics a convex combination, or meeting point, between dynamics under unconstrained/linear MD and FD cases, or do they display emergent properties, and why? *Second*, and based on the answer to the former, where this leaves the design of optimal monetary-fiscal rules in such contexts. In particular, circling back to the key debate on policy design under Fiscal Dominance (cf. Kumhof et al., 2010), the desirability or stronger rather than weaker inflation targeting and optimal monetary-fiscal interactions, and the possibility of alternative solutions for achieving inflation targeting without compromising optimality.

While we believe the paper offers theoretical insights to complement pure-form, perfectly-flexible models fiscal dominance and fiscal theories by considering scenarios with endogenously arising non-linearities in the form of regime switches to the effect of relaxing the assumption of an unconditionally passive monetary side, we think that the positive and normative applications of the paper are also strongly motivated on the back of real world issues; by the same coin, that the exercise can shed light on real world dynamics and optimal policy benchmark amid an alleged transition to a greater role of fiscal dominance across economies. In particular, with fiscal dominance plausibly on the rise in advanced economies, and already an endemic reality in many developing ones, understanding dynamics and optimal policy benchmarks in these scenarios is essential for modern macroeconomic policy. Fiscal dominance models, however, are fundamentally premised on a perfectly passive monetary policy, i.e. such to enable any level of debt or bond sales backing the active surplus/revenue policy. But both in principle (say, a ”limitedly tolerant or cooperative” monetary authority concerned with solvency or willing to allow for money growth to accommodate fiscal policy up to but not past some point) and as an empirical norm, as is the case for legislated or debated ceilings on debt, there might be in general less than perfect willingness to support overshooting some bounded quantity of debt. Finally, the constraint on new borrowing or money growth beyond some ceiling can be the product of ”original sin” and ”debt intolerance” phenomena highlighted by the monetary economics literature in developing economies. In these contexts, it hence seems natural – as a an empirically motivated extension and as a robustness check on the frictionless FD case – what happens, and with what implications, when economies operating under or transitioning to an FD regime simultaneously are unwilling flexibly or passively accept, beyond some measure, the debt position sanctioned by according priority to fiscal considerations. Particularly in the context of the institutional organisation of developing economies, the proposed analysis of transitional dynamics and the ensuing optimal policy design stand as a contribution to the literature on macroeconomic transition and design of monetary-fiscal policy in developing economies to meet stabilisation objectives.

As we see it, considering the model and questions we propose, by relaxing the assumption of a perfectly passive monetary policy side, becomes key to bridging the gap between fiscal dominance in the real world and the theoretical tools at our disposal for studying dynamics and optimal policy in such contexts. We develop further on potential causes of fiscal dominance and occasional inflexibilities in advanced and developing economies in the literature review in section 2.

1.2 Methodology and Modelling

1.2.1 Modelling FD with and without Occasional Inflexible Monetary Policies

As we outlined, building on the FD modelling literature we work with a micro-founded sticky-price model of a monetary economy similar to the representative agent New-Keynesian model with a forward-looking Phillips Curve. The latter, e.g. in the canonical three-equations setup, represent however a specialisation of the more general model we consider to a case of Monetary Dominance. In particular, as stated, the model we work with is such to admit both Monetary Dominance (or RANK) and Fiscal Dominance (á-la-Kumhof, henceforth FD) as analytically obtained and theoretically consistent specialisations of the generalised, unifying framework we propose. Clarifying how indeterminacy (rather than asymptotic price stability) is key problem of the forward looking, sticky-price workhorse model economy, bridging computational FD models and works on FTPL, we explicitly conceptualise and allow to model MD and FD as second-order devices for addressing such equilibrium indeterminacy problem through explosive threats. In particular, MD and FD assign to either monetary or fiscal behaviour the unique role of providing such explosive threat, familiar from BK conditions in DSGE modelling, in the form of an explosive root in the relevant laws of motion that they govern; such explosive threat pins down a determinate equilibrium inflation path as the ”jumping” forward solution to the relevant equation in the presence of standard transversality or boundary conditions ruling out expectations of asymptotic explosions.

Based on the unifying framework for alternative dominance regimes provided by the above model, we then move to consider a relaxation of FD with perfectly flexible passive monetary policy to the weaker assumption of occasional inflexibility. Specifically, as motivated, we are interested in cases isomorphic to the introduction of a ceiling or intolerance threshold on allowed debt or bonds issuance,[3] such that monetary policy accommodates, through bond sales or implicitly growth of the money supply between periods redeeming the debt, the preferred debt position of the government under its committal fiscal policy as far as the economy lies within such threshold – a regime we call ”normal times” – but no further, a ”constrained times” regime in which the debt position is stuck at the occasionally binding inflexibility point.

[3]Structurally, the debt ceiling could be modelled as a function of different steady state or long-run output economic variables, such as GDP growth, fiscal policy, inflation or political factors. We remain moot on this point as similar treatments are isomorphic to the general one we propose.

1.2.2 The Endogenous Regime Switching Problem and Solution Algorithm

In line with the nonlinear DSGE literature, this can be viewed as the economy endogenously switching between a pure FD with perfectly passive policy case, corresponding to the ”normal times” regime, and the ”constrained times” regime. Relaxing the assumption is hence isomorphic to augmenting the baseline FD case to a case with endogenous regime switches.

The technical difficulty with the analysis of endogenous regime-switching DSGE models is twofold. First, because the regime is effectively a discrete state variable, the rational expectations DSGE system will be nonlinear and, more importantly, non-differentiable in the state. Hence, standard perturbation methods in the state space are generally unavailable. In principle, perturbation methods could be applied to an augmented system, leading to standard state-space solution through undetermined coefficient methods, by modelling the regime-switching as governed by a Markov Chain (cf. Bianchi, 2013). This strategy offers a good methodological route when we can an assume, in the time series vernacular, that the regime state variable is *strongly exogeneous* in the model.[4] This assumption is obviously not appropriate in the present context.[5] Anticipating such criticism and discarding such route, to solve the model in the case with occasional inflexibility on the monetary policy side, we hence draw from the recent methodological literature on simulation-based *piece-wise linear root finding algorithms* in the sequence-space (cf. Guerrieri and Iacoviello, 2015) to deal with regime witching that occur with endogenous timing or transition probabilities. The algorithm fixes a typical shock-path of interest, making it part of the information set at the origin, and solves endogenously for the unknown regime sequence and associated equilibrium path through a guess and verify method. The verification condition to be met at convergence of the algorithm is slackness of the constraint, or reversion to normal times after the last period in which the constrained is guessed to bind. At any iteration of the regime sequence guess, the equilibrium path is solved for by means of a perturbation in the sequence space keeping the shock sequence at the initialised value, corresponding to a linear algebraic problem.

1.2.3 Macroeconomic Transmission and Policy Optimisation Analyses

Using the above solution routine for the occasionally constrained FD case, which allows to also solve for the perfectly flexible FD case (as the unconstrained solution), and for the MD case through altering accordingly the parametrisation of the model, we are able to use simulation based equilibrium solutions across the three different

[4]Weak exogeneity does not suffice, as implicit in the solution of the system is a prediction or forecasting problem

[5]There is a separate question, which we leave open for further work on the topic, on whether exogenous regime switching can approximate well and under what conditions the endogenous regime switching model we study

models to investigate the transmission of typical demand side shocks coming from financial or IS curve shocks and fiscal policy shocks. The simultaneous solution of the three models allows for direct comparisons between the three scenarios, and hence to compare how the FD case with occasional inflexibility compares to replicated results in the MD and perfectly flexible FD literature. We do not examine monetary policy shocks and cost-push shocks to inflation but these can be easily accommodated in the model. This analysis form the basis of our answer to the first research question on dynamics in the FD economy with occasional inflexibility on the monetary side.

To answer the second question we raise, we then study optimal fiscal-monetary policy design comparatively across the perfectly flexible and occasionally inflexible FD economy, taking a simulation-based route consistent with the above solution procedure. In this model, the space of fiscal and monetary policy rules, under the feasibility constraints dictated by the FD device, naturally take the form of "simple rules" – that is, a stricter version of a Ramsey Instrument, which is only allowed to adjust in response to a limited number of variables, following the methodological literature pioneered by Schmitt-Grohé et al (2005) alternative to Ramsey instruments with unrestricted updating. Optimisation can be thus carried out (ad no added cost) directly with respect to the parameters of the monetary and fiscal rules already at play in the benchmark models. We choose to focus on a standard "robust" objective, akin to macroeconomic stabilisation, cast in terms of minimisation of price and output volatility. We focus on a multivariate optimisation problem with respect to the *joint/simultaneous* choice of feasible simple monetary and tax revenue updating rules, and hence are able to account directly for and comment interactions of monetary-fiscal rules in the optimisation exercise. To do this, we use a double-loop algorithm: the outer loop iterates initialisations of the inner loop over points in the monetary-fiscal policy space, while the inner loop (1) solves the model of FD with and without occasional inflexibility on the monetary policy side for transitional dynamics in response typical shocks (and their interaction) using the piece-wise linear root-finding algorithm described above, and (2) evaluates the robust planner objective along such equilibrium dynamics.

Based on the results from the optimisation exercise and to further elaborate on them, the last part of the paper, finally, proposes a game-theoretic analysis of monetary reform in FD economies subject to an occasionally inflexible monetary policy side that endogenises the choice of the planner modelled above. Through this, we higlight how occasional inflexibility might, under gradualist reform, trap an economy initialised at globally suboptimal monetary-fiscal policy rules as the only rational expectations equlibrium of the reform game. Using the optimal policy routine in the context of interacting fiscal news and monetary-fiscal policy pairs, we propose a solution to the above emergent problem.

1.3 Main Contributions and Results

The paper makes both substantive contributions to monetary macroeconomics and optimal monetary-fiscal policy, and to the fiscal dominance methodological literature.

On the substantive side, the theoretical and policy-applicable contributions of the paper are thus to the theory and modelling of fiscally dominant economies as theoretical and empirically motivated alternatives to benchmark NK or monetary dominance models informing much of modern economic analysis and to the unconventional and policy interactions literature, with mandated ceilings or bounds to the activity of the fiscal authority might represent a policy possibility in FD regimes. In particular, by enabling to account for the possibility of occasional inflexibility on the passive monetary policy side in the form of weak inequalities on the quantity of debt or bond issuance at which the monetary authority (or other institutionally relevant agents) are willing to operate at any point in time. The modelling and ensuing policy optimisation exercise proposed enables to examine the implications for fiscal dominance of relaxing the assumption of an unconditionally passive monetary policy. This is appealing both on theoretical grounds, as it constitutes a robustness check on FD modelling, and on empirical grounds: either because explicit or implicit boundaries on the degree of subservience of the monetary authority to the fiscal one constitute empirical and policy possibilities, especially in developing economies, insofar as they might arise from legislation or political pressure, or because (as perhaps austerity threats and structural adjustment programmes best teach) the fuse of the monetary authority, while long, is probably still of finite-length.

By means of this extension, we identify a number of what we think to be interesting results to the above literature. On the positive side, transitional dynamics in the case with occasional inflexibility are **not** – as one might guess to be intuitively – a convex combination of dynamics under pure FD and MD; rather, dynamics under occasional inflexibility display emergent properties, and fundamentally reflect the highly nonlinear nature of the endogenous regime switching problem emerging in such context. In particular, we show that in the context of occasional inflexibility, the response to positive aggregate demand shocks – whether structurally induced, in the form of shocks to inter-temporal preferences, or policy-wise by expansionary fiscal policy – result in (a) generally more dragged out or sluggish dynamics, and (b) in a protracted recessionary-deflationary phase, accompanied by low nominal rates, down the line. As we explain, and circle back to when motivating the finding on optimal policy, we believe this to be the result of the endogenous interaction emerging between the unwillingness to acquiesce to debt positions in violation of the inflexibility point or threshold, and the inflation-targeting objective built into the feasible monetary rule. Pursuing simultaneously both objectives can only be achieved, in the constrained case, by penalising inflation expectations down the line by keeping nominal rates, hence threatening a recession-deflation sequence, down the line. For a consistent or committal monetary policy rule and with rational private sector expectations, the price of pursuing the twofold objective in the constrained case through such means is actually delivering such recession-deflation sequence.

In the policy optimisation exercise, they key upshot of the above typical dynamics for the optimal policy literature (and in FD specifically) is that the upper end of the monetary rule set, corresponding to stronger, Taylor-approaching inflation-targeting rules become severely sub-optimal, and so at any feasible dominant fiscal rule. The optimal monetary-fiscal policy pair assigns low reactivity to nominal rates in current inflation (around 0.6 per inflation point), a result which, considering the optimality of stronger inflation targeting under perfectly flexible FD, is entirely driven by the presence occasional inflexibility. A second upshot, which we elaborate through the analysis of the proposed monetary reform game, is that gradualist reforms to monetary policy rules might fail to move an initially excessively inflation-targeting FD economy to the global optimum on the policy space. The upshot of the analysis for the inflation-targeting literature are clear: in general, FD economies display a trade-off or fork between gradualism of reform, the ability to enforce ceilings or bound fiscal activity, and the optimality of stronger inflation targeting, implying one of the three should be abandoned. Based on the conviction that neither gradualism or inflation targeting are unlikely to go, we propose, however, a solution to the dilemma to the effect of restoring optimality of inflation targeting in contexts with occasional inflexibility, founded on the use of contractionary fiscal innovations (on top of monetary-fiscal interaction) to meet positive aggregate demand shocks.

Undergirding the above substantive contributions are two methodological-applied ones. First, we interface the modelling of fiscally dominant economies and simple optimal fiscal-monetary rules therein, with the use of sequence-space perturbation-based piece-wise rootfinding algorithms (Guerrieri and Iacoviello, 2018) to solve for equilibrium dynamics in models with endogenous regime switches emerging from occasional inflexibility on the monetary policy side and giving rise to non differentiabilities in the state space. To the best of our knowledge, the closest exercise in the literature on non-linear FD and occasionally binding constraints is proposed by Schmidt (2024). In case, however, the relevant inflexibility is provided by the unwillingness of a passive fiscal authority to raise fiscal surplus beyond some upper ceiling. While similar in drawing attention to the role of less-than-perfect flexibility on the passive policy side for ensuing dynamics, we view our work as complementing this recent contribution, by studying a case symmetric to the one the latter proposes. In particular, the "normal times" regime considered by Schmidt is a monetary-dominant or "orthodox" economy which, by virtue of the occasionally binding constraint, endogenously switches for some time to a fiscally-led regime: in our case, we study economies in which rules or institutional constraints are such that fiscal dominance is the norm, and the occasional inflexibility point comes from a passive monetary authority, rather than the passive fiscal authority. As such, our contribution appears complementary to this recent exercise by studying the role of inflexible passive policies in economies in which fiscal dominance constitutes the norm, such as chiefly many developing countries.

Second, in order to construct the FD or ”normal times” benchmark for the implementing the above procedure, we provide a unifying framework treating Monetary Dominance and Fiscal Dominance as alternative ”second-order” devices that achieve equilibrium determinacy in the workhorse new Keyenesian sticky-price macroeconomic model. While in terms of end product, we take a similar, calibration-based approach to qualitatively differentiating the two alternative regimes to the one in alia Kumhof et al (2010), contrary to the latter, we link the use of such approach to Cochrane’s work on equilibrium determinacy as opposed to inflation stabilisation as the central problem of New Keynesian modelling, providing a sounder and more economically consistent logic underlying the operation of the MD and FD regimes than the one proposed by Kumhof. Additionally, rather than focusing on the non-responsivity of tax setting to debt levels as the mark of fiscal dominance, we provide and implement explicitly a conceptual way for generalising the choice of coefficients in a way consistent with distinguishing qualitatively between monetary and fiscal dominance regimes. In particular, by posing MD and FD devices as similarly picking equilibria by carrying a threat – through a Taylor Rule for MD device and through sufficiently weak reaction of a tax revenue rule to extant debt for the FD device – to de-stabilise the economy should the inflation path fail to converge to the unique value consistent with forward solution to the relevant explosive equation coupled with a *boundary* or *transversality* condition. This has the benefits of offering a straightforward, analytical, and economic-theory based route to the calibration of the model as opposed to purely numerical routines; of making monetary dominance and fiscal dominance immediately comparable, and finally to clarify, on the one hand, the link between Fiscal Dominance and Fiscal Theories of the Price Level, and on the other the misguided concern for ”price instability” (rather than more or less aggressive inflation targeting) under FD regimes.

1.3.1 Software and Computation

All simulation-based sequence-space solutions and optimisation routines forming the analysis are implemented in MATLAB r2024a. Verification of equilibrium determinacy conditions for the normal times unconstrained Fiscal Dominance – technically required as the occasionally inflexible case reverts to a the slack or unconstrained scenario under the assumption supporting the solution algorithm – and exploration of the benchmark FD and MD economies is carried out in DYNARE. Both the computation of equilibrium solution and policy optimisation routines can be parallelised across the three unconstrained MD, unconstrained FD, and FD with occasional inflexibility regimes.

1.4 Organisation of the paper

The rest of the paper unpacks the above and is organised as follows. Section 2 reviews and contextualises our work in the relevant background literature. Section 3 develops the micro-founded workhorse model unifying the treatment of the MD, FD, and FD with occasionally inflexible monetary policy; this model is essentially a standard sticky-price model, and can be skipped to rather focus on the conceptualisation and modelling of MD and FD as alternative solutions to the indeterminacy problem of the model – this is done in section 4. The same section also outlines and discussed the solution procedure, and based on the computations earlier described the transitional dynamics across the three economies, focusing on the first of the two questions we raise. Section 5, toward answering the second question that we have raised, discusses the policy optimisation exercise and its results, together with the closing game-theoretic analysis of complications to gradual reform toward weaker-inflation targeting optimal rules, and solutions to the problem. Ulterior and complementary materials, signposted throughout the text, is available in the Appendix.

2 Background literature

2.1 Causes of Fiscal Dominance and Occasional Policy Inflexibility

Fiscal Dominance, generically referring to economies with institutional organisation such that fiscal considerations take precedence over conventional inflation-targeting[6] in the conduit of monetary-fiscal policy, has faced resurgent interest both as a topic in contemporary macroeconomics research and for its applications to rethinking benchmarks for current and future macroeconomic policy.

In part, motivating this is a prevalent sense that the traditional ”passive” role ascribed to fiscal policy in New Keynesian theory has been or will be swapped for a dominant role in the face of recent and under-way real world challenges to sticking to institutional configurations that keep fiscal considerations in the backseat. In advanced economies, the extraordinary fiscal stimulus introduced globally during the COVID-19 pandemic has led to significant changes in the economic landscape of various countries. Between January 2019 and the same month in 2020, OECD countries experienced an average increase in the debt-to-GDP ratio of 21.26 percentage points. Although these aggressive fiscal interventions might

[6]Technically, in the perspective taken by this paper, Taylor-principled monetary rules. We think it incorrect or unfair to motivate the issue, as much of the literature has done, with alarming calls they take precedence over ”price-stability” (cf. Schmidt, 2024), as in the workhorse NK model the core problem addressed by aggressive inflation targeting in is never stability, but rather determinacy of the inflation path. Aggressive (more than proportional) inflation targeting, corresponding to a Monetary Dominance device, is surely an alternative device to Fiscal Dominance, but they still serve the same goal of solving the indeterminacy problem. Inflation paths are stable to begin with. Our paper devotes significant effort to settling this mischaracterisation through the developed framework.

have been necessary to mitigate the economic setbacks caused by the pandemic, they also raise concerns about the long-term sustainability of public finances if monetary policy is to stick to the aggressive inflation-target as in a Monetary Dominant regime.

In developing economies, similar challenges to maintaining a commitment to fiscal passivity in the face of ongoing and protracted crises, ranging from the need to contain rising inequality through fiscal action, to that of speeding up a sluggish post-pandemic recovery relative to their advanced counterparts, and finally to managing adaptation to the climate crisis. As some authors have noted, besides an exacerbation due to the above challenges, Fiscal Dominance might be in some sense endemic to the broader institutional organisation of many developing economies: restricted revenue basins and large informal sectors, a difficulty in controlling tax evasion and weak tax collection systems, overspending at the regional or below-government level (Kumhof et al., 2010), and finally the greater reliance on less flexible sources of revenue – such as property tax.

As pointed out, a final, much more ordinary cause for a transition to fiscal dominance regimes in both advanced and developing economies is, finally, an unwillingness on the fiscal policy-making side to subjugating tampering and adjustment of net surpluses to the inflation-targeting goals of the monetary authority (cf. Schmidt, 2024) – this might itself be more serious in institutional environments, more prevalent in middle income countries, with higher dependence of the central bank on the government. Due to these changes, macroeconomic modelling and policy design must plausibly contend with the Fiscal Dominance as a norm in empirical institutional organisation alternative to the Monetary Dominance benchmark around which much of the theory and policy advice have been built.

Another reason for the surging relevance of Fiscally-Dominance, separate from its current and future empirical applicability to the institutional configuration of economies, is theoretical. In particular, largely due to the seminal work by Cochrane (2023, 2022, and 2011), there is renovated interest in *Fiscal Theories of the Price Level* (FTPL) as an alternative, achieved essentially through the implementation of an FD regime, to Taylor Rules to solve the indeterminacy problem of the inflation path in sticky-price, monetary economies with forward looking price setting. Such interest, in particular, may be motivated by an allegedly more realistic and intuitive picture that these closures provide in terms of the functioning of monetary policy (Ibid.). Our paper thus taps into a both empirically and theoretically motivated interest in Fiscally Dominant economies, particularly in relation to the ordinary occurrence of Fiscal Dominance in the context developing economies.

A key element in fiscal dominance theories (cf. Kumhof et al., 2010) is the (more or less implicit) assumption that the monetary policy side is perfectly flexible. Specifically, that it will accommodate, through the passive nominal rate rule at equilibrium inflation and inflation expectations, the preferred debt or bond issuance position of the government consistent with the evolution of government balances under the active fiscal rule. From a monetary point of view, this is akin to a situation in which the monetary authority (or other relevant institutional actors), flexibly or agrees – as a yes-man – to print as much money or allow for as much money growth to accommodate the evolution of the government debt position between periods. However, a potential issue which can plausibly arise in FD settings is that the willingness to cooperate or the feasibility of cooperation might be conditional on the debt position of the government, giving rise to scenarios in which monetary policy is willing to allow, through its activity, bond sales or money printing as far as fiscal activity does not cross some line, after which cooperation is suspended. This debt-ceiling phenomenon, to the effect of relaxing the assumption of a perfectly flexible monetary policy side in favour of an occasionally inflexible one, mirrors symmetric concerns already raised in MD economies, e.g. the unwillingness to assist active monetary authorities with tax collection policy above some ceiling (cf. Schmidt, 2024), a question which has shifted attention to the role of nonlinear interactions between the dominant policy or dominance device and a limitedly cooperative passive policy side – in some sense, the paper contributes to advancing such work by interfacing it with the modelling of increasingly prevalent fiscal dominance scenarios.

Contemplating this theoretical possibility, in addition to extending the theory of dynamics under FD, thus responds to the necessity of accounting for key, potential frictions on the functioning of the FD device in empirical Fiscally-led regimes. In practice, as we see them similar ceilings or threshold points under girding occasional inflexibility can have different causes. There are three broad reasons why such frictions may arise.

First, they may arise as exogenous constraints on the leeway accorded to the dominant fiscal authority, and imposed *inter alia* by domestic or supra-national legislation, participation in monetary and (relevant to the current EU debate on common debt) fiscal-monetary unions, and participation in structural adjustments and conditional aid programmes. Some countries already have a debt ceiling either in total (e.g., Denmark and the United States) or as a percentage of output (e.g., European Union countries); similarly, they constitute currently debated, realistic policy options – especially in the context of an underway transition to fiscally-led economies with monetary policy in the backseat – contemplated as unconventional monetary policies to constrain an excessively imprudent fiscal side in fiscally led regimes. Second, even if some countries do not have debt ceiling legislation, more or less explicit limitations on new borrowing might be the result of exogenous political considerations and pressure, or responding to the concerns for fiscal sustainability of the private sector or other institutional actors (chiefly, but not limited to, monetary authorities) in systems in which democratic accountability or institutional concertations are necessary. For example, the new labour government in the United Kingdom, in its review of fiscal rules and targets and emphasis on fiscal prudence, has explicitly announced the intention to keep in check new borrowing (Commons

Research Briefing, 2024). In the Indian economy, similarly, a current discussion exists at the sixteenth financial commission relative to the operationalisation of limits on new borrowing at Union and state level. Similarly, in the "inflexible monetary authority" scenario naming this paper, the ceiling or friction might directly stem from a monetary authority that, in a Fiscal Dominance system, agrees (perhaps through gritted and is rationally expected to passively accommodate fiscal policy by allowing bond sales or more money-printing up to some point, but no further. Third, there might be structural or physiological limits to the amount of borrowing in economies operating under Fiscal Dominance – chiefly, well documented "debt intolerance" and "original sin" institutional pheoneomna in developing economies with weak fiscal systems; this case is particularly interesting in the sense that the same causes of fiscal dominance might also undergird the occasional inflexibility problem (Mehl and Reinaud, 2005; Eichengreen et al., 2003).

Whether as the object of (a) empirical legislations and debated reform, (b) political pressure and concertation, (c) widespread beliefs on debt tolerance, or (d) structural impediments to borrowing domestically, similar devices are isomorphic to introducing an upper boundary on the amount of new debt or money at which a Fiscally Dominant economy is willing to operate at any point in time. Accounting for occasional inflexibilities in FD regimes caused by similar debt-ceiling phenomena introduces non-linearities into the picture in the form of an occasionally binding constraint on the debt position. In particular, since this result as noted in an model with endogenously-timed switches between a "normal times" and "constrained times" regime in which the occasionally binding inflexibility constraint holds respectively slack and tight, their analysis draws on recent work over the last decade on solving similar, models falling short of being everywhere-differentiable.

2.2 Occasionally Binding Constraints in DSGE Models

Guerrieri and Iacoviello (2015) developed a state space method (labeled as **OccBin algorithm**) to solve models with OBCs, demonstrating their importance in understanding economic responses to policy interventions and shocks. Also Holden and Paetz (2012) provide a solution method that allows for OBCs. In contrast with Guerrieri and Iacoviello (2015), the two author decided to introduce the OBCs using anticipated shocks. The paths for the endogenous variables are identical to the ones of OccBin algorithm but it has a drawback, the choice of anticipated shocks that mimic OBCs is specific to each model and is not amenable to a general specification.

McGrattan (1996), Preston and Roca (2007), and Kim et al. (2010) give us an alternative way of thinking the discontinuity implied by occasionally binding constraints. The insight is to penalize agents' utility when a particular constraint is hit. While this method has the advantage of converting a model with occasionally binding constraints into a model that is solvable by perturbation methods, it suffers from undesirable drawbacks. The solution could change with the size and the shape of the penalty (the barrier parameter). Moreover, any high-order perturbation method will generate a smooth solution that in some instances will violate the inequality constraint.

The integration of the FTPL with models incorporating OBCs offers, in our view, an innovative approach to understanding the interaction between policies and macroeconomic constraints. This combination allows for the examination of scenarios where fiscal policy could have a direct impact on price levels, while being subject to binding fiscal rules or limits, in crisis or distress circumstances occasionally. Simply with this approach, a policy maker/advisor can have a broader perspective.

2.3 Fiscal Dominance works

According to Cochrane (2021), the history of the Fiscal dominance began in 1981 with "Some unpleasant monetarist arithmetic" by Sargent and Wallace, who demonstrated the importance of the interaction between fiscal and monetary policy for inflationary outcomes. The two economists argue that if there was no reform of the US deficit in the early 1980s, it would be sooner or later monetized. Monetary policy alone can only act by exchanging lower inflation today for higher inflation tomorrow, but it cannot avoid inflationary phenomena. It is now well understood that the disinflationary process of the 1980s was also the result of, at least, two elements: (i) restrictive monetary policy and; (ii) an increase in the present value of surpluses, following tax policy reforms and regulatory reforms toward more prudent, and fiscally restrained or passive benchmarks (Visco, 2023, 2022). Sargent and Wallace considered however the interaction between monetary policy and fiscal policy only from the seigniorage point of view.

A turning point in the debate was certainly represented by Leeper (1991), who overcame this narrow point of view by presenting a model with a target interest-rate expressed in the same way as in the well-known New-Keynesian sticky-price models. In 1994, Sims built a non-linearized model where the strict control of the money supply does not lead to the full control of the price level. In this work Sims also focuses on stability and determinacy, that is guaranteed by an interest rate peg supporting the anchoring of future fiscal expectations, a result that was later confirmed by Woodford (2001).

The study by Kumhof et al. (2010) questions whether interest rates following the Taylor principle are feasible in countries suffering from Fiscal dominance. Even if the central bank responds to government debt, the model may produce unique equilibria associated with highly volatile inflation. There are two key insights from their work. The first is that Taylor-type inflation targeting rules are infeasible in Fiscal Dominance regimes, and a conclusion that inflation-targeting might not be desirable in FD regimes. We do not agree and show that, in a generalisation of their model that recognises indeterminacy rather than price stability to be the problem to be addressed by rules, this is not the case in general. Second, that the welfare gain from responding to public debt is minimal compared to the welfare gain from eliminating fiscal dominance altogether.

Although the work of Kumhof et al. provides some useful hints in thinking about fiscal dominance regimes, optimal policy in such contexts, and the role of concerns on the debt position, it has two shortfalls that our work addresses. First, it fails to define well jow fiscal and monetary dominance regimes are attained and the economic logic at play in them, as pointed out in the previous section and, as a consequence, offers a misguided interpretation of the role of the Taylor Rule in monetary dominance scenarios as a comparative benchmark and hence of the undesirability of inflation targeting in FD regimes. While numerically they achieve the modelling of fiscal dominance regimes by correctly identifying explosivity of the recursive equilibrium debt equation as the mark of FD regimes, they do not link this to the resulting FTPL mechanism at play, and to how FD and MD alternative devices for addressing core indeterminacy in the workhorse sticky-price model. Second, while they find Taylor-type monetary rules infeasible in FD regimes on equilibrium determinacy grounds (which reflects, in our view and as we show, standard BK conditions in DSGE analysis), this is different from a finding that inflation-targeting is suboptimal or should be limited – which is the question motivating their paper. For example, given feasibility/equilibrium determinacy constraints imposed by the assumption of an FD regime, are stronger inflation targeting or weaker inflation targeting rules better?

Part of the confusion underlying such conclusions, we think, arises from the undue concern with price instability, and hence the need to rein in inflation, as the problem addressed by the Taylor Rule. Our paper preliminarily settles these issues in constructing a coherent and consistent benchmark for MD, FD, and constrained FD regimes. In particular, we show that subject to feasibility constraints ruling out Taylor rule (which at any rate do not work in the stabilising way underlying Kumhof et al's commentart), stronger inflation-targeting is still optimal in unconstrained FD economies. This is no longer true, however, in FD economies constrained by occasional inflexibility. In this sense, we provide a more nuanced answer than the one they propose. Second, we see our work on debt ceilings and occasional inflexibility as complementing theirs; as opposed to having the bank caring about debt in an exogenous or parametric way through an augmented feasible monetary policy rule, we endogenise ”prudent” or ”inflexible” phases to the crossing of some threshold. In their model, to put it prosaically, concerns with debt enter linearly; in ours, they enter nonlinearly.

The fiscal dominance, and the linked FTPL strand of literature, failed however to impose itself immediately. For example, Buiter (1999) , Buiter (2002) and Buiter (2017) define the Fiscal Theory of the Price Level as ”fatally flawed” and a ”fallacy” to mistreat a ”budget constraint”. Yet, replying to these claims, in ”Money as a stock” Cochrane proved that the Fiscal Theory of the Price Level is based on a valuation equation, an equilibrium condition and not a ”budget constraint”.[7]

[7]We circle back to this point when showing how the FD model we construct in fact implies (and

Based on the empirically motivated resurgence of interest in fiscal dominance models, and a stronger theoretical basis to defend FTPL/FD from undue criticisms of the above sort, several subsequent studies have considered the inextricable link between monetary and fiscal policy, considering occasional shifts in monetary and fiscal policy regimes (e.g. Bianchi and Ilut (2017), Bianchi and Melosi (2017,2019)). Locating ourselves at an interface between works on fiscal dominance and nonlinear-non differentiable DSGE modelling, we believe the paper is able to contribute to growing literature on FD economies by offering theoretical insights to complement and check for robustness pure-form, perfectly-flexible models of fiscal dominance and fiscal theories by considering scenarios with endogenously arising when relaxing, as motivated earlier, the assumption of an unconditionally passive monetary side.

To the best of our knowledge, the closest exercise to our work in the literature on non-linear FD and occasionally binding constraints is proposed by Schmidt (2024). In case, however, the relevant inflexibility is provided by the unwillingness of a passive fiscal authority to raise fiscal surplus beyond some upper ceiling. While similar in drawing attention to the role of less-than-perfect flexibility on the passive policy side for ensuing dynamics, we view our work as complementing this recent contribution, by studying a case symmetric to the one the latter proposes. In particular, the ”normal times” regime considered by Schmidt is a monetary-dominant or ”orthodox” economy which, by virtue of the occasionally binding constraint, endogenously switches for some time to a fiscally-led regime [8]. Basically when the starting public debt is high, a sufficiently large inflationary shock, could led the surplus to a ceiling and the economy will transit to the fiscally dominant regime. In our case, we study economies in which rules or institutional constraints are such that fiscal dominance is the norm, and the occasional inflexibility point comes from a passive monetary authority, rather than the passive fiscal authority. As such, our contribution appears complementary to this recent exercise by studying the role of inflexible passive policies in economies in which fiscal dominance constitutes the norm, such as chiefly many developing countries.

3 Benchmark Sticky-Prices Model

3.1 Households

The rapresentative agent maximizes a lifetime expected utility function, which is:

works) by implying such an equilibrium pricing condition – such equilibrium pricing condition, which solves the determinacy problem, is coterminuous to the solution to the forward equation defined by explosive threat under a parametrisation consistent with fiscal dominance and a boundary or terminal value condition on expectations on asymptotics of the economy

[8] The model is solved using a collocation method. See Appendix A of Shmidt’s article for an insight into the underlying iterative solution method.

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{h_t^{1+\phi}}{1+\phi} \right) \right\} \tag{1}$$

$\mathbb{E}_0$ is the expectations operator conditional to the information set at time zero, β denotes the subjective intertemporal discount factor, c_t denotes the consumption, and h_t is the amount of hours worked. The factor σ is the coefficient of the relative risk aversion and ϕ is the inverse of the Frish elasticity. Lowercase letters denote individual quantities and uppercase letters denote aggregate quantities. The utility function is a standard concave, twice continuously differentiable function that satisfies the Inada conditions in c_t.

c_t represents a composite consumption good, described by the following C.E.S. (constant elasticity of substitution) function:

$$c_t = \left[\int_0^1 c_{tj}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{2}$$

where:

- c_{tj} denotes the individual good produced by the j-th firm at time t;
- θ indicates the constant elasticity of substitution among individual goods. If $\theta \to \infty$, then goods will be perfect substitutes, while if $\theta \to 0$, they will be perfect complements.

The household's consumption problem is divided into two stages:

1. given any level of consumption c_t, the household decides, minimizing expenditure, how to allocate it among the different goods c_{jt};
2. given the expenditure associated with each level of c_t, the household determines the total consumption amount c_t at time t (along with other optimal quantities of h_t, b_t).

The first stage addresses the following minimization problem:

$$\min_{c_{jt}} \int_0^1 p_{tj} c_{tj} dj \quad s.t. \quad c_t = \left[\int_0^1 c_{tj}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}. \tag{3}$$

Given the Lagrangian of the minimization problem (3), and rearranging the partial derivative $\frac{\partial L}{\partial c_{jt}}$, we obtain the demand for the individual good:

$$c_{jt} = \left(\frac{p_{jt}}{\psi_t} \right)^{-\theta} c_t. \tag{4}$$

Substituting (4) into (2), we can demonstrate that the multiplier ψ_t is an appropriate measure of the general price index of consumption goods. Therefore, we can define:

$$\psi_t = \left[\int_0^1 {p_{jt}}^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \equiv P_t \tag{5}$$

from which the demand for the individual good becomes:

$$c_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta} c_t. \tag{6}$$

Equation (6) tells us that the share of c_{jt} in the total demanded goods c_t is inversely related, through the elasticity of substitution, to the ratio between the price of the individual good p_{jt} and the general price index P_t.

The second stage consists of the following optimisation problem:

$$\begin{aligned} &\max_{c_t,h_t,b_t} \quad E_0\left\{\sum_{t=0}^{\infty}\beta^t\left(\frac{(c_t)^{1-\sigma}}{1-\sigma} \quad - \frac{(h_t)^{1+\phi}}{1+\phi}\right)\right\} \\ &\text{s.v.} \quad c_t + \frac{b_t}{P_t} = \frac{W_t h_t}{P_t} - T_t + (1+i_{t-1})\frac{b_{t-1}}{P_t} + \Pi_t \end{aligned} \tag{7}$$

The intertemporal optimization problem corresponds to maximizing the utility function (1), subject to the intertemporal real budget constraint[9].

- b_t is the amount of bonds issued by the government and held by households at time t, assumed a non-negative RV i.e. weakly above zero almost surely or with probability one (WP1). This financial asset lasts one period and at the beginning of the next period, it yields income equal to $(1+i_{t-1})$ (where i_{t-1} is the interest rate at time $t-1$), which can be used by the consumer to purchase new bonds or other consumption goods. The existence of bonds allows for consumption smoothing over time;

- W_t is the nominal wage;

- Π_t are the net real profits of all firms. Π_t appears in the households' budget constraint because we are assuming that this economy is a private ownership economy, so we are assuming that households own firms in the aggregate. See section 3.2 on firms for pinning down profits and implications under the closure we employ.

[9]The *No Ponzi game condition* must hold:

$$\lim_{T\to\infty} E_t(b_T) \geq 0, \quad \forall_t. \tag{8}$$

Note that in this form (with inequality) the No-Ponzi game condition is not actually imposing anything new/carrying any information by virtue of the fact that bonds are non-negative RVs, i.e. are weakly bounded below by zero with probability one.

Given the Lagrangian of the maximization problem (7), we obtain:

1. the first-order condition (F.O.C.) with respect to consumption $\frac{\partial L}{\partial c_t}$:

$$\Lambda_t = \beta^t (c_t)^{-\sigma}; \tag{9}$$

2. the F.O.C. with respect to labour $\frac{\partial L}{\partial h_t}$:

$$-\beta^t (h_t)^{\phi} + \Lambda_t \frac{W_t}{P_t} = 0; \tag{10}$$

3. the F.O.C. with respect to bonds $\frac{\partial L}{\partial b_t}$:

$$-\frac{\Lambda_t}{P_t} + (1+i_t) E_t \left(\frac{\Lambda_{t+1}}{P_{t+1}} \right) = 0; \tag{11}$$

From the F.O.C. in consumption (9) and in labour (10), we obtain the labour supply:

$$\frac{h_t^{\phi}}{c_t^{-\sigma}} = \frac{W_t}{P_t}. \tag{12}$$

Note that equation (12) equates the marginal rate of substitution between labor and consumption to the ratio of prices, which in this circumstance is the real wage $\frac{W_t}{P_t}$.

Considering the F.O.C. in bonds (11), that in consumption (9), and defining the real interest rate as:

$$R_t = (1+i_t) \frac{P_t}{P_{t+1}} \tag{13}$$

we arrive at the equation describing the dynamics of consumption c_t, the Euler equation:

$$E_t \left[\beta R_t \left(\frac{c_t}{c_{t+1}} \right)^{\sigma} \right] = 1. \tag{14}$$

3.2 Firms

Assume there is a continuum of sectors with unit mass over the interval $[0,1]$ and monopolistic or monopolistically competitive market structure. For each of this sector, there is a representative firm assumed to operate as a sectoral monopolist, and the continuum of sectors and firms are thus interchangeable. Each firm produces a single good using H_t as the only input. The production function of the j-th firm is assumed to be linear in labour:

$$Y_{jt} = H_t. \tag{15}$$

The function is homogeneous of degree one, hence displaying constant returns to scale. Assuming the technique is given, the firms:

1. Demand labor H_t in a competitive market with fully flexible wages.
2. Set the price of their produced good p_{jt}, taking into account the demand for the individual good (see equation 6, on the rapresentative agent assumption see 3.6).

In the economy with perfect price flexibility so that aggregate demand does matter (by construction of the price setting problem and symmetry), i.e. the neutral launchpad, the labor demand is of course derived from the following minimization problem:

$$\min_{H_t} \frac{W_t}{P_t} H_t \quad s.t. \quad Y_{jt} = H_t \tag{16}$$

which leads to the following equation upon resolution:

$$\frac{W_t}{P_t} = \varphi_t. \tag{17}$$

Equation (17) equates the real wage W_t/P_t to the real marginal cost φ_t. For all practical purposes the above launchpad does not apply and dynamics are – as standard – studied in the context of a short-run with frictions. Under our model of the looking Phillips curve discussed in the next section based on Calvo Pricing, in the short run nominal price rigidities occur and price re-setting is either directly unavailable to some firms or fails to converge, on the grounds of forward-looking optimal behaviour, to the price that would be setby a monopolistically competitive producer facing a zero probability of not being able to reset prices at any point in the future and hence solving the standard static optimisation problem. Under such circumstances, the aggregate price level is no longer necessarily consistent with market clearing; under our standard Keynesian closure, the equilibrium output (and hence, interchangeably due to linearity of the deterministic production function) and aggregate labour are those required to meet the aggregate demand determined by private and government consumption, while the flexible wage adjusts to clear the labour market a such demand. Finally, we assume household ownership of firms so that aggregate profits are redistributed in a lump-sum to households.[10]

3.3 Price Setting

Consider any of the ex-ante identical firms. Let $\Omega \equiv J$ and $S : J \times T \to \{0, 1\}$ be defined as:

$$s : (\omega, t) \to s(\omega, t) = s_t(\omega)$$

[10]Hence households real income in a period coincides with nominal aggregate demand $P_t Y_t$, as this is, under standard distributive condition, the sum of nominal wage income and nominal profits $P_t \Pi_t + w_t N_t = P_t(1 - w_t/P_t)Y_t + w_t Y_t = P_t Y_t$, both of which flow to the household.

where $s_t(\omega)$, $s_t : \Omega \to \{0,1\}$, is a random variable on Ω denoting whether (the same) randomly picked firm in period t is able to reset its price ($s = 1$) or not ($s = 0$). The process can be cast in discrete or continuous time (as a stochastic jump process) alike. Here we focus on the discrete time case. Because the firms are ex-ante identical, i.e., are assumed to draw from the same distribution of pricing-flexibility processes, we can just focus on a representative one. By assumption, the process is a countable collection or family of random variables, hence of measurable functions on the probability space obtained by equipping Ω with the appropriate sigma field (Borel-field). We can use the Lebesgue Measure and the associated properties to quantify the probability of facing any sequence of periods with inflexible pricing for the representative firm. This requires formalising further the relevant probability space and properties of the firm-level process.

Appropriate Filtered Measured Space and Densities for the Calvo-Pricing Process

The probability space on which the stochastic Calvo pricing process at the firm level is defined is then $(\Omega, \mathcal{F}, P)$. Curly $\mathcal{F}$ is the sigma-field limiting the filtration introduced below, and equivalently represents the smallest sigma-field with respect to which all RVs in the infinite sequence defining the stochastic Calvo Pricing process are measurable (the induced field). The measure P (or P_λ) is a distribution or probability measure on the realisations of the process — it has a simple expression, under standard assumptions in Calvo Models, in terms of the probability measure of the single RVs/period draws.

The latter probability measure will be given by the Lebesgue measure introduced earlier, which is the natural probability measure mapping Borel sets in the Borel field on $[0,1]$ equipping the space onto the $[0,1]$ interval. We further equip this measured space $(\Omega, \mathcal{F}, P)$ with a standard filtration with limit set the Borel field on Ω, consisting of the Borel-fields generated by finite-dimensional truncations of the process, i.e., a filtration $\mathcal{F} = \{\mathcal{F}_t\}_{t\geq 0}$, where $\mathcal{F}_t \equiv \sigma(\{s_r\}_{0\leq r\leq t}) \subset \mathcal{B}_t$ is the time-t information set. We assume that the process is non-anticipating or adapted to the filtration — i.e., future information available for $s > t$ is irrelevant to determining the state of the process at t. Technically, this implies that each RV forming the process s_t is measurable with respect to the information set at time t, i.e., $\mathbb{E}_t\{s_t\} = s_t$. This allows us to use standard operators on functions of the stochastic process, i.e., conditional expectation operators and their properties.

Let for all times t, $\{\Omega_0, \Omega_1\}$ be a partition of the set Ω between firms that adjust and firms that fail to readjust (a set from which everyone with full information, including us modelers and the firms, pick at random), where one of the two sets is, by convenience, open — say Ω_0. By construction of the partition and definition of a sigma-field (including the Borel Field), both sets are measurable when s_t is measurable with respect to $\mathcal{F}_t$, which is guaranteed under the standard non-anticipating/adapted process assumption.

Proof. Note that partition $\{\Omega_0, \Omega_1\}$ consists of the pullback sets of $s_t = 0$ and $s_t = 1$. Recall $\mathcal{F}_t$ is defined as the induced sigma field $\mathcal{F}_t \equiv \sigma(\{s_r\}_{0\leq r\leq t})$ under the non-anticipating assumption. By definition of the induced sigma-field, this is the smallest field with respect to which all random variables s_t forming the finite dimensional/truncated collection at t are measurable. By definition of measurability of $s_t : \Omega \to \{0,1\}$, the pullback sets $\{\Omega_0, \Omega_1\}$ must be in $\mathcal{F}_t$. Hence the measures $P(\Omega_i), i \in \{0,1\}$ are well-defined in the above space. The above is given by the Lebesgue measure (i.e., the intra-period relevant fraction of firms).

Finally, in line with Calvo Pricing, we specialize the measure so that the draws s_t are independent over time. Then:

$$P(s_t = i, s_t = j) = P(s_t = i)P(s_t = j) \quad \forall r, t \in T, \forall (i,j) \in \{0,1\} \times \{0,1\}$$

Hence, a sequence of pricing-inflexibility outcomes $(0,0,0,\dots)$ has probability:

$$P(0,0,0,\dots) = \lambda_0 \lambda_1 \lambda_2 \cdots = \lambda^t$$

The Flexible Price Limit or Neutral Launchpad and Rational Beliefs

How large can the set of firms constrained in the long run be? We have a positive answer on that thanks to measure-theoretic introduction of the Calvo Pricing rigidity. Where the second equality uses a stationary measure assumption implicit in all Calvo models with a constant success/failure probability. Clearly,

$$\lambda^t \to 0 \text{ as } t \to \infty$$

i.e., unless the set of firms facing inflexibility has stationary measure one, the event of getting stuck along a perpetual inflexibility sequence has measure zero and rational firms know this. In other words, all firms rationally expect full flexibility in the long run.

Forward-looking Phillips curve with Quadratic Loss Function

Consider now firm with price setting status – governed by the above process – allowing for resetting. The forward-looking firm will make its choice anticipating that it might end up stuck at the chosen price, in the face of changing fundamentals and optimal pricing benchmarks, for some period of time. Under ownership assumptions and no principal-agent problem (e.g. less risk-averse managers under owner liability), we assume firms share the same discount factors as households. Under these circumstances, it is a well known results that firms will choose its current price strategy to minimize the expected present value of the loss function (i.e. maximize the expected present value of profits under suboptimal pricing) associated to sequences of pricing statuses.

To formally derive this and use this to introduce a forward-looking or NK Phillips Curve, we assume thus that the firm thus chooses $p \in \mathbb{R}$ (note no restriction on price updating relative to the last period price prior to re-setting) to solve (note the loss already includes a negative sign, hence we must be maximizing)

$$\max_{p_t \in \mathbb{R}^+} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^{t+j} \mathcal{L}(s_{t+j})$$

where the loss function $\mathcal{L}(s(\omega, t))$ is a random variable / a function of the stochastic process, depending on the value attained by the Calvo pricing process at time t when the loss is recorded.

The loss function is given by:

$$\mathcal{L}(s(\omega, t+j)) = \mathcal{L}(s(\omega, j)) = \begin{cases} -\frac{1}{2}h(p - p^*_{t+j})^2 & \text{for } s(t+j) = 0 \\ 0 & \text{for } s(t+j) = 1 \end{cases}$$

The firm maximizes the following:

$$\max_{p_t \in \mathbb{R}^+} -h\frac{1}{2}\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\lambda)^{t+j} (p_t - p^*_{t+j})^2$$

For the above to be bounded and the problem well-defined, we require that the scaled discount factor $\beta\lambda$ lies below one.[11] Then

$$\beta\lambda < 1 \quad \Rightarrow \quad \lambda < \frac{1}{\beta}$$

The first-order condition of the problem is as follows:

$$-2h\frac{1}{2}\sum_{j=0}^{\infty} (\beta\lambda)^{t+j} (p_t - p^*_{t+j}) = 0$$

$$\sum_{j=1}^{\infty} (\beta\lambda)^{t+j} p_t = \mathbb{E}_t \sum_{j=0}^{\infty} (\beta\lambda)^{t+j} p^*_{t+j}$$

Using the measurability of p_t with respect to the information set at t and the Neumann Series Lemma (NSL):

$$\frac{1}{1-\beta\lambda} p_t = \mathbb{E}_t \sum_{j=0}^{\infty} (\beta\lambda)^{t+j} p^*_{t+j}$$

[11] Note the assumption is not actually restrictive relative to admissible distributions of firm-level rigidities, under standard assumptions on the discount factor used to ensure boundedness of the solution to the household problem, and the fact λ is bounded within $[0,1]$ by definition of probability measure.

$$p_t = (1-\beta\lambda)\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\lambda)^{t+j} p^*_{t+j}$$

Note that the above corresponds to the bounded forward solution obtained by integrating forward the recursive difference equation — a standard technique in solving for the path traversed by jump-variables in terms of recursive updating of the condition associated to the explosive blocks of linear rational expectation models admitting a bounded solution (cf. Cochrane, 2023):

$$p_t = \beta\lambda\mathbb{E}_t p_{t+1} + (1-\beta\lambda)p^*_t$$

Together with a transversality condition or boundary condition ruling out explosive price paths, i.e., such that

$$\lim_{k\to\infty} (\beta\lambda)^k \mathbb{E}_t p_{t+k} = 0$$

The interpretation of the above equation — analogous to the Forward-Looking Phillips Curve — is that the pricing decision is forward-looking and displays stickiness, i.e., prices do not adjust to the optimal markup over marginal costs due to the fact that they partly respond to rational expectations on what future optimal resetting behavior would call for, precisely because this is barred from being enacted for a firm moving along a sequence of inflexibility in its own price schedule.

Define the general price level aggregator for the period:

$$p_t = \lambda p_{t-1} + (1-\lambda)p_t$$

Inverting the law of motion implies an aggregate consistency condition:

$$p_t = \frac{p_t - \lambda p_{t-1}}{1-\lambda}$$

Taking conditional expectation, using the measurability of the conditional expectation random variable, and substituting the above in the first-order condition. Using the observation that the optimal price solution consists of the forward-solution of the given difference equation, and then substituting in the above aggregate consistency condition:

$$p_t = (1-\beta\lambda)\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\lambda)^{t+j} p^*_{t+j}$$

$$p_t = \beta\lambda\mathbb{E}_t p_{t+1} + (1-\beta\lambda)p^*_t$$

$$p_t - \lambda p_{t-1} = \beta\lambda\mathbb{E}_t (p_{t+1} - \lambda p_t) + (1-\lambda)(1-\beta\lambda)p^*_t$$

Note that:

$$p_t - \lambda p_{t-1} = \lambda \pi_t + (1-\lambda) p_t$$

where π_t is the inflation rate. Thus:

$$\lambda \pi_t = \beta \lambda \mathbb{E}_t \left(\lambda \pi_{t+1} + (1-\lambda) p_{t+1} \right) + (1-\lambda)(1-\beta\lambda) p_t^* - (1-\lambda) p_t$$

After some algebraic manipulation, we obtain:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-\lambda)(1-\beta\lambda)}{\lambda} (p_t^* - p_t)$$

Note that:

$$p_t^* - p_t = \ln\left(\frac{P_t^*}{P_t}\right) = \ln\left(\frac{P_t^* \bar{P}}{P_t \bar{P}}\right) = \tilde{p}_t^* - \tilde{p}_t$$

So that the difference between the logarithm of the prices has a comparable expression in terms of the difference log-deviations from the stationary price level.The difference of the log-deviations of the optimal price and the prevailing price from the price along the stationary equilibrium path can in turn be written, by a first order perturbation of the optimal pricing or markup condition in levels for the firm that gets to reset the price (letting $\tilde{x}$ denote the log deviation from the steady-state value as usual), as:

$$P_t^* = \mu Q_t$$

$$\tilde{p}_t^* = \tilde{q}_t$$ [12]

By the definition of the optimal pricing schedule along the stationary equilibrium path:

$$\tilde{p}_t^* = \tilde{q}_t$$

By virtue of our argument, in a sufficiently small neighbourhood of the stationary equilibrium path where a first-order or linear perturbation offers an accurate approximation to nonlinearity, the rightmost term in the curve can be written as:

$$p_t^* - p_t = \tilde{p}_t^* - \tilde{p}_t = \tilde{q}_t - \tilde{p}_t.$$

This can be interpreted, around the steady state, as a measure of real marginal costs faced by the firm.

Closing the forward-looking Phillips curve model requires a way of pinning down the above marginal costs, since as we've seen these approximating up to a first

[12] Economically, the interpretation is that with stationary substitution parameter or preference for variety, so that the optimal markup factor μ is constant, deviations of the optimal price (reset by the optimal firm or implied at the equilibrium output level via firms' supply curve) from the steady state one react one-to-one to changes in the marginal costs of production.

order perturbation the key differential that $p^* - p$ forward-looking firms target with their price-resetting decisions when rationally anticipating firm-level price rigidity. The closure comes from economic rather than mathematical arguments, and lies at the heart of the notion of aggregate demand having macroeconomic consequences in NK models with price stickiness as opposed to flexible price models.[13]. In particular, since price updating, and hence period price level in this model do not converge, as the derivation of the forward-looking pricing tell us, to period optimal prices but partially respond to forecasts of future optimal prices,[14] then in general there is by construction no guarantee that aggregate demand and supply at the prevailing price will intersect at a market clearing equilibrium.

What happens at such disequilibrium point, which is key both for micro-foundations of the models and to pinning down the marginal costs faced by the firms and the Phillips Curve, we recognise to be a matter of debate: depending on what side aggregate supply decisions at the prevailing ”sticky” price fall relative to the projection of the price onto the demand space through the inverse demand curve, we might face ”rationing” or ”over-filled shelves.”

A standard economic assumption implicit in New Keynesian models of the forward-looking Phillips Curve to pin down such real marginal costs is, however, that at the prevailing sticky prices firms instead expand or contract output to meet, at possible sub-optimal behaviour, the aggregate demand schedule. Technically, this means that the marginal cost function in the Phillips Curve is evaluated at period aggregate demand. We further assume reception of signals from ”rationing” and ”over-filled shelves” and adjustments in response to them to be instantaneous or in logical time.[15] This assumption of demand-side determined output decisions, on the back of dispelling disequilibrium phenomena emerging on the back of price stickiness, is how aggregate demand comes to matter and cojointly pins down

[13]To be sure, this does not include only RBC-type models with perfectly competitive/price-taking firms in which prices are always flexible enough to ensure the economy lies along its (stochastic) potential output curve and absorb firm level output. It also includes NK models without nominal rigidities as a source of short-run macroeconomic frictions, the so called launchpad economy or long-run neutral benchmark (cf. Cochrane, 2023). In such cases as well, the presence of monopolistic firms (i.e. rationally believing they face a downward sloping demand curve) is not enough on its own to guarantee that aggregate demand bites on macro dynamics – this is because, with fully flexible price setting at the firm level and rational forecasts of the inverse demand curve, market clearing at potential output is always encoded (as a constraint) in the firm's pricing solution and ensured by letting the optimal markup update accordingly. The markup condition, in other words, tells us precisely that aggregate demand (hence equilibrium aggregate demand) always matches aggregate supply decisions as it is the one induced by optimal prices consistent with the projection of the chosen aggregate supply level on the inverse demand curve computed by the firm. Hence short run price inflexibility, rather than the market structure in the goods market, is the source of Keynesianisms, i.e. the demand-side nature of the analysis proposed in the model

[14]That is, they cannot fully slide along the demand curve to meet aggregate supply decisions.

[15]For a critique of this type of assumptions and the discussion of real-time alternatives for Aggregate-Demand signal-reception and output-adjustment in Keynesian models, see the monograph by Marglin (2021).

prevailing marginal costs. Thus, the marginal costs are those implied by the goods market-clearing condition $C_t + G_t = Y_t$. In particular, since the only real marginal cost faced by firms is the real wage rate and there are no real wage rigidities (real wages are flexible), then we can use the household labour supply condition to pin down the marginal costs in terms of the marginal value from an extra unit of leisure attained by the household evaluated at the level of labour employed to meet aggregate demand Y_t.

With the above observation, we can find an expression for these real marginal costs based on the household labor supply condition, appropriately log-linearized around the steady state, by inverting the labor supply curve to yield the equilibrium log-deviation of the real wage:

$$\tilde{q}_t - \tilde{p}_t = \tilde{w}_t = \sigma \tilde{c}_t + \phi \tilde{h}_t$$

The Forward-Looking, New Keynesian Phillips Curve follows:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}(\sigma \tilde{c}_t + \phi \tilde{h}_t)$$

$$\kappa = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}$$

And under our closure of the model:[16]

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa(\sigma \tilde{c}_t + \phi \tilde{y}_t)$$

3.4 Central Bank

The policy tool of the central bank is represented by the nominal interest rate i_t, which for simplicity will be equal to the interest rate on bonds. The central bank implements the following policy rule:

$$\frac{1+i_t}{1+\rho} = \left(\frac{1+\pi_t}{1+\pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y^*}\right)^{\phi_y}$$

where ϕ_π, ϕ_y are the weights assigned respectively to the inflation, defined as $(1+\pi_t) = \frac{P_t}{P_{t-1}}$ and output. π^* and Y^* are respectivily the long run values for inflation and output.

3.5 Government

The Government, like households, is characterized by a real budget constraint, described by the equation:

$$\frac{B_t}{P_t} = (1+i_{t-1})\frac{B_{t-1}}{P_t} + G_t - T_t. \tag{18}$$

[16]See Appendix 7.1. for a discussion of the issue of core indeterminacy

where B_t is subjected to an inequality constraint such that:

$$B_t \leq B_{max} \tag{19}$$

So there is an occasionally inflexible passive policies which take the form of an occasionally binding constraint on the amount of nominal debt consistent with passive behaviour by the monetary authority. For convenience, we define:

$$A_{t-1} = (1 + i_{t-1}) \frac{B_{t-1}}{P_{t-1}}. \tag{20}$$

Government spending G_t is exogenous, unproductive, and follows an exponentially decaying process. The policy tool of the government is represented by the tax rate τ_t, which will be equal to T_t due to the non-distorsive nature of taxation (lump-sum tax assumption). τ_t follows this dynamics:

$$\tau_t = \tau^* + \phi_\tau (A_{t-1} - A^*). \tag{21}$$

3.6 Sequence-Space Rational Expectations Equilibrium

A rational expectations, competitive equilibrium in the model in the space of sequences, for the economy defined in the previous sections is defined as a vector of prices $\{P_t, i_t, W_t\}$, inflation rates $\{\pi_t\}$ aggregate allocations $\{C_t, Y_t, H_t, B_t, \tau_t, \varphi_t\}$, collectively referred to as aggregates, individual allocations by the household $\{c_t, h_t, b_t\}$ and a process for the exogenous state variables $\{G_t, u_t\}$, and corresponding beliefs on the sequence of aggregates, such that:

- Given the forecasts or beliefs on the sequences of aggregates, households are optimising relative to the choice of consumption, bond-holding sequences, and labour supplies.
- Given their beliefs on the sequences of aggregates, forward-looking firms set prices optimally, i.e. the forward-looking Phillips Curve holds, and demand labour to meet aggregate demand.
- The government follows a fiscal policy rule consistent with the FD/MD device.
- The central bank follows a (passive/active) monetary policy rule.
- Goods and Labour Markets clear, implying the bonds market clears as well (see below).
- Beliefs are rational, so that forecasts on aggregates coincide with the aggregate sequences realising ex post when agents act with such beliefs.
- Given the representative agent assumption, aggregate consistency must holds ex post, i.e. aggregate sequences in (market clearing) consumption, bonds, and labour supply match their household level counterpart

Market clearing requires that the output produced is also consumed (and under our closure, pins it down as mathching aggregate demand), the labour supply is equal to labour demand, and the bonds supply is equal to bonds demand:

$$Y_t = C_t + G_t \tag{22}$$

$$\frac{H_t^{\phi}}{C_t^{-\sigma}} = \varphi_t \tag{23}$$

$$\frac{B_t}{P_t} = (1 + i_{t-1})\frac{B_{t-1}}{P_t} + G_t - T_t. \tag{24}$$

Note that by Walras' Law closure of the bonds market is implied by the government budget constraint alone (which is thus to be read as an equilibrium condition pinning down prices given aggregates), without the need to include the redundant budget constraint for the household to pin down bond demand, under the first two market clearing equations.[17]

The rational expectations and aggregate consistency conditions effectively allows us to treat (ex post, i.e. after individual level problems have been solved) forecasts on aggregates and individual choices as the same stochastic sequences.

4 A Fiscal Dominance Device with and without perfectly flexible passive monetary policy

We now proceed to outline the joint model of fiscal and monetary policy we employ to generalise the benchmark economy with alternative MD and FD regimes. These are construed in order to accommodate, respectively, a New-Keynesian (NK) and Fiscal Theory (FTPL) mechanism for price level and inflation path determination. In particular, this is achieved by means of implied restrictions on the admissible action sets by the monetary and fiscal regulators when choosing the conduit of the respective policy through the applicable instrument, in the appropriate sense.

The benchmark economy we have modelled is a New Keynesian economy characterised by core indeterminacy; that is, such that, without a specification of the monetary and fiscal policy blocks such to provide necessary boundary conditions, the equilibrium price and inflation path is indeterminate. Against this well-known indeterminacy of the core, which is discussed in 4.1.1, we introduce a mechanism-design characterisation of monetary dominance or fiscal dominance as ex-ante alternative, second-order devices through which the economy subsequently addresses the equilibrium-picking or equilibrium-trimming problem once initialised

[17]And noting, to derive from the above the budget constraint on the household and hence bonds market clearing, that profits also flow to the household under ownership assumptions, to the effect that the households nominal income from factor payments is as noted $P_t Y_t$.

at time 0. In constructing MD and FD as essentially alternative second-order ex-ante devices to address core indeterminacy, we hence not only depart from the traditional mootness of NK theories on such second-order problem, but also from the analysis of monetary policy and appropriate monetary objectives stressed in recent contributions to the FTPL and fiscal dominance models in particular. Specifically, as a methodological point, we highlight how consistently viewing MD with "passive" fiscal policy and FD with "passive" monetary policy as alternative solutions to an *indeterminacy* rather than instability problem affecting equilibrium inflation paths, some classical concerns with inconsistencies and trade-offs between fiscal dominance and traditional inflation-targeting by the central bank that have come to define much of the research agenda on optimal policy in FD (cf. Leeper, 2005; Kumhof et al, 2010), are easily exposed as fundamentally misguided by an inccorrect rationalisation of MD (or FD) solutions and assessment of implications. The aggressive reactivity of nominal rates prohibited under FD regimes is not motivated nor functions at any rate, in a NK economy with a forward looking Phillips Curve, to stabilise the inflation path – on the contrary, it purposefully threatens instability to induce a unique equilibrium path. Passive monetary policies falling short of the celebrated Taylor rule are perfectly consistent (in fact, are required) to ensure a stable inflation path by tying in inflation expectations.

It follows that, in an internally-consistent mechanism-design view we propose of FD with passive monetary policy as an alternative device for addressing equilibrium indeterminacy, expected inflation-targeting or stabilisation are by construction objectives perfectly coherent with fiscally dominant regimes. As such, by virtue of the framework we employ, considerations on the inconsistency of inflation targeting objectives with FD animating some well known contributions are outright discarded as partially misguided. We instead draw attention, in our welfare simulations and analyses in section 5, to the fact that in FD models with occasional inflexible monetary policy, a more substantial inconsistency and departure from both monetary policy theory and policy guidance emerges when inflation-targeting and stabilisation objectives (through *passive* monetary policy) are pursued through more "aggressive/hawkish" rather than more "dovish" nominal rate hikes (to borrow the vernacular from Bianchi, 2008). Hence, while rejecting through our framework questions of inconsistency of inflation-targeting objectives of the sort raised by Kumhof et al (2010) as misplaced, we think similar concerns are half-right when it comes to the practical implications they bear for nominal rate policy; in particular, the undesirability, in FD with occasionally inflexible policy, of the aggressive nominal rate hikes generally adopted by modern monetary regulators. One of the achievements of the modelling we propose, as we see it, is thus to provides a DSGE-based justification and analysis of why this is the case, leaving aside misplaced concerns with the alleged infeasibility of inflation-targeting as an objective and instead correctly pinpointing the inefficiency or departure from optimality associated to attaining it through traditional "more aggressive is better" monetary rules.

The specialisation of the model to the FD as an ex ante device for equlibrium-piking will serve, naturally, as a benchmark against which we study an economy with insubordinate or occasionally inflexible "passive" monetary policy of central interest to the theoretical and applied concerns of the paper.[18]

In 4.2. we outline the solution techniques employed and associated computational procedure for simulation of the benchmark FD economies with and without occasionally inflexible monetary policy. This draws in particular from the recent literature on piece-wise root-finding algorithms for obtaining sequence-space solutions of models with occasionally binding constraints (Occ Bin) giving rise to non-differentiabilities and nonlinearities making a perturbation, state-space approach infeasible.

Building on that, section 4.3. outlines key aspects of economic dynamics in the modelled economy in response to various sources of orthogonal shocks or "economic news" of interest; we consider in particular financial or IS curve shocks, and fiscal policy (construed as mildly persistent deficit-funded government expenditure) shocks.[19] We also consider policy transmission in the form of orthogonal policy shock in response to contemporaneous structural ones, in particular the transmission of traditional stabilisation programmes organised around a contractionary (expansionary) fiscal policy impulse in response to a negative (positive) "financial" shock to the IS curve

4.1 A Mechanism-Design View of MD and FD as 2nd Order Solutions for Core Indeterminacy in NK Models

4.1.1 Prelude to the MD and FD Devices: Core Indeterminacy

Consider a re-arranged version of our forward-looking Phillips curve modelling the dynamics of current prices and inflation. Note the way we read the curve is not changed, i.e. this is not saying that expected inflation responds or is caused to current inflation. In the forward-looking model we consider, it is the other way around, with current inflation responding to inflation expectations on the grounds of (forecast) price stickiness. In this form, the curve is best seen as defining a locus of points in the space of inflation sequences.

$$E_t\hat{\pi}_{t+1} = \frac{1}{\beta}\pi_t - \frac{1}{\beta}\kappa(\sigma\hat{c}_t + \phi\hat{y}_t) = \frac{1}{\beta}\pi_t - \frac{1}{\beta}\kappa(\sigma + \phi)\hat{c}_t$$

[18]To clear away frequent misnomers, the sense in which we ascribe notions of activity and passivity to the sub-components of the monetary-fiscal policy block, as well as the relationship of "fiscal activity" in FD economies with FTPLs á-la-Cochrane, are also concomitantly made clear. This is the topic addressed by 4.1.

[19]monetary and inflation news... (?). (**Comment:** Monetary policy shock in response to inflation news? Not too sure, see debate on the interpretation of this kind of naive excercise in NK models by Cochrane)

The second equality uses the aggregate demand definition at market clearing, neglecting for simplicity the role of (exogenous) government expenditure, which is irrelevant to our present point. Using the equilibrium dynamic IS to substitute out consumption, and neglecting dependence on expectations of future consumption flows again for simplicity, we obtain the recursion:

$$E_t\hat{\pi}_{t+1} = \frac{1}{\beta}\pi_t + \frac{1}{\beta}\kappa(\sigma+\phi)\frac{1}{\sigma}i_t - \frac{1}{\beta}\kappa(\sigma+\phi)E_t\hat{\pi}_{t+1} - \frac{1}{\beta}\kappa(\sigma+\phi)\rho$$

Note, inter alia, the usual pro-cyclicality of nominal rates and expected inflation along equilibrium sequences – this is the well known trade-off between reducing unexpected inflation news or output news through nominal rate rises and reducing inflation and expansionary output expectations characterising the NK Phillips curve when correctly interpreted. Re-arraging the equation, we obtain

$$E_t\hat{\pi}_{t+1} = \frac{1}{\beta+\kappa(\sigma+\phi)}\pi_t + ...$$

The point is seen most easily for values of the discount factor close to one, but is generalised (cf. Cochrane, 2023). In particular, with

$$E_t\hat{\pi}_{t+1} = \frac{1}{1+\kappa(\sigma+\phi)}\pi_t + ...$$

it is clear that a sequence of equilibrium inflation rates satisfying the forward-looking Phillips curve is such to define a stable auto-regressive process in the inflation rate. In other words, even without having yet discussed any aspect of monetary policy or fiscal policy formation – to the effect that the fiscal side is moot and the nominal rate a peg, the inflation path is stable, and never risks spiralling away on the grounds of past high equilibrium inflation. The absence of the Old Keynesian instability concern – here cashed out in terms of the stability of the process to the origin – is naturally a consequence of the fact that inflation formation does not respond to past inflation rates (as might be the case under old-Keynesian adaptive expectations).

By the same coin, if price instability is not a concern, determinacy is under the moot monetary and fiscal policy side we are presently assuming. In particular, as Cochrane (2023) puts it, while expected inflation is tied down, no information is available from the Phillips Curve to pin down where around such stable average it ends up batting. Another way of seeing this problem, through Blanchard-Khan conditions, is that – since in the above univariate AR(1) process the unique eigen-value of the transition matrix or root of the process is trivially the coefficient on the auto-regressive part, such eigen-value is within the unit circle, implying the benchmark economy displays too much stability. We lack (ut nunc) enough explosive eigenvalues to pin down/solve for the inflation path as a determinate, bounded sequence traversed by a jump variable.

This problem we refer to as *Core Indeterminacy* of the NK economy. It is clear from this explosive-root provision perspective that, insofar as both monetary dominance (view Taylor rule based threats of price explosions) and fiscal dominance (via a sufficiently non-reactive taxation policy and threats of real debt explosion) are able to provide the extra explosive root needed to solve for the determinate inflation paths on the above lines, the problem faced at the origin is a non-trivial mechanism design one: i.e. how we choose who gets to subsequently pick the equilibrium, and with what implications for macroeconomic transmission and policy design.

4.1.2 The Mechanism-Design View

The unified view of MD and FD as alternative specialisations of a second-order decision-mechanism consistent with subsequent selection of a determinate equilibrium path that we articulate below has a number of benefits. First, it rationalises from a theoretical economic viewpoint numerical calibration available procedures based on targeting determinacy regions over the fiscal and monetary policy parameter space (Kumhof et al, 2010). While adopted in the FD modelling space, the economic intuition and theory behind this have been somewhat obscure and geared to the calibrations specific to the undertaken modelling exercise (Ibid.). Here, the proposed model purports to offer a generalisation of use not only to the undertaken modelling exercise and extensions to other types of rules or NK economies, but also to the broader theory of FD models and how they mirror the MD device implicitly present, but seldom explicitly recognised, in the vernacular of standard NK models. Second, it makes explicit the fact that such selection mechanism is also present, albeit latent, in well-defined NK economies – in principle, such models are solved with an assumption of MD as the solution to the second-order selection problem, but this need not be in principle the case.

Conceptually, the fundamental idea we propose and exploit, generalising from the theoretical viewpoint a parametrisation procedure disovered in Kummhof et al. (2010) and linking it to the seminal analysis in Cochrane (2023), is that Monetary Dominance and Fiscal Dominance constitute alternative *equilbrium trimming or pruning devices* to deal with the core property of indeterminacy of the equilibrium price level and inflation paths in the NK economy. Viewed from such mechanism design angle, we subscribe to the view that MD and FD regimes (and ultimately the FD regime with occasional inflexibility we set to study) are best conceptualised and modelled as solutions to the second-order, mechanism design problem, that is *ex-ante second-order selection devices* (targetable by optimisation protocols over a first-order, time 0 institutional design case). Essentially, by means of pre-allocating admissible action sets (i.e. sets defining feasible instrument updates, such an nominal rate policy), they determine who gets to subsequently (ex-post) pick which equilibrium path the economy.

Why is such a second order-selection device necessary, and how do consequently MD and FD operate as alternative solutions to the design problem? In other words, how do they represent alternative solutions for equilibrium-picking or trimming in the benchmark indeterminate NK economy? The idea is that, conditional on such second-order (possibly suboptimal) design choice being made at the time origin, the behavioural rule of the dominant agent is such to induce a specific equilibrium path to prune-down to determinacy the infinite number of sequences consistent with the equilibrium definition, while the non-dominant or passive agent behaves in such a way that the determinate equilibrium picked by the dominant one prevails. The case with insubordinate monetary policy or occasional monetary policy inflexibility we consider can be viewed as endogenising departures or relaxations of this passivity requirement. Hence, key in this modelling approach unifying the traditional MD/canonical NK and in the FD/FTPL case as alternative solutions to an underlying mechanism design-problem, is what kinds of behaviour are consistent with the above pruning exercise and based on what economic intuition. This is the first-order selection mechanism, i.e. conditionally on having chosen who gets to pick the equilibrium, it specifies how the equilibrium path is picked. As we formally show in the derivation specific to the FD case, mirroring a correct intepretation of Taylor rule principles in the MD/canonical NK case, the idea is simple.

The dominant agent in each regime adopts – up to further, quantitative optimisation choices considered in section 5 – behaviour, defined by the path traced by its individual instrument[20] such that, *if* the current price level or inflation rate[21] deviate from the unique equilibrium associated with a stable, bounded solution, *then* the economy enters an unstable spiral. In other words, conditionally on the second-order mechanism selection between MD and FD (and FD with occasionally inflexible policy), equilibrium prices and inflation rates at time t are time-t jump variables that adjust to a level consistent with ensuring stability of the equilibrium path and a bounded solution and the threat never actually is observed as being implemented. This is a technical rendering of the standard notion that in NK models with core indeterminacy, the Taylor rule – which in the generalised/unified case we address constitute the central bank behaviour under MD – does not purport to stabilise the economy as in Old Keynesian models with backward-looking Phillips Curves and no determinacy problems (cf. Cochrane, 2010). Rather, it purposefully acts a threat-mechanism to force convergence – assuming asymptotic instability to be undesirable – to the desired price and inflation outcomes. The model of the FD case exploits the same threat-based[22] first order selection mechanism familiar from

[20]These are the nominal rate for the monetary authority, and tax revenue for the government.

[21]Equivalently, by definition of measurable conditional expectations, the linear innovation against the conditionally expected value with respect to the previous period information set/unexpected inflation/linear inflation news. See Appendix 7.1

[22]To dispel immediately with a standard, incorrect criticism often brought to bear on the kind of explosivity threat conformable to the FD case, note, echoing points similar to those raised by Cochrane (2023) in his version of FTPL, that this threat does not ever require a threat to violate a budget constraint on the government part. The government real budget constraint, improperly defined so as it is in fact a market clearing or equlibrium equation, holds alike at on and off equilibrium path –

the canonic NK literature – unsurprisingly so, as the latter is actually an analysis of the economy with an MD solution to the second-order mechanism design problem.

4.1.3 Formal Framework and Specialisation to FD Solutions to the Mechanism Design-Problem

Our model of the monetary and fiscal policy processes generalised to equilibrium with MD and FD is thus as follows. Let $d \in 0, 1$ an indicator function taking value 0 when the second-order selection mechanism is monetary dominance and 1 for fiscal dominance. We assume throughout that the equilibrium sequence $\{d_t\}_{t \in T}$ is constant and this is known/measurable information with respect to the information set at the origin.[23]

Monetary and Fiscal policy are assumed to be jointly modelled by the nonlinear matrix measurement equation, with design matrix sequence $\Gamma(d_t)$:

$$\begin{bmatrix} (1+\rho)^{-1}(1+i_t) \\ \tau_t - \tau^* \end{bmatrix} = \Gamma(d_t) \begin{bmatrix} (1+\pi^*)^{-1}(1+\pi_t) \\ Y^{*-1} Y_t \\ L(A_t) - A^* \end{bmatrix},$$

$$\Gamma(d_t) = \begin{bmatrix} \phi_\pi(d_t) & \phi_y(d_t) & \phi_{L(A)}(d_t) \\ 0 & \psi_y(d_t) & \psi_{L(A)}(d_t) \end{bmatrix},$$

Where $A_t = (1+i_t)B_t/P_t$ is the extant level/outstanding gross real debt at time t, i.e. real debt inclusive on interest repayments. $L(A_t) = A_{t-1}$ is the lag operator $L^k(x_t) = x_{t-k}$ defined by the linear recursive equation $x_t = L^{-1}x_{t-1}$. A widespread assumption in fiscal policy is that fiscal policy updating against new information is more sluggish, hence the reaction of tax revenue (and nominal rates) to the lag of A_t (cf. Kumhof et al., 2010).

Other variables as standard: the first equation block corresponds to the matrix version of a Taylor Rule with destabilising monetary policy (with Monetary Dominance) or, with Fiscal Dominance, to pro-stabilisation policy conformable to both a single price stability mandate (with $\phi_y = 0$), e.g. European Central Bank (ECB), or a dual price and output-stability mandate ($\phi_y > 0$), e.g. US Federal Reserve (FED), or a dual price-stability and growth mandate, e.g. the Reserve Bank of India (RBI) ($\phi_y < 0$).[24] The second block of the matrix equation corresponds to a standard

hence independently of the threat mechanism. Insolvency (in the benchmark case) is never threatened. The threat refers to a *solvent* government moving along an unstable real debt spiral.

[23]Naturally, relaxing this assumption – which are to the effect of making the regime/selection-mechanism constant – can be relaxed to a more general exogenous regime switching model of the kind considered by Leeper.

[24]We could allow, in principle for the bank to also care about influencing the path of debt, on top of disallowing deviations above the occasional inflexibility point. For example, by considering a triple mandate with containment of the debt path, by means of introducing a negative response of the nominal rate to lagged debt. Similar augmentations to traditional dual or single stabilisation mandates is explored in Kumhof et al (2010). Albeit outside the scope of the paper, this extension and

tax-revenue rule with lagged adjustment.

We now discuss the choice of admissible parameter sets consistent with FD as the solution to the second-order, mechanism design problem. The case for MD is well known as implicit in the standard determinate solution to the NK model, and corresponds to the ”Taylor principle”, hence the focus on the FD case.

Recall that, conditional on the (possibly suboptimal) solution to the second-order mechanism design problem, the solution to the indeterminacy problem consists of the dominant agent acting, via update of its instrument according to a time-invariant rule, in order to threaten a (rationally expected) unstable spiral or explosion should the current price level and inflation rate fail to adjust to the picked equilibrium value. Technically, this is equivalent to the price level, and corresponding point traversed by the inflation path, updating to solve at all times t[25] the equation obtained by forward-integrating the stochastic difference equation in the policy instrument together with a boundary (or transversality) condition that ties down expectations on the long-run of the economy (this and its implications for reading price and inflation determination are made clearer below).

Trimming down the set of equilibrium sequences to determinacy is thus, in principle, achieved by a rule against explosions together with a credible threat, encoded in instrument update, that the economy will explode whenever prices or inflation rate fail to jump at the implicitly pinned down equilibrium values consistent with fending off explosions. For the MD (i.e. the canonical NK case), the relevant instrument is the nominal rate adhering to a Taylor-rule principle, which provides the necessary explosive root to solve the forward-looking Phillips Curve forward for a bounded solution. For the FD case, where we employ as outlined earlier a tax revenue rule as the policy tool, we consider as the implicit instrument the law of motion for real debt under the adopted tax revenue policy – technically, the one obtained by substituting the equilibrium tax revenue rule into the debt/bond supply condition viewed, recall, as an equlibrium market clearing condition, and not as a budget constraint.

As such, the strength or weakness of the response of tax revenue to lagged debt constitutes the critical feature of the behaviour of fiscal authority responsible under FD for providing (or failing to provide consistently with MD) the necessary explosive root or eigenvalue to solve the indeterminacy problem. In the modern fiscal policy

any interactions it might lead to with the occasionally inflexible stance are left to further work and analysis on the backbone of the endogenous regime switching model and welfare analyses for such economies of interest that we develop here.

[25] Hence the notion of ”jumping” from DSGE and more pertinently the state-space modelling and state-space solutions vernacular. Note that in fact the solution implied by the forward-integrated solution with its associated transversality condition – i.e. the price level and inflation rate prevailing along an equilibrium path supported by rational beliefs that the government will blow up the real debt – is the same as the one we could obtain by assuming equilibrium price dynamics are recursively generated as a function of state variables measurable in the information set, i.e. the jump or measurement variable in a state-space representation of the model.

and macroeconomic stabilisation literature with fiscal expenditure financed through bonds emissions and tax revenue as substitutes (cf. Schmitt-Grohé et al, 2005), the strength of the response of the fiscal instrument to (lagged) is effectively an inverse metric of the strength of the response of new debt emission to extant debt. As such, the strength of the response of the tax revenue can be structurally interpreted, in policy applications, as a metric for the volatility of debt relative to taxes or, equivalently, a defining a convex-combination of revenue-funding (taxes today) and deficit-funding (taxes tomorrow) underlying budgetary rules. Offering a structural interpretation for determinacy cutoffs for FD as shown below, weaker responses of the tax instrument to lagged debt[26] are indicative of more deficit-based funding and volatile debt. It is, as an intuitive prelude for the forward bounded solution procedure adopted below, precisely a *weak enough* response of tax revenue to lagged debt – i.e. sufficient predominance of deficit-funding in the budget and hence debt volatility – that provides the explosive root allowing to solve for the equilibrium price level and inflation rate as the bounded solution to the forward-integrated law of motion for real debt.

Substituting the tax revenue rule from above into equation (18 or 2X), we obtain the following recursion in real government debt:[27]

$$\frac{B_t}{P_t} = (1+i_{t-1})\frac{B_{t-1}}{P_t} + G_t - \tau^* - \psi_{L(A)}(1+i_{t-1})\frac{B_{t-1}}{P_{t-1}} - \psi_{L(A)}(1+\rho)\frac{B^*}{P^*}.$$

$$\frac{B_t}{P_{t+1}}(1+\pi_{t+1}) = (1+i_{t-1})\Big(1-\psi_{L(A)}(1+\pi_t)\Big)\frac{B_{t-1}}{P_t} + G_t - Z^*, \qquad \textit{where}$$

$$Z^* = \tau^* + \psi_{L(A)}(1+\rho)\frac{B^*}{P^*}$$

The tax-revenue rule, along an equilibrium sequence, defines a law of motion for equilibrium real debt, given by the above equation. Suppose, further, that we are sufficiently close to the stationary equilibrium path (with the equilibrium picking device under FD hence valid only locally, but w.l.o.g. for our purposes since the model is subsequently studied using a perturbation around such steady state). Then, with the nominal rate and gross inflation rate (with asymptotic zero inflation equilibria) being approximated respectively by $i_t \approx i^* = \rho$ and $(1+\pi_t) \approx (1+\pi^*) = 1$ in such neighbourhood, and exploiting time-t measurability, the above stochastic difference equation in the instrument can be written as

$$\frac{B_t}{P_{t+1}} = (1+\rho)\Big(1-\psi_{L(A)}\Big)\frac{B_{t-1}}{P_t} + G_t - Z^*$$

[26]Note that our model of the fiscal policy block, since it pins down taxes based on lagged debt, implies that orthogonal fiscal expenditure or fiscal news g_t are initially fully deficit-funded.

[27]An equivalent procedure is available to obtain a difference equation and jumping-solution directly in the inflation rates rather than based on the definition of inflation and solution in terms of price levels. These are however equivalent.

$$\mathbb{E}_t \frac{B_t}{P_{t+1}} = (1+\rho)\left(1-\psi_{L(A)}\right)\mathbb{E}_t \frac{B_{t-1}}{P_t} + G_t - Z^*.$$

Under a weak enough linear response of the current real debt to the lagged real debt – corresponding to more hawkish tax revenue rule, see the parametrisation below, and hence to a lesser incidence of deficit-funding – the above law of motion is easily seen isomorphic to a stationary Markov process or a stationary and indeterministic autoregressive AR(1) process centered at Z^*. In other words, there is stability at the point Z^* (the centred origin) when fiscal rules are sufficiently passive to adjust taxation to always ensure that the real debt converges to such target no matter where the law of motion is initialised. This paradoxially more active, "mindful" adjusting of tax revenue is coterminous with the notion of passive fiscal policy: at the end of the day, fiscal rules are such to guarantee sufficient flexibility of the revenue so that a target real debt level prevails independently of where on the real debt space and orthogonal fiscal news the transitional dynamics are initialised. The stability at the (centred) origin of real debt under "passive fiscal policy" is consistent with a regime of monetary dominance: this is because, precisely because the convergence of the real debt process is guaranteed everywhere on the state space, then in principle any price level (since B_{t-1} is pre-determined at time t, being t-1 measurable) is consistent with eventual convergence to the steady state debt path $\{Z^*\}$. The reaction of fiscal revenue, prosaically, is strong enough so that whatever price and hence real debt happens to prevail now, a solvent, explosive debt path is never a threat, hence the price level can be initialised anywhere in line with price-stability objectives. Naturally, it will be picked by an appropriately calibrated Taylor rule.

Conversely, toward establishing and discussing an FD rather than MD device, assume the equation can be integrated and solved forward. This will be the case if the process has an unstable root or AR coefficient outside the unit circle. In particular, sufficient for this is the boundary – which we employ to parametrise fiscal dominance:

$$(1+\rho)(1-\psi_{L(A)}) \geq 1 \rightarrow \psi_{L(A)} \leq \frac{\rho}{1+\rho}$$

Under this assumption, we can rearrange the equilibrium law of motion for debt, letting $\xi = [(1+\rho)(1-\psi_{L(A)}]^{-1}$ with $\xi < 1$ under the above explosive-root condition characterising the FD device, as

$$\frac{B_{t-1}}{P_t} = \xi \left[\mathbb{E}_t \frac{B_t}{P_{t+1}} - G_t + Z^*\right],$$

and solve for real debt today as the realisation of a jumping variable pinned down by the right-hand side, by recursively iterating forward by substituting the next-period debt (in turn pinned down as a jumping variable etc), under the assumption rational expectations, to yield:

$$\frac{B_{t-1}}{P_t} = \lim_{s\to\infty}\left\{\mathbb{E}_t \xi^s \frac{B_{t+s}}{P_{t+s+1}}\right\} - \mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \left(G_{t+k} - Z^*\right)$$

Solving the above requires, naturally, a boundary condition on the time-t conditional expectations on long-run real debt. Imposing such boundary condition, together with the explosive behaviour threatened by FD such that terminal value conditions matter, lies at the core of FD and MD as equilibrium picking devices against NK indeterminacy. As standard, toward constructing the bounded solution, we disallow explosions by assuming prevailing rational expectations that the real-debt grows at most at a rate such that the limit equals zero. Note, as Cochrane (2011) suggests, that the requisite boundary condition on expectations under the FD device is in some sense more credible or empirically plausible than the corresponding requirement with an MD device if we take the question of hyper-inflations and rational expectations seriously. Specifically, the boundary condition here disallows *real* explosions, i.e. expected explosions in the real debt, as opposed to disallowing nominal explosions, i.e. expected explosions in the inflation rate, as in the MD device employed in canonic NK modelling. Subject to such boundary condition, *extant* the current value of real debt is then a jump variable determined and updating based on time-t measurable information, for all t. In particular:

$$\frac{B_{t-1}}{P_t} = -\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \left(G_{t+k} - Z^*\right)$$

The economics of equilbrium picking under the FD device and disallowance of real explosions can be summarised as follows, and we think it is best understood with reference to the passive fiscal policy case outlined earlier. Under sufficiently weak tax revenue reactivity, i.e. sufficiently strong dominance of deficit-funding, it won't be the case in general that a stationary level of real debt eventually prevails *independently* of where we initialise the economy over the real debt space – by predetermination/(t-1)-measurablity of the nominal debt extant at t, this is of course equivalent to initialisation over the current price space. Taxation adjusts too weakly for convergence to be guaranteed independently of initial conditions. Hence initial conditions matter, since for most price levels/initial real debts but one, the economy will embark on a (solvent), unstable debt spiral and agents rationally forecast this. If such spirals are to be ruled out, then real-debt must jump at the unique value consistent with fending off such explosions and their rational anticipation by agents.

Under the assumption[28] that the government fiscal expenditure process is stable – here encoded in the stationary AR(1) process model – we can solve analytically for the price level and inflation rate as a function of current state variables. The fiscal dominance model hence provides, as expected, a Fiscal Theory of the Price Level (cf.

[28]Necessary, under BK-Uhlig standard stablity assumptions on the law of motion for the exogenous block for a well-behaved solution to the DSGE system, cf. Uhlig, (1999)

Appendix). Note that government fiscal expenditure shocks enter with a negative sign, i.e. affect negatively the bounded solution. This is the mark of the FTPL at play with FD and essential to pinning down how the stream of surplus, under the weak response of taxation with and FD device, picks the innovation of price level and inflation rate against prevailing expectation: recalling that the extant nominal debt is pre-determined, this cannot be adjusting at time t. Hence the negative sign on fiscal expenditure reflects the fact that the adjustment (a reduction) in the real debt comes from price inflation. That is, expansionary fiscal news or contractionary shocks to the stream of surpluses are partly financed through current inflation. To fend off real explosions in debt, nominal debt gets inflated away, so that, exploiting this inflating-away mechanism, fiscal policy picks the determinate equilibrium price level and rate of inflation.

Table 1: MD vs FD/Policy Inflexibility

Parameters	**MD**	**FD/Policy Inflexibility**
ϕ_τ	$> \frac{\rho}{1+\rho}$	$\leq \frac{\rho}{1+\rho}$
ϕ_π	> 1	≤ 1

4.2 Solution method for FD with occasional inflexibility: OccBin algorithm in the Sequence Space

The presence of occasional inflexibility on the monetary policy side – entering in the form of weak inequality (19) on the debt position prevailing under fiscally dominant rules in the outlined model – introduces as noted an occasionally binding constraint in the model. It is well known that a models with occasionally bindings constraints cannot be solved with standard linear or higher-order perturbation methods. The technical difficulty with the analysis of endogenous regime-switching DSGE models is twofold. First, the model equations are not differentiable at the occasionally binding point, unless the constraint always binds (in which case the debt position is essentially a constant) or never binds (in which case, no occasional inflexibility is present by construction).. Models with occasionally bindings constraints can be seen as regimes switches models, with a ”normal times” and ”constrained times” regimes. In principle, hence perturbation methods could be applied to an augmented system *despite* a discrete, leading to standard state-space solution through undetermined coefficient methods, by modelling the regime-switching as governed by a Markov Chain (cf. Bianchi, 2013). This strategy offers a good methodological route when we can an assume, in the time series vernacular, that the regime state variable is *strongly exogeneous* in the model.[29] This assumption is obviously not appropriate

[29]Weak exogeneity does not suffice, as implicit in the solution of the system is a prediction or forecasting problem

in the present context; in this types of models, timing of transitions or transition probabilities ($\mathbf{P}$) – hence the forecast on the future state – are not exogenous, as in strictly defined regimes switches models, but depend on position of the economy.

Building on the sequence-space piece-wise linear rootfinding algorithms literature (cf. Guerrieri and Iacoviello, 2015), the simulation-based approach we have followed is to consider a deterministic version of the model. Consider the model presented in section 3, the log-linearized version based on a first-order perturbation around the steady state can be written in Sequence Space, as:

$$\mathbb{E}_t\{\mathbf{F}_x\mathbf{x} + \mathbf{F}_u\mathbf{u}\} = 0 \tag{25}$$

where $\mathbf{F}_x$ is the matrix of derivatives of the log-linearised system $\mathbf{f}(\cdot)$ w.r.t. $\mathbf{x}$ (evaluated at steady state) or more simply the Jacobian evaluated at the steady-state. Because the system is a linear form at the first order perturbation, this is simply the matrix collecting the coefficients on the log-deviations in the first-order Taylor Series expansion of the original, nonlinear equilibrium function around the steady state. As usual, $\mathbf{x} \equiv \{x_0, x_1, \ldots, x_t\}$ is the vector of the endogenous variables, $\mathbf{F}_u$ is the matrix of derivatives of $\mathbf{f}(\cdot)$ w.r.t. $\mathbf{u}$ (evaluated at steady state), $\mathbf{u} \equiv \{u_0, u_1, \ldots, u_t\}$ is the vector of the exogenous shock.

The sequence of shocks, in line with the perfect-foresight assumption in the sequence-space solution method is assumed to be consists of family of random variables jointly measurable with respect to the information set at the origin or – to the same effect of eliminating stochasticity/uncertainty and hence turning the stochastic-expectational difference equation system into a linear-algebraic problem – that the sequence $\mathbf{u} = \{u_t\}$ has a probability measure one conditional on the information set at the origin. To solve the above linear system we need to solve for the path of $\mathbf{x}$. Assuming the Jacobian $\mathbf{F}_x$ to be invertible, which means that $\mathbf{x}$ can be obtained as:

$$\mathbf{x} = -(\mathbf{F}_x)^{-1}\mathbf{F}_u\mathbf{u} \tag{26}$$

Let's jump into the main idea of the OccBin methods. The OccBin algorithm, for a given path of shocks $\mathbf{u}$, consists in guessing in which periods the constraint is binding. Given the guess, we need to find the path for $\mathbf{x}$ solving the linearized system:

$$\mathbf{F}_x\mathbf{x} = \mathbf{b} \rightarrow \mathbf{x} = (\mathbf{F}_x)^{-1}\mathbf{b} \tag{27}$$

where $\mathbf{b} \equiv -\mathbf{f}(\mathbf{x}, 0)$, and $B = B_{max}$ when the underlying regime is guessed to be "constrained times" regime while $B < B_{max}$ must hold, at the prevailing B, when the economy is guessed to have reverted to "normal times". The algorithm hence works by keeping the shocks path fixed at some typical sequence of interest, making it part of the information set at the origin, and providing an initial guess of the sequence of endogenously-switching regimes. Based on such iteration of the guess, it solves endogenously for the unknown regime sequence and associated equilibrium path through a guess and verify method. The verification condition to be met at convergence of the algorithm is slackness of the constraint, or reversion to normal times after the last period in which the constrained is guessed to bind. At any iteration of the regime sequence guess, the equilibrium path is solved for by means of a perturbation in the sequence space keeping the shock sequence at the initialised value, corresponding to the linear algebraic problem given above.

In principle, there are two ways of implementing the above linear-algebraic step in the OccBin algorithm. The first is to recognise that the imposition of the constraint results in an over-determined system, which can be solved through a least squares approach (allowing for residual diagnostics). The other approach is to guess that one equilibrium equation – generally one of the two policy rules – can be consistently suppressed (as redundant information) to accommodate the inflexibility point, using the unconstrained case as an inexact counterfactual scenario (e.g. the monetary rule in a ZLB model). This leads to an exact solution to a slightly different system. We follow the first approach; however, in the appendix we also solve through the second approach by suppressing the monetary policy rule (assuming that in the neighbourhood of the occasional inflexibility point, constraining the debt position takes priority over inflation targeting) – as we point out, the insights for optimal policy do not change.

4.3 Macroeconomic Transmission with and without Occasional Policy Inflexibility

We now present, comment, and rationalise the comparative results for the simulation based, sequence-space solutions across the three models: a Monetary Dominance device, a Fiscal Dominance device, and a Fiscal Dominance facing occasionally inflexible passive policy, and comment on how the occasional inflexibility produces emergent dynamics in the latter case. [30]. We considered two sources of shocks: structural shocks in the IS curve and expansionary fiscal policy shocks triggering an expansion in aggregate demand and, through the Phillips Curve, inflation. The reason we consider these shocks, in particular, is that they are especially likely to trigger occasional policy inflexibility insofar as they aggravate, under FD, the debt position: directly, as for the unfunded shock (since tax revenue has lagged reaction) to public expenditure, or indirectly (for both shocks) via the implied reaction of nominal rates under an inflation targeting (or dual) mandate.

As a sanity check on the underlying unconstrained MD and FD regimes[31] forming the basis of the comparisons, these replicate standard behaviour respectively found in the literature on MD or RANK economies and on FD economies with perfectly flexible passive monetary policy at comparable parametrisations. Following either shocks, the inflation-targeting monetary rule reacts by raising nominal

[30] Code for the state space solutions for the former two on DYNARE is also available.

[31] And hence on the slack phase of the model with occasional inflexibility on which the solution for the tight phase is based

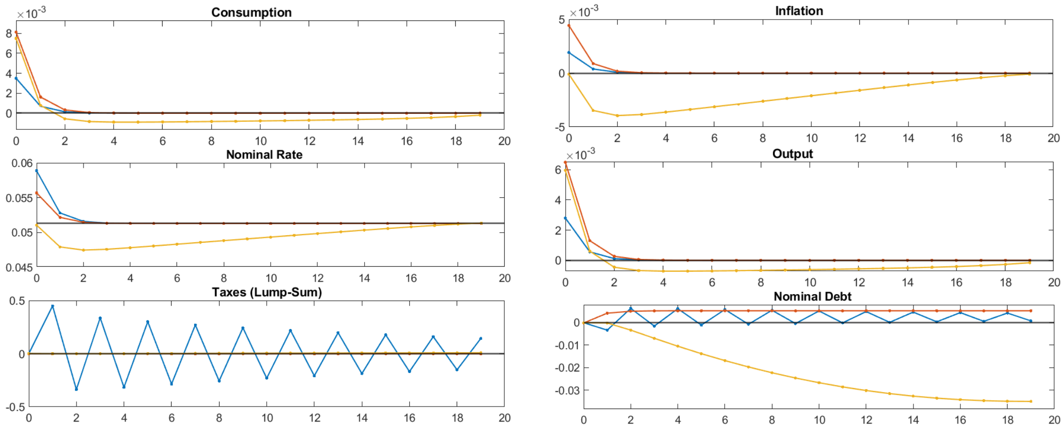


Figure 1: IRF to a positive IS shock (positive shock to β). The blue line represents the MD case, the red one the FD case and the yellow one the case of policy inflexibility. ϕ_π and ϕ_τ for FD and PI cases, are at their upper boundaries values (7). In MD case $\phi_\pi = 2.5$, $\phi_y = 1.0$ and $\phi_\tau = 2.0$.

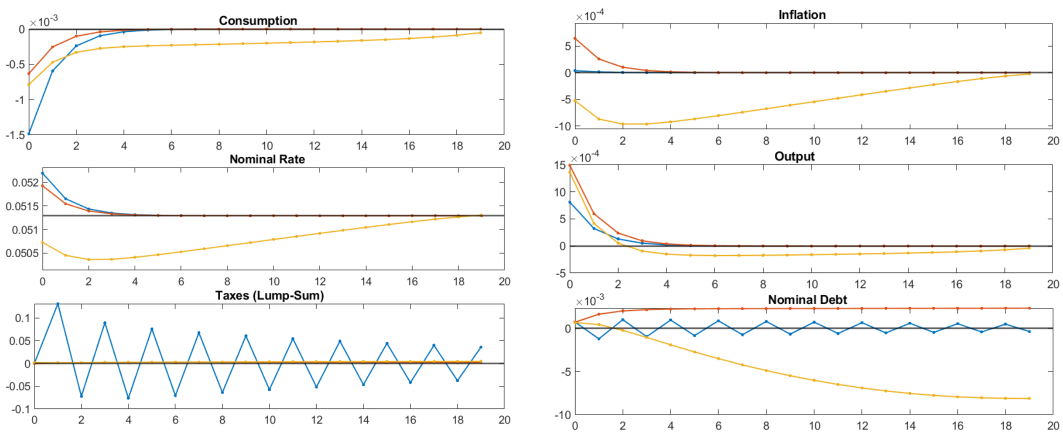


Figure 2: IRF to a positive AD shock (positive shock to G_t). The blue line represents the MD case, the red one the FD case and the yellow one the case of policy inflexibility.ϕ_π and ϕ_τ for FD and PI cases, are at their upper boundaries values (7). In MD case $\phi_\pi = 2.5$, $\phi_y = 1.0$ and $\phi_\tau = 2.0$.

rates, which – given consistently formed inflation expectations – in turn discourages consumption to smooth the inflationary shock by spreading it over the following periods through the standard Ricardian substitution effect along the equilibrium IS curve. The mechanism, as expected, delivers a stronger or more persistent inflation in the unconstrained FD than in the MD case. This is due to two reasons. First, by construction of the FD and MD devices, the feasible reaction of monetary policy to inflation is weaker under FD than MD, resulting in higher expected inflation under FD than under MD. Second, in the unconstrained FD case the innovation against expected or average inflation is determined by the fiscal side as opposed to by explosive monetary policy. In particular, combinations of higher beginning of period nominal debt, occurring for both set of shocks, and unchanged long-run revenue (IS shock case) or reduced long-run revenue (expansionary fiscal policy case) imply as shown an inflationary pressure on the ”jumping” price level. Because nominal rates must respond positively under inflation targeting, this results in a pair of higher time zero inflation and post-time zero expected (and realised) inflation than in the MD case.

When we move to occasionally inflexibility case, a reasonable guess on the impact of the added friction might be that the resulting dynamics will behave as a convex combination of dynamics in the FD and MD case. After all, we might think about the monetary authority retaking, albeit temporarily, control of the debt position as the economy approaches the inflexibility point from below. We found it interesting that this is not the case: in particular, after a brief phase of *convexity*, transitional dynamics display emergent properties relative to the two benchmark models.[32] In the spirit of dissecting such non-convexity, two emergent features, in particular, we think merit the reader's consideration in an analysis of the nonlinear interactions in force.

First, dynamics in the FD case are much more sluggish or dragged-out than under the unconstrained operation of the alternative FD and MD mechanisms. Even as the endogenous regime reverts to normal times after the occasional inflexibility initially kicks in, the economy takes longer to return or resettle on the stationary equilibrium path. This, naturally, implies the FD economy with occasionally inflexible policy displays higher volatility in response to typical shocks than the MD and FD benchmarks. There are two upshots from this. First, that the friction or non-linearity introduced by the occasional policy inflexibility is a cause for *endogenous* volatility or *macroeconomic risk*– in the DSGE vernacular – offers an amplification, fully endogenously triggered mechanism mediating the transmission of exogenous demand-side shocks to aggregates. Second, that such endogenous volatility makes comparable levels of macroeconomic stabilisation more difficult to achieve in the case woth occasional inflexibility than in unconstrained FD (and MD) regimes at *comparable monetary-fiscal rules*. This positive limitative result begets but leaves open for now the normative question, which we directly address later, of whether *different* rules – specifically in terms of inflation targeting – might attain comparable stabilisation results.

Second, when we consider the form such endogenous volatility and sluggish transitional dynamics actually take in simulations, especially interesting is the fact that, associated to an initial ”containment” or ”convexification” of the dynamics that keep the reaction of the constrained FD economy bounded between that of an unconstrained FD and MD economies, is a protracted *recessionary-deflationary* phase, with output and inflation falling before converging to the stationary path from below. The non-linearity or amplification mechanism introduced by the occasional policy

[32] This was verified to hold, to ensure robustness, for different initialisations of the initial shock-/shock sequence and feasible rule parameters.

inflexibility hence is able to generates (or explain) endogenous cycles around the stationary equilibrium path in response to *positive* aggregate demand shocks. This endogenous business cycle phenomenon, absent in the constrained FD and MD benchmarks, also displays a structure close to the asymmetries noted in empirical business cycles, in which an ”expansion” phase in response to news is followed by a sharp burst and a protracted, asymmetric recovery.

Hence, relative to the unconstrained FD and MD benchmarks four main features appear to emerge in response to a relaxed environment that introduces the possibility of an occasionally inflexible passive monetary policy side: (i) the presence of greater, and endogenous, aggregate volatility or risk at similar shocks-initialisations; (ii) the achievement of lower macroeconomic stabilisation at similar inflation-monetary rules pairs; (iii) the presence of endogenous and asymmetric cycles in response to initial positive aggregate-demand shocks; (iv) related, a protracted *recessionary-deflationary* phase after the initial burst.

While providing a fully-fledged theory to account for the above endogenous features at equilibrium transitional dynamics is outside the scope of this paper, and constitutes the bulk of ongoing research work, we propose two plausible (and interacting) mechanisms at play that may at least in part account for the above results, guided by our knowledge of the constrained and benchmark models and experimental simulations with counterfactual scenarios. In particular, we wish to highlight two ”second best” channels: *inflation-targeting with forward-guidance (ITFG)* and *inefficient Ricardianism (IR)*. At the heart of both channels, consistent with the existence of a more or less strong inflation-targeting objective, is the *credible* threat – hence delivered at a rational expectations equilibrium under committal simple monetary policy rules – of a deflationary-recessionary phase to enable time-0 inflation/inflation news targeting when the occasionally inflexible policy (and hence the inability or unwillingness to tolerate more debt or money growth) is binding.

We think these are best viewed, recalling the severe nonlineraity present in the constrained problem, as a reflection of the Lipsey-Lancaster theory of *second-best* (1956) familiar from the microeconomics and welfare economics literature in the presence of constraints from frictions. In particular, given the inability to achieve inflation-targeting in the same way the normal-times unconstrained FD economy (and of course the MD economy), as this would imply, ceteris paribus, overshooting of the occasionally inflexibility point, then maintenance of the inflation-targeting objectives at the occasionally binding constraint requires an adjustment of all other variables away from the path they would follow along an unconstrained equilibrium sequence.

The two mechanisms we higlight capture or rationalise such second-best adjustment. Regarding the ITFG channel, the intuition develops on two standard ideas. First, that in a forward-looking model expected inflation or inflation tomorrow is pro-cyclical to the nominal rate (Cochrane, 2023, 2022).[33] Second, which is the

[33]This is the case with a forward-looking Phillips curve, but also in the neutral benchmark or

case in MD and occasionally becomes the case, as shown in appendix A3, in the constrained FD case, inflation expectations contribute to pinning down inflation today. Then, it follows that a credible threat to keep equilibrium nominal rates below the long run rate as the economy converges to the steady state can be used to engineer lower equilibrium inflation today, specifically by inducing expectations that deflation will prevail tomorrow on the back of the threatened ”recessionary” rate policy. This represents the ”forward guidance” element. The credibility of the threat, with committal rules as the monetary policy rules employed in the model, translates into the fact that such threat materialises along the equilibrium sequence. Under such forward-guidance mechanism, inflation targeting can thus be achieved today at lower nominal rates and, hence, bond-issuance/new borrowing or money-growth. Circling back to the point borrowed from the theory of second *second-best*, with occasional policy inflexibility suddenly binding, simultaneously maintaining an inflation targeting objective built-in the committal monetary rule, and respecting the binding constraint on the new debt position is thus achieved (under time-0 rational expectations and committal rules) by having the equilibrium path followed by the economy deviate from the first-best benchmark of unconstrained or frictionless FD and MD cases. In these cases, underlying the time-0 equilibrium (higher) inflation rate is a sharp increase of the nominal rate, paid for with higher inflation down the line. In the constrained FD case with lower equilibrium time 0 inflation, in the absence the ability to freely tap into a similar exchange due to the occasional inflexibility point, inflation targeting is enabled and paid for in terms of lower inflation/deflation down the line.

The alternative solution approach for transitional dynamics available to the OccBin algorithm outlined in 4.2. – i.e. the solution of an approximate system along the constrained sequence by guessing that the monetary rule can be suppressed consistently – provides, in our view, a useful counterfactual or experimental test for the plausibility of the above mechanism. In particular, since it eliminates the clash between the inflation-targeting objective and the occasionally binding inflexibility constraint, it can be viewed as shedding light on dynamics in the absence of the conditions we guessed to lead to a second-best scenario. The results are, as said, in Appendix A4.[34]. In the counterfactual scenario provided by construction of the alternative solution approach, the constrained economy takes the bulk of the shock at time zero, resulting in much higher inflation today than in the unconstrained FD and MD cases, and the recessionary-deflationary phase disappears. In our view, this is precisely because, giving up on inflation-targeting when the occasional inflexibility constraint binds, the need to engineer low inflation through the committal monetary rules disappears. The second-best result, instead, takes the form of nomi-

launchpad economy with perfect price flexibility, in which the response is one-to-one leading to sharp but for our purposes pro-cyclical adjustments

[34]The same endogenous volatility and most optimal policy results, with the only exception being the failure of reform under gradualism, carry over to this alternative solution as well

nal rates immediately falling relative to the unconstrained FD and MD systems. For these reasons, we think it a plausible explanation (if interim), that driving emergent properties (i)-(iv) of transitional dynamics under occasional inflexibility is precisely the clash, leading to a second-best equilibrium, of an inflation-targeting objective through committal monetary policy rules on the one hand, and the occasional inflexibility in terms of the ceiling on new debt or money-growth. In particular, the best (of different perhaps) adjustments consistent with such second-best equilibrium is precisely the recessionary-deflationary phase and protracted undershooting of nominal rates that transitional dynamics display.

This explanation is, note, further consistent with two important and aspects of the analysis. First, it rationalises the result in the policy optimisation exercise we conduct in the next section, namely that in FD with occasional inflexibility, weaker inflation-targeting is – under general circumstances and barring unconventional solutions of the type we highlight at the end of the paper – more desirable for macroeconomic stabilisation objectives to stronger inflation targeting, under any feasible fiscal rule. It is consistent with this result as, with weaker-inflation targeting objectives, the need for a second-best solution is attenuated. Second, it is consistent with the hybrid theory of inflation and price determination given in A3; namely, that at the occbin point in which the standard FTPL from the unconstrained FD case does not apply, inflation today can be reduced through deflation tomorrow.

The second ”IR” channel can be viewed as a consistency check in the light of the above. In particular, given deflationary expectations in the constrained FD case, as opposed to inflationary expectations in the unconstrained FD (and MD) case, the only way of achieving weakly lower new debt (and respecting the ceiling) in the former scenario than in the latter (which is clearly the case from the IRFs presented) is through a lower expected real rate. Hence, lower nominal rates (and equilibrium time 0 inflation) are needed relative to the unconstrained FD case. This constitutes an ”inefficient” Ricardian mechanism because, rather than using higher time-0 nominal rates to smooth the initial inflation shock through the IS curve, the economy resorts to using a credible threat on post-time 0 rates, to the effect of penalising expectations on future consumption after time 0 through the deflationary-recessionary mechanism: it is as if, barred the ability to tap into the substitution effect through real rate targeting when occasionally inflexibility binds, this is instead ”mimicked” through tapping into the channel provided by the income effect.

To sum up, our analysis of transitional dynamics in the FD economy with occasional inflexibility highlights tha these are not a convex combination or weighted average of the equilibrium dynamics observable under unconstrained FD and MD, as an initial educated guess might lead to suppose. Rather, the nonlinearity introduced by the inflexibility constraint generates emergent dynamics relative to the unconstrained FD and MD benchmarks. These are: (i) the presence of greater, and endogenous, aggregate volatility or risk at similar shocks-initialisations; (ii) the achievement of lower macroeconomic stabilisation at similar inflation-monetary rules pairs; (iii) the presence of endogenous and asymmetric cycles in response to initial positive aggregate-demand shocks; (iv) related, a protracted *recessionary-deflationary* phase after the initial burst, accompanied by undershooting of nominal rates. As we see them, these are best understood as defining the second-best equilibrium attained in these economies in which a clash, to the effect of preventing the first-best equilibrium observed in the other cases, exists between inflation targeting objectives and the non-linearity introduced by occasional inflexibility of the monetary side.

Based on this positive model of transitional dynamics in the three models, we now proceed to discuss its relationship and implications for the design of optimal stabilisation policy in FD cases with occasional policy inflexibility, and how this differs from the unconstrained FD case.

5 Optimal Monetary and Fiscal Rules under a Robust Stabilisation Objective

5.1 The policy-optimisation exercise at a glance

To answer the second question we raise, we then study optimal fiscal-monetary policy design comparatively across the perfectly flexible and occasionally inflexible FD economy, taking a simulation-based route fully integrated with the above solution procedure. In this model, the space of fiscal and monetary policy rules, under the feasibility constraints dictated by the FD device, naturally take the form of ”simple rules” – following the methodological literature pioneered by Schmitt-Grohé et al (2005), these are robuster, real world policy-oriented, and structurally interpretable alternatives to Ramsey instruments with unrestricted updating, insofar as they additional require that feasible policy instruments only respond to a set number of variables. Optimisation can be thus carried out (ad no added cost) directly with respect to the parameters of the monetary and fiscal rules already at play in the benchmark models.

We choose to focus on a standard ”robust” objective, akin to macroeconomic stabilisation, cast in terms of minimisation of price and output volatility. We focus on a multivariate optimisation problem with respect to the *joint/simultaneous* choice of feasible simple monetary and tax revenue updating rules, and hence are able to account directly for and comment interactions of monetary-fiscal rules in the optimisation exercise.

5.2 Welfare Criterion

The objective to be minimised is conformable to a robust, standard macroeconomic stabilisation objective of the form

$$\mathcal{W} = \min \mathbb{E}_0 \left\{ \sum_{i=0}^{\infty} \beta^i \left[\alpha y_{t+i}^2 + (1-\alpha)\pi_{t+i}^2 \right] \right\} \tag{28}$$

The formula above represents the expected present value of Quadratic Costs (QCs) under a pair of (interacting) monetary-fiscal rules, conditional on the time-zero information set. Costs to stability accrue from output deviations and inflation. QCs can be interpreted as conditionally expected percentage losses relative to the steady state, in which shocks are zero by construction. In this sense, they offer a measure of costs from instability. Parameters α and $(1-\alpha)$ with $\alpha \in [0,1]$ are weighting or distributional parameters reflecting different "convex" mandates (e.g. the single-mandate ECB vs. the dual-mandate FED).[35]. But how do we construct the appropriate expectation based on the available data from the piecewise linear solution (occ-bin) in the sequence space? The key idea is that that, in the sequence space solution we employ, there is perfect foresight/no uncertainty. Expected utility conditional on the time-zero information set is hence identical to utility conditional on the full sigma-field: by measurability, it equals ex-post utility along the shock sequence. Hence, ex-post/realised stream of quadratic costs under a rule is mathematically identical (not just a proxy for) to the current value of its time-zero conditional expectation. This allows us to use, iteratively at each point in the monetary-fiscal rules space, the solution-by-simulation approach employed to study transitional dynamics, to appropriately evaluate the objective.

The optimisation exercise can thus be conducted numerically through the outlined double-loop algorithm: the outer loop iterates initialisations of the inner loop over points in the monetary-fiscal policy space, while the inner loop (1) solves the model of FD with and without occasional inflexibility on the monetary policy side for transitional dynamics in response typical shocks (and their interaction) using the piece-wise linear root-finding algorithm described above, and (2) evaluates the robust planner objective along such equilibrium dynamics. Fifteen alternative policies are considered, uniformly distributed on the $[0.6, \phi_{\pi}^{ub})$ and $[0, \phi_{\tau}^{ub})$ interval, resulting in a grid of 225 points on which the piece-wise linear root finding algorithm is looped and the evaluation the attained planner objective consistently evaluated. The current runtime, including computation of the 3D graphics, is around 50 seconds on an iOS system with 8 GB RAM. [36]

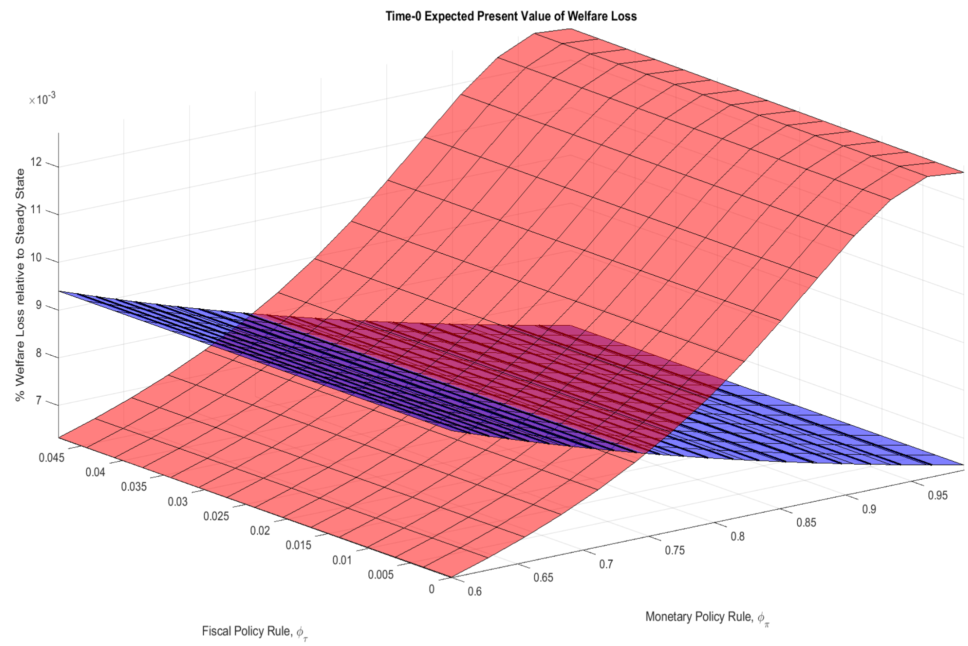


Figure 3: Expected welfare loss at t_0 after a positive IS shock. The red figure denotes the expected welfare loss in the constrained case, while the blue figure is the unconstrained case.

5.3 Main Results

The welfare analysis tells us that the welfare loss (**WL**) after a positive IS shock, in the constrained solution, dominates the unconstrained one for a monetary policy reaction parameter $\phi_\pi > 0.75$ and for each fiscal policy reaction parameter ϕ_τ. The maximum walfare loss is reached for a monetary policy reaction parameter $\phi_\pi = 0.95$. In this region of the interaction space between monetary and fiscal policy, a central bank adopting occasional inflexibility worsens the results. For values of the monetary policy reaction parameter $\phi_\pi < 0.75$, we have an opposite situation: a weak central bank reaction in the inflation-constrained solution will provide a lower welfare loss than in the unconstrained case. A central bank in a constrained solution with a reaction parameter $\phi_\pi < 0.6$, beats in terms of WL a central bank following a Taylor principle policy rule in the unconstrained solution. The minimum WL for an inflexible central bank requires a lower responsiveness of nominal rates to inflation than that of a flexible central bank under fiscal dominance. Note that the WL in the constrained case is non-monotonic. The WL analysis sheds light on the value of the optimal response below inflation in the two solutions. Tightening monetary

[35] Accommodating different dual mandates, e.g. the RBI's price-stability and growth (rather than output stabilisation) mandate necessitates a different objective

[36] The values for ϕ_π^{ub} and ϕ_τ^{ub} are reported in table 7.

rules beyond a certain point is harmful; a stronger response to inflation increases the strokes of inflexibility, leading to a greater welfare loss. In other words, we can say that there is a trade-off between inflexibility and aggressive inflation targeting. Inflexibility is more prone to welfare losses.

Lower values of ϕ_π, given the fiscal policy reaction parameter ϕ_τ, in the constrained solution, will reduce WL. As we preliminarily pointed out in our discussion of endogenous volatility and second-best nature of transitional dynamics, this is because, with occasional inflexibility, weaker inflation-targeting is more desirable for macroeconomic stabilisation because, implying weaker-inflation targeting objectives, the need for a second-best solution is attenuated. In other words, aggregate demand shocks will trigger to a lower degress the tradeoff between inflation targeting objectives and satisfaction of the occasionally binding constraint imposed by the presence of policy inflexibility. At the lower boundary of monetary and fiscal policy, the economy will converge back to steady state in eleven periods. In contrast, at the upper limit (see Figures 1 and 2), the economy will converge to steady state only in the 19th period. The endogenous volatility, resulting from a more persistent recessionary-deflationary mechanism at a second-best equilibrium, causes greater social losses. For low values of ϕ_π, given the reaction parameter of fiscal policy ϕ_τ, output in the early periods, where it weighs more in terms of WL, expands more than in the unconstrained solution, causing lower welfare losses. Adding to the output path, for lower values of ϕ_π, given the reaction parameter of the fiscal policy ϕ_τ, in the early periods the economy will enter a deflationary phase, while in the unconstrained solution there is an inflationary phase, causing less welfare losses.
In the presence of a positive fiscal shock, welfare analysis tells us that WL in the constrained solution dominates, for each reaction parameter of monetary policy ϕ_π and fiscal policy ϕ_τ, the unconstrained one. As in the case of the IS shock, the WL in the constrained case is not monotonic. Once again, monetary rules embedding stronger inflation-targeting objectives do unambiguously worse than weaker inflation targeting ones. The maximum welfare loss is reached for a monetary policy response parameter $\phi_\pi = 0.95$. The explanation for the welfare loss results after a positive government shock is similar to that of the IS shock, and similarly taps into the second-best equilbrium path logic we outlines in the previous section. Lower values of ϕ_π, given the fiscal policy reaction parameter ϕ_τ, in the constrained case will reduce WL because the recessionary process, imposed by the central bank, will last less. At the lower limit of monetary and fiscal policy, the economy will converge back to steady state in eleven periods. In contrast, at the upper limit (see Figures 1 and 2), the economy will converge to steady state only in the 19th period. The lasting recession causes greater social losses. WL in the constrained solution dominates, for each reaction parameter of monetary policy ϕ_π and fiscal policy ϕ_τ, the unconstrained one, because higher deviations in output and inflation are achieved.

A secondary result of the modelling undertaken for optimal Monetary and

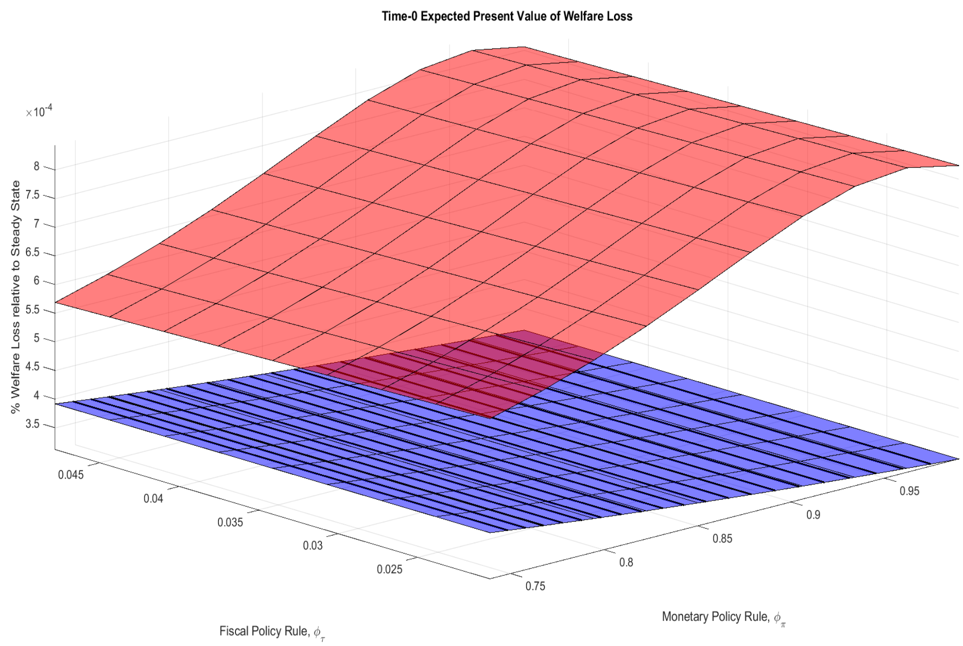


Figure 4: Expected welfare loss at t_0 after a positive fiscal shock.

Fiscal policy rules design in FD with and without occasionally inflexible policies is that the presence of an occasionally inflexible monetary authority notably affects curvature properties, or more technically the value of the gradient, of the welfare loss that a rational, pro-stabilisation planner forecasts in designing optimal rules. In particular, relative to the case with perfectly flexible policy or always-slack inflexibility constraint, we find the Welfare Loss Function associated to the FD device with occasionally inflexible policy exhibits local minima coexisting as distinct points from the global one, due to gradient of the loss function over the feasible monetary-fiscal policy space for optimisation crossing the zero line from below. Compared with the fully flexible or control case, this "local valleys" phenomenon is entirely driven by the presence of inflexible stances that might occasionally constrain the operation of the FD device. This is, further, the case under both simulation scenarios for the considered demand-side shocks. The non-monotonicity in the constrained solution, and in particular the presence of local optima regardless of the type of the shock raises some questions about the possibility that economies initialised at typical or mainstream strong inflation-targeting rules might find themselves (when faced with occasional policy inflexibility) trapped at a globally suboptimal fiscal-monetary rules pair. This hypothesis emerging on the back of the policy optimisation routine will be explored further through the game-theoretic analysis proposed in the next section.

5.3.1 Policy traps and gradualism-optimality tradeoffs in FD with occasionally inflexible policy through a simple game of monetary policy reform

If we take, as we think we should based on our motivation of the paper, an occasionally inflexible stance of the monetary authority as a realistic or unlikely-to-go constraint faced by economies operating under effective fiscal dominance as the default institutional environment[37] and as a serious feasibility constraint to be taken into account by the FTPL theorist and its vocational application to advocacy of institutional reform toward a more active fiscal side, then we submit that this endogenous property of the planning problem has positive and normative implications for the *reform* of monetary policy rules toward the optimal benchmark in FD economies with occasionally inflexible policy. In particular, we think it has implications for the relationship between gradualism required from monetary policy reform and the ability of the reform to attain the optimal benchmark (or gradually move closer to it).

Informally put, gradualism can be viewed as a way of organising searches of an optimum when the objective function is unknown and must hence be sampled. Then, gradualism requirements might be a good idea when the criterion between the initial policy point and the optimal benchmark is monotonic, so that the gradient in a neighbourhood of the initial policy point targeted by a gradual search offers a good idea of the behaviour of the criterion as one moves toward the optimal benchmark. In such case, which is conformable to a gradient descent problem on monotonic objective function, gradualism offers a route through which policy "prudently" converges to the optimal benchmark. Monetary reform in FD without occasional inflexible policies conforms to this case, as we see from figures (3)-(4). When non-monotonicity kicks in, as displayed by FD with occasionally inflexible policy, this is no longer the case, and gradualism might imply optimal searches confine the economy to the initial, suboptimal policy point.

This trade-off between the gradualism and welfare-enhancing ability of monetary policy reforms endogenously occurring in the design problem associated to the FD device with occasional inflexibility has two upshots. First, in gradualist-reform environments it might make starting conditions, in terms of initialisation of the economy on the monetary-policy rules space, matter for the feasibility of reform programmes. Hence, endogenous curvature properties offer some explanatory power for the lack of policy reform as an endogenously occurring trap: in particular, how reform to address the distortions caused by an occasionally inflexible monetary stance is made more difficult by the occasionally inflexible stance itself. Second, and related, it more importantly suggests that breaking away with gradualism requirements might be necessary to break away from traps confining the economy to globally sub-optimal (albeit locally optimal) policy regimes.

[37] Such as many developing countries or advanced economies requiring undergoing some phase of fiscal exceptionalism.

We capture and elaborate on this idea more formally by means of a small game-theoretic model of monetary policy reform in which a planner or a government sharing the planner objective and information set is allowed to search and implement a new policy point located weakly within a sphere with radius depending on gradualism requirements and centred at the initial monetary-fiscal policy pair. The ex-post payoff received by the planner depends on whether a representative voter decides to punish and vote the planner/government out, in which case the planner is evicted and receives a zero gain, or acquiescing (perhaps though gritted teeth), in which case the planner receives a gain equal to the difference between the criteria pre- and post-reform.

To introduce some uncertainty or leeway for reforms, we model eviction and acquiescence as an ex ante probabilistic phenomenon at the planner's decision node, due to random sources of bias in the ex-post assessment carried out by the representative voter.[38] To do this, we assume that the representative voter's assessment of welfare after the reform is (and the standard exogenous shock sequence drawn) is randomly biased by some *sunspot* or *animal spirits* variable with unconditional mean zero, entering as linear noise on the true welfare loss/criterion at the new policy point, and conditionally independent of the information set of the planner. Hence, while in expectation – if we were to average the assessment over the whole sigma-field induced by sunspots/animal spirits – the representative voter correctly assesses the policy, i.e. the estimator underlying any contingent assessment is unbiased (and consistent) in the econometric sense, there is some probability that it yields a biased estimate contingent on a single repetition of the policy experiment, and hence that the representative voter fails to punish a planner choosing a (locally) suboptimal policy point. For all our purposes, these families of *sunspots* or *animal spirits* variables can be thought of as "fake news" or "white lies" skewing the assessment of how much worse or better off the representative voter is at the new policy point relative to the pre-reform benchmark. Naturally, at a rational expectations equilibrium of the reform game, the planner must be correctly anticipating the probabilities of evictions and acquiescence at the new policy point when optimally choosing such point (if such point exists).

Based on this simple game solved via backward induction, we argue that, for stringent enough gradualism requirements and sunspots variance, the FD economy with occasionally inflexible monetary authorities fails to display reform toward

[38]This is inessential to the main point, as the limit of the ex ante payoff/gain of the planner as the distribution of the "animal spirits" or sunspot variable collapses on a dirac-delta distribution centred at zero i.e. at a no-uncertainty/noise case, is the criterion used for figures 3-4 minus the loss at the initial policy point. In such case, the main points an be made simply with reference to figures 3-4. However, introducing such sunspots/animal spirits in the assessments of the private sectors (forecast by the planner), in addition to being perhaps more realistic or at the very least less defeatist on the viability of reform, also opens up to implications on the varying importance attached role of communication and guidance in the reform process depending on initial conditions in the economies under study.

optimal policy as an equilibrium of the game. That is, the combination of occasionally inflexible policy and gradualism lock the FD economy into a policy trap. For an economy at a sub-optimal monetary policy point, one of three elements must be hence sacrificed, defining a menu of tradeoffs for institutional reform: abandon the occasionally inflexible stance, abandon gradualism, or abandon stronger macro-stabilisation as an objective.[39]

Toward formalising this, assume the fiscal policy rule is given and the economy to be initialised at some suboptimal monetary policy rule denoted ϕ_π^0.[40]

A Metric Model of Gradualism in Reform

To provide a rigorous notion of gradualism constraining optimal searches by the planner, we equip the monetary policy rules targettable by reform with a suitable metric notion of distance. To do this, we define the mapping $d_{\phi_\pi^0} \equiv^\Delta d : \Phi_\pi \rightarrow \mathbb{R}+$ as the non-negative real valued "distance function" mapping points in the subset $\Phi_\pi \equiv [0, ubm] \in \mathbb{R}$, which is an inner product (hence vector) space on itself and hence such that norms-based distance metrics are well defined, onto the real line. Since the vector field in question is the set of real numbers, the euclidean distance L^2 or taxicab/Manhattan distance L^1 yield equivalent models for the above function. In particular, we define

$$d(\phi'_\pi) = \|\phi'_\pi - \phi_\pi^0\|_{L^1} = \|\phi'_\pi - \phi_\pi^0\|_{L^2} = |\phi'_\pi - \phi_\pi^0|.$$

Since the above distance function is continuous on its domain, and the domain is a closed and bounded set (under our construction of the FD device with passive monetary policy) and hence compact by the *Heine-Borel Theorem*, it follows from the *Weierstrass Theorem* that the distance metric is also compact-valued, taking value in a compact set D.[41] Since it is real-valued, then its range is closed and bounded, implying it attains a maximum and a minimum on the set of feasible monetary policy rules. The implication of this is that searches in a neighbourhood of the initial policy point of radius equal to the distance function will be on bounded and closed (hence compact) subsets as well. Let the accordingly well-defined maxima

[39] In fact, in the next section we argue that there is a potential solution to the trilemma, available to FD economies with occasionally inflexible monetary authorities starting at the inefficient, upper boundary of the monetary rules space. This solution is to exploit aspects of fiscal-monetary policy interaction endogenous to the presence of occasional policy inflexibility to restore optimality inflation targeting. Whether or not this restorative us more desirable than optimally departing from gradualism or uncompromising stances on the need for inflation targeting is a question that we leave open for further work.

[40] We suggest the upper boundary on the monetary policy rules space, corresponding to the traditional "inflation targeting" norm in modern monetary policy, and found to depart from the optimal benchmark in the case with occasionally binding inflexible stances.

[41] I.e. every sequence in the compact set has at least one convergent subsequence that converges to a point in the set. The proof is standard hence omitted.

and minima attained by the function, respectively $d_{min} = 0$ and $d_{max} = max_d \in D$.[42] The gradualism requirement is a choice $d' \in D$, hence ranging from 0 (at which no reform or search is allowed) to $\max|\phi_\pi - \phi_\pi^0|$, corresponding to the case in which any feasible rule under FD can be targeted through the reform. More formally, we model an arbitrary gradualism requirement as a restriction or mapping $d' : \Phi_\pi \to \tilde{\Phi}_\pi$ on the set on which the optimal control problem is defined, such to send the original policy set to a *compact subset of itself* of radius d' and centred and the initial policy point. That is, under arbitrary gradualism the feasible (compact[43]) search set or action set open to the planner for picking the reform will be

$$\tilde{\Phi}_\pi(d') \equiv \Phi_\pi \cap [\phi_\pi^0 - d', \phi_\pi^0 + d'].$$

In solving the game for backward induction under a rational expectations equilibrium assumption, gradualism is taken as initially given. However, in drawing normative conclusions, we comment on endogenising the choice of gradualism in the FD case with occasionally inflexible policy.

Equilibrium Reform under Exogenous Gradualism

Assume that some arbitrary gradualism requirement d' has been chosen. The planner/government can choose to implement *new* policy point to implement via the reform from the set $\tilde{\Phi}_\pi(d') - \{\phi_\pi^0\}$, or to stick to the current policy ϕ_π^0 (in which case, no reform and assessment take place).

Since the planner in principle has access to the full information set, and hence the welfare loss criterion, up to some degree gradualism generally prevents the planner from implementing immediately the (known) global optimum. Knowing this, the planner can only attempt to move toward the global optimum by gradually getting closer to it; we assume, critically, that out of multiple possible progressive moves, the planner picks a policy or reform as long as it minimises the ex ante or expected gain from the move, which depends on ex post assessment and choices by the representative voter, and it beats the current policy. This is critical, as it means that when initial gradualism confines the planner to search within a neighbourhood of the current policy such that the minimum ex ante payoff from reform lies above maintaining the current policy, then no equilibrium with reform exists.

To define the ex ante gain, we note that the ex-post gain from implementing a reform *and remaining in office* is naturally given by the difference of the welfare loss at the two policy points, i.e. we let the payoff from a reform $v(\phi_\pi) = WL(\phi_\pi) - WL(\phi_\pi^0)$.[44] Note that, because the underlying objective is a loss function, this is

[42] We can also normalise the distance function by re-scaling through the monotonic or rank-preserving linear transformation $m = \frac{1}{d_{max}} d$. This yields a standardised distance metric $m : \Phi_\pi \to [0,1] \equiv M$ on the monetary policy rules set, inheriting the properties noted for d.

[43] Trivial to show.

[44] We assume in what follows that the WL, of which only a discretised version can be obtained computationally, is continuous over Φ_π. While this condition is not verified analytically, our simulations

negative-valued at "stabilisation-improving" or "welfare-improving" reforms that criterion-dominate the current policy (if at all), hence the reason for expected gain minimisation. If evicted from office, the reform is reverted and the planner receives a payoff of zero. At the optimal choice ϕ_π^*, the planner attains the minimum expected or ex ante gain defining the planner objective, given the planners (rational) beliefs on probabilities of eviction $1-\theta^e$ and acquiescence θ^e at the new reform point. The solution is thus defined by the conditions:

$$\begin{cases} J(\phi_\pi^*;\theta^e,d') = \max_{\phi_\pi\in\tilde{\Phi}_\pi(d')-\{\phi_\pi^0\}} -\theta^e\left(WL(\phi_\pi^*)-WL(\phi_\pi^0)\right) \iff J(\phi_\pi^*;\theta^e,d')>0 \\ \phi_\pi^*=\phi_\pi^0 \iff J(\phi_\pi^*;\theta^e,d')=0 \end{cases}$$

subject to beliefs θ^e satisfying a rational expectations assumption pinned down below. The former condition pins down an "interior" solution as the one inducing the minimum value of the objective over the set of policy points with reform, and the second one pins down the continuation of the current policy (no reform) as a corner solution to the case minimisation of the objectives over the feasible set of monetary policy rules under gradualism fails to beat the current regime.

Rational Expectations Equilibrium[45]

Clearly, the planner's choice (and our solution to the above problem) is determined, hence strategic interdependence, by its forecasts of subsequent actions by the representative voter. At a rational expectations equilibrium or Nash equilibrium, such forecasts or beliefs must be consistent with the actual probabilities of eviction and acquiescence at the new policy point (prior to the draw of the sunspot), and are hence endogenous – i.e. we focus on an equilibrium reform in which the beliefs of the planner match the average ex-post outcome from reform when optimising based on such beliefs.

More formally, let

$$H:\Theta^e\times\mathcal{U}\to\{0,1\}$$

the (complex) *law-of-motion operator* mapping Planners' forecasts or beliefs $\theta^e\in\Theta^e$, via the induced optimal policy choice, and ex-post sunspots/animal spirits draw $u\in\mathcal{U}$ to the set of post-reform eviction or acquiescence outcomes, respectively $H(\theta^e,u)=0$ and $H(\theta^e,u)=0$ which is assumed – conditional on a belief point – measurable with respect to the field $\sigma(\mathcal{U})$. Because the law of motion depends on the sunspot draw, which is orthogonal to the planner information set, the planner with knowledge of the model can at most forecast an average law of motion, i.e. whereby all uncertainty due to the subspot has been averaged out in the unconditional

with different grid sizes do not detect any behaviour consistent with there being discontinuity points as the set of points in the range increases

[45]This leads to a solution isomorphic, once we impose rational beliefs, to the solution of an unconstrained nonlinear bounded minimisation problem over a compact set.

expectation sense[46]. Suppose for a moment that we (and the planner) know the operator H (this will be pinned down below when we discuss voter behaviour conditional on sunspot and reformed policy). Averaging over unknown u implies taking the unconditional expectation of the operator as a function of u, and by a change of variable theorem:

$$\mathbb{E}H(u) = \int_{\Omega} H(u)P(du) = \int_{\{0,1\}} hPH^{-1}(dh) = 1 \times PH^{-1}(\{1\}) + 0 \times PH^{-1}(\{0\}) = \theta$$

Where θ is the actual probability of acquiescence at the new policy point, or measure, conditional on the implemented policy point, of the pullback set of $u \in \mathcal{U}$ consistent with acquiescence. At a rational expectations equilibrium, we require beliefs to be consistent with the average law of motion resulting from the planner acting in accordance to such beliefs. That is, at a rational expectations equilibrium, beliefs (or the law of motion) satisfy:

$$\theta^e = \theta(\theta^e) = \int_{\Omega} H(\theta^e, u)P(du) = \mathbb{E}H(\theta^e, u)$$

In other words, the beliefs supporting a candidate equilibrium are rational whenever, on average (if we were to repeat the reform experiment conditional on each point in the sunspot space and average results) they match the proportion of acquiescence and eviction outcomes as a result of the policy equilibrium they support.

A Discrete Choice Model for voter behaviour and endogenous rational beliefs

As noted, subject to the rational belief assumption and knowledge of the mapping H, we can solve for the equilibrium policy point by backward induction in terms of the solution to the nonlinear minimisation problem obtained by swapping beliefs as a free-parameter with one consistent with the optimal policy chosen based on it. Such beliefs, in particular, satisfy the last equality above. This requires specifying the mapping H, i.e. we need a model for voter behaviour in response to the new policy implemented by the reform and the sunspot draw. Assume we are at a rational expectations equilibrium with reform, i.e. such that an optimal policy $\phi^*_\pi(\eta^e, d') \neq \phi^0_\pi$ has been implemented. Conditional on the sunspot or animal spirits draw $u \in U$, we assume that the representative voter assesses the *gain from the reform relative to the pre-reform benchmark* $\hat{v}$ via a noisy linear estimator:

$$\hat{G}(\phi^*_\pi, u) = WL(\phi^*_\pi) - WL(\phi^0_\pi) + u \equiv v(\phi^*_\pi) + u,$$

where our second equality follows under our definition of the true gain. Recall, as we said, that contingent on the "white lies" or "fake news" modelled as the

[46] Since, with u orthogonal to the information set of the planner, then for a given belief point, the $H(\theta^e, \cdot)$ is also orthogonal to the information set hence $E(Hu|...) = E(Hu)$

random noise u, the estimate is biased. However, in expectation it converges to the true gain, implying an (ex ante) unbiased estimator over counterfactual histories/possible worlds. Subject to this assessment, we assume the voter receives payoff $\hat{G}$ if acquiescing, or 0 if voting out the government (as this reverses the reform). Clearly, they decide to acquiesce whenever $\hat{G} < 0$, corresponding to a biased assessment of improved macroeconomic stabilisation. Hence, *conditional on the subspot draw* at a rational expectations equilibrium policy, we have a natural model for law of motion:

$$H(\theta^e, u) = \mathbf{1}\{\hat{G}(\phi_\pi^*(\eta^e, d')) < 0\} \quad with \quad \eta^e = \mathbb{E}H$$

From an econometric or statistical perspective, note that conditional on u the law of motion is akin to a nonlinear model of the type arising from discrete-choice or discrete-demand models, with $\hat{G}$ as the latent variable or unobservable underlying utility. Using the definition of $\hat{G}$, and our formula for the average law of motion to integrate out unknown u at the planner stage, we obtain a general solution, subject to distributive/parametric assumptions, for the ex-ante probability contingent on the implemented optimal policy, and hence for endogenous rational expectations. Specifically, integrating out u, we obtain:

$$\theta^e = \theta(\theta^e) = \mathbb{E}\{\mathbf{1}\{\hat{G} > 0\}\} = \Pr(u < -v(\phi_\pi^*)) = P(u < -v(\phi_\pi^*)) \approx P(u \leq -v(\phi_\pi^*))$$

Where the first and second equality follows from the rational expectation assumptions and definition of law of motion, and the second one from the properties of the integral with respect to a measure.

The above solution for endogenous rational expectations and average true law of motion holds at a rational expectations equilibrium in general. However, solving a specific model requires assumptions on the distribution of the sunspot or animal spirits noise entering the voter's estimator of post-reform payoffs.

Natural closures for these models are provided by an assumption that the sunspot is a sum of two normally distributed variables (and itself normally distributed), leading to the *Probit* model endogenous rational expectations, or an assumption that it is distributed logistically, leading to a closed form, Logit case. We refer the reader back to the work on Train (2003) and McFadden (2001) for a discussion of implications and limitations of such parametric closures for DCM-s/Nonlinar Probability Models. We focus on the logistic case for three reasons. *First*, it leads to close form expressions for the probabilities and hence expected gain objective, greatly aiding analysis without the need for simulation-based evaluation of the choice probability/rational expectations integrals. *Second* the underlying distributional assumption is virtually indistinguishable empirically from normality, yet better suited to modelling events that are extreme in some respect.[47]. *Third*, it is

[47] For example, if we think that – due to demonstration or snowballing effects – the representative voter tend to respond form its biased assessment in response to the "largest" white lie or fake news, when events in turn draw from a normal distribution

well known that generalisations of the baseline model proposed to include random effects or unobserved heterogeneity (i.e. leading to a Mixed Logit Model) offers a good approximation to any partly-random utility/payoff function (Train, 2003). We thus assume that the noise is logistically distributed i.e. $f(u) = e^{-u}/(1+e^{-u})^2$. This is, in particular, consistent with assuming that separate sub-sunspots (alternative specific lies to acquiescing with the reform or reverting it) are distributed *Gumbel* or *Extreme-Value Type 1*. Under either, we can directly (or indirectly) obtain a closed form solution for endogenous rational expectations at the equilibrium solution:

$$\theta^e = \theta(\theta^e) = \frac{1}{1+e^{-(-v(\phi_\pi^*))}} = \frac{1}{1+e^{v(\phi_\pi^*)}}$$

Clearly, the probability of acquiescing converges to zero and hence converges to the true law of motion ($\theta^e \to H$) when the gain grows unbounded in the positive direction (i.e. at infinitely more macroeconomically-destabilising policies) – cf. Train, 2002 on this property of logistic closures for DCMs. More generally, rational beliefs forecast higher probability of acquiescing at more relative than less pro-stabilisation policies. This intuition under the present closure is important, as it means that sufficient monotonicity in the true gain is preserved for the equilibrium of the game to preserve the informal insights we pointed out in our visual examination of figures (3) and (4).

Hence, equipped with our closed-form solution of the average law of motion rationally forecast by the planner at a solution, the rational expectations equilibria of the monetary policy reform game with gradualism is a policy point $\phi'_\pi \in \tilde{\Phi}_\pi(d')$ satisfying:

$$\phi_\pi^* = \arg\max_{\Phi_\pi(d')-\{\phi_\pi^0\}} -\frac{1}{1+e^{v(\phi_\pi^*)}} v(\phi_\pi^*) \quad \textit{iff} \quad J(\phi_\pi^*;\theta^e,d') > 0$$

$$\phi_\pi^* = \phi_\pi^0 \quad \text{iff} \quad J(\phi_\pi^*;\theta^e,d') = 0$$

We call an equilibrium with policy point pinned down by the first condition an ”equilibrium with reform”, and an equilibrium with the policy point met by the second condition an ”equilibrium without reform”. Clearly, given the initialised policy point and information set of the planner for forming rational beliefs on the ex ante gain, gradualism requirement matters in determining which of the two equilibria applies by varying the size of the search set.

Reformless Gradualism with Occasional Inflexibility

Given an initialised policy point, we can use the above equilibrium definition to simulate the outcome of the reform game at different gradualism requirements, using the computed welfare loss function and the rational beliefs formula to extract expected gains and hence applicable equilibria. Prior to carrying out such computational exercise, we provide a pair of lemmas to filter and better understand the results of the game, by exploiting concavities of the WL function in a neighbourhood of the upper bound of the monetary policy set, which corresponds to the mainstream inflation-targeting objective, and found to constitute a feasible but sub-optimal benchmark for passive MP rules in the case of FD with occasionally inflexible policy.

Lemma 5.2.1. *Assume the economy initially lies at a local optimum π_0. If the gradualism requirement restricts the set of admissible reforms $\tilde{\Phi}_\pi(d')$ to a compact pullback set $A \subset WL(\Phi_\pi)$ such that (a) the WL function is weakly quasi-concave on A, and (b) $WL(\max(A)) \geq WL(\phi_\pi^0) \leq WL(\min(A))$ then the set of equilibria with reform for the game with a gradualism requirement d' is empty.*

Proof. By assumption, $WL(\max(A)) \equiv^\Delta WL(\bar{A}) \geq WL(\phi_\pi^0)$, so that the extrema of set A, which are well defined under compactness of the set, lie weakly above the initial policy point. Because the set of feasible reform points A is strictly convex (under our construction of the restriction d'), then linear combinations of the initial policy point and the extrema, of the form $\alpha\pi_0 + (1-\alpha)\bar{a}$ for $\alpha \in (0,1)$ will be admissible reform points. We now use curvature, i.e. weak quasi-concavity, to rule out that any such linear combination can beat the initial policy point.[48]. Toward showing this, by weak quasi-concavity, it follows that in general $WL(\alpha\pi_0 + (1-\alpha)\bar{a}) \geq \min\{WL(\bar{a}), WL(\phi_\pi^0)\} = WL(\pi_0)$, which is to say that the welfare loss at linear combinations of the initial policy point and "furthest" reform under gradualism, that are always feasible under convexity, will lie weakly above the welfare loss at the original policy point. Hence by this result and assumption (b), we have that $WL(\pi_\pi^0) \leq WL(\phi_\pi'), \forall \pi' \in \tilde{\Phi}_\pi(d')$ in general. Because, as a result, the gain function $v(\phi_\pi)$ is weakly positive over the feasible set of reforms under gradualism, it follows under our definition of the equilibrium conditions for the game that $\phi_\pi^* \neq \phi_\pi^0$ cannot be an equilibrium of the game: in particular, assuming that there the set of equilibria with reform is non-empty (there is some optimal new policy point $\phi_\pi^* \neq \phi_\pi^0$) contradicts the equilibrium condition when, as shown, (a) and (b) are true.

Lemma 5.2.2. *Assume the economy initially lies at a local optimum π_0. If there is some gradualism requirement d' such to meet conditions (a) and (b) in Lemma 5.2.1, and hence such that the set of equilibria with reforms is empty, then stricter gradualism requirements $0 \leq d < d'$ also lead to an equilibrium without reform.*

Proof. To show this, assume that some arbitrary d' exists satisfying conditions (a) and (b). Then there is, by Lemma 5.2.1. and the definition of an equilibrium of the game, no point in $\tilde{\Phi}_\pi(d')$ such that the value function attains a positive value J. Now, by construction of the gradualism mapping, clearly $\tilde{\Phi}_\pi(d_i) \subset \tilde{\Phi}_\pi(d_j)$ whenever

[48]Such curvature property (a) is critical, as a number counterexamples exists leading to equilibria with reform when only condition (b) is met.

$d_i \leq d_j$. Hence, no point in $\Phi_\pi(d)$, with $d \in [0, d']$ can also be such that the value function is positive, nor thus consistent with an equilibrium with reform.

Based on the above two lemmas, the key insight from the computation of the Welfare Loss function(s) given in figures (3) and (4) for the FD case with occasional policy inflexibility is that they meet the concavity, and hence quasi-concavity requirement, in a small neighbourhood of the upper end of admissible monetary policy rules. Critically, these policy points correspond to the most-hawkish or most-inflation targeting rules that are admissible under the normal-times or slack fiscal dominance regime to which the initially constrained economy reverts back over time, and with the mainstream understanding of monetary policy objectives – as they tend, in the limit, to the characteristic one-to-one or inflation-targeting response of the Taylor rule mainstreamed into prevalent monetary policy mandates. We think it plausible – or at least useful as an empirically-relevant exercise – to suppose that most economies operating de-facto under fiscal dominance, but committed to aggressive inflation targeting objectives mimicking those implicit in NK models with a *Monetary Dominance* device, might be thought of as located initially at such maximal feasible inflation targeting local optimum.

In such contexts, our game-theoretic analysis based on the endogenous differentials in the WL functions for the FD economies with and without occasional policy inflexibility, suggests that occasional policy inflexibility coupled with gradualism might introduce precisely a policy trap whereby such economies *fail* to move away through reform from the initial globally suboptimal monetary policy and to convergence to the more doveish or weaker inflation targeting benchmark identified by the policy optimisation exercise. This *reformless gradualism* trap, we stress, is an endogenous outcome or an emergent property of friction introduced by an occasionally inflexible passive monetary policy relative to the FD benchmark with perfectly passive, flexible conduit of monetary policy. Such failure at reform, to the effect of combining an economy with excessively strict gradualism requirements to global sub-optimality of stabilisation policy, can be thought of as a complex outcome: in particular, the fact that the kicking-in of occasionally inflexible policy changes the optimal policy benchmark toward more doveish monetary rules or weaker commitments to inflation targeting through traditional nominal rate policy (on the grounds of the considerations on positive dynamics in 5.1.2.), but also simultaneously constrains the planner's ability to update or shape stabilisation policy to accommodate the optimal benchmark.

In addition to the obvious upshot in recommending a relaxation of gradualism requirements in monetary policy reform emerging from the analysis, the positive applications of the theory appear appealing. In particular, if we take gradualism as an unlikely-to-go requirement, the highly non-linear nature of the stabilisation policy rules design problem under occasionally inflexible policy thus contains in itself indications consistent – as exploited by the game theoretic exercise we proposed – with a positive theory for the lack of reform and stickiness of suboptimal stabilisation rules design concomitantly emerging in such contexts. While the central insight of the paper for the design of stabilisation policy, in contexts with frictions conformable to the occasional policy inflexibility constraining the operation of the FD device, remains the weakening of inflation targeting objectives in designing monetary rules, it is natural to suppose whether alternative or unconventional solutions might subsist to the effect of bridging the gap between inflation targeting and optimal stabilisation policy.

Hence, addressing this question in the last section on endogenous or emergent policy interactions, we show that the strategic interaction or coordination of inflation-targeting rules with *contractionary fiscal policy* in response to positive IS shocks has the ability, in FD economies with occasionally inflexible policy, of restoring optimality of conventional inflation targeting. In other words, that an unlikely-to-go commitment to inflation targeting built in Taylor-mimicking monetary rules in FD economies with occasional inflexible policy presents a case, on stabilisation policy grounds, for moving to a more uncoventional, strategic coordination of fiscal policy innovation with inflation targeting.

5.3.2 Endogenous Interaction between Policies

An interesting aspect emerging from the optimisation routine for fiscal-monetary policy interaction, and regardless of the type of shock, is that moving to the occasionally inflexibility solution not only changes optimal interaction, but also the way the two policies interact.

One can easily note that surfaces in the unconstrained cases are traced by sliding the same curve over the fiscal policy space – in other words, by joining a dense set of parallel lines. Conditional on monetary policy, fiscal policy rules do not effectively matter for welfare as long as they ensure determinacy of the equilbrium (i.e. the FD condition, see table **??**). As long as this requirement is met, any variation in welfare is from the monetary policy response and orthogonal to fiscal rule adopted. A key result is that the interaction in these settings is rather limited.

Moving to the policy inflexibility case, this is no longer true. The occasionally binding constraint introduces an endogenously (as the bindingness is) occurring extra source of interaction between the prevailing fiscal rule and the monetary one.

Another important result emerges comparing the positive IS shock case with and without fiscal intervention (see figure 3 and 5) .

Fiscal intervention seems to flatten the right tail of the WL[49] in the case without interaction, indicating that more aggressive fiscal rules (mimicking Taylor/traditional or better said nominal rate targeting) become feasible without incurring inconsistencies with the welfare criterion/stabilisation objective. The endogenous interaction that emerges is that in the constrained case, the nominal rate targeting (or more commonly called inflation targeting) traditionally adopted is (optimally)

[49]For a better visualisation of figures 3, 4 and 5, see appendix 7.

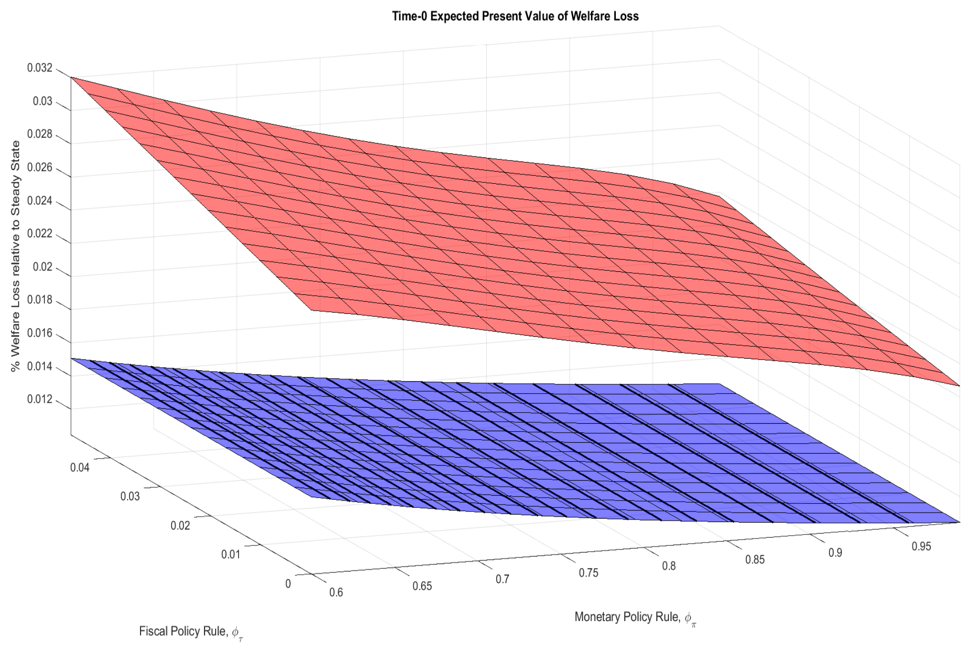


Figure 5: Expected welfare loss at t_0 after a negative fiscal shock and a positive IS shock. The red figure denotes the expected welfare loss in the constrained case, while the blue figure is the unconstrained case.

pursuable only when the government systematically intervenes with fiscal shocks in response to positive IS shocks. With a reduction in government spending, aggregate demand is discouraged by going to reduce labor demand and thus expected inflation and hence inflation. More aggressive rate responses, on average, are less able to bring the economy close to the point of inflexibility. In other words, we can afford more aggressive responses, thus more able to dampen the positive IS shock, to current inflation (higher inflation targeting), resulting in a reduction in WL.

Essentially, at any implied inflation rate the contractionary fiscal shock makes facilitates inflation targeting through lower positive deviations in the end of period nominal debt. This is the result of two interacting channels that the proposed "coordinated" stabilisation programme taps into:

1. Positive Substitution Effect through Fiscal-Monetary Coordination (IS)

Given (endogenous) inflation expectations, and given consumption choices, the reduction in government expenditure reduces aggregate demand and hence the current marginal costs faced by (price-resetting) firms. Through the Phillips-Curve, this reduces current inflation in the same way a *negative* cost-push shock does. Hence inflation targeting implicit in the monetary rule, ceteris paribus, can be conducted at lower nominal rates, and consequently lower deviations in bonds balances or nominal debt relative to the case in which no fiscal contraction takes place. This is a substitution effect founded in the interaction of the fiscal side with the monetary policy rule: the contractionary shock to fiscal expenditures essentially orchestrates a substitution toward consumption by making consumption today relatively cheaper than consumption tomorrow.

2. Negative Real Income Effect through Contractionary Fiscal Policy

The reduction in aggregate demand, at any nominal rate and inflation expectations pair, implies through goods market clearing some combination of an upward movement consumption and a lower adjustment in period output. Hence, a combination of higher aggregate consumption and reduced aggregate output. Because aggregate output flows, via factor payments and profits, entirely to households, then the reduced real fiscal expenditure essentially acts as an orthogonal/unpredictable tax on the household's real resources, with a consequent reduction in the demand for bonds at the prevailing nominal rate.

6 Alternative Implementation of OccBin Algorithm and Solution

Equilibrium equations along the constrained state subsequence holds only approximately. The system of equations to be satisfied at a perturbation in the sequence space is either overdetermined, or holds exactly for an approximation to the model through suppression of one of the endogenously holding equilibrium equations (IS Curve, Monetary Policy Rule, etc). Importantly the latter guess is not verifiable from the reduced model. We note, given that the methodological literature on the approach is still in develpment, that this point and consequent choice applies to the sequence space Occ. Bin approach in general as opposed to the application to solving for equilibria in the model we develop.

Currently we use the former approach, hence obtaining solution to the approximate or overdetermined system that is guessed to hold, with endogenous timing, when the economy lies in the constrained state, i.e. stuck (in a small neighbourhood) of the debt ceiling. Residuals computations indicate that at the typical solution we achieve a sum of square residuals

$$\text{SSR} = (\mathbf{F}_x\mathbf{x} - \mathbf{b})^T(\mathbf{F}_x\mathbf{x} - \mathbf{b}) \tag{29}$$

of order between $1E^{-}5$ to $1E^{-}8$. An alternative approach is to guess that the system can be reduced to an exactly determined system (i.e. by suppression of an equilibrium equation) for the number of periods in which the economy is guessed to be constrained following the initial shock, again with endogenous timing. In this case, we can use the FD solutions to the original or true system as a counterfactual – albeit inexact since produced by what is effectively a different model of the constrained economy – of how the equilibrium equation suppressed in the reduced system would react.

Note that this approach itself is only approximate since, even though we guess that some equation can be eliminated by virtue of the occasionally binding state of the economy (i.e. it would imply behaviour inconsistent with the constraint), once this is eliminated the model effectively changes. We can use as said FD to verify the counterfactual case, but that would be — naturally — based on an unconstrained solution; hence we have at most an approximate verification that indeed the system can be reduced as guessed at a constrained state. The Least Squares solution preserves efficiency (doesn't throw out information or equilibrium equations) at the cost of accuracy of the solution; the second solution preserves accuracy of the solution by slightly changing the problem to be solved, under the guess (not exactly verified) that the information that gets thrown away does not matter at the constrained solution.

Here we compare the IRFs obtained under either solution, i.e. the Least Squares Approximation we employ and the alternative Endogenous System Approximation. This second approach is based on disallowing the Monetary Policy Rule along the sequence of constrained states (to be solved for endogenously). In other words, contrary to the approach in the benchmark model, we are assuming that in the proximity of the occasionally binding debt-ceiling the Central Bank temporarily gives up its normal time behaviour (i.e. inflation targeting via the policy rate instrument) and passively adjust the policy rate in order to ensure a non-violation of the constraint. This approach is consistent with the main narrative rationalising the LS approximation one: in the analysis of the dynamics in the paper, it was pointed out how central to the result was a trade-off between the incentive of the Central Bank to pursue inflation targeting by raising rates and the commitment to occasional policy inflexibility in the proximity of the debt ceiling to ensure the equilibrium debt does not overshoot such barrier. In this alternative solution approach, the conflict of incentives is resolved in favour of giving up on the former objective at a constraint.[50]

The take-home for the endogenous volatility and consequently optimal stabilisation policy remain unchanged, as clear from the Loss Functions. The modelled transmission mechanism changes by construction of this alternative solution protocol (since the Central Bank temporarily gives up commitment to its passive policy rate rule), but provides a sanity check on the logic employed in analysing the transmission underlying the IRFs in the text. In particular, without the need to pursue simultaneously both commitments and consequently being tolerant to any equilibrium inflation in the proximity of the binding debt ceiling, the bulk of the aggregate

[50]Importantly, in the second case, since the Monetary Policy Rule is absent in the constrained model, we use the FD solution as a counterfactual to initialise the economy at a (counterfactually) consistent shock. I.e. such that, if the constraint is binding in the initial period in the Occ Bin solution, the FD solution does not imply slackness of the constraint. Again, because the Monetary Policy Rule is ultimately absent for the constrained model but not for the unconstrained, the counterfactual provided unconstrained FD case offers useful indications for consistent simulations, but cannot be used to verify the initial guess on the redundancy of the monetary policy rule/behaviour.

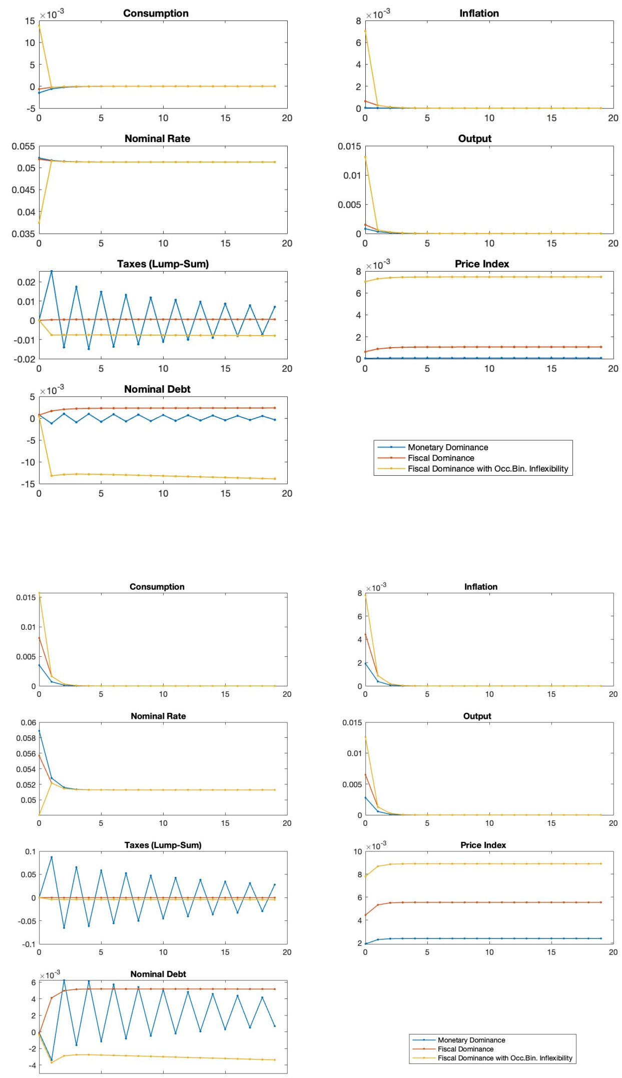


Figure 6: IRFs under Alternative Solution Protocol. Top is Fiscal Shock, bottom is IS shock.

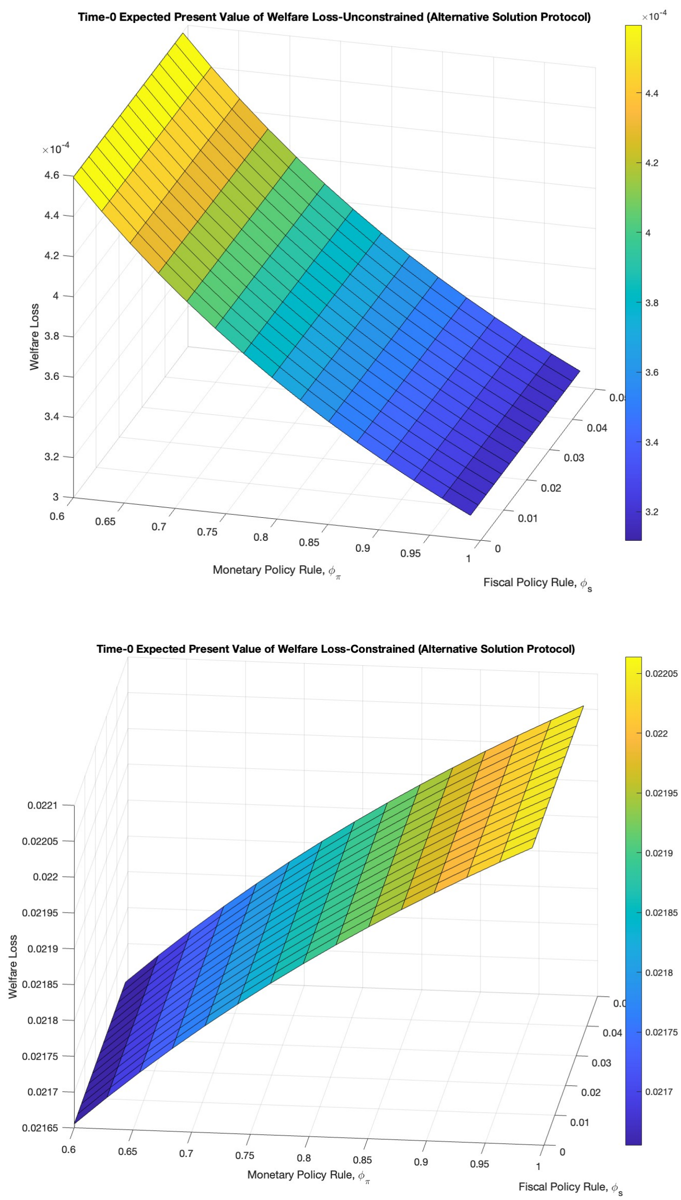
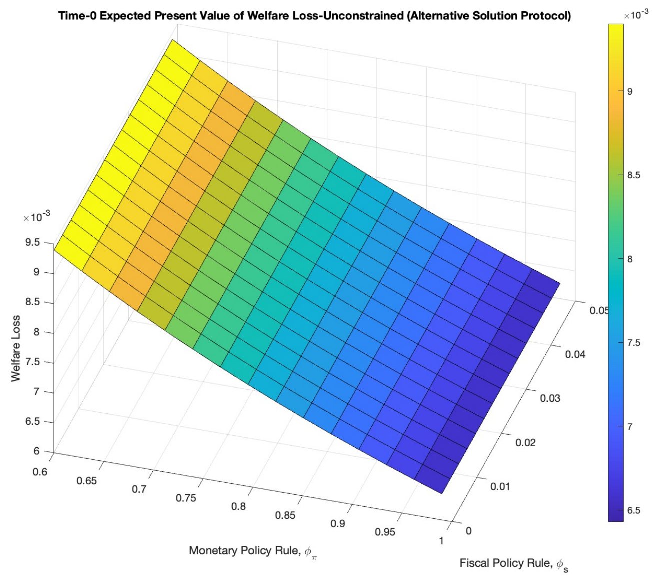


Figure 7: WL with Alternative Solution Protocol Fiscal Shock. Top Figure is unconstrained FD. Bottom figure is FD with Policy Inflexibility



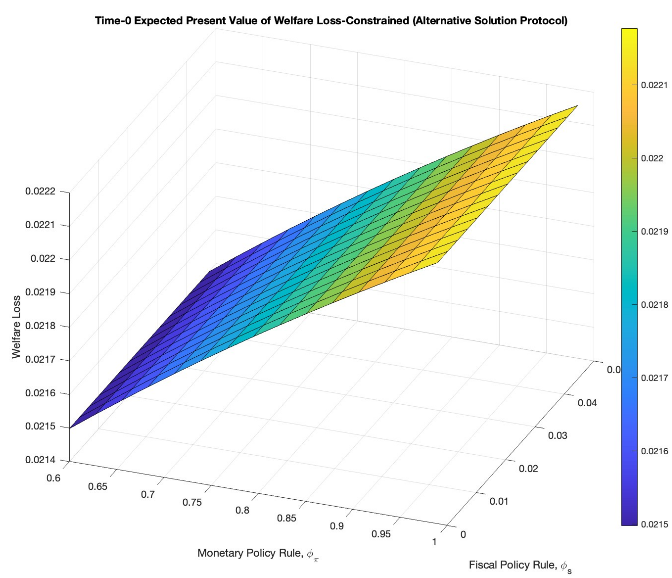


Figure 8: WL with Alternative Solution Protocol, IS Shock.op Figure is unconstrained FD. Bottom figure is FD with Policy Inflexibility

fluctuation and volatility is incurred in the first period and the need to employ forward guidance along the equilibrium sequence mitigated. The commitment to both objectives highlighted in the analysis of the IRFs for the model solved with the LS approach and absent in the present case as a consequence gets corroborates as the source of the highly distortionary and deflationary dynamics in the former.

7 Conclusion

With fiscal dominance plausibly on the rise in advanced economies, and already an endemic reality in many developing ones, understanding dynamics and optimal policy benchmarks in these scenarios is essential for modern macroeconomic policy. Fiscal dominance and FTPLs models, however, are fundamentally premised on a perfectly passive monetary policy, i.e. such to enable any level of debt or bond sales backing the active surplus/revenue policy. But both in principle, say with a ”limitedly tolerant or cooperative” monetary authority concerned with solvency or willing to allow for money growth to accommodate fiscal policy up to but not past some point, and as an empirical norm, as is the case for legislated or debated ceilings on debt, there might be in general less than perfect willingness to support overshooting some bounded quantity of debt. Finally, the constraint on new borrowing or money growth beyond some ceiling can be the product of ”original sin” and ”debt intolerance” phenomena highlighted by the monetary economics literature in developing economies. Our work thus addressed as empirically and theoretically motivated extension to the theory of dynamics and optimal policy in fiscally dominant economies.

The paper offers unifying framework for raising and addressing these questions. The framework we propose, building on recent contributions and unifying NK, FD, and FTPL, is a DSGE sticky-price model admitting both MD and FD as alternative (mutually exclusive) solutions to indeterminacy of the equilibrium inflation path, and extended to an endogenous regime switching case cause by the presence of an occasionally binding upper boundary to quantity of debt or money at which the economy is willing to operate. We study transitional dynamics in response to demand-side shocks, as well as optimal monetary-fiscal rules.

On the theoretical side, we believe that this paper is able to contribute methodologically and with its applied findings to the growing literature on FD economies and the optimality of inflation targeting (with substantially different results, for example, relative to the benchmark FD in both our model and Kumhof et al, 2010) by offering theoretical insights to complement and check for robustness pure-form, perfectly-flexible models of fiscal dominance and fiscal theories by considering scenarios with endogenously arising when relaxing, as motivated earlier, the assumption of an unconditionally passive monetary side.

To the best of our knowledge, the closest exercise to our work in the literature on non-linear FD and occasionally binding constraints is proposed by Schmidt

(2024). In case, however, the relevant inflexibility is provided by the unwillingness of a passive fiscal authority to raise fiscal surplus beyond some upper ceiling in a ”normal times” monetary-dominance economy. While similar in drawing attention to the role of less-than-perfect flexibility on the passive policy side for ensuing dynamics, we view our work as complementing this recent contribution, by studying a concern and relaxation symmetric to the one the latter proposes in FD regimes. In our case, we study economies in which rules or institutional constraints are such that fiscal dominance – rather than monetary dominance is the norm – and the occasional inflexibility point comes from a passive monetary authority, rather than the passive fiscal authority. As such, our contribution appears complementary to this recent exercise by studying the role of inflexible passive policies in economies in which fiscal dominance constitutes the norm, such as chiefly many developing countries.

In terms of applied contributions, we highlighted a number of results on transitional dynamics and associated optimal policy benchmarks for FD economies with occasionally inflexible monetary authorities isomorphic to the introduction of debt ceilings: (i) dynamics display emergent properties, in the form of endogenous volatility and cycles, due to the second-best nature of the equilibrium path; (ii) at stronger inflation targeting stabilisation is more difficult to achieve than with a perfectly flexible passive policy; (iii) more dovish monetary rules are better, in terms of welfare loss, than more hawkish responses; (iv) gradualism in policy reform could lead to a lower Pareto solution; and (v) more systematic fiscal policy intervention could approximate the optimal regime, opening the way to uncoventional fiscal-monetary coordination.

Appendix

A1. Equilibrium Trimming: Equivalence of Paths-Selection and Innovation/News Selection

Let inflation at time (t+1) π_{t+1}. Then time t+1 inflation expectations at time t, labelled $\pi^e_{t+1} := \mathbb{E}_t \pi_{t+1}$ with $\mathbb{E}_t : \mathcal{V} \times \Omega \to \mathbb{R}$ the mathematical conditional expectations operator with respect to the sigma-field/information set at time t, with $\mathcal{V}$ the set of random variables on Ω, are by definition of the conditional expectation and a standard, weakly increasing filtration of information sets to model the resolution of uncertainty over time: (a) measurable functions of time t information, and (b) measurable with respect to time t+1 information. Then, consider a candidate decomposition of inflation at t+1 into its (measurable) projection on the time t-information set and some correction/updating term v with unknown timing (of the first time it becomes measurable):

$$\pi_{t+1} = \pi^e_{t+1} + v_{t+s},$$

where $s \in 0,1 \quad \forall t$, is a timing variable to denote whether v is known/measurable at time t or t+1. To solve consistently the timing problem based on the information data and belief structure, take conditional expectations on either side to obtain

$$\mathbb{E}_t \pi_{t+1} = \mathbb{E}_t \pi^e_{t+1} + \mathbb{E}_t v_{t+s}$$

$$E_t v_{t+s} = 0.$$

Here we have used the measurability of the projection π^e_{t+1} on the time t information set when rational beliefs are modelled as mathematical conditional expectations. Hence the linear updating term is conditional mean zero relative to the information set at time t. It also must have zero unconditional mean, since, when integrating/averaging over the event space (with well defined P-measure on it as per the standard definition of sigma-field and P-measure):

$$\int_\Omega \pi_{t+1} P(d\omega) = \int_\Omega \pi^e_{t+1} P(d\omega) + \int_\Omega v_s P(d\omega)$$

$$\int_\Omega v_s P(d\omega) = 0 = E[v_s|\Omega] = E v_s$$

Where the second equality uses the fundamental equation of conditional expectations over measurable sets. We thus have that since $E_t v_s = E v_s$ the linear update v_s is orthogonal/conditionally mean independent from information at time t, for all t, and hence consistently timed v_{t+1}. Then, because $E_t \pi_t$ is known/measurable at t and, with weakly resolving uncertainty/weakly increasing filtration, at $t+1$, it must be the case that when picking the current inflation rate, with prior (rational/full information) conditional expectations formed at t constant by construction from t onwards, the economy must be picking the innovation against efficient conditional expectations/beliefs from last period. In this sense, MD and FD as second order selection mechanisms can be equivalently viewed, along a rational expectation equilibrium sequence, as selecting who gets to pick *inflation or price level news*.

A2. Equilibrium Equations at a First-Order Perturbation

$$\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma}(i_t - \hat{\pi}_{t+1} - \rho + u_t) \tag{30}$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa(\sigma \hat{c}_t + \phi \hat{y}_t) \tag{31}$$

$$i_t = \rho + \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t \tag{32}$$

$$\hat{\tau}_t \tau_{\text{ss}} = \phi_\tau (1+\rho) A(\hat{b}_{t-1} + i_{t-1} - \hat{p}_{t-1} - \rho) \tag{33}$$

$$A\hat{b}_t = -\hat{\tau}_t \tau_{ss} + \gamma_g g_t + A(\hat{b}_{t-1} + i_{t-1} - \rho) + A\rho(\hat{b}_{t-1} + i_{t-1} - \rho - \hat{p}_t) \tag{34}$$

$$\hat{\pi}_t = \hat{p}_t - \hat{p}_{t-1} \tag{35}$$

$$\hat{y}_t = \gamma_c \hat{c}_t + \gamma_g g_t \tag{36}$$

Note that the above perturbation applies to the normal times economy or economy in which the occasional inflexibility constraint $b \leq bmax$ holds with slackness. At the endogenously occurring inflexible stance regime (i.e. whenever the nominal debt is not an interior point of the admissible set of nominal debts under an inflexible monetary authority $b_t \leq b_{max}$) holds, the previously slack inequality constraint holds now tightly. This regime necessitates that equilibrium nominal debt hits and gets stuck at the boundary point $b_t = b_{max}$.

This constrained case is exploited to solve the model in the sequence space, through the outlined OccBin or linear piece-wise rootfinding algorithm, under a verifiable guess that the economy eventually reverts to normal times, implying that the above equations describe dynamics after the last period in which the constraint holds tightly, and can be used to solve backward for dynamics in the constrained case under the assumption b_t is stuck at b_{max} boundary. Naturally, the timing of reversion to normal times, and hence point from which we solve backwards, is endogenous and solved by iterating to convergence of the guess on the terminal period in which the constrained state applies. Section A3.2 discusses the implications for the determination of the price level and inflation path in such case. Two alternative procedures are in principle available for constructing the constrained economy along the sequence of periods in which the constraint is guessed to hold tightly. For details on this, see appendix A4.

A3. Fiscal Theory of the Price Level implied by FD Device

We discuss here the FTPL at play in the model and the price level and inflation path determination with fiscal dominance when the passive monetary policy is perfectly flexible; we then compare it to FD with occasionally inflexible passive policy. This allows us to formulate a general FTPL for the FD models under study.

A3.1. FTPL with Perfectly Flexible Passive Monetary Authority

Consider the expression for the equilibrium price level as a jump-variable updating to match the bounded forward solution for the price level under the FD device and perfectly flexible passive monetary authority. We now show that the FD device implies a FTPL, similar to the one obtained by Cochrane, characterised by two essential and linked features. First, as we have already seen, the fact that conditional expectations on next period debt, as long as rational in the sense of correctly forecasting that beginning of next period real debt will in turn be pinned down as a jumping solution to the forward equation, i.e. picked by the FD device, that such expectations do not directly matter to price and inflation path determination conditional on the forecast of the fiscal policy process. This is a technical condition mirroring the fact that, under the FD device with a fully flexible monetary policy, the latter, which works through real debt forecasts, does not matter to equilibrium picking. Second and immediately related, exploiting the well known link between bounded solutions to forward-integrated stochastic difference equations and (bounded by construction) undetermined coefficients solutions for jump variables, that the FD device works by effectively making the fiscal policy process (the deficit-funded fiscal policy shock) the only state variable relevant to equilibrium picking. Both conditions cease to be true at the point of occasional policy inflexibility, providing a straightforward and economically meaningful way of understanding the impact of endogenous regime switching on equilibrium picking in the generalised model of FD we study.

Letting again ξ the AR coefficient on debt, which we recall to be strictly lower than one, induced by the sufficiently weak updating of the tax revenue instrument:

$$\frac{B_{t-1}}{P_t} = -\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k G^* e^{g_{t+k}} + Z^* \sum_{k=0}^{\infty} \xi^k$$

As already noted, with B_{t-1} pre-determined and measurable wrt to the information set $t-1$ and hence at t, the above equation determines the price level which raises, ceteris paribus, for expansionary fiscal policy shocks and decreases for higher Z^*. Under our assumption that we are in the vicinity of the stationary equilibrium path, and approximating the nonlinear government expenditure shock by a first-order perturbation around that:

$$\frac{B_{t-1}}{P_t} = -\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k G^* (1 + g_{t+k}) + Z^* \sum_{k=0}^{\infty} \xi^k$$

$$\frac{B_{t-1}}{P_t} = -\mathbb{E}_t G^* \sum_{k=0}^{\infty} \xi^k - G^* \sum_{k=0}^{\infty} \xi^k g_{t+k} + Z^* \sum_{k=0}^{\infty} \xi^k.$$

Under our standard assumption that the government expenditure shock is governed by a stationary and stable-at-the-origin Markov process (i.e. a univariate AR(1) process with stable root):

$$g_{t+1} = \rho_g g_t + \epsilon_{g,t+1} \quad with \quad |\rho_g| \leq 1$$

The noise $\epsilon_{g,t+1}$ is a Gaussian noise process orthogonal to the time-t information set, hence measuring fiscal policy news or innovations against the conditional expected value. We can give for all $k \in \mathbb{N}$ a standard Moving Average (MA) representation to the stochastic sequence g_{t+k} through a backward integration of the law of motion – such that in the limit of time tending to infinity the displacement traced by the process is approximated by white noise around the stationary zero average, and this is independent of initialisation conditions (i.e. the process is in fact indeterministic). In particular, this yields:

$$g_{t+k} = \rho^k g_t - \epsilon_{g,t} + \sum_{s=0}^{k} \rho^s \epsilon_{g,t+s}.$$

Plugging such sequence of MA representations, for all k, in into the equilibrium price level equation:

$$\frac{B_{t-1}}{P_t} = -\mathbb{E}_t G^* \sum_{k=0}^{\infty} \xi^k - G^* \sum_{k=0}^{\infty} \xi^k \left(\rho^k g_t - \epsilon_{g,t} + \sum_{s=0}^{k} \rho^s \epsilon_{g,t+s} \right) + Z^* \sum_{k=0}^{\infty} \xi^k.$$

$$\frac{B_{t-1}}{P_t} = -\mathbb{E}_t G^* \sum_{k=0}^{\infty} \xi^k - G^* \sum_{k=0}^{\infty} \left((\xi\rho)^k g_t - \xi^k \epsilon_{g,t} + \xi^k \sum_{s=0}^{k} \rho^s \epsilon_{g,t+s} \right) + Z^* \sum_{k=0}^{\infty} \xi^k.$$

$$\frac{B_{t-1}}{P_t} = -\mathbb{E}_t G^* \sum_{k=0}^{\infty} \xi^k - G^* g_t \sum_{k=0}^{\infty} (\xi\rho)^k + G^* \epsilon_{g,t} \sum_{k=0}^{\infty} \xi^k - G^* \sum_{k=0}^{\infty} \xi^k \sum_{s=0}^{k} \rho^s \epsilon_{g,t+s} + Z^* \sum_{k=0}^{\infty} \xi^k.$$

Passing the conditional expectation operator and using measurability and Gaussianism of the fiscal news process:

$$\frac{B_{t-1}}{P_t} = -G^* \sum_{k=0}^{\infty} \xi^k - G^* g_t \sum_{k=0}^{\infty} (\xi\rho)^k + Z^* \sum_{k=0}^{\infty} \xi^k.$$

Applying the Neumann Series Lemma (NSL) based on an observation that the coefficients in which power series are defined are all strictly below one so that all infinite summations converge, we obtain our FTPL equation for the equilibrium, jumping price variable as a function of time-t measurable information:

$$\frac{B_{t-1}}{P_t} = \frac{1}{1-\xi}(Z^* - G^*) - \frac{1}{1-\xi\rho} G^* g_t.$$

$$\frac{B_{t-1}}{P_t} = \frac{1}{1-\xi}(Z^* - G^*) + \frac{1}{\xi\rho - 1} G^* g_t.$$

Let

$$\frac{1}{1-\xi} + \frac{1}{\xi\rho - 1} = \frac{(\xi\rho - 1) + (1-\xi)}{(1-\xi)(\xi\rho - 1)} = \frac{\xi\rho - \xi}{(1-\xi)(\xi\rho - 1)} := \delta \geq 0.$$

Then

$$\frac{1}{1-\xi} = \delta\frac{1}{\delta(1-\xi)} = \delta\left(1 - \frac{1}{\delta(\xi\rho - 1)}\right)$$

Hence

$$\frac{B_{t-1}}{\tilde{P}_t} = \left(1 - \frac{1}{\delta(\xi\rho - 1)}\right)(Z^* - G^*) + \frac{1}{\delta(\xi\rho - 1)}G^* g_t.$$

$$\frac{B_{t-1}}{\tilde{P}_t} = (1 - \lambda_{(\xi,\rho)})(Z^* - G^*) + \lambda_{(\xi,\rho)}G^* g_t.$$

Where $\tilde{P}_t = h(P_t) = \delta P_t$ is a monotonicity-preserving transformation generated by the operator $h(\delta) : \mathcal{P} \rightarrow \mathcal{P}$ on the state space/range of price levels traversed by the stochastic sequence P_t. Hence, the FTPL implied by the FD device with perfectly flexible passive policy implies that, given the extant amount of nominal debt to be inflated or deflated away, the price level is thereby determined as a statistic of a weighted average of the long run real debt prevailing in the FD economy (the $Z^* - G^*$ term) and the current fiscal policy shock that is measurable/observable for the first time in the current period.

Effectively (cf. Cochrane, 2011), as is generally the case for equilibrium jumping-variable pinned down as the bounded solution to the forward integrated equation with disallowed explosions – i.e. through the FD threat-device here – these effectively converge to sequences generated by a measurement equation in the aggregate variables that the device picks as the true "state" vector for the economy as far as price determination is concerned. The effect of FD through the outlined mechanism can be read from the above equation, in this sense, as effectively making the fiscal expenditure process the single state variable of import to price-level determination. A similar derivation for the MD case with a noisy Taylor rules similarly shows that the single state variable in such case would be the monetary policy shock. Because, finally, the price is effectively defined, from a time series viewpoint, as a filter on the policy shock, its properties in the time and frequency domains are inevitably tied to those. We note, in particular, pace those insisting that FTPLs of the kind delivered by our FD model imply a (solvent, as already discussed) unstable real debt spiral, that this is not the case. In particular, as long as the fiscal policy process g_t is stable (which is warranted under the necessary, standard BK-Uhlig assumptions to solve DSGE models), it easy to verify that, in the limit $t \rightarrow \infty$, the real debt converges to a bounded, stationary value given by the term in brackets. The threat mechanism underlying equilibrium picking under an FD device (or an MD device, as a parallelism) never resolves in the threats being actually carried out – this is because equilibrium picking is precisely enforced by the need to avoid a credible, but then by construction off-equilibrium, threat.

A3.2. FTPL with Occasionally Inflexible Monetary Authority as an Endogenous Regime-Switching Model

How does the picture change when the FD device operates under occasionally inflexible policy? The case is rather straightforward to derive as a limitative case of the FTPL derived above. In particular, note that the above FTPL implicitly rests on recursively pinning down (endogenous) rational expectations on beginning-of-next-period real debt,together with the noted non-explosion boundary conditions, such that the equilibrium extant real debt at the start of the period / period price level is pinned down as the usual jumping, bounded solution to the forward-integrated stochastic difference equation generated by the weak tax revenue instrument. Under occasionally inflexible monetary policy, this is importantly not the case and hence forecast by rational agents. Technically, the key point is that we can't (consistently) integrate forward the target equation as as the conditional expectation of next period real debt will be pinned down by the conditional expectation of real debt at the inflexibility point as opposed to, as usually under "normal times", in terms of the usual solution/jumping variable. Rather, the emission of nominal debt is capped at the inflexibility point, yielding:

$$\frac{B_{t-1}}{P_t} = \xi \mathbb{E}_t \frac{B_{max}}{P_{t+1}} - \xi G_t + \xi Z^*$$

Here, fiscal policy still contributes to the determination of the price level via the term $G_t \approx G^*(1+g_t)$, but ceases to be only relevant state variable. What is the economic intuition supporting the above technical condition? The idea is that, under the occasionally inflexible policy, nominal debt is by construction prevented from following the explosive path coherent with "sufficiently weak" tax revenue rules. Since agents have rational expectations and correctly anticipate this, this removes the key threat-based mechanism for equilibrium picking under the FD device. As such, absent a credible threat-based mechanism, fiscal policy fails to unilaterally induce the usual equilibrium behaviour of prices and hence next period extant real debt such to fend off explosions. This is rationally anticipated by agents, implying that the forecast of extant real debt at the inflexibility point, i.e. of prices tomorrow or equivalently of the inflation rate, also matters to the determination of the price level and inflation rate today. In particular, we revert to a part-Phillips-Curve scenario at which inflationary expectations (rather than expectations on fiscal shocks only as in the FD case in A3.1) drive inflation today. As we remain by construction, despite occasional inflexibility, in the realm of a passive monetary rule falling short of a Taylor principle (and hence consistent, contra erroneous interpretations, with expected inflation targeting), monetary policy rules hence concurrently matters to current price level and inflation rate determination through using its nominal rate instrument to tie down inflationary expectations. At an occasionally inflexible policy point, we interpret the above to mean that price level and inflation rate determination is achieved through a hybrid mechanism in which monetary and fiscal policy interact up to satisfying the above equilibrium equation. As we outlined in the benchmark results section, in particular, this can be exploited by the central bank at the occasionally inflexible policy by using deflationary policy in the next period to reduce, with disinflationary or deflationary implications relative to the unconstrained benchmark, the price level today – as such, to keep in check the price that would be picked by fiscal policy alone.

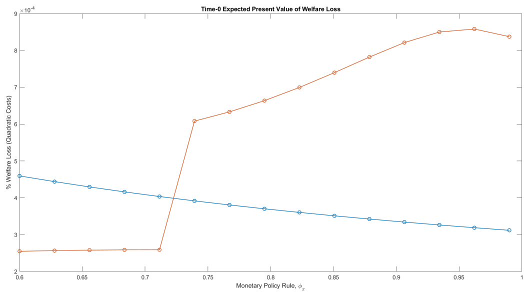
The analyses we propose based on the above model hence can be viewed as largely exploring the implications for macroeconomic and policy transmission and the design of optimal, interacting fiscal and monetary rules for macro-stabilisation when the above hybrid-mechanism for equilibrium selection occasionally kicks in with endogenous timing, on the grounds of endogenously occurring inflexibility of the passive monetary authority, and thereby supplants the mechanism outlined in A3.1. In other words, to link back to the original regime-switching and non-linearities literature, when along an equilibrium path there is some *endogenous transition probability* of moving from a pure FD regime representing the default or "normal times" state of the economy, in which only the stream of deficit-funded fiscal policy matters for equilibrium picking, to the hybrid mechanism in which fiscal and monetary policy compete in determining the price level and inflation path.

A4. Calibration

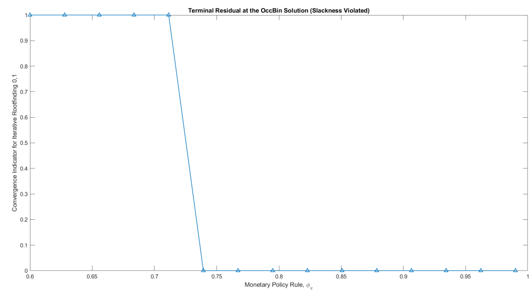
Table 2: General Parameters

Parameter	Description	Value
β	Households Discount factor	0.95
ρ	Long run interest rate along stationary path	$-\log(\beta)$
κ	Slope of the Phillips curve (microfoundable)	0.17
σ	Intertemporal elasticity of substitution of private consumption	1
ϕ	Inverse Frisch elasticity of labour supply	2
γ_c	Percentage of private consumption	0.8
γ_g	Percentage of government consumption	0.2
ρ_g	AR parameter for governemnt consumption	0.4
ρ_u	AR parameter for β shock	0.2
$\bar{B}$	Long run real debt to real output ratio	2.5
$\bar{P}$	Long run price index	1
A	Long run gross real liabilities	$(1+\rho)\cdot\frac{\bar{B}}{\bar{P}}$
τ_{ss}	Long Run taxation	$\gamma_g + \rho \cdot A$
$\bar{B}_{\max}$	$\bar{B}$ at the OccBin solution	2.501
$b_{\max}$	Bound on deviation of nominal debt with occ. inflex.	$\frac{(\bar{B}_{\max}-\bar{B})}{\bar{B}}$
ϕ_{π}^{ub}	Elasticity of i_t to inflation at upper boundary (FD/PI)	0.99
ϕ_{τ}^{ub}	Elasticity of τ_t to lagged liabilities at upper boundary (FD/PI)	$\frac{\rho}{1+\rho}$

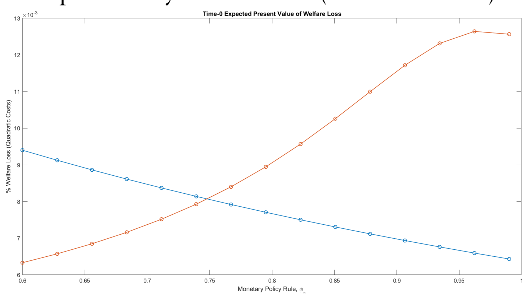
A5. Welfare Loss Planar Projections



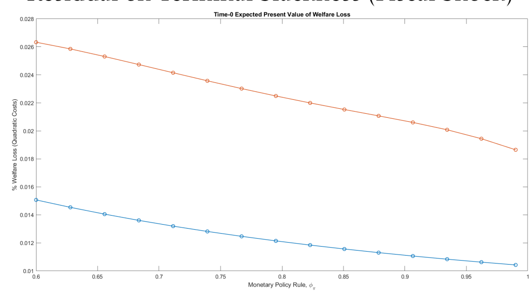
Expansionary Fiscal Shock (Deficit-Funded)



Residual on Terminal Slackness (Fiscal Shock)



IS/Financial shock (Expansionary)



IS/Financial Shock + Fiscal Contraction

Figure 9: Projections for expected Welfare Loss at t_0 for different values for ϕ_π on the plane at $\phi_\tau = 0$.

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