Modern Universal Growth Theory (MUGT): A comprehensive upgrade to Solow

de la Fonteijne, Marcel R.

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A Comprehensive Upgrade to Solow

The foundation of economic growth theory

Marcel R. de la Fonteijne

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Abstract

For several decades, it has been recognized that the implementation of capital and labor augmented technical progress, as is done to date, leads to a theoretical paradox: either the CES production function has to be Cobb-Douglas or there exists labor augmented technical progress only. This so-called "Cobb-Douglas or labor augmented technical progress only paradox" continues to appear in economic models despite its inconsistency. In this paper, we reject the conventional approach, i.e., all kind of neutral and non-neutral capital and labor augmented technical progress and propose a revised implementation of technical progress that resolves the paradox. Economic growth is modeled as partly exogenous, driven by technical change, and partly endogenous, driven by capital accumulation. We provide formulas to translate total factor productivity (TFP) into economic growth to show the connection, thereby clarifying the link between TFP and output dynamics. This approach offers a new perspective on the Solow model and opens alternative paths for investigating endogenous growth mechanisms.

Keywords: Capital and Labor Augmented Technical Progress, Growth Model, Maximum Profit Condition, Production Functions, General Technological Progress, Capital-Labor-mix, Elasticity of Substitution, DSGE, Total Factor Productivity, Solow model, Hicks, Harrod

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Content

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Contact information:

- Marcel R. de la Fonteijne
- DLF Macro Economic Research
- Montgomerylaan 218
- 2625 PV Delft
- The Netherlands
- +31621885101
- marcel@dlfeconomics.com
- https://dlfeconomics.com

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1 Introduction

The realm of economic theory is characterized by its dynamic nature, continually evolving to address the complex interplay of factors that drive economic growth and development. At the forefront of this evolution stands the growth theory, a pivotal framework that attempts to elucidate the mechanisms behind the expansion of economies over time. Rooted in historical context and enriched by the contributions of visionary economists, the growth theory has undergone significant transformations, with the works of Robert Solow, Hirofumi Uzawa, John Hicks, and Sir Roy Harrod serving as milestones in its development.

The growth theory, as envisaged by Robert Solow in the mid-20th century, emerged during a period of post-World War II recovery and reconstruction. Solow's pathbreaking research laid the foundation for understanding the drivers of economic growth by introducing the concept of technological progress as a central determinant. His seminal model, often referred to as the Solow-Swan model, highlighted the roles of capital accumulation and technological advancements in fostering sustained economic growth. By distinguishing between short-term fluctuations and long-term trends, Solow's work established a framework that would inspire subsequent economists to delve deeper into the intricate dynamics of growth.

Building upon Solow's work, Hirofumi Uzawa ventured into the realm of endogenous growth theory, which sought to explain the sources of technological progress itself. Uzawa's groundbreaking contributions illuminated the role of human capital and education in propelling economies forward. He postulated that investments in education and research could lead to self-sustaining growth, where the pursuit of knowledge fuels innovation and productivity enhancements. Uzawa's insights challenged the conventional wisdom of exogenous technological progress and spurred a new wave of research into the determinants of innovation-driven growth.

In parallel, John Hicks and Sir Roy Harrod enriched the growth theory by introducing concepts that delved into the nuances of economic instability and fluctuations. Hicks' theory of capital utilization and its dynamic adjustment in response to changes in demand provided a lens through which economists could understand the cyclical nature of growth. Harrod, on the other hand, delved into the intricacies of economic instability arising from the mismatch between savings and investment. His work highlighted the potential for instability even within a framework of long-term growth, emphasizing the need for policy interventions to mitigate economic fluctuations.

The amalgamation of these visionary contributions not only expanded the scope of the growth theory but also paved the way for a more comprehensive understanding of the intricate forces at play in the realm of economic growth. From Solow's fundamental insights into capital accumulation and technological progress to Uzawa's emphasis on human capital and endogenous innovation, and from Hicks' and Harrod's analysis of economic fluctuations to their implications for policy, these economists collectively wove a tapestry of theories that continues to shape modern discussions on economic development.

It is at this point that our contribution to the theory begins. To be more specific, in order to mathematically describe economic growth through production functions, factors for labor and capital improvement were introduced to represent technical progress.

In the 1960s, pioneering economists like Solow employed capital and labor factors within Cobb-Douglas and CES production functions to model economic growth. Although these models gained wide acceptance, they never fully dispelled concerns about an underlying inconsistency. Even at the time, economists debated key aspects of the approach, sensing that something fundamental was missing. This theoretical inconsistency has persisted, yet the model remains widely accepted and continues to appear in standard textbooks on modern growth theory. This inconsistency was addressed in De la Fonteijne (2018), which demonstrated how technical progress must be implemented in a two-factor, homogeneous degree 1 CES production function to allow for a balanced growth path (BGP) with a constant capital-to-income ratio.

We will use the term Modern Universal Growth Theory to refer to the corrected version of the growth theory originally introduced by Solow, among others, in the in1960s of the 20th century, in which the factors of technical progress have been adapted in order to solve the inconsistency. You can find detailed information about the proof of mentioned inconsistency in De la Fonteijne (2018).

In Section 2, we introduce a simple economy model.

In Section 3, we elucidate that in the old theory we are left with a choice: either the CES production function must be Cobb-Douglas, or it must rely solely on labor-augmenting technical progress. From an economic perspective, neither option is realistic. We therefore reject both and conclude that the root of the problem lies in the way technical progress is incorporated. The only viable resolution is to revise the implementation of technical progress itself.

Moving to Section 4, we show how technical progress is implemented in our Modern Universal Growth Theory (MUGT). We introduce Total Factor Productivity (TFP) in the production function and demonstrate what has to be done to make a Balanced Growth Path possible. Moreover, we show the relation between TFP and productivity growth in a CES production function.

Concluding our discussion in Section 5, we offer a summary of key insights and remarks drawn from our analysis.

2 The equations of the economic system

The system under consideration consists of the following equations:

The production function expressed in its base point (Y_0, K_0, L_0) with parameters α and γ

$$Y = Y_0 \left[\alpha \left(\frac{K}{K_0}\right)^{\gamma} + (1-\alpha)\left(\frac{L}{L_0}\right)^{\gamma}\right]^{1/\gamma} \qquad (1)$$

which is a homogeneous degree 1 production function, α is the capital-labor-mix and σ is the elasticity of substitution

$$\sigma = \frac{1}{1-\gamma} \tag{2}$$

$$Y = C + I \tag{3}$$

$$C = c_1 Y \tag{4}$$

c_1 is the part of income not being depreciation.

$$\dot{K} = I - \delta K \tag{5}$$

Additionally, we have the equations with the wages, profit and depreciation

$$Y = wL + (r+\delta)K \tag{6}$$

The derivatives are

$$\frac{\partial Y}{\partial K} = \frac{\alpha Y}{\alpha\left(\frac{K}{K_0}\right)^{\gamma} + (1-\alpha)\left(\frac{L}{L_0}\right)^{\gamma}} \left(\frac{K}{K_0}\right)^{\gamma-1} \frac{1}{K_0} \tag{7}$$

$$\frac{\partial Y}{\partial L} = \frac{(1-\alpha) Y}{\alpha\left(\frac{K}{K_0}\right)^{\gamma} + (1-\alpha)\left(\frac{L}{L_0}\right)^{\gamma}} \left(\frac{L}{L_0}\right)^{\gamma-1} \frac{1}{L_0} \tag{8}$$

Using the maximum profit condition, we require

$$\frac{\partial Y}{\partial K} = r + \delta \tag{9}$$

and

$$\frac{\partial Y}{\partial L} = w \tag{10}$$

Notice that with arbitrary choice $c_1 \in (0,1)$ the system has a unique and stable solution.

3 The rejection of the implementation of the Solow Growth Theory

In general, the Solow approach takes the following form

$$Y = Y_0 \left[\alpha_0 \left(\frac{A_K K}{K_0}\right)^{\gamma} + (1-\alpha_0)\left(\frac{A_L L}{L_0}\right)^{\gamma}\right]^{1/\gamma} \tag{11}$$

$$\sigma = \frac{1}{1-\gamma} \tag{12}$$

Y is income and K is capital.

The production function is described in its base point (Y_0, K_0).

A_K and A_L are factors for Capital and Labor technical progress, respectively.

Apart from the Cobb-Douglas case ($\sigma = 1$) for every choice of $A_K \neq 1$ you will eventually end up with a labor only or capital only economy. However, this does not correspond to what we observe in reality. Therefore, it is commonly assumed that $A_K = 1$, meaning that technical progress is labor-augmenting only.

In total, only two options remain:

1. The elasticity of substitution equals 1 (i.e., the production function is Cobb-Douglas), or
2. Technical progress is labor-augmenting only (the Harrod-neutral case).

Both options allow for the possibility of achieving a Balanced Growth Path (BGP), while maintaining the same characteristics of the production function at the new base point.

The case that an arbitrary production process should always be Cobb-Douglas is very unlikely and not realistic. Even so, assuming that technical progress is only labor augmented does not match reality.

We therefore reject both and conclude that the root of the problem lies in the way technical progress is incorporated. The only viable resolution is to revise the implementation of technical progress itself.

What is most problematic is the placement of technical progress within the production function, specifically, directly in front of the capital and labor inputs. This position is typically reserved for scaling the *quantities* of those inputs. If the goal is to represent improvements due solely to technical progress, this should be done using a factor placed *outside* the production function, leaving the input quantities unchanged. This is because the production function itself already represents the transformation of inputs into output. The argument applies equally to both capital- and labor-augmenting technical progress.

In the next section, we adopt this revised approach.

4 The implementation of TFP growth in a CES production function

Consider a homogeneous degree 1, CES type production function.

Because the production function is homogeneous degree 1, we can write the production function in the intensive form with a technical progress term

$$y = \xi_{TFP} y_0 \left[\alpha_1 \left(\frac{k}{k_0}\right)^{\gamma} + (1 - \alpha_1)\right]^{1/\gamma} \tag{13}$$

$$\alpha_1 = \frac{\alpha_0}{\xi_{TFP}{}^{\gamma}} \tag{14}$$

$$\sigma = \frac{1}{1-\gamma} \tag{15}$$

y denotes income per capita or per hour worked—that is, labor productivity—while k represents capital deepening.

The production function is described in its base point (y_0, k_0).

Note that the technical progress term ξ_{TFP} is now placed outside the production function.

ξ_{TFP} expresses the factor of total factor productivity, which is the increase of productivity by technical progress only, expressed by moving from point (k_0, y_0) to point $(k_1, y_1) = (k_0, \xi_{TFP} y_0)$. If TFP increases e.g. 2 % then $\xi_{TFP} = 1.02$.

In contradiction of its misleading name, the factor ξ_{TFP} expresses only technical progress due to innovations, education, labor improvement, capital improvement, etc. and not through capital increase.

The relation between productivity growth ξ_g and TFP growth ξ_{TFP} is

$$\xi_g = \left(\frac{{\xi_{TFP}}^{\gamma} - \alpha_0}{1-\alpha_0}\right)^{1/\gamma} \tag{16}$$

ξ_g is the factor increase of productivity.

We will show that the productivity will indeed increase with ξ_g, i.e., $y_2 = y_0 \xi_g$.

If we take $\xi_{TFP} = 1$, then so is $\xi_g = 1$ and the production function at the beginning is

$$y = y_0 \left[\alpha_0 \left(\frac{k}{k_0}\right)^{\gamma} + (1-\alpha_0)\right]^{1/\gamma} \tag{17}$$

Now with ξ_{TFP} and after an increase of capital with factor ξ_g, i.e. $k_2 = \xi_g k_0$ the final result will be

$$y_2 = \xi_{TFP} y_0 \left[\frac{\alpha_0}{{\xi_{TFP}}^{\gamma}} \left(\frac{\xi_g k_0}{k_0}\right)^{\gamma} + \left(1 - \frac{\alpha_0}{{\xi_{TFP}}^{\gamma}}\right)\right]^{1/\gamma} = y_0 \xi_g \left[\alpha_0 + ({\xi_{TFP}}^{\gamma} - \alpha_0)/(\xi_g)^{\gamma}\right]^{1/\gamma} =$$

$$= y_0 \xi_g [\alpha_0 + (1-\alpha_0)]^{1/\gamma} = y_0 \xi_g \tag{18}$$

The capital to income ratio

$$\beta = \frac{k}{y} \tag{19}$$

If ξ_g is the total increase of productivity due to TFP and capital increase and we move from point $(k_1, y_1) = (k_0, \xi_{TFP} y_0)$ along the production function to point $(k_2, y_2) = (\xi_g k_0, \xi_g y_0)$ then the capital to income ratio β_2 will obviously remain the same as the original β_0 where we started with.

In the same way you can show that the new production function in its new base point $(k_{2,} \, y_2) = (\xi_g k_{0,} \, \xi_g y_0)$ has the same α_0 as the production function we started with. Of course, by solving the system under maximum profit conditions.

So, we start with equation 13 and use equation 14

$$y = \xi_{TFP} y_0 \left[\frac{\alpha_0}{\xi_{TFP}{}^\gamma}\left(\frac{k}{k_0}\right)^\gamma + \left(1 - \frac{\alpha_0}{\xi_{TFP}{}^\gamma}\right)\right]^{1/\gamma} = \xi_g y_0 \left[\frac{\alpha_0}{\xi_g{}^\gamma}\left(\frac{k}{k_0}\right)^\gamma + (\,\xi_{TFP}{}^\gamma - \alpha_0)/\,\xi_g{}^\gamma\right]^{1/\gamma} =$$

$$= \xi_g y_0 \left[\alpha_0 \left(\frac{k}{\xi_g k_0}\right)^\gamma + (\,\xi_{TFP}{}^\gamma - \alpha_0)/\,\xi_g{}^\gamma\right]^{1/\gamma} \qquad (20)$$

Using equation 16 the production function results in

$$y = y_2 \left[\alpha_0 \left(\frac{k}{k_2}\right)^\gamma + (1 - \alpha_0)\right]^{1/\gamma} \qquad (21)$$

which shows that with this implementation of technical progress the condition of a BGP is fulfilled. The new production function is described in its new basepoint and has the same capital-labor-mix α_0.

5 Concluding remarks on growth regarding homogeneous degree 1 CES production functions

Within the MUGT framework we employ three core parameters: the technical growth factor ξ_{TFP}, the capital-labor-mix α and the elasticity of substitution σ. When dealing with growth dynamics, it becomes essential to adjust the capital-labor-mix α with the factor $\left(\frac{1}{\xi_{TFP}}\right)^\gamma$ in $\alpha_1 = \alpha_0 \left(\frac{1}{\xi_{TFP}}\right)^\gamma$ to make a Balanced Growth Path (BGP) with constant capital to income ratio and constant capital-labor-mix possible.

In our view, this adjustment is not just a methodological choice, it is the only consistent way to incorporate technical progress. For formal derivations and proof, see De la Fonteijne (2018, 2024). A key implication is that the initial capital-labor-mix α_0 must be actively adapted throughout the growth process. However, the precise mechanisms driving these adjustments remain largely unexplained.

As a consequence, it is **never** possible to achieve a BGP without adapting α_0, apart from the Cobb-Douglas case.

The decision to opt for a final solution that maintains a constant capital-labor-mix might seem somewhat arbitrary, and in a certain way it is. On the other hand, it is the only viable choice possible if you require a BGP with a constant capital-labor-mix.

Relaxing this requirement opens up alternative growth trajectories. One could, for instance, define $\alpha_1 = \alpha_2 \left(\frac{1}{\xi_{TFP}}\right)^{\gamma}$ to express total change due to technical progress, where the difference between α_2 and α_0 is varying around zero, mirroring real-world scenarios. It results in a BGP with constant capital to income ratio and varying capital-labor-mix.

Moreover, if predictive or policy tools exist to anticipate or influence the evolution of either the capital-labor-mix α or the elasticity of substitution, you can leverage these insights to shape the economic growth trajectory accordingly.

For foundational perspectives, see Jones (2013) and Acemoglu (2009).

For further implications of the MUGT framework, refer to De la Fonteijne (2023).

The MUGT framework challenges the validity of an estimated 40% of the existing growth theory, which will need to be reconsidered or reformulated in light of the theoretical inconsistency it resolves.

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