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## Persistent Global Growth Differences and Euro Area Adjustment: Real Activity, Trade and the Real Exchange Rate

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Based on an estimated two-region dynamic general equilibrium model, we show that the persistent productivity growth differential between the Euro Area (EA) and rest of the world (RoW) has been a key driver of the EA trade surplus since the launch of the Euro. A secular decline in the EA's spending home bias and a trend decrease in relative EA import prices account for the stability of the EA real exchange rate, despite slower EA output growth. By incorporating trend shocks to growth and trade, the analysis departs from much of the open-economy macroeconomics literature which has focused on stationary disturbances. Our results highlight the relevance of non-stationary shocks for the analysis of external adjustment.

JEL Classification: F4, F3, E2, E3, C5

Keywords: global growth divergences, trade balance, real exchange rate, estimated DSGE model, Euro Area, demand and supply shocks, persistent growth shocks

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#### 1. Introduction

Over the past decades, the Euro Area (EA) has exhibited persistently lower output and productivity growth than its global competitors, notably the United States and China. Since the launch of the Euro in 1999, average annual real GDP growth has been 1.26% in the EA compared to 3.28% in an aggregate of the rest of the world (RoW) (1999–2023); GDP per person in the labor force—a proxy for aggregate productivity—grew by 0.70% per year in the EA, compared to 1.95% in RoW. This growth gap is projected to persist into the medium and long term (IMF WEO, 2025).

Over the same period, the euro area (EA) experienced a steady increase in trade openness vis-àvis the rest of the world (RoW), as measured by the ratio of extra-EA exports and imports divided by EA GDP. This EA foreign trade share rose from approximately 15% in 1999 to 30% in 2023. In contrast, the RoW trade share (EA–RoW trade relative to RoW GDP) remained broadly stable.

In striking contrast to the diverging EA and RoW growth trends, and to the trend in EA trade openness, the EA trade balance/GDP ratio and the EA real exchange rate (RER) have remained stable, and do not exhibit pronounced trends. Since the launch of the Euro, the EA has experienced a persistent trade balance (TB) surplus, averaging 2.6% of GDP.

Our paper's main contribution is to quantify the links between the EA-RoW growth differential, trade, and the RER. We develop a rich two-region New Keynesian dynamic stochastic general equilibrium (DSGE) model, and we estimate the model with Bayesian Maximum Likelihood methods, using EA and RoW data since the launch of the Euro. Productivity growth in each region is modeled as the sum of two components: a persistent autoregressive process capturing shocks to the trend growth rate, and a transitory component capturing short-run supply disturbances.<sup>2</sup> To account for the secular rise in the EA's trade share, the model incorporates permanent downward shifts in spending home bias (i.e., increased preference for imported goods), and in the relative price of EA imports, consistent with rapid productivity growth in RoW export sectors. In addition to these permanent disturbances, the framework includes a broad set of transitory shocks to aggregate supply, demand, and financial risk premia. The estimation sample encompasses several major macroeconomic and financial disturbances, including the global financial crisis, and the Covid pandemic. A model structure, such as ours, with a wide array of shocks and transmission channels, is thus needed to isolate and quantify the role of permanent productivity and trade shocks, which are the central focus of our analysis. A more restrictive model, with fewer alternative disturbances, could risk overstating the trend shocks.

Our estimated model suggests that highly persistent, region-specific productivity growth shocks have been key drivers of the significant and prolonged GDP growth differential between the EA and RoW. These shocks have also been an important source of the persistent EA TB surplus, accounting for 1/4 to 1/2 of the TB, in most periods. We find that the EA TB and RER were driven more by RoW productivity shocks than by EA productivity shocks. A positive shock to the RoW productivity trend growth rate signals a steady future increase in the *level* of RoW productivity

<sup>1</sup> Recent policy reports and debates, including the Draghi report (2024), underscore the urgency with which European leaders are approaching the EA vs. RoW growth gap.

<sup>&</sup>lt;sup>2</sup> Empirical support for this specification can be found, among others, in Aguiar and Gopinath (2007) and Fernald et al. (2017).

and GDP. In our estimated model, this boosts RoW consumption and investment demand which, in turn, triggers a *depreciation* of the EA RER and an improvement of the EA TB, in the short to medium run. The rise in RoW absorption crowds out EA investment, which depresses EA GDP. Over time, as the anticipated RoW productivity gains materialize, however, the EA TB deteriorates, and the EA RER appreciates, reversing the initial response. This response pattern explains why a succession of positive RoW growth rate shocks – such as those observed since the launch of the Euro – can generate a persistently elevated EA TB surplus, alongside appreciation pressures on the EA RER.

Our results suggest that the observed long-term stability of the EA RER, despite the large and persistent productivity and output growth differential vis-à-vis RoW, primarily results from shifts in EA trade patterns, namely a declining home bias in EA goods spending and a sustained decline in the relative price of EA imports. These trade shocks have increased EA demand for RoW goods, offsetting the long-run appreciation pressure on the RER stemming from the RoW-EA productivity growth differential. In sum, productivity growth shocks alone cannot account for the joint dynamics of relative output, the TB, and the trendless RER. These findings underscore the importance of incorporating structural trends in preferences for imports and in import prices into models of EA macroeconomic adjustment.

While the literature on TB and RER dynamics is too extensive to review here, it has established that a wide range of macroeconomic, financial, and trade shocks can influence both variables (see, e.g., Obstfeld, 2025, for a recent in-depth discussion of the U.S. TB). Consistent with this literature, our estimated model incorporates a broad set of structural shocks. The distinguishing feature of our analysis lies in the explicit treatment of *non-stationary* shocks to productivity growth and trade structure—features largely absent from open-economy DSGE models, which have focused on persistent but stationary disturbances (e.g., Backus, Kehoe, and Kydland, 1992; Obstfeld and Rogoff, 1995). Our empirical findings underscore the importance of non-stationary shocks in shaping external adjustment dynamics.

The theoretical prediction that a country's TB improves in response to faster foreign productivity growth is a feature of basic models with optimizing forward-looking agents (see Obstfeld and Rogoff, 1996, for a textbook treatment). A small number of previous studies have offered quantitative empirical analyses of this effect, but they relied on simpler models and differ in several key respects from the approach taken here. For example, Kollmann (1998) showed that a calibrated two-region RBC model with a widening productivity gap between the U.S. and the rest of the G7 can explain the rising U.S. trade deficit of the 1980s. Related explanations of the US TB deficit (in later sample periods) were provided by Engel and Rogers (2006), who used a model of endowment economies, and by Hoffmann et al. (2017), who estimated a two-country RBC model incorporating survey-based expectations of future growth.<sup>3</sup> In both models, a fully anticipated persistent rise in foreign growth induces a sharp but short-lived rise in the TB – a pattern driven by the immediate surge in foreign absorption. Both studies argue that this model-predicted front-loaded TB adjustment is at odds with the gradual TB dynamics observed in the data, and advocate models with imperfect information, in which agents gradually learn the persistence of productivity shocks—yielding a smoother TB response. By contrast, the present paper assumes full information

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<sup>&</sup>lt;sup>3</sup> See also Aguiar and Gopinath (2007) who show that a small-open economy RBC model with persistent shocks to the productivity growth rate captures the dynamics of the TB and consumption in emerging market economies.

regarding the persistence of growth shocks but introduces adjustment frictions in aggregate spending and trade flows (following, e.g., Auclert et al., 2024). We show that such real frictions are sufficient to generate gradual and persistent TB responses to productivity shocks, without the need to invoke imperfect informational.

The present paper is also related to a growing literature that studies the macroeconomic effects of trade shocks. For example, Clancy et al. (2024) present a calibrated DSGE model with reshoring shocks that reduce the import content of exports. Consistent with our findings, they show that a decline in home bias improves the TB in the short run and induces a persistent RER depreciation. Kollmann (2017) and Bodenstein et al. (2024) examine stylized calibrated models with *stationary* shocks to the import content of consumption, and find that such shocks account for a substantial share of RER fluctuations at business-cycle frequencies, reducing the explanatory role of uncovered interest parity (UIP) shocks for RER dynamics. In contrast to these studies, our estimated model incorporates *permanent* home bias shocks that simultaneously affect the import content of consumption, exports, and production. This richer propagation structure allows the model to capture long-run trade trends.

The persistent EA TB surplus vis-à-vis RoW has received little attention in the research literature. By contrast, a substantial body of work has analyzed *intra*-EA trade imbalances, particularly those associated with the boom-bust dynamics in Southern European economies around the global financial crisis (e.g., Kollmann et al. (2014), Philippon and Martin (2017), Cardani et al. (2022)). These studies stress the role of resource misallocation (Fernandez-Villaverde and Ohanian (2015), Gopinath et al. (2017)) and of financial market imperfections (Kollmann et al. (2016), Kemal Ozhan (2019), Jaccard and Smets (2020)) for TB adjustment. Given the economic weight of Southern Europe, such mechanisms likely contribute also to the EA's external position. Complementing this academic literature, policy reports, such as Draghi (2024), have argued that financial frictions have contributed to persistently weak investment in parts of the EA, thereby sustaining the region's trade surplus. In light of this literature, our estimated model incorporates financial frictions and a broad set of savings and investment shocks, enabling a comprehensive account of EA external adjustment.

Few other studies have *estimated* DSGE models of a comparable scale and empirical richness. This aspect is central to our contribution. We use Bayesian methods, drawing on 37 time series on prices and quantities across the EA and RoW. This setup allows us to test competing hypotheses on the drivers of the TB and RER and to assess their relative importance over time. Existing large-scale estimated models differ in both specification and focus. Coenen et al. (2018) examine productivity shocks within the EA. In contrast, our framework incorporates persistent growth and trade shocks in *both* the EA and the RoW. Other estimated EA models emphasize cyclical financial shocks (Kollmann et al., 2016) or commodity price shocks (Giovannini et al., 2019), while Cardani et al. (2022, 2023) study output and inflation dynamics during the COVID-19 pandemic. None of these studies analyze the joint behavior of the TB and the RER within a unified framework that incorporates both transitory and permanent shocks to trade and productivity.<sup>4</sup>

<sup>&</sup>lt;sup>4</sup> Our framework thus builds on earlier work by authors of the present paper (see also the European Commission's Global-Multicountry Model (Albonico et al., 2019)), but differs considerably in both specification and research focus. For example, our paper embeds generalized permanent growth-rate processes in both regions (for intermediates

## 2. EA and RoW growth trends and external adjustment

Our paper provides a quantitative framework to assess the role of persistent EA vs. RoW growth differentials in explaining three key empirical patterns: a trendless EA RER, a persistent EA trade surplus, and a rising EA import share. This Section documents these empirical facts.

Our model-based econometric analysis will focus on the period 1999-2023, i.e. it considers a sample that starts with the launch of the Euro (1999). The theoretical model assumes that the EA region has a common monetary policy, and thus it is natural to focus the empirical analysis on the period since 1999. Nevertheless, it is instructive to place the post-1999 period in perspective by reviewing growth trends over a longer horizon, 1960-2023.5

#### 2.1. Growth trends: 1960-2023

This Section documents large low-frequency changes in the growth rates of aggregate and per capita GDP for EA and RoW.6 To account for these persistent shifts, the structural model introduced below incorporates persistent shocks to productivity growth.

Panel a of Fig. 1 plots annual EA and RoW growth rates of real GDP, over the period 1960–2023. The solid blue and red lines depict year-on-year (YoY) growth for the EA and RoW, respectively. The dashed lines represent trend growth rates, computed as Hodrick-Prescott (HP) trends of the respective YoY growth rates (smoothing parameter:  $\lambda = 400$ ). In most years, the GDP growth rate was larger in RoW than in EA. The EA trend growth rate has declined steadily since the 1960s, from close to 6% p.a. to about 1%-1.5% p.a. after 2010. RoW trend growth also fell during the period 1960-90, from 6% to 3%, but rose slightly during the period 1990-2005, before gradually slowing thereafter.

The higher average growth rate in RoW partly reflects faster population growth (1.6% in RoW vs 0.4% in EA on average, 1960-2023). Panel b of Fig. 1 shows the growth rate of per capita GDP, which serves as a crude proxy for aggregate productivity growth. The trend growth rate of EA per capita GDP too has been declining since the 1960s (from close to 5% p.a. to about 1% in the 2000s). Interestingly, EA per capita trend GDP growth exceeded RoW growth until the late 1990.

productivity, export sector productivity and home bias) and estimate the steady state global growth rate. We also estimate time-varying input-demand elasticities. The RoW block and dataset are enriched with investment and a financial wedge, providing a more richer external environment than in previous models (e.g. Kollmann et al., 2016; Giovannini et al., 2019).

<sup>&</sup>lt;sup>5</sup> Data source: World Bank Development Indicators (WDI). As of this writing, the WDI provides annual data for 1960-

<sup>&</sup>lt;sup>6</sup>The EA series considered here pertain to the EA19, backcast prior to the euro's launch. Data for EA members Croatia, Estonia, Latvia, Lithuania, Slovakia and Slovenia only begin in 1990, but have negligible impact on aggregate EA19 growth. In this Section, RoW is defined as WDI 'world' minus EA. WDI world includes China, India, and other major emerging economies throughout (1960-2023), but former Communist countries are only included from 1989/1990. Maddison (2001) provides estimates of pre-1990 world GDP that includes Communist countries. WDI and Dennison pre-1990 world growth series are very similar.

<sup>&</sup>lt;sup>7</sup>The GDP-to-employment ratio would be a more accurate productivity measure, however comprehensive employment data are not available for RoW. The growth of GDP/(working-age population) (ages 15-64) tracks capita GDP growth very closely.

RoW trend growth fell during 1960-90, but rose during the period 1990-2005, before gradually slowing thereafter; the post-1990 resurgence largely reflects the rapid transformation and expansion of China and other emerging economies, driven by pro-market reforms and deeper integration into the global trading system. The second half of the 1990s marked an inflection point, after which the trend growth of EA per capita GDP growth fell below that of RoW. Since 2010, there has been a modest convergence in trend growth between the two regions.

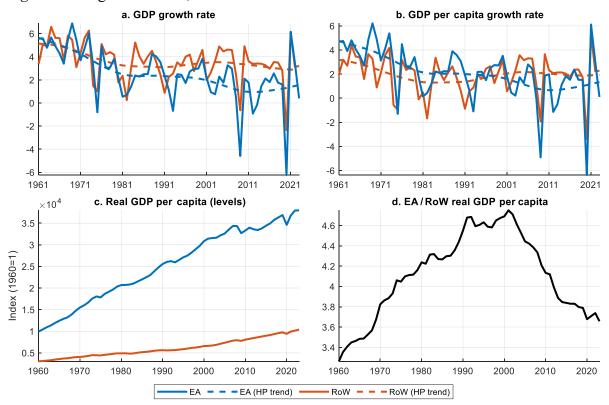


Fig 1: Global growth trends, EA and RoW

*Notes*: Panel a shows year-on-year GDP growth rates for the EA and RoW; Panel b shows year-on-year per capita GDP growth rates (%). Solid blue and red lines depict actual YoY growth for EA and RoW, respectively; dashed lines trace the Hodrick–Prescott (HP) trends. Panel c plots per capita real GDP in the EA and RoW, measured in units of 10<sup>4</sup> constant 2015 USD. Panel d presents the ratio of EA to RoW per capita real GDP.

The persistent growth gap between the rest of the world and the Euro Area (EA) led to a steady and substantial decline in the EA's share of global real GDP, from 25% in 1960 to 14% in 2023.

Table 1 reports autocorrelations (lags 1 to 8) of EA-RoW GDP per capita growth differential over the period 1960–2023. The slow decay of the autocorrelogram indicates a high degree of persistence in the growth differential.

Table 1. Autocorrelations of EA-RoW GDP per capita growth differential (annual), 1960-2023

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
Autocorr.	0.51	0.33	0.40	0.33	0.24	0.35	0.33	0.38

It is important to note that, despite the persistently low growth of the EA economy, the *level* of per capita GDP remains much larger in EA than in RoW (see Fig. 1c, d). The relative per capita real GDP between the EA and RoW showed a secular rise until about 2000, but has subsequently declined significantly, from a ratio of about 4.6 in 2000 to 3.6 in 2023.

The remainder of the paper focuses on the period 1999-2023. During this time, real GDP in the EA grew by 36% (from 9.6 trillion constant USD (2015) to 13.1 tn USD) compared to 117% real GDP growth in RoW (from 3.6 tn USD to 7.9 tn USD). The EA population increased by 8% (from 325 million to 350 mn), while the RoW population grew by 33% (from 5.7 billion to 7.6 bn).

## 2.2. EA-RoW trade and RER: 1999-2023

While EA GDP growth decelerated after the 1990s, trade between the EA and RoW grew steadily and significantly between 1999 and 2023, with only temporary contractions during the global financial crisis (2008) and the COVID-19 pandemic (2020). Over this period, the ratio of EA nominal exports and imports to nominal GDP doubled, rising from approximately 15% to almost 30% (see Fig. 2a). Due to the strong growth of RoW GDP, the rise in EA-RoW trade relative to RoW GDP has been more modest; the ratio of RoW imports and exports (i.e., EA exports and imports) to RoW GDP rose before the global financial crisis (GFC), it then fell and stabilized at about 3% to 4%.

In striking contrast to the gap between EA and RoW growth trends, and to the trend in EA trade openness, the EA TB/GDP ratio has remained relatively stable. The EA has maintained a TB surplus every year since 1999, with an average surplus of 2.6% of EA GDP over 1999–2023. The TB/GDP ratio increased by nearly 2 percentage points during the European sovereign debt crisis and has remained elevated since, except in 2022 (Fig. 2c).

Figure 2d plots the EA-RoW RER, expressed as a percentage deviation from its 2010 level. Throughout the paper, the RER is constructed using GDP deflators (rather than consumer price indices, due to limited CPI coverage for RoW). An increase in the RER represents a depreciation from the perspective of the EA. After the launch of the Euro, the EA RER initially depreciated (for about one year), and then appreciated by about 20%–25% until the GFC. Following the GFC, the RER depreciated by a similar magnitude and subsequently stabilized near the levels observed at the Euro's introduction. While the RER experienced substantial medium-term fluctuations, it thus displayed no clear trend over the full 1999–2023 period.

<sup>&</sup>lt;sup>8</sup> Over the same period, the EA labor force expanded by 15% (from 149 mn to 172 mn), compared to a 36% rise in the RoW labor force (from 2.5 bn to 3.4 bn).

<sup>&</sup>lt;sup>9</sup> The international trade data presented in this paper encompass flows of both goods and services. EA (RoW) exports and imports refer exclusively to transactions between the EA and the RoW, excluding intra-EA (intra-RoW) trade flows.

That absence of a long-term RER trend stands in stark contrast to the pronounced trends in relative EA-RoW output and productivity. This observation aligns with the findings of Krugman (1989) and Gagnon (2008), who documented, for a broader sample of countries, that secular RER trends are generally much more muted than those in relative (domestic/foreign) output.

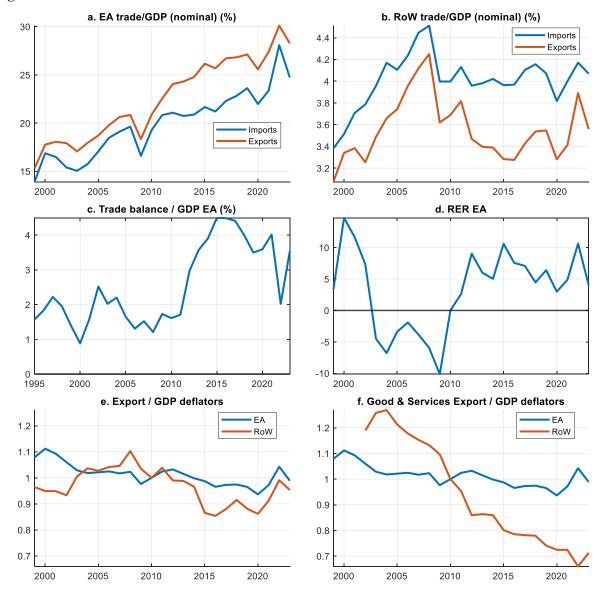


Fig. 2. EA-RoW trade and RER data

Notes: Panels a, b: Total imports and exports as a share of nominal GDP for EA (a) and RoW (b). Panel c: EA trade balance over GDP (%). Panel d: EA RER. Panel e: Total export deflators relative to GDP deflators. Panel f: Export of good and services deflators (excluding commodities) relative to GDP deflators.

Despite the long-term stability of the EA-RoW RER, the sample period has witnessed substantial shifts in relative export prices. Figure 2e plots each region's export deflator divided by its GDP deflator (deflators are normalized at unity in 2010). Since the launch of the euro, the EA's

exports/GDP deflator ratio has declined modestly but steadily. In contrast, the RoW ratio shows no clear trend but displays substantial medium-term fluctuations that are largely driven by the wide fluctuations in the prices of commodities exported by RoW to EA.<sup>10</sup> Figure 2f plots noncommodity export deflators, again normalized by regional GDP deflators. Here, a distinct pattern emerges: RoW non-commodity export prices have declined persistently relative to the RoW GDP deflator—by more than 40% since 1999. This suggests that productivity growth in RoW's noncommodity export sector has outpaced productivity growth in the rest of the RoW economy. The downward trend for RoW non-commodity relative export prices has been offset, however, by a rising trend in the relative price of RoW commodity exports (not shown in Figure), leading to the absence of a trend in the overall RoW export-to-GDP deflator ratio documented in Figure 2e. RoW export prices, when expressed in Euros, correspond to EA import prices. The long-run stability of the EA–RoW RER (based on GDP deflators) thus implies that the deflator for EA non-commodity imports (relative to the EA GDP deflator) has followed a similar downward trend (not displayed in Figure 2).

Guided by these empirical patterns, the theoretical model (next Section) includes a commodity-producing sector in RoW, and it allows for distinct stochastic productivity trends in non-commodity exporting sectors.

## 3. Model description

This Section presents our structural model. We focus here on the essential elements; a full description is provided in the Not-for-Publication Appendix.<sup>11</sup> The model's detailed structure matters for quantitatively disentangling the drivers and interactions across the two regions. It is also necessary to match the large set of observable time series used in our estimation. Our main methodological contribution lies in the joint estimation of trend and cyclical factors within a unified framework.

Overview. The model comprises two regions, the EA and RoW. In both regions, long-term productivity growth arises from technical progress in the intermediates goods sector, where monopolistically competitive firms employ domestic labor and capital. Perfectly competitive firms combine domestically produced intermediates with imported inputs to generate final output. Bonds denominated in RoW currency serve as the vehicle for cross-region financial flows.

Each region includes two representative household types: (i) 'Ricardian' households, who own local firms and have access to financial markets; (ii) 'Hand-to-mouth' households, who consume their disposable wage and transfer income each period. Wage setting is governed by monopolistic trade unions. Wages and intermediate-goods prices exhibit nominal stickiness.

The following subsections outline the main features of the EA model block. The RoW block shares the same general structure, with one key difference: RoW exports commodities that are exclusively consumed in the EA. The EA is assumed not to produce commodities.

Due to limited RoW data availability, fiscal policy is modeled for the EA. While the EA and RoW blocks are structurally similar, parameter values are allowed to differ between regions.

<sup>&</sup>lt;sup>10</sup> For example, the pre-GFC and post-Covid commodity price surges correspond to sharp increases in the RoW export deflator during those episodes.

<sup>&</sup>lt;sup>11</sup> For example, we do not explicitly describe public investment and capital, overhead labor, fixed costs, and include only the main exogenous shocks as identified in our estimation (see Section 5).

#### 3.1. Exogenous shocks

The model incorporates both non-stationary and stationary exogenous processes to capture dynamics at both trend and business-cycle frequencies. A generic logged exogenous variable  $\ln(A_t^q)$  (e.g., productivity in sector q) is specified as the sum of a stochastic trend variable,  $T_t^q$ , whose first difference is a stationary serially correlated AR(1) process, and of a cyclical component  $S_t^q$  that is an AR(1) in levels:

$$\ln(A_t^q/A^q) = T_t^q + S_t^q, \quad \text{where } A^q \text{ is a constant,} \quad \text{and} \quad (1)$$

$$T_t^q - T_{t-1}^q \equiv g_t^q = \rho^q g_{t-1}^q + (1 - \rho^q) g^q + \varepsilon_t^q, \quad \text{with } 0 < \rho^q < 1,$$
 (2)

$$S_t^q = \lambda^q S_{t-1}^q + \eta_t^q, \quad \text{with } 0 \le \lambda^q \le 1.$$
 (3)

 $\eta_t^q$  and  $\varepsilon_t^q$  are orthogonal *i.i.d.* white noise innovations.  $g^q$  is the 'drift', i.e. the unconditional mean of the (log) growth rate of  $A_t^q$ . Innovation  $\varepsilon_t^q$  induces a serially correlated but mean-reverting change in the growth rate of  $A_t^q$  and leads to a permanent level shift. We refer to  $g_t^q$  as the "trend growth rate" of  $A_t^q$  and to  $\varepsilon_t^q$  as a "trend growth rate shock".

## 3.2. Multi-stage production

Production follows a multi-stage process. Final goods are produced by perfectly competitive firms combining domestic and imported intermediate goods, and in the EA, imported commodities. Intermediate goods are produced by monopolistically competitive firms using domestic capital and labor.

## 3.2.1. Final goods (aggregate demand components)

Final goods, used for domestic private consumption (C), public consumption (G) and domestic physical investment (I) as well as exports (X), are produced by perfectly competitive firms. The model assumes that a share of imports is re-exported, reflecting the EA's role in global value chains. Final good production functions for final goods  $D \in \{C, G, I, X\}$  are of the following CES (constant elasticity of substitution) form:

$$D_{t} = A_{t}^{D} \left( \left( 1 - s_{t}^{M,D} \right)^{\frac{1}{\sigma_{z}}} O_{t}^{D}^{\frac{\sigma_{z} - 1}{\sigma_{z}}} + \left( s_{t}^{M,D} \right)^{\frac{1}{\sigma_{z}}} M_{t}^{D}^{\frac{\sigma_{z} - 1}{\sigma_{z}}} \right)^{\frac{\sigma_{z} - 1}{\sigma_{z} - 1}}$$
(4)

where  $O_t^D$  is an aggregate of domestically-produced intermediates, while  $M_t^D$  denotes imports.  $\sigma^Z$  is the elasticity of substitution between domestic output and imports.  $s_t^{M,D}$  is a time-varying exogenous parameter that governs foreign content, and thus determines home bias  $(1 - s_t^{M,D})$ .  $A_t^D$  is an exogenous productivity shock that is specific to the production of final good D. Steady-state home bias differs across final goods, but stochastic shocks to home bias are common across sectors.

The final good deflators corresponding to (4) is

$$P_t^D = \left[ \left( 1 - s_t^{M,D} \right) (P_t^O)^{1 - \sigma^Z} + s_t^{M,D} (P_t^M)^{1 - \sigma^Z} \right]^{\frac{1}{1 - \sigma^Z}} / A_t^D,$$

 $<sup>^{12}</sup>$  The good-specific  $A_t^D$  shocks are assumed to capture empirical fluctuations in ratios of C,G,I,X deflators divided by the GDP deflator, which are included in the set of observables (see Appendix).

where  $P_t^O$  and  $P_t^M$  are price indices of domestic and imported intermediate inputs, respectively. Note that the technology shock  $A_t^D$  drives a wedge between the deflator of final good D and the prices of the inputs used to produce D.

Time-varying intermediate input elasticities. The estimated model enriches the final good technology (4) by including adjustment frictions. Following Auclert et. al. (2024), we incorporate delayed substitution into intermediate input demand functions. Final good firms adjust their domestic and foreign inputs with probability  $(1 - \rho^z)$  each period. With probability  $\rho^z$ , firms keep their input ratios fixed and adjust only total expenditure. This mechanism mirrors Calvo-style price-setting but applies to input composition: firms reset their inputs bundles based on current and expected future relative prices. As only a fraction  $(1 - \rho^z)$  adjust each period, aggregate input ratios evolve gradually over time (see Not-for-Publication Appendix for details).

## 3.2.2. Intermediate goods

In the EA, the domestic intermediate aggregate  $O_t$  is itself a CES aggregate of EA domestic value added,  $Y_t$ , and industrial supplies  $IS_t$  (a bundle of energy and non-energy commodities imported from RoW):

$$O_t = ((1-s_t^{is})^{1/\sigma_z}(Y_t)^{(\sigma_o-1)/\sigma_o} + (s_t^{is})^{1/\sigma_o}(IS_t)^{(\sigma_o-1)/\sigma_o})^{\sigma_o/(\sigma_o-1)},$$

with  $\sigma_o > 0$ . Perfectly competitive "packers" aggregate a continuum of differentiated local

intermediates using a CES technology with substitution elasticity  $\sigma_y$ :  $Y_t = \left[ \int_0^1 Y_{i,t}^{\frac{\sigma_{y-1}}{\sigma_y}} di \right]^{\frac{\sigma_y}{\sigma_{y-1}}}$ . 13

The production function for intermediate good i is

$$Y_{i,t} = (A_t^Y N_{i,t})^{\alpha} (c u_{i,t} K_{i,t-1})^{1-\alpha},$$
(5)

where  $A_t^Y$  is an exogenous productivity parameter that is common across firms;  $N_{i,t}$ ,  $K_{i,t-1}$  denote labor and capital in period t;  $cu_{i,t}$  is an endogenous rate of capacity utilization. The capital stock accumulates according to  $K_{i,t} = K_{i,t-1}(1-\delta) + I_{i,t}$ , with  $0 < \delta < 1$ , where  $I_{i,t}$  is gross investment.

The firm's dividend in period t is:  $div_{i,t} = P_{i,t}Y_{i,t} - W_tN_{i,t} - P_t^KI_{i,t} - P_t\Gamma_{i,t}$ , where  $P_{i,t}$  is the firm's output price,  $P_t^K$  is the investment price, and  $\Gamma_{i,t}$  summarizes nominal and real adjustment costs. Price adjustment costs follow  $\Gamma_{i,t}^P \equiv \frac{1}{2}\gamma(P_{i,t} - (1+\pi)P_{i,t-1})^2/P_{i,t}$  where  $\pi$  is the steady-state inflation rate. Up to a linear approximation, this gives rises to a standard New-Keynesian Phillips curve,  $\pi_t - \pi = \beta_{t,t+1}E_t(\pi_{t+1} - \pi) + \vartheta^j\left(P_{i,t}/MC_{i,t} - \frac{\sigma_y}{\sigma_y - 1}\right)$ , where  $\pi_t$  is intermediate good inflation,  $MC_{i,t}$  is the marginal cost of intermediate good firms and  $\sigma_y/(\sigma_y - 1)$  is the

<sup>&</sup>lt;sup>13</sup> In the RoW, we set  $O_{RoW,t} = Y_{RoW,t}$ .

 $<sup>^{14}</sup>$   $u_{i,t}$  is assumed to capture the cyclicality of the intermediate sector Solow residual; e.g., King and Rebelo (1999).

<sup>&</sup>lt;sup>15</sup>Real adjustment costs for investment, labor inputs, and capacity utilization follow standard quadratic forms, as described in the Not-for-publication Appendix.

steady-state mark up.  $\beta_{t,t+1}$  is the households' subjective discount factor (see below). The slope coefficient  $\theta_i > 0$  depends on the degree of price rigidity.

#### 3.2.3. Forcing processes in production sectors

Secular growth in real activity is assumed to originate from trend productivity growth in the intermediate goods sector (as mentioned above). Accordingly,  $\ln(A_t^Y)$  (see (5)) follows a unit root process with positive drift ( $g^Y > 0$ ). As empirical import shares and the relative price of export goods (compared to the GDP deflator) too exhibit trends (see Sect. 2), we also allow for a unit root (but assume zero drift) in the foreign content parameters ( $s_t^{M,D}$ ) of the final good technology (4), and in export sector productivity ( $A_t^X$ ). <sup>16</sup> All other exogenous variables in the production process, and in other model blocks discussed below, follow stationary processes in levels ( $T_t^q = 0 \ \forall t; \ 0 \le \lambda^q < 1$ ). <sup>17</sup>

#### 3.3. Households and unions

Household welfare depends on consumption and hours worked. Each household j = r, h(r): Ricardian, h: hand-to-mouth) has period utility function

$$U_{j,t} \equiv \frac{(C_{j,t} - hC_{j,t-1})^{1-\theta}}{1-\theta} - s_t^N \cdot (C_t)^{1-\theta} \frac{(N_{j,t})^{1+\theta^N}}{1+\theta^N} \quad \text{with } 0 < \theta, \theta^N, s_t^N \text{ and } h \in (0,1)$$

where  $C_{j,t}$  and  $N_{j,t}$  denote consumption and labor hours, respectively. The parameters  $\theta$ ,  $\theta^N$ , and h govern risk aversion, labor disutility, and external consumption habit formation, respectively.  $s_t^N$  captures an exogenous shock to labor disutility. <sup>18</sup>

Households maximize expected life-time utility  $V_{j,t} = U_{j,t} + E_t \beta_{t,t+1} V_{j,t+1}$ , where the subjective discount factor  $0 < \beta_{t,t+1} < 1$  fluctuates exogenously.

#### 3.3.1. Ricardian households

Ricardian households own domestic firms, hold domestic government bonds (denominated in local currency and not traded internationally) and internationally traded bonds. Their period *t* budget constraint is:

$$(1+\tau^{c})P_{t}^{c}C_{t}^{r}+B_{t+1}^{r}=(1-\tau^{N})W_{t}N_{t}^{r}+B_{t}^{r}(1+i_{t}^{r})+div_{t}+T_{t}^{r},$$

<sup>&</sup>lt;sup>16</sup> Kollmann (2019) and Bodenstein et al. (2024) develop DSGE models with stationary home-bias shocks affecting household consumption. By contrast, the present paper incorporates permanent home-bias shocks, allowing the model to capture the observed long-run trends in EA and RoW import shares. In addition, the shocks here are assumed to affect not only private consumption, but also government consumption, investment, and exports.

<sup>&</sup>lt;sup>17</sup> Productivity in the final consumption good and investment sectors  $A_i^C$ ,  $A_i^G$ ,  $A_i^G$ ,  $A_i^G$  (see (4)) is assumed to be stationary. Allowing for non-stationary processes for these productivity parameters does not affect the estimation results discussed below.

<sup>&</sup>lt;sup>18</sup>To allow for balanced growth, the disutility of labor features the multiplicative term  $(C_t)^{1-\theta}$  that depends on aggregate consumption (treated as exogenous by an individual household).

where  $P_t^C$ ,  $W_t$ ,  $div_t$  and  $T_t^r$  are the consumption (final good) price, the nominal wage rate, dividends generated by domestic firms, and government transfers received by Ricardian households.  $B_{t+1}^r$  denotes the Ricardian households' total asset (bonds and stocks) holdings at the end of period t, and  $t_t^r$  is the nominal return on the households' portfolio between periods t-1 and t (net of tax).  $\tau^C$  and  $\tau^N$  are (constant) consumption and labor tax rates, respectively.

Bonds and stocks are subject to exogenous stochastic convenience yields that affect the households' perceived returns on these assets. Pricardian households' Euler equations imply that the expected returns on stocks is equated (up to a linear approximation) to bond returns adjusted for an investment risk premium  $\varepsilon_t^S$  (reflecting convenience yields) that is assumed to follows an AR(1) process:

$$E_t\left(\frac{P_{t+1}^S + div_t}{P_t^S}\right) = 1 + r_t + \varepsilon_t^S,\tag{6}$$

where  $P_t^s$  is the stock price and  $r_t$  is risk-free bond interest rate.<sup>20</sup>

#### 3.3.2. Hand-to-mouth households

Hand-to-mouth households do not participate in asset markets. Their consumption equals disposable income each period:  $(1 + \tau^C)P_tC_t^h = (1 - \tau^N)W_tN_t^h + T_t^h$ .

## 3.3.3. Wage setting

A trade union transforms homogeneous labor hours supplied by the two domestic household types into imperfectly substitutable labor services. These differentiated services are sold to intermediate goods producers, who use a CES aggregator to combine them into the composite labor input  $N_t$  in their production function.

The trade union sets the wage as a mark-up  $\mu_t^W$  over the marginal rate of substitution between leisure and consumption. This target wage mark-up is inversely related to the elasticity of substitution across labor varieties. Wage adjustment costs introduce frictions into wage setting, causing the wage mark-up to fluctuate.

Following Blanchard and Gali (2007), we incorporate real wage inertia by modeling the current-period real wage as a weighted average of the union's desired wage and last period's real wage.

The real wage equation is:  $\frac{W_t}{P_t} = ((1 + \mu_t^W) - mrs_t)^{1-\gamma^{wr}} \left(\frac{W_{t-1}}{P_{t-1}}\right)^{\gamma^{wr}}$ , where  $mrs_t$  is the (weighted) marginal rate of substitution between consumption and leisure across households, and  $\gamma^{wr} \in (0,1)$  governs the degree of wage rigidity. Real wage inertia helps the model replicate the persistence of employment fluctuations observed in the data.

#### 3.4. Monetary policy

The monetary policy (nominal) interest rate  $i_t$  is set at date t by the central bank according to the interest rate feedback rule

$$i_{t} = (1 - \rho^{i})\overline{i} + \rho^{i}i_{t} + (1 - \rho^{i})\{\eta^{\pi}[ln(P_{t}^{C}/P_{t-4}^{C}) - \pi] + \eta^{Y}[ln(Y_{t} / Y_{t-1}) - g^{Y}]\} + \varepsilon_{t}^{i},$$

<sup>&</sup>lt;sup>19</sup> For a micro foundation, see Fisher (2015). Cardani et al. (2023) apply a similar approach to a model of the US economy. See also the Not-for-publication Appendix.

<sup>&</sup>lt;sup>20</sup> We also allow for exogenous fluctuations in government bond premia.

where  $\pi$  is the steady state inflation rate;  $0 \le \rho^i < 1$  is an interest rate smoothing parameter;  $\eta^{\pi}$  and  $\eta^{Y}$  are policy response coefficient to CPI inflation and to GDP growth;  $\varepsilon_t^i$  is a white noise policy shock.

#### 3.5. International financial markets

The only internationally traded asset is a one-period bond denominated in RoW currency. Ricardian households face a quadratic adjustment cost when their net foreign bond holdings, relative to nominal GDP, deviate from a predetermined target (these costs are rebated to households as lump-sum transfers). As a result, the interest rate spread between domestic and foreign assets depends on the level of foreign bond holdings, consistent with the framework in Kollmann (2002, 2004, 2005) and Itskhoki and Mukhin (2021).

The Ricardian household's first-order conditions for domestic and foreign bonds yield the following (linearizes) uncovered interest parity (UIP) condition

$$1 + i_{EA,t} = \left(1 + i_{RoW,t}\right) \frac{E_t(\mathcal{E}_{t+1})}{\mathcal{E}_t} exp\left[\alpha^{BW} \left(\frac{NFA_t}{P_t^Y Y_t} - \overline{b^*}\right) + \varepsilon_t^{BW}\right],\tag{7}$$

where,  $i_t$  and  $i_t^*$  are domestic and RoW nominal interest rates, respectively.  $\mathcal{E}_t$  is the nominal exchange rate expressed as Euros per unit of RoW currency.  $\frac{NFA_t}{P_t^Y Y_t}$  is the EA's net foreign bond position normalized by nominal GDP;  $\overline{b^*}$  denotes the target level of foreign bond holdings.  $\alpha^{BW}$  indexes the curvature of the bond-holding cost (and the degree of international bond market integration).  $\varepsilon_t^{BW}$  is a serially correlated UIP shock that represents an exogenous shift in the marginal cost of holding foreign bonds. (Departures from interest parity are also induced by the endogenous term on the right-hand side of (7) that depends on NFA; according to the estimated model, that term fluctuates less than  $\varepsilon_t^{BW}$ .)

Total net foreign assets (NFA) evolve as:

$$NFA_{EA,t} = (1 + i_{ROW,t})NFA_{EA,t-1} + TB_{EA,t} + \varepsilon_{EA,t}^{NFA}$$
(8)

where  $\varepsilon_{EA,t}^{NFA}$  is an exogenous serially correlated international transfer. This shock captures international remittances, transfers and other current account components that are needed to reconcile the observed net foreign asset path with the model's current account equation.

## 3.6. Region-specific features

## 3.6.1. EA fiscal policy

We model EA fiscal policy using detailed data and targeted policy rules. EA real government consumption,  $G_t$ , is set according to the rule  $c_t^G = \overline{c^G} + \varepsilon_t^G$ , where  $c_t^G \equiv P_t G_t / (P_t^y Y_t^{pot})$  is government consumption normalized by domestic potential output (with steady state  $\overline{c^G}$ );  $\varepsilon_t^G$  is a serially correlated exogenous variable. Government investment and transfers follow similar rules. Government investment builds a public capital stock, which enters private sector production (see the Not-for-Publication Appendix for details). Lump-sum taxes follow a feedback rule that links them to the government budget deficit and public debt levels.

## 3.6.2. RoW commodity supply

In RoW, a competitive sector supplies two distinct commodities, energy and non-energy materials, to domestic and foreign firms. Commodity prices are flexible. The real commodity price (in units of RoW GDP) follows a stationary exogenous process.

#### 4. Econometric approach

#### 4.1. Data

Our estimation sample is the period from 1999Q1 to 2024Q2. We use quarterly time series for 32 EA variables, and 5 annual series for the Rest of the World (mixed frequency estimation). For the EA, data are drawn from Eurostat, World Bank Development Indicators, and the IMF International Financial Statistics and World Economic Outlook databases.<sup>21</sup> The observables employed in estimation together with details on data construction are presented in Appendix B.

#### 4.2. Calibration

We calibrate the steady-state annual growth rates of population and prices (common across regions) to 0.34% and 2% respectively. The steady-state share of EA in world GDP is set to 18% while the trade shares (openness) for EA and RoW are set to 18% and 5%. The subjective rate of time preferences is set to 0.25% per quarter. The share of Ricardian households is set to 2/3 for both regions. We set the steady-state ratios of various GDP components to match their sample means. The steady-state government debt/GDP and fiscal deficit/GDP ratios are set to 77% (annual terms) and 3%, respectively. We report the full list of calibrated parameters in the Appendix (Table C3).

## 4.3. Model solution and approximation

We normalize all variables by their respective deterministic balanced growth paths, and we linearize the detrended system. In our framework, the non-stationary (trend) shocks induce permanent *level* deviations from the deterministic balanced growth path, while preserving the stationarity of the growth rates of all variables. The Appendix details the stationarization technique, and compares linearized and nonlinear model versions.<sup>22</sup>

#### 4.4 Estimation procedure

We use a Bayesian Maximum Likelihood estimation method that combines prior information with observed data to estimate the model parameters. We estimate the model in levels, imposing a common steady-state trend growth rate for GDP per capita across the EA and RoW. This assumption ensures the existence of a balanced growth path. It is consistent with the fact that long-run (1960-2023) average GDP per capita growth rates were roughly similar across the EA and RoW (see Section 2). The model has the property that regions will converge to the common per

<sup>&</sup>lt;sup>21</sup> The RoW dataset aggregates data from 57 countries.

<sup>&</sup>lt;sup>22</sup> The analysis confirms that the linear approximation provides an accurate representation of both simulated dynamics and impulse responses, including those triggered by permanent shocks, with relatively small approximation errors arising during the COVID period due to the magnitude and nature of the shocks.

capita steady state growth rate in the (very) long run. The global steady-state growth rate of intermediate good productivity is estimated jointly with other structural parameters.

To account for the sharp contraction in economic activity during the Covid pandemic, we follow Cardani et al. (2022, 2023) and assume pandemic-specific heteroskedastic shocks.<sup>23</sup> These pandemic shocks are jointly estimated alongside the other structural parameters using a diffuse and heteroskedastic Kalman filter. Posterior distributions are sampled using Gibbs sampling, specifically the slice sampler (Neal, 2003). Point estimates are derived from the posterior mode.

#### 5. Estimation results

This Section first presents the estimated model parameters, followed by model-predicted business cycle statistics, impulse response functions and historical shock decompositions.

#### **5.1. Posterior parameter estimates**

Table 2 reports estimates of the key parameters driving the persistent processes central to our analysis; the full list of estimated parameters is provided in Appendix C. The estimated steady-state annual global growth rate of output is 1.76%. This implies a quarterly steady-state productivity growth rate of 0.3% (given a steady-state population growth rate of 0.085%, see Appendix E). The estimated persistence of the trend components of productivity and import shares growth rates (parameter  $\rho^q$  in (2)) is high in both regions, between 0.96 and 0.98 (the standard deviations of the estimates of  $\rho^q$  are relatively small). This enables the model to capture the persistent cross-region trend growth differences and the import share trends observed in the data.<sup>24</sup> By contrast, the persistence of the export sector productivity trend growth shocks is lower, around 0.5.

Figure 3 displays smoothed estimates of intermediate sector productivity trend growth rates (quarterly). The estimates reveal a persistent and time-varying differential between EA and RoW productivity trend growth. At the start of the sample, the RoW experienced strong productivity growth well above the steady-state trend growth rate. Although RoW trend growth gradually slowed, it remained consistently above steady state. In contrast, EA productivity growth dropped below the steady-state rate in the early 2000s and remained subdued thereafter, reflecting a sequence of negative productivity shocks until the global financial crisis. These opposing dynamics led to a widening productivity gap between the RoW and the EA until around 2005. The gap then narrowed temporarily before diverging again after 2019.

Turning to the other parameters presented in Appendix C, the estimation indicates a relatively high degree of habits in consumption for EA (0.84) and RoW (0.66), trade elasticities in line with standard empirical estimates (2.6/1.8 for EA/RoW), higher risk aversion coefficient for EA (1.5)

<sup>&</sup>lt;sup>23</sup> All our results are robust to excluding the Covid pandemic from the estimation sample.

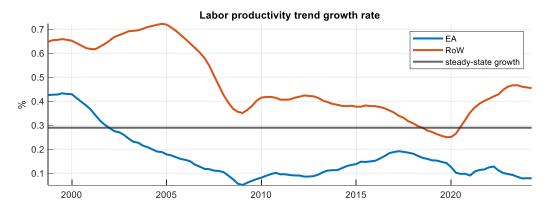
<sup>&</sup>lt;sup>24</sup> We have also estimated a model version in which home-bias shocks are assumed stationary. This yields a significantly lower marginal likelihood (log data density: 10851) than the baseline model with permanent home-bias shocks (log data density: 10864), indicating that the data strongly support the permanent home-bias shocks.

compared to RoW (1.3), nominal rigidities for EA in line with previous studies and similar degrees of real wage rigidities.

Table 2. Key Estimated parameters

Description	Prior type	Prior Mean	Prior Std.	Posterior Mode	Posterior Std.
		-	-		
Persistence intermed. productiv. growth rate EA	Beta	0.85	0.1	0.99	0.012
Persistence intermed.productiv. growth rate RoW	Beta	0.85	0.1	0.98	0.005
Persistence foreign content growth EA	Beta	0.85	0.1	0.94	0.072
Persistence foreign content growth RoW	Beta	0.85	0.1	0.93	0.106
Persistence export sector productiv. growth EA	Beta	0.85	0.1	0.64	0.060
Persistence export sector productiv. growth RoW	Beta	0.85	0.1	0.66	0.039
Std. technology growth shocks EA (%)	Gamma	0.03	0.002	0.03	0.002
Std. technology growth shocks RoW (%)	Gamma	0.03	0.002	0.03	0.002
Std. foreign content growth shock EA (%)	Gamma	0.05	0.02	0.05	0.021
Std. foreign content growth shock RoW (%)	Gamma	0.05	0.02	0.05	0.022
Std. export sect. productiv. growth shock EA (%)	Gamma	0.10	0.04	0.5	0.041
Std. export sect. productiv. growth shock RoW(9	Gamma	0.10	0.04	0.59	0.011
Std. transitory technology shocks EA (%)	Gamma	0.1	0.01	0.09	0.009
Std. transitory technology shocks RoW (%)	Gamma	0.1	0.01	0.11	0.011
Steady-state GDP quarterly growth rate (%)	Beta	0.4	0.04	0.44	0.017

Fig. 3. Smoothed estimates of productivity trend growth rates



## 5.2. Model-predicted and historical business cycle statistics

We report key simulated moments and compare them with their empirical counterparts to assess the model's ability to replicate salient features of the data. The model reproduces well the main patterns in observed volatilities and co-movements of key macroeconomic variables. Notably, consistent with the data, it captures the weak international correlation of GDP growth rates and generates a RER that is roughly twice as volatile as GDP. It also matches the low correlation

between the consumption growth differential and the RER (consumption – real exchange rate disconnect; Kollmann, 1991, 1995; Backus and Smith, 1993).

Table 3. Data and model moments

Moments	EA	<b>\</b>	RoW					
	data	model	data	model				
Volatility								
$std(\Delta c)/std(\Delta y)$	0.68	0.81	0.73	0.76				
$std(\Delta inv)/std(\Delta y)$	3.02	3.47	2.29	3.49				
Domestic correlations								
$corr(\Delta y, \Delta c)$	0.74	0.57	0.88	0.53				
$corr(\Delta y, \Delta inv)$	0.81	0.63	0.88	0.78				
International statistics								
$corr(\Delta y^{EA}, \Delta y^{ROW})$	0.39	0.23						
$corr(\Delta \mathcal{E}^{EA}, \Delta RER^{EA}))$	0.95	0.69						
$corr(\Delta TBY^{EA}, \Delta RER^{EA})$	-0.20	-0.11						
$corr(\Delta c^{EA} - \Delta c^{ROW}, \Delta RER^{EA})$	-0.06	-0.016						
$std(\Delta RER^{EA})/std(\Delta y^{EA})$	2.60	2.19						
$std((\Delta RER^{EA})/\Delta E^{EA})$	1.00	0.90						

Note:Statistics are computed at annual frequency; data moments are computed for the sample 1999-2019 while model moments are computed as mean across 1000 simulations of the same length as the sample using the covariance matrix of the smoothed shocks and excluding temporary shocks associated with the COVID period.

#### **5.3.** Impulse responses

This Section presents impulse response functions for key technology, trade, and aggregate demand shocks to illustrate the model's dynamics, given the estimated parameters. For each shock, we examine its role in explaining key empirical patterns, specifically, the joint behavior of GDP, the RER, and the TB.

The left panel of Fig. 4 shows dynamic responses to an *intermediate goods productivity trend* growth rate shock in RoW (i.e. a trend growth shock to the RoW counterpart of  $A_t^Y$  in EA eq. (5)). The shock raises RoW productivity and GDP, on impact. This is followed by strong and long-lasting future productivity and GDP increases, due to the high autocorrelation of the RoW productivity trend growth rate (0.98). Fig. 4 shows that this strongly boosts RoW consumption, on impact, as RoW Ricardian households consume more, in response to their increased permanent income. The rise in RoW aggregate demand boosts the EA TB, and it crowds out EA consumption and investment, which causes a persistent decline in EA GDP. The strong immediate boost to RoW absorption also improves the RoW terms of trade, and it appreciates the RoW RER, i.e. the EA RER depreciates. Over the longer term, when the anticipated higher RoW productivity and output materializes, EA consumption and investment rise above pre-shock levels, the EA RER appreciates, and the EA TB falls below its pre-shock level. Note, however that this EA TB

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<sup>&</sup>lt;sup>25</sup> Note that the RoW productivity growth rate shock raises inflation (CPI), and the real interest rate in both regions-during the adjustment, the RoW real interest rate exceeds the EA rate, which also helps to understand the gradual appreciation of the EA RER in the periods after the shock.

reversal occurs only more than 100 quarters after the RoW productivity growth rate shock. In other terms, a positive RoW productivity growth rate shock triggers a very persistent EA TB improvement. This extended TB response is driven by the very gradual estimated adjustment of EA absorption. As mentioned above, the model assumes real frictions that delay the adjustment of consumption and investment to shocks (those frictions are estimated alongside the other model parameters). Below we show that model versions that do not feature those adjustment frictions predict a much more front-loaded TB adjustment to a productivity growth shock (Sect. 6).

RoW aggregate productivity trend growth rate shock RoW transitory aggregate productivity shock **GDP** Consumption چ 0.3 د Investment **GDP** Consumption Investment 1.5 trom ss dev. from ss from ss S 1.5 dev. from ss 0.1 dev. from 8 9.2 <u>ل</u> 0.1 % dev. fr 0.05 0.1 % dev. 9.0 0.5 0 % % % % 0 50 50 100 0 100 0 100 50 100 50 100 100 0 TB/GDP **RER EA** Import Share TB/GDP Import Share pp change from ss **RER EA** pp change from ss from ss 0.03 0.2 from ss dev. from ss change from 0 0.02 0.05 change -0.2 -0.05 0.01 dev. -0.4 % -0.6 % -0.01 -0.1 dd 50 0 50 0 50 100 100 50 100 50 100 productivity growth **Real Interest Rate** Inflation pp change from ss pp change from ss **Real Interest Rate** Inflation productivity growth pp change from ss from ss pp change from ss pp change from ss 0.15 0.1 bb change fr -0.02 -0.02 0.05 -0.04 0 -0.05 50 50 0 100 50 100 0 0 50 100 50 100 0 50 100 RoW EΑ EΑ RoW

Fig. 4. Dynamic responses to aggregate (intermediate goods) productivity shocks in RoW

*Notes:* The shock size corresponds one estimated standard deviation of the exogenous innovation. Periods correspond to quarters. An upward movement of the RER indicates a depreciation from the EA viewpoint.

In sum, the RoW productivity trend growth shock generates a persistently positive EATB response but implies a long-lasting appreciation of the EARER. In the long term, the shock raises the EA import share (due to a fall in the relative price of RoW exports), but the effect on the import share is muted. This suggests that while RoW productivity trend growth shocks have the potential to explain the persistent EATB surplus observed since the launch of the Euro, those shocks alone fail to account for the absence of a secular trend in the RER and the secular rise in the EA import share.

It is insightful to contrast the persistent RoW growth shock to a *transitory RoW intermediate goods productivity* rate shock;<sup>26</sup> see right panel in Fig. 4. While a trend growth rate shock induces a sharp immediate increase in RoW absorption (see above), the transitory growth rate shock induces a much more muted response of absorption, and it hence implies a stronger co-movement of GDP and absorption. Consequently, the transitory growth rate shock has only negligible effects on the TB. In the short run, habit persistence and investment adjustment costs slow the response of RoW aggregate demand, which lowers the RoW real interest rate and triggers an initial Euro appreciation. Higher RoW income boosts RoW import demand, immediately improving the EA TB.

The left panel of Fig. 5 shows dynamic responses to a shock to **productivity trend growth in the RoW export sector.** That shock permanently raises the efficiency with which domestic and imported intermediates are transformed into exports (parameter  $A_t^X$  in eq. (4) for sector D = X)). The right panel of Fig. 5 shows responses to a permanent positive shock to the **foreign content** of EA final goods (i.e. a negative shock to EA home bias); we assume that the shock increases the parameter  $S_t^{M,D}$  by the same relative amount in the EA production functions of all final good types,  $C_1I_1G_1X$ .

The *RoW export sector productivity trend growth shock* raises RoW GDP through increased export demand. Simultaneously, the EA benefits from lower prices for imported consumption and investment goods. Unlike the productivity trend growth shock in the RoW intermediate goods sector, the exports sector shock generates positively synchronized GDP responses across regions. By reducing EA import, the RoW export shock reduces EA CPI inflation, and triggers a persistent depreciation of the EA RER. In the short to medium run, cheaper imports also raise the EA import share and worsen the EA TB.

Similarly, the *permanent negative shock to EA home bias* triggers a gradual rise in the EA import share. The resulting decline in relative demand for EA intermediates lowers EA GDP, consumption and investment, while boosting foreign output. The shift in demand toward imports also drives a depreciation of the EA RER and an initial deterioration in the TB. Over time, the TB improves, as households anticipate higher future import spending and increase savings.

We conclude the discussion of impulse responses by considering two shocks originating in the EA: a shock to **EA intermediate goods productivity trend growth** (left panel of Fig. 6) and a positive (transitory) shock to the **EA investment risk premium** (right panel).

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<sup>&</sup>lt;sup>26</sup> I.e. a shock  $\eta_t^q$  to the cyclical component  $S_t^q$  of RoW intermediates productivity (see (3)).

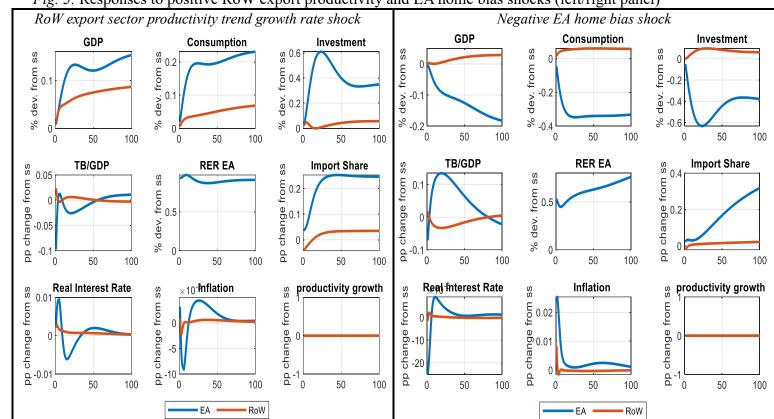


Fig. 5. Responses to positive RoW export productivity and EA home bias shocks (left/right panel)

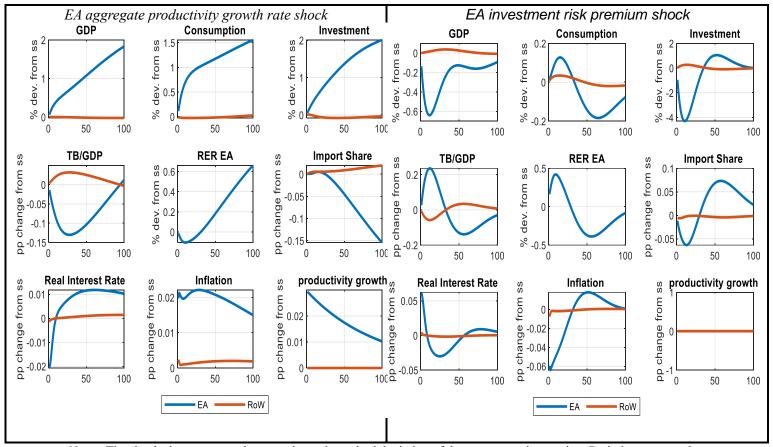
*Notes:* The shock size corresponds one estimated standard deviation of the exogenous innovation. Periods correspond to quarters. An upward movement of the RER indicates a depreciation from the EA viewpoint.

The effects of a positive persistent *EA intermediate goods productivity growth shock* largely mirror those of the RoW productivity growth shock considered in Fig. 4 (left panel). EA GDP rises, while the EA TB declines persistently. Thus, a *negative* EA productivity growth shock, as identified in the estimation sample, is predicted to improve the EA TB persistently. Interestingly, although both a negative EA and a positive RoW productivity shock widen the productivity gap, their effects on the EA RER differ: the RER appreciates more sharply but briefly after an EA shock. This asymmetry reflects stronger estimated habit formation and lower shock persistence in the EA which dampen the domestic demand response.

As discussed further below, historical shock decompositions show that EA aggregate demand shocks, particularly shocks to the investment risk premium  $\varepsilon_t^S$  played a key role in driving EA real activity around the time of the global financial crisis. The risk premium shock drives a wedge between the marginal product of capital and the interest rate earned by savers (see (6)). Fig. 6 shows that a positive EA investment risk premium shock reduces EA investment and output, improves the EA TB, and leads to a depreciation of the RER lasting around 8 to 10 years. In the short and medium term, the shock thus triggers EA RB and RER responses that are similar to the predicted responses to positive RoW productivity growth shocks. However, due to its transitory

nature, the investment risk-premium shock does not exert long-lasting effects on the RER. Additionally, as a contractionary demand shock, it reduces domestic inflation and initially lower the EA import share.

Fig. 6. Dynamic responses to aggregate (intermediate goods) productivity growth shock in EA (left panel) and to EA investment risk premium shock (right panel)



*Notes:* The shock size corresponds one estimated standard deviation of the exogenous innovation. Periods correspond to quarters. An upward movement of the RER indicates a depreciation from the EA viewpoint.

Summary of main insights from shock responses. The dynamic responses suggest that a combination of positive RoW productivity growth rate shocks and of negative EA growth rate shocks can jointly account for two key empirical patterns: (i) the persistent deviation between RoW and EA output growth trends and (ii) the persistent increase in the EA TB. However, these shocks also predict a medium- to long-term appreciation of the EA RER, a trend not observed in the historical data since the launch of the Euro. This discrepancy implies that, in the data, productivity growth rate shocks must have been offset by other forces that stabilized the RER.

The trade shocks considered in Fig. 5, namely improved RoW export sector productivity and increased imported content in EA final goods, induce a persistent depreciation of the EA RER. As such, they are plausible candidates for offsetting the appreciation effects of diverging RoW vs. EA

productivity trends. These shocks are also consistent with the observed secular rise in the EA import share. <sup>27</sup>

Persistent adverse EA aggregate demand shocks, such as the investment risk premium shock in Fig. 6, are other possible drivers of the EA TB surplus. However, their impact on the EA TB is less long-lasting, and they do not explain the long-term increase in the EA import share.

In sum, the dynamic shock responses indicate that a combination of persistent supply-side shocks, both domestic and external, combined with structural trade shocks might jointly account for the evolution of EA output, TB, RER, and import share dynamics since the launch of the Euro. The next Section turns to the historical shock decompositions to assess the empirical relevance of these mechanisms over time.

#### 5.4. Historical shock decompositions

This Section presents historical shock decompositions (SDs), which quantify the contribution of each exogenous shock to the historical paths of the model's endogenous variables (based on the estimated model parameters and smoothed estimates of the exogenous variables).

The discussion here focuses on SDs of *quarterly* time series of EA log real GDP, EA TB/GDP and log RER. SDs of other variables are reported in Appendix A.<sup>28</sup> In each figure, the thick black line shows the historical time series of a variable in deviation from its deterministic trend that is the balanced growth path it would have followed absent any stochastic shocks. For example, a value of -0.1 in the SD for EA real GDP at a given date indicates that GDP was 10% below the steady-state trend at that date. (The model is calibrated so that the steady-state TB is zero. The steady-state RER is normalized at unity.)

Each sub-plot of the SD Figures displays the contribution of a specific shock (or group of shocks), while the grey area shows the contribution of all remaining shocks. Bars above the horizontal axis (abscissa) represent positive shock contributions, while bars below the horizontal axis show negative shock contributions. All shocks together recover the detrended empirical time series.

Given the large number of shocks, we group together the contributions of related shocks. Specifically, the following shocks (or groups of shocks) are considered: (1) productivity trend growth shocks in the EA intermediate goods sector; (2) productivity trend growth shocks in the RoW intermediate goods sector; (3) other EA aggregate supply shocks (including cyclical productivity shocks, as well as price and wage markup shocks); (4) EA aggregate demand shocks (including shocks to the household rate of time preference and investment risk premium shocks); (5) other RoW aggregate supply shocks; (6) RoW aggregate demand shocks; (7) EA home bias shocks; (8) RoW home bias shocks; (9) productivity trend growth shocks in the EA and RoW export sectors; (10) shocks to EA and RoW monetary and fiscal policy rules; (11) international

Fig. F3-F5 in Appendix F), supporting their empirical relevance <sup>28</sup> Because the SDs are computed for quarterly data, the plotte

<sup>&</sup>lt;sup>27</sup> The effects of the trade shocks are statistically significant when accounting for posterior parameter uncertainty (see Fig. F3-F5 in Appendix F), supporting their empirical relevance.

<sup>&</sup>lt;sup>28</sup> Because the SDs are computed for quarterly data, the plotted series appear more jagged than those in Section 3, where annual data were used to allow for longer sample.

financial markets shocks (UIP shocks and international transfer (NFA) shocks); (12) all remaining shocks (including RoW commodity shocks) and initial conditions.

## 5.4.1 Historical shock decomposition of EA GDP (Fig.7)

Figure 7 shows that, according to the model estimates, weak EAGDP growth was primarily driven by negative EA productivity trend growth shocks (intermediate sector). Positive RoW productivity trend growth shocks had a small but noticeable negative on EAGDP, by inducing capital outflows from the EA, and depressing EA investment. From 2015 onward, the secular decline in EA absorption home bias exerted increasing downward pressure on EAGDP. This drag was partly offset by positive RoW export sector productivity shocks which boosted EAGDP by stimulating EA investment. <sup>29</sup>

Cyclical fluctuations in EA GDP were shaped mainly by EA aggregate demand shocks (especially by investment risk premium shocks), and by "other aggregate supply disturbances EA" (notably EA markup shocks). Falling investment risk premia contributed to the EA GDP expansion before the global financial crisis (GFC), while a rise in investment risk premia during the GFC, was a key driver of the post-GFC slump. The 2011–12 period saw a renewed decline in aggregate demand, linked to the EA sovereign debt crisis. While other supply-side shocks (such as shocks to wage markups) supported activity before 2008, their contribution turned negative toward the end of the sample. Fiscal and monetary policy shocks (deviations from estimated policy rules) were broadly neutral, with limited stabilizing effects.

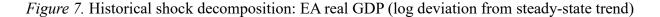
Other EA supply shocks, while mildly supportive before 2008 (e.g. wage markups), turn negative toward the end of the sample. Fiscal and monetary policy shocks remained broadly neutral, with a weak stabilizing effect.

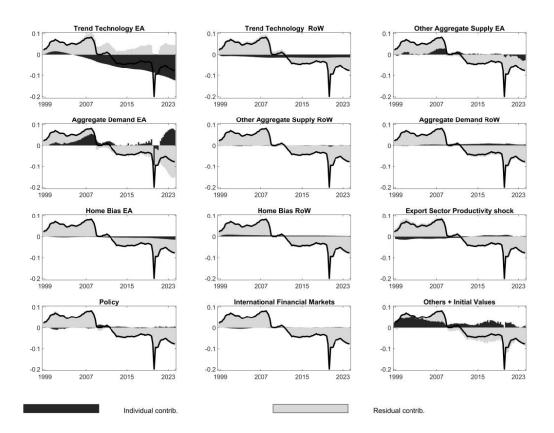
The same forces that shaped EA GDP dynamics also drove the evolution of EA consumption and investment (Appendix, Figure A2). Overall, the results here indicate that domestic shocks were the dominant source of fluctuations in EA GDP and final demand components.

RoW GDP dynamics (Appendix, Figure A3) were likewise primarily determined by domestic shocks—most notably, positive productivity trend shocks. These favorable supply developments were partially offset by negative RoW aggregate demand shocks. EA-specific disturbances had negligible effects on RoW GDP.

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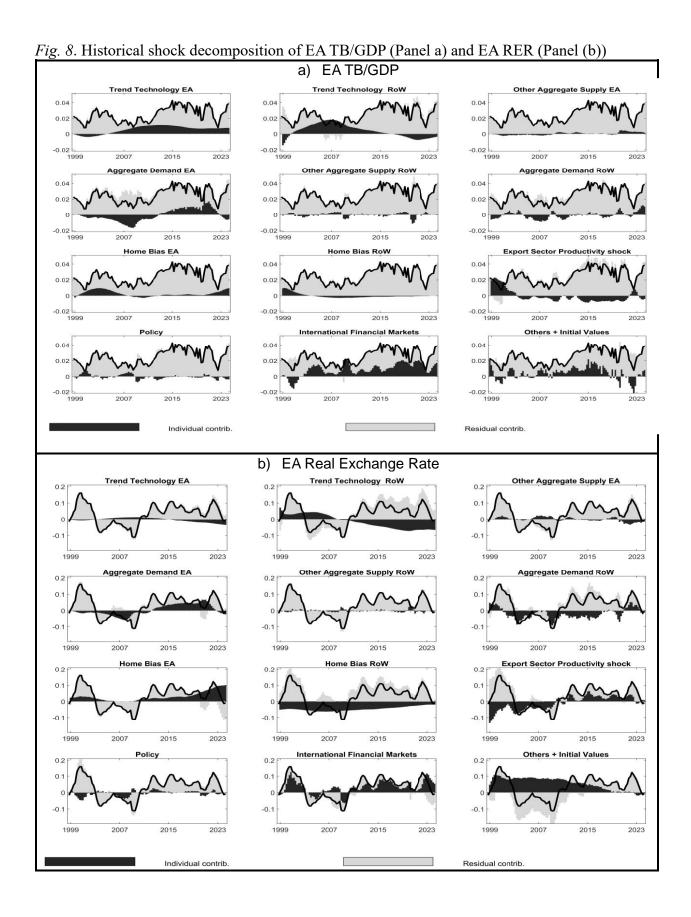
<sup>&</sup>lt;sup>29</sup> RoW export sector productivity shocks had a markedly greater effect on EA GDP than EA export sector shocks (due to limited space, Fig. 7 plot the combined effect of those EA and RoW shocks).





5.4.2 Historical shock decompositions of EA trade balance/GDP ratio and EA RER (Figure 8) Figure 8a shows that strong RoW trend productivity growth in the intermediate goods sector was a major contributor to the EA's TB surplus at the start of the sample (see Panel labelled "Trend Technology RoW"). This contribution fades over time as the RoW intermediates productivity growth rate converged toward its steady-state level (see Figure 3). In contrast, EA productivity growth in intermediates remained persistently below steady state for most of the sample, thus exerting sustained upward pressure on the EA TB by compressing absorption relative to output. Persistent productivity growth shocks in both regions thus emerge as key drivers of the enduring EA TB surplus, jointly accounting for roughly 1/4 to 1/2 of the TB surplus in most periods.

Positive RoW export sector productivity shocks, by contrast, had a consistently negative effect on the EA TB—particularly in the later years—as their impact on EA absorption via cheaper imports outweighed their support to EA supply. However, this negative effect was weaker than that of the persistent intermediate goods productivity growth shocks. Other supply shocks (such as markup shocks), in both EA and RoW, played only a minor role for the TB.



EA and RoW aggregate demand shocks had a noticeable, but more cyclical impact on the TB. Before the GFC, positive EA aggregate demand shocks (see discussion above) depressed the EA TB by up to 1.5% of EA GDP. After the GFC, weaker EA aggregate demand had a gradually more positive effect on the EA TB, with a marked increase during the COVID-19 recession. RoW aggregate demand shocks induced smaller and more short-lived effects on the EA TB. Fiscal and monetary policy shocks (deviations from the estimated policy rules) in both regions had a negligible influence on the TB overall, though they mattered modestly in some episodes. Over the full sample, EA and RoW aggregate demand shocks made an average net contribution to the EA TB that was close to zero.

Finally, international financial markets shocks (interest parity disturbances and shocks to the net foreign asset position) made a persistent and positive contribution to the TB, especially after 2010.<sup>30</sup> The secular decrease in EA goods home bias too had a small but discernible positive impact on the EA TB.

Figure 8b presents the historical decomposition of the EA RER. The productivity trend growth rate differential between the RoW and the EA exerted appreciation pressure on the RER, while the long-run decline in EA goods home bias pushed in the opposite direction, especially after the GFC. Home bias shocks influenced the RER more strongly than the TB. In addition, persistent productivity shocks in the export sector—more pronounced in the RoW than in the EA—also contributed to trend downward pressure on the RER, and thereby helped offset the trend appreciation induced by the intermediates productivity growth differential.

Other shocks too had noticeable effects on the RER, but those effects mainly operated at a short-to medium-term frequency, and thus did not affect the RER trend. This holds, for example, for EA and RoW aggregate demand shocks. Positive EA aggregate demand shocks before the GFC contributed to a marked RER appreciation in the run-up to the GFC (and supported EA GDP, see above), with peak effects exceeding 5 percent. The collapse in EA aggregate demand during the GFC, followed by persistently weak aggregate demand, induced RER depreciation as economic slack weighed on EA inflation. Persistent RoW aggregate demand weakness (see above) also contributed to an appreciated EA RER for much of the sample, though without inducing a sustained trend.

As in the case of the TB, international financial market shocks accounted for a sizable share of short-run RER fluctuations. Other factors, including "other supply disturbances" (e.g., mark-up shocks) and policy shocks, made only modest contributions to RER movements.

Taken together, the different forces largely offset each other on average, helping to explain the overall stability of the EA RER during the sample period, despite notable shorter-term fluctuations.

-

<sup>&</sup>lt;sup>30</sup> The mean contribution of the international financial market component to the EA trade balance is positive, reflecting systematic net transfers from the EA to the RoW (including government development aid and private remittances). High-frequency fluctuations in this component primarily reflect UIP shocks.

Filtered shock estimates and parameter uncertainty. Appendix D presents SDs based on filtered estimates of the shocks (using information only up to the date of the shock) -- as an alternative to the smoothed shock estimates (based full-sample information) employed for Figures 7 and 8. This is particularly relevant given the presence of several large persistent shocks in our sample, which can cause filtered and smoothed shocks to diverge and potentially affect conclusions about the effects of growth shocks. Figures D1 and D2 in the Appendix confirm that our main results are robust to using filtered shocks. In addition, Appendix F quantifies the role of model parameter estimation uncertainty for SDs. Accounting for parameter uncertainty preserves our main insights, although the contribution of home bias shocks is more uncertain than that of other groups.

## 6. Gradual trade balance adjustment to persistent productivity growth shocks

This Section further investigates the mechanism through which a persistent productivity growth rate shock affects the TB. In the estimated model, such a shock—despite inducing a large and persistent divergence between EA and RoW GDP—generates only a gradual and relatively muted TB response. This contrasts with the results of Hoffmann et al. (2019), whose theoretical two-country, one-good DSGE model predicts a large, front-loaded TB adjustment to a trend shock, under full information.

We now show that the gradual TB adjustment predicted by our model stems from two features: imperfect substitution between domestic and foreign traded goods, and real adjustment frictions. Figure 9 illustrates dynamic responses of key variables to a persistent RoW intermediates productivity growth rate shock, across three model variants: (i) the estimated baseline model (blue lines); (ii) a one-traded-good version (perfect substitution between domestic and foreign tradables, all other parameters kept at baseline values; red); and (iii) a version of (ii), with all real adjustment frictions removed (yellow).<sup>31</sup>

With perfect substitutes (red), the absence of terms of trade and RER adjustment amplifies the response of EA absorption and of the EA TB, relative to the baseline. Eliminating real frictions further amplifies the TB response (yellow). Hoffmann et al. (2019) argue for a theoretical setting with imperfect information—where agents gradually learn about the persistence of shocks—as a mechanism for generating smoother TB responses to trend growth shocks (see also Engel and Rogers, 2006). By contrast, our model assumes full information, yet still produces gradual TB dynamics. Figure 9 shows that this outcome can be attributed to imperfect substitution and real frictions, without relying on imperfect information

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<sup>&</sup>lt;sup>31</sup> Namely, consumption habits, trading frictions of international assets costs, "hand-to-mouth" households, and delayed input substitution were removed.

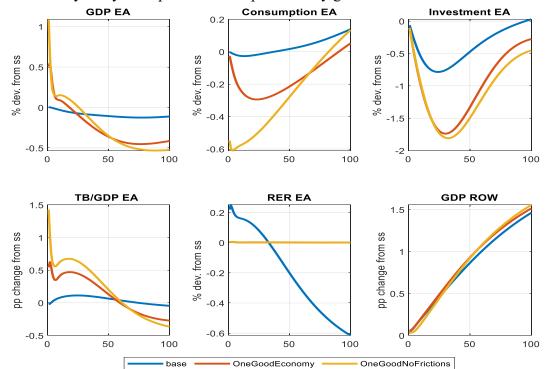


Fig. 9. Sensitivity analysis of positive RoW productivity growth rate shock

Note: The blue line ("base") depicts the dynamics from the estimated model, the red line shows a one-good economy version of the model (assuming perfect substitutability between domestic and foreign intermediates) while the yellow line presents a one-good economy version of the model without the following frictions: consumption habits, trading frictions of international assets costs, "hand-to-mouth" households, delayed input substitution

## 7. Conclusion

This paper analyzes the effect of global growth differences on real activity, trade, and the real exchange rate (RER). Based on an estimated large-scale two-region DSGE model, we show that the persistent productivity growth differential between the Euro Area (EA) and rest of the world (RoW) has been a key driver of the EA's trade surplus. A secular decline in the EA's spending home bias and a trend decrease in relative EA import prices account for the stability of the EA real exchange rate, despite slower EA output growth. By incorporating trend shocks to growth and trade, the analysis departs from much of the open-economy macroeconomics literature which has focused on stationary disturbances. Our results highlight the relevance of non-stationary shocks for the analysis of external adjustment.

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# **Appendix**

## **Appendix A. Additional Results**

Figure A1. Historical shock decompositions of EA Consumption (left) and Investment (right)

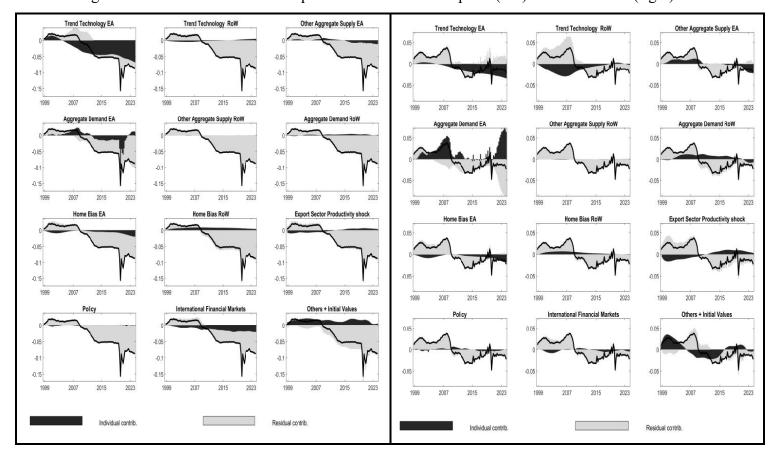
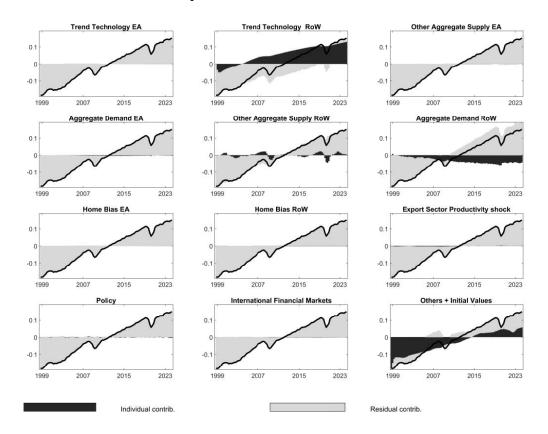


Figure A2. Historical shock decomposition, GDP RoW



# **Appendix B. Data Sources**

The analysis uses quarterly and annual data for the period 1998q4 to 2024q2 based on the data set of the European Commission's Global Multi-country Model (Albonico et al., 2019). Data for the Euro Area aggregate (EA20) are taken from Eurostat (in particular, from the European System of National Accounts). The Rest of the World (RoW) data are annual data and are constructed using IMF International Financial Statistics (IFS) and World Economic Outlook (WEO) databases.

Series for GDP and prices in RoW start in 1999 and are constructed on the basis of data for the following 57 countries: Albania, Algeria, Argentina, Armenia, Australia, Azerbaijan, Belarus, Brazil, Bulgaria, Canada, Chile, China, Colombia, Czech Republic, Denmark, Egypt, Georgia, Hong Kong, Hungary, Iceland, India, Indonesia, Iran, Israel, Japan, Jordan, Korea, Lebanon, Libya, FYR Macedonia, Malaysia, Mexico, Moldova, Montenegro, Morocco, New Zealand, Nigeria, Norway, Philippines, Poland, Romania, Russia, Saudi Arabia, Serbia, Singapore, South Africa, Sweden, Switzerland, Syria, Taiwan, Thailand, Tunisia, Turkey, Ukraine, United Arab Emirates, United Kingdom, United States and Venezuela.

When quarterly-frequency data is not available, mixed frequency data estimation is used. In this case, we observe annual RoW data in the last quarters of each year (as quarterly annualized variables) and allow the Kalman smoother to interpolate the annual data in the other three quarters in a model-consistent manner.

Table B1 lists the observed time series. GDP deflators and relative prices of aggregates are computed as the ratios of current price value to chained indexed volume.

We make a few transformations to the raw investment series. We compute the deflator of public investments based on annual data and then obtain its quarterly frequency counterpart through interpolation. This series together with nominal public investments is then used to compute real quarterly public investments. In order to assure consistency between nominal GDP and the sum of the nominal components of aggregate demand, we impute change in inventories to the series of investments.

## Table B1. Data used for estimation

#### Euro area

Nominal short-term interest rate

Log of GDP

Log of GDP price

Log of nominal consumption share

Log of nominal gov. consumption share

Log of nominal gov. investment share

Log of nominal investment share

Log of nominal total import share

Log of nominal export share

Log of nominal industrial supply import share

Log of consumption price final to observed GDP price

Log of export price to GDP price

Log of import price to observed GDP price

Log of gov. observed price to observed GDP price

Log of govt. investment price to observed GDP price

Log of observed total investment price to observed GDP price

Price of industrial supply

Oil price

Log of employment

Log of hours

Log Nominal wage share

Log of nominal gov. bonds share

Log of nominal gov. interest payments share

Log of nominal gov. transfers share

Log effective nominal exchange rate

Log of population

Log of active rate population

Log of hours per employee

Log of productivity

## RoW

RoW nominal Interest rate

Log of population

Log of GDP price

Log of GDP

Log of nominal investment share

# **Appendix C. Estimation results**

Table C1. Prior and posterior distributions of key estimated model parameters

	•					
	Prior Distribution			<b>Posterior Distribution</b>		
	Type	Mean	Std.	Mode	Std.	
EA parameters						
Consumption habit	Beta	0.50	0.10	0.886	0.036	
Risk aversion	Gamma	1.50	0.20	1.498	0.213	
Inverse of Frisch elasticity	Gamma	2.50	0.50	2.177	0.548	
Import elasticity common to all final goods	Gamma	2.00	0.40	2.387	0.437	
Constant elasticity of total Consumption bundle	Beta	0.10	0.04	0.083	0.036	
Preference for international bonds	Uniform	0.50	0.29	0.000	0.001	
Price adj. cost	Gamma	20.00	12.00	24.975	5.112	
Wage adj. cost	Gamma	40.00	24.00	18.687	7.970	
Real wage rigidity	Beta	0.85	0.06	0.873	0.034	
Hours quadratic adj. cost	Gamma	20.00	12.00	5.334	0.945	
Employment quadratic adj. cost	Gamma	20.00	12.00	15.044	2.343	
Investment quadratic adj. cost	Gamma	200.00	150.00	939.517	264.828	
Capacity Utilization quadratic adj. cost 1	Gamma	0.03	0.01	0.005	0.002	
Capacity Utilization quadratic adj. cost 2	Beta	0.30	0.12	0.274	0.102	
J-curve inertia parameter	Beta	0.50	0.20	0.866	0.053	
Taylor rule output growth gap coefficient	Beta	0.05	0.02	0.083	0.012	
Taylor rule inflation coefficient	Beta	1.70	0.15	1.535	0.125	
Interest rate inertia Taylor rule parameter	Beta	0.85	0.08	0.898	0.013	
Deficit coefficient in the transfers fiscal rule	Beta	0.03	0.01	0.025	0.008	
Persistence trend growth population	Beta	0.85	0.08	0.960	0.010	
Fixed costs in production	Beta	0.05	0.02	0.034	0.014	
SS Labor hoarding	Beta	0.05	0.02	0.027	0.012	
RoW parameters						
Consumption habit	Beta	0.70	0.10	0.673	0.098	
Risk aversion	Gamma	1.50	0.20	1.240	0.115	
Inverse of Frisch elasticity	Gamma	2.50	0.50	2.203	0.418	
Constant import elasticity	Gamma	2.00	0.40	1.630	0.233	
Price adj. cost	Gamma	20.00	12.00	0.633	0.353	
Real wage rigidity	Beta	0.50	0.20	0.910	0.036	
Hours adj. cost	Gamma	20.00	12.00	3.260	1.689	
Investment quadratic adj. cost	Gamma	200.00	150.00	21.734	134.649	
Capacity Utilization quadratic adj. cost 1	Gamma	0.03	0.01	0.050	0.012	
Capacity Utilization quadratic adj. cost 2	Beta	0.30	0.12	0.248	0.087	
Taylor rule inflation coefficient	Beta	1.70	0.15	1.715	0.135	
Taylor rule output growth gap coefficient	Beta	0.20	0.08	0.200	0.067	
Interest rate inertia Taylor rule	Beta	0.85	0.08	0.909	0.019	
J-curve inertia parameter	Beta	0.50	0.20	0.924	0.269	
Persistence trend growth population	Beta	0.85	0.08	0.989	0.001	

Table C2. Prior and posterior distributions of estimated shock processes (in %)

	Prior	Prior Distribution			Posterior Distribution	
	Туре	Mean	Std.	Mode	Std.	
Persistence						
UIP shock EA	Beta	0.50	0.20	0.756	0.073	
NFA shock EA	Beta	0.50	0.20	0.854	0.038	
Governmental consumption shock EA	Beta	0.70	0.10	0.943	0.016	
Governmental investment shock EA	Beta	0.70	0.10	0.931	0.020	
Transfers shock EA	Beta	0.70	0.10	0.868	0.038	
Tax shock EA	Beta	0.85	0.06	0.875	0.038	
Real inventories shock EA	Beta	0.85	0.05	0.881	0.035	
Private consumption specific productivity shock EA	Beta	0.50	0.20	0.912	0.035	
Government consumption specific productivity shock EA	Beta	0.50	0.20	0.931	0.025	
Private investment specific productivity shock EA	Beta	0.50	0.20	0.951	0.018	
Labor demand shock EA	Beta	0.50	0.20	0.679	0.064	
Consumption preference shock EA	Beta	0.50	0.20	0.782	0.068	
Preferences in investing in bonds EA	Beta	0.50	0.20	0.950	0.024	
Risk premium shock EA	Beta	0.85	0.08	0.922	0.035	
Stationary home bias shock EA	Beta	0.50	0.20	0.677	0.126	
Autoregressive stationary export price EA	Beta	0.50	0.20	0.695	0.152	
Risk shock RoW	Beta	0.50	0.20	0.784	0.079	
Stationary home bias shock RoW	Beta	0.50	0.20	0.669	0.076	
Stationary export price shock RoW	Beta	0.50	0.20	0.502	0.197	
Standard deviation (in %)						
UIP shock EA	Gamma	1.00	0.40	0.481	0.155	
NFA shock EA	Gamma	1.00	0.40	1.773	0.126	
Governmental consumption shock EA	Gamma	1.00	0.40	0.132	0.011	
Governmental investment shock EA	Gamma	1.00	0.40	0.075	0.005	
Transfers shock EA	Gamma	1.00	0.40	0.165	0.013	
Tax shock EA	Gamma	1.00	0.40	0.570	0.038	
Real inventories shock EA	Gamma	0.50	0.20	0.194	0.015	
Private consumption specific productivity shock EA	Gamma	1.00	0.40	0.261	0.023	
Government consumption specific productivity shock EA	Gamma	1.00	0.40	0.652	0.054	
Private investment specific productivity shock EA	Gamma	1.00	0.40	0.330	0.037	
Labor demand shock EA	Gamma	0.50	0.20	1.305	0.166	
Consumption preference shock EA	Gamma	2.00	1.15	1.144	0.754	
Wage markup shock EA	Gamma	1.00	0.40	1.229	0.545	
Preferences in investing in bonds EA	Gamma	1.00	0.40	0.098	0.007	
Risk premium shock EA	Gamma	0.75	0.43	0.361	0.356	
Price markup shock EA	Gamma	2.00	0.80	1.970	0.840	
Taylor rule shock EA	Gamma	1.00	0.40	0.086	0.007	
Taylor rule shock RoW	Gamma	1.00	0.40	0.175	0.033	
Risk shock RoW	Gamma	1.00	0.40	0.391	0.252	
Preference shock RoW	Gamma	1.00	0.40	0.664	0.174	
Price markup shock RoW	Gamma	1.00	0.40	1.392	0.760	
COVID-specific shock variances (in %)						
Labor supply shock EA	Gamma	10.00	4.00	9.362	8.867	
Labor shock EA	Gamma	0.50	0.20	0.421	0.187	
Forced savings (Preference shock) EA	Gamma	1.00	0.20	3.170	0.187	
Risk premium shock EA	Gamma	10.00	4.00	8.481	3.620	
Labor demand shock RoW	Gamma	5.00	2.00	10.441	3.377	
Forced savings (Preference shock) RoW	Gamma	5.00	2.00	5.400	1.808	

**Table C3. Calibrated parameters** 

Description Preferences	Parameter or ratio	Source
Intertemporal discount factor	0.999	annual discount rate of 1%
Weight of disutility of labor EA	9.79	endogenized in steady state
Weight of disutility of labor RoW	2.77	endogenized in steady state
Share of Ricardian households	0.67	Data (survey)
Degree of openness EA	0.18	data
Degree of openness RoW	0.05	data
Production		
Cobb-Douglas labor share	0.65	data
Depreciation of private and public capital	0.0143	data
Fiscal policy		
Consumption tax EA	0.20	data
Corporate profit tax EA	0.30	data
Labor tax EA	0.43	endogenized in steady state
Global excise duty EA	0.12	,
Deficit target EA	0.03	data
Debt target (annual) EA	3.1	data
Public capital share EA	0.1	data
Steady state ratios		
Private nominal consumption share EA	0.57	data
Private investment share EA	0.18	data
Gov't consumption share EA	0.21	data
Gov't investment share EA	0.032	data
Share of private consumption imports in the total demand EA	0.10	data
Share of private investment imports in the total demand EA	0.15	data
Share of government investment imports in the total demand EA	0.15	data
Share of imports in exports EA	0.14	data
Share of imports in production EA	0.06	data
Private investment share RoW	0.26	data
Share of private investment imports in the total demand RoW	0.03	data
Share of private consumption imports in the total demand RoW	0.05	data
Share of imports in exports RoW	0.15	data
Others		
Population trend growth (annual)	0.34%	data
Price level trend growth (annual)	2%	monetary target
Size of EA (% of world)	18%	data

## Appendix D. Historical Decompositions using real-time filtered shocks

The historical decompositions from the main body of the paper such as the ones presented in Figure 8 are computed using smoothed shocks. Given that the model includes several permanent shocks, filtered shocks can deviate from the smoothed ones which might influence our conclusions related to the effects of permanent growth shocks on the RER and the TB/GDP. To check this possibility, we redo our analysis from Figure 8 by computing the real-time shock decompositions which capture the contributions of each category of shocks with information up to time *t* instead of using information for the entire sample. Figures D1 and D2 show that our results remain robust to considering the effects of filtered shocks instead of smoothed ones.

This form of one-sided shock decomposition is done by computing rolling window smoothers and shock decompositions spanning the entire observation time interval t = 1, ..., T. For each time t, we compute smoother and shock decomposition for the time interval 1, ..., t and store the shock contributions for the last period t. Finally, we plot all these last period shocks contributions, for each period t = 1, ..., T, to produce Figures D1 and D2, which then show shock contributions that only account for past information and not full sample (i.e. proper one-sided shock decompositions).

Figure D1. Trade Balance

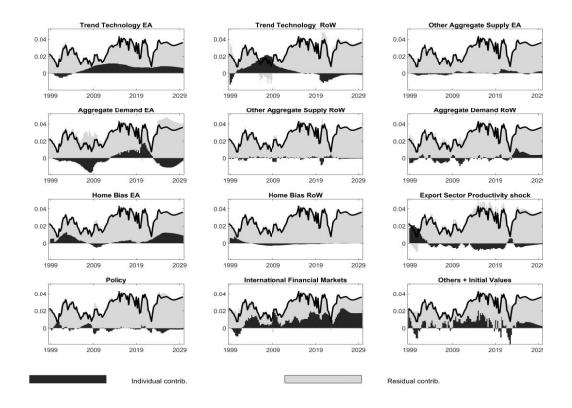
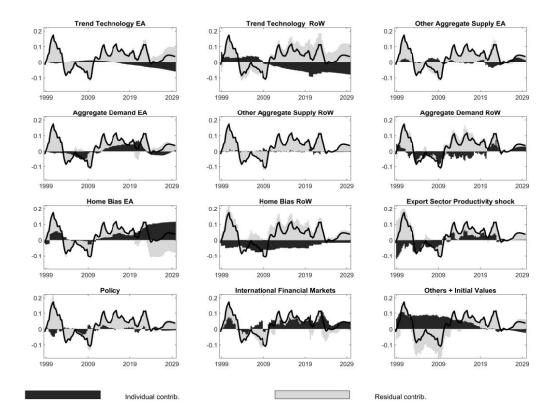


Figure D2. Real Exchange Rate



# Appendix E. Model solution and approximation

The model features a long run balanced growth path: there are global (common to all regions) deterministic trends for population, labor productivity, prices: global inflation rate is 2% annual, population growth gpop is set to euro area average (0.34% annual), while labour productivity growth ga is estimated (posterior mode  $\sim 1.2\%$  annual).

For each country *j*, the intermediate sector productivity parameter evolves as:

$$A_{j,t}^{y} = (A^{y})^{t} exp(T_{j,t}^{y} + S_{j,t}^{y}),$$

where  $A^{y} = \exp(ga)$  is the global deterministic (log-linear) growth of productivity,  $T_{j,t}^{y}$  and  $S_{j,t}^{y}$  denote the non-stationary and stationary country specific productivity shocks.

Population in each country *j* evolves as:

$$Pop_{j,t} = Pop^t exp(T_{j,t}^{pop}),$$

where  $Pop = \exp(gpop)$  is the global population log-linear growth trend.

Country specific stochastic trends of productivity  $T_{j,t}^{y}$  and population  $T_{j,t}^{pop}$  do not have any additional drift term, hence only the global productivity and population trends matter for determining balanced growth path of the model, which we can derive starting from the production function (that also includes public capital):

$$Y_{j,t} = (A_{j,t}^{y} \cdot N_{j,t})^{\alpha} K_{j,t}^{1-\alpha} K G_{j,t}^{\alpha g}$$

Imposing common growth for GDP, private and public capital and remembering that hours grow with population, we get

$$Y = (A^{y} \cdot Pop)^{\alpha} Y^{1-\alpha} Y^{\alpha^{g}}$$

from which the balanced real GDP growth rate is:

$$gy = \frac{\alpha}{\alpha - \alpha^g}(gpop + ga)$$

Global GDP annual growth gy is ~1.76% at posterior mode, given calibrated labor share a=0.65 and public capital share  $a^g=0.1$ .

The de-trended model around this balanced growth path provides the baseline stationary solution for all model variables (levels and growth rates). The model is linearized around this baseline stationary solution. For convenience and without loss of generality, the model also features some normalization assumptions:

- baseline GDP level is normalized to 1 for all regions
- baseline price levels are normalized to 1 for all regions, and so are the nominal exchange rates

- country size weights used in all cross-country market clearing equations are calibrated based on the average detrended real GDP data weighted by the respective USD/national currency (NC) exchange rate in base year (for proper comparability, all GDP sizes need to be converted into a common currency).

Consistently with model balanced growth and normalization assumptions, data series are detrended using deterministic balanced growth trends. GDP size for each country is set equal to average of detrended real GDP weighted by USD/NC exchange rate in base year. Then, all real variables are normalized by the GDP scale, so that data have the same normalization of the model.

Similarly, all price levels are first detrended by the 2% global inflation rate and then rescaled to be 1 in the base year. All exchange rates are also normalized to be 1 in the base year.

#### The model has 11 unit roots:

- 2 population trends and one active population shock for euro area
- 2 productivity trend (both RW and AR(1) growth shocks)
- 2 export productivity shocks
- 2 import share shocks
- P level EA
- P level RoW

All growth rates, inflation rates and interest rates are stationary: this implies that, whatever non stationary shock occurs, the model will ultimately converge to the assumed/estimated balanced growth path. At the same time, unit roots imply that all level variables (real/nominal) are non-stationary, i.e. such variables will converge to a new level relative to the balanced growth path. Hence price ratios, the RER, real/nominal ratios will permanently adjust after such unit root shocks. Co-integration relationship among nominal variables still occur via stationarity of TB and NFA to nominal GDP shares (so that the RER permanently adjusts to ensure this).

The new level reached by the linearized model is of course an approximation of the one that would be obtained via a perfect foresight simulation using the original non-linear model. Such an approximation is measurable and testable, comparing impulse-response functions of the linear and non-linear models.

## Simulation approximation error

Using estimated standard deviations for the innovations of such non-stationary processes, linear and non-linear results are almost identical. It is interesting to understand key permanent adjustments associated to such non stationary shocks, and whether the linearized model is able to capture the correct changes with respect to the nonlinear one. These are shown in Table F1, where we simulate the model using innovations ten times larger than the estimated standard deviations, in order to increase the effect of the nonlinearity.

*Table E1.* Table of multipliers of GDP RoW/EA and RER EA after 1% permanent level shocks.

	linear			nonlinear positive (negative) shock			
	GDP EA	GDP RoW	RER EA	GDP EA	GDP RoW	RER EA	
persistent productivity EA	1.2	0.01	0.5	1.2 (1.1)	0.01	0.4 (0.5)	
level productivity EA	1.2	0.01	0.4	1.2	0.01	0.4 (0.5)	
persistent productivity RoW	0.1	1.2	-0.4	0.1	1.2	-0.4 (-0.5)	
level productivity RoW	0.1	1.2	-0.4	0.1	1.2	-0.4	
export productivity EA	0.1	0.02	-0.2	0.1	0.02	-0.2	
export productivity RoW	0.1	0.05	0.4	0.1	0.05 (0.06)	0.5 (0.4)	
import share EA	-0.1	0.01	0.3	-0.1	0.01	0.3	
import share RoW	0.1	-0.008	-0.4	0.1	-0.012 (-0.005)	-0.4	
population EA	1.2	0.01	0.5	1.2	0.01	0.5	
population RoW	0.1	1.2	-0.4	0.1	1.2	-0.4	
active population rate EA	1.2	0.01	0.5	1.2	0.01	0.4 (0.5)	

*Note:* For nonlinear simulations the multiplier of a negative shock is reported in parenthesis when it differs from positive shock.

Using innovations ten times larger than the estimated standard deviations, deterministic and linear simulations are still the same in the short run, while the terminal level may differ up to 10-15%, i.e. the linear model still provides an excellent approximation to the nonlinear one even in the presence of sizeable permanent level shifts.

# Key adjustments to permanent shocks:

- Permanent productivity shocks in EA (RoW) produce permanent positive effect to domestic GDP and a permanent real depreciation (appreciation). Long run spillovers to EA are positive and significant for RoW shocks, while spillover to RoW for EA is positive but modest (due to relative size).
- Population (active population) shocks feature very similar long run effects as the productivity ones, both qualitatively and quantitatively.
- Export productivity shocks in EA/RoW provide the same small positive effect for EA GDP (long run multiplier 0.1) while the effect is smaller for RoW. Euro area RER *appreciates* (*depreciates*) after a positive permanent productivity shock in EA (RoW) exports.
- A permanent increase in the import share in EA (RoW) triggers a permanent drop (rise) in EA GDP (long run multiplier 0.1) and a permanent real depreciation (appreciation) of the euro. GDP effects for RoW are more modest but qualitatively the same.

## Data filtering approximation error

One may also measure the approximation error of data filtering using a linearized model around balanced growth path. To do so, we perform counterfactual non-linear simulations using the linear smoother to set initial state variables and historical shocks, with a staggered type of algorithm:

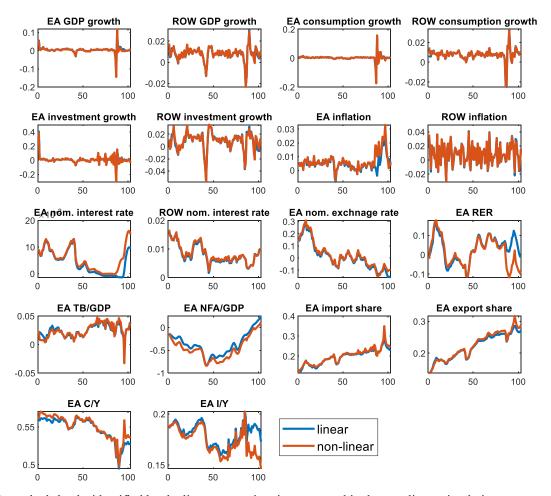
for all periods t = 1, ..., T

- 1) given previous period states  $s_{t-1}$  and unexpected shocks in current period  $\varepsilon_t$ , one may perform deterministic perfect foresight simulations for a large number of periods ahead until new equilibrium is reached.
- 2) from this multi period ahead simulations, current period simulated values provide the updated variables and states in t:  $y_t$  and  $s_t$ . These values can be used to initialize states and do the simulation in period t + 1.
- 3) repeat the same procedure in 1) and 2) for all periods t = 1, ..., T.

We report results of this exercise in Figure F1: the non-linear simulations broadly reproduce the correct historical patterns. One notable exception regards inflation response in euro area during pandemic: indeed the latter shocks are several times larger than usual business cycle shocks and hence push (temporarily) the model very far from the baseline approximation region.

Would these discrepancies be reduced by using stationary persistent shocks in place of the non-stationary processes? *The answer is NO*, *since the approximation error depends on how much the data deviate from the balanced growth path*, independently on the shocks that generate these deviations. The nature of the shocks only affect the forecast error and the long run implications of those shocks, but cannot change the deviation of the data with respect to the assumed balanced growth.

Figure E1. Counterfactual non-linear simulations vs linear approximation

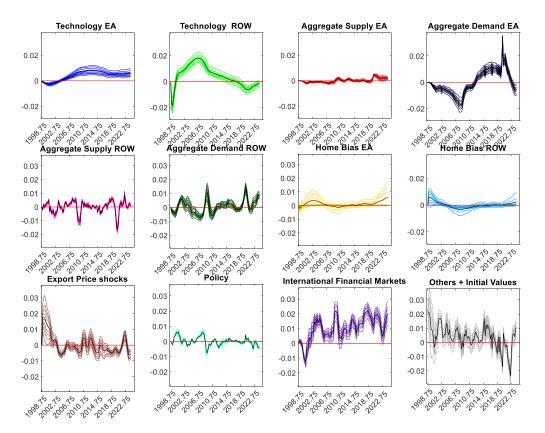


Notes: Smoothed shocks identified by the linear approximations are used in the non-linear simulation.

# **Appendix F. Parameter Uncertainty**

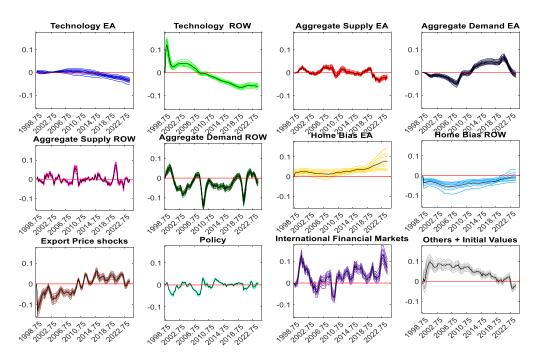
Figures F1 and F2 illustrate how much each structural shock category has driven the trade-balance and RER over time and how confident we can be in those estimates by plotting their time-varying posterior contributions alongside associated uncertainty bands (across deciles). Figures F3-F5 present impulse-response functions to main permanent shocks with 90% confidence intervals considering posterior parameter uncertainty.

Figure F1. Trade Balance



Note: Each panel presents the posterior uncertainty (deciles) of the historical contribution of a category of shocks in the shock decomposition of the trade balance together with the mean contribution (black line).

Figure F2. Real Exchange Rate



# Uncertainty of transmission of permanent shocks

Figure F3. Productivity Shocks

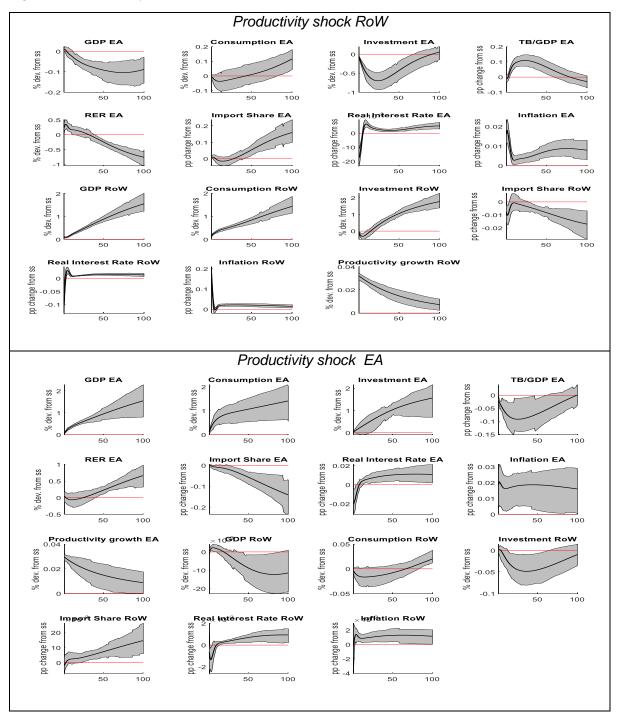


Figure F4. Home Bias Shocks

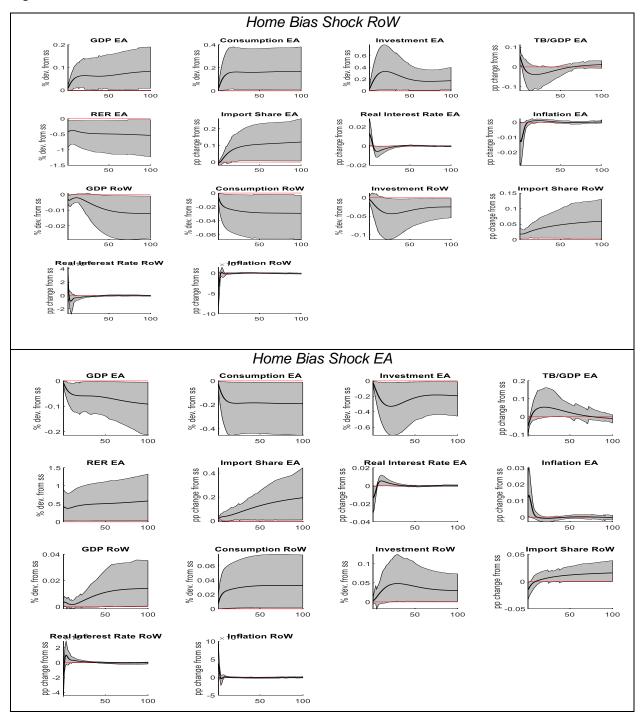
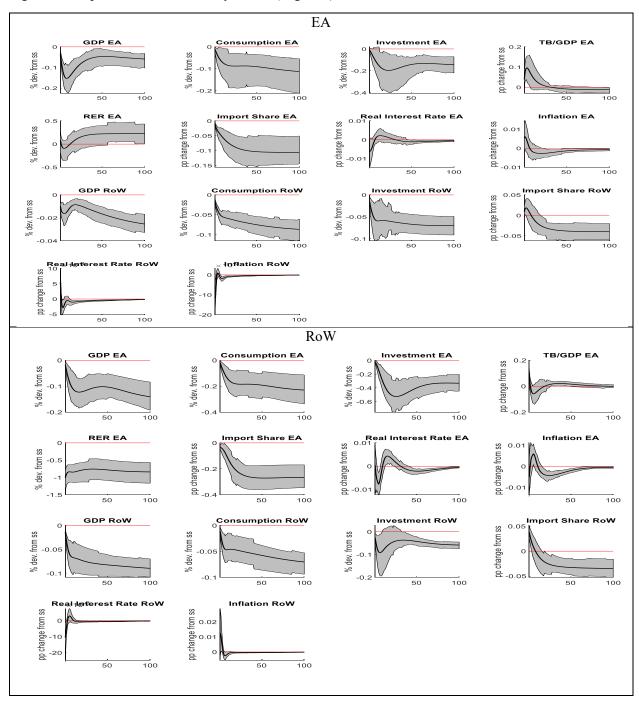


Figure F5. Export Sector Productivity Shock (negative)



# Not For Publication Appendix: Model Details

# Persistent Global Growth Differences and Euro Area Adjustment: Real Activity, Trade, and the Exchange Rate \*

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This appendix provides additional model details omitted in the main text. The model shares many standard elements with Albonico et al. (2019), and we also refer to the model description contained therein.

# 1 Households

# 1.1 Savers: Problem and Asset Choices

A fraction  $\omega^s$  of households are savers (s). These households own domestic firms, participate in asset markets, consume, and receive wage income for  $N_{j,t}^{s,\mathrm{paid}}$  hours worked, where  $N_{j,t}^{s,\mathrm{paid}}$  is set by unions and not chosen by households. Let  $P_t^{C,\mathrm{vat}} = (1+\tau^C)P_t^C$  denote the price of the consumption basket including VAT.

**Financial assets.** Savers allocate wealth across several assets: private risk-free bonds in zero net supply, government bonds, internationally traded bonds denominated in the currency of the rest of the world, and shares in domestic firms. In what follows, the superscript Q identifies the asset class, with rf denoting risk-free private bonds, G for government bonds, g for international bonds, and g for firm shares (equity). For use of more compact notation below, we define equity as  $g_t^S = g_t^S$ ,  $g_{tt}$ , where  $g_t^S$  is the nominal price of shares and  $g_{tt}$  the number of shares held by the household. Moreover,  $g_{tt}^{bw} = \mathcal{E}_t g_{tt}^W$  is the nominal balance of foreign-denominated bonds (expressed in domestic currency). Total nominal financial wealth is:

$$B_{jt} = B_{jt}^{rf} + B_{jt}^g + B_{jt}^{bw} + B_t^S, (1)$$

**Optimization.** Each period, the representative saver j chooses consumption  $C_{j,t}^s$  and next-period asset holdings  $B_{j,t+1}^{\mathcal{Q}}$ , where  $\mathcal{Q}$  takes values in  $\{rf, G, bw, S\}$ , to maximise expected lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t^t \left\{ \frac{\left( C_{j,t}^s - \varepsilon_{t-1}^{tC} - h(C_{t-1}^s - \varepsilon_t^{tC}) \right)^{1-\theta}}{1-\theta} - \frac{\overline{\lambda}_t^s}{P_t^{C,\text{vat}}} \sum_{\mathcal{Q}} B_{j,t}^{\mathcal{Q}} \left( \alpha^{\mathcal{Q}} - \varepsilon_t^{\mathcal{Q}} \right) \right\}. \tag{2}$$

Here,  $C_{j,t}^s$  is the consumption of saver j at time t, h is the external habit parameter, and  $\theta>0$  is the inverse elasticity of intertemporal substitution. The time discount factor is given by  $\tilde{\beta}_t=\beta\xi_t$ , where  $\xi_t$  captures stochastic disturbances to the discount factor  $\beta$ .  $\xi_{t+1}/\xi_t=\exp\left(\varepsilon_t^C\right)$  implies that the Euler equations are affected by the time t cyclical saving shock process  $\varepsilon_t^C$ . The shock  $\varepsilon_t^{tC}$  represents a transitory, COVID-specific "forced savings" shock that shifts consumption independently of the habit term. The term  $\overline{\lambda}_t^s$  is a time-varying scaling factor for marginal utility of asset positions, included for balanced growth. For each asset,  $\alpha^Q$  is the steady-state risk premium and  $\varepsilon_t^Q$  is the risk premium shock at time t.

The budget constraint is

$$P_{t}^{C,\text{vat}}C_{j,t}^{s} + \sum_{\mathcal{Q}} B_{j,t+1}^{\mathcal{Q}} = W_{t}N_{j,t}^{s,\text{paid}} + div_{t}^{TOT} + \sum_{\mathcal{Q}} R_{t}^{\mathcal{Q}}B_{j,t}^{\mathcal{Q}} + T_{j,t}^{s} - \varphi^{bw} \frac{1}{2} \frac{\left(B_{j,t+1}^{bw}\right)^{2}}{GDP_{t}^{N}}.$$
 (3)

In this constraint,  $W_t$  is the nominal wage,  $div_t^{TOT}$  is the total dividend payout from domestic firms, and  $R_t^{\mathcal{Q}}$  is the gross nominal return on asset  $\mathcal{Q}$  between t-1 and t. The term  $T_{i,t}^s = 1$ 

 $TR_{j,t}^s - tax_{j,t}^s - \tau^N W_t N_{j,t}^{s,\mathrm{paid}} - \tau^C P_t^{C,\mathrm{vat}} C_{j,t}^s$  collects lump-sum transfers, lump-sum taxes, labor taxes (at rate  $\tau^N$ ), and consumption taxes (at rate  $\tau^C$ ). The parameter  $\varphi^{bw}$  governs the size of the quadratic adjustment cost on foreign bonds (as a share of Y), which is rebated lump-sum.

The first-order conditions for the household's problem consist of the consumption Euler equation and the asset FOCs, which are linked by marginal utility. Marginal utility of consumption is given by

$$\lambda_t^s = (C_t^s - \varepsilon_t^{tC} - h(C_{t-1}^s - \varepsilon_{t-1}^{tC}))^{-\theta},$$

and the intertemporal marginal rate of substitution is

$$\Lambda_{t,t+1}^s = E_t \left[ \tilde{\beta}_{t+1} \frac{\lambda_{t+1}^s}{\lambda_t^s} \right].$$

Using this definition, the first-order conditions for assets Q in  $\{rf, G, S, bw\}$  are

$$1 = E_t \left[ \Lambda_{t,t+1}^s \frac{R_t^{rf}}{1 + \pi_{t+1}^{c,\text{vat}}} \right] \tag{4}$$

$$1 = E_t \left[ \Lambda_{t,t+1}^s \frac{R_t^g - \varepsilon_t^B - \alpha^{bG}}{1 + \pi_{t+1}^{c,\text{vat}}} \right]$$
 (5)

$$1 = E_t \left[ \Lambda_{t,t+1}^s \frac{R_{t+1}^S - \varepsilon_t^S - \alpha^S}{1 + \pi_{t+1}^{c,\text{vat}}} \right]$$
 (6)

$$1 = E_t \left[ \Lambda_{t,t+1}^s \frac{R_t^W \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} - \varepsilon_t^{bw} - \alpha^{bw} - \varphi^{bw} \frac{\mathcal{E}_t B_t^W}{P_{t-1}^Y Y_{t-1}}}{1 + \pi_{t+1}^{c,\text{vat}}} \right]$$
(7)

where  $R^{\mathcal{Q}}$  is the (gross) nominal return and  $\pi_{t+1}^{C,\text{vat}}$  is the gross inflation rate for the VAT-inclusive consumption basket. The return on firm shares is defined as

$$R_{t+1}^S = \frac{P_{t+1}^S + div_{t+1}}{P_t^S},$$

where  $P_t^S$  is the equity price and  $div_{t+1}$  is the dividend payout from domestic differentiated intermediate producers.

## 1.2 Hand-to-mouth Households

The remaining households, with a population share  $1 - \omega^s$ , are so-called *hand-to-mouth* consumers (denoted c), who face a zero-borrowing constraint and do not participate in asset markets. Each period, these households consume all of their current disposable wage and transfer income. Their consumption is determined by

$$P_t^{C,\text{vat}}C_{j,t}^c = W_t N_{j,t}^{c,\text{paid}} + T_{j,t}^c - P_t^{C,\text{vat}} \left( \varepsilon_t^{tC} - \frac{1}{6} \sum_{i=8}^{13} \varepsilon_{t-i}^{tC} \right). \tag{8}$$

Here,  $P_t^{C,\text{vat}}$  is the consumption price index including VAT,  $W_t$  is the nominal wage,  $N_{j,t}^{c,\text{paid}}$  denotes paid hours worked, and  $T_{j,t}^c$  collects net transfers, taxes, and social contributions. The terms  $\varepsilon_t^{tC}$  and  $\varepsilon_{t-i}^{tC}$  represent COVID-19-related forced savings shocks. During the pandemic, these households accumulate forced savings, which are gradually spent as the pandemic ends.

# 1.3 Aggregation

Total household consumption (per capita) is given by

$$C_t = (1 - \omega^s)C_t^c + \omega^s C_t^s, \tag{9}$$

where  $C_t^c$  and  $C_t^s$  are average consumption of hand-to-mouth and saver households, respectively. The labor union (see below) sets hours worked for both groups hence  $N_t = N_t^c = N_t^s$ , where  $N_t^c$  and  $N_t^s$  denote average paid hours for each group.

# 2 Production

# 2.1 Forcing Processes

We estimate our model using non-stationary data and, accordingly, include both trend and stationary exogenous variables to capture fluctuations at different frequencies. A generic logged exogenous variable  $\ln A_t^q$  (e.g., productivity in sector q) is specified as the sum of a stochastic trend component  $T_t^q$  and a cyclical component  $S_t^q$ , as follows:

$$\ln\left(\frac{A_t^q}{A^q}\right) = T_t^q + S_t^q,\tag{10}$$

where  $A^q$  is a constant.

The trend component is modeled such that its first difference  $g_t^q \equiv T_t^q - T_{t-1}^q$  follows an AR(1) process:

$$g_t^q = \rho^q g_{t-1}^q + (1 - \rho^q) \bar{g}^q + \varepsilon_t^q, \quad 0 < \rho^q < 1,$$
 (11)

and the cyclical component follows a stationary AR(1) process in levels:

$$S_t^q = \lambda^q S_{t-1}^q + \eta_t^q, \quad 0 \le \lambda^q \le 1. \tag{12}$$

The innovations  $\varepsilon^q_t$  and  $\eta^q_t$  are orthogonal i.i.d. white noise processes. When  $\lambda^q < 1$ ,  $S^q_t$  is stationary; if  $\lambda^q = 1$ , it follows a random walk. Innovation  $\varepsilon^q_t$  induces a persistent but mean-reverting change in the growth rate of  $A^q_t$  with drift  $\bar{g}^q$ , resulting in a permanent shift in the level of  $A^q_t$ .

# 2.2 Differentiated Intermediate Goods

Each firm  $i \in [0, 1]$  produces a variety of the domestic good, which is an imperfect substitute for varieties produced by other firms. Firms are monopolistically competitive and face a downward-sloping demand function for their individual varieties.

Differentiated goods are produced using capital  $K_{i,t-1}$  and labour  $N_{i,t}$ , combined in a Cobb-

Douglas production function:

$$Y_{i,t} = \left(A_t^Y(N_{i,t} - FN)\right)^{\alpha} \left(cu_{i,t}K_{i,t-1}\right)^{1-\alpha} \left(K_{i,t-1}^G\right)^{1-\alpha_G} - \left(A_t^Y\right)^{\frac{\alpha}{\alpha + \alpha_G - 1}} \Phi,\tag{13}$$

where  $\alpha$  is the steady-state labour share,  $A_t^Y$  is labour-augmenting productivity (common across firms), FN is a a parameter governing overhead labor,  $cu_{i,t}$  is firm-specific capital utilisation, and  $\Phi$  captures fixed costs in production.  $\alpha_G$  denotes the output elasticity with respect to public capital.  $A_t^Y$  follows a non-stationary process defined above.

Total hours paid are given by:

$$N_{i,t}^{\text{paid}} = Empl_{i,t} \cdot Hpere_{i,t},$$

and firms can adjust at both the extensive and intensive labour margins, subject to adjustment costs. During the COVID-19 pandemic, we introduce a labour hoarding shock  $\varepsilon_t^{LU}$  to capture the wedge between paid hours and effective hours worked:

$$\frac{N_{i,t}}{N_{i,t}^{\text{paid}}} = 1 - \varepsilon_t^{LU}. \tag{14}$$

Firms maximise the real market value of the firm, which is the discounted stream of expected profits, subject to the demand function  $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\sigma^y} Y_t$ , the technology constraint (13), and the capital accumulation equation  $K_{i,t} = I_{i,t} + (1 - \delta)K_{i,t-1}$ .

The firm's optimisation problem is

$$\max_{\substack{P_{i,t}, Hpere_{i,t}, Empl_{i,t}, I_{i,t}, \\ cu_{i,t}, K_{i,t}}} \sum_{s=t}^{\infty} \Lambda_{t,s} \cdot div_{i,s}, \tag{15}$$

where  $\Lambda_{t,s}$  is the stochastic discount factor from time t to s, and

$$\Lambda_{t,s} = \prod_{\tau=t}^{s-1} \Lambda_{\tau,\tau+1}.$$

The intermediate input price index is:

$$P_t = \left(\int_0^1 P_{i,t}^{1-\sigma^y} di\right)^{1/(1-\sigma^y)}.$$
 (16)

The firm's real dividend is:

$$\frac{div_{i,t}}{P_t} = (1 - \tau^K) \left( \frac{P_{i,t}}{P_t} Y_{i,t} - \frac{W_t}{P_t} N_{i,t}^{\text{paid}} \right) + \tau^K \delta \frac{P_t^I}{P_t} K_{i,t-1} - \frac{P_t^I}{P_t} I_{i,t} - \Gamma_{i,t},$$
(17)

where  $\tau^{K}$  is the corporate tax rate, and  $\delta$  is the depreciation rate.

Firms face quadratic adjustment costs:

$$\Gamma_{i,t} = \Gamma_{i,t}^P + \Gamma_{i,t}^E + \Gamma_{i,t}^H + \Gamma_{i,t}^I + \Gamma_{i,t}^{cu}, \tag{18}$$

<sup>&</sup>lt;sup>1</sup>We assume that the total number of shares  $S_t^{\text{tot}} = 1$ .

with

$$\Gamma_{i,t}^{P} = \sigma^{y} \frac{\gamma^{P}}{2} Y_{t} \left( \frac{P_{i,t}}{P_{i,t-1}} - \exp(\bar{\pi}) \right)^{2}, \tag{19}$$

$$\Gamma_{i,t}^{E} = \frac{\gamma^{E}}{2} \left( \frac{Empl_{i,t}}{Empl_{i,t-1}} - \exp(g^{pop}) \right)^{2}, \tag{20}$$

$$\Gamma_{i,t}^{H} = \frac{W_t}{P_t} Empl_{i,t} Hpere_t^{trend} \left[ \gamma^{H,1} (Hpere_{i,t} - 1) + \frac{\gamma^{H,2}}{2} (Hpere_{i,t} - 1)^2 \right], \tag{21}$$

$$\Gamma_{i,t}^{I} = \frac{P_t^{I}}{P_t} \left[ \frac{\gamma^{I,2}}{2} \frac{(I_{i,t} - I_{i,t-1} \exp(g^Y))^2}{K_{t-1}} \right], \tag{22}$$

$$\Gamma_{i,t}^{cu} = \frac{P_t^I}{P_t} K_{i,t-1} \left[ \gamma^{cu,1} (cu_{i,t} - 1) + \frac{\gamma^{cu,2}}{2} (cu_{i,t} - 1)^2 \right].$$
 (23)

Trend terms  $g^{pop}$ , and  $g^Y$ , and are the trends in population, and GDP, respectively.  $\delta_t^K \neq \delta$  adjusts depreciation to remove trend-path adjustment costs:<sup>2</sup>

The first-order conditions (FOCs) for the firm's choice variables, hours per employee  $Hpere_t$ , employment  $Empl_t$ , investment  $I_t$ , and capital utilisation  $cu_t$ , are:

$$\alpha \frac{\mu_t^y P_t Y_t}{W_t Empl_t Hpere_t^{\text{trend}}} = Hpere_t \left[ \gamma^{H,1} + \gamma^{H,2} (Hpere_t - 1 + \varepsilon_t^{Hpere}) \right], \tag{24}$$

$$(1 - \tau^K) \frac{W_t}{P_t} = \alpha (\mu_t^y - \varepsilon_t^{ND}) \frac{Y_t}{Empl_t - FN} - \frac{\partial \Gamma_t^E}{\partial Empl_t} + E_t \left[ \Lambda_{t,t+1} \frac{\partial \Gamma_{t+1}^E}{\partial Empl_t} \right], \quad (25)$$

$$Q_{t} = E_{t} \left[ \Lambda_{t,t+1} \frac{P_{t+1}^{I}}{P_{t+1}} \frac{P_{t}}{P_{t}^{I}} \left( \tau^{K} \delta^{K} - \frac{\partial \Gamma_{t}^{cu}}{\partial K_{t-1}} + Q_{t+1} (1 - \delta) + (1 - \alpha) \mu_{t+1}^{y} \frac{P_{t+1}}{P_{t+1}^{I}} \frac{Y_{t+1}}{K_{t}} \right) \right], \tag{26}$$

$$Q_t = 1 + \gamma^{I,2} \frac{(I_t - I_{t-1} \exp(g^Y))}{K_{t-1}}$$
(27)

$$-E_{t} \left[ \Lambda_{t,t+1} \frac{P_{t+1}^{I}}{P_{t+1}} \frac{P_{t}}{P_{t}^{I}} \exp(g^{Y}) \gamma^{I,2} \frac{(I_{t+1} - I_{t} \exp(g^{Y}))}{K_{t}} \right], \tag{28}$$

$$\mu_t^y(1-\alpha)\frac{Y_t}{cu_t}\frac{P_t}{P_t^I} = K_{t-1}\left[\gamma^{u,1} + \gamma^{u,2}(cu_t - 1)\right],\tag{29}$$

where  $Q_t = \mu_t^y \frac{P_t}{P_t^I}$ , and  $\Lambda_{t,t+1}$  is the stochastic discount factor from t to t+1.

In a symmetric equilibrium  $(P_{i,t} = P_t)$ , the price-setting FOC yields a New Keynesian Phillips Curve:

$$\mu_t^y \sigma^y = (1 - \tau^K)(\sigma^y - 1) + \sigma^y \gamma^P \frac{P_t}{P_{t-1}} (\pi_t - \bar{\pi}) - \sigma^y \gamma^P E_t \left[ \Lambda_{t,t+1} \frac{P_{t+1}}{P_t} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - \bar{\pi}) \right] + \sigma^y \varepsilon_t^\mu, \tag{30}$$

<sup>&</sup>lt;sup>2</sup>We specify  $\delta_t^K = \exp(g^Y) - (1 - \delta)$  so that  $\frac{I}{K} - \delta^K = 0$  on trend.

where  $\varepsilon_t^{\mu}$  is a white noise markup shock.

# 2.3 Intermediate Goods Bundling

The composite intermediate good  $Z_t$  is produced by bundling domestically produced goods  $Y_t$  and a commodity input  $IS_t$ :

$$Z_{t} = \left[ (1 - s_{t}^{IS})^{1/\sigma^{o}} Y_{t}^{(\sigma^{o} - 1)/\sigma^{o}} + (s_{t}^{IS})^{1/\sigma^{o}} (IS_{t})^{(\sigma^{o} - 1)/\sigma^{o}} \right]^{\sigma^{o}/(\sigma^{o} - 1)}, \tag{31}$$

where  $s_t^{IS}$  is the time-varying share parameter and  $\sigma^o>1$  is the elasticity of substitution.

Firms choose  $Y_t$  and  $IS_t$  to maximise profits:

$$\max_{Y_t, IS_t} \left[ P_t^Z Z_t - P_t Y_t - P_t^{IS} I S_t \right],$$

where  $P_t^Z$  is the price of the composite  $Z_t$ , and  $P_t$ ,  $P_t^{IS}$  are the prices of  $Y_t$  and  $IS_t$ . The first-order conditions yield the demand functions:

$$Y_t = (1 - s_t^{IS}) \left(\frac{P_t}{P_t^Z}\right)^{-\sigma^o} Z_t, \tag{32}$$

$$IS_t = s_t^{IS} \left(\frac{P_t^{IS}}{P_t^Z}\right)^{-\sigma^o} Z_t. \tag{33}$$

The CES price index is:

$$P_t^Z = \left[ (1 - s_t^{IS})(P_t)^{1 - \sigma^o} + s_t^{IS}(P_t^{IS})^{1 - \sigma^o} \right]^{1/(1 - \sigma^o)}.$$
 (34)

# 2.4 Intermediate Inputs

Intermediate inputs  $O_t$  are a CES aggregate of domestic intermediates  $Z_t$  and imported intermediates  $MZ_t$ :

$$O_t = \left[ (1 - s_t^{MZ})^{1/\sigma^z} Z_t^{(\sigma^z - 1)/\sigma^z} + (s_t^{MZ})^{1/\sigma^z} M Z_t^{(\sigma^z - 1)/\sigma^z} \right]^{\sigma^z/(\sigma^z - 1)}, \tag{35}$$

where  $s_t^{MZ}$  is the stochastic share of imported intermediates, and  $\sigma^z$  is the elasticity of substitution.

The corresponding demand functions are:

$$Z_t = (1 - s_t^{MZ}) \left(\frac{P_t^Z}{P_t^O}\right)^{-\sigma^z} O_t, \tag{36}$$

$$MZ_t = s_t^{MZ} \left(\frac{P_t^{MZ}}{P_t^O}\right)^{-\sigma^z} O_t. \tag{37}$$

The CES price index for  $O_t$  is

$$P_t^O = \left[ (1 - s_t^{MZ})(P_t^Z)^{1 - \sigma^z} + s_t^{MZ}(P_t^{MZ})^{1 - \sigma^z} \right]^{1/(1 - \sigma^z)}.$$
 (38)

# 2.5 Final Goods

The final good  $D_t$  is produced by combining aggregate intermediate inputs  $O_t$  and imported final inputs  $M_t$ , also using a CES technology:

$$D_t = A_t^p \left[ (1 - s_t^M)^{1/\sigma^z} O_t^{(\sigma^z - 1)/\sigma^z} + (s_t^M)^{1/\sigma^z} M_t^{(\sigma^z - 1)/\sigma^z} \right]^{\sigma^z/(\sigma^z - 1)}, \tag{39}$$

where  $A_t^p$  is a time-varying technology or preference shifter,  $s_t^M$  is the stochastic import share in final goods, and  $\sigma^z$  is the elasticity of substitution. As discussed in the main text,  $s_t^M$  is subject to cyclical and trend shocks and the same for all components.

Profit maximisation gives the input demand functions:

$$O_t = (A_t^p)^{\sigma^z - 1} (1 - s_t^M) \left(\frac{P_t^O}{P_t^D}\right)^{-\sigma^z} D_t, \tag{40}$$

$$M_t = (A_t^p)^{\sigma^z - 1} s_t^M \left(\frac{P_t^M}{P_t^D}\right)^{-\sigma^z} D_t, \tag{41}$$

with the corresponding price index

$$P_t^D = (A_t^p)^{-1} \left[ (1 - s_t^M)(P_t^O)^{1 - \sigma^z} + s_t^M (P_t^M)^{1 - \sigma^z} \right]^{1/(1 - \sigma^z)}.$$
 (42)

Total intermediate output is equal to the sum of all domestic output components:

$$O_{t} = O_{t}^{C} + O_{t}^{I} + O_{t}^{G} + O_{t}^{IG} + O_{t}^{X} + \frac{INV_{t}}{P_{t}^{O}},$$

where  $O_t^C$  etc. denote output supplied to each use, and  $INV_t$  is nominal inventory investment (residual from the resource constraint).

Total imports are defined below.

# 2.6 Delayed Substitution

We follow Auclert et al. (2021) in modeling delayed substitution. The standard CES demand functions are log-linearized and extended to incorporate forward-looking dynamics and inertia, allowing for sluggish adjustment over time. Consider the optimal demands for the domestic and imported components in the bundle:

$$\frac{O_t^{\mathcal{D}}}{\mathcal{D}_t} = (A_t^{p^{\mathcal{D}}})^{\sigma^z - 1} \left( 1 - s^{M, \mathcal{D}} \right) \left( \frac{P_t^O}{P_t^{\mathcal{D}}} \right)^{-\sigma^z} \tag{43}$$

$$\frac{M_t^{\mathcal{D}}}{\mathcal{D}_t} = (A_t^{p^{\mathcal{D}}})^{\sigma^z - 1} s^{M, \mathcal{D}} \left(\frac{P_t^M}{P_t^{\mathcal{D}}}\right)^{-\sigma^z} \tag{44}$$

Taking logs gives:

$$\log\left(\frac{O_t^{\mathcal{D}}}{\mathcal{D}_t}\right) = (\sigma^z - 1)\log(A_t^{p^{\mathcal{D}}}) + \log(1 - s^{M,\mathcal{D}}) + \sigma^z\log\left(\frac{P_t^{\mathcal{D}}}{P_t^{\mathcal{D}}}\right) \tag{45}$$

$$\log\left(\frac{M_t^{\mathcal{D}}}{\mathcal{D}_t}\right) = (\sigma^z - 1)\log(A_t^{p^{\mathcal{D}}}) + \log(s^{M,\mathcal{D}}) + \sigma^z\log\left(\frac{P_t^{\mathcal{D}}}{P_t^M}\right)$$
(46)

Defining deviations from steady state shares as

$$d\log\left(\frac{O_t^{\mathcal{D}}}{\mathcal{D}_t}\right) \equiv \log\left(\frac{O_t^{\mathcal{D}}}{\mathcal{D}_t}\right) - \log(1 - s^{M,\mathcal{D}}), \quad d\log\left(\frac{M_t^{\mathcal{D}}}{\mathcal{D}_t}\right) \equiv \log\left(\frac{M_t^{\mathcal{D}}}{\mathcal{D}_t}\right) - \log(s^{M,\mathcal{D}}),$$

we obtain:

$$\log\left(\frac{O_t^{\mathcal{D}}}{\mathcal{D}_t}\right) - \log(1 - s^{M,\mathcal{D}}) = (\sigma^z - 1)\log(A_t^{p^{\mathcal{D}}}) + \sigma^z\log\left(\frac{P_t^{\mathcal{D}}}{P_t^{\mathcal{D}}}\right)$$
(47)

$$\log\left(\frac{M_t^{\mathcal{D}}}{\mathcal{D}_t}\right) - \log(s^{M,\mathcal{D}}) = (\sigma^z - 1)\log(A_t^{p^{\mathcal{D}}}) + \sigma^z\log\left(\frac{P_t^{\mathcal{D}}}{P_t^M}\right)$$
(48)

Letting  $\mathcal{XOD}_t = \frac{O_t^{\mathcal{D}}}{\mathcal{D}_t}$  and  $\mathcal{XMD}_t = \frac{M_t^{\mathcal{D}}}{\mathcal{D}_t}$ , we can rewrite:

$$\log(\mathcal{XOD}_t) - \log(1 - s^{M,\mathcal{D}}) = (\sigma^z - 1)\log(A_t^{p^{\mathcal{D}}}) + \sigma^z \log\left(\frac{P_t^{\mathcal{D}}}{P_t^{\mathcal{D}}}\right)$$
(49)

$$\log(\mathcal{XMD}_t) - \log(s^{M,\mathcal{D}}) = (\sigma^z - 1)\log(A_t^{p^{\mathcal{D}}}) + \sigma^z \log\left(\frac{P_t^{\mathcal{D}}}{P_t^M}\right)$$
 (50)

To introduce sluggishness, we allow the shares to adjust according to:

$$\log(\mathcal{XOD}_{t}) - \log(1 - s^{M,\mathcal{D}}) = (1 - \tilde{\beta}_{t}\rho^{z}) \left[ (\sigma^{z} - 1)\log(A_{t}^{p^{\mathcal{D}}}) + \sigma^{z}\log\left(\frac{P_{t}^{\mathcal{D}}}{P_{t}^{\mathcal{O}}}\right) \right]$$

$$+ \tilde{\beta}_{t}\rho^{z} \left[ \log(\mathcal{XOD}_{t+1}) - \log(1 - s^{M,\mathcal{D}}) \right]$$

$$\log(\mathcal{XMD}_{t}) - \log(s^{M,\mathcal{D}}) = (1 - \tilde{\beta}_{t}\rho^{z}) \left[ (\sigma^{z} - 1)\log(A_{t}^{p^{\mathcal{D}}}) + \sigma^{z}\log\left(\frac{P_{t}^{\mathcal{D}}}{P_{t}^{M}}\right) \right]$$

$$+ \tilde{\beta}_{t}\rho^{z} \left[ \log(\mathcal{XMD}_{t+1}) - \log(s^{M,\mathcal{D}}) \right]$$

$$(52)$$

The actual bundle shares then evolve according to:

$$\log\left(\frac{O_t^{\mathcal{D}}}{\mathcal{D}_t}\right) = (1 - \rho^z)\log(\mathcal{XOD}_t) + \rho^z\log\left(\frac{O_{t-1}^{\mathcal{D}}}{\mathcal{D}_{t-1}}\right) - \frac{s^{M,\mathcal{D}}}{1 - s^{M,\mathcal{D}}}\log(u_t^M)$$
 (53)

$$\log\left(\frac{M_t^{\mathcal{D}}}{\mathcal{D}_t}\right) = (1 - \rho^z)\log(\mathcal{XMD}_t) + \rho^z\log\left(\frac{M_{t-1}^{\mathcal{D}}}{\mathcal{D}_{t-1}}\right) + \log(u_t^M)$$
(54)

In this framework, the input elasticity is time-varying, and  $\rho_k^z$  and  $\tilde{\beta}_{kt}$  govern the speed and degree of delayed adjustment to price changes in demand for imports and domestic goods.

# 2.7 Import Sector

# 2.7.1 Non-commodity imports

Imported goods are assembled by monopolistically competitive firms that purchase products at the foreign export price  $P_t^{X,*}$ , converted into domestic currency at the nominal exchange rate  $\mathcal{E}_t$ . These firms set their import prices subject to quadratic adjustment costs and face CES demand for their individual variety.

The profit of an import firm is

$$\operatorname{div}_{it}^{M} = \frac{P_{it}^{M}}{P_{t}^{Y}} M_{it} - \mathcal{E}_{t} \frac{P_{t}^{X,*}}{P_{t}^{Y}} M_{it} - \operatorname{adj}_{it}^{PM},$$

where  $P_t^Y$  is the domestic output deflator. The adjustment cost is

$$\operatorname{adj}_{it}^{PM} = \frac{\gamma^{p^M}(\sigma^M - 1)}{2} \frac{P_t^M}{P_t^Y} M_t \left( \frac{P_{it}^M}{P_{it-1}^M} - \exp(\bar{\pi}) \right)^2.$$

The firm maximizes dividends by choosing its price as:

$$\max_{P_{it}^M} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \operatorname{div}_{it}^M,$$

where the choice variable is  $P_{it}^{M}$ , and  $\Lambda_{0,t}$  is the stochastic discount factor.

The demand for each imported variety is:

$$M_{it} = \left(\frac{P_{it}^M}{P_t^M}\right)^{-\sigma^M} M_t,$$

with  $P_t^M$  the aggregate import price and  $M_t$  total import demand.

In symmetric equilibrium  $(P_{it}^M = P_t^M)$ , the optimal price satisfies

$$P_{t}^{M} = \mathcal{E}_{t} P_{t}^{X,*} - \gamma^{pM} \frac{P_{t}^{M}}{P_{t-1}^{M}} P_{t}^{M} \left( \frac{P_{t}^{M}}{P_{t-1}^{M}} - \exp(\bar{\pi}) \right)$$
$$+ \gamma^{pM} E_{t} \left[ \Lambda_{t,t+1} \frac{P_{t+1}^{M}}{P_{t}^{M}} P_{t+1}^{M} \frac{M_{t+1}}{M_{t}} \frac{P_{t}^{Y}}{P_{t+1}^{Y}} \left( \frac{P_{t+1}^{M}}{P_{t}^{M}} - \exp(\bar{\pi}) \right) \right].$$

If there are no adjustment costs ( $\gamma^{p^M} = 0$ ), the optimal import price equals the foreign export price adjusted for the exchange rate (law of one price).

## 2.7.2 Commodity Importers

Commodity importers are monopolistically competitive firms. Each importer sets its price  $P_{i,t}^{IS}$  to maximise the expected present value of dividends, taking into account price adjustment costs

and CES demand for its variety:

$$\max_{P_{i,t}^{IS}} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[ \left( \frac{P_{i,t}^{IS}}{P_t} - \frac{P_t^{IS,M}}{P_t} - \tau^{IS} \right) IS_{i,t} - \frac{\gamma^{P,IS} (\sigma^{IS} - \exp(\bar{\pi}))}{2} IS_t \frac{P_t^{IS}}{P_t} \left( \frac{P_{i,t}^{IS}}{P_{i,t-1}^{IS}} - \exp(\bar{\pi})^2 \right) \right],$$

where the choice variable is  $P_{i,t}^{IS}$ . Here  $P_t$  is the domestic output deflator,  $IS_{i,t}$  is the demand for variety i,  $P_t^{IS,M}$  is the world price,  $\tau^{IS}$  is the excise duty,  $\gamma^{P,IS}$  parametrizes price rigidity,  $\sigma^{IS}$  is the elasticity of substitution, and  $\Lambda_{0,t}$  is the stochastic discount factor.

The demand for each variety is:

$$IS_{i,t} = \left(\frac{P_{i,t}^{IS}}{P_t^{IS}}\right)^{-\sigma^{IS}} IS_t,$$

with  $P_t^{IS}$  the aggregate price and  $IS_t$  total commodity imports.

In a symmetric equilibrium, the optimal price satisfies:

$$P_{t}^{IS} = P_{t}^{IS,M} + \tau^{IS} P_{t} - \gamma^{P,IS} \frac{(P_{t}^{IS})^{2}}{P_{t-1}^{IS}} \left( \frac{P_{t}^{IS}}{P_{t-1}^{IS}} - \exp(\bar{\pi}) \right)$$
$$+ \gamma^{P,IS} E_{t} \left[ \Lambda_{t,t+1} \frac{IS_{t+1}}{IS_{t}} \frac{P_{t}}{P_{t+1}} \frac{(P_{t+1}^{IS})^{2}}{P_{t}^{IS}} \left( \frac{P_{t+1}^{IS}}{P_{t}^{IS}} - \exp(\bar{\pi}) \right) \right].$$

# 2.8 Total imports

Total imports are defined as:

$$P_t^{Mtot}M_t^{tot} = P_t^M M_t + P_t^{IS} I S_t. (55)$$

Non-commodity imports are defined as the sum of imported components for different uses:

$$P_{t}^{M}M_{t} = P_{t}^{M}\left(M_{t}^{C} + M_{t}^{I} + M_{t}^{G} + M_{t}^{IG} + M_{t}^{Z} + M_{t}^{X}\right),$$

where the superscripts C, I, G, IG, Z, and X denote imports used in consumption, investment, government consumption, government investment, intermediate goods, and exports, respectively.

# 3 Policy

# 3.1 Fiscal Policy

The government collects taxes on labour income  $(\tau^N)$ , corporate profits  $(\tau^K)$ , consumption  $(\tau^C)$ , and levies lump-sum taxes  $(tax_t)$ . It also imposes a fixed tax on commodity imports  $(\tau^{IS})$  and pays a labour hoarding subsidy  $(\tau^{LU})$ . The government issues one-period bonds  $B_t^G$  to finance government consumption  $G_t$ , public investment  $I_t^G$ , transfers  $T_t$ , and the servicing of outstanding debt. The government budget constraint is:

$$B_t^G = (1 + i_{t-1}^G)B_{t-1}^G - R_t^G + P_t^G G_t + P_t^{IG} I_t^G + T_t P_t,$$
(56)

where total nominal government revenues  $R_t^G$  are given by:

$$R_t^G = \tau^K \left( P_t Y_t - W_t N_t^{paid} - P_t^I \delta K_{t-1} \right) + \tau^N W_t N_t^{paid} + \tau^C P_t^C C_t$$
$$+ \tau^{IS} P_t I S_t + tax_t P_t Y_t + \tau^{LU} W_t (N_t^{paid} - N_t).$$
 (57)

The budget is closed via lump-sum taxes following:

$$tax_{t} = \rho^{\tau} tax_{t-1} + \eta^{d} \left( \frac{\Delta B_{t-1}^{G}}{Y_{t-1}P_{t-1}} - d\bar{e}f \right) + \eta^{B} \left( \frac{B_{t-1}^{G}}{Y_{t-1}P_{t-1}} - B\bar{G} \right) + \varepsilon_{t}^{tax}, \tag{58}$$

where  $d\bar{e}f$  and  $\bar{B}G$  are the targets for the government deficit and debt, with coefficients  $\eta^d$  and  $\eta^B$ ;  $\rho^\tau$  is the persistence parameter and  $\varepsilon^{tax}_t$  a white noise shock.

Government capital evolves as:

$$K_t^G = (1 - \delta)K_{t-1}^G + I_t^G, \tag{59}$$

where  $\delta$  is the depreciation rate.

We use the following fiscal rules for government consumption,  $G_t$ , investment,  $I_t^G$ , and transfers,  $T_t$ :

$$\frac{G_t P_t^G}{Y_t^{pot} P_t^Y} = \bar{G} + \varepsilon_t^G \tag{60}$$

$$\frac{I_t^G P_t^{IG}}{Y_t^{pot} P_t^Y} = \bar{I^G} + \varepsilon_t^{IG} \tag{61}$$

$$\frac{T_t P_t^Y}{Y_t^{pot} P_t^Y} = \bar{T} + \varepsilon_t^T \tag{62}$$

where  $\bar{G}$ ,  $barI^G$  and barT denote their respective steady-state shares.  $\varepsilon_t^G$ ,  $\varepsilon_t^{IG}$ ,  $\varepsilon_t^T$  represent serially correlated shocks to government consumption, investment and transfers, respectively.

#### 3.2 Potential Output

Potential output,  $Y_t^{pot}$ , is defined using a production function approach:

$$Y_t^{pot} = \left( A_t^Y (N_t^{pot} - FN) \right)^{\alpha} (K_{t-1})^{1-\alpha} \left( K_{t-1}^G \right)^{1-\alpha_G} - \left( A_t^Y \right)^{\frac{\alpha}{\alpha + \alpha_G - 1}} \Phi, \tag{63}$$

where  $A_t^Y$  is trend total factor productivity,  $N_t^{pot}$  is potential labor input (i.e., labor supply in the absence of wage adjustment frictions),  $K_{t-1}$  is private capital at full utilization, and  $K_{t-1}^G$  is public capital.

# 3.3 Monetary Policy

The central bank sets the nominal interest rate according to

$$i_{t} = (1 - \rho^{i})\bar{i} + \rho^{i}i_{t-1} + (1 - \rho^{i})\left[\eta^{\pi}\left\{\frac{1}{4}\ln\left(\frac{P_{t}^{C,vat}}{P_{t-4}^{C,vat}}\right) - \pi\right\} + \eta^{Y}g_{t}^{Y}\right] + \varepsilon_{t}^{i},$$
 (64)

where  $\rho^i$  is the interest rate smoothing parameter,  $\bar{i}$  is the steady-state nominal interest rate,  $\eta^{\pi}$  and  $\eta^{Y}$  are the response coefficients to annual inflation and output growth, respectively. The term  $\frac{1}{4} \ln \left( \frac{P_t^{C,vat}}{P_{t-4}^{C,vat}} \right)$  denotes annualized inflation and  $\pi$  the steady-state inflation target.  $\varepsilon^i_t$  is a white noise monetary policy shock. It is assumed that the risk-free rate is equal to the policy rate:  $(1+i_t) = (1+i_t^{rf}) = R^{rf}$ .

# 4 External balances

**Trade balance.** The trade balance is defined as:

$$TB_{t} = P_{t}^{X} X_{t} - \mathcal{E}_{t} P_{t}^{M,*} M_{t}^{*} - P_{t}^{IS} IS_{t}, \tag{65}$$

where  $X_t$  are exports to the foreign country,  $M_t^*$  are imports from the foreign country (i.e., home exports),  $P_t^{M,*}$  is the price of foreign imports in foreign currency,  $\mathcal{E}_t$  is the nominal exchange rate (units of home currency per unit of foreign currency).

Net foreign asset accumulation. Net foreign assets (NFA) for the EA evolve according to:

$$\mathcal{E}_t B_t^W = R_{t-1}^{bw} \mathcal{E}_t B_{t-1}^W + T B_t + \varepsilon_t^{NFA}, \tag{66}$$

where  $B_t^W$  is the stock of net foreign assets,  $R_{t-1}^{bw}$  is the gross return on international bonds, and  $\varepsilon_t^{NFA}$  is a net foreign asset shock. This shock captures international remittances, transfers and other current account components that are needed to reconcile the observed net foreign asset path with the model's accounting identity.

Global consistency. In the two-country world, the NFA positions sum to zero:

$$sizeNFA_t + size^*NFA_t^* = 0, (67)$$

ensuring global closure.

# 5 Rest of the world

The RoW final good. Similar to EA region, final goods producers combine domestic output,  $Y_t^{int,*}$ , and imported goods,  $M_t^*$ , in a CES production function:

$$Z_{t}^{*} = A_{t}^{Z,*} \left[ \left( 1 - s_{t}^{M,Z,*} \right)^{\frac{1}{\sigma^{Z,*}}} \left( O_{t}^{Z,*} \right)^{\frac{\sigma^{Z,*} - 1}{\sigma^{Z,*}}} + \left( s_{t}^{M,Z,*} \right)^{\frac{1}{\sigma^{Z,*}}} \left( M_{t}^{Z,*} \right)^{\frac{\sigma^{Z,*} - 1}{\sigma^{Z,*}}} \right]^{\frac{\sigma^{Z,*} - 1}{\sigma^{Z,*} - 1}}, \tag{68}$$

where  $Z_t^* \in \{C_t^*, I_t^*, X_t^*\}$  denotes the demand for final goods by households, private investors, and exporters of final goods, respectively.  $A_t^{Z,*}$  denotes a productivity shock in sector Z, and  $0 < s_t^{M,Z,*} < 1$  is the stochastic import share associated with the different components of final demand. This is given by  $s_t^{M,Z,*} = s^{M,Z,*} \exp(\varepsilon_t^{M,*})$ , where  $s^{M,Z,*}$  denotes the steady-state import share of the demand component Z, and  $\varepsilon_t^{M,*}$  is an import demand (preference) shock. The parameter  $\sigma^{z,*} > 0$  is the elasticity of substitution between domestic output and imports in

the assembly of the final good. This elasticity is assumed to be common across all final demand components.

**Output.** Perfectly competitive firms produce output  $(O_t)$  by combining domestic value added  $(Y_t)$  and imported industrial supplies  $(IS_t)$  in a CES production function:

$$O_t^* = \left[ \left( 1 - s_t^{IS,*} \right)^{\frac{1}{\sigma^{O,*}}} (Y_t^*)^{\frac{\sigma^{O,*} - 1}{\sigma^{O,*}}} + \left( s_t^{IS,*} \right)^{\frac{1}{\sigma^{O,*}}} (IS_t^*)^{\frac{\sigma^{O,*} - 1}{\sigma^{O,*}}} \right]^{\frac{\sigma^{O,*} - 1}{\sigma^{O,*} - 1}}, \tag{69}$$

where  $s_t^{IS,*}$  is the RoW share of commodities use.<sup>3</sup> The specification in eq. (69) leads to optimality conditions for the demand for commodities as in the detailed regions of the model.

**Intermediate goods.** The intermediate good producers use labour and capital to manufacture domestic goods (non-commodity output) according to a Cobb-Douglas production function and are subject to a standard CES demand function of RoW output packers (analogously to the EA block):

$$Y_{i,t}^* = A_t^{Y,*} \left( c u_{i,t}^* K_{i,t-1}^* \right)^{1-\alpha} \left( N_t^* \right)^{\alpha}, \tag{70}$$

where  $A_t^{Y,*}$  captures a trend in the productivity, and  $N_t^* = Actr_t^* Pop_t^*$  is the active population in the economy.  $K_{i,t-1}^*$  denotes capital. It is utilised at rate  $cu_{i,t}^*$  and follows a law of motion analogous to the detailed regions in the model. RoW adjustment costs are given by:

$$\Gamma_{i,t}^{P,*} = \frac{\sigma^{Y,*} \gamma^{P,*}}{2} Y_t^* \left( \frac{P_{i,t}^*}{P_{i,t-1}^*} - 1 - \pi^* \right)^2 \tag{71}$$

$$\Gamma_{i,t}^{cu,*} = \frac{P_t^{I,*}}{P_t^*} K_{i,t-1}^* \left( \gamma_0^{u,*} \left( c u_{i,t}^* - 1 \right) + \frac{\gamma_1^{u,*}}{2} \left( c u_{i,t}^* - 1 \right)^2 \right)$$
 (72)

$$\Gamma_{i,t}^{I,*} = \frac{\gamma_1^{I,*}}{2} \frac{P_t^{I,*}}{P_t^*} \frac{\left(I_{i,t}^* - I_{i,t-1}^* \exp\left(g^Y + g^{P^I,*}\right)\right)^2}{K_{t-1}^*}$$
(73)

The first-order condition with respect to  $I_{i,t}^*$  reads:

$$Q_{t}^{*} = \left(1 + \gamma_{1}^{I,*} \frac{\left(I_{t}^{*} - I_{t-1}^{*} \exp\left(g^{Y} + g^{P^{I}}\right)\right)}{K_{t-1}^{*}}\right)$$
$$-E_{t} \left[\frac{\lambda_{t+1}^{*}}{\lambda_{t}^{*}} \frac{P_{t+1}^{I,*}}{P_{t}^{I,*}} \frac{P_{t}^{*}}{P_{t+1}^{*}} \gamma_{1}^{I,*} \frac{\left(I_{t+1}^{*} - I_{t}^{*} \exp\left(g^{Y} + g^{P^{I}}\right)\right)}{K_{t}^{*}} \exp\left(g^{Y} + g^{P^{I}}\right)\right], \tag{74}$$

 $<sup>^3</sup>$ Unlike the EA, the RoW region does not feature energy commodities as an additional (direct) element in the households' consumption good.

where  $Q_t^* \equiv \frac{\mu_{i,t}^*}{\frac{P_t^{I,*}}{P_t^*}}$  is Tobin's marginal Q. The first-order condition with respect to  $K_{i,t}^*$  solves:

$$Q_{t}^{*} = E_{t} \left[ \frac{\lambda_{i,t+1}^{*}}{\lambda_{i,t}^{*}} \frac{P_{t+1}^{*}}{P_{t+1}^{*}} \frac{P_{t}^{*}}{P_{t}^{I,*}} \left( \left( -\gamma_{0}^{u,*} \left( cu_{i,t+1}^{*} - 1 \right) - \frac{\gamma_{1}^{u,*}}{2} \left( cu_{i,t+1}^{*} - 1 \right)^{2} \right) + (1 - \delta^{*}) Q_{t+1}^{*} + (1 - \alpha^{*}) \mu_{t+1}^{Y,*} \frac{P_{t+1}^{*}}{P_{t+1}^{I,*}} \frac{Y_{i,t+1}^{*}}{K_{i,t}^{*}} \right) \right].$$

$$(75)$$

The first-order condition with respect to  $cu_{i,t}^*$  yields:

$$\frac{P_t^{I,*}}{P_t^*} K_{i,t-1}^* \left( \gamma_0^{u,*} + \gamma_1^{u,*} \left( c u_{i,t}^* - 1 \right) \right) = \mu_t^{Y,*} \left( 1 - \alpha^* \right) \frac{Y_{i,t}^*}{c u_{i,t}^*}. \tag{76}$$

Price setting for non-oil output follows a New Keynesian Phillips curve:

$$\pi_t^{Y,*} - \bar{\pi}^{Y,*} = \beta_t^* \frac{\lambda_{t+1}^*}{\lambda_t^*} \left[ (\pi_{t+1}^{Y,*} - \bar{\pi}^{Y,*}) \right] + \phi^{y,*} \log \frac{Y_t^*}{\bar{Y}^*} + \varepsilon_t^{Y,*}, \tag{77}$$

where  $\lambda_t^* = (C_t^* - h^* C_{t-1}^*)^{-\theta^*}$  is the marginal utility of consumption, and  $\varepsilon_t^{Y,*}$  is a cost push shock.

RoW commodity supply. In the RoW, a competitive sector supplies two distinct commodities, namely oil (o, Brent) and non-oil commodities (no, e.g. natural gas and materials) to domestic and foreign firms. There is a supply disturbance  $\varepsilon^{IS,*}$  that captures exogenous commodity supply shocks, such as the discovery of new raw material deposits. Demand for commodities is determined by final good producers from the two regions (see above). The producer combines oil  $(o^*)$  and non-oil  $(no^*)$  commodities into the CES bundle  $IS^*$  that is exported to EA or used locally with price  $P_t^{IS,*}$ :

$$P_t^{IS,*} = \varepsilon_t^{IS,*} \left[ s^* (P_t^{o,*})^{1 - \sigma^{COMM,*}} + (1 - s^*) (P_t^{no,*})^{1 - \sigma^{COMM,*}} \right]^{1/(1 - \sigma^{COMM,*})}.$$
 (78)

Commodity prices are exogenous in this model specification, i.e.:

$$P_t^{o,*} = \frac{P_t^*}{A_t^{o,*}},\tag{79}$$

where  $A_t^{o,*}$  is the exogenous oil-specific productivity technology (analogously for prices of non-oil commodities).

The total supply of commodities by RoW is residual, i.e. it satisfies global demand.

**Consumption-savings choices.** RoW households maximise utility subject to the aggregate budget constraint:

$$P_t^* Y_t^* + div_t^* = P_t^{C,*} C_t^* + P_t^{C,*} I_t^* + T B_t^*,$$
(80)

where  $div_t^*$  are dividends from intermediate good producers, and  $TB_t^*$  are net exports.  $I_t^*$  denotes investment. The consumption Euler equation is:

$$1 = E_t \left[ \Lambda_{t,t+1}^* \frac{R_t^*}{1 + \pi_{t+1}^{C,*}} \right], \tag{81}$$

where  $\Lambda_{t,t+1}^*$  is defined as in the EA economy.

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