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Abstract

This paper extends Armstrong, Vickers, and Zhou (2007) to the case with multiple prominent firms. All consumers first search among prominent firms, and if their products are not satisfactory, they continue to search among non-prominent ones. Prominent firms will charge a lower price than their non-prominent rivals as in the case with a single prominent firm, but relative to the situation without any prominent firm, the presence of more than one prominent firm can induce *all* firms to raise their prices. We also characterize how market prices and welfare vary with the number of prominent firms.

Keywords: consumer search, marketing, prominence, product differentiation
JEL classification: D43, D83, L13

1 Introduction

In many markets, not as most of the search literature assumes, the order in which consumers search for products is non-random, and it is often influenced by sellers' marketing activities or framing effects. For example, when using the online search engine, people might first click through the links displayed on the top of a page; in a

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supermarket or a bookstore, those products displayed at the entrance or other attention-grabbing positions might get noticed first by consumers; in a restaurant, customers may also first consider those dishes recommended by the waiter or offered in a special menu. In all these examples, options are not randomly presented, and some options are more prominent than others such that they will get considered by consumers prior to others.[1]

Moreover, lots of evidence shows that prominent options could be favored disproportionately. For example, Madrian and Shea (2001) identify the default effect in 401(k) participation and saving behavior. They find that participation is significantly higher under automatic enrolment and a substantial fraction of participants hired under automatic enrolment stick to both the default contribution rate and the default fund allocation. Ho and Imai (2006) and Meredith and Salant (2007) both point out that ballot order affects election outcomes: being listed first can significantly increase vote shares. Einav and Yariv (2006) present evidence that economists with surname initials earlier in the alphabet have more successful professional outcomes.

Sellers in the market also realize the importance of prominence in affecting buyers' choices, and are willing to pay for their products to be displayed prominently. For example, internet search engines make money through selling sponsored links, more prominent adverts are more expensive in yellow page directories, manufacturers pay supermarkets for access to prominent display positions, and eBay offers sellers the option to list their products prominently in return for an extra fee.

The above discussion suggests that prominence plays an important role in the market, and its impact on market performance deserves investigation. Arbatskaya (2007), and Armstrong, Vickers, and Zhou (2007) (AVZ thereafter) have made progress in this direction. Arbatskaya considers a search model with homogeneous products in which consumers only search for price in an exogenously specified order. In equilibrium, the prices decline with the rank of products. (Otherwise, no consumer would have incentive to sample products in unfavorable positions.) AVZ consider a search model with horizontally differentiated products where consumers search both for price and product fitness. They introduce a prominent firm by supposing that all consumers will visit it first in their search process. They find that the prominent firm will charge a lower price

[1] To notice the prominent option first seems to be a natural tendency of people. For example, Lohse (1997) reports an experiment which shows that, in a yellow page directory, those adverts which are colorful, with graphics, with larger sizes, or near the beginning of a heading, are much more likely to catch subjects' attention. In psychological literature, it is actually well documented that a salient stimulus can more effectively catch people's attention, and this reaction, to some degree, is independent from the economic importance of the stimulus. See, for example, Fiske and Taylor (1991).

than its prominent rivals, and making one firm prominent will usually increase industry profit but lower consumer surplus and total welfare.

This paper extends AVZ by allowing for multiple prominent firms. Specifically, all consumers are assumed to first search among prominent firms randomly, and if their products are not satisfactory, they continue to search among non-prominent ones. Our main purpose is to examine how market prices and welfare may vary with the number of prominent firms. We find that prominent firms still charge a lower price than their non-prominent rivals, which generalizes the price result in AVZ. However, relative to the situation without any prominent firm, the presence of more than one prominent firm can induce *all* firms to raise their prices. This result will never happen in the case with a single prominent product where making one product prominent will always lower its price. We also find that the price of non-prominent products tends to increase with the number of prominent products, while the price of prominent products may not. We further show that the relationship between welfare and the number of prominent products is non-monotonic. This is because the case without any prominent product is the same as that where all products are prominent. We characterize this relationship when the search cost is small: industry profit will first increase and then decrease with the number of prominent products, and it will reach its maximum when about half of products become prominent; while consumer surplus and total welfare will vary in the opposite way.[2]

This paper draws on the rich literature on consumer search in the market. In particular, our model is related to the branch on search with differentiated products, which is initiated by Wolinsky (1986) and developed further by Anderson and Renault (1999).[3] Both of them consider random consumer search, while we introduce non-random search to model the concept of prominence. Except Arbatskaya (2007) and AVZ, an earlier paper on non-random search and firm competition is Perry and Wigderson (1986). But their setup is very different from ours: it does not allow buyers to go back to previously visited sellers, and consumers' valuation of the product is determined before starting

[2] There are also two technical complications caused by the presence of multiple prominent firms. First, consumers' optimal stopping rule is no longer stationary. Specifically, the reservation surplus level they apply when search among prominent products is different from that they apply when search among non-prominent products. Second, with more than one prominent product, the form of consumers' stopping rule crucially depends on their expectation of whether prominent products are cheaper or more expensive than non-prominent products. So we need to deal with the issue of multiple equilibria.

[3] Weitzman (1979) is an earlier paper which studies the general optimal search among options with stochastic match values. But there is no supply side in his model.

searching and so there is no scope for searching for match values as in our model. In equilibrium, the observed prices could be non-monotonic in the rank order of sellers. As Arbatskaya (2007), they did not discuss the impact of non-random search on welfare.

Prominence in the market is often related with advertising. In the advertising literature, there is a small branch on advertising and consumer search order. In Bagwell and Ramey (1994), though advertising does not directly influence consumers' search order, it can coordinate their behavior in the following sense: consumers buy immediately from the firm which advertises most heavily, and due to economies of scale, this firm has a lower cost and does offer a lower price than its rivals. Thus, the consumer response to advertising is indeed rational. In our model, the consumer response to prominence is also rational, but the driving force is very different. More recently, and closer in spirit to our approach, Hann and Moraga-Gonzalez (2007) propose a search model *a la* Wolinsky (1986) in which a consumer's likelihood of sampling a firm is proportional to that firm's advertising intensity. They show that, in equilibrium all firms advertise with the same intensity and set the same price, and consumers end up searching randomly.[4]

Finally, our work is related to the literature on auctions for being listed prominently on online search engines. The two papers by Chen and He (2006) and Athey and Ellison (2007) are especially relevant, since they include in the consumer side a formal model of the interaction between sponsor links and consumer search.[5] In their model, high-quality sellers will buy top links, and consumers will rationally click through those links first. Therefore, prominence can signal quality in equilibrium and so improve overall efficiency. Nevertheless, there is no price competition in their models, and so they do not discuss the impact of prominence on market prices, which, however, is our focus.[6]

[4] Wilson (2008) proposes another model with endogenous consumer search order. In his model, products are homogenous, but before the price competition, each firm can choose the search cost that consumers must incur to inspect its product. Based on their observation of each product's search cost and their expectation of equilibrium prices, consumers choose their optimal search orders. In equilibrium, firms differentiate their search costs to avoid intense price competition and consumer search order is non-random.

[5] See also Borgers et al. (2007), Edelman et al. (2007), and Varian (2007) for online paid-placement auctions. But these papers do not have a formal search model in the consumer side.

[6] Chen and He (2006) do have prices charged by advertisers, but the structure of consumer demand in their model means that the Diamond Paradox is present, and all firms set monopoly prices.

2 The Model

Our model generalizes AVZ to allow for more than one prominent product. There are $n \geq 2$ firms, each of them supplying a single product at a constant unit cost which we normalize to zero.

There are a large number of consumers with measure normalized to one. Each consumer has a unit demand, and the value of a firm's product is idiosyncratic to consumers. Specifically, $(u_1, u_{2,} \cdots, u_n)$ are the values attached by a consumer to different products, and u_i is assumed to be independently drawn from a common distribution $F(u)$ on $[u_{\min}, u_{\max}]$ which has a positive and differentiable density function $f(u)$. We also assume that all match utilities are realized independently across consumers. The surplus from buying one unit of firm i's product at price p_i is $u_i - p_i$. If all match utilities and prices are known, a consumer will choose the product providing the highest surplus. If $u_i - p_i < 0$ for all i, she will leave the market without buying anything.

Initially, however, we assume consumers have imperfect information about the actual price and match utility of each product, but they can gather information through a sequential search process: a consumer can find out a product's price and match utility by incurring a search cost $s > 0$, and she can stop searching whenever she wants. Following the tradition in the search literature, we assume that the sampling process is without replacement and there is costless recall (i.e., a consumer can return to any option she has sampled without extra cost).

Although there are no systematic quality differences among products, some products are assumed to be more prominent than others. Without loss of generality, let $A = \{1, \cdots, m\}$ be the set of prominent products and $B = \{m+1, \cdots, n\}$ be the set of non-prominent products. The effect of prominence on consumer behavior is reflected through consumers' search order: consumers will always sample those prominent products first.[7] But no matter among prominent products or among non-prominent ones, consumers sample products randomly. When A or B is empty, all products are equally prominent

[7] There are at least three ways to think about our assumption about prominence. First, consumers may be exposed to options in an exogenously restricted order, and they have no ability to avoid prominent products. For instance, if a consumer goes to a travel agent to buy airline tickets or a financial advisor to buy a savings product, the advisor may reveal some options prior to others. Second, consumers could suffer from bounded rationality of some form and be susceptible to manipulation by marketing ploys. Third, consumers could be fully rational: they choose to visit prominent firms first because they expect these firms to make the best offers, and this expectation is correct in equilibrium. Our approach is largely neutral with respect to these three possibilities.

and our model degenerates to Wolinsky (1986); our model with $m = 1$ is just the case considered by AVZ.

Firms maximize their profit, and they simultaneously set prices p_i $(i = 1, 2, \cdots, n)$ conditional on the relative prominence between their products and their expectations of consumer behavior.

3 The Equilibrium

Since all prominent products and all non-prominent products are symmetric, we focus on the equilibrium where they are charged at p_A and p_B, respectively. Denote by $\Delta = p_B - p_A$ the price difference (if any) between them.

We first consider consumers' optimal stopping rule. Let a solve

$$\int_a^{u_{\max}} (u - a)dF(u) = s.$$

Thus, if there is no price difference among products and if a consumer has found a product with utility a, she is indifferent between buying this product and sampling one more product. As long as the search cost is not too high, a exists uniquely and decreases with s. Throughout this paper, we assume the search cost is relatively small such that both p_A and p_B are no greater than a in equilibrium and the search market is active.[8]

When $m \geq 2$, the optimal stopping rule crucially depends on whether consumers expect $p_A < p_B$ or $p_A > p_B$. If $p_A < p_B$, as we shall show below, the stopping rule is actually stationary within each product group (but not across groups). Nevertheless, if $p_A > p_B$, the stopping rule in the prominent group is nonstationary. This is because, the more a consumer approaches to the end of the prominent group, the more attractive the low price in the non-prominent group is, and so the less willing she is to stop searching. As a result, when $m \geq 2$ we may have multiple equilibria depending on consumers' expectation of prices. However, as we shall show below, in the uniform-distribution setting which most of our following analysis will focus on, $p_A > p_B$ cannot be an

[8]When consumers expect $p_A \leq a$, we have $\int_{p_A}^{u_{\max}} (u - p_A)dF(u) \geq s$, and so they are willing to participate in the market. When consumers expect $p_B \leq a$, there also exist some consumers who will search beyond prominent firms. However, as usual in search models, there are uninteresting equilibria where consumers expect all firms to set very high prices such that participating in the market is not worthwhile at all, or consumers expect non-prominent firms to set too high prices such that they will never search beyond prominent firms. We do not consider these equilibria further.

equilibrium outcome. Therefore, from now on we focus on consumers' expectation of $p_A \le p_B$. Let

$$z_A \equiv a - p_A \ge z_B \equiv a - p_B.$$

They are interpreted as the cutoff reservation surplus levels in group A and B, respectively.

The Optimal Stopping Rule:

Phase 1: In the prominent group A, stop searching if the surplus of the best offering so far has been no less than z_A; otherwise, search on whenever there are prominent products remained unsampled.

Phase 2: After sampling all prominent products, if the highest available surplus has been no less than z_B, then buy the best prominent product. Otherwise, keep searching in non-prominent group B.

Phase 3: In the group B, stop searching whenever the highest surplus so far has been no less than z_B. Otherwise, search on if there are non-prominent products remained unsampled.

Phase 4: After searching all products, if the highest surplus is non-negative, then go back to buy the best product. Otherwise, leave the market without buying anything.

The stopping rule among non-prominent products is standard, and here we explain the stopping rule among prominent products. Denote by v_i the highest net surplus after sampling $i \le m$ products in A. If a consumer comes to the last product in A and finds out $v_m < z_B$, then entering B is always desirable because the benefit from searching one more product in group B is larger than the unit search cost. (Recall the definition of a.) If $v_m \ge z_B$, she should not enter B according to her stopping rule in B. Thus, the consumer should enter group B if and only if $v_m < z_B$. Now consider the situation when the consumer comes to the penultimate product in A. If she finds out $v_{m-1} < z_A$, sampling the last product in A is always desirable. Otherwise, she should stop searching now, because even if she searched on, she would not enter B since $z_A \ge z_B$. This argument can go backward further and explain the stopping rule in A.

We now derive demand functions. We claim that a prominent firm's demand, if it deviates to a price p while other firms keep charging their equilibrium prices, is

$$q_A(p) = h_A \cdot [1 - F(a - p_A + p)] + \hat{r}_A(p) + r_A(p), \tag{1}$$

where

$$h_A = \frac{1 - F(a)^m}{m\,(1 - F(a))}$$

is the number of consumers who come to this firm for the first time,

$$\hat{r}_A(p) = \int_{a-\Delta}^{a} F(u)^{m-1} f(u + p - p_A) du$$

is the number of consumers who return to this firm after sampling all *prominent* products, and

$$r_A(p) = \int_{p_B}^{a} F(u - \Delta)^{m-1} F(u)^{n-m} f(u + p - p_B) du$$

is the number of consumers who return after sampling all products.

To understand (1), consider three possible sources of a prominent firm's demand. Let i be this firm's index. (i) A consumer may come to firm i after searching $k \leq m-1$ prominent products but without finding a satisfactory one (i.e., all of them have net surplus less than $z_A = a - p_A$). This probability is $\frac{1}{m}F(a)^k$.[9] Summing up these probabilities over $k = 0, \cdots, m-1$ leads to h_A. For such a consumer, she will buy at firm i immediately if $u_i - p \geq z_A$, of which the probability is $1 - F(a - p_A + p)$. This explains the first term in (1). We call this portion of firm i's demand the "fresh demand". (ii) If this consumer finds that all prominent products' net surplus less than z_A but product i is the best one and has net surplus greater than z_B, then she will return to buy it without searching on among non-prominent firms. (If firm i happens to be the last firm in group A, she just buys at it immediately.) The probability of this event is

$$\begin{aligned} &\Pr\left(\max_{j \neq i, j \in A} \{z_B, u_j - p_A\} < u_i - p < z_A\right) \\ = &\int_{p+z_B}^{p+z_A} F(u - p + p_A)^{m-1} dF(u), \end{aligned}$$

which is equal to $\hat{r}_A(p)$ by changing the integral variable. We call this portion of demand the "midway returning demand". (iii) The last possibility is, after sampling all products (which requires that each product has net surplus less than z_B), this consumer goes back to firm i if its product has the highest positive surplus. The probability of this

[9] Notice that $1/m$ is just the probability that this prominent product is on the $(k+1)_{\text{th}}$ position in the consumer's search process.

event is

$$\Pr\left(\max_{j\neq i, j\in A, l\in B}\{0, u_j - p_A, u_l - p_B\} < u_i - p < z_B\right)$$
$$= \int_p^{p+z_B} F(u-p+p_A)^{m-1}F(u-p+p_B)^{n-m}dF(u),$$

which equals $r_A(p)$ by changing the integral variable. We call this portion of demand the "final returning demand".

Secondly, we claim that a non-prominent firm's demand, if it deviates to a price p while other firms stick to their equilibrium prices, is

$$q_B(p) = h_B \cdot [1 - F(a - p_B + p)] + r_B(p), \tag{2}$$

where

$$h_B = F(a-\Delta)^m \frac{1 - F(a)^{n-m}}{(n-m)(1-F(a))}$$

is the number of consumers who come to this non-prominent firm for the first time, and

$$r_B(p) = \int_{p_B}^{a} F(u-\Delta)^m F(u)^{n-m-1} f(u+p-p_B)du$$

is the number of consumers who return to it after sampling all products.

The explanation goes as follows. Let j be this non-prominent firm's index. For a typical consumer, she will come to firm j fresh if she has left all prominent firms (which requires that each prominent product has net surplus less than z_B) and has sampled $k \leq n-m-1$ non-prominent products in B but has not found a satisfactory one. This probability is $F(a-\Delta)^m \frac{1}{n-m} F(a)^k$. Summing up these probabilities over $k = 0, \cdots, n-m-1$ yields h_B. Then she will buy at firm j immediately if $u_j - p > z_B$, of which the probability is $1 - F(a - p_B + p)$. This consumer will return to firm j if all products' net surplus is less than z_B but product j offers the highest positive surplus. The probability of this event is

$$\Pr\left(\max_{l\neq j, i\in A, l\in B}\{0, u_i - p_A, u_l - p_B\} < u_j - p < z_B\right)$$
$$= \int_p^{p+z_B} F(u-p+p_A)^m F(u-p+p_B)^{n-m-1}dF(u),$$

which is equal to $r_B(p)$ by changing the integral variable.

A useful observation is that how a firm's returning demand varies with its *actual* price crucially depends on the density function f. In particular, for the uniform distribution, a firm's returning demand is independent of its actual price, and so the fresh demand is more price responsive than the returning demand. All else equal, a higher fraction of returning demand makes a firm more want to raise its price.

We now derive equilibrium prices by assuming the uniform valuation distribution on $[0, 1]$. (In Appendix A.8, we will extend our main price result to the setting with more general distributions.) In this case, a is the solution to

$$\int_a^1 (u-a)du = s,$$

and so $a = 1 - \sqrt{2s}$. Throughout this paper, we keep the following condition which ensures that equilibrium prices p_A and p_B are less than a and so an active search market exists:

$$0 < s < \frac{1}{8}, \text{ or } \frac{1}{2} < a < 1. \tag{3}$$

According to (1), a prominent firm's demand, when it charges p, is now

$$q_A(p) = h_A \cdot (1 - a + p_A - p) + \hat{r}_A + r_A,$$

where

$$h_A = \frac{1-a^m}{m(1-a)},\ \hat{r}_A = \int_{a-\Delta}^{a} u^{m-1}du,\ r_A = \int_{p_B}^{a} (u-\Delta)^{m-1}u^{n-m}du.$$

Notice that both returning demands are independent of the actual price and so less price sensitive than the fresh demand.[10] Profit maximization implies the first-order condition

$$h_A \cdot (1 - a - p_A) + \hat{r}_A + r_A = 0. \tag{4}$$

According to (2), a non-prominent firm's demand, when it charges p, is now

$$q_B(p) = h_B \cdot (1 - a + p_B - p) + r_B,$$

where

$$h_B = \frac{(a-\Delta)^m (1-a^{n-m})}{(n-m)(1-a)},\ r_B = \int_{p_B}^{a} (u-\Delta)^m u^{n-m-1}du.$$

[10] If p is too high, then the fresh demand will be zero and the returning demand will depend on p, which makes the demand function no longer globally concave. However, by using the similar arguments as in AVZ (footnotes 15 and 18), we can show that the equilibrium derived below will not be overturned by the global deviation problem.

The first-order condition is then

$$h_B \cdot (1 - a - p_B) + r_B = 0. \tag{5}$$

In general, the system of equations (4)–(5) has no analytical solution, but the solution exists.

Proposition 1 *Under condition (3), on the area* $[0, a]^2$, *(4)–(5) have a unique solution* $(p_A, p_B) \in (1 - a, \frac{1}{2})^2$, *and* $p_A < p_B$.

Proof. We prove the existence and uniqueness in Appendix A.1. To show $p_A < p_B$, notice that

$$\Delta = p_B - p_A = \frac{r_B}{h_B} - \frac{\hat{r}_A + r_A}{h_A} > \frac{1}{h_A}(r_B - \hat{r}_A - r_A).$$

The second equality follows from the first-order conditions (4)–(5), and the inequality is because in equilibrium $h_A > h_B$ (i.e., a consumer who comes to a non-prominent firm must have visited a prominent firm). While

$$\hat{r}_A + r_A - r_B = \int_{a-\Delta}^{a} u^{m-1} du + \int_{p_B}^{a} \Delta (u - \Delta)^{m-1} u^{n-m-1} du$$

has the sign of Δ given $p_B < a$. Therefore, Δ must be positive. ■

Due to the consumer search order, each prominent firm's demand consists of more fresh demand proportionally than each non-prominent firm, and as we have known, the fresh demand is more price sensitive than the returning demand in the uniform-distribution setting.[11] Therefore, prominent firms have incentive to charge a lower price.

Before proceeding, we discuss the issue of multiple equilibria. Our analysis so far is predicated on consumers' expectation of $p_A < p_B$, and we have confirmed that $p_A < p_B$ is indeed an equilibrium outcome. Nevertheless, we have not yet discussed the other possible equilibrium with $p_A > p_B$. The following proposition excludes this possibility in our uniform-distribution setting. (All omitted proofs are presented in the Appendix.)

[11] In effect, the result that the fresh demand is more price sensitive then the returning demand will hold in a more general setting (see Appendix A.8). The intution is as follows. When a firm raises its price, its fresh demand will decrease for sure since more consumers will then search on. But part of these consumers will become returning consumers, so raising a firm's price has a potential positive effect on its own returning demand.

Proposition 2 *In the uniform-distribution setting, there is no equilibrium in which prominent products are more expensive than non-prominent products.*

Several polar cases also deserve mention: (i) When $n \to \infty$, both prices p_A and p_B converge to $1-a$.[12] (ii) It is also straightforward to verify that, when $a \to 1$ (i.e., when the search cost tends to zero), both prices converge to the full information price $\bar{p}$ that satisfies $n\bar{p} = 1 - \bar{p}^n$. (iii) When $a \to \frac{1}{2}$ (i.e., when the search cost is sufficiently high but restricted by condition (3)), both prices approach to the monopoly price $\frac{1}{2}$ since both of them lie between $1-a$ and $\frac{1}{2}$.[13] Moreover, in these three polar cases, p_A and p_B also tends to the price when there are no prominent firms at all. This means that prominence has little impact on market prices in these polar cases.

AVZ have shown that making one firm become prominent will induce all non-prominent firms to raise their price but induce the prominent one to lower its price. However, we will show below that, when more than one firm is made prominent, *all* firms may increase their prices. That is, prominence can be totally anti-competitive.

We first introduce a useful result and define the equilibrium price p_0 when there is no prominent firm. From the first-order conditions (4)–(5), we can see that equilibrium demands for a prominent product and a non-prominent product are $q_A = h_A p_A$ and $q_B = h_B p_B$, respectively. Thus, equilibrium total demand is $mh_A p_A + (n-m)\, h_B p_B$. On the other hand, since the number of consumers who eventually leave the market without buying anything is $p_A^m p_B^{n-m}$, total demand should be also equal to $1 - p_A^m p_B^{n-m}$. Therefore, in equilibrium the following equality must hold:

$$\frac{1-a^m}{1-a} p_A + \frac{1-a^{n-m}}{1-a} (a-\Delta)^m p_B = 1 - p_A^m p_B^{n-m}. \tag{6}$$

We can then define p_0 as the solution to

$$\frac{1-a^n}{1-a} = \frac{1-p_0^n}{p_0} \tag{7}$$

by letting $m = n$ and $p_A = p_B = p_0$ in (6).

[12] From (5), we have $p_B = 1 - a + \frac{r_B}{h_B}$. Notice that $\frac{r_B}{h_B} < \int_{p_B}^{a} \left(\frac{u}{a}\right)^{n-m-1} du$, and the right-hand side tends to zero as $n \to \infty$. Thus, p_B tends to $1-a$. Since $1 - a < p_A < p_B$, we also have p_A tends to $1-a$ as $n \to \infty$.

[13] The intuition is that a high search cost will make consumers willing to stop searching whenever she finds a product with positive surplus, and so each firm acts as a monopoly.

Proposition 3 *After some firms are made prominent, all non-prominent firms will raise their price (i.e., $p_B > p_0$), but prominent firms may raise or reduce their price. In particular, $p_A > p_0$ if $n \geq 4$, $2 \leq m \leq n-1$ and the search cost is relatively high such that a is close to $\frac{1}{2}$; and $p_A < p_0$ if $m = 1$ or the search cost is sufficiently low such that a is close to one.*

Non-prominent firms raise their prices because the presence of prominent firms makes their demand include more returning demand proportionally. But how prominent firms adjust their price seems more complicated. In general both $p_A < p_0$ and $p_A > p_0$ are possible, but when the search cost tends to be extreme, the relationship between p_A and p_0 is unambiguous. A little surprising result is that introducing more than one prominent firm can lead *all* firms to raise their prices. As numerical simulations show, this result can even take place under milder conditions (see a numerical example presented in Figure 1 below).

4 The Impact of the Number of Prominent Firms

This section examines how the number of prominent firms affects market prices and welfare. A general comparative static analysis with respect to m is intractable, but we can conduct it in the limit case when the search cost is close to zero (i.e., when a tends to one).[14]

We first investigate how market prices vary with the number of prominent firms.[15]

Proposition 4 *When the search cost is sufficiently small, both p_A and p_B increase with m at $1 \leq m \leq n-2$.*

For intermediate a, numerical simulations suggest that p_B still increases with m but p_A may not. The following graph presents an example of how prices vary with m when $a = 0.7$ and $n = 8$. The horizontal dashed line is p_0, the upper solid line is p_B, and the lower solid line is p_A. We can see that p_A changes with m non-monotonically.

[14] For fixed m, if we let n tend to infinity, then the situation is similar to the case with just one prominent firm.

[15] When $m = 0$ or n, the market price should degenerate to p_0. Hence, in general both market prices p_A and p_B must vary with m non-monotonically. But our equilibrium conditions of p_A and p_B are only valid for $1 \leq m \leq n-1$.

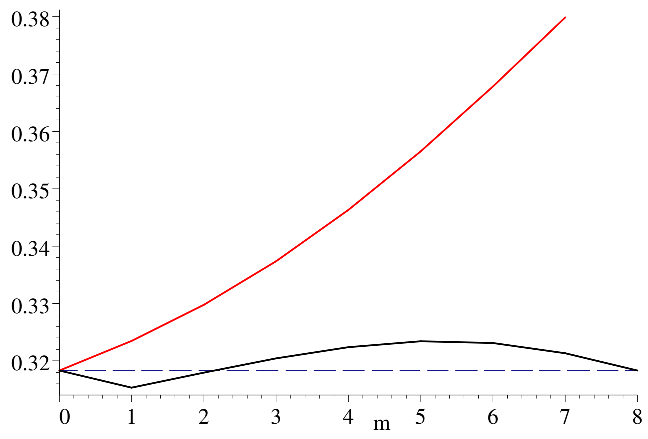

Figure 1: An Example of Equilibrium Prices and m

We then investigate how welfare vary with the number of prominent firms. Total output is

$$Q_m = 1 - p_A^m p_B^{n-m},$$

and so industry profit is

$$\Pi_m = p_B Q_m - \Delta \cdot m h_A p_A,$$

where $mh_A p_A$ is the output supplied by all prominent firms. We subtract the second term because prominent firms are charging a lower price than others. As we show in Appendix A.7, total welfare is

$$\begin{aligned} W_m &= a(1-a^n) - a(1-a^{n-m})\left[a^m - (a-\Delta)^m\right] \\ &\quad + m\int_{a-\Delta}^{a} u^m du + n\int_{p_B}^{a} (u-\Delta)^m u^{n-m} du. \end{aligned}$$

Consumer surplus is then $V_m = W_m - \Pi_m$.

A simple observation is that making all firms prominent is the same as no prominent firm at all, so all welfare variables must vary with m non-monotonically. We characterize this non-monotonic relationship below when the search cost is sufficiently small.

Proposition 5 *When the search cost is sufficiently small, total output decreases with m first and then increases. In particular, it must decrease with m at $1 \leq m \leq \frac{n}{2}$ and increase with m at $\frac{\sqrt{2}}{1+\sqrt{2}}n < m \leq n-1$.*

The intuition of this result is as follows. First, as we have shown, when the search cost is small, both p_A and p_B increase with m. Hence, larger m will reduce total output.

Second, larger m shifts more consumers to prominent firms. Since prominent firms are charging a lower price, the rise of m also has a positive effect on total output. Our non-monotonic result just reflects the combination of these two opposite effects.

Proposition 6 *When the search cost is sufficiently small, industry profit increases with m and total welfare and consumer surplus decrease with m if and only if total output decreases with m.*

Total welfare is mainly determined by total output and the price difference Δ. On the one hand, since the production cost is zero, every consumer should be served. Hence, higher total output means higher efficiency. On the other hand, consumers' search behavior is socially efficient when the market has a uniform price, but now $p_A < p_B$ makes too few consumers search beyond, and too many consumers return to, the group of prominent firms. Thus, larger Δ tends to result in less efficient search behavior. When the search cost is sufficiently small, our results indicate that the search effect is negligible relative to the output effect.

We present an example in the two graphs below where $a = 0.7$ and $n = 8$. The first graph describes how industry profit (the lower curve) and consumer surplus (the upper curve) vary with m, and the second graph describes how total welfare changes with m. In effect, further numerical simulations suggest that this kind of pattern is widespread.

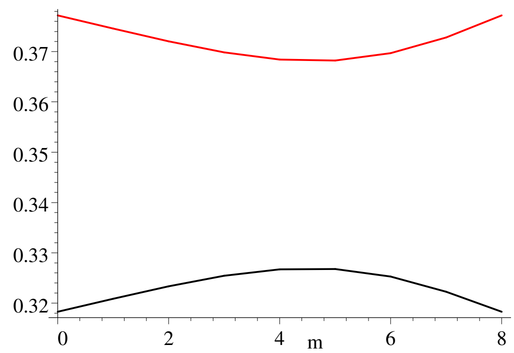

Figure 2: Industry Profit, Consumer Surplus, and m

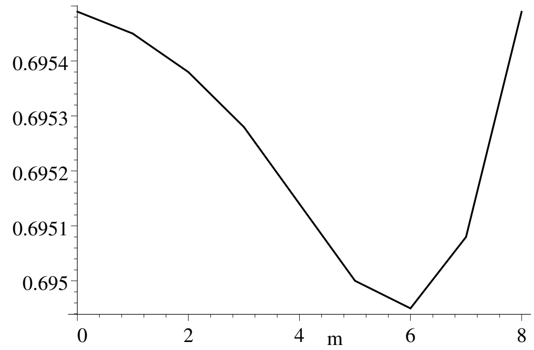

Figure 3: Total Welfare and m

A corollary of Propositions 5–6 is that, when the search cost is small, industrial profit will reach its maximum and total welfare and consumer surplus will reach their minimums when m is between $\frac{n}{2}$ and $\frac{\sqrt{2}n}{1+\sqrt{2}} \approx 0.586n$. This result suggests that, if there is a platform (for example, a search engine or a yellow page directory) through which all firms sell their products, and if the platform can extract the whole (or a fixed proportion of) industry profit, then it will make about half of firms prominent when the search cost is small.[16]

5 Conclusion

This paper extends Armstrong, Vickers, and Zhou (2007) to the case with multiple prominent firms. Prominent firms still charge a lower price than their non-prominent rivals as in the case with a single prominent firm, but relative to the situation without prominence, the presence of more than one prominent firm can induce *all* firms to raise their prices. We also show that, at least when the search cost is small, industry profit first rises and then goes down with the number of prominent products, and it reaches its maximum when about half of products become prominent; while consumer surplus and total welfare vary in the opposite way.

Several issues related with prominence deserve future study. First, it may be desirable to consider the impact of prominence on firms' quality choices. In particular,

[16]Numerical simulations also suggest that the optimal number of prominent positions m^* does not vary by too much even if the search cost becomes higher. Of course, m^* will become smaller if we consider the cost of establishing prominent positions, and it might also be restricted by the available space for prominent positions. In addition, the effect of prominence on guiding consumers' search may also become weaker when more prominent positions are present, which will further reduce m^*.

will prominent firms provide higher or lower quality products than their non-prominent rivals? Second, it may also be interesting to consider the implications of prominence when consumers need to search to find a satisfactory choice within a multi-product seller (for example, a restaurant or a supermarket). Third, beyond the product market, prominence may also play a significant role in the labor market. For example, some employers are more "famous" than others and job seekers may apply for their vacancies first; and some job candidates are more prominent than others and so they are more likely to be considered first by employers. Hence, two-sided prominence can exist in a search labor market.

A Appendix

A.1 Existence and uniqueness of equilibrium

Existence: for expositional convenience, define

$$K_i \equiv \frac{1-a^i}{i\,(1-a)}.$$

Rewrite the first-order condition (5) as $p_B = 1 - a + t_B$, where

$$t_B \equiv \frac{r_B}{h_B} = \frac{1}{K_{n-m}} \int_{p_B}^{a} \left(\frac{u-\Delta}{a-\Delta}\right)^m u^{n-m-1} du.$$

t_B is a decreasing function of p_B on $[0, a]$ since $\frac{u-\Delta}{a-\Delta}$ decreases with p_B when $u < a$. If $p_B = 1-a$, then condition (3) implies $t_B > 0$ and so $p_B < 1-a+t_B$. If $p_B = 1/2$, then $p_B > 1-a+t_B$. This is because $t_B < a - p_B$ by realizing $K_{n-m} > a^{n-m-1}$. Therefore, for any fixed p_A, on the range of $[0, a]$ the first-order condition (5) has a unique solution $p_B = b_B(p_A) \in (1-a, 1/2)$.

Rewrite the first-order condition (4) as $p_A = 1 - a + t_A$, where $t_A \equiv \frac{\hat{r}_A + r_A}{h_A}$. We first show that, given $p_B \in [0, a]$, t_A is a decreasing function of p_A. Notice that

$$\begin{aligned}\frac{\partial r_A}{\partial p_A} &= (m-1)\int_{p_B}^{a} (u-\Delta)^{m-2} u^{n-m} du \\ &< a^{n-m}\left[(a-\Delta)^{m-1} - p_A^{m-1}\right],\end{aligned}$$

and so

$$\begin{aligned}\frac{\partial(\hat{r}_A + r_A)}{\partial p_A} &< a^{n-m}\left[(a-\Delta)^{m-1} - p_A^{m-1}\right] - (a-\Delta)^{m-1} \\ &= (a^{n-m}-1)(a-\Delta)^{m-1} - a^{n-m} p_A^{m-1} < 0.\end{aligned}$$

Since the best response $b_B(p_A) \in (1-a, 1/2)$, we can focus on $p_B \in (1-a, 1/2)$. Then, if $p_A = 1-a$, we have $p_A < 1-a+t_A$. This is because $r_A > 0$ and $1-p_B < a$ also implies $\hat{r}_A = \int_{1-p_B}^{a} u^{m-1} du > 0$. On the other hand, if $p_A = 1/2$, then $p_A > 1-a+t_A$ if $t_A < a - 1/2$. We now show it is actually true. When $p_A = 1/2$,

$$\begin{aligned} r_A &= \int_{p_B}^{a} (u - p_B + 1/2)^{m-1} u^{n-m} du \\ &< \frac{a^{n-m}}{m} \left[(a - p_B + 1/2)^m - 1/2^m \right], \end{aligned}$$

and

$$\hat{r}_A = \frac{1}{m} \left[a^m - (a - p_B + 1/2)^m \right].$$

They imply

$$t_A < \frac{1}{m h_A} (a^m - 1/2^m) = \frac{1-a}{1-a^m} (a^m - 1/2^m) < a - 1/2.$$

Therefore, for any $p_B \in (1-a, 1/2)$, (4) has a unique solution $p_A = b_A(p_B) \in (1-a, 1/2)$.

The continuity of $b_A(p_B)$ and $b_B(p_A)$ is no problem. Hence, the Brower fixed point theorem implies that, on the area $(0, a)^2$, the system of the first-order conditions has at least one solution $(p_A, p_B) \in (1-a, 1/2)^2$.

Uniqueness: we first show $b'_A(p_B) \in (0, 1)$. Note that

$$b'_A(p_B) = \frac{\partial t_A / \partial p_B}{1 - \partial t_A / \partial p_A},$$

where

$$\frac{\partial t_A}{\partial p_A} = \frac{1}{K_m} \left[(m-1) \int_{p_B}^{a} (u - \Delta)^{m-2} u^{n-m} du - (a - \Delta)^{m-1} \right]$$

and

$$\frac{\partial t_A}{\partial p_B} = \frac{1}{K_m} \left[(a - \Delta)^{m-1} - (m-1) \int_{p_B}^{a} (u - \Delta)^{m-2} u^{n-m} du - p_A^{m-1} p_B^{n-m} \right].$$

It is clear that $1 - \frac{\partial t_A}{\partial p_A} > \frac{\partial t_A}{\partial p_B}$. Moreover, $\frac{\partial t_A}{\partial p_B} > 0$ since the square-bracket term is greater than

$$(a - \Delta)^{m-1} - a^{n-m} \left[(a - \Delta)^{m-1} - p_A^{m-1} \right] - p_A^{m-1} p_B^{n-m} > 0$$

given $p_B < a$. Hence, we have $b'_A(p_B) \in (0, 1)$.

Substituting $b_A(p_B)$ into the first-order condition (5), we obtain

$$p_B = 1 - a + \frac{1}{K_{n-m}} \int_{p_B}^{a} \left(\frac{u - p_B + b_A(p_B)}{a - p_B + b_A(p_B)} \right)^m u^{n-m-1} du.$$

$b'_A(p_B) \in (0,1)$ implies that $b_A(p_B) - p_B$ decreases with p_B, and so the term in the bracket is decreasing in p_B. Therefore, the whole right-hand side of the above equation is a decreasing function of p_B and our solution is unique.

A.2 Proof of Proposition 2

The rough idea of this proof is simple: we will show that, if consumers hold expectation of $p_A > p_B$, then a prominent firm will have more fresh demand but less returning demand than a non-prominent firm. Since returning demand is less price sensitive, prominent firms tend to charge a lower price, which contradicts consumers' expectation. The proof consists of several steps:

Step 1: The stopping rule with $p_A > p_B$. If consumers expect $p_A > p_B$ but their search order is still restricted, then what is their optimal stopping rule? We keep the notation $\Delta = p_B - p_A$. First of all, once a consumer enters B, her stopping rule is the same as in the case with $p_A < p_B$. Now consider the situation before she enters B. Denote by z_k ($k \leq m$) the reservation surplus level when she visits the k_{th} firm in her search process. That is, she will buy at the k_{th} firm immediately if and only if this firm provides surplus greater than z_k. According to Kohn and Shavel (1974), these z_k are well defined and unique in our setup. We further claim that $a - p_A \leq z_1 \leq \cdots \leq z_m = a - p_B$. Here $z_m = a - p_B$ is easy to understand. Now consider a consumer who has visited the $(m-1)_{\text{th}}$ firm and been ensured a surplus v_{m-1}. If $v_{m-1} < a - p_A$, then searching the last prominent firm is always desirable. If $v_{m-1} \geq z_m = a - p_B$, then she should stop searching now since $a - p_B > a - p_A$ and she would never enter B. Therefore, $a - p_A \leq z_{m-1} \leq z_m$.[17] Similarly, we can prove $a - p_A \leq z_{m-2} \leq z_{m-1}$ and others. The intuition is, when a consumer more approaches the end of pool A, she has more incentive to search on in pursuit of the lower price in B. Let us summarize the consumer's stopping rule with expectation of $p_A > p_B$:

Among prominent firms, stop at the k_{th} firm if and only if the highest available surplus so far is no less than z_k, where $a - p_A \leq z_1 \leq \cdots \leq z_m = a - p_B$;[18] *among non-prominent firms, stop searching if and only if the highest available surplus is no*

[17] More precisely, z_{m-1} can be defined as follows: let $\mu \equiv \max(u_m - p_A, z_{m-1})$ and $G(\mu)$ be its distribution function. Then $z_{m-1} = \int_{z_m}^{\infty} \mu dG(\mu) + \int_{-\infty}^{z_m} V_B(\mu) dG(\mu) - s$, where $V_B(x)$ is the expected surplus from entering B when the consumer has been ensured a surplus x. We can recursively define other z_i.

[18] The case with strictly increasing z_i and $z_1 > a - p_A$ can actually take place. For example, when $n = 3$, $m = 2$, $a = 0.6$, $p_A = 0.45$, and $p_B = 0.4$, one can show that $z_1 \approx 0.17$ and $z_2 = 0.2$.

less than $z_B = a - p_B$; after searching all firms, return to the firm providing the highest non-negative surplus (if any).

Step 2: The demand system. Since now there is no midway returning demand, each firm's demand consists of two parts: fresh demand and (final) returning demand. We consider the returning demands r_A and r_B first. One can show that a firm's returning demand is independent of its actual price (for local deviation) and $r_A \leq r_B$.[19] The intuition for $r_A \leq r_B$ is simple: when a consumer leaves a prominent firm and a non-prominent firm, the former's product on average has a lower net surplus since the reservation surplus $z_i \leq z_m$ for $i \leq m$, and so it wins the consumer back less likely.

Now we are ready to write down demand functions. For a prominent firm, if it charges p while other firms stick to their equilibrium prices, its demand is

$$q_A(p) = \frac{1}{m} \sum_{k=1}^{m} \left[(1 - z_k - p) \prod_{i=1}^{k-1} (z_i + p_A) \right] + r_A.$$

Here $\prod_{i=1}^{k-1} (z_i + p_A) / m$ is the probability that a consumer will visit this prominent firm as the k_{th} firm in her search process, and $1 - z_k - p$ is the conditional probability that this consumer will buy immediately.[20] For a non-prominent firm, if it charges p while others keep charging their equilibrium prices, its demand is

$$q_B(p) = (1 - z_B - p) K_{n-m} \prod_{k=1}^{m} (z_k + p_A) + r_B.$$

Here $K_{n-m} \prod_{k=1}^{m} (z_k + p_A)$ is the likelihood that a consumer will come to this non-prominent firm as a fresh consumer.

Step 3: $p_A > p_B$ is incompatible with equilibrium conditions. Define $\alpha_k \equiv \prod_{i=1}^{k-1} (z_i + p_A)$ and notice $\alpha_k \geq \alpha_{k+1}$. Then the first-order conditions are

$$\frac{1}{m} \sum_{k=1}^{m} \alpha_k (1 - z_k - 2p_A) + r_A = 0, \tag{8}$$

[19] One can check that

$$\begin{aligned} r_A &= \frac{1}{m} \sum_{k=1}^{m} \int_{p_A}^{p_A + z_k} (u + \Delta)^{n-m} u^{m-k} \prod_{i=1}^{k-1} \min(z_i + p_A, u) du, \\ r_B &= \int_{p_B}^{a} u^{n-m-1} \prod_{i=1}^{m} \min(z_i + p_A, u - \Delta) du. \end{aligned}$$

The details for $r_A \leq r_B$ are available on request.

[20] More precisely, all $z_i + p_A$ terms should be replaced by $\min(1, z_i + p_A)$ because of the boundary problem. But this does not change our following analysis.

and

$$K_{n-m}\alpha_{m+1}(1-a-p_B)+r_B=0, \tag{9}$$

where we have used $z_B = a - p_B$. Suppose $p_A > p_B$ is the solution. Then we must have $p_A > p_B > 1-a$, where the later inequality is from (9). Since $z_k \geq a - p_A$, we have

$$1 - z_k - 2p_A \leq 1 - a - p_A < 0.$$

Then (8) implies

$$\frac{1}{m}\sum_{k=1}^{m}\alpha_k\,(1-a-p_A)+r_A \geq 0.$$

So

$$K_{n-m}\alpha_{m+1}\,(1-a-p_A)+r_B > 0$$

since $K_{n-m} < 1$, $\sum_{k=1}^{m}\alpha_k/m > \alpha_{m+1}$, and $r_B \geq r_A$. This, however, contradicts with (9) when $p_A > p_B$. Therefore, consumers' initial expectation of $p_A > p_B$ cannot be sustained in equilibrium.

A.3 Proof of Proposition 3

(i) Since $\Delta > 0$, the left-hand side of (6) is less than

$$\frac{1-a^m}{1-a}p_B + \frac{1-a^{n-m}}{1-a}a^m p_B = \frac{1-a^n}{1-a}p_B,$$

while the right-hand side of (6) is greater than $1-p_B^n$. Thus,

$$\frac{1-a^n}{1-a} > \frac{1-p_B^n}{p_B}.$$

Comparing it to (7) yields $p_0 < p_B$.

(ii) When $a \to \frac{1}{2}$, we have $\Delta \to 0$, and so

$$(a-\Delta)^m p_B \approx (a^m - ma^{m-1}\Delta)(p_A+\Delta) \approx a^m p_A + a^{m-1}(a-mp_A)\Delta$$

and

$$p_A^m p_B^{n-m} = p_A^m(p_A+\Delta)^{n-m} \approx p_A^n + p_A^{n-1}(n-m)\Delta.$$

Substituting them into (6) yields

$$\frac{1-a^n}{1-a}p_A - (1-p_A^n) \approx \Delta\left[\frac{1-a^{n-m}}{1-a}a^{m-1}(mp_A-a)-(n-m)p_A^{n-1}\right].$$

When $a \to \frac{1}{2}$ (and so $p_A \to \frac{1}{2}$), the square-bracket term approaches to

$$\frac{m-1}{2^{m-1}} - \frac{n-1}{2^{n-1}}.$$

When $2 \le m \le n-1$ and $n \ge 4$, this is positive and so

$$\frac{1-a^n}{1-a} > \frac{1-p_A^n}{p_A}.$$

Comparing it to (7) yields $p_A > p_0$.

(iii) We first show that $(a-\Delta)^m p_B > a^m p_A$ is a sufficient condition for $p_A < p_0$. When this condition holds, the left-hand side of (6) is greater than $\frac{1-a^n}{1-a}p_A$. Meanwhile, the right-hand side of (6) is less than $1-p_A^n$ since $p_A < p_B$. As a result,

$$\frac{1-a^n}{1-a} < \frac{1-p_A^n}{p_A},$$

and so $p_A < p_0$. When $m = 1$, the sufficient condition is equivalent to $p_B < a$ which must be true. When $a \to 1$, p_B tends to the full-information equilibrium price $\bar{p} = (1-\bar{p}^n)/n < a/m$. So when $a \to 1$, we have

$$1 - \frac{\Delta}{p_B} < 1 - \frac{m\Delta}{a} < \left(1 - \frac{\Delta}{a}\right)^m,$$

which just implies the above sufficient condition.

A.4 Proof of Proposition 4

We first approximate equilibrium prices as the search cost is close to zero.

Claim 1 *Define*

$$\varphi_A \equiv \frac{(\theta - \bar{p})\, m + 1 + \bar{p}^{n-1}}{2 - m\bar{p} + \theta},\ \varphi_B \equiv \frac{(\theta - \bar{p})\, m}{2 - m\bar{p} + \theta},$$

where $\bar{p}$ is the full-information equilibrium price satisfying $n\bar{p} = 1 - \bar{p}^n$ and $\theta \equiv \frac{1-\bar{p}^{n-1}}{n-1}$. When a is close to one, equilibrium prices can be approximated as

$$p_i \approx \bar{p} + k_i \varepsilon,\ i = A, B$$

where $\varepsilon = 1 - a$ and

$$k_i = \frac{\bar{p}}{2(1+\bar{p}^{n-1})}\left[n(1-\varphi_i) + m - 1\right].$$

Proof. When $a = 1$, equilibrium price is $\bar{p}$. So we can approximate p_i as $\bar{p} + k_i\varepsilon$ as $a \to 1$, where $\varepsilon = 1 - a$ and k_i needs to be determined. Use Taylor expansion to extend the first-order conditions (4)–(5) around $a = 1$ and discard all terms of higher than first order. We get a system of equations about k_A and k_B. (The details of approximation are available on request.) Then we can solve

$$\begin{aligned}(1+\bar{p}^{n-1})k_A &= \frac{n+m-1}{2}\bar{p} - \left[(\theta-\bar{p})\,m + 1 + \bar{p}^{n-1}\right]k_\Delta, \\ (1+\bar{p}^{n-1})k_B &= \frac{n+m-1}{2}\bar{p} - (\theta-\bar{p})\,mk_\Delta,\end{aligned}$$

where

$$k_\Delta \equiv k_B - k_A = \frac{n\bar{p}}{2(2-m\bar{p}+\theta)}.$$

Using the notation we have introduced, we have

$$2(1+\bar{p}^{n-1})k_i/\bar{p} = n(1-\varphi_i) + m - 1.$$

■

We now show equilibrium prices rise with m in this limit case. For p_i to be increasing with m, it suffices to show that $\frac{\partial\varphi_i}{\partial m} < \frac{1}{n}$. Notice that

$$\begin{aligned}\frac{\partial\varphi_B}{\partial m} &< \frac{\partial\varphi_A}{\partial m} = \frac{\partial\varphi_B}{\partial m} + \frac{\bar{p}(1+\bar{p}^{n-1})}{(\theta+2-m\bar{p})^2} \\ &< \frac{1}{(\theta+1)^2}\left[(\theta-\bar{p})(\theta+2) + \bar{p}(1+\bar{p}^{n-1})\right] \qquad (10)\end{aligned}$$

since $\frac{\partial\varphi_B}{\partial m} = \frac{(\theta-\bar{p})(\theta+2)}{(\theta+2-m\bar{p})^2}$ and $\theta + 2 - m\bar{p} > \theta + 1$ (which is because $m\bar{p} < n\bar{p} = 1 - \bar{p}^n < 1$). Using the definition of θ, one can show that (10) is less than $\frac{1}{n}$ if and only if $(n+1)\bar{p} + \bar{p}^{n-1}/n < 2$. This must be true since $\bar{p} < \frac{1}{n}$. Therefore, both p_A and p_B increase with m when the search cost is close to zero.

A.5 Proof of Proposition 5

When $a \to 1$, using the approximated equilibrium prices, we can approximate total output as

$$Q_m \approx 1 - \bar{p}^n - (nk_B - mk_\Delta)\bar{p}^{n-1}\varepsilon.$$

So total output and $nk_B - mk_\Delta$ vary with m in the opposite direction. Using the results in Claim 1, we have

$$nk_B - mk_\Delta = \frac{n(n+m-1)\bar{p}}{2(1+\bar{p}^{n-1})} - \left(\frac{n(\theta-\bar{p})}{1+\bar{p}^{n-1}} + 1\right)mk_\Delta,$$

and so

$$\frac{\partial}{\partial m}\left(nk_B - mk_\Delta\right) = \frac{n\bar{p}}{2(1+\bar{p}^{n-1})} - \left(\frac{n(\theta-\bar{p})}{1+\bar{p}^{n-1}}+1\right)\frac{(\theta+2)n\bar{p}}{2(\theta+2-m\bar{p})^2}.$$

By using the definition of θ, a lengthy algebra manipulation shows that this expression has the sign of

$$m^2(n-1)\bar{p} + (n-2m)(2n-1-\bar{p}^{n-1}). \tag{11}$$

When $2m \leq n$, this is clearly positive, so total output decreases with m. When $2m > n$, the opposite result can happen. Using $2n-1-\bar{p}^{n-1} > 2n(n-1)\bar{p}$ (which is implied by $\bar{p} < 1/n$ and $\bar{p}^{n-1} < 1$), a sufficient condition for (11) to be negative (and so total output increases with m) is $2(n-m)^2 < m^2$ or $m > \frac{\sqrt{2}}{1+\sqrt{2}}n$. Also notice that (11) decreases with m, and so total output rises with m first and then goes down.

A.6 Proof of Proposition 6

When $a \to 1$, using the approximated equilibrium prices, we can approximate industry profit as

$$\Pi_m \approx n\bar{p}^2 + \left(nk_B - mk_\Delta\right)(\bar{p}-\bar{p}^n)\varepsilon$$

and total welfare as

$$W_m \approx \frac{n}{n+1}(1-\bar{p}^{n+1}) + \left(mk_\Delta - nk_B\right)\bar{p}^n\varepsilon.$$

(The details of approximation are omitted since the procedure is standard.) We have known that, as $a \to 1$, $\frac{\partial Q_m}{\partial m}$ has the sign of $\frac{\partial}{\partial m}\left(mk_\Delta - nk_B\right)$. Therefore, in this limit case, how industry profit, consumer surplus, and total welfare vary with m is totally determined by how total output varies with m.

A.7 Deriving total welfare

We derive the expression for total welfare for general distributions. Define two new random variables:

$$v_A \equiv \max\{u_1, \cdots, u_m\},\ v_1 \equiv \max\{u_1+\Delta, \cdots, u_m+\Delta, u_{m+1}, \cdots, u_n\}.$$

Their distribution functions are $F_A(u) = F(u)^m$ and $F_1(u) = F(u-\Delta)^m F(u)^{n-m}$, respectively. Let $F \equiv F(a)$. Then we claim that total welfare is

$$\begin{aligned} W_m &= [(1-F^m) + F(a-\Delta)^m\left(1-F^{n-m}\right)]E(u|u\geq a) \\ &\quad + \int_{a-\Delta}^{a} udF_A(u) + \left[\int_{p_B}^{a} udF_1(u) - \Delta m r_A\right] - sT_m, \end{aligned}$$

where

$$T_m = \frac{1}{1-F}\left[1 - F^m + F(a-\Delta)^m\left(1 - F^{n-m}\right)\right]$$

is a consumer's expected number of searches.[21]

A consumer will end up as a fresh buyer in the prominent group with probability $1-F^m$, and she will end up as fresh buyer in the non-prominent group with probability $F(a-\Delta)^m\left(1-F^{n-m}\right)$. Hence, the first term reflects the expected gross surplus (excluding the search cost) from all fresh buyers. The second term is the expected gross surplus from those midway returning consumers, and the third one is from those final returning consumers. The reason why we subtract $\Delta m r_A$ is that in the order statistics v_1, each prominent product's utility is augmented by Δ. So when they win back returning buyers (of which the probability is mr_A), Δ should be subtracted in surplus calculation.

Using the fact that $E(u|u \geq a) - \frac{s}{1-F} = a$ (which is from the definition of a), one can show

$$\begin{aligned} W_m &= a[1 - F^m + F(a-\Delta)^m\left(1 - F^{n-m}\right)] + \int_{a-\Delta}^{a} u dF_A(u) \\ &\quad + \int_{p_B}^{a} (u-\Delta)F(u)^{n-m} dF(u-\Delta)^m + \int_{p_B}^{a} uF(u-\Delta)^m dF(u)^{n-m}. \end{aligned}$$

The expression in the uniform-distribution setting then follows immediately.

A.8 Equilibrium prices with general distributions

We aim to show that the result $p_A < p_B$ holds for more general valuation distributions. For expositional convenience, let $\phi(p,x) \equiv 1 - F(p+x) - pf(p+x)$. We keep the following two assumptions:

Assumption 1 *$f(u)$ is logconcave, and $a > \frac{1-F(a)}{f(a)}$.*

Assumption 2 *$\phi_2(p,x) \leq 0$ for positive p and x.*

The second part of Assumption 1 requires a relatively small search cost, and it corresponds to condition (3) in the main text. Assumption 2 is a reasonable restriction.

[21] In each step of the search process (no matter among the prominent firms or among the non-prominent firms), a consumer has a probability of $1-F$ to be a fresh buyer. While the likelihood of becoming a fresh buyer in the whole search process is the square-bracket term. Therefore, the expected number of searches is T_m.

It means that, if a monopoly firm supplies a product for which consumers' valuation distribution is $F(u)$ and consumers' reservation utility is x, then the optimal monopoly price decreases with x since $\phi(p,x)$ is just the firm's marginal profit at price p.

The first-order conditions are[22]

$$h_A\phi(p_A, a-p_A) + \hat{R}_A + R_A = 0, \tag{12}$$

and

$$h_B\phi(p_B, a-p_B) + R_B = 0, \tag{13}$$

where $\hat{R}_A$, R_A, and R_B are equilibrium marginal profits from a prominent firm's midway returning demand, its final returning demand, and a non-prominent firm's returning demand, respectively. Assumption 2 ensures that all these marginal profits are positive (i.e., returning demand is less price "sensitive" than fresh demand in equilibrium).

Our demand functions are predicated on consumers' expectation of $p_A < p_B$, and we now confirm that this is indeed an equilibrium outcome.

Claim 2 *Given Assumptions 1–2, if the system of the first-order conditions has solutions, then one solution must specify $p_A < p_B$.*[23]

Proof. Denote by $\zeta(p_A, p_B)$ the left-hand side of (12). As we will show below, if all solutions to the system of (12)–(13) satisfied $p_A \geq p_B$, then we would have $\zeta(p_B, p_B) < 0$, and so the equation $\zeta(p_A, p_B) = 0$ would have a solution $p_A < p_B$ since $\zeta(0, p_B) > 0$ is always true. This is a contradiction.

We now show that $\zeta(p_B, p_B) < 0$ if $p_A \geq p_B$. Notice

$$\begin{aligned}\zeta(p_B, p_B) &= h_A\phi(p_B, a-p_B) - \int_{p_B}^{a} F(u)^{n-1}\phi_2(p_B, u-p_B)du \\ &= -\frac{h_A}{h_B}R_B - \int_{p_B}^{a} F(u)^{n-1}\phi_2(p_B, u-p_B)du.\end{aligned}$$

The second equality is because of (13). Since $\phi_2(p_B, u-p_B) \leq 0$ for $u \in [p_B, a]$ due to Assumption 2 and

$$R_B = -\int_{p_B}^{a} F(u-\Delta)^m F(u)^{n-m-1}\phi_2(p_B, u-p_B)du,$$

[22] One can show that our profit functions are actually concave under Assumptions 1–2 if the support of $F(u)$ is unbounded. Hence, in that case the first-order conditions are also sufficient for equilibrium prices.

[23] The technique used in the uniform setting relies on $\hat{R}_A + R_A - R_B$ having the sign of $\triangle$, and does not apply in this general setting.

a sufficient condition for negative $\zeta(p_B, p_B)$ is $F(u)^{n-1} < h_A F(u-\Delta)^m F(u)^{n-m-1}/h_B$ for $u \in [p_B, a]$, or equivalently,

$$\frac{F(u)^m}{F(u-\Delta)^m} < \frac{K_m}{K_{n-m}F^m}\frac{F^m}{F(a-\Delta)^m},$$

where $F = F(a)$ and $K_i = \frac{1-F^i}{i(1-F)}$. Since $K_m > K_{n-m}F^m$, it suffices to have

$$\frac{F(u)}{F(u-\Delta)} \leq \frac{F(a)}{F(a-\Delta)} \text{ for } u \in [p_B, a]. \tag{14}$$

Assumption 1 implies logconcave $F(u)$ and so decreasing $\frac{f(u)}{F(u)}$. If $p_A \geq p_B$ (and so $\Delta \leq 0$), we have $\frac{f(u)}{F(u)} \geq \frac{f(u-\Delta)}{F(u-\Delta)}$, which implies that $\frac{F(u)}{F(u-\Delta)}$ is an increasing function and so condition (14) holds.[24] ■

References

Anderson, S., and R. Renault (1999): "Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model," *RAND Journal of Economics*, 30(4), 719–735.

Arbatskaya, M. (2007): "Ordered Search," *RAND Journal of Economics*, 38(1), 119–126.

Armstrong, M., J. Vickers, and J. Zhou (2007): "Prominence and Consumer Search," *Rand Journal of Economics*, forthcoming.

Athey, S., and G. Ellison (2007): "Position Auctions with Consumer Search," mimeo.

Bagwell, K., and G. Ramey (1994): "Coordination Economies, Advertising, and Search Behavior in Retail Markets," *American Economic Review*, 84(3), 498–517.

Borgers, T., I. Cox, M. Pesendorfer, and V. Petricek (2006): "Equilibrium Bids in Auctions of Sponsored Links: Theory and Evidence," mimeo.

[24] We have seen that our result only needs $\phi_2(p_B, u-p_B) \leq 0$ for $u \in [p_B, a]$, which is weaker than Assumption 2. In fact, if n is sufficiently large, this condition is ensured by Assumption 1. The details are available on request.

Chen, Y., and C. He (2006): "Paid Placement: Advertising and Search on the Internet," mimeo.

Edelman, B., M. Ostrovsky, and M. Schwarz (2007): "Internet Advertising and the Generalized Second Price Auction: Selling Billions of Dollars Worth of Keywords," *American Economic Review*, 97(1), 242–259.

Einav, L., and L. Yariv (2006): "What's in a Surname? The Effects of Surname Initials on Academic Success," *Journal of Economic Perspective*, 20(1), 175–188.

Fiske, S. T., and S. E. Taylor (1991): *Social Cognition*. McGraw-Hill, New York, second edn.

Hann, M., and J. Moraga-Gonzalez (2007): "Advertising for Attention," mimeo.

Ho, D., and K. Imai (2006): "Randomization Inference With Natural Experiments: An Analysis of Ballot Effects in the 2003 California Recall Election," *Journal of the American Statistical Association*, 101(475), 888–900.

Kohn, M. G., and S. Shavell (1974): "The Theory of Search," *Journal of Economic Theory*, 9(2), 93–123.

Lohse, G. L. (1997): "Consumer Eye Movement Patterns on Yellow Pages Advertising," *Journal of Advertising*, 26(1), 61–73.

Madrian, B. C., and D. F. Shea (2001): "The Power of Suggestion: Inertia in 401(K) Participation and Savings Behavior," *Quarterly Journal of Economics*, 116(4), 1149–1187.

Meredith, M., and Y. Salant (2007): "The Causes and Consequences of Ballot Order-Effects," mimeo.

Perry, M., and A. Wigderson (1986): "Search in a Known Pattern," *Journal of Political Economy*, 94(1), 225–230.

Varian, H. R. (2007): "Position Auctions," *International Journal of Industrial Organization*, 25(6), 1163–1187.

Weitzman, M. L. (1979): "Optimal Search for the Best Alternative," *Econometrica*, 47(3), 641–654.

Wilson, C. (2008): "Ordered Search and Equilibrium Obfuscation," mimeo.

Wolinsky, A. (1986): "True Monopolistic Competition as a Result of Imperfect Information," *Quarterly Journal of Economics*, 101(3), 493–511.