Indirect tax evasion, shadow economy, and the Laffer curve: A theoretical approach

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Indirect tax evasion, shadow economy, and the Laffer curve: A theoretical approach*

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Abstract

This paper provides new theoretical insights into the causes and consequences of indirect tax evasion. I propose a decision-making framework that contemplates biased perceptions of apprehension probabilities, which are affected by the environment where the agents operate. This micro-founded formulation allows for the analysis of how taxation affects tax evasion (and vice versa) in the aggregate, emphasizing the existing relationships between the relative size of the shadow economy, tax rates, and government revenue. It is shown that a traditional Laffer curve (inversely U-shaped and with a unique maximum) can only exist under certain conditions. The maximum government revenue attainable turns out to be, in any case, lower than in the absence of tax evasion. Nevertheless, evasion control policies are proven to be always effective in increasing government revenue.

Keywords: Indirect tax evasion; Law and Economics; Biased perceptions.

JEL Classification: D80, H26, K42.

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1 Introduction

Since Becker's (1968) seminal work, economists have studied criminal behavior using the methodology he proposed. Potential offenders are considered rational agents who operate in a context of risk or uncertainty regarding the possible outcomes of their offenses (they *might* be detected, apprehended, and punished), choosing the level of delinquent activity that maximizes their expected utility.

Tax evasion models first appeared in the 1970s with Allingham and Sandmo's (1972) pioneering paper on income tax evasion, which was the first application of Becker's approach to this specific type of felony. It was only several years later that indirect tax evasion models began to emerge (Cremer & Gahvari, 1993; Marrelli & Martina, 1988; Marrelli, 1984; Virmani, 1989). In the empirical realm, compelling evidence[1] has suggested that potential tax evaders tend to overestimate the objectively small probability of being caught, which results in lower levels of tax evasion than predicted by traditional models.

Tax evasion is a crucial consideration in the design of tax policies.[2] Therefore, to develop "better" tax policies, grounded in a solid theoretical foundation, we should try to account for the factors that cause this discrepancy in the models eventually employed.

Individual decisions regarding tax evasion can be significantly influenced by the social environment (shaped by both taxpayers and tax authorities) in which they are made. This environment affects not only the perceived probability distributions over outcomes (Bergman & Nevarez, 2005; Cooter et al., 2008; Scholz & Pinney, 1995; Sheffrin & Triest, 1992) but also the preferences[3] over them (Alm & McClellan, 2012; Bergman & Nevarez, 2006; Frey & Feld, 2002; Scholz & Pinney, 1995; Torgler et al., 2008). Additionally, considering that these decisions are made under uncertainty, biases and heuristics can affect the perceived probability distributions over outcomes.[4]

Recently, many income tax evasion models have intended to address those factors. Some of them focus on the interactions between agents (degl'Innocenti & Rablen, 2017; Di Gioacchino & Fichera, 2020, 2022; Fortin et al., 2007; Traxler, 2010), others emphasize the impact of tax audits (Advani et al., 2023; Kirchler et al., 2008; Levaggi & Menoncin, 2016; Ma et al., 2021), and there is a group that analyze tax evasion decisions using a non-expected utility framework (Bernasconi, 1998; Bernasconi & Zanardi, 2004; Frey & Torgler, 2007; Hokamp & Pickhardt, 2010; Yaniv, 1999). In contrast, most of the modern theory of indirect tax evasion (Arias, 2011; Besfamille et al., 2009, 2013; Buccella et al., 2024; Fanti & Buccella, 2021; Goerke, 2017; Goerke & Runkel, 2011; López, 2017) is still mainly concerned with the same issues as in its early stages; this

[1] For comprehensive surveys of the literature, see Alm (2019) or Andreoni et al. (1998).

[2] Recently, the US annual gross tax gap for 2014-2016 was estimated at $496 billion (IRS, 2022).

[3] For instance, the utility level for a certain amount of successful tax evasion may depend on factors such as how taxpayers are treated by the tax authority or how much trust the government has earned.

[4] See Kahneman et al. (1982) or Thaler and Sunstein (2008) for a more in-depth discussion.

is, the study of the relationships between tax evasion, output, prices, and production efficiency under a determinate market structure. Few of them have incorporated social interactions (Advani et al., 2023; Bayer & Cowell, 2009; Cason et al., 2016), yet not as richly as in income tax evasion models.

It might be argued that this split is because the commented empirical findings are valid for individuals but not for firms. However, decisions within firms are made by individuals operating within an organizational structure (Coase, 1937, 1988). Therefore, it could not be appropriate to claim that individuals are perfectly rational and well-informed when making indirect tax evasion decisions for a company but not when evading income taxes on their personal earnings.[5]

The purpose of the paper is to develop an indirect tax evasion model with a decision-making framework that accounts for some of the empirical observations often overlooked by the theoretical literature. It is structured as follows: Section 2 develops the model at a single-agent level, Section 3 analyzes tax evasion in the aggregate, Section 4 simulates an economy with heterogeneous agents to evaluate the effects of different policy tools, and Section 5 provides a conclusion.[6]

2 The model

2.1 Some previous considerations

If we accept that the aforementioned empirical insights ought to be considered when modeling indirect tax evasion, a significant matter to overcome is the incongruity between tax evasion and production decisions that arises when we take into account "irrational" probability perceptions. It would be inconsistent to accept that tax evasion decisions are made under total uncertainty, leading to the formation of biased probabilities of detection on which an agent's behavior relies, whilst production decisions take place in a context of perfect and/or complete information (as assumed by traditional market structures).

One potential solution is to isolate tax evasion decisions from production decisions, thereby focusing on the former. This may well imply the absence of a market structure, but it does not necessarily mean that revenue has to be taken as given. It can be treated as an "endogenous" variable influenced by tax rates. Furthermore, this approach would allow for a broader aggregate analysis, as it overcomes certain limitations imposed by each market structure.[7]

This would conflict with the well-known *separability* conclusions of indirect tax evasion theory (Arias, 2011; Yaniv, 1995) if the probability of detection or the costs of concealment vary with firm size. However, there is an important factor that we should take into account: Tax evasion decisions and production decisions occur at different times, not simultaneously as commonly assumed.

[5] As an example, Busenitz and Barney (1997) show that both managers of large corporations and sole entrepreneurs, yet to a different extent, are prone to misperceive probabilities due to behavioral biases when facing uncertainty.

[6] The Annex explains in detail the notation.

[7] For instance, it would allow for the existence of heterogeneous agents, which is impossible in a market of perfect competition.

Therefore, it might not be incorrect to focus on the second stage of the problem, even though both types of decisions may, in the last instance, affect one another.[8]

2.2 Exogenous revenue approach

In a one-period time horizon, a rational[9] and risk-neutral[10] agent whose gross sales revenue is given by y must choose the proportion a of that revenue to be concealed from the tax authority, by whom is levied a tax $\tau = ty$, $t \in [0, 1]$.[11]

The agent perceives a probability $p = p(a, y, \upsilon, \omega, \gamma)$ of being detected, where $0 \leq p \leq 1$. The variables υ, ω, and γ represent, respectively, the influence of behavioral biases, the perceived behavior of other agents, and the perceived behavior of the tax authority on this probability. Each of these variables can be thought of as an index that quantitatively summarizes the influence of the respective item on the probability of detection. I assume that p is a decreasing function of these indexes, so that $p_\upsilon, p_\omega, p_\gamma < 0$. I also suppose that $p_y > 0$.[12] Finally, $p_a > 0$ and $p_{aa} \geq 0$ are assumed.[13]

When evasion is a positive quantity, the agent must face concealment costs, denoted by $\varsigma = c\Lambda(a, y)$, where $c > 0$ and $\Lambda(0, y) = 0$. It is assumed that $\varsigma_a, \varsigma_y > 0$ and $\varsigma_{aa} \geq 0$.[14] If caught when evading, the agent is fined a sum $\chi = xay$ for the offense, where $x > 0$ is the fine to be paid per unit of concealed revenue.[15] We have $\chi_a, \chi_y, \chi_x > 0$ and $\chi_{aa} = 0$.

Subjective expected net revenue after evading can be expressed as

$$y^e = p\left[(1-t)\,y - \chi - \varsigma\right] + (1-p)\left[(1-t)\,(y - ay) + ay - \varsigma\right]$$

$$= (1-t)\,y + t\,(1-p)\,ay - p\chi - \varsigma.$$

[8]See Besfamille et al. (2009) for a two-stages model.

[9]In the sense that the agent maximizes its subjective expected utility, as axiomatically defined by Savage (1954). Although decision-making frameworks such as Prospect Theory or Bounded Rationality (in any of its forms) are available, I believe that the complexity involved in using them outweighs the benefits.

[10]It is common knowledge that agents tend to be risk-averse, but Arias (2005) has shown that assuming risk neutrality and incorporating concealing costs (which act like a risk premium) leads to similar results as those models that assume risk aversion.

[11]Note that this tax is expressed in terms of the after-tax output value. If a pre-tax charge ρ is modeled, as would be the case with a retail sales tax, replace with $t = \rho/(1+\rho)$.

[12]A higher gross sales revenue should lead to a higher perceived probability of detection because the tax authority has greater incentives to monitor you, which you are aware of.

[13]It is widely accepted that the perceived probability of detection should grow with the proportion of concealed revenue. Yet it remains unclear whether this increase occurs at an increasing or decreasing rate. The assumption of convexity is made for simplicity; however, as shown later, it is not truly necessary.

[14]Concealing the same proportion of y should be less costly for a small firm if compared with a big one. In addition, the greater the proportion of concealed revenue, the higher the cost of concealing it ought to be. Last but not least, it is assumed that the "marginal cost of concealment" cannot be a decreasing function of a.

[15]We will not be modeling a fine that is based on the amount of evaded tax (e.g., Yitzhaki (1974)), such as $\chi = xtay$. This is harmless, given the purpose of this paper. It can be proven that a fine of that nature would only lead to an additional (and rather unnecessary) source of ambiguity in the relationship between tax rates and tax evasion, given the one we will be having in the endogenous revenue approach.

Subtracting $(1-t)y$ from both sides and denoting $\pi = y_e - (1-t)y$ as the expected tax evasion profit, we get:

$$\pi = t\,(1-p)\,ay - p\chi - \varsigma. \tag{1}$$

Intuitively, Equation (1) shows that only three elements determine how profitable tax evasion is: The expected additional revenue (first term), the expected fine (the second term), and the costs of concealment (last term). Note that π=0 if $a=0$. The agent will choose a^* so as to maximize π. If the agent considers that the decision will neither affect other agents' behavior nor the tax authority's behavior, by maximizing π with $0 \leq a \leq 1$ and $\pi \geq 0$ as constraints, we obtain the following solution (its derivation can be found in the Appendix):

$$a^* = \begin{cases} 1 & if \quad \pi_a > 0\ \forall a \in (0,1) \\ \theta & if \quad \pi_a = 0\ for\ some\ a \in (0,1) \\ 0 & if \quad \pi_a < 0\ \forall a \in (0,1) \end{cases} \tag{2}$$

where

$$\theta = \frac{1}{p_a}\left[(1-p) - \frac{\varsigma_a + p_a\chi + p\chi_a}{ty}\right].$$

Of course, as y is fixed, the total *amount* of tax evasion will be given by

$$z^* = a^* y. \tag{3}$$

Regarding comparative statics, let

$$F(\cdot) = a - \frac{1}{p_a}\left[(1-p) - \frac{\varsigma_a + p_a\chi + p\chi_a}{ty}\right]$$

and

$$\kappa = \frac{\partial F}{\partial a} = 2 + \frac{p_{aa}}{p_a}\theta + \frac{1}{p_a}\frac{\varsigma_{aa} + p_{aa}\chi + 2p_a\chi_a}{ty}.$$

Under our assumptions, $\kappa > 0$. For any other variable than y, evaluated in the interior solution, the results are the following:

$$\frac{\partial a^*}{\partial x} = -\frac{1}{\kappa p_a}\left(\frac{p_a\chi_x + p\chi_{ax}}{ty}\right) < 0, \tag{4a}$$

$$\frac{\partial a^*}{\partial c} = -\frac{1}{\kappa p_a}\left(\frac{\varsigma_{ac}}{ty}\right) < 0, \tag{4b}$$

$$\frac{\partial a^*}{\partial t} = \frac{1}{\kappa p_a}\left(\frac{\varsigma_a + p_a\chi + p\chi_a}{t^2 y}\right) > 0, \tag{4c}$$

$$\frac{\partial a^*}{\partial \upsilon} = -\frac{1}{\kappa p_a}\left(p_{a\upsilon}\theta + p_\upsilon + \frac{p_{a\upsilon}\chi + p_\upsilon\chi_a}{ty}\right) > 0, \tag{4d}$$

$$\frac{\partial a^*}{\partial \omega} = -\frac{1}{\kappa p_a}\left(p_{a\omega}\theta + p_\omega + \frac{p_{a\omega}\chi + p_\omega\chi_a}{ty}\right) > 0, \tag{4e}$$

$$\frac{\partial a^*}{\partial \gamma} = -\frac{1}{\kappa p_a}\left(p_{a\gamma}\theta + p_\gamma + \frac{p_{a\gamma}\chi + p_\gamma\chi_a}{ty}\right) > 0. \tag{4f}$$

As standard in the literature, under risk neutrality, tax evasion tends to increase in response to more lenient penalties, lower concealment costs, and higher tax rates. When one of the indexes increases, the agent perceives a lower probability of detection for each a, making tax evasion more appealing and thus encouraging it.

Without stronger assumptions, the effects of y on a^* and z^* are ambiguous and might have different directions.[16] We have:

$$\frac{\partial a^*}{\partial y} = -\frac{1}{\kappa p_a}\left(p_{ay}\theta + p_y + B\right) \tag{5a}$$

$$\frac{\partial z^*}{\partial y} = -\frac{y}{\kappa p_a}\left(p_{ay}\theta + p_y + B\right) + \theta, \tag{5b}$$

where B is equal to

$$\frac{\Big(\varsigma_{ay} + p_{ay}\chi + p_a\chi_y + p_y\chi_a + p\chi_{ay}\Big)\, y - \left(\varsigma_a + p_a\chi + p\chi_a\right)}{ty^2}.$$

There could be, simultaneously, a decrease in the *proportion* of the gross sales revenue concealed and an increase in the *amount* hidden (or vice versa). Intuitively, for any value of a, when y grows, both the expected fine and the costs of concealment will be higher; however, the expected additional revenue may also increase.

2.3 Endogenous revenue approach

So far, it has been assumed that the tax rate does not influence gross sales revenue. This assumption may be unrealistic, primarily because it overlooks the possibility of tax shifting. To address this issue without employing a specific market framework, I will consider a scenario where gross sales revenue is affected by the tax rate (i.e., by tax policy) but not by the choices of potential tax evaders.

Let $y = y(t, \varepsilon)$ denote the gross sales revenue as a function of the tax rate t and all other non-tax-related factors ε affecting it. The main implication is that now we have $p = p[a, y(t), \upsilon, \omega, \gamma]$, $\chi = xay(t)$, and $\varsigma = c\Lambda[a, y(t)]$. Considering the interior solutions of (2) and (3), the new results for t are, respectively:

$$\frac{\partial a^*}{\partial t} = -\frac{1}{\kappa p_a}\left[\left(1+\theta\right) p_y y_t + D y_t - \frac{\varsigma_a + p_a\chi + p\chi_a}{t^2 y}\left(1 + y_t\frac{t}{y}\right)\right] \tag{6a}$$

$$\frac{\partial z^*}{\partial t} = -\frac{y}{\kappa p_a}\left[\left(1+\theta\right) p_y y_t + D y_t - \frac{\varsigma_a + p_a\chi + p\chi_a}{t^2 y}\left(1 + y_t\frac{t}{y}\right)\right] + \theta y_t, \tag{6b}$$

where

$$D = \frac{\varsigma_{ay} + p_{ay}\chi + \chi_y + p_y\chi_a + p\chi_{ay}}{ty}.$$

[16] In Section 4.1, I introduce a reduced version of the model to study these relationships in further detail.

The effects on both variables of a variation in t are unclear, as they largely depend on y_t, for which we cannot state any justified assumption. Even assuming that no variations in relative prices take place when t increases or decreases (symmetric tax shifting across all commodity markets), y will still be affected in two opposite directions because quantities sold will decrease and after-tax prices will increase. As no particular market structure has been assumed, it is not possible to determine which effect, if any, will dominate.

A sufficient condition for $a_t^* > 0$ is $-1 < \eta < 0$, where $\eta = y_t(t/y)$ denotes the elasticity between gross sales revenue and tax rates. Only if $-1 < \eta < 0$ and $\theta/(1-\theta) < p_y y/p_a$ we can ensure $z_t^* > 0$.

As it is shown in the following section, in a context of heterogeneous agents, these ambiguities are far from irrelevant. They shape the relationships between the relative size of the shadow economy, government revenue, and tax rates. Moreover, to a great extent, they determine whether or not a "traditional" Laffer curve (inversely U-shaped and with only one maximum) arises.

3 Aggregate analysis

3.1 Assumptions

Consider a closed economy with n agents, where the total gross sales revenue is denoted by $Y = \sum_{i=1}^{n} y_i$ and a tax $T = tY$ is levied on it. The total amount of evasion is represented by $Z = \sum_{i=1}^{n} z_i$. Evasion is partially discovered by the tax agency, resulting in a sum $t\sum_{i=1}^{n-j} z_i + \sum_{i=1}^{n-j} C_i$ of additional government revenue, where $n-j$ is the number of agents caught and fined. In order to focus on taxation, I assume that this sum is allocated in its totality to cover the costs of the audits.

3.2 Government revenue in the presence of tax evasion

Under the assumptions of Section 3.1, government revenue can be expressed as $R = t\,(Y - Z)$. Multiplying by Y and defining $\phi = Z/Y$ as the relative size of the shadow economy (discovered or not), we can rewrite R as follows:

$$R = tY\,(1-\phi)\,. \tag{7}$$

Regarding the properties of ϕ as a function, its first and second derivative with respect to t are, respectively:

$$\frac{\partial \phi}{\partial t} = \frac{1}{Y}\,(Z_t - \phi Y_t) \tag{8a}$$

$$\frac{\partial^2 \phi}{\partial t^2} = \frac{1}{Y}\left[-\frac{Y_t}{Y}\,(Z_t - \phi Y_t) + (Z_{tt} - \phi Y_{tt}) - \phi_t Y_t\right]. \tag{8b}$$

If t is taken as given, the sufficient condition for $\phi_t > 0$ is $Z_t > \phi Y_t$, since $0 \leq \phi \leq 1$. According to Equations (6a) and (6b), it is guaranteed if $0 < \eta_i < 1$

and $\theta_i/(1-\theta_i) < p_{y(i)}y_i/p_{a(i)}$ $\forall i = 1, 2, ..., n$. Regarding $\phi_{tt} > 0$, ensuring it requires $Z_t > \phi Y_t$, $Z_{tt} > Y_{tt}$, and $Y_t \leq 0$.

3.3 First-order condition, second-order condition and uniqueness of the maximum

The first and second-order conditions that a tax rate t^* must satisfy in order to maximize $R = t(Y - Z)$ are, respectively:

$$\frac{\partial R}{\partial t} = [(Y - Z) + t\,(Y_t - Z_t)] = 0$$
$$\frac{\partial^2 R}{\partial t^2} = [2\,(Y_t - Z_t) + t\,(Y_{tt} - Z_{tt})] < 0.$$

Note that $Y \geq Z$ $\forall t \in [0, 1]$. Now, if $Z_t > Y_t$ and $Z_{tt} > Y_{tt}$ are satisfied $\forall t \in [0, 1]$, then both expressions $Y - Z$ and $-t(Y_t - Z_t)$ will always be non-negative, but the former will decrease and the latter will increase as t grows. Under those circumstances, $-t(Y_t - Z_t)$ will be zero when $t = 0$ and positive when $t = 1$. Provided $Y - Z$ positive when $t = 0$ and null when $t = 1$, there can only exist one critical point $t^* \in (0, 1)$ where the first-order condition is met, which will also be a maximum since the second-order condition will also be satisfied. Therefore, a traditional Laffer curve for government revenue from gross sales revenue will exist.[17] Of course, without further assumptions about the individual functions involved or the distribution of the agents, it is not possible to ensure that those conditions will be held.

Furthermore, it is possible to rewrite the first-order condition more elegantly by solving for t and considering $\phi = Z/Y$:

$$t^* = \frac{Y\,(1 - \phi)}{Z_t - Y_t}.$$

Replacing with this solution in Equation (7) yields R^*, which is the maximum potential government revenue in nominal terms:

$$R^* = \frac{Y^2\,(1 - \phi)^2}{(Z_t - Y_t)}.$$

3.4 Addressing the nominality issue

In could be argued that t^* may not maximize government revenue in real terms due to the impact of tax rate adjustments on prices. Although the model does not deal with prices, we can maximize government revenue as a proportion of the total gross sales revenue of the economy (henceforth in proportional terms).

[17] Intuitively, these conditions can be summarized as follows: i) Aggregate tax evasion increases more rapidly than aggregate gross sales revenue for any t. ii) When $t = 0$, aggregate gross sales revenue is positive (null tax evasion if there is no taxation). iii) When $t = 1$, aggregate tax evasion is equal to aggregate gross sales revenue (no one is paying confiscatory taxes).

Consider $r = R/Y = t(1-\phi)$. In order to maximize this function, the following first and second-order conditions must be satisfied:

$$\frac{\partial r}{\partial t} = [(1-\phi) - t\phi_t] = 0$$
$$\frac{\partial^2 r}{\partial t^2} = -(2\phi_t + t\phi_{tt}) < 0.$$

Provided $Y > Z$ if $t = 0$ and $Y = Z$ if $t = 1$, then $1 - \phi > 0$ when $t = 0$ and $1 - \phi = 0$ when $t = 1$. If in addition $\phi_t > 0$ and $2\phi_t > (-t\phi_{tt})$ $\forall t \in [0,1]$, which requires $Y_t < 0 < Z_t$ and $Z_{tt} > Y_{tt}$ whenever $0 \leq t \leq 1$ (see Equations (8a) and (8b)), r will have a unique maximum $t^{**} \in (0,1)$.[18] It can be expressed as:

$$t^{**} = \frac{(1-\phi)}{\phi_t}.$$

In this scenario, since $r = t(1-\phi)$, the maximum potential government revenue in proportional terms is given by:

$$r^{**} = \frac{(1-\phi)^2}{\phi_t}.$$

3.5 Implications

Various results from the aggregate analysis, valid in both nominal and proportional terms, ought to be highlighted. Each respective proof can be found in the Appendix.

Proposition 1 *In the presence of tax evasion, a shadow economy whose relative size increases whenever the tax rate is raised is a necessary condition to guarantee the existence of a traditional Laffer curve, yet not a sufficient one.*

Proposition 2 *In the presence of tax evasion, for any given tax rate, government revenue is always lower than in a no-evasion situation. Additionally, it tends to decrease as tax evasion increases due to non-revenue-related factors. Therefore, the position of the curves $R(t)$ and $r(t)$ can be affected by the evasion control policy.*

Proposition 3 *In the presence of tax evasion, if the existence of a traditional Laffer curve is guaranteed, then both the tax rate for which it is maximum and the maximum potential government revenue are lower compared to the no-evasion situation.*

[18] If $Y_t < 0 < Z_t$ and $Z_{tt} > Y_{tt}$ are satisfied $\forall t \in [0,1]$, then both expressions $1-\phi$ and $t\phi_t$ will always be non-negative, but the former will decrease and the latter will increase as t grows. Under those circumstances, $t\phi_t$ will be zero when $t = 0$ and positive when $t = 1$. Provided $1-\phi$ positive when $t = 0$ and null when $t = 1$, there can only exist one critical point $t^{**} \in (0,1)$ where the first-order condition is met, which will also be a maximum because the second-order condition will also be satisfied.

Those results have been partially provided, either explicitly or implicitly, by some models for both indirect tax evasion (Kanniainen & Pääkkönen, 2004; Kotamäki, 2017; Palda, 1998; Vasilev, 2018; Vogel, 2012) and income tax evasion (Besfamille, 2008; Busato & Chiarini, 2013; Chang et al., 1999; Feige & McGee, 1983; Méder et al., 2012; Peacock & Shaw, 1982; Ricketts, 1984), assuming a linear income tax in the case of the latter group. However, to the best of my knowledge, a systematic and simultaneous presentation of them have not taken place until now.

4 Simulation

In this section, we will further develop some intuitions about the model by simulating a heterogeneous-agent economy. This simulation should help illustrate some of the policy implications that can be derived from the paper.

4.1 A particular version of the model

To avoid unnecessary complexity, a simplified version of the model will be used. Despite its stronger assumptions, it allows for both an explicit solution and an interesting interaction between y and γ. For each agent, let $p_i = z_i/(\upsilon_i \omega_i y_i^{\gamma} + z_i)$ be the perceived probability of detection and $\varsigma_i = c_i a_i y_i$ be the concealment costs, where $x, \upsilon_i, \omega_i, c_i > 0$, and $\gamma < 1$. Note that p_i is bounded between 0 and 1. In addition, the assumption $p_{aa(i)} \geq 0$ for all $i = 1, 2, ..., n$ has been relaxed.

Why $\gamma < 1$? In this simplified version of the model, the partial derivative of p_i with respect to y_i is:

$$\frac{(1-\gamma)\, a_i \upsilon_i \omega_i y_i^{\gamma}}{\left(\upsilon_i \omega_i y_i^{\gamma} + a_i y_i\right)^2}.$$

As the assumption $p_{y(i)} < 0 \; \forall i = 1, 2, ..., n$ ought not to be forgone, the condition $\gamma < 1$ must be satisfied. Keeping it seems reasonable because it implies that the perceived likelihood of being caught for a fixed *proportion* of evasion increases as gross sales revenue raises.[19] It is worth noting that if $\gamma < 0$, the perceived likelihood of being caught for a fixed *amount* of evasion increases as gross sales revenue raises.[20]

Replacing in Equation (1) with these stronger assumptions yields the expression $\pi_i = (t - c_i) a_i y_i - [(t + x) a_i^2 y_i^2]/(\upsilon_i \omega_i y_i^{\gamma} + a_i y_i)$.[21] By doing the same

[19] Due to this assumption, when a small business and a *blue chip* company conceal the same proportion of their revenues, all other variables equal, the former will always perceive a lower probability of detection than the latter.

[20] Denoting those elements as $\tilde{z}_i$ and $\tilde{p}_i$ to avoid any possible misunderstanding, we have $\partial \tilde{p}_i / \partial y_i = -\gamma \left(\upsilon_i \omega \tilde{z}_i y_i^{\gamma}\right) / y_i \left(\upsilon_i \omega_i y_i^{\gamma} + \tilde{z}_i\right)^2$. Intuitively, $\gamma < 0$ means that a small business will always perceive a lower probability of detection than a *blue chip* company were they to conceal the same amounts of money from the tax authority, all other variables equal.

[21] Under these assumptions and taking h_i as defined in the following page, $\pi_{a(i)} > 0 \; \forall a \in (0,1)$ only if $t > h_i$, $\pi_{a(i)} < 0 \; \forall a \in (0,1)$ only if $t < c_i$, and $\exists! a \in (0,1)$ such that $\pi_{a(i)} = 0$ only if $c_i < t < h_i$. Note that if $t = c_i$, then $\pi_{a(i)} = 0$ when $a = 0$, and if $t = h_i$, then $\pi_{a(i)} = 0$ when $a = 1$. Additionally, $\pi_{aa(i)} < 0 \; \forall a \in [0,1]$.

replacement in Equation (3) we obtain:

$$z_i^* = \begin{cases} y_i & if & t \geq h_i \\ \psi_i & if & c_i < t < h_i \\ 0 & if & t \leq c_i \end{cases}$$

where

$$\psi_i = \left(\sqrt{\frac{x+t}{x+c_i}} - 1 \right) \upsilon_i \omega_i y_i^{\gamma}$$

and

$$h_i = (x + c_i) \left(\frac{y_i^{1-\gamma}}{\upsilon_i \omega_i} \right)^2 + 2 (x + c_i) \left(\frac{y_i^{1-\gamma}}{\upsilon_i \omega_i} \right) + c_i.$$

If $0 < \gamma < 1$, the optimal amount of evasion decreases in relative terms but increases in absolute terms as gross sales revenue does. If $\gamma < 0$, the optimal amount of evasion decreases in both relative and absolute terms as gross sales revenue increases. If $\gamma = 0$, the optimal amount of tax evasion in absolute terms is independent of gross sales revenue, thus, in relative terms, the former decreases when the latter increases.

4.2 Distributional assumptions

For every agent $i = 1, 2, ..., n$, the relationship between gross sales revenue and the tax rate is assumed to be given by the quadratic expression $y_i = q_i t^2 + s_i t + \tilde{y}_i$, where s_i and $\tilde{y}_i$ are positive but q_i can be greater or lower than zero.

We assume that the variables $\tilde{y}_i$, υ_i, ω_i, and c_i follow a multivariate normal distribution $\mathbf{W} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where

$$\mathbf{W} = \begin{pmatrix} \tilde{y}_i \\ \upsilon_i \\ \omega_i \\ c_i \end{pmatrix}, \qquad \boldsymbol{\mu} = \begin{pmatrix} 5000 \\ 6000 \\ 1800 \\ 0.01 \end{pmatrix}, \qquad \boldsymbol{\Sigma} = VarCov(\mathbf{W}).$$

For a meaningful characterization of $\boldsymbol{\Sigma}$, define $\boldsymbol{\sigma}(\mathbf{W})$ as the vector of standard deviations of $\mathbf{W}$ and $Corr(\mathbf{W})$ as the correlation matrix of $\mathbf{W}$. Therefore, we have:

$$\boldsymbol{\sigma}(\mathbf{W}) = \begin{pmatrix} 1000 \\ 500 \\ 300 \\ 0.001 \end{pmatrix}, \qquad Corr(\mathbf{W}) = \begin{pmatrix} 1 & -0.6 & -0.4 & 0.7 \\ -0.6 & 1 & 0.8 & -0.2 \\ -0.4 & 0.8 & 1 & -0.3 \\ 0.7 & -0.2 & -0.3 & 1 \end{pmatrix}.$$

Three different cases are presented. In the first one (case 1), it is assumed that $q_i < 0$, so that sales revenue a concave function of t. In the second one (case 2), the assumption is $q_i > 0$, hence sales revenue is a convex function of t. In the third one (case 3), a sinusoidal term is added, so that the equation for sales revenue becomes $y_i = q_i t^2 + s_i t + k_i \sin(l_i t) + \tilde{y}_i$, with $q_i < 0$, $k_i > 0$, and $l_i > 0$.[22] Regarding the distributions of the parameters: For case 1, $-q_i \sim logN\,(1, 0.25)$ and $s_i \sim logN\,(1, 0.25)$. For case 2, $q_i \sim logN\,(1, 0.25)$ and $s_i \sim logN\,(1, 0.25)$. For case 3, $-q_i \sim logN\,(1, 0.25)$, $s_i \sim logN\,(1, 0.25)$, $k_i \sim U\,(2000, 2500)$, and $l_i \sim U\,(0.5, 1.5)$.

4.3 Results

$n = 1000$ has been set for data generation. Descriptive statistics of the parameters are presented in Table 1.

Figure 1 shows the relationship between total gross sales revenue and tax rates for each case. In case 1, Y decreases monotonically whenever t increases, while the opposite occurs in case 2. In contrast, case 3 shows both increasing and decreasing intervals throughout the domain of t. Although these curves determine, to a great extent, how Z, ϕ, R, and r are related to t, note that these relationships cannot be affected by changes in the parameters of the evasion control policy (γ and x).

In Figure 2, the results for case 1 are shown. The level of tax evasion is inversely related to the tax rate, following an inverse U-shaped pattern. The share of the shadow economy tends to increase as the tax rate raises, and traditional Laffer curves in both nominal and proportional terms are observed despite the sufficient conditions for their existence not being met.

Figure 3 allows us to see the results for case 2. As the tax rate increases, the level of tax evasion also increases. The relative size of the shadow economy shows an inverse U-shaped relationship with the tax rate, and no Laffer curve arises in either nominal or proportional terms.

Figure 4 contains the results for case 3. Traditional Laffer curves do not emerge, whether in nominal or proportional terms. Instead, we observe multiple local maxima and minima for R and r. The consequence of this is far from trivial: There is no unique threshold after which a small raise (or decline) in t will always increase or decrease government revenue. Therefore, if t is fixed on the neighborhood of a local minimizer, government revenue could potentially grow with either higher or lower tax rates.

It should be highlighted that, in the three cases, for *any* given tax rate, a reduction in γ or an increase in x can simultaneously reduce total tax evasion, decrease the share of the shadow economy, and increase government revenue in both nominal and proportional terms.

[22]Whether or not this functional form is realistic, it is intended solely for illustrative purposes.

Descriptive Statistics -				
Variable	Mean	SD	Min	Max
tilde_y_i	5,009.044	978.244	2,291.587	8,216.046
upsilon_i	5,978.199	520.115	4,334.780	7,715.148
omega_i	1,790.747	310.661	717.115	2,795.972
c_i	0.010	0.001	0.007	0.013
|q_i|	1.024	0.262	0.456	2.367
s_i	1.040	0.261	0.442	2.532
k_i	2,249.847	145.327	2,000.251	2,498.903
l_i	0.990	0.285	0.500	1.499

Table 1: Mean, standard deviation, and range of the behavioral parameters generated.

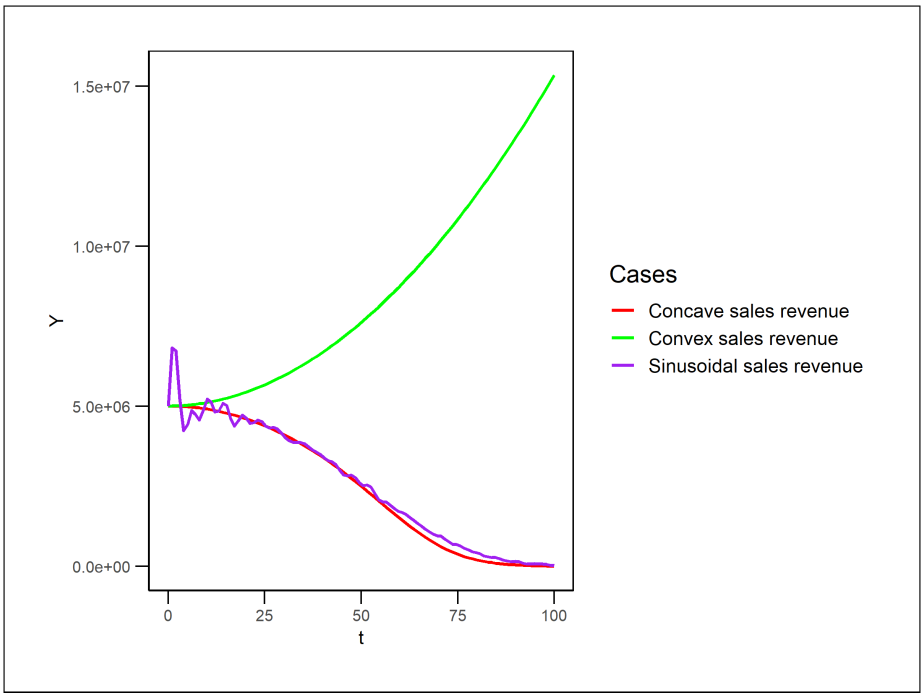
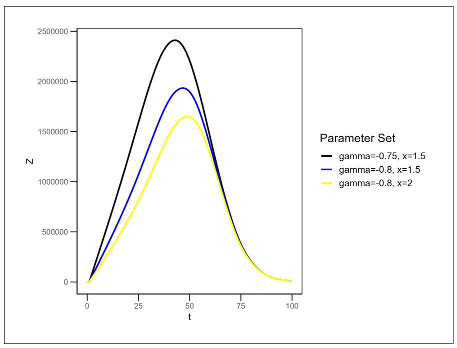
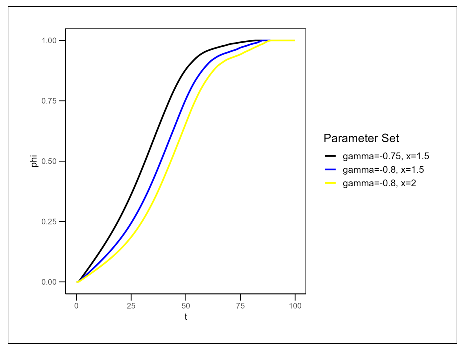


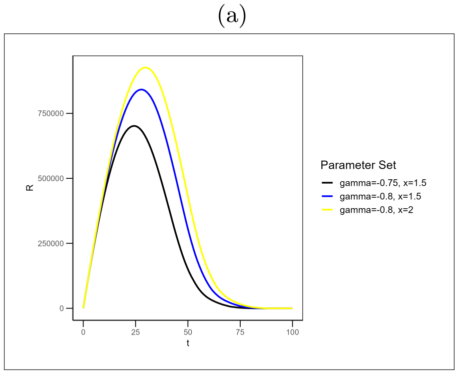
Figure 1: Aggregate gross sales revenue as a function of the tax rate for each of the cases.



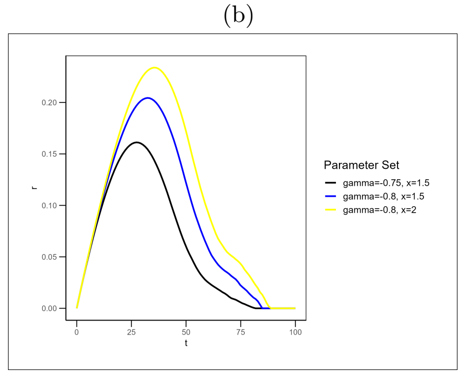
(a)



(b)

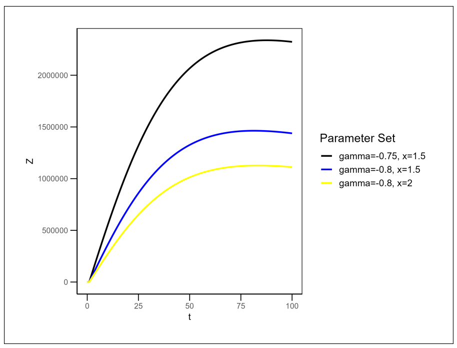


(c)

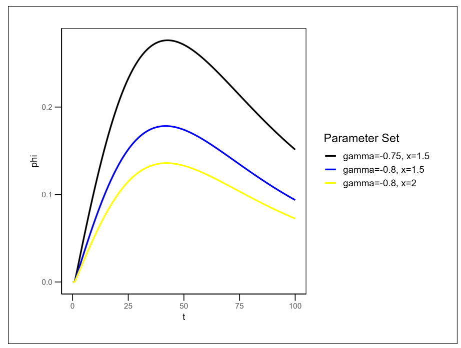


(d)

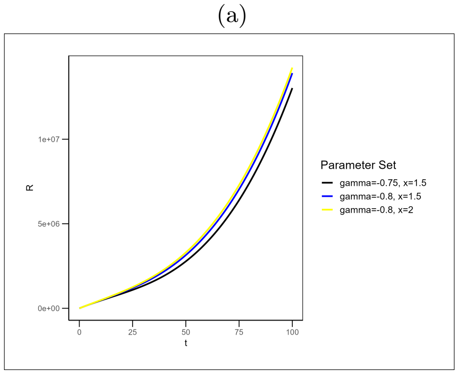
Figure 2: Aggregate level of tax evasion (a), share of the shadow economy (b), nominal government revenue (c), and proportional government revenue (d) as a function of the tax rate for case 1.



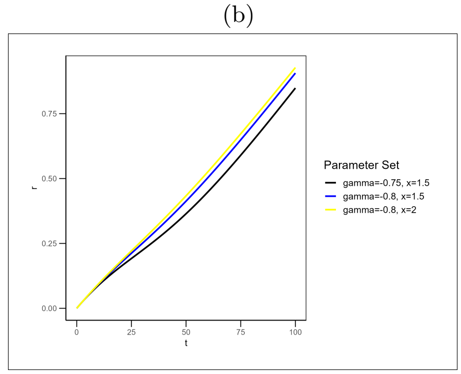
(a)



(b)



(c)



(d)

Figure 3: Aggregate level of tax evasion (a), share of the shadow economy (b), nominal government revenue (c), and proportional government revenue (d) as a function of the tax rate for case 2.

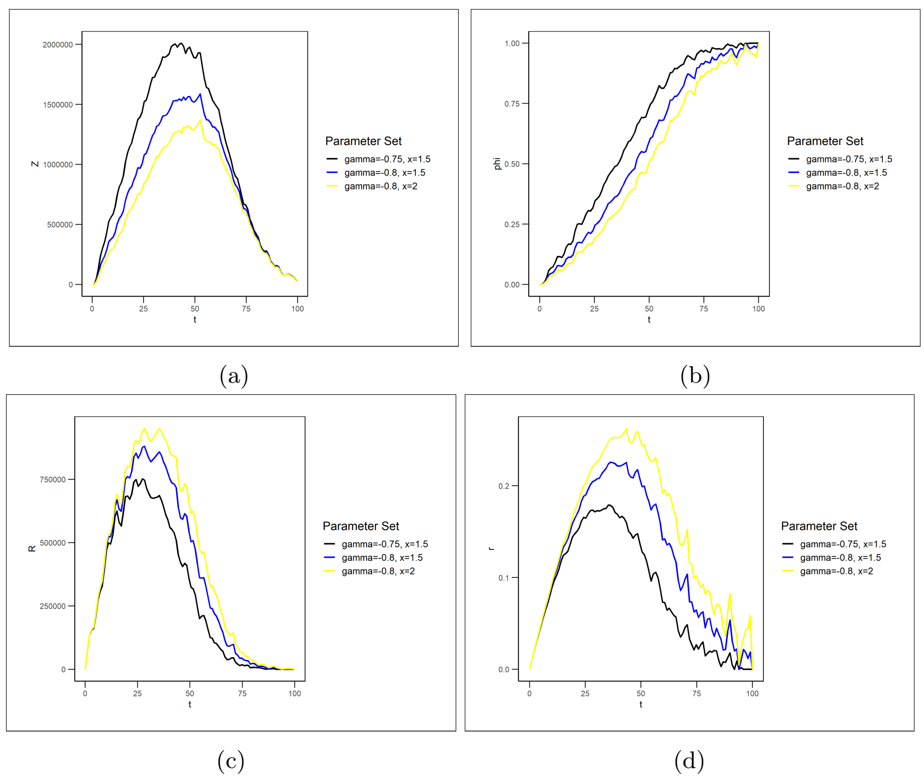


Figure 4: Aggregate level of tax evasion (a), share of the shadow economy (b), nominal government revenue (c), and proportional government revenue (d) as a function of the tax rate for case 3.

5 Concluding remarks

At a single-agent level, a model that recognizes the uncertainty faced by the decision-maker when trying to evade a sales tax has been developed. Beliefs about the probability of being detected and caught are influenced by behavioral biases, government activity, and other agents' behavior. In the aggregate, the effects of the tax rates on the shadow economy and government revenue have been analyzed. Special attention has been paid to the conditions under which a traditional Laffer curve (in either nominal or proportional terms) emerges as well as the implications of its existence.

It is important to recognize that certain issues have remained beyond the scope of this mathematical formalization. The potential impact of tax evasion on gross sales revenue, either through adjustments in prices or output, has not been considered. The framework does not go beyond one single period of time, and it neglects the influence of the social environment (for instance, through moral values) on the preferences of the decision-maker. It also ignores the tax compliance costs that taxpayers usually face.[23] Substitution effects between tax avoidance and tax evasion are also overlooked.[24] Lastly, oversimplifying assumptions might have been made about tax audits, such as binary outcomes for each level of evasion or perfect coverage of their cost by the additional income that they entail.[25]

Notwithstanding the limitations of this theory, its validity is subject to future empirical testing. However, it will be challenging in a field where the lack of data and its reliability have always been a problem (Alm, 2019; Andreoni et al., 1998; Cowell, 1985). One potential approach for future empirical research is to collect data through experimental economics.

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[23] Das-Gupta (2004) shows how these costs could be modeled in an economy where an income tax and a sales tax coexist.

[24] Recently, degl'Innocenti and Rablen (2017) and degl'Innocenti et al. (2022) formalize simultaneous decisions of tax evasion and tax avoidance. This may be an interesting extension.

[25] Sanyal et al. (2000) present a simple framework in which corruption leads to non-binary outcomes, while in Arozamena et al. (2008), the audit cost determines whether or not the government commits to enforcing the tax law. Future extensions in any of these directions seem both plausible and worthwhile.

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Annex

Individual variables

y: Gross sales revenue.
τ: Tax levied on gross sales revenue.
t: Tax rate as a proportion of gross sales revenue (in after-tax terms).
a: Proportion of gross sales revenue concealed.
a^*: Optimal proportion of gross sales revenue concealed.
z: Amount of gross sales revenue concealed.
z^*: Optimal amount of gross sales revenue concealed.
ς: Concealment costs.
c: Relevance of concealment costs for a and y given.
p: Perceived probability of detection.
υ: Influence of behavioral biases on the perceived probability of detection.
ω: Influence of other agents' perceived behavior on the perceived probability of detection.
γ: Influence of the perceived behavior of the tax authority on the perceived probability of detection.
χ: Fine imposed if evasion is detected.
x: Fine to be paid per unit of concealed revenue.
y^e: Expected net revenue.
π: Expected tax evasion profit.
η: Elasticity of gross sales revenue with respect to the tax rate.
ε: Non-tax related factors affecting gross sales revenue.

Aggregate variables

Y: Total gross sales revenue.
T: Total tax levied on gross sales revenue.
Z: Total amount of tax evasion.
ϕ: Relative size of the shadow economy.
R: Government revenue.
R^*: Maximum potential government revenue in nominal terms.
t^*: Tax rate that maximizes government revenue in nominal terms.
r: Government revenue as a proportion of total gross sales revenue.
r^{**}: Maximum potential government revenue as a proportion of total gross sales revenue.
t^{**}: Tax rate that maximizes government revenue as a proportion of total gross sales revenue.

Appendix

Kuhn-Tucker Optimization Problem

The problem is:

$$\max_{a} \quad \pi = t(1-p)ay - p\chi - \varsigma \quad \text{s. t.} \quad 0 \leq a \leq 1, \quad \pi \geq 0.$$

The Lagrangian function that we need to maximize is written in the following way:

$$\mathcal{L}(a, \lambda_1, \lambda_2, \lambda_3) = (1+\lambda_3)\pi + \lambda_1 a + \lambda_2(1-a),$$

where λ_1, λ_2, and λ_3 are the Lagrange multipliers associated with each of the constraints.

The conditions of stationarity, primal feasibility, dual feasibility, and complementary slackness are, respectively:

$$\frac{\partial \mathcal{L}}{\partial a} = (1+\lambda_3)\pi_a + \lambda_1 - \lambda_2 = 0 \tag{A1}$$

$$0 \leq a \leq 1, \quad \pi \geq 0 \tag{A2}$$

$$\lambda_1 \geq 0, \quad \lambda_2 \geq 0, \quad \lambda_3 \geq 0 \tag{A3}$$

$$\lambda_1 a = 0, \quad \lambda_2(1-a) = 0, \quad \lambda_3 \pi = 0. \tag{A4}$$

This problem has three possible solutions:

i) Interior solution: It occurs when $\pi_a = 0$ for some $a \in (0,1)$. Considering (A3) and (A4), we require $\lambda_1, \lambda_2 = 0, \lambda_3 \geq 0$. Note that (A1) will be satisfied $\forall \lambda_3 \in \mathbb{R}_+$. As demonstrated later on, it consists on a unique maximum where $\pi > 0$ is satisfied.

ii) Total evasion solution: It arises if $\pi_a > 0 \ \forall a \in (0,1)$. Given the constraints from (A2), the maximum feasible value of π is reached when $a = 1$, where $\pi > 0$ is ensured since $\pi = 0$ when $a = 0$. Due to (A3) and (A4), we need $\lambda_1, \lambda_3 = 0, \lambda_2 \geq 0$. Regarding (A1), as $\pi_a \geq 0$ for $a = 1$, $\exists \lambda_2 \in \mathbb{R}_+$ such that $\pi_a - \lambda_2 = 0$.

iii) No-evasion solution: In this case, $\pi_a < 0 \ \forall a \in (0,1)$. Considering the constraints from (A2), the maximum feasible value of π is reached when $a = 0$, resulting in $\pi = 0$. Because of (A3) and (A4), it is necessary that $\lambda_2 = 0$ and $\lambda_1, \lambda_3 \geq 0$. In reference to (A1), given that $\pi_a \leq 0$ for $a = 0$, $\exists \lambda_1, \lambda_3 \in \mathbb{R}_+$ such that $(1+\lambda_3)\pi_a + \lambda_1 = 0$.

Now, I prove that if an interior solution θ exists, it is a unique maximum that satisfies $\pi > 0$. Taking into account that

$$\frac{\partial \pi}{\partial a} = ty(1 - p - p_a a) - p\chi_a - p_a\chi - \varsigma_a.$$

We have

$$\frac{\partial \pi}{\partial a} = 0 \iff a = \theta = \frac{1}{p_a}\left[(1-p) - \frac{\varsigma_a + p_a\chi + p\chi_a}{ty}\right]. \tag{A5}$$

Also considering that

$$\frac{\partial^2 \pi}{\partial a^2} = -ty\left(2p_a + p_{aa}a\right) - \left(2p_a\chi_a + p_{aa}\chi + \varsigma_{aa}\right) < 0 \quad \forall \theta \in (0,1),$$

it is clear that (A5) defines a maximum. This maximum is unique because if two critical points θ_1, θ_2 were to exist, at least one of them should be either a minimum or an inflection point, which would imply $\pi_{aa} \geq 0$ for some $\theta \in (0,1)$ (i.e., a contradiction). Recalling that $\pi = 0$ when $a = 0$, it is easy to conclude that $\pi > 0$ if $a = \theta$.

Finally, the solution to the Kuhn-Tucker optimization problem can be expressed in the following way:

$$a^* = \begin{cases} 1 & if \quad \pi_a > 0 \ \forall a \in (0,1) \\ \theta & if \quad \pi_a = 0 \ for\ some\ a \in (0,1) \\ 0 & if \quad \pi_a < 0 \ \forall a \in (0,1) \end{cases}$$

where

$$\theta = \frac{1}{p_a}\left[(1-p) - \frac{C_a + p_a X + pX_a}{ty}\right].$$

Proof of Propositions 1-3

Proof of Proposition 1

In nominal terms: *Assume* $Y - Z > 0$ *when* $t = 0$, *and* $Y = Z = 0$ *when* $t = 1$. *The condition* $Z_t > Y_t \ \forall t \in [0,1]$ *necessarily implies* $Z_t > \phi Y_t \ \forall t \in [0,1]$ *since* $0 \leq \phi \leq 1$, *which means* $\phi_t > 0$ *for all* t *such that* $0 \leq t \leq 1$. *However, this cannot ensure* $Z_{tt} > Y_{tt} \ \forall t \in [0,1]$, *thus* R *may have more than one critical point in* $0 < t < 1$.

In proportional terms: *The demonstration is essentially the same.* $\phi_t > 0$ $\forall t \in [0,1]$ *can only satisfy the first-order condition, not the second-order one.*

Proof of Proposition 2

In nominal terms: *Considering* $R = t(Y - Z)$, *it is clear that* $tY > t(Y - Z)$ *as long as* $0 < Z \leq Y$. *We also have* $R_Z = -t < 0$. *Without loss of generality, consider the case for* γ. *As we have stated,* $z^*_{\gamma(i)} > 0 \ \forall i = 1, 2, .., n$, *hence* $Z_\gamma = \sum_{i=1}^{n} z^*_{\gamma(i)} > 0$. *Using the chain rule,* $R_\gamma = -tZ_\gamma < 0$.

In proportional terms: *The demonstration is analogous since* $r = t(1 - Z/Y)$.

Proof of Proposition 3

In nominal terms: *The no-evasion situation is a particular case in which ensuring the existence of a traditional Laffer curve requires* $Y_t, Y_{tt} < 0 \ \forall t \in [0,1]$ *since* $Z, Z_t, Z_{tt} = 0 \ \forall t \in [0,1]$. *For the comparison of this case to another one* *where* $Z > 0$ $\forall t \in [0, 1]$ *we need* $Z_t, Z_{tt} > 0$ $\forall t \in [0, 1]$*, otherwise we could not ensure the existence of a traditional Laffer curve. Now I prove the proposition for:*

i) t^**: Let* $(t^*)_0 = -Y_0/(Y_t)_0$ *if* $Z = 0$*, and* $(t^*)_1 = Y_1(1-\phi)/[(Z_t)_1 - (Y_t)_1]$ *if* $Z > 0$*. Assume* $(t^*)_1 > (t^*)_0$*. Since* $Y_t < 0$ *and* $0 < \phi \leq 1$ $\forall t \in [0, 1]$*, we have* $Y_0 > Y_1(1-\phi)$*. Therefore,* $-(Y_t)_0 > (Z_t)_1 - (Y_t)_1$ *necessarily needs to be met, which is equivalent to* $-(Y_t)_0 + (Y_t)_1 > (Z_t)_1$*. Because* $Y_{tt} < 0$ $\forall t \in [0, 1]$*,* $-(Y_t)_0 + (Y_t)_1 < 0$ *holds. In consequence,* $(Z_t)_1 < 0$ *is required for* $(t^*)_1 > (t^*)_2$*, which is a contradiction.*

ii) R^**: Define* $(R^*)_0 = -(Y_0)^2/1(Y_t)_0$ *for the no-evasion situation and* $(R^*)_1 = (Y_1)^2(1-\phi)^2/1[(Z_t)_1 - (Y_t)_1]$ *for the other case. The same previous contradiction will be encountered if it is assumed that* $(R^*)_2 > (R^*)_1$*.*

In proportional terms: *A traditional Laffer curve in proportional terms cannot exist in the no-evasion situation because* $\phi, \phi_t, \phi_{tt} = 0$ $\forall t \in [0, 1]$*, thus there are corner solutions for both* t^{**} *and* r^{**}*. Having said that, I prove the proposition for:*

i) t^{**}*: Let* $(t^{**})_0 \approx 1$ *if* $Z, Z_t, Z_{tt} = 0$ $\forall t \in [0, 1]$*, and* $(t^{**})_1 = (1-\phi)/\phi_t$ *for any other case where a traditional Laffer curve in proportional terms exists. Solving the algebra,* $(t^{**})_0 > (t^{**})_1$ *only if* $(1-\phi)/\phi_t < 1$*, which is necessarily true when a traditional Laffer curve in proportional terms exists since it implies* $(t^{**})_1 = [(1-\phi)/\phi_t] \in (0, 1)$*.*

ii) r^{**}*: Let* $(r^{**})_0 \approx 1$ *denote the no-evasion situation and* $(r^{**})_1 = (1-\phi)^2/\phi_t$ *any other case where a traditional Laffer curve in proportional terms exists.* $(r^{**})_0 > (r^{**})_1$ *holds as long as* $1/(1-\phi) > (1-\phi)/\phi_t$*, which is always satisfied when* $0 < (1-\phi)/\phi_t < 1$ *because* $0 < \phi \leq 1$*.*