The Value of Anonymous Option

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Abstract. Privacy regulations require that sellers obtain explicit consumer consent before collecting personal data. We offer a novel analysis of this requirement by introducing an *anonymous option* into a repeated-purchase model with limited commitment in which consumers need to be incentivized to disclose their data. We show that despite full surplus extraction through data collection, a monopolist generally benefits from offering the option, as it changes market segmentation, credibly supports a high second-period uniform price, and mitigates the ratchet effect. However, the option may reduce consumer surplus and social welfare due to higher average prices and lower aggregate demand.

Keywords: anonymous option; data disclosure; personalized pricing; privacy regulations

JEL Codes: D4, D8, L1

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1 Introduction

In repeated interactions, sellers often collect consumer data during the initial period and use it for price discrimination in subsequent periods.[1] A common practice among online platforms is to incentivize first-time consumers to create accounts through discounts while tailoring prices for repeat consumers based on their historical transaction details. Meanwhile, consumers are becoming increasingly concerned about the economic exploitation through data disclosure and potential harms from data breaches, and may act strategically to protect their privacy and resist disclosing personal data during transactions.[2]

To address widespread consumer privacy concerns, policymakers globally have implemented data protection regulations, such as the EU's General Data Protection Regulation (GDPR). These regulations require sellers to obtain explicit consumer consent before collecting and processing their personal data. In compliance, many sellers now offer an anonymous option, allowing consumers to choose between opting in for data collection or opting out to remain anonymous during transactions. For example, Amazon permits purchases through guest accounts, and Taobao offers features like "buying anonymously" and "third-party payment" to safeguard consumer privacy. When consumers opt out, sellers are prevented from tracking consumer information, including their purchase and search history, payment details, etc. Without such information, firms may not be able to learn and predict consumer behavior and target their prices accordingly.[3]

This paper contributes to the ongoing debate on the effect of privacy regulations by building a two-period model in which a monopolist under limited commitment interacts with consumers repeatedly and may offer an *anonymous option* allowing the consumers to keep their anonymity in the initial period. We offer the key insight that the anonymous option modifies the second-period market segmentation and provides the seller a commitment device which credibly sustains a high second-period uniform price and mitigates consumers' ratcheting incentives which are usually harmful to the seller.

In the model, consumers are fully aware that their first-period choices signal their valuations and the information will be used by the seller for price discrimination in the second period, while

[1] See, e.g., Villas-Boas (1999, 2004), Taylor (2004), Acquisti, et al. (2016) for foundational work on such dynamics.
[2] Aridor, et al. (2023) provide rich evidence that consumers value their privacy.
[3] Goldberg, et al. (2024) evaluate the change in EU user website page views and website revenues using data from Adobe's website analytics platform after the GDPR's enforcement deadline.

the seller takes consumers' strategic choices fully into account when making pricing decisions in the two periods. Anticipating the usage of data for price discrimination makes the consumers unwilling to disclose such information. Moreover, consumers inherently dislike sharing their data (for fear of data breach, for example) and incur a direct utility loss, measured by $K \geq 0$, when they opt into disclosure.[4] As a result, consumers withhold their data unless adequately compensated in the initial period and the ratcheting forces determine the level of indirect compensation consumers demand for disclosing data.[5]

In the baseline analysis, we compare the regime without and with anonymous option assuming that K is publicly observable. This allows us to illustrate the effect of anonymous option in a crystal way. Without anonymous option, the seller operates either in the *no-disclosure mode*, selling without collecting any consumer data, or in the *disclosure mode*, selling only to consumers who disclose their data. Under the no-disclosure mode, the model resembles repeated static monopoly problem, with the seller charging the uniform static monopoly price in each period. Under the disclosure mode, consumers purchase and disclose in the first period if their valuations are sufficiently high. This divides the second-period market into two segments: a recognized segment of high-valuation consumers, whose consumer surplus is fully extracted, and an unrecognized segment of consumers whose valuations are known to be below a threshold. The seller caters to the unrecognized segment by a relatively low uniform price at $t = 2$. This leads to ratcheting incentives among consumers: anticipating this lower price and accounting for the utility loss from disclosure, some consumers strategically delay their first-period consumption, even if they receive a positive utility from making a purchase at $t = 1$.

In the regime with anonymous option, the seller offers consumers a price menu of three choices: the *anonymous option*, the *disclosure option*, and *not purchasing*. The menu provides the consumers the freedom of purchasing the product without disclosing data, reflecting the intention of privacy regulations. Note that if the anonymous and disclosure options specify the same price,

[4]We follow the literature (eg. Lin, 2022) that consumers exhibit two-dimensional privacy concerns: *instrumental* and *intrinsic* privacy concerns. The former is captured by consumers' strategic response to the anticipation that their data will be used for personalized pricing, while the latter is captured by the disutility K when disclosing data.

[5]In our model, consumer data is generated endogenously through the strategic interaction between the seller and consumers over the two periods. This distinguishes our paper from most existing works which assume that the seller has data as an endowment (Rhodes and Zhou, 2024a), or obtains data from intermediaries (Ichihashi, 2021) or can pay consumers directly for their data (e.g. Choe, et al., 2025). Consumer data also arises endogenously in Argenziano and Bonatti (2024), like in our paper, but their focus is on the formation of data linkage.

all consumers will choose the anonymous option, as it spares them both the intrinsic utility loss from disclosure and the instrumental utility loss from future price discrimination. Therefore, for the consumers to opt in, it must provide them with a (weakly) higher utility than both the anonymous option and not purchasing.

We show that the regime with anonymous option has a unique seller-optimal partial-disclosure equilibrium when K is small. In this equilibrium, high-valuation consumers choose the anonymous option, intermediate-valuation consumers choose the disclosure option, and low-valuation consumers refrain from purchasing at $t = 1$. It follows that the second-period market segmentation differs drastically from the regime without anonymous option: the recognized segment now consists of intermediate-valuation consumers who opted in to disclose data, and the unrecognized segment consists of high-valuation consumers who chose the anonymous option and low-valuation consumers who did not purchase at $t = 1$. Because high- and low-valuation consumers are pooled in the unrecognized segment, the seller optimally sets a high uniform price to maximize second-period profit.

Since data disclosure enables maximum surplus extraction at $t = 2$, one might expect that offering the anonymous option would lower the seller's profit. Surprisingly, the opposite is true. Anonymous option affects the seller's profit through three channels: a *strategic delay effect* which captures the consumers' incentives to delay initial consumption anticipating a lower future price, a *privacy cost effect* which captures consumers' intrinsic utility loss from disclosing data, and a *surplus extraction effect* which captures the benefits the seller receives from implementing personalized pricing among recognized consumers at $t = 2$. In the regime without anonymous option, the positive surplus extraction effect dominates the first two when K is small, making the disclosure mode optimal for the seller. In the regime with anonymous option, because high- and low-valuation consumers are pooled together in the unrecognized segment, the seller has no incentive to lower the price to cater to low-valuation consumers, and this eliminates the consumers' ratcheting incentives. When K is small, the anonymous option helps the seller to avoid the strategic delay effect and reduce the privacy cost without significantly reducing surplus extraction from the recognized consumers, and as a result the seller's profit is strictly higher than in the regime without anonymous option.

Does the anonymous option reduce data generation? Our analysis indicates that with the anonymous option, more consumers disclose their data when K is moderate. Without the anonymous option, the seller chooses the no-disclosure mode and sell the product to the consumers without collecting any data, while with the anonymous option some consumers opt in and disclose their data in the partial-disclosure equilibrium.

Does the anonymous option increase consumer surplus and social welfare? Not necessarily. Consumers are prone to be harmed by the anonymous option. In the partial-disclosure equilibrium, high-valuation consumers benefit from the option by receiving a positive surplus at $t = 2$ and avoiding the intrinsic utility loss associated with disclosure. However, their anonymity raises the uniform price for unrecognized consumers at $t = 2$, imposing a negative externality on low- and intermediate valuation consumers. These consumers can no longer take advantage of a lower second-period price by strategically withholding their first-period purchases. As a result, the anonymous option can reduce overall consumer surplus by inducing higher average prices and lower aggregate demand relative to the regime without the option. Since it affects the seller's profit and consumer surplus in opposite directions, the anonymous option can increase or decrease social welfare, depending on the change in aggregate demand and privacy loss.

Our analysis provides rich policy implications. First, mandating the adoption of anonymous option may not be necessary. A monopolist seller interacting repeatedly with consumers should be willing to offer such an option voluntarily. Second, the good intention of privacy regulation may achieve an adverse outcome: consumers face higher average prices after the option is in place, and they are more likely being harmed rather than being protected by the regulation. Moreover, the partial-disclosure equilibrium provides a compelling explanation for the "privacy paradox"[6]: while some consumers express concerns about privacy, they still choose to disclose personal data even when anonymity is an option. Anticipating a high future uniform price that exceeds their valuations, consumers gain no advantage from strategically withholding their initial purchases or concealing their identities. Instead, when offered a sufficiently large discount upfront, they weigh the benefits of immediate consumption against the potential privacy loss and opt in to disclose if the upfront discounts are lucrative.

We then examine a scenario of *partial anonymity* where the seller can track consumers' purchasing histories even if they choose the anonymous option. This alters the market segmentation

[6]See Norberg, et al. (2007) for a discussion of the privacy paradox.

in both regimes in the baseline analysis. Without anonymous option, the interaction between the seller and consumers under the no-disclosure mode resembles that in classic behavior-based pricing models. In the anonymous option regime, the second-period market is divided into three segments: high-valuation consumers who chose the anonymous option, intermediate-valuation consumers who chose the disclosure option, and low-valuation consumers who refrained from purchasing. At $t = 2$, since it is optimal for the seller to set different uniform prices for the low- and high-valuation segments, the strategic delay effect cannot be eliminated as in the baseline case of *full anonymity*. As a result, the anonymous option benefits the seller for a narrower range of parameters. Moreover, the seller's profit under partial anonymity is strictly lower than that under full anonymity, further highlighting the importance of the commitment effect that can be achieved under full anonymity.

Finally, we consider an extension where a consumer possesses private information about both her valuation (v_i) and privacy type (K_i). In the unique partial-disclosure equilibrium, consumers with intermediate v_i and low K_i disclose their data, while others either purchase anonymously or refrain from purchasing at $t = 1$. A second-period uniform price higher than the first-period prices is uniquely sustained in equilibrium, and no consumers strategically delay their consumptions to the second period. Reinforcing the main results of the baseline analysis, the seller is strictly better off, while both consumer surplus and social welfare decline with the anonymous option.

The remainder of this section connects our work to the existing literature. Section 2 introduces the model setup. Section 3 derives the equilibrium in the benchmark regime without the anonymous option. Section 4 fully characterizes the equilibrium outcome in the anonymous option regime. Section 5 compares the two regimes to draw welfare implications, provides an illustration through a uniform distribution example, and discusses the scenario of partial anonymity. Section 6 examines the extension of unobservable privacy types. Concluding remarks are in Section 7. Proofs for Propositions 1 and 4 and Lemma 3-4 are substantiated in the main text. All the remaining proofs are relegated to the Appendix.

Related Literature. Our paper contributes to two strands of literature: privacy regulation and behavior-based pricing with the ratcheting effect as the core issue.

The existing literature on privacy regulation largely focuses on information externalities generated by a consumer's privacy decision, as information revealed by one consumer can be used to predict the behavior of another consumer. Choi, et al. (2019) study the data externalities among consumers with correlated preferences under different privacy rules. Bergemann, et al. (2022) show that the social dimension of the individual data generates a data externality that can reduce the intermediary's cost of acquiring the information. Rhodes and Zhou (2024b) study how personalization interacts with consumers' privacy choices when a privacy-choice externality exists across consumers. Aridor, et al. (2023) provide empirical evidence that privacy-conscious consumers exert privacy externalities on opt-in consumers, making them more predictable. Our model offers a diagonal analysis of privacy regulation in a repeated-purchase model under limited commitment, where the core economic force is a commitment effect which eliminates the consumers' ratcheting incentives. An externality across consumers occurs through a price effect: opting-out consumers change the composition of the unrecognized market segment, which raises the second-period uniform price and hurts the consumers who do not purchase and who disclose data at $t = 1$.[7]

Our setup builds on and modifies behavior-based pricing (BBP) models to accommodate the consent requirement of the privacy regulations. The BBP literature focuses on the relationship between sharing of personal data and dynamic pricing, and explores the trade-off between consumer recognition and price discrimination in repeated interactions, emphasizing the key insight that, owing to the ratcheting effect, a monopolist will never find it optimal to condition prices on consumers' purchasing histories. For comprehensive reviews, see Acquisti, et al. (2016) and Goldfarb and Tucker (2019).[8] Driven by instrumental privacy concerns, consumers can also actively manage their privacy using modern technologies. In Conitzer, et al. (2012) and Belleflamme and Vergote (2016), the seller and the consumers play a hide-and-seek game in which consumers can hide their identities at a cost. Dengler and Prüfer (2021) establish a microfoundation for consumer's privacy choices when the seller and the consumers have different levels of sophistication. In Ke and Sudhir (2023), consumers can erase their data after learning their valuations for personalized goods in the second period, and thus an anonymous option offered in the first period is never

[7] A couple of papers examine the optimal consumer information structure. In a multi-product and Bayesian persuasion setting, Ichihashi (2020) demonstrates that consumers may benefit from pre-committing to withhold certain information when purchasing from a multi-product monopolist. Ali, et al. (2023) show that richer and more sophisticated information disclosure can benefit consumers.

[8] Classic contributions include Taylor (2004), Acquisti and Varian (2005), Fudenberg and Villas-Boas (2006) and Calzolari and Pavan (2006). For more recent contributions refer to Li and Zhang (2025).

used in equilibrium.[9] Lagerlöf (2023) examines the social efficiency of consumer hiding behavior when a monopolist sets prices based on purchasing histories. Argenziano and Bonatti (2023, 2024) analyze a data market where consumers distort their purchases from a data-collecting firm to manipulate a data-using firm's beliefs about their willingness to pay. We contribute to the BBP literature by offering a fresh, nuanced perspective on the ratchet effect. Our model clarifies the role of the anonymous option and refines the insights from the BBP literature by embedding the classic models in the regime of no anonymous options.

Finally, our model also relates to the more broad dynamic pricing literature under limited commitment. In a sequential screening framework, Courty and Hao (2000) show that optimal mechanisms depend on the informativeness of consumers' initial knowledge about their valuations. Skreta (2006) demonstrates that posting a price in each period is a revenue-maximizing allocation mechanism in a finite-period model without commitment. Doval and Skreta (2022) examine the seller's trade-off between revealing consumer information through product line choices and extracting rent via second-period price discrimination, showing that limiting the number of product lines in period 1 allows the seller to commit to not learning consumer information. In our analysis, the seller benefits from offering consumers an anonymous option in the initial period, which acts as a credible commitment to a high second-period uniform price, suggesting that the optimal mechanism may involve partial data disclosure in a repeated-purchase setting under limited commitment.

2 The Model

We consider a two-period model where a monopoly seller produces and sells a non-durable good to a unit mass of consumers in each period. A consumer demands at most one unit of the product in each period. A consumer's private valuation for the product, $v_i \in [0, \bar{v}]$, follows a distribution $F(x)$ with density $f(x)$ and does not change across periods. We assume that $F(x)$ is continuous, three-

[9] Cong and Matsushima (2023) study the welfare effects of consumers' withdrawal of personal data in a competitive environment.

times differentiable and weakly convex.[10] Production has zero fixed cost and constant marginal cost which is normalized to zero. There is no discounting.[11]

The seller possesses a technology that reveals to him a consumer's valuation (v_i) if she allows the seller to track her personal data. A consumer suffers a direct utility loss $K \in [0, \bar{v})$ when she discloses data, which captures her intrinsic privacy concerns. At $t = 1$, all consumers are anonymous to the seller and the seller offers the same menu to all consumers. At $t = 2$, the seller posts second-period prices using his updated information about consumers' valuations. The seller has limited commitment power, and the prices he chooses in each period must be time-consistent. Thus, the second-period prices are discriminatory if the seller has learned some consumers' valuations through their first-period choices and data disclosure.

The timing of the game is as follows:

1. At $t = 1$, the seller offers a menu $M \equiv \{(A, p_a), (D, p_d), (N, 0)\}$, and consumers make their purchase decisions after observing the menu. A consumer pays price p_a and keeps her anonymity if she chooses A (the *anonymous option*), pays p_d and discloses her data if she chooses D (the *disclosure option*), and pays 0 and keeps her anonymity if she chooses N (*not purchasing*).

2. At $t = 2$, the seller learns the valuations of the consumers who chose D at $t = 1$ and posts personalized prices $p_{2i} = v_i$ to these recognized consumers. The seller posts a uniform price p_{2u} to unrecognized consumers who chose either A or N at $t = 1$. Consumers make their second-period purchasing decisions after observing the prices.

A consumer is fully rational and maximizes her expected utility when making purchase decisions in each period. At $t = 2$, the seller posts personalized prices to consumers who disclosed their data at $t = 1$. Anticipating this, consumers strategically adjust their first-period purchasing behavior, reflecting their instrumental privacy concern — their concern about the seller's strategic use of personal data for price discrimination.

[10]The assumption of weak convexity ensures that it is not optimal for the seller to set a very low first-period price to induce data disclosure from all consumers. Examples with this property include power distributions ($F(x) = ax^b$, $a > 0$ and $b \geq 1$), Beta distributions ($f(x; \alpha, \beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1} du}$, $\alpha \geq 1$ and $\beta \leq 1$). The assumption of weak convexity is not a necessary condition for our main results as shown in the uniform distribution example. A discussion on this can be found in footnote 18.

[11]Incorporating discounting does not alter our main results.

We focus on the case of *full anonymity*: consumers who choose A and N are not distinguishable to the seller in the subsequent period.[12] Thus, the consumers' first-period choices divide the second-period market into two segments: a *recognized segment* consisting of consumers who chose D and an *unrecognized segment* consisting of consumers who chose A and N at $t = 1$. Note that no consumer chooses D if $p_d > p_a$ because a consumer suffers triple losses from data disclosure: (i) she pays a higher price for the same product at $t = 1$, (ii) she suffers a privacy loss K, and (iii) her consumer surplus will be fully extracted at $t = 2$ when she makes a repeat purchase. Thus, a consumer needs to be compensated through a lower p_d for her to disclose data to the seller. Without loss of generality, we focus on price menus with $p_d \leq p_a$ in the subsequent analysis.

Any menu M posted by the seller at $t = 1$ starts a proper subgame. Thus, the equilibrium concept is Subgame Perfect Equilibrium (SPE). In the following, we first solve for the equilibrium in the benchmark case without anonymous option. We then proceed to the main case that the seller includes an anonymous option in the first-period price menu. When there are multiple equilibria, we focus on those that maximize the seller's payoff.

3 Benchmark without Anonymous Option

Without anonymous option, the seller chooses between the *disclosure mode* and the *no-disclosure mode*. Under the no-disclosure mode, the seller offers $\{(A, p_a), (N, 0)\}$ and does not collect any consumer data at $t = 1$. Under the disclosure mode, the seller offers $\{(D, p_d), (N, 0)\}$, and a consumer has to disclose her data if she makes a purchase at $t = 1$.

No-Disclosure Mode. Under the *no-disclosure* mode, no consumer data is collected at $t = 1$. All consumers remain anonymous at $t = 2$, and the second-period market is identical to the first-period market. The game degenerates into a repeated static monopoly problem, and it is optimal for the seller to post the static monopoly price which maximizes his per-period profit:

$$\tilde{p} \equiv \arg\max_{p} \pi(p) = \arg\max_{p} \left(1 - F(p)\right)p.$$

[12] In subsection 5.2, we consider a case of partial anonymity where the seller can keep a record of the consumers' purchasing histories and can thus distinguish consumers who chose A and N at $t = 1$.

The weak convexity of $F(x)$ ensures that $g(x) \equiv \frac{1-F(x)}{f(x)}$ is well-defined, differentiable, and strictly decreasing. Thus, the static monopoly price $\tilde{p} \in (0, \bar{v})$ is uniquely defined by

$$\tilde{p} = g(\tilde{p}). \tag{1}$$

The seller's profit under the no-disclosure mode is given by $2\tilde{\pi} \equiv 2\pi(\tilde{p}) = 2\Big(1 - F(\tilde{p})\Big)\tilde{p}$, which provides a lower bound on the seller's total expected profit if the seller collects some consumer data at $t = 1$. We summarize the outcome of the no-disclosure mode in Remark 1.

Remark 1. *Under the no-disclosure mode, the seller chooses the static monopoly price in each period,* $p_a^{nd} = p_{2u}^{nd} = \tilde{p}$. *Consumers with* $v_i \in [\tilde{p}, \bar{v}]$ *purchase the product in both periods and consumers with* $v_i \in [0, \tilde{p})$ *do not consume in either period. The seller's profit is* $\Pi^{nd} = 2\tilde{\pi}$ *and the total consumer surplus is* $CS^{nd} = 2\int_{\tilde{p}}^{\bar{v}} (v - \tilde{p})\mathrm{d}F(v)$.

Disclosure Mode. Under the *disclosure mode*, the seller offers $\{(D, p_d), (N, 0)\}$. A consumer pays price p_d and discloses her data if she makes a purchase at $t = 1$. In the subsequent period, she faces a personalized price $p_{2i} = v_i$ when she makes a repeat purchase. If she does not purchase at $t = 1$, the consumer remains anonymous and faces a uniform price p_{2u} at $t = 2$.

Observing the first-period menu, a consumer makes a purchase if and only if $v_i \geq \hat{v}$, where $\hat{v}$ is the valuation of the marginal consumer who is indifferent between purchasing and not purchasing at $t = 1$. Following this, $\hat{v}$ divides the second-period market into two segments: a recognized segment where consumers' valuations satisfy $v_i \in [\hat{v}, \bar{v}]$ and an unrecognized segment where consumers' valuations satisfy $v_i \in [0, \hat{v})$.

At $t = 2$, the seller optimally posts personalized prices ($p_{2i} = v_i$) to consumers in the recognized segment. Meanwhile, the seller chooses the second-period uniform price to maximize his profit from the unrecognized segment, $\pi_{2u}(p) = \Big(F(\hat{v}) - F(p)\Big)p$, requiring $p_{2u} = \frac{F(\hat{v}) - F(p_{2u})}{f(p_{2u})}$. Let $\phi(x) \equiv f(x)x + F(x)$ for $x \in [0, \bar{v}]$. By the weak convexity of $F(x)$, $\phi(x)$ is monotonically increasing in x. Then the second-period optimal uniform price p_{2u} depends on $\hat{v}$ in the following way,

$$\phi(p_{2u}) = F(\hat{v}) \Leftrightarrow p_{2u}(\hat{v}) = \phi^{-1}(F(\hat{v})). \tag{2}$$

A consumer rationally anticipates that her surplus will be fully extracted at $t = 2$ if she chooses D at $t = 1$. However, if she does not purchase at $t = 1$ and remains anonymous, she avoids the direct utility loss K, and faces the uniform price p_{2u} which may bring her a positive surplus at $t = 2$. Thus, a consumer chooses D at $t = 1$ if and only if the first-period price p_d is sufficiently low. Note that if $p_d + K \geq \tilde{p}$, $\hat{v} = \bar{v}$ and no consumer chooses D at $t = 1$, while if $p_d + K \leq 0$, $\hat{v} = 0$ and all consumers would choose D at $t = 1$. For the seller, a menu with either $p_d + K \geq \tilde{p}$ or $p_d + K \leq 0$ is dominated.

Given $0 < p_d + K < \tilde{p}$, a marginal consumer is indifferent between purchasing at price p_d at $t = 1$ (and facing $p_{2i} = v_i$ at $t = 2$) and not purchasing at $t = 1$ (but facing price p_{2u} at $t = 2$). Thus, $\hat{v}$ satisfies

$$\hat{v} - p_d - K + \max\{\hat{v} - p_{2i}, 0\} = \max\{\hat{v} - p_{2u}, 0\}, \tag{3}$$

leading to $p_{2u} = p_d + K$. Combining this with (2), we can uniquely define $\hat{v}(p_d)$ by

$$F(\hat{v}(p_d)) = \phi(p_d + K) \in (0,1) \Leftrightarrow \hat{v}(p_d) = F^{-1}(\phi(p_d + K)) \in (0, \bar{v}). \tag{4}$$

At $t = 1$, the seller chooses $p_d \in (-K, \tilde{p} - K)$ to maximize his total expected profit:

$$\Pi^d(p_d) = \Big(1 - F(\hat{v}(p_d))\Big)p_d + \int_{\hat{v}(p_d)}^{\bar{v}} v \mathrm{d}F(v) + \Big(F(\hat{v}(p_d)) - F(p_d + K)\Big)(p_d + K), \tag{5}$$

in which the three terms on the right-hand-side are respectively the seller's profit from selling to a mass of $1 - F(\hat{v}(p_d))$ consumers at price p_d at $t = 1$, from selling to the recognized consumers at their valuations at $t = 2$, and from selling at the uniform price $p_{2u} = p_d + K$ to the unrecognized consumers at $t = 2$. The optimal first-period price $\tilde{p}_d$ is thus determined by

$$1 - \phi(\tilde{p}_d + K) - (F^{-1}(\phi(\tilde{p}_d + K)) - K)\phi'(\tilde{p}_d + K) = 0, \tag{6}$$

leading to $\Pi^d(\tilde{p}_d)$, seller profit under the disclosure mode. Remark 2 characterizes the equilibrium outcome under the disclosure mode.

Remark 2. *Under the disclosure mode, the seller posts* $\{(D, \tilde{p}_d), (N, 0)\}$ *with* $\tilde{p}_d$ *in* (6) *at* $t = 1$*; consumers with* $v_i \in [\hat{v}(\tilde{p}_d), \bar{v}]$ *purchase and disclose their data, and consumers with* $v_i \in [0, \hat{v}(\tilde{p}_d))$ *do not purchase* *at $t = 1$. At $t = 2$, the seller posts $\tilde{p}_{2u} = \tilde{p}_d + K$ to the unrecognized consumers and charges $p_{2i} = v_i$ to recognized consumers. The seller's profit and the consumer surplus are, respectively, $\Pi^d(\tilde{p}_d)$ and $CS^d(\tilde{p}_d) = \int_{\tilde{p}_d+K}^{\bar{v}}(v - \tilde{p}_d - K)\mathrm{d}F(v)$.*

3.1 Equilibrium in the Benchmark

Without anonymous option, the seller chooses the disclosure mode over the no-disclosure mode if and only if collecting some consumer data brings a strictly higher profit. Intuitively, when consumers' intrinsic privacy loss K is low, the seller does not need to significantly sacrifice first-period profit by lowering the first-period price to incentivize a sufficient mass of consumers to disclose their personal data. In such cases, the disclosure mode is more profitable for the seller.

Proposition 1. *Without anonymous option, there exists a $K_o \in [0, \tilde{p})$ such that the seller chooses the no-disclosure mode over the disclosure mode iff $K \geq K_o$. The seller's profit and consumer surplus are*

$$\Pi^{NA} = \begin{cases} \Pi^d(\tilde{p}_d) & \text{if } K < K_o \\ \Pi^{nd} & \text{if } K \geq K_o \end{cases}, \qquad CS^{NA} = \begin{cases} CS^d(\tilde{p}_d) & \text{if } K < K_o \\ CS^{nd} & \text{if } K \geq K_o \end{cases}.$$

Proof of Proposition 1. The seller chooses the disclosure mode over the no-disclosure mode if and only if $\Pi^d(\tilde{p}_d) > \Pi^{nd} = 2\tilde{\pi}$. Note that $\Pi^d(\tilde{p}_d)$ is continuous and strictly decreasing in K. Remark 2 shows that the optimal $\tilde{p}_d$ satisfies $\tilde{p}_d + K < \tilde{p}$. When $K = \tilde{p}$, $\tilde{p}_d < 0$, and the seller's profit is strictly lower than $\int_0^{\bar{v}} v dF(v) \leq 2\tilde{\pi}$.

Let $K_o \in (0, \tilde{p})$ be the unique solution to $\Pi^d(\tilde{p}_d) = 2\tilde{\pi} = \Pi^{nd}$ when $\Pi^d(\tilde{p}_d)|_{K\to 0} > 2\tilde{\pi}$, and $K_o = 0$ when $\Pi^d(\tilde{p}_d)|_{K\to 0} \leq 2\tilde{\pi}$. If follows that $\Pi^{nd} \geq \Pi^d(\tilde{p}_d)$ if and only if $K \geq K_o$. □

In the repeated interaction between the seller and consumers, there are three effects affecting the seller's profit: (1) *Strategic delay effect*. If a consumer anticipates a lower second-period price than the effective first-period price she needs to pay when she signals a low valuation, she has an incentive to delay her first-period consumption. This puts a downward pressure on the effective first-period price. (2) *Privacy cost effect*, determined jointly by the number of consumers who disclose their data and the intrinsic privacy loss K. (3) *Surplus extraction effect*, determined jointly by the number of consumers who disclose their data and their individual valuations.

Under the no-disclosure mode, all three effects are absent, while the disclosure mode introduces all of them into the analysis. Under the disclosure mode, for any $K \geq 0$, $\hat{v}(p_d) > p_d + K$ and some consumers with $v_i < \hat{v}(p_d)$ strategically delay their consumption until the second period even though their valuations are above $p_d + K$, the effective first-period price. To incentivize the consumers to purchase the product and disclose their data, $p_d + K$ needs to be lower than $\tilde{p}$, the optimal per-period price under the no-disclosure mode. As a result, the strategic delay effect and the privacy cost effect hurt the seller's first-period profit. On the other hand, as $p_d + K$ gets lower, $\hat{v}(p_d)$ also gets lower. The seller's second-period profit increases both from a larger second-period demand and through full surplus extraction from the recognized segment.

When K is small, the positive surplus extraction effect dominates the negative strategic delay effect and the privacy cost effect, and the seller benefits from collecting consumer data. When K is large, the two negative effects dominate, and the seller refrains from collecting consumer data. Thus, it is optimal for the seller to choose the disclosure mode when K is small and choose the no-disclosure mode otherwise. We close this section with an example of uniform distribution which will be used throughout the paper for illustration.

3.2 An Example with Uniform Distribution

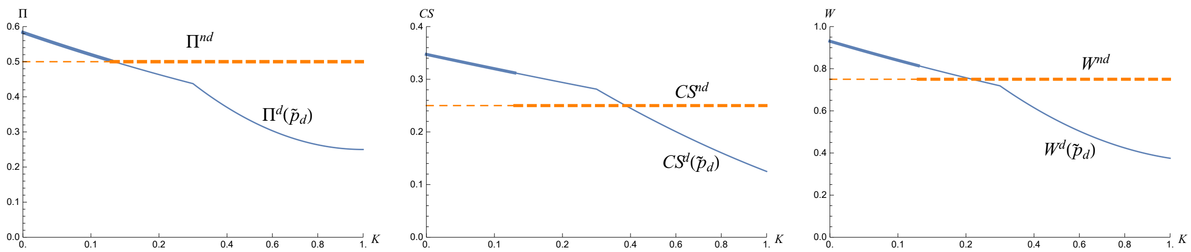
Suppose v_i is uniformly distributed on $[0,1]$ such that $\bar{v} = 1$ and $F(v) = v$. Under the no-disclosure mode, the optimal prices are $p_a^{nd} = p_{2u}^{nd} = \tilde{p} = \frac{1}{2}$. The seller's total profit and the consumer surplus are, respectively, $\Pi^{nd} = \frac{1}{2}$ and $CS^{nd} = \frac{1}{4}$.

Under the disclosure mode, the valuation of the marginal consumer in (4) becomes $\hat{v}(p_d) = 2(p_d + K)$, and the seller's profit (5) becomes

$$\Pi^d(p_d) = p_d - 3(p_d)^2 + \frac{1}{2} - 4Kp_d - K^2,$$

leading to the optimal first-period price $\tilde{p}_d = \frac{1-4K}{6}$. It follows that $\hat{v}(\tilde{p}_d) = \frac{1+2K}{3}$, and $\tilde{p}_{2u} = \frac{1+2K}{6}$. The seller profit and consumer surplus are, respectively, $\Pi^d(\tilde{p}_d) = \frac{4K^2-8K+7}{12}$ and $CS^d(\tilde{p}_d) = \frac{(5-2K)^2}{72}$.

Comparing $\Pi^d(\tilde{p}_d)$ and Π^{nd} shows that $K_o = \frac{2-\sqrt{3}}{2}$. The seller chooses the disclosure mode over the no-disclosure mode *iff* $K < \frac{2-\sqrt{3}}{2}$. Figure 1 illustrates how the seller's profit, consumer surplus, and social welfare vary with K when there is no anonymous option.



(a) Seller Profit (b) Consumer Surplus (c) Social Welfare

Figure 1: Seller profit, consumer surplus and total welfare under the disclosure mode (the solid curve) and the no-disclosure mode (the dashed curve) for $F(v) = v$ on $v \in [0,1]$. The thick part indicates the equilibrium outcome when there is no anonymous option.

4 Anonymous Option

In this section, we analyze the main case that the seller includes an anonymous option in the first-period menu, $M = \{(A, p_a), (D, p_d), (N, 0)\}$. Observing M, a consumer can choose A, purchasing the product at price p_a while maintaining anonymity, or choose D, purchasing the product at price p_d while disclosing her data, or choose N, not purchasing the product at $t = 1$. In making her first-period choice, a consumer is fully rational, anticipating a personalized price $p_{2i} = v_i$ at $t = 2$ after choosing D, and a uniform price p_{2u} at $t = 2$ if she keeps her anonymity by choosing A or N at $t = 1$. A consumer's expected utility when her valuation is v_i and she chooses option $m \in M$ at $t = 1$ is

$$u_i(v_i, m) = \begin{cases} v_i - p_a + \max\{v_i - p_{2u}, 0\} & \text{if } m = A \\ v_i - p_d - K + \max\{v_i - p_{2i}, 0\} = v_i - p_d - K & \text{if } m = D \cdot \\ \max\{v_i - p_{2u}, 0\} & \text{if } m = N \end{cases}$$

Note that a consumer with a higher valuation has a stronger incentive to choose A over D. This is because a marginal increase in the consumer's valuation increases her benefit from D only through her first-period consumption, while by choosing A, her payoff increases both from her first-period and second-period consumptions if she anticipates the second-period uniform price, p_{2u}, to be lower than her valuation v_i. In Lemma 1, we establish the ranking of consumer valuations when they make distinct choices, which is useful in subsequent analysis.

Lemma 1. *Suppose in equilibrium, some consumers with v_1, v_2, and v_3 choose, respectively, option A,*

D, and N at $t = 1$. It holds that 1) $p_{2u} \geq p_d + K$, 2) $v_1 > v_2 > v_3$.

Let v_m (respectively v_h) be the valuation of a marginal consumer who is indifferent between options N and D (respectively between options D and A).[13] Formally,

$$\max\{v_m - p_{2u}, 0\} = v_m - p_d - K, \quad v_h - p_d - K = v_h - p_a + \max\{v_h - p_{2u}, 0\}. \tag{7}$$

One can rank the (inferred) consumers' valuations on the basis of their first-period choices. By Lemma 1, if a consumer with v_m chooses N, then all consumers with $v_i < v_m$ will choose N; if a consumer with v_h chooses A, all consumers with $v_i > v_h$ will also choose A.

Note that the seller can always post a price menu that induces $v_h \geq \bar{v}$ through a prohibitively high p_a. Such an anonymous option is trivial because it has no extra effect on the interaction between the seller and the consumers relative to that under the disclosure mode in the benchmark. In the following analysis, we focus on price menus that induce $v_h < \bar{v}$ in equilibrium so that the anonymous option is nontrivial and a positive mass of consumers choose option A at $t = 1$.

4.1 Nontrivial Anonymous Option

The interaction between the seller and consumers can be analyzed as a game in which the seller chooses p_d and p_a at $t = 1$ to induce two marginal valuations, v_m and v_h, followed by the second-period uniform price p_{2u}. By sequential rationality, it is optimal for the seller to charge p_{2u} to consumers in the unrecognized segment at $t = 2$, and all players rationally anticipate the optimal p_{2u} at $t = 1$. In Lemma 2, we show that in equilibrium, the anonymous option is always nontrivial.

Lemma 2. *In the anonymous option regime, $0 < v_m \leq v_h < \bar{v}$ holds in any equilibrium.*

We prove Lemma 2 by showing that for any price menu M that induces $v_m = 0$ (or $v_h = \bar{v}$ respectively), there exists an alternative price menu with $v_m > 0$ (or $v_h < \bar{v}$, respectively) that leads to higher total profit for the seller. Thus, $v_m = 0$ and $v_h = \bar{v}$ cannot be supported in an equilibrium.

[13]Following the literature, e.g., Villas-Boas (1999), Conitzer, et al. (2012), Dengler and Prüfer (2021), we assume that when a consumer is indifferent between D and N, her choices are determined by the limit of her behavior as the discounting factor approaches one; when a consumer is indifferent between D and A, she chooses D. The first part would produce a strict preference order between D and N, and the second part is justified because the seller can always charge a personalized price marginally below a consumer's valuation. The uniqueness of v_m and v_h is further pinned down by necessary equilibrium conditions.

Lemma 2 implies that in the anonymous option regime, there is a positive mass of consumers choosing option A and N in equilibrium. However, there might be no consumers choosing option D at $t = 1$. Thus, we can classify the outcomes of all possible equilibria into two categories:

1. No-disclosure outcome ($0 < v_m = v_h < \bar{v}$). Consumers with $v_i \in [v_h, \bar{v}]$ choose option A, consumers with $v_i \in [0, v_h)$ choose option N, and no consumer chooses option D at $t = 1$.

2. Partial-disclosure outcome ($0 < v_m < v_h < \bar{v}$). Consumers with $v_i \in (v_h, \bar{v}]$ choose option A, consumers with $v_i \in [v_m, v_h]$ choose option D, and consumers with $v_i \in [0, v_m)$ choose option N at $t = 1$.

In an equilibrium with partial-disclosure outcome, $0 < v_m < v_h < \bar{v}$ holds and the second-period market is divided into two segments: an unrecognized segment with $v_i \in [0, v_m) \cup (v_h, \bar{v}]$ and a recognized segment with $v_i \in [v_m, v_h]$. Consumers with $v_i \in (v_h, \bar{v}]$ who choose A and consumers with $v_i \in [0, v_m)$ who do not purchase at $t = 1$ are pooled in the unrecognized segment and face a uniform price p_{2u} at $t = 2$. Consumers with $v_i \in [v_m, v_h]$ choose D at $t = 1$, form the recognized segment, and face personalized prices $p_{2i} = v_i$ at $t = 2$.

In an equilibrium with no-disclosure outcome, since no consumers choose D at $t = 1$, all consumers are anonymous to the seller at $t = 2$. Thus, it is optimal for the seller to post $p_{2u}^{nd} = \tilde{p}$ to all consumers. Moving backwards to $t = 1$, it is optimal for the seller to post a menu with $p_a^{nd} = \tilde{p}$. Moreover, $p_d^{nd} \geq p_a^{nd} - K$ must hold so that no consumer strictly prefers option D over A or N. The seller's total profit is the same as in the no-disclosure mode in the benchmark. We summarize this observation in Lemma 3.

Lemma 3 (No-disclosure Outcome)**.** *Suppose the seller offers a menu with an anonymous option. In an equilibrium with no-disclosure outcome, $p_a^{nd} = p_{2u}^{nd} = \tilde{p}$, $p_d^{nd} \in [\tilde{p} - K, \tilde{p}]$. Consumers with $v_i \in [\tilde{p}, \bar{v}]$ choose option A at $t = 1$ and purchase again at price p_{2u}^{nd} at $t = 2$. Consumers with $v_i \in [0, \tilde{p})$ do not purchase in either period. No consumer chooses option D at $t = 1$. The seller's profit is $\Pi^{nd} = 2\tilde{\pi}$.*

4.2 Partial-disclosure Outcome

In this section, we characterize the equilibrium with partial-disclosure outcome. We solve the game by backward induction, first characterizing the optimal p_{2u} that may occur in equilibrium (for consumers in the recognized segment the optimal price is always $p_{2i} = v_i$), then pinning down the feasible sets of v_m and v_h that can be supported in an equilibrium for a given price menu, and finally solving for the optimal first-period prices.

First, consider $t = 2$ and take (p_d, p_a) and the induced (v_m, v_h) as given. Note that it is never optimal for the seller to choose $p_{2u} \in [v_m, v_h)$ because such a price is dominated by $p_{2u} = v_h$. Thus, there are two candidates for the optimal second-period uniform price: (i) a high price with $p_{2u} \geq v_h$, with which the seller excludes consumers with $v_i \in [0, v_m)$ from purchasing at $t = 2$; or (ii) a low price with $p_{2u} < v_m$, with which the seller sells to some consumers with $v_i \in [0, v_m)$ at $t = 2$.

In case (i), the optimal price is given by $p^h_{2u} = \max\{v_h, \tilde{p}\}$, leading to the second-period profit from the unrecognized segment

$$\pi^h_{2u}(v_h) = \Big(1 - F(\max\{v_h, \tilde{p}\})\Big)\max\{v_h, \tilde{p}\}.$$

In case (ii), the optimal price solves $\max_p \Big(1 - F(v_h) + F(v_m) - F(p)\Big)p$, leading to

$$\phi(p^\ell_{2u}) = 1 - F(v_h) + F(v_m), \quad \text{and} \quad \pi^\ell_{2u}(v_m, v_h) = \Big(1 - F(v_h) + F(v_m) - F(p^\ell_{2u})\Big)p^\ell_{2u}. \tag{8}$$

Given (v_m, v_h), the low price p^ℓ_{2u} defined in the first part of (8) is supported in equilibrium if and only if $p^\ell_{2u} \in [0, v_m)$ and the seller's second-period profit is higher with p^ℓ_{2u} than with p^h_{2u}, that is, $\pi^\ell_{2u}(v_m, v_h) > \pi^h_{2u}(v_h)$. Otherwise, p^h_{2u} is supported in equilibrium. We summarize this observation in Lemma 4.

Lemma 4. *Given v_m and v_h, the optimal uniform price at $t = 2$ is $\hat{p}_{2u} = p^h_{2u} = \max\{v_h, \tilde{p}\}$ iff*

$$(i)\ p^\ell_{2u} \geq v_m, \quad \text{or} \quad (ii)\ p^\ell_{2u} < v_m \quad \text{and} \quad \pi^h_{2u}(v_h) \geq \pi^\ell_{2u}(v_m, v_h), \tag{9}$$

and the optimal uniform price is $\hat{p}_{2u} = p^\ell_{2u}$ otherwise.

Whether the uniform price at $t = 2$ is high or low has important implications: when $\hat{p}_{2u} = p^\ell_{2u}$, some consumers with $v_i \in [0, v_m)$ may choose not to purchase at $t = 1$, even if $v_i > p_d + K$, because they anticipate a lower price at $t = 2$ if they delay their purchase. In contrast, when $\hat{p}_{2u} = p^h_{2u}$, such strategic delay is not beneficial, as consumers who cannot afford the product at $t = 1$ will also find it unaffordable at $t = 2$. In sum, a low p_{2u} induces some consumers to strategically withhold their first-period consumption, while a high p_{2u} eliminates such incentives.

Moving back to $t = 1$. Given (p_d, p_a), what are the feasible marginal valuations that induce p_{2u}^h and p_{2u}^ℓ respectively? First, consider an equilibrium in which p_{2u}^h is induced. Observing p_a and p_d, anticipating that $p_{2i} = v_i$ and $p_{2u}^h = \max\{v_h, \tilde{p}\} > v_m$, a marginal consumer with v_m is indifferent between options D and N while a marginal consumer with v_h is indifferent between D and A. Making use of (7), we get

$$v_m - p_d - K = 0, \quad v_h - p_a + \max\{v_h - \max\{v_h, \tilde{p}\}, 0\} = v_h - p_d - K,$$

leading to $p_a = p_d + K = v_m$. For p_{2u}^h to be supported in equilibrium, it is necessary that the effective prices of options A and D, p_a and $p_d + K$, are the same. Moreover, $v_m = p_d + K$ and $v_h > p_a$ must hold so that no consumer finds it beneficial to strategically delay her initial consumption until the second period.

An equilibrium that supports $p_{2u}^h = v_h$ exists only when $p_a = p_d + K$.[14] In such an equilibrium, a consumer with $v_i \in [v_m, v_h]$ receives the same expected payoff from options D and A at $t = 1$. On the other hand, consumers with $v_i \in (v_h, \bar{v}]$ are strictly better off choosing option A over D, and consumers with $v_i \in [0, v_m)$ are strictly better off choosing option N over D.

Second, consider an equilibrium in which p_{2u}^ℓ is induced. At $t = 1$, consumers rationally anticipate the second-period uniform price to be $p_{2u} = p_{2u}^\ell < v_m$. The indifference conditions of marginal consumers with v_m and v_h lead to

$$v_m - p_d - K = v_m - p_{2u}^\ell \quad \text{and} \quad v_h - p_a + v_h - p_{2u}^\ell = v_h - p_d - K,$$

implying $v_h = p_a$ and $p_{2u}^\ell = p_d + K$. Thus, for p_{2u}^ℓ to be supported in equilibrium, it is necessary to have $v_h = p_a > v_m > p_d + K$.

An equilibrium with p_{2u}^ℓ exists only when $p_a > p_d + K$. In such an equilibrium, a consumer with $v_i \in [v_m, v_h]$ receives the same expected payoff from choosing options D and N. A consumer with $v_i \in (v_h, \bar{v}]$ is strictly better off choosing A over D and N. Note that consumers with $v_i \in [p_d + K, v_m)$

[14] In Lemma 5 below, we show that $v_h > \tilde{p}$ always holds in equilibrium, and thus $p_{2u}^h = \max\{v_h, \tilde{p}\} = v_h$.

strategically postpone their first purchase to $t = 2$ even though they can get a positive payoff by purchasing at $t = 1$ given $v_i > p_d + K$.

Combining the above discussions with Lemma 4, we are able to characterize the necessary conditions about (p_d, p_a) and the feasible sets of v_m and v_h so that p^h_{2u} and p^ℓ_{2u} can be, respectively, supported in a partial-disclosure equilibrium.

Lemma 5. *In a partial-disclosure equilibrium in which $\hat{p}_{2u} = p^h_{2u}$ is optimal at $t = 2$, the first-period price menu M_h satisfies $p_d = p_a - K$; moreover $v_m = p_a$ hold and $v_h \in (p_a, \bar{v})$ satisfies the following constraints*

$$\begin{aligned} &(i) \quad F(v_h) \le 1 - f(p_a)p_a, \quad \textit{or} \\ &(ii) \quad F(v_h) > 1 - f(p_a)p_a \quad \textit{and} \quad \big(1 - F(v_h)\big)v_h \ge \pi^\ell_{2u}(p_a, v_h). \end{aligned} \tag{10}$$

In a partial-disclosure equilibrium in which $\hat{p}_{2u} = p^\ell_{2u}$ is optimal at $t = 2$, the first-period price menu M_ℓ with (p_d, p_a) satisfies $p_a > p_d + K$; moreover, $p^\ell_{2u} = p_d + K < v_m$ and $v_h = p_a$ hold, and v_m satisfies the following constraints

$$F(p_a) > 1 - f(v_m)v_m, \quad \textit{and} \quad \big(1 - F(p_a)\big)p_a < \pi^\ell_{2u}(v_m, p_a). \tag{11}$$

Lemma 5 provides the conditions under which p^h_{2u} or p^ℓ_{2u} can be supported in a partial-disclosure equilibrium. In particular, given a price menu with $p_a = p_d + K$, p^h_{2u} can be supported in equilibrium only if $v_m = p_a$ hold and the feasible set of v_h is nonempty, that is, there exists a $v_h > p_a$ satisfying condition (10), in which $\pi^\ell_{2u}(p_a, v_h)$ is the seller's profit from choosing p^ℓ_{2u}. On the other hand, given a price menu with $p_a > p_d + K$, p^ℓ_{2u} can be supported in equilibrium only if $p^\ell_{2u} = p_d + K$ and $v_h = p_a$ hold and there exists a $v_m > p_d + K$ satisfying condition (11), in which $\pi^\ell_{2u}(v_m, p_a)$ gives the associated seller's profit from choosing p^ℓ_{2u}.

The next step is to solve for the first-period price menu and fully characterize the partial-disclosure equilibrium. Before proceeding to that, we first state two results in Lemma 6 that help to simplify the analysis dramatically.

Lemma 6. *(i) There exists no subgame-perfect equilibrium in which p^ℓ_{2u} is the optimal uniform price at $t = 2$. (ii) There is a threshold $\hat{p}_a \in (\tilde{p}, \bar{v})$ such that the feasible set of v_h in condition (10) is nonempty iff*

$p_a \le \hat{p}_a$. When $p_a \le \hat{p}_a$, condition (10) is equivalent to

$$\tilde{p} \le v_h \le \bar{v}_h(p_a) \quad \text{if } p_a < \tilde{p}; \quad \underline{v}_h(p_a) \le v_h \le \bar{v}_h(p_a) \quad \text{if } \tilde{p} \le p_a \le \hat{p}_a, \tag{12}$$

where $\bar{v}_h(p_a)$ and $\underline{v}_h(p_a)$ are the solutions to $\big(1 - F(v_h)\big)v_h = \pi^{\ell}_{2u}(p_a, v_h)$.[15]

Lemma 6 (i) is established by showing that for any price menu M_ℓ that induces p^{ℓ}_{2u}, there exists an alternative price menu M_h that induces p^h_{2u} and strictly brings the seller higher expected profits. Thus, no M_ℓ can form a SPE of the whole game. This allows us to focus on price menus that induce p^h_{2u} in subsequent analysis.

Lemma 6 (ii) implies that a partial-disclosure equilibrium with p^h_{2u} exists only if the first-period price p_a is sufficiently low. This is intuitive because at $t = 2$, the unrecognized segment contains consumers with $v_i \in [0, v_m) \cup (v_h, \bar{v}]$, there must be a sufficient number of consumers with high valuations in this segment for p^h_{2u} to be optimal. Given the restriction that $v_h > p_a$ must hold, p_a cannot be very high for such an equilibrium to exist. In fact, the feasible set of v_h under condition (10) is non-empty and a partial-disclosure equilibrium with p^h_{2u} exists only if $p_a \le \hat{p}_a$.

When a partial-disclosure equilibrium exists, it will not be unique. By Lemma 5, given a price menu M_h with $p_d = p_a - K$ while inducing $v_m = p_a$ and $p^h_{2u} = v_h$, any $v_h \in (p_a, \bar{v})$ satisfying condition (10) can be supported in a partial-disclosure equilibrium. Thus, Lemma 5 and 6 together imply that when $p_a \le \hat{p}_a$, the feasible set of v_h in condition (10) is non-empty, and given the same M_h, multiple v_h can be supported in equilibrium. We focus on the one that maximizes the seller's total profit.[16]

Making use of Lemma 5 and 6, in particular $p_d = p_a - K$, $v_m = p_a$, $p^h_{2u} = v_h$, and the feasible set of v_h for given p_a, we formalize the seller's maximization problem at $t = 1$ as choosing $p_a \le \hat{p}_a$ to maximize

$$\Pi^h(p_a) = (F(v_h) - F(p_a))(p_a - K) + (1 - F(v_h))p_a + \int_{p_a}^{v_h} v\,\mathrm{d}F(v) + \big(1 - F(v_h)\big)v_h, \tag{13}$$

[15] The equation $\big(1 - F(v_h)\big)v_h = \pi^{\ell}_{2u}(p_a, v_h)$, where $v_h \in (F^{-1}(1 - f(p_a)p_a), \bar{v})$, has one unique solution when $p_a < \tilde{p}$, and two solutions when $\hat{p}_a \ge p_a \ge \tilde{p}$. We use $\underline{v}_h(p_a)$ to denote the smaller solution and $\bar{v}_h(p_a)$ to denote the larger solution or the unique solution.

[16] Given a price menu M_h with $p_a = p_d + K = v_m$, although different $v_h \in (p_a, \bar{v}]$, and thus different $p^h_{2u} = v_h$, can be supported in equilibrium, the seller can coordinate on a particular one by announcing an unbinding uniform price p^h_{2u} at $t = 1$, and such an announcement turns out to be self-sustained because p^h_{2u} is optimal at $t = 2$.

subject to condition (12), the feasible set of v_h.

Solving the above constrained maximization problem gives us the optimal p_a and the best v_h (among the multiple v_h that can be induced by price menu M_h) that can be supported in a partial-disclosure equilibrium. The first-order conditions of $\Pi^h(p_a)$ with respect to p_a and v_h are respectively

$$\frac{d\Pi^h(p_a)}{dp_a} = (-f(v_h)K + 1 - F(v_h))\frac{dv_h}{dp_a} + 1 - F(p_a) + f(p_a)K - 2p_a f(p_a) = 0, \tag{14}$$

$$\frac{d\Pi^h(p_a)}{dv_h} = 1 - F(v_h) - f(v_h)K = 0. \tag{15}$$

Define the unconstrained solutions (p_a^*, v_h^*) by

$$\frac{1 - F(v_h^*)}{f(v_h^*)} = K, \quad p_a^* = \frac{1 - F(p_a^*)}{2f(p_a^*)} + \frac{K}{2}. \tag{16}$$

If (p_a^*, v_h^*) satisfy condition (12), they naturally form the optimal solutions because v_h^* lies within the feasible set of v_h induced by p_a^* and maximizes the seller's expected profit among all v_h that can be supported in a partial-disclosure equilibrium.

If (p_a^*, v_h^*) do not satisfy (12), the inequalities in (12) become binding at the solutions. By (15), when $K \le \tilde{p}$, $\frac{d\Pi^h(p_a)}{dv_h} > 0$ at feasible v_h, implying that $v_h = \bar{v}_h(p_a)$ holds at the optimum. In this case, (14) together with $v_h = \bar{v}_h(p_a)$ yield the optimal solutions. When $K > \tilde{p}$, the feasible set of v_h defined by condition (12) is empty for a given p_a, and no partial-disclosure equilibrium exists. In Lemma 7, we establish that the seller optimally chooses $p_a \le \hat{p}_a$ if and only if $K < \tilde{p}$, which implies that the partial-disclosure outcome can be supported in equilibrium only when $K < \tilde{p}$.

Lemma 7 (Partial-disclosure Outcome)**.** *When the seller uses an anonymous option, the partial-disclosure outcome can be supported in equilibrium iff $K < \tilde{p}$.*

1. *In a partial-disclosure equilibrium, the price menu M_h satisfies $p_d = p_a - K$ and induces $v_m = p_a < v_h$ and $p_{2u} = v_h$.*

2. *There exists a threshold K_a such that when $K \in [K_a, \tilde{p})$, $(p_a, v_h) = (p_a^*, v_h^*)$ are uniquely supported in the seller-optimal partial-disclosure equilibrium.*

3. *When $K \in [0, K_a)$, p_a^{**} which is the solution to*

$$1 - F(p_a) - 2f(p_a)p_a + Kf(p_a) + \left[1 - F(\bar{v}_h(p_a)) - Kf(\bar{v}_h(p_a))\right]\frac{\mathrm{d}\bar{v}_h(p_a)}{\mathrm{d}p_a} = 0,$$

and $v_h^{**} = \bar{v}_h(p_a^{**})$ are supported in the seller-optimal partial-disclosure equilibrium.

Lemma 7 fully characterizes an equilibrium with the partial-disclosure outcome. In this equilibrium, $p_a = p_d - K$, $v_m = p_a$, and $p_{2u} = v_h$ hold, and no consumers benefit from strategically delaying their first-period consumption to the second period. Consumers with low valuations ($v_i \in [0, v_m)$) do not consume in either period, while those with high and intermediate valuations ($v_i \in [v_m, v_h] \cup (v_h, \bar{v}]$) consume twice.

Relative to the no-disclosure outcome in Lemma 3, $p_a^{**} < p_a^* < p_a^{nd} = \tilde{p}$. Two forces drive down the first-period prices to induce data disclosure: (i) prices must be sufficiently low to compensate consumers for their intrinsic privacy loss, and (ii) to sustain a high-second period uniform price that deters strategic consumption delay, prices must be low enough to ensure a sufficient number of consumers choose the anonymous option at $t = 1$ and purchase again at p_{2u} at $t = 2$. While low (p_d, p_a) reduce the seller's first-period profit relative to the no-disclosure outcome, the prospect of a high uniform price for the unrecognized consumers and perfect surplus extraction from recognized consumers increases the seller's second-period profit. However, when K is very large, the first-period profit loss is too significant, making it unprofitable for the seller to induce data disclosure, and the partial-disclosure outcome can no longer be supported in equilibrium.

4.3 SPE with Anonymous Option

By Lemma 3, the seller can achieve the no-disclosure outcome through a price menu M with $p_a^{nd} = \tilde{p}$ and $p_d^{nd} \in [\tilde{p} - K, \tilde{p}]$, while inducing $p_{2u}^{nd} = \tilde{p}$. This outcome can be supported in equilibrium for any $K \geq 0$. On the other hand, when $K < \tilde{p}$, the seller can induce a partial-disclosure outcome using a price menu M_h with $p_d = p_a - K$ and $p_a = p_a^*$ or $p_a = p_a^{**}$, as characterized in Lemma 7. In this case, $v_m = p_a < v_h$ and $p_{2u} = v_h$.

Comparison of these two outcomes determines the seller's optimal first-period price choice and, consequently, the subgame perfect equilibrium of the entire game. Since the seller's total profit is strictly higher in the partial-disclosure outcome than in the no-disclosure outcome, the game with anonymous option has a unique (seller-optimal) SPE: when $K < \tilde{p}$, the equilibrium features the partial-disclosure outcome; when $K \geq \tilde{p}$, the equilibrium features the no-disclosure outcome.

Proposition 2. *Suppose the seller adopts an anonymous option at $t = 1$.*

1. *When $K \in [0, \tilde{p})$, there exists a unique SPE with partial-disclosure outcome, in which*

$$p_a^A = p_d^A + K = v_m^A = \begin{cases} p_a^* & \text{if } K \in [K_a, \tilde{p}) \\ p_a^{**} & \text{if } K \in [0, K_a) \end{cases}, \quad v_h^A = p_{2u}^A = \begin{cases} v_h^* & \text{if } K \in [K_a, \tilde{p}) \\ \bar{v}_h(p_a^{**}) & \text{if } K \in [0, K_a) \end{cases}.$$

At $t = 1$, consumers with $v_i \in [0, v_m^A)$ do not purchase, consumers with $v_i \in [v_m^A, v_h^A]$ choose D and disclose their data, and consumers with $(v_h^A, \bar{v}]$ choose A and keep their anonymity. At $t = 2$, the seller posts p_{2u}^A to unrecognized consumers and posts $p_{2i} = v_i$ to recognized consumers.

2. *When $K \geq \tilde{p}$, there exists a unique equilibrium with no-disclosure outcome, in which*

$$p_a^A = p_{2u}^A = \tilde{p}, \quad p_d^A \in [0, \tilde{p}].$$

At $t = 1$ consumers with $v_i \in [\tilde{p}, \bar{v}]$ choose option A and consumers with $v_i \in [0, \tilde{p})$ do not purchase, and no consumers choose option D. At $t = 2$, all consumers are unrecognizable to the seller and face the uniform price $p_{2u}^A = \tilde{p}$.

Proposition 2 fully characterizes the SPE when the seller adopts an anonymous option in the first-period price menu, identifying conditions on K under which the partial-disclosure outcome emerges. In Section 5, we first compare the equilibrium outcomes across the two regimes to analyze welfare implications of the anonymous option. We then illustrate the main insights using an example with a uniform distribution. At the end of Section 5, we examine the value of the anonymous option when the seller can observe a consumer's purchasing history even if she has chosen the anonymous option.

5 Welfare Effects of Anonymous Option

From Propositions 1 and 2, when $K \geq \tilde{p}$, the seller adopts the no-disclosure mode without anonymous option; with anonymous option the equilibrium outcome also features no disclosure. Since there is no data disclosure in either regime, the seller's profit and the consumer surplus are identical with and without anonymous option.

When $K < \tilde{p}$, without anonymous option, the seller adopts the disclosure mode if $K \in [0, K_o)$ and the no-disclosure mode if $K \in [K_o, \tilde{p})$; with anonymous option, the equilibrium features the partial-disclosure outcome. Comparing the seller's profit, consumer surplus and social welfare in the two regimes leads to Proposition 3 on the welfare effects of anonymous option.

Proposition 3. *Suppose $K < \tilde{p}$ so that the equilibrium with anonymous option features the partial-disclosure outcome. (i) The seller's profit is strictly higher with anonymous option. (ii) More personal data is disclosed with anonymous option when $K \in [K_o, \tilde{p})$. (iii) There exists a non-empty set of K in $[\max\{K_o, K_a\}, \tilde{p})$ such that consumer surplus is strictly lower with anonymous option.*[17] *(iv) There exists a threshold K_w such that social welfare is strictly lower with anonymous option when $K \in (K_w, \tilde{p})$.*

Recall the three effects that affect the seller's profit in the repeated interaction: the strategic delay effect, the privacy cost effect, and the surplus extraction effect. These forces collectively determine the profitability of an anonymous option to the seller.

When $K < K_o$, the equilibrium in the absence of an anonymous option is characterized by the disclosure mode, where all three effects are active. By introducing an anonymous option, the seller reshapes the second-period market segmentation, enabling a high uniform price to be sustained for the unrecognized segment at $t = 2$. This eliminates consumers' incentives for strategic delay. Moreover, when high-valuation consumers opt for option A instead of D, they avoid incurring the privacy loss K as well as full surplus extraction at $t = 2$. On balance, the positive benefits of eliminating the strategic delay effect and lowering privacy costs outweigh the negative impact of less surplus extraction from high-valuation consumers. Consequently, the anonymous option strictly enhances the seller's profit compared to the scenario without it.

When $K \in [K_o, \tilde{p})$, the equilibrium without an anonymous option features the no-disclosure mode. In this case, there is no data disclosure, no strategic delay in consumption, and no surplus extraction via personalized pricing at $t = 2$. The anonymous option boosts the seller's profit by permitting limited data disclosure and personalized pricing, while avoiding the adverse strategic delay effect and not incurring too much privacy cost for consumers.

[17] There exist sufficient conditions under which the consumer surplus is always lower with anonymous option. Instead of presenting the complicated conditions here, we illustrate such a case with the uniform example below.

In contrast to the seller, who invariably benefits from the anonymous option, consumers are often adversely affected. When the pricing regime shifts from no anonymous option to one with an anonymous option, high-valuation consumers benefit at $t = 2$, but they impose a negative externality on others through higher prices. Without the option, consumers with lower valuations can signal their low willingness to pay by abstaining from purchasing at $t = 1$, thereby securing the product at a reduced price at $t = 2$. However, the anonymous option disrupts this mechanism: the anonymity of high-valuation consumers leads to a higher second-period uniform price, rendering the product unaffordable for others. Moreover, the equilibrium first-period prices for all consumers are increased by the option. Although market demand expands in the first period, high-valuation consumers benefit only in $t = 2$ at the expense of their lower-valuation counterparts. As a result, aggregate consumer surplus often declines with the adoption of the anonymous option.

Proposition 3(iv) demonstrates that the anonymous option strictly reduces social welfare for intermediate values of K. Since the anonymous option affects the seller's profit and consumer surplus in opposite directions, its net impact on social welfare depends on the interplay between changes in aggregate demand and the total privacy cost. The latter is jointly determined by the number of consumers disclosing data and the magnitude of K. When the benchmark equilibrium involves disclosure, the introduction of the anonymous option tends to reduce social welfare due to lower aggregate demand. Conversely, when the benchmark equilibrium features no disclosure and K is not excessively large, the anonymous option can increase aggregate demand. This effect of demand expansion may outweigh the negative impact of increased intrinsic privacy loss, resulting in a net benefit. As K increases further, these welfare gains diminish. In the uniform example presented in Section 5.1, we show that the change in social welfare follows an inverted U-shape as the pricing regime shifts from no anonymous option to one with an anonymous option.

Recent global privacy regulations mandate that consumers must consent before their data can be collected, primarily with the aim of protecting consumer privacy and social welfare. In a repeated interaction setting, where a monopolist incentivizes data disclosure through initial discounts and subsequently exploits the data for surplus extraction, as analyzed above, the seller has an inherent incentive to obtain such consent by offering an anonymous option to screen the consumers. Thus, mandating this requirement is redundant for the seller. However, such regulations may prove counterproductive in protecting consumer welfare, as consumers are often harmed by the anonymous option.

5.1 Example with Uniform Distribution

We revisit the example in Section 3.2 in which a consumer's valuation v_i is uniformly distributed on $[0,1]$ such that $\bar{v}=1$, $F(v)=v$, and $\tilde{p}=\frac{1}{2}$. In the regime of anonymous option, by Proposition 2, when $K\in[0,\frac{1}{2})$, the unique equilibrium features partial disclosure. By Lemma 7, $K_a=\frac{1}{26}$.

When $K\in[\frac{1}{26},\frac{1}{2})$, the optimal first-period prices are $p_a^A=p_a^*=\frac{1+K}{3}$ and $p_d^A=\frac{1-2K}{3}$; the induced valuations of the marginal consumers are $v_h^A=1-K>v_m^A=\frac{1+K}{3}$; the second-period uniform price for the unrecognized segment is $p_{2u}^A=v_h^A=1-K$. The seller's total profit is $\Pi^A=\frac{2}{3}(1-K+K^2)$.

When $K\in[0,\frac{1}{26})$, $p_a^A=p_a^{**}<\frac{1+K}{3}$ in which p_a^{**} is the unique solution to

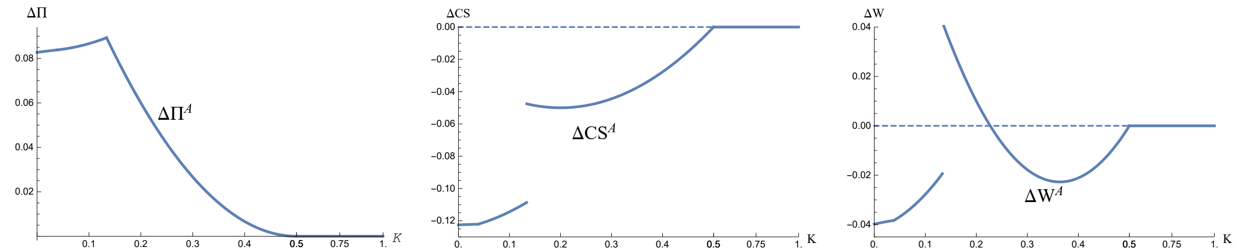
$$(1+K-3p_a)+\Big(1-K-\bar{v}_h(p_a)\Big)\frac{1}{5}\Bigg(1-\frac{1+2p_a}{\sqrt{1-p_a-p_a^2}}\Bigg)=0,$$

and $p_d^A=p_a^{**}-K$, $v_h^A=\bar{v}_h(p_a^{**})=\frac{3+p_a^{**}+2\sqrt{1-p_a^{**}-p_a^{**2}}}{5}>v_m^A=p_a^{**}$, $p_{2u}^A=v_h^A=\bar{v}_h(p_a^{**})$. The seller's profit is given by $\Pi^A=-\frac{1}{2}\bar{v}_h(p_a^{**})^2+(1-K)\bar{v}_h(p_a^{**})+(1+K)p_a^{**}-\frac{3}{2}p_a^{**2}$.

When $K\geq\frac{1}{2}$, the unique equilibrium features no disclosure. In this case, $p_a^A=\frac{1}{2}$, $p_d^A\in[0,\frac{1}{2}]$, and $p_{2u}^A=\frac{1}{2}$. Consumers with $v_i\in[\frac{1}{2},1]$ choose A, and consumers with $v_i\in[0,\frac{1}{2})$ choose N at $t=1$. The seller's profit is $\Pi^A=\Pi^{nd}=\frac{1}{2}$.

Figure 2 illustrates the change in the seller's profit, consumer surplus, and total welfare when the pricing regime moves from no anonymous option to one with anonymous option. For all K, the seller is better off but consumers are worse off. Since $\Delta\Pi^A$ and ΔCS^A take different signs, the change in social welfare is non-monotonic: $\Delta W^A>0$ for some intermediate K.[18]

[18] Since the seller's incremental profit, $\Delta\Pi^A=\Pi^A-\Pi^{NA}$, is strictly positive with the anonymous option under uniform distribution, it will not be difficult to find examples where the distribution of v_i violates weak convexity and the results in Propositions 1-3 hold.



(a) $\Delta\Pi^A$ (b) ΔCS^A (c) ΔW^A

Figure 2: Welfare Effects of Anonymous Option, for $\bar{v} = 1$ and $F(v) = v$

Discussion of $K = 0$. A positive K is not essential for the anonymous option to be beneficial for the seller. When $K = 0$, the seller chooses the disclosure mode in the regime without anonymous option. In the partial-disclosure equilibrium with anonymous option, $p_a^A = p_d^A = 0.33$, following which $v_m^A = 0.33 < v_h^A = 0.96$, and $p_{2u}^A = v_h^A = 0.96$.

The seller's total profit is $\Pi^A = 0.66$, which is 13.14% higher than the profit in the no anonymous option regime, $\Pi^{NA} = \frac{7}{12}$. This example shows that a small mass of consumers choosing option A at $t = 1$ is sufficient to support a high price for the unrecognized segment at $t = 2$.

Discussion of $K = \frac{1}{4}$. In the benchmark without anonymous option, the seller chooses the no-disclosure mode and receives $\Pi^{NA} = \frac{1}{2}$. In the regime with anonymous option, the equilibrium is a partial-disclosure one with $p_a^A = \frac{5}{12}$, $p_d^A = \frac{1}{6}$, and following these prices, $v_m^A = \frac{5}{12} < v_h^A = \frac{3}{4} = p_{2u}^A$. The seller's equilibrium profit is $\Pi^A = \frac{13}{24}$, higher than Π^{NA} by 8.33%.

5.2 Partial Anonymity

So far we have considered the case of *full anonymity*, where consumers who choose the anonymous option are indistinguishable from those who do not purchase at $t = 1$. With the rapid development of the digital economy, many transactions occur online, with products delivered to consumers at specified addresses. In such settings, even when consumers opt for option A, the seller may still be able to keep a record of their purchasing histories and infer some information about their valuations. Consequently, the seller can distinguish between repeat purchasers and new purchasers at $t = 2$ and practice (third-degree) price discrimination accordingly. We term this scenario *partial anonymity* (PA).

Under partial anonymity, there are two key changes in the analysis: First, without anonymous option, the game under no disclosure is now identical to a behavior-based pricing model. Second, with anonymous option, the second-period market is divided into three segments, allowing the seller to practice third-degree price discrimination in addition to personalized pricing.

In the absence of an anonymous option, the game under the disclosure mode remains unchanged from the analysis in Section 3, with the equilibrium characterized as in Remark 2. Under the no-disclosure mode, when a consumer purchases through option A, she signals that her valuation is at least as high as the first-period price. Then at $t = 2$ the seller can distinguish these consumers from those who did not purchase at $t = 1$ and charge them different prices. Comparing the seller's profit under the disclosure and no-disclosure mode in the context of partial anonymity reveals an equilibrium outcome mirroring the pattern observed under full anonymity. Specifically, there exists a threshold value K_o^{PA} (where $K_o^{PA} > K_o$) such that the seller chooses the disclosure mode if and only if $K < K_o^{PA}$.

In the anonymous option regime, Lemma 1 remains valid: if some consumers with v_1, v_2 and v_3 choose options A, D, and N respectively, it can be inferred that $v_1 > v_2 > v_3$. Based on consumers' first-period choices, the second-period market is segmented into three groups (as opposed to two under full anonymity): consumers with $v_i \in [0, v_m)$ who did not purchase, consumers with $[v_m, v_h]$ who chose D and disclosed their valuations, and consumers with $v_i \in (v_h, \bar{v}]$ who chose A at $t = 1$ and kept their anonymity. At $t = 2$, the seller posts a uniform price p_{2n} for consumers who did not purchase, personalized prices $p_{2i} = v_i$ for consumers who disclosed their data, and a uniform price $p_{2r} = \max\{v_h, \tilde{p}\}$ for consumers who purchased through option A. Recall that under full anonymity it is optimal for the seller to set a single high second-period uniform price to exclude consumers who did not purchase at $t = 1$ from consumption. However, such pricing strategy is no longer time-consistent under partial anonymity. At $t = 2$, it is optimal for the seller to set two distinct uniform prices for the group that chose A and the group that chose N at $t = 1$, as these two groups are now distinguishable.

Following a similar analytical procedure as under full anonymity, we characterize the unique seller-optimal SPE under partial anonymity: (i) When $K = 0$, the equilibrium outcome coincides with that of the disclosure mode without anonymous option. (ii) When $K \in (0, K_a^{PA})$, where $K_a^{PA} \in (0, \tilde{p})$, the equilibrium outcome features partial disclosure. In this case, high-valuation consumers choose option A, consumers with intermediate valuations choose D, and consumers with low valuations choose N at $t = 1$. (iii) When $K \geq K_a^{PA}$, the equilibrium outcome is the same as that of the no-disclosure mode without anonymous option. The welfare implications under partial anonymity mirror those under full anonymity. When the equilibrium with anonymous option involves partial disclosure, the seller is strictly better off, while consumer surplus and social welfare can be strictly lowered by the anonymous option.[19]

When the seller possesses information about consumers' purchasing histories, at $t = 2$, the seller cannot avoid charging different prices to consumers who chose option A and those who chose N at $t = 1$. Consequently, it is no longer optimal for the seller to maintain a single high second-period uniform price to exclude consumers with low valuations. Under partial anonymity, the commitment channel that deters strategic delay under full anonymity is eliminated. As a result, the benefits of the anonymous option to the seller arise solely from the trade-off between the *privacy cost effect* and the *surplus extraction effect*. When $K = 0$ or $K \in (K_a^{PA}, \tilde{p})$, the seller cannot benefit from the anonymous option under partial anonymity, in contrast to the outcome under full anonymity. Moreover, for $K \in (0, K_a^{PA})$, the seller's profit under partial anonymity is strictly lower than that under full anonymity, further highlighting the importance of the commitment effect that can be achieved through anonymous option.

Finally, we illustrate the equilibrium under partial anonymity using the example with uniform distribution. In the no anonymous option regime, $K_o^{PA} = 0.225$. When $K < 0.225$, the seller chooses the disclosure mode with $\tilde{p}_d = \frac{1-4K}{6}$, and $\hat{v}(\tilde{p}_d) = \frac{1+2K}{3}$, $\tilde{p}_{2u} = \frac{1+2K}{6}$, $p_{2i} = v_i$. When $K \geq 0.225$, the seller chooses the no-disclosure mode with $\tilde{p}_a^{nd} = \frac{3}{10}$, $\hat{v}(\tilde{p}_a^{nd}) = \frac{3}{5}$, and the second-period prices are $\tilde{p}_{2r}^{nd} = \frac{3}{5}$ and $\tilde{p}_{2n}^{nd} = \frac{3}{10}$. The seller's profit and consumer surplus are

$$\widetilde{\Pi}^{PA} = \begin{cases} \frac{7-8K+4K^2}{12} & \text{if } K < K_o^{PA} \\ \frac{9}{20} & \text{if } K \geq K_o^{PA} \end{cases}, \qquad \widetilde{CS}^{PA} = \begin{cases} \frac{25-20K+4K^2}{72} & \text{if } K < K_o^{PA} \\ \frac{13}{40} & \text{if } K \geq K_o^{PA} \end{cases}.$$

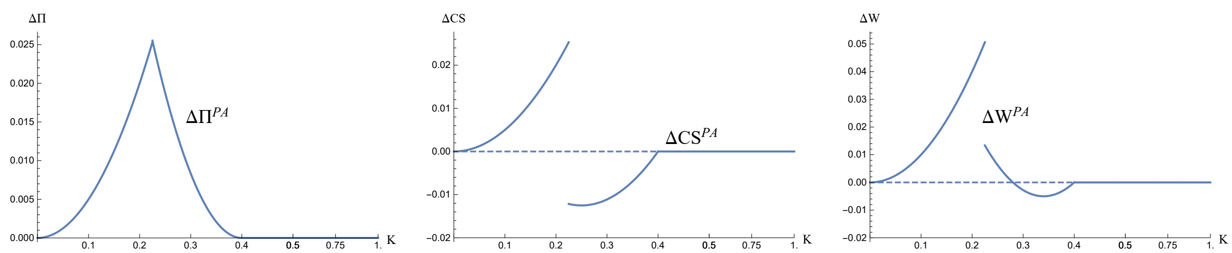
Now consider the anonymous option regime. When $K \in (0, \frac{2}{5})$, the equilibrium features partial disclosure, with $p_a^{PA} = \frac{1+2K}{6}$, $p_d^{PA} = \tilde{p}_d = \frac{1-4K}{6}$, $v_m^{PA} = \hat{v}(\tilde{p}_d) = 2(\tilde{p}_d + K) = \frac{1+2K}{3}$, $v_h^{PA} = 1 - K$, and the second-period uniform prices are $p_{2r}^{PA} = 1 - K$ and $p_{2n}^{PA} = \frac{1+2K}{6}$. When $K = 0$ and $K \geq \frac{2}{5}$,

[19] The full analysis is available upon request. We illustrate the equilibrium with uniform distribution below.

the equilibrium outcomes are, respectively, the same as those of the disclosure mode and the no-disclosure mode without anonymous option. The seller's profit and consumer surplus are respectively

$$\Pi^{PA} = \begin{cases} \frac{7-8K+10K^2}{12} & \text{if } K < \frac{2}{5} \\ \frac{9}{20} & \text{if } K \geq \frac{2}{5} \end{cases}, \qquad CS^{PA} = \begin{cases} \frac{25-20K+40K^2}{72} & \text{if } K < \frac{2}{5} \\ \frac{13}{40} & \text{if } K \geq \frac{2}{5} \end{cases}.$$

Figure 3 illustrates the welfare implications of anonymous option under partial anonymity. When $K = 0$ and $K \geq \frac{2}{5}$, the outcomes are identical in the two regimes. When $K \in (0, \frac{2}{5})$, the seller is strictly better off with the anonymous option. However, there exist intervals of K under which consumer surplus and social welfare are strictly lower with anonymous option.



(a) $\Delta\Pi^{PA}$ (b) ΔCS^{PA} (c) ΔW^{PA}

Figure 3: Welfare Effects of Anonymous Option under Partial Anonymity, for $\bar{v} = 1$ and $F(v) = v$.

6 Unobservable Privacy Types

So far we have assumed that the intrinsic privacy loss K is publicly observed and identical across all consumers. However, in reality, consumers may differ in the extent to which they experience privacy loss, and such information is privately held by the consumers. In this section, we extend the baseline model to incorporate two-dimensional private information and examine how information asymmetry on consumers' intrinsic privacy types affects the value of anonymous option. Specifically, we assume that v_i and K_i are uniformly and independently distributed on $[0, 1]$, and the realizations of both random variables are consumers' private information. The timing of the game remains unchanged from the baseline model. We demonstrate that, in this two-dimensional private information framework, the anonymous option remains advantageous for the seller while being detrimental to consumers.

6.1 Benchmark without Anonymous Option

No-Disclosure Mode. The seller offers the menu $\{(A, p_a), (N, 0)\}$ at $t = 1$ and does not collect any consumer data. At $t = 2$, the seller sets a uniform price p_{2u} for all consumers. As in Section 3, the seller's per-period price choice is the same as that in a repeated static monopoly setting. The seller's optimal prices satisfy $\hat{p}_a^{nd} = \hat{p}_{2u}^{nd} = \tilde{p} = \frac{1}{2}$. Consumers with $v_i \in [\frac{1}{2}, 1]$ purchase the product in both periods, while those with $v_i < \frac{1}{2}$ abstain from consumption in either period. The seller's overall profit is $\widehat{\Pi}^{nd} = \frac{1}{2}$, and total consumer surplus is $\widehat{CS}^{nd} = \frac{1}{4}$.

Disclosure Mode. The seller offers the menu $\{(D, p_d), (N, 0)\}$ at $t = 1$, and a consumer discloses her data by purchasing through option D. At $t = 2$, the seller charges personalized prices $p_{2i} = v_i$ to consumers who chose option D and a uniform price p_{2u} to those who chose option N at $t = 1$.

Upon observing p_d and anticipating the second-period uniform price p_{2u} (which must be consistent with the seller's optimal second-period choice on the equilibrium path), a consumer makes a purchase at $t = 1$ if and only if:

$$v_i - p_d - K_i + 0 \geq \max\{v_i - p_{2u}, 0\} \Leftrightarrow \; v_i \geq p_d + K_i \text{ and } K_i \leq p_{2u} - p_d. \tag{17}$$

This implies that a consumer purchases at $t = 1$ *iff* her first-period net valuation $v_i - K_i$ is sufficiently high and her intrinsic privacy type K_i is sufficiently low. Note that the equilibrium prices must satisfy $p_d \leq p_{2u}$. If this condition does not hold, even consumers with $K_i = 0$ would delay their purchase until the second period, which cannot be optimal for the seller. In Remark 3 below, we characterize the equilibrium outcome under the disclosure mode without anonymous option.

Remark 3. *Suppose (v_i, K_i) are consumers' private information. Under the disclosure mode, the seller posts $\{(D, \hat{p}_d), (N, 0)\}$ with $\hat{p}_d = \frac{2-\sqrt{3}}{4}$ at $t = 1$. A consumer chooses D at $t = 1$ iff $v_i \geq K_i + \frac{2-\sqrt{3}}{4}$ and $K_i \leq \frac{\sqrt{3}}{4}$, and chooses N otherwise. At $t = 2$, the seller posts $\hat{p}_{2u} = \frac{1}{2}$ to the unrecognized consumers and $p_{2i} = v_i$ to the recognized consumers. The seller's total profit is $\widehat{\Pi}^d = \frac{4+\sqrt{3}}{16}$.*

Because consumers with high K_i do not purchase at $t = 1$, the unrecognized market segment at $t = 2$ contains consumers with all possible valuations and $K_i > \frac{\sqrt{3}}{4}$, as well as consumers with $K_i \leq \frac{\sqrt{3}}{4}$ and $v_i < K_i + \frac{2-\sqrt{3}}{4}$. This mixture of both high- and low-valuation consumers makes a relatively high second-period uniform price optimal. Moreover, some consumers with $K_i > \frac{\sqrt{3}}{4}$ strategically delay their consumption to the second period, even if their valuations exceed $p_d + K_i$ at $t = 1$. In equilibrium, consumers with $K_i > \frac{\sqrt{3}}{4}$ and $v_i \geq \frac{1}{2}$ purchase their first units at $t = 2$.

Figure 4 illustrates consumers' equilibrium choices under the no-disclosure mode (left panel) and the disclosure mode (right panel). Under the no-disclosure mode, consumers with $v_i \in [\frac{1}{2}, 1]$ (represented by the yellow rectangle) consume in both periods, while the remaining consumers do not consume in either period. Under the disclosure mode, consumers with $v_i \geq K_i + \frac{2-\sqrt{3}}{4}$ and $K_i \leq \frac{\sqrt{3}}{4}$ (represented by the yellow trapezoid) purchase at $t = 1$. These consumers also constitute the recognized segment at $t = 2$. Consumers with $K_i > \frac{\sqrt{3}}{4}$ and $v_i \geq \frac{1}{2}$ (represented by the red rectangle) purchase their first units at $t = 2$.

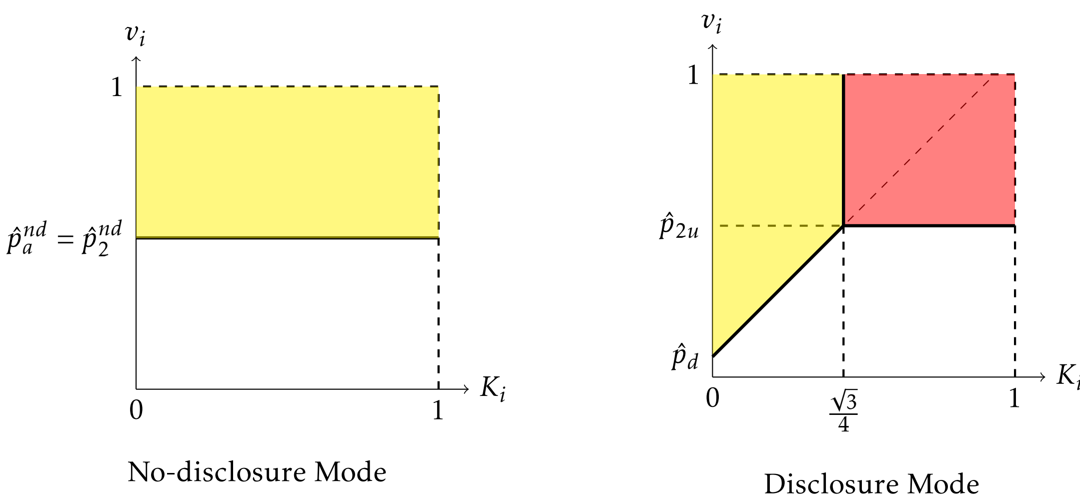


Figure 4: Consumers' choices in the no anonymous option regime when (v_i, K_i) are private information. In the left panel, $\hat{p}_a^{nd} = \hat{p}_2^{nd} = \frac{1}{2}$. In the right panel, $\hat{p}_d = \frac{2-\sqrt{3}}{4}$ and $\hat{p}_{2u} = \frac{1}{2}$.

Equilibrium in the benchmark. Comparing the seller's profits under the disclosure and no-disclosure modes reveals that the seller is better off choosing the *no-disclosure mode* in the regime without anonymous option. We summarize the unique subgame perfect equilibrium in Proposition 4.

Proposition 4. *Suppose (v_i, K_i) are consumers' private information. Without anonymous option, the seller chooses the no-disclosure mode and sets prices $\hat{p}_a^{nd} = \hat{p}_{2u}^{nd} = \frac{1}{2}$ in equilibrium. Consumers with $v_i \in [\frac{1}{2}, 1]$ purchase the product in both periods and consumers with $v_i < \frac{1}{2}$ do not consume in either period. The seller's overall profit is $\widehat{\Pi}^{nd} = \frac{1}{2}$, and total consumer surplus is $\widehat{CS}^{nd} = \frac{1}{4}$.*

Under the disclosure mode, the seller must set a low first-period price ($\hat{p}_d < \hat{p}_a^{nd}$) to incentivize some consumers to disclose their data, accounting for both the strategic delay effect and the intrinsic privacy loss. The total demand at $t = 1$ is also lower compared to the no-disclosure mode. At $t = 2$, the seller can extract full surplus from consumers in the recognized segment. However, the additional benefits from full surplus extraction are insufficient to offset the profit losses from the lower first-period price and reduced demand relative to the no-disclosure mode. As a result, it is optimal for the seller to choose the no-disclosure mode.

When information about K is public, as analyzed in Section 3, the seller prefers the disclosure mode for small K without anonymous option. However, this is no longer the case when K_i is also private information for consumers. The consumers' incentive to strategically delay consumption under the disclosure mode is exacerbated by the uncertainty about K. Specifically, all consumers with $K_i > \frac{\sqrt{3}}{4}$, even those with very high valuations, wait until the second period to purchase their first units. This reinforced strategic delay effect renders the disclosure mode inferior to the no-disclosure mode, despite the fact that data disclosure enables full surplus extraction from the recognized consumers at $t = 2$.

6.2 Anonymous Option

With the anonymous option, at $t = 1$, the seller offers a price menu $M = \{(A, p_a), (D, p_d), (N, 0)\}$, allowing consumers to choose freely: they can disclose data by choosing D, purchase anonymously by choosing A, or abstain from purchasing. At $t = 2$, the unrecognized segment comprises consumers who chose A or N, while the recognized segment consists of those who chose D, as in Section 4. Consequently, the seller sets a uniform price p_{2u} for the unrecognized segment and charges personalized prices $p_{2i} = v_i$ to the recognized segment.

Observing price menu M and anticipating the second-period price p_{2u} (which must be consistent with the seller's second-period optimal price), a consumer with valuation v_i and privacy type K_i chooses option D at $t = 1$ if and only if

$$v_i - p_d - K_i \geq \max\Big\{v_i - p_a + \max\{v_i - p_{2u}, 0\}, \max\{v_i - p_{2u}, 0\}\Big\},$$

which is equivalent to

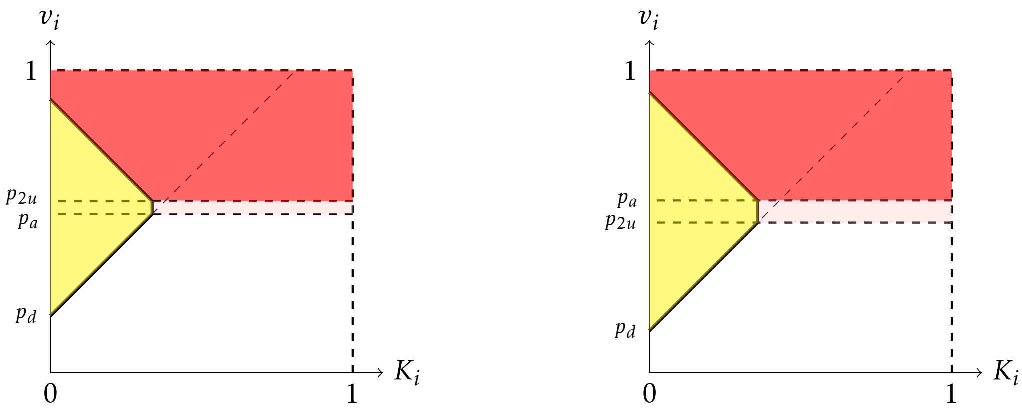
$$p_d + K_i \leq v_i \leq p_a - p_d + p_{2u} - K_i, \quad K_i \leq p_a - p_d, \text{ and } K_i \leq p_{2u} - p_d. \tag{18}$$

If $p_d > p_a$ or $p_d > p_{2u}$, no consumer chooses option D at $t = 1$. Then the game degenerates to the no-disclosure mode in the benchmark. By setting $p_d > p_a = p_{2u} = \frac{1}{2}$, the seller secures a total profit of $\frac{1}{2}$, which serves as a lower bound on the seller's profit under anonymous option. Alternatively, by posting a menu with $p_d \leq p_a$ and $p_d \leq p_{2u}$, the seller may incentivize some consumers to choose option D and disclose their data at $t = 1$, potentially achieving a profit higher than $\frac{1}{2}$. Two cases arise depending on whether $p_a \leq p_{2u}$ or $p_a > p_{2u}$.

Case 1: $p_d \leq p_a \leq p_{2u}$**.** No consumer purchases her first unit at $t = 2$: since $p_a \leq p_{2u}$, buying a single unit at price p_{2u} at $t = 2$ is dominated by purchasing through option A at $t = 1$. By Condition (18), consumers with $p_d + K_i \leq v_i \leq p_a - p_d + p_{2u} - K_i$ and $K_i \leq p_a - p_d$ choose option D, while others choose A if $v_i \geq p_a$ and N if $v_i < p_a$. Among those who choose A at $t = 1$, consumers with $v_i \in [p_a, p_{2u}]$ do not purchase again at $t = 2$.

Case 2: $p_d \leq p_{2u} < p_a$**.** Some consumers purchase their first units at $t = 2$. By Condition (18), a consumer chooses option D if $p_d + K_i \leq v_i \leq p_a - p_d + p_{2u} - K_i$ and $K_i \leq p_{2u} - p_d$, and the remaining consumers choose A if $v_i \geq p_a$ and N otherwise. Among the consumers who choose N, those with $v_i \in [p_{2u}, p_a]$ purchase their first units at $t = 2$.

A key difference between the two cases lies in whether there are consumers who purchase their first units at $t = 2$. Figure 5 illustrates consumer choices in both cases. The yellow trapezoids in both panels represent consumers who choose D at $t = 1$, forming the recognized segment and paying personalized prices at $t = 2$. The red areas depict consumers who choose option A at $t = 1$ and purchase again at p_{2u} at $t = 2$. In the left panel, the pink area represents consumers who choose option A at $t = 1$ but do not purchase again at $t = 2$; in the right panel, it represents consumers who do not purchase at $t = 1$ but purchase at p_{2u} at $t = 2$. The uncolored areas correspond to consumers who do not purchase in either period.



Case 1: $p_d \leq p_a \leq p_{2u}$. Case 2: $p_d \leq p_{2u} < p_a$.

Figure 5: Consumers' choices under anonymous option when (v_i, K_i) are private information.

Solving for the seller's optimal prices in both cases and comparing their outcomes reveal that the subgame perfect equilibrium under anonymous option features partial disclosure, with $p_d \leq p_a \leq p_{2u}$.

Proposition 5. *Suppose (v_i, K_i) are consumers' private information. With anonymous option, the seller posts $\{(A, \hat{p}_a^A), (D, \hat{p}_d^A), (N, 0)\}$ with $\hat{p}_a^A = 0.5251$ and $\hat{p}_d^A = 0.1881$ at $t = 1$. Consumers with $v_i \in [0.1881 + K_i, 0.9042 - K_i]$ and $K_i \leq 0.337$ choose option D, others with $v_i \geq \hat{p}_a^A = 0.5251$ choose option A, and those with $v_i < \hat{p}_a^A = 0.5251$ do not purchase at $t = 1$.*

At $t = 2$, the seller posts $\hat{p}_{2u}^A = 0.5672$ to consumers in the unrecognized segment and charges $p_{2i} = v_i$ to consumers in the recognized segment.

With anonymous option, the seller's total profit is $\widehat{\Pi}^A = 0.5192$, and total consumer surplus is $\widehat{CS}^A = 0.2215$. The anonymous option increases the seller's profit but lowers both consumer surplus and total welfare.

In equilibrium, $\hat{p}_d^A \leq \hat{p}_a^A \leq \hat{p}_{2u}^A$ holds. Consumers with intermediate v_i and low K_i disclose their personal data, those with high valuations choose the anonymous option, and those with low valuations do not purchase. The second-period uniform price is sufficiently high that no consumer strategically delays consumption at $t = 1$: all consumers with $v_i \geq K_i + \hat{p}_d^A$ purchase at $t = 1$, either through option D or A. Notably, some consumers (with $v_i \in [\hat{p}_a^A, \hat{p}_{2u}^A]$ and $K_i \geq \hat{p}_a^A - \hat{p}_d^A$) make only one purchase at $t = 1$ due to the higher second-period price. No consumers delay their purchase to $t = 2$, reinforcing that the optimal prices in the repeated game under limited commitment are designed to minimize consumers' incentives to delay their first-period consumption.

7 Conclusion

Privacy regulations often require that sellers obtain explicit consumer consent before collecting personal data. We formalize this requirement by introducing an *anonymous option*, which allows consumers to maintain anonymity during transactions. In a two-period model of repeated purchases under limited commitment, we examine a monopolist's strategic incentives to offer this option. We find that despite the potential for full consumer surplus extraction in the second period through data collection, the seller generally benefits from providing the option, which enables the seller to credibly commit to a high second-period price for unrecognized consumers and mitigates consumers' incentives to strategically delay initial consumption. However, the option can reduce consumer surplus and social welfare due to higher average prices and lower aggregate demand, running counter to the intent of the privacy regulations. The welfare implications of anonymous option extend to scenarios where the seller can keep track of consumers' purchasing history and practice third-degree price discrimination accordingly, and where both valuations and privacy types are consumers' private information.

We assumed positive K throughout the analysis. In practice, some consumers may enjoy sharing personal data ($K < 0$) which implies a data-sharing benefit instead of a privacy cost.[20] The anonymous option may still be profitable for the seller in this scenario, as consumers' instrumental privacy concerns are still present. Without anonymous option, strategic delay effect still exists under the disclosure mode, though alleviated by the consumers' data-sharing benefits. In the anonymous option regime, it gets harder to induce the consumers to choose the anonymous option. When $|K|$ is large, the data-sharing benefits make it optimal for the seller to adopt the disclosure mode. However, when $|K|$ is relatively small, the data-sharing benefits are not pivoting. The balance between the strategic delay effect and the surplus extraction effect makes it optimal for the seller to induce some high-valuation consumers to purchase anonymously, supporting the profitability of the anonymous option.

One natural extension for future research is to examine the impact of the anonymous option in

[20] Some consumers may experience psychological benefits from sharing data. See, for example, Tamir and Mitchell (2012).

a competitive environment. When multiple sellers possess data about the same set of consumers, they compete in a Bertrand fashion for individual consumers, driving prices down to marginal production costs. Consequently, sellers face incentives either to avoid collecting consumer data or to differentiate their target consumer groups for data collection. Besides the three effects that determine the value of the anonymous option in a monopoly setting, the additional competition effect may further enhance the desirability of the anonymous option for sellers.

Appendix

This appendix contains the proofs to Remark 2-3, Lemma 1-2, Lemma 5-7, Propositions 2-3 and 5 that are not included in the main text.

Proof of Remark 2. When choosing $-K < p_d < \tilde{p} - K$, the seller's expected profit is given by (5). Its derivative with respect to p_d is

$$\begin{aligned}\frac{d\Pi^d(p_d)}{dp_d} &= 1 - F(\hat{v}) - f(\hat{v})\hat{v}'(p_d)p_d + (-\hat{v}f(\hat{v})\hat{v}'(p_d)) + (p_d + K)f(\hat{v})\hat{v}'(p_d) + F(\hat{v}) - \phi(p_d + K) \\ &= 1 - \phi(p_d + K) - f(\hat{v})\hat{v}'(p_d)(\hat{v} - K),\end{aligned}$$

where the first line makes use of $\phi(p_d+K) = F(p_d+K)+(p_d+K)f(p_d+K)$. By (4), $\hat{v}(p_d) = F^{-1}(\phi(p_d+K))$ and $\phi'(p_d + K) = f(\hat{v}(p_d))\hat{v}'(p_d)$. Thus,

$$\frac{d\Pi^d(p_d)}{dp_d} = 1 - \phi(p_d + K) - (F^{-1}(\phi(p_d + K)) - K)\phi'(p_d + K). \tag{19}$$

Note that $\phi(0) = 0$, $F^{-1}(\phi(0)) = 0$, and $\phi'(0) = 2f(0)$; $\phi(\tilde{p}) = 1$, $F^{-1}(\phi(\tilde{p})) = \bar{v}$ and $\phi'(\tilde{p}) = \tilde{p}f'(\tilde{p}) + 2f(\tilde{p})$. Thus, the following conditions hold

$$\frac{d\Pi^d(p_d = -K)}{dp_d} = 1 + 2Kf(0) \geq 1 > 0, \quad \frac{d\Pi^d(p_d = \tilde{p} - K)}{dp_d} = -(\bar{v} - K)\big(\tilde{p}f'(\tilde{p}) + 2f(\tilde{p})\big) < 0,$$

where $\tilde{p}$ is the static monopoly price in (1). Therefore, by continuity, there exists a $\tilde{p}_d$ such that (19) equals to zero, and the seller gets the highest profit choosing $\tilde{p}_d$. Moreover, the seller's profit at $\tilde{p}_d$, $\Pi^d(\tilde{p}_d)$, is strictly higher than $\tilde{\pi}$ which is the seller's maximal profit choosing $p_d \geq \tilde{p} - K$. Thus, $\tilde{p}_d$ defined in (6) is the seller's optimal first-period price.

Given $\tilde{p}_d$, the valuation of the marginal consumer, $\hat{v}(\tilde{p}_d)$, and the optimal second-period uniform price, $\tilde{p}_{2u}$, follow directly from (3) and (4). Consumers with $v_i \in [\hat{v}(\tilde{p}_d), \bar{v}]$ receive a positive surplus from their first-period purchase at price $\tilde{p}_d$, while those with $v_i \in [\tilde{p}_d + K, \hat{v}(\tilde{p}_d))$ receive a positive surplus from their second-period purchase at price $\tilde{p}_{2u} = \tilde{p}_d + K$. Thus, total consumer surplus is given by $CS^d(\tilde{p}_d) = \int_{\tilde{p}_d+K}^{\bar{v}}(v - \tilde{p}_d - K)\mathrm{d}F(v)$. □

Proof of Lemma 1. 1) Suppose $p_{2u} < p_d + K$ instead. Then $u_i(v_i, N) = \max\{v_i - p_{2u}, 0\} > u_i(v_i, D) =$

$v_i - p_d + K$ for all v_i, and option N always dominates D at $t = 1$ for all v_i. Therefore, no consumer will choose D, leading to a contradiction. Thus, for a positive mass of consumers to choose D in equilibrium at $t = 1$, it must hold that $p_{2u} \geq p_d + K$.

2) Anticipating a uniform price $p_{2u} \geq p_d + K$ for unrecognized consumers at $t = 2$, a consumer with v_2 chooses D instead of A and N at $t = 1$, implying that

$$u_i(v_2, D) \geq \max\{u_i(v_2, A), u_i(v_2, N)\} \Leftrightarrow \; v_2 - p_d - K \geq \max\Big\{v_2 - p_a + \max\{v_2 - p_{2u}, 0\}, \max\{v_2 - p_{2u}, 0\}\Big\}.$$

It follows that $\max\{v_2 - p_{2u}, 0\} \leq p_a - p_d - K$.

We now show that consumers with v_1 choosing option A implies $v_1 > v_2$. Suppose $v_1 < v_2$ instead. Then we have

$$\max\{v_1 - p_{2u}, 0\} \leq \max\{v_2 - p_{2u}, 0\} \leq p_a - p_d - K,$$

which implies

$$v_1 - p_a + \max\{v_1 - p_{2u}, 0\} \leq v_1 - p_d - K.$$

If $v_1 - p_d - K \geq 0$, then $v_1 - p_d - K \geq \max\{v_1 - p_{2u}, 0\}$, thus,

$$v_1 - p_d - K \geq \max\Big\{v_1 - p_a + \max\{v_1 - p_{2u}, 0\}, \max\{v_1 - p_{2u}, 0\}\Big\} \Leftrightarrow u_i(v_1, D) \geq \max\{u_i(v_1, A), u_i(v_1, N)\},$$

and the consumer with v_1 also chooses option D instead of A, which is a contradiction. If $v_1 - p_d - K < 0$, then $v_1 - p_d - K < \max\{v_1 - p_{2u}, 0\}$, and the consumer with v_1 must choose N instead of A, which again is a contradiction. Thus, $v_1 > v_2$ holds.

Consumers with v_2 and v_3 choosing D and N respectively implies

$$v_2 - p_d - K \geq \max\{v_2 - p_{2u}, 0\}, \qquad \text{and} \quad v_3 - p_d - K \leq \max\{v_3 - p_{2u}, 0\},$$

leading to $v_2 > v_3$. □

Proof of Lemma 2. We proceed with the proof by showing in sequence that, under weak convexity of $F(x)$, a price menu inducing either $v_m = 0$ or $v_h = \bar{v}$ is dominated, and thus $v_m > 0$ and $v_h < \bar{v}$

hold in equilibrium.[21]

(i) Suppose price menu $M = \{(A, p_a), (D, p_d), (N, 0)\}$ induces $v_m = 0$. We show that there exists an alternative price menu M' inducing $v_m > 0$ leads to a higher overall profit for the seller.[22]

Under price menu M with $v_m = 0$, a consumer with $v_i = 0$ makes a purchase at price p_d at $t = 1$, implying $p_d + K \leq 0$. We show that $v_h < \bar{v}$ must hold. Suppose $v_h = \bar{v}$ instead. Then all consumers choose D in the first period, and the seller's total profit $\Pi(M)$ is bounded above by $\int_0^{\bar{v}} v \mathrm{d}F(v) - K$. Consider an alternative price menu M' with $p'_d \in [\tilde{p} - K, \tilde{p}]$, $p'_a = \tilde{p}$. It follows that $v'_m = \tilde{p}$, $v'_h = \tilde{p}$, and $p'_{2u} = \tilde{p}$. Consumers with $v_i \geq \tilde{p}$ purchase the product in both periods and consumers with $v_i < \tilde{p}$ do not consume, and the seller's total profit is $\Pi(M') = 2\tilde{\pi}$. Since $\int_0^{\bar{v}} v \mathrm{d}F(v) \leq 2\tilde{\pi}$, $v_h = \bar{v}$ cannot be optimal and $v_h < \bar{v}$ must hold when $v_m = 0$.

Next we show that under price menu M with $v_m = 0$, $v_h = 0$ cannot be supported in equilibrium either. Suppose $v_h = 0$ holds, all consumers purchase at price p_a at $t = 1$, which implies p_a must be equal to 0 and the seller earns zero first-period profit. The seller's maximal second-period profit is $\tilde{\pi}$ by setting $p_{2u} = \tilde{p}$, leading to a total profit $\tilde{\pi}$. Obviously M is dominated by the alternative menu M' with $p'_d \in [\tilde{p} - K, \tilde{p}]$ and $p'_a = p'_{2u} = \tilde{p}$, which brings a total profit $2\tilde{\pi}$. This implies that if $v_m = 0$, $0 < v_h < \bar{v}$ must hold, and consumers with $v_i \in [0, v_h]$ choose option D while consumers with $v_i \in (v_h, \bar{v}]$ choose option A.

Now we show that a price menu M with p_d and p_a inducing $v_m = 0$ is dominated by an alternative price menu M' that induces $v'_m > 0$. Under price menu M, with $v_h \in (0, \bar{v})$, the seller's total profit is

$$\Pi(M) = \Big(1 - F(v_h)\Big)p_a + F(v_h)p_d + \int_0^{v_h} v \mathrm{d}F(v) + \max\Big\{\Big(1 - F(v_h)\Big)v_h, \tilde{\pi}\Big\}.$$

Consider a price menu M' with $p'_d = p_d + \epsilon$ and $p'_a = p_a + \epsilon$ in which ϵ is an infinitely small positive number. Now $v'_m = \epsilon$ and $v'_h = v_h > 0$. For infinitely small ϵ, $p'_{2u} = p_{2u} = v_h$. The seller's total profit

[21] Here we abuse the notations a little bit: by $v_m = 0$ we refer to the case that no consumers choose option N, and by $v_h = \bar{v}$ we refer to the case that no consumers choose option A.

[22] To avoid excessive notations, we denote the constructed alternative price menu as M' in multiple cases in this proof. In each individual case, we will specify carefully the corresponding (p'_d, p'_a) and the induced v'_m, v'_h and p'_{2u}.

under price menu M' is

$$\Pi(M') = \Big(1 - F(v_h)\Big)p'_a + \Big(F(v_h) - F(\epsilon)\Big)p'_d + \int_{\epsilon}^{v_h} v \mathrm{d}F(v) + \max\Big\{\Big(1 - F(v_h)\Big)v_h, \tilde{\pi}\Big\}.$$

Price menu M' brings the seller a strictly higher profit because

$$\Pi(M') - \Pi(M) = \epsilon\Big(1 - F(\epsilon)\Big) - p_d F(\epsilon) - \int_0^{\epsilon} v \mathrm{d}F(v) \geq \epsilon\Big(1 - F(\epsilon)\Big) - \int_0^{\epsilon} v \mathrm{d}F(v) > \epsilon\Big(1 - 2F(\epsilon)\Big) > 0,$$

where $\epsilon\Big(1 - F(\epsilon)\Big)$ is the lower bound of profit increase at $t = 1$, and $\int_0^{\epsilon} v \mathrm{d}F(v)$ is the profit decrease at $t = 2$ when the price menu changes from M to M'. We have $\Pi(M') - \Pi(M) > 0$. Therefore, $v_m > 0$ holds in equilibrium.

(ii) We show that a price menu M inducing $v_h = \bar{v}$ in equilibrium is dominated by an alternative price menu M' that induces $v_h < \bar{v}$.

Suppose $v_h = \bar{v}$ holds instead. Then no one chooses option A at $t = 1$. The equilibrium is trivial in the sense that the outcome is the same as that under the disclosure mode in the benchmark. Since $v_m > 0$, consumers with $v_i \in [v_m, \bar{v}]$ choose option D and consumers with $v_i \in [0, v_m)$ choose option N at $t = 1$. Then $\hat{v}(p_d) = v_m$ and $p_{2u} = p_d + K$, in which $\hat{v}(p_d)$ is defined in (4), consumers with $v_i \in [p_{2u}, v_m)$ purchase their first units at $t = 2$.

Now consider an alternative menu M' with $p'_a = p'_d + K = p_d + K$. Let $\eta \in (\tilde{p}, \bar{v})$ be defined by $1 - F(\eta) = F(v_m) - F(p_d + K)$ in which $v_m = \hat{v}(p_d)$. Then $v'_m = p'_d + K$ and $v'_h = p'_{2u} = \eta$ are supported in equilibrium under M'. To see this, note that anticipating $p'_{2u} > \tilde{p}$, the valuations of marginal consumers, v'_h and v'_m, satisfy

$$v'_h - p'_a + v'_h - p'_{2u} = v'_h - p_d - K; \quad v'_m - p_d - K = 0.$$

Thus, under M', consumers with $v_i > p'_{2u}$ choose option A and consumers with $v_i \in [p'_d + K, p'_{2u}]$ choose option D. For the unrecognized segment with $v_i \in [0, p'_d + K) \cup (p'_{2u}, \bar{v}]$, p'_{2u} is the optimal uniform price for the seller at $t = 2$. To see this, recall $1 - F(\eta) + F(p_d + K) = F(v_m)$. This means the optimal price that maximizes $\Big(1 - F(\eta) + F(p_d + K) - F(p)\Big)p$ equals $p_d + K$. When the market consists of consumers with $v_i \in [0, p_d + K) \cup (\eta, \bar{v}]$, clearly choosing price $p_{2u} = \eta$ is optimal on the range $[\eta, \bar{v}]$ and it strictly dominates $p_{2u} = p_d + K$.

We now compare the seller's profits from menu M and M':

$$\Pi(M) = \Big(1 - F(v_m)\Big)p_d + \Big(F(v_m) - F(p_{2u})\Big)p_{2u} + \int_{v_m}^{\bar{v}} v \mathrm{d}F(v);$$

$$\Pi(M') = \Big(F(p'_{2u}) - F(p'_a)\Big)p'_d + \Big(1 - F(p'_{2u})\Big)p'_a + \int_{p'_a}^{p'_{2u}} v \mathrm{d}F(v) + \Big(1 - F(p'_{2u})\Big)p'_{2u}$$

$$= \underbrace{\Big(F(p'_{2u}) - F(p_{2u})\Big)}_{=1-F(v_m)} p_d + \underbrace{\Big(1 - F(p'_{2u})\Big)}_{=F(v_m)-F(p_d+K)} p_{2u} + \int_{p'_a}^{p'_{2u}} v \mathrm{d}F(v) + \Big(1 - F(p'_{2u})\Big)p'_{2u}.$$

Thus, we have

$$\Pi(M') - \Pi(M) = \Big(1 - F(p'_{2u})\Big)p'_{2u} + \int_{p_d+K}^{v_m} v \mathrm{d}F(v) - \int_{p'_{2u}}^{\bar{v}} v \mathrm{d}F(v)$$

$$= \int_{p_d+K}^{v_m} v \mathrm{d}F(v) - \int_{p'_{2u}}^{\bar{v}} (v - p'_{2u}) \mathrm{d}F(v) > \int_{p'_{2u}}^{\bar{v}} \Big(p_d + K + p'_{2u} - v\Big) \mathrm{d}F(v).$$

By the convexity of $F(v)$ and the inequality $p'_{2u} > p_d + K$, we have

$$f(p'_{2u})(\bar{v} - p'_{2u}) \leq 1 - F(p'_{2u}) = F(v_m) - F(p_d + K) = f\Big(p_d + K\Big)(p_d + K) \leq f(p'_{2u})(p_d + K).$$

Thus, $p_d + K + p'_{2u} \geq \bar{v}$. Therefore, we have $\Pi(M') - \Pi(M) > 0$ and $v_h < \bar{v}$ must hold in equilibrium. □

Proof of Lemma 5. First, consider a price menu M_h that induces p^h_{2u} at $t = 2$. From the previous discussions, $v_m = p_a = p_d + K$ hold. Moreover, in a partial-disclosure equilibrium, $v_h > v_m$ thus $v_h \in (p_a, \bar{v})$. To establish the equivalence between condition (9) and condition (10), we still need to show the equivalence between $p^\ell_{2u} \geq v_m$ and $F(v_h) \leq 1 - f(p_a)p_a$, and prove $\max\{v_h, \tilde{p}\} = v_h$.

By (8), we have $\phi(p^\ell_{2u}) = 1 - F(v_h) + F(v_m)$. Recall that $\phi(x) = f(x)x + F(x)$. Because $\phi(x)$ monotonically increases in x, condition $p^\ell_{2u} < v_m$ implies $1 - F(v_h) + F(v_m) = \phi(p^\ell_{2u}) < \phi(v_m) = \phi(p_a) = f(p_a)p_a + F(p_a)$, which in turn implies $F(v_h) > 1 - f(p_a)p_a$. This establishes the equivalence between $p^\ell_{2u} \geq v_m$ and $F(v_h) \leq 1 - f(p_a)p_a$.

We now show that $v_h < \tilde{p}$ cannot occur in equilibrium so that $p^h_{2u} = \max\{v_h, \tilde{p}\} = v_h$. Suppose $v_h < \tilde{p}$ holds and $p^h_{2u} = \tilde{p}$. At $t = 1$, consumers with $v_i \in [p_a, v_h]$ purchase at $p_d = p_a - K$, and consumers with $v_i \in (v_h, \bar{v}]$ purchase at p_a. The seller's first-period maximization problem can be written as

$$\max_{p_a} \Pi(p_a) = (F(v_h) - F(p_a))(p_a - K) + (1 - F(v_h))p_a + \int_{p_a}^{v_h} v\mathrm{d}F(v) + \tilde{\pi}, \tag{20}$$

in which the first two terms are the seller's first-period profit and the last two terms represent the seller's second-period profit. Since $\frac{\partial \Pi(p_a)}{\partial v_h} = (-K + v_h)f(v_h)$, $\Pi(p_a)$ in (20) decreases with v_h when $v_h < K$ and increases with v_h when $v_h > K$. Thus, on the interval $v_h \in (p_a, \tilde{p})$, when $v_h < K$, the seller's profit $\Pi(p_a)$ is bounded above by $\Pi(p_a)\mid_{v_h \to p_a} = (1 - F(p_a))p_a + \tilde{\pi} \le 2\tilde{\pi}$. Such outcome is obviously dominated by the no-disclosure equilibrium characterized in Lemma 3. When $v_h > K$, for $v_h < \tilde{p}$ satisfying (9), $v_h + \epsilon$ also satisfies (9) for infinitely small positive ϵ, thus can also be supported in an equilibrium, and leads to a larger profit than v_h. Therefore, $v_h < \tilde{p}$ cannot constitute a seller-optimal equilibrium. Thus, condition (9) can be rewritten as condition (10).

Second, consider a price menu M_ℓ that induces p_{2u}^ℓ at $t = 2$. From the previous discussions, $p_{2u}^\ell = p_d + K$ and $v_h = p_a$ hold. Moreover, $v_m < v_h$ holds in a partial-disclosure equilibrium. Using $\phi(p_{2u}^\ell) = 1 - F(v_h) + F(v_m)$, $p_{2u}^\ell < v_m$ implies $1 - F(p_a) + F(v_m) = \phi(p_{2u}^\ell) < \phi(v_m) = f(v_m)v_m + F(v_m)$, which further reduces to $F(p_a) > 1 - f(v_m)v_m$.

We still need to show $v_h > \tilde{p}$. Suppose $v_h \le \tilde{p}$ holds instead. Then $\big(1 - F(\max\{v_h, \tilde{p}\})\big)\max\{v_h, \tilde{p}\} = (1 - F(\tilde{p}))\tilde{p} \ge \max_p (1 - F(p) - F(v_h) + F(v_m))p = \pi_{2u}^\ell(v_m, v_h)$, implying that p_{2u}^ℓ cannot be optimal. Thus, for p_{2u}^ℓ to be optimal, $v_h > \tilde{p}$ must hold. Thus, $\pi_{2u}^h = (1 - F(v_h))v_h = (1 - F(p_a))p_a$, and condition (11) is derived. □

Proof of Lemma 6. (i) Consider price menu M_ℓ that induces p_{2u}^ℓ at $t = 2$. In the following, we will first establish an upper bound on the seller's expected profit from a price menu M_ℓ, and then construct an alternative price menu $\bar{M}$ that induces p_{2u}^h and leads to an expected profit higher than the upper bound.

By Lemma 5, $p_{2u}^\ell = p_d + K$ and $v_h = p_a$ hold in an equilibrium that supports p_{2u}^ℓ. At $t = 1$, the seller chooses p_a and p_d to maximize

$$\Pi^\ell(p_a, p_d) = \Big(F(p_a) - F(v_m)\Big)p_d + (1 - F(p_a))p_a + (1 - F(p_a) + F(v_m) - F(p_d + K))(p_d + K) + \int_{v_m}^{p_a} v\mathrm{d}F(v), \tag{21}$$

in which the first two terms are the seller's first-period profits from consumers with $v_i \in [v_m, v_h]$ and $(v_h, \bar{v}]$ respectively, and the last two terms are the seller's expected profits at $t = 2$, respectively, from the unrecognized segment and the recognized segment.

Recall the definition of $\hat{v}(p_d)$ in (4). By $\phi(p_{2u}^{\ell}) = \phi(p_d + K) = F(\hat{v}(p_d))$, and making use of (8), we get $F(v_m) = F\big(\hat{v}(p_d)\big) + F(p_a) - 1$. Thus, (21) can be written as

$$\Pi^{\ell}(p_a, p_d) = \big(1 - F[\hat{v}(p_d)]\big)p_d + (1 - F(p_a))p_a + \pi_{2u}^{\ell}(v_m, p_a) + \int_{v_m}^{p_a} v\mathrm{d}F(v).$$

Since $p_{2u}^{\ell} = p_d + K$ maximizes $(1 - F(v_h) + F(v_m) - F(p))p$, by the envelope theorem, $\frac{d\pi_{2u}^{\ell}(v_m, p_a)}{dp_a} + \frac{d\pi_{2u}^{\ell}(v_m, p_a)}{dv_m}\frac{dv_m}{dp_a} = 0$. Thus, the first-order condition of $\Pi^{\ell}(p_a, p_d)$ with respect to p_a can be written as

$$\begin{aligned}\frac{\partial \Pi^{\ell}}{\partial p_a}(p_a, p_d) &= 1 - F(p_a) - f(p_a)p_a + p_a f(p_a) - v_m f(v_m)\frac{\mathrm{d}v_m}{\mathrm{d}p_a} \\ &= 1 - F(p_a) - v_m f(v_m)\frac{f(p_a)}{f(v_m)} < v_m f(v_m) - v_m f(p_a) \le 0,\end{aligned}$$

in which $\frac{dv_m}{dp_a} = \frac{f(p_a)}{f(v_m)}$ holds because $F(v_m) = F(\hat{v}(p_d)) + F(p_a) - 1$, the first inequality holds because $F(p_a) > 1 - f(v_m)v_m$ in condition (11), and the second inequality holds because $f'(t) \ge 0$ and $p_a = v_h > v_m$.

Since $\Pi^{\ell}(p_a, p_d)$ decreases in p_a, $\Pi^{\ell}(p_a, p_d)$ takes the maximum value when $F(p_a)$ approaches $1 - f(v_m)v_m$ from above asymptotically. This implies $p_{2u}^{\ell} = p_d + K$ approaches v_m from below. Since $p_{2u}^{\ell} = p_d + K < v_m$ implies $p_d < v_m - K$, we have

$$\begin{aligned}\Pi^{\ell}(p_a, p_d) =& (F(v_h) - F(v_m))p_d + (1 - F(v_h))p_a + \int_{v_m}^{v_h} v\mathrm{d}F(v) + (1 - F(v_h) + F(v_m) - F(p_d + K))(p_d + K) \\ <& (F(v_h) - F(v_m))(v_m - K) + (1 - F(v_h))v_h + \int_{v_m}^{v_h} v\mathrm{d}F(v) + (1 - F(v_h))v_m, \end{aligned} \tag{22}$$

in which (22) gives an upper bound on $\Pi^{\ell}(p_a, p_d)$ under price menu M_{ℓ}. Note that this upper bound is obtained when $F(p_a)$ approaches $1 - f(v_m)v_m$ asymptotically from above, following which $p_{2u}^{\ell} = p_d + K \to v_m$, and $v_h = p_a$. When $F(p_a) = 1 - f(v_m)v_m$, M_{ℓ} can no longer be supported in equilibrium because the optimal second-period uniform price is $p_{2u}^{h} = v_h$ instead of $p_{2u}^{\ell} = p_d + K$.

Take v_m and v_h induced by M_{ℓ} with p_a and p_d. Now consider an alternative price menu $\bar{M}$ with $\bar{p}_d + K = \bar{p}_a = v_m$. It follows that $v_m = \bar{p}_a$ and $\bar{p}_{2u} = v_h > \bar{p}_a$ are supported in equilibrium by construction. The seller's expected profit in this case is

$$\Pi^h(\bar{p}_a,\bar{p}_d) =(F(v_h)-F(v_m))\bar{p}_d+(1-F(v_h))\bar{p}_a+\int_{v_m}^{v_h} v\mathrm{d}F(v)+(1-F(v_h))\bar{p}_{2u}$$
$$=(F(v_h)-F(v_m))(v_m-K)+(1-F(v_h))v_m+\int_{v_m}^{v_h} v\mathrm{d}F(v)+(1-F(v_h))v_h>\Pi^\ell(p_a,p_d),$$

in which the second line makes use of the upper bound derived in (22). Thus, a price menu M_ℓ with (p_a,p_d) that induces p_{2u}^ℓ is dominated by an alternative price menu that induces p_{2u}^h.

(ii) In a partial-disclosure equilibrium in which p_{2u}^h is supported, $v_m=p_a$ and $v_h\in(p_a,\bar{v})$ hold by Lemma 5. When $v_m=p_a$, let $p_{2u}^\ell\equiv\delta$. By (8),

$$\delta f(\delta)+F(\delta)=1-F(v_h)+F(p_a),\quad \pi_{2u}^\ell(p_a,v_h)=(1-F(v_h)+F(p_a)-F(\delta))\delta=\delta^2 f(\delta). \tag{23}$$

Let $\big(1-F(v_h)\big)v_h-\pi_{2u}^\ell(p_a,v_h)\equiv\Delta(v_h)$. In the following, we show that when $p_a<\tilde{p}$, condition (10) can be rewritten as the first part of (12), and there always exists a v_h satisfying the condition; when $p_a>\tilde{p}$, (10) is equivalent to $\Delta(v_h)\geq 0$, which can be rewritten as the second part of (12) and a v_h satisfying condition (10) exists if and only if $p_a<\hat{p}_a$ ($\hat{p}_a$ will be defined shortly).

Note that

$$\frac{d\Delta(v_h)}{dv_h}=1-F(v_h)-v_hf(v_h)-\Big(-f(v_h)\delta+\big(1-F(v_h)+F(p_a)-F(\delta)-f(\delta)\delta\big)\frac{d\delta}{dv_h}\Big)$$
$$=1-F(v_h)-(v_h-\delta)f(v_h).$$

Since $\delta f(\delta)+F(\delta)=1-F(v_h)+F(p_a)$ implying $\frac{d\delta}{dv_h}<0$, we have

$$\frac{d^2\Delta(v_h)}{(dv_h)^2}=-f(v_h)-f'(v_h)(v_h-\delta)-(1-\frac{d\delta}{dv_h})f(v_h)<0.$$

Thus, $\Delta(v_h)$ is concave and $\frac{d\Delta(v_h)}{dv_h}$ changes sign at most once on $v_h\in(p_a,\bar{v})$. Note that at $v_h\uparrow\bar{v}$, $\Delta(v_h)$ is negative and decreasing. Thus, whether there exists a $v_h\in(p_a,\bar{v})$ such that $\Delta(v_h)\geq 0$ is crucial for part (ii) of condition (10) to be satisfied.

Since the sign of $\Delta(v_h)$ and $\frac{d\Delta(v_h)}{dv_h}$ at certain v_h depend on whether p_a is higher or lower than $\tilde{p}$, we discuss these two cases $p_a<\tilde{p}$ and $p_a\geq\tilde{p}$ in turn. Recall $\tilde{p}=\frac{1-F(\tilde{p})}{f(\tilde{p})}=g(\tilde{p})$ and $g(\cdot)$ is a decreasing function. It follows that when $p_a \lesseqgtr \tilde{p}$, $F^{-1}\Big(1-f(p_a)p_a\Big) \gtreqless \tilde{p}$ holds.

When $p_a < \tilde{p}$, with $v_h \in (p_a, \bar{v})$ and $v_h \geq \tilde{p}$, part (i) of (10) is equivalent to $v_h \in \Big[\tilde{p}, F^{-1}(1-f(p_a)p_a)\Big]$. Moreover, since $\Delta(v_h)$ is positive at $v_h = F^{-1}\Big(1-f(p_a)p_a\Big)$, $\Delta(v_h) = 0$ has a unique solution on $\Big(F^{-1}\Big(1-f(p_a)p_a\Big), \bar{v}\Big)$. Let $\bar{v}_h(p_a) \in \Big(F^{-1}\Big(1-f(p_a)p_a\Big), \bar{v}\Big)$ be this unique solution. Part (ii) of condition (10) then simplifies to $F^{-1}(1-f(p_a)p_a) < v_h \leq \bar{v}_h(p_a)$. Combining the two intervals of v_h, we have the equivalence of condition (10) and $\tilde{p} \leq v_h \leq \bar{v}_h(p_a)$ for the case $p_a < \tilde{p}$. Furthermore, a v_h satisfying condition (10) always exists.

When $p_a \geq \tilde{p}$, $F^{-1}\Big(1-f(p_a)p_a\Big) \leq \tilde{p}$. With $v_h \in (p_a, \bar{v})$ and $v_h > \tilde{p}$, $1-f(p_a)p_a \leq F(\tilde{p}) < F(v_h)$ holds, and condition (10) reduces to $\Delta(v_h) \geq 0$. Since $\Delta(v_h) < 0$ at $v_h \downarrow p_a$, there are either two solutions or no solutions to $\Delta(v_h) = 0$ on $v_h \in (p_a, \bar{v})$. (i) When $\Delta(v_h) = 0$ does not have a solution, there exists no v_h that satisfies (10) for given p_a. (ii) When there are two solutions to $\Delta(v_h) = 0$, let $\underline{v}_h(p_a) \leq \bar{v}_h(p_a)$ be the solutions, and (10) becomes $v_h \in [\underline{v}_h(p_a), \bar{v}_h(p_a)] \subset (p_a, \bar{v})$. Thus, when $p_a \geq \tilde{p}$, there exists a v_h satisfying condition (10) (equivalently, $\Delta(v_h) \geq 0$) if and only if the set $[\underline{v}_h(p_a), \bar{v}_h(p_a)]$ is nonempty.

Since $\Delta(v_h)$ is concave, is negative and decreasing at $v_h \uparrow \bar{v}$, the set $[\underline{v}_h(p_a), \bar{v}_h(p_a)]$ is non-empty if and only if $\Delta(v_h)$ is increasing at $v_h \downarrow p_a$, and positive at $\frac{d\Delta(v_h)}{dv_h} = 0$, which are equivalent to

$$\frac{d\Delta(v_h)}{dv_h}\Big|_{v_h \downarrow p_a} = 1 - F(p_a) - (p_a - \tilde{p})f(p_a) > 0, \text{ and} \tag{24}$$

$$\Delta(v_h) \geq 0 \quad \text{at} \quad 1 - F(v_h) - (v_h - \delta)f(v_h) = 0, \tag{25}$$

in which (24) makes use of $\delta \uparrow \tilde{p}$ when $v_h \downarrow p_a$. Note that $p_a - g(p_a) < \tilde{p}$ follows from inequality (24). Thus, the set $[\underline{v}_h(p_a), \bar{v}_h(p_a)]$ is non-empty if and only if inequality (25) holds given p_a satisfying $p_a - g(p_a) < \tilde{p}$. Define $\hat{p}_a$ is as $\hat{p}_a \equiv F^{-1}\left(\phi(t - g(t)) + F(t) - 1\right)$, where t is the solution to $(1-F(t))t - f(t-g(t))(t-g(t))^2 = 0$. In the following we show that there exists a v_h satisfying (25) if and only if $p_a \leq \hat{p}_a$ by showing the maximal value of $\Delta(v_h)$ is nonnegative if and only if $p_a \leq \hat{p}_a$.

At $\frac{d\Delta(v_h)}{dv_h} = 1 - F(v_h) - (v_h - \delta)f(v_h) = 0$, $v_h - g(v_h) = \delta$. In combination with the definition of δ, $\phi(\delta) = 1 - F(v_h) + F(p_a)$, the maximizer $v^o(p_a) \in (p_a, \bar{v})$ of $\Delta(v_h)$ for a given p_a can be uniquely pinned down by

$$v^o \equiv v^o(p_a) = \arg\max_{v_h} \Delta(v_h) \Leftrightarrow \phi\Big(v^o - g(v^o)\Big) = 1 - F(v^o) + F(p_a),$$

and the associated maximal value of $\Delta(v_h)$ is given by

$$\bar{\Delta}(p_a) \equiv \Delta(v^o(p_a)) = (1-F(v^o))v^o - f(v^o - g(v^o))(v^o - g(v^o))^2.$$

Since v_o increases in p_a and $\Delta(v^o)$ decreases in v^o, it follows that $\bar{\Delta}(p_a)$ decreases in p_a. Moreover, when $p_a \downarrow \tilde{p}$, $\bar{\Delta}(p_a) \geq \Delta(v_h \downarrow \tilde{p}) > 0$; while when $p_a - g(p_a) \uparrow \tilde{p}$, $\bar{\Delta}(p_a) = \Delta(v_h \downarrow p_a) = (1-F(p_a))p_a - f(\tilde{p})\tilde{p}^2 < 0$. Thus, by the definition of $\hat{p}_a$, we have $v^o(p_a) > v^o(\hat{p}_a)$ and $\bar{\Delta}(p_a) < \bar{\Delta}(\hat{p}_a) = 0$ when $p_a > \hat{p}_a$. On the other hand we have $v^o(p_a) < v^o(\hat{p}_a)$ and $\bar{\Delta}(p_a) > \bar{\Delta}(\hat{p}_a) = 0$ when $p_a < \hat{p}_a$.

Thus, $\bar{\Delta}(p_a) \geq 0$ if and only if $p_a \leq \hat{p}_a$. This concludes the proof of part (ii) of Lemma 6. □

Proof of Lemma 7. The proof of Lemma 7 consists of Claim 1 and Claim 2 below and their respective proofs. Claim 1 characterizes the seller optimal v_h, among all v_h that satisfies condition (12), for every $p_a \leq \hat{p}_a$. In Claim 2 we characterize the seller's optimal first-period price p_a and the associated seller-optimal v_h in a partial-disclosure equilibrium. □

Claim 1. *Suppose $p_a \leq \hat{p}_a$. For every p_a, there exists a v_h that satisfies Condition (12) and is supported in a partial-disclosure equilibrium. In particular, for $p_a \leq \hat{p}_a$,*

$$v_h(p_a) = \begin{cases} v_h^* & \text{if } g\big(\bar{v}_h(p_a)\big) \leq K < \min\Big\{\tilde{p}, g\big(\underline{v}_h(p_a)\big)\Big\} \\ \bar{v}_h(p_a) & \text{if } K < g\big(\bar{v}_h(p_a)\big) \\ \max\{\underline{v}_h(p_a), \tilde{p}\} & \text{if } K \geq \min\Big\{\tilde{p}, g\big(\underline{v}_h(p_a)\big)\Big\} \end{cases}. \qquad (26)$$

Proof of Claim 1. Recall the definition of v_h^* in (16), which is equivalent to $v_h^* = g^{-1}(K)$. By (15), the seller's objective function (13) increases in v_h for $v_h < v_h^*$, and decreases in v_h for $v_h > v_h^*$. When $v_h = v_h^*$ satisfies Condition (12), it naturally forms the seller's optimal choice; otherwise, the optimal v_h that maximizes the seller's expected profit for given p_a is either $\bar{v}_h(p_a)$ or $\max\{\underline{v}(p_a), \tilde{p}\}$, depending on whether the seller's objective increases or decreases in v_h.

Since $g(x) = \frac{1-F(x)}{f(x)}$ is a decreasing function and $\tilde{p} = g(\tilde{p})$, it follows that $v_h^* > \tilde{p}$ is equivalent to $K < \tilde{p}$. Moreover, $\bar{v}_h(p_a)$ decreases in p_a. To see this, recall that $\bar{v}_h(p_a)$ is either the larger solution (when $\tilde{p} \leq p_a \leq \hat{p}_a$) or the unique solution (when $p_a < \tilde{p}$) to $\Delta(v_h) = 0$, and $\Delta(v_h) < 0$ if $v_h > \bar{v}_h(p_a)$. Consider price p_a and the associated $\bar{v}_h(p_a)$, we have $\Delta(v_h) = (1-F(v_h))v_h - \pi_{2u}^{\ell}(p_a, v_h) \leq 0$ for $v_h \geq \bar{v}_h(p_a)$. Now consider a marginally higher price $p_a' = p_a + \epsilon$. Since $\pi_{2u}^{\ell}(t, v_h)$ increases in t

by (23), it holds that $\Delta(v) = (1-F(v))v - \pi^{\ell}_{2u}(p'_a+\epsilon, v) < 0$ for $v \geq \bar{v}_h(p_a)$. Thus, $\bar{v}_h(p'_a) < \bar{v}_h(p_a)$ for $p'_a > p_a$.

In line with Condition (12), we consider respectively $p_a < \tilde{p}$ and $\tilde{p} \leq p_a \leq \hat{p}_a$.

1. Suppose $p_a < \tilde{p}$. Condition (12) reduces to $\tilde{p} \leq v_h \leq \bar{v}_h(p_a)$, which is non-empty as shown in the proof Lemma 6. Because $g(\cdot)$ is a decreasing function, we have $g(\bar{v}_h(p_a)) < \tilde{p}$.

When $g(\bar{v}_h(p_a)) \leq K \leq \tilde{p}$, $\tilde{p} \leq v^*_h = g^{-1}(K) \leq \bar{v}_h(p_a)$ holds. Since v^*_h satisfies condition (12), it is uniquely optimal for the seller.

When $K < g(\bar{v}_h(p_a))$, $v^*_h = g^{-1}(K) > \bar{v}_h(p_a)$. Since seller's profit increases in v_h on $v_h \leq v^*_h$, the seller's optimal v_h is uniquely given by $v_h = \bar{v}_h(p_a)$.

When $K > \tilde{p}$, $v^*_h = g^{-1}(K) < \tilde{p}$. Since seller's profit decreases on $v_h \geq v^*_h$, the seller's optimal v_h is uniquely given by $v_h = \tilde{p}$.

2. Suppose $\tilde{p} \leq p_a \leq \hat{p}_a$. Condition (12) reduces to $\underline{v}_h(p_a) \leq v_h \leq \bar{v}_h(p_a)$. By Lemma 6, $[\underline{v}_h(p_a), \bar{v}_h(p_a)]$ is non-empty for $p_a \in [\tilde{p}, \hat{p}_a]$.

When $g(\bar{v}_h(p_a)) \leq K \leq g(\underline{v}_h(p_a))$, $\underline{v}_h(p_a) \leq v^*_h = g^{-1}(K) \leq \bar{v}_h(p_a)$. Since v^*_h satisfies condition (12), the seller's optimal v_h is uniquely given by $v_h = v^*_h$.

When $K < g(\bar{v}_h(p_a))$, $v^*_h = g^{-1}(K) > \bar{v}_h(p_a)$. Since seller's profit increases on $v_h \leq v^*_h$, the seller's optimal v_h is uniquely given by $v_h = \bar{v}_h(p_a)$.

When $K > g(\underline{v}_h(p_a))$, $v^*_h = g^{-1}(K) < \underline{v}_h(p_a)$. Since seller's profit decreases on $v_h \geq v^*_h$, the seller's optimal v_h is uniquely given by $v_h = \underline{v}_h(p_a)$.

Combining the above two cases, (26) fully characterizes the induced v_h in a partial-disclosure equilibrium for every $p_a \leq \hat{p}_a$. □

Claim 2. *A seller-optimal partial-disclosure equilibrium exists if and only if $K < \tilde{p}$. There exists a $K_a \in (0, g(\bar{v}_h(\hat{p}_a)))$ such that*

1. *when $K \in [K_a, \tilde{p})$, p^*_a and v^*_h constitute the seller-optimal partial-disclosure equilibrium.*

2. *when $K \in [0, K_a)$, $p_a = p^{**}_a < p^*_a$ in which p^{**}_a is the solution to*

$$1 - F(p_a) - 2f(p_a)p_a + Kf(p_a) + \Big[1 - F(\bar{v}_h(p_a)) - Kf(\bar{v}_h(p_a))\Big]\frac{\mathrm{d}\bar{v}_h(p_a)}{\mathrm{d}p_a} = 0,$$

and $v^{**}_h = \bar{v}_h(p^{**}_a)$ constitute the seller-optimal partial disclosure equilibrium.

Proof of Claim 2. Equation (26) in Claim 1 has characterized the seller-optimal v_h, among all v_h that satisfies condition (12), for every $p_a \le \hat{p}_a$. It remains to solve for p_a that maximizes (13), subject to (26). Recall the unconstrained solution p_a^* defined in (16), which can be written as $p_a^* = \frac{1}{2}g(p_a^*) + \frac{1}{2}K$. When p_a^* induces a v_h that satisfies condition (12), p_a^* must be the seller's optimal choice. Notice that p_a^* is increasing in K, combined with $g(\tilde{p}) = \tilde{p}$, when $K < \tilde{p}$, it holds that $p_a^* < \tilde{p}$.

By (26), when p_a^* satisfies $g(\bar{v}_h(p_a^*)) \le K < \min\left\{\tilde{p}, g\left(\underline{v}_h(p_a^*)\right)\right\}$, p_a^* and v_h^* are uniquely supported in a partial-disclosure equilibrium. Since $\underline{v}_h(p)$ is increasing in p and $\underline{v}_h(\tilde{p}) = \tilde{p}$, it follows that $g\left(\underline{v}_h(p_a^*)\right) > \tilde{p}$ for $p_a^* < \tilde{p}$. Thus, the condition $g(\bar{v}_h(p_a^*)) \le K < \min\{\tilde{p}, g(\underline{v}_h(p_a^*))\}$ simplifies to $g\left(\bar{v}_h(p_a^*)\right) \le K < \tilde{p}$, which is equivalent to $p_a^* \le \bar{v}_h^{-1}\left(g^{-1}(K)\right)$ and $K < \tilde{p}$. Define K_a to be the solution to

$$p_a^* = \bar{v}_h^{-1}\left(g^{-1}(K)\right). \tag{27}$$

In the following, the proof proceeds in three steps. Step 1, we establish the existence and uniqueness of K_a. Step 2, we show that when $K \in [K_a, \tilde{p})$, (p_a^*, v_h^*) constitute the unique seller-optimal partial disclosure equilibrium; when $K < K_a$, (p_a^{**}, v_h^{**}) constitute the seller-optimal partial-disclosure equilibrium. Step 3, we show that no partial-disclosure equilibrium exists when $K > \tilde{p}$.

Step 1: we establish the existence and uniqueness of K_a.

Notice that when $K \to 0$, $p_a^* = \frac{1}{2}g(p_a^*) > 0$, $g^{-1}(K) = \bar{v}$ and $\bar{v}_h^{-1}\left(g^{-1}(K)\right) = 0$. Thus, $p_a^* > \bar{v}_h^{-1}\left(g^{-1}(K)\right)$. When $K \to g(\bar{v}_h(\hat{p}_a))$, $\bar{v}_h^{-1}\left(g^{-1}(K)\right) = \hat{p}_a > \tilde{p}$. And since $g(\bar{v}_h(\hat{p}_a)) < \tilde{p}$, it follows that $p_a^* < \tilde{p}$. Thus, $p_a^* < \bar{v}_h^{-1}\left(g^{-1}(K)\right)$. Thus, there exists a $K_a \in \left(0, g(\bar{v}_h(\hat{p}_a))\right)$ that satisfies (27).

We prove the uniqueness of K_a by showing that $\frac{d\left(\bar{v}_h(p_a^*) - g^{-1}(K)\right)}{dK} > 0$ holds for any K satisfying (27). Suppose not, and there exists a K_a such that $\left.\frac{d\left(\bar{v}_h(p_a^*) - g^{-1}(K)\right)}{dK}\right|_{K=K_a} \le 0$. Then, for $K' = K_a + \epsilon$, where ϵ is infinitely small positive number, $\bar{v}_h(p_a^{*'}) - g^{-1}(K') \le 0$, where $p_a^{*\prime}$ is defined by $p_a^{*\prime} = \frac{1}{2}g(p_a^{*\prime}) + \frac{1}{2}K'$, similarly to p_a^* in (16).

Since $g^{-1}(K)$ decreases in K, there exists $K'' \ge K'$ such that $g^{-1}(K'') = \bar{v}_h(p_a^{*\prime}) \le g^{-1}(K')$. Let $p_a^{*\prime\prime}$ defined by $p_a^{*\prime\prime} = \frac{1}{2}g(p_a^{*\prime\prime}) + \frac{1}{2}K''$. And since $\bar{v}_h(p_a^*)$ decreases in K, $\bar{v}_h(p_a^{*\prime\prime}) \le \bar{v}_h(p_a^{*\prime})$. It follows that $\bar{v}_h(p_a^{*\prime\prime}) - g^{-1}(K'') \le 0$. For any $K^c \in [K', K'']$, $g^{-1}(K'') \le g^{-1}(K^c) \le g^{-1}(K')$, thus $g^{-1}(K^c) \ge \bar{v}_h(p_a^{*\prime}) \ge \bar{v}_h(p_a^{*c})$, where $p_a^{*c} = \frac{1}{2}g(p_a^{*c}) + \frac{1}{2}K^c$. Therefore, $\bar{v}_h(p_a^*) - g^{-1}(K) \le 0$ for all $K \in [K', K'']$.

We can also show that given $\bar{v}_h(p_a^{*''}) - g^{-1}(K'') \le 0$, $\bar{v}_h(p_a^*) - g^{-1}(K) \le 0$ for all $K \in [K'', K''']$, where

$K''' \geq K''$ is defined by $g^{-1}(K''') = \bar{v}_h(p_a^{*''})$. Then by induction, $\bar{v}_h(p_a^*) - g^{-1}(K) \leq 0$ for all $K \in [K_a, \tilde{p})$. However, recall that at $K \to g(\bar{v}_h(\hat{p}_a)) < \tilde{p}$, $p_a^* < \bar{v}_h^{-1}\left(g^{-1}(K)\right)$, so that $\bar{v}_h(p_a^*) - g^{-1}(K) > 0$, which is a contradiction. This establishes the uniqueness of K_a.

We conclude that $p_a^* \leq \bar{v}_h^{-1}\left(g^{-1}(K)\right)$ if and only if $K \in [K_a, \tilde{p})$.

Step 2: we confirm that (p_a^*, v_h^*) and (p_a^{**}, v_h^{**}) are supported in a partial-disclosure equilibrium, respectively, when $K \in [K_a, \tilde{p})$ and $K \in [0, K_a)$.

1. When $K \in [K_a, \tilde{p})$, a price menu M_h with $p_a^* = p_d + K$, following which $v_m = p_a^*$ and $v_h^* > v_m$, and $p_{2u} = v_h^*$ and $p_{2i} = v_i$ at $t = 2$ is uniquely supported in a partial-disclosure equilibrium. Recall that p_a^* and v_h^* are defined in (16).

Observing price menu M_h, consumers with $v_i \in [p_a^*, v_h^*]$ choose (D, p_d), while those with $v_i \in (v_h^*, \bar{v}]$ choose (A, p_a^*), anticipating the second-period uniform price $p_{2u} = v_h^*$ and personalized prices $p_{2i} = v_i$. Given consumers' first-period choices, it is optimal for the seller to post $p_{2u} = v_h^*$ to unrecognized consumers and $p_{2i} = v_i$ to recognized consumers at $t = 2$.

2. When $K < K_a$, $p_a^* > \bar{v}_h^{-1}\left(g^{-1}(K)\right)$, and by (26), $v_h(p_a^*) = \bar{v}_h(p_a^*)$. The seller's problem of choosing $p_a \leq \hat{p}_a$ to maximize (13) reduces to choosing $p_a \in \left(\bar{v}_h^{-1}(g^{-1}(K)), \hat{p}_a\right]$ to maximize

$$\Pi^h(p_a) = (F(\bar{v}_h(p_a)) - F(p_a))(p_a - K) + (1 - F(\bar{v}_h(p_a)))p_a + \int_{p_a}^{\bar{v}_h(p_a)} v \mathrm{d}F(v) + (1 - F(\bar{v}_h(p_a)))\bar{v}_h(p_a). \quad (28)$$

The first-order-condition is

$$\frac{\mathrm{d}\Pi^h(p_a)}{\mathrm{d}p_a} = 1 - F(p_a) - 2f(p_a)p_a + Kf(p_a) + [1 - F(\bar{v}_h(p_a)) - Kf(\bar{v}_h(p_a))]\frac{\mathrm{d}\bar{v}_h(p_a)}{\mathrm{d}p_a} = 0. \quad (29)$$

Note that

$$\frac{\mathrm{d}\Pi^h(p_a)}{\mathrm{d}p_a}|_{p_a \downarrow \bar{v}_h^{-1}(g^{-1}(K))} = 1 - F(p_a) - 2f(p_a)p_a + Kf(p_a) > 0,$$
$$\frac{\mathrm{d}\Pi^h(p_a)}{\mathrm{d}p_a}|_{p_a \geq p_a^*} = \underbrace{1 - F(p_a) - 2f(p_a)p_a + Kf(p_a)}_{\leq 0} + \underbrace{[1 - F(\bar{v}_h(p_a)) - Kf(\bar{v}_h(p_a))]}_{>0}\underbrace{\frac{\mathrm{d}\bar{v}_h(p_a)}{\mathrm{d}p_a}}_{<0} < 0.$$

In the last inequality, $1 - F(p_a) - 2f(p_a)p_a + Kf(p_a) \leq 0$ follows directly from $p_a \geq p_a^*$; since $p_a \geq p_a^* > \bar{v}_h^{-1}\left(g^{-1}(K)\right)$, $\bar{v}_h(p_a) < g^{-1}(K) = v_h^*$ holds, and it follows that $1 - F(\bar{v}_h(p_a)) - Kf(\bar{v}_h(p_a)) > 0$.

Moreover, since $\frac{\mathrm{d}\bar{v}_h(p_a)}{\mathrm{d}p_a} < 0$, we get $\frac{\mathrm{d}\Pi^h(p_a)}{\mathrm{d}p_a}|_{p_a \geq p_a^*} < 0$. Therefore, there exists a p_a^{**} which solves (29) and maximizes the seller's objective (28).

Following a price menu M_h with $p_d = p_a^{**} - K$, the partial-disclosure outcome is supported in equilibrium. Let $v_h^{**} \equiv \bar{v}_h(p_a^{**})$. Observing a price menu with p_a^{**} and $p_d = p_a^{**} - K$, it is optimal for consumers with $v_i \in [p_d + K, v_h^{**}]$ to choose (D, p_d) and consumers with $v_i \in (v_h^{**}, \bar{v}]$ to choose (A, p_a^{**}), anticipating that $p_{2u} = v_h^{**}$ and $p_{2i} = v_i$. Then it is optimal for the seller to set $p_{2u} = v_h^{**}$ and $p_{2i} = v_i$, respectively, for unrecognized and recognized consumers at $t = 2$.

Step 3: we show that the partial-disclosure equilibrium does not exist when $K \geq \tilde{p}$. When $K \geq \tilde{p}$, by (26), $v_h(p_a) = \max\{\underline{v}_h(p_a), \tilde{p}\}$. Moreover, $v_h(p_a) = \tilde{p}$ when $p_a \leq \tilde{p}$ and $v_h(p_a) = \underline{v}_h(p_a)$ when $p_a > \tilde{p}$. The maximization program Γ reduces to the choice of $p_a \leq \hat{p}_a$ to maximize

$$\Pi^h(p_a) = \begin{cases} \tilde{\pi} + (1 - F(p_a))\, p_a + \int_{p_a}^{\tilde{p}} (v - K)\mathrm{d}F(v) & \text{if } p_a \leq \tilde{p} \\ (1 - F(p_a))\, p_a + (1 - F(\underline{v}_h(p_a)))\, \underline{v}_h(p_a) + \int_{p_a}^{\underline{v}_h(p_a)} (v - K)\mathrm{d}F(v) & \text{if } p_a \in (\tilde{p}, \hat{p}_a] \end{cases}. \tag{30}$$

Firstly, by (30) $\Pi^h(p_a \uparrow \tilde{p}) = \Pi^h(p_a \downarrow \tilde{p}) = 2\tilde{\pi}$. Therefore, $\Pi^h(p_a)$ is continuous at $p_a = \tilde{p}$.

Next, consider $K = \tilde{p}$. When $p_a < \tilde{p}$, $1 - \phi(p_a) > 0$, it follows that $\frac{\mathrm{d}\Pi^h(p_a)}{\mathrm{d}p_a} = 1 - F(p_a) - f(p_a)p_a + (\tilde{p} - p_a)f(p_a) > 0$. When $p_a > \tilde{p}$, we have

$$\frac{\mathrm{d}\Pi^h(p_a)}{\mathrm{d}p_a} = \underbrace{\frac{\partial \Pi^h(p_a)}{\partial p_a}}_{<0} + \underbrace{\Big(1 - F(\underline{v}_h(p_a)) - f(\underline{v}_h(p_a))\underline{v}_h(p_a) + (\underline{v}_h(p_a) - K)f(\underline{v}_h(p_a))\Big)}_{=1-F(\underline{v}_h(p_a))-Kf(\underline{v}_h(p_a))<0} \underbrace{\frac{\mathrm{d}\underline{v}_h(p_a)}{\mathrm{d}p_a}}_{>0} < 0$$

where $\frac{\partial \Pi^h(p_a)}{\partial p_a} = 1 - F(p_a) - f(p_a)p_a - (p_a - \tilde{p})f(p_a) < 0$, and moreover, $1 - F(\underline{v}_h(p_a)) - Kf(\underline{v}_h(p_a)) < 0$ since $1 - F(v_h^*) - Kf(v_h^*) = 0$ and $\underline{v}_h(p_a) > v_h^*$. (By similar arguments that $\bar{v}_h(p_a)$ decreases in p_a as shown in the Proof of Claim 1, $\underline{v}_h(p_a)$ increases in p_a). Therefore, at $K = \tilde{p}$, $\Pi^h(p_a)$ increases in p_a when $p_a < \tilde{p}$ and decreases in p_a when $p_a > \tilde{p}$, and $\Pi^h(p_a)$ takes its maximum as $p_a \to \tilde{p}$.

Then, notice that when $K > \tilde{p}$, for all prices $p_a \neq \tilde{p}$, the seller's profit $\Pi^h(p_a)$ is strictly lower than that under $K = \tilde{p}$. Thus, the unique solution for all $K \geq \tilde{p}$ features $p_a \to \tilde{p}$, following which $v_h \to \tilde{p}$. However, with $v_m = v_h$, no consumers choose option D at $t = 1$, and there exists no partial-disclosure equilibrium.

Finally, since p_a^{**} solves (29) and the last term of equation (29) is negative, it follows that

$1-\phi(p_a^{**})-(p_a^{**}-K)f(p_a^{**})>0$ which rewrites as $p_a^{**}<\frac{1}{2}g(p_a^{**})+\frac{1}{2}K$. Comparing this with $p_a^*=\frac{1}{2}g(p_a^*)+\frac{1}{2}K$, by monotonicity of $p-\frac{1}{2}g(p)$, we have $p_a^*>p_a^{**}$. □

Proof of Proposition 2. From Lemma 3 and Lemma 7, when $K\geq\tilde{p}$, the no-disclosure outcome is uniquely supported in an equilibrium. When $K<\tilde{p}$, both the no-disclosure outcome in Lemma 3 and the partial-disclosure outcome in Lemma 7 are supported. In the following, we show that, when $K<\tilde{p}$, the seller's profit is strictly higher in the partial-disclosure outcome than in the no-disclosure counterpart, and thus constitutes a unique subgame perfect equilibrium of the game. Recall that the seller's profit in the no-disclosure equilibrium is $\Pi^{nd}=2\tilde{\pi}$. In the following, we show that the seller's profit in a partial-disclosure equilibrium, $\Pi^h(p_a^A)$ decreases in K and is bounded below by $2\tilde{\pi}$ as $K\to\tilde{p}$. Firstly, $\Pi^h(p_a^A)$ by (13) decreases in K since,

$$\frac{\mathrm{d}\Pi^h(p_a^A)}{\mathrm{d}K}=\begin{cases}-\Big(F(\bar{v}_h(p_a^{**}))-F(p_a^{**})\Big)<0 & \text{if } K\in[0,K_a]\\ -\Big(F(v_h^*)-F(p_a^*)\Big)<0 & \text{if } K\in(K_a,\bar{v}]\end{cases}.$$

In addition, when $K\uparrow\tilde{p}$, $v_h^*\downarrow\tilde{p}$ and it is optimal for the seller to choose $p_a^*\uparrow\tilde{p}$, leading to

$$\Pi^h(p_a^A)\Big|_{K\to\tilde{p}}=2\tilde{\pi},$$

which gives a lower bound of the seller's profit in a partial disclosure equilibrium. Thus, $\Pi^h(p_a^A)>\Pi^{nd}=2\tilde{\pi}$ when $K<\tilde{p}$. The equilibrium prices and consumers' choices follow directly from Lemma 3 and Lemma 7. □

Proof of Proposition 3. (i) By Lemma 2 the seller's profit in a partial-disclosure equilibrium (in which $v_h<\bar{v}$) is strictly higher than that in a disclosure equilibrium (in which $v_h=\bar{v}$). Proposition 2 shows that the equilibrium with anonymous option features partial disclosure (instead of no disclosure) when $K<\tilde{p}$. It follows that the seller is strictly better off with anonymous option.

(ii) The claim follows directly from Proposition 1 and 2. Without anonymous option, when $K\in[K_o,\tilde{p})$, the seller chooses the no disclosure mode rather than the disclosure mode; while in the partial-disclosure equilibrium with anonymous option, consumers with $v_i\in[v_m^A,v_h^A]$ choose option D and disclose their personal information.

(iii) For $K \in [\max\{K_o, K_a\}, \tilde{p})$, consumer surplus without anonymous option is

$$CS^{NA} = CS^{nd} = 2\int_{\tilde{p}}^{\bar{v}} (v - \tilde{p})\mathrm{d}F(v).$$

Using the equilibrium outcome in Proposition 2, consumer surplus with anonymous option is

$$CS^{A} = \int_{p_a^*}^{\bar{v}} (v - p_a^*)\mathrm{d}F(v) + \int_{v_h^*}^{\bar{v}} (v - v_h^*)\mathrm{d}F(v),$$

where the two terms are respectively the first- and the second-period consumer surplus. Let

$$\Delta CS^A \equiv CS^A - CS^{NA} = \int_{p_a^*}^{\tilde{p}} (1 - F(v))\mathrm{d}v - \int_{\tilde{p}}^{v_h^*} (1 - F(v))\mathrm{d}v.$$

It follows that,

$$\frac{d\Delta CS^A}{dK} = -(1 - F(p_a^*))\frac{dp_a^*}{dK} - (1 - F(v_h^*))\frac{dv_h^*}{dK} = -(1 - F(p_a^*))\frac{1}{2 - g'(p_a^*)} - (1 - F(v_h^*))\frac{1}{g'(v_h^*)}.$$

At $K \uparrow \tilde{p}$, p_a^* and v_h^* both converge to $\tilde{p}$. Thus, $\Delta CS^A\big|_{K\uparrow\tilde{p}} = 0$ and

$$\frac{d\Delta CS^A}{dK}\Big|_{K\uparrow\tilde{p}} = -(1 - F(\tilde{p}))\frac{2}{g'(\tilde{p})(2 - g'(\tilde{p}))} > 0.$$

Since ΔCS^A converges to zero and is locally increasing when $K \uparrow \tilde{p}$, there exists a non-empty set of K in the interval $[\max\{K_o, K_a\}, \tilde{p})$ where $CS^A < CS^{NA}$.

(iv) With anonymous option, when $K < \tilde{p}$, consumers with $v_i \geq v_m^A = p_d^A + K$ consume the product in both periods, while consumers with $v_i \in [v_m^A, v_h^A]$ where $v_h^A = p_{2u}^A$ suffer an intrinsic privacy loss K when disclosing their information at $t = 1$. Thus, social welfare can be written as

$$W^A = CS^A + \Pi^A = 2\int_{p_d^A + K}^{\bar{v}} v\mathrm{d}F(v) - \Big(F(p_{2u}^A) - F(p_d^A + K)\Big)K.$$

The derivative of W^A with respect to K gives us

$$\begin{aligned}\frac{\mathrm{d}W^A}{\mathrm{d}K} &= \Big(-2(p_d^A+K)f(p_d^A+K)+Kf(p_d^A+K)\Big)\frac{1}{2-g'(p_d^A+K)} - Kf(p_{2u}^A)\frac{1}{g'(p_{2u}^A)} - \Big(F(p_{2u}^A)-F(p_d^A+K)\Big)\\ &= -(2p_d^A+K)\frac{f(p_d^A+K)}{2-g'(p_d^A+K)} - Kf(p_{2u}^A)\frac{1}{g'(p_{2u}^A)} - \Big(F(p_{2u}^A)-F(p_d^A+K)\Big).\end{aligned}$$

At $K \uparrow \tilde{p}$, W^A is locally increasing with K because

$$\frac{\mathrm{d}W^A}{\mathrm{d}K}\bigg|_{K\uparrow\tilde{p}} = (K-2\tilde{p})\frac{f(\tilde{p})}{2-g'(\tilde{p})} - Kf(\tilde{p})\frac{1}{g'(\tilde{p})} = \frac{2f(\tilde{p})}{(2-g'(\tilde{p}))g'(\tilde{p})}\Big((K-\tilde{p})g'(\tilde{p})-K\Big) > 0.$$

On the other hand, in the no anonymous option regime, the seller chooses the no-disclosure mode when $K \geq K_o$ in which $K_o \in [0,\tilde{p})$, which leads to a level of social welfare W^{NA} that is continuous and constant in K. Moreover, W^A is continuous at $K \to \tilde{p}$ since both $p_d^A + K$ and p_{2u}^A converge to $\tilde{p}$, thus,

$$W^A\Big|_{K\uparrow\tilde{p}} = 2\int_{\tilde{p}}^{\bar{v}} v\mathrm{d}F(v) - (F(\tilde{p})-F(\tilde{p}))K = 2\int_{\tilde{p}}^{\bar{v}} v\mathrm{d}F(v) = W^{NA}.$$

Therefore, there exists a threshold $K_w \in (K_o, \tilde{p})$ such that $W^A < W^{NA}$. □

Proof of Remark 3. We prove the remark by differentiating two cases, $p_d > p_{2u}$ and $p_d \leq p_{2u}$, where p_{2u} is the anticipated second-period uniform price. We first show that the optimal second-period price satisfies $\hat{p}_{2u} = \frac{1}{2}$ in both cases, and in equilibrium, $p_d \leq p_{2u}$ holds. We then solve for the optimal first-period price p_d, and compute the seller's expected profit.

Suppose $p_d > p_{2u}$. By (17), no consumer makes a purchase at $t = 1$. As a result, all consumers remain unrecognized at $t = 2$. The optimal second-period uniform price is given by $\hat{p}_{2u} = \frac{1}{2}$, leading to the seller's total expected profit $\Pi^d = \frac{1}{4}$.

Suppose $p_d \leq p_{2u}$. By (17), a consumer with (v_i, K_i) purchases at $t = 1$ if and only if $v_i \geq p_d + K_i$ and $K_i \leq p_{2u} - p_d$. At $t = 2$, the unrecognized segment contains consumers with either $v_i \leq p_d + K_i$ or $K_i \geq p_{2u} - p_d$. When the second-period uniform price p is larger than the anticipated p_{2u}, that is $p \geq p_{2u}$, among the consumers with $K_i \geq p_{2u} - p_d$, those with $v_i \geq p$ will purchase at the uniform price p. This leads to a second-period demand $D_{2u}(p) = (1-p)(1-p_{2u}+p_d)$ from the unrecognized segment. When $p < p_{2u}$, the demand in the unrecognized segment is higher, because consumers with $v_i \leq p_d + K_i$ and $K_i < p_{2u} - p_d$ in the unrecognized segment will purchase at $t = 2$ if $v_i \in (p, p_{2u})$.

Therefore, the seller's maximization problem at $t = 2$ for the unrecognized segment can be formalized as choosing uniform price p to maximize

$$\pi_{2u}(p) = D_{2u}(p)p = \begin{cases} (1-p)(1-p_{2u}+p_d)p & \text{if} \quad p \geq p_{2u} \\ \left[(1-p)(1-p_{2u}+p_d) + \frac{1}{2}(p_{2u}-p)^2\right]p & \text{if} \quad p_d \leq p < p_{2u} \end{cases}.$$

The derivative with respect to p is

$$\frac{d\pi_{2u}(p)}{dp} = \begin{cases} (1-p_{2u}+p_d)(1-2p) & \text{if} \quad p \geq p_{2u} \\ (1-p_{2u}+p_d)(1-2p) + \frac{1}{2}(p_{2u}-p)(p_{2u}-3p) & \text{if} \quad p_d \leq p < p_{2u} \end{cases}.$$

In equilibrium, the anticipated second-period price must be consistent with the seller's optimal second-period choice. Thus, the following must hold at $p = p_{2u}$

$$\frac{d\pi_{2u}(p)}{dp} \leq 0 \text{ if } p \geq p_{2u}, \quad \text{and} \quad \frac{d\pi_{2u}(p)}{dp} \geq 0 \text{ if } p_d \leq p < p_{2u}.$$

Thus, it is necessary that $\hat{p}_{2u} = \frac{1}{2}$ in equilibrium with $p_d \leq p_{2u}$.

Taking into account that a consumer with (v_i, K_i) purchases at $t = 1$ *iff* $v_i \geq p_d + K_i$ and $K_i \leq \hat{p}_{2u} - p_d$, the seller's expected profit when setting a price $p_d \leq \frac{1}{2}$ is then written as follows:

$$\begin{aligned} \widehat{\Pi}^d(p_d) &= \underbrace{\frac{1}{2}(\frac{3}{2}-p_d)(\frac{1}{2}-p_d)}_{D_1}p_d + \underbrace{\frac{1}{2}(\frac{1}{2}+p_d)}_{D_{2u}(\hat{p}_{2u})}\frac{1}{2} + \int_0^{\frac{1}{2}-p_d}\int_{p_d+K}^{1} v\mathrm{d}v\mathrm{d}K \\ &= \frac{17+6p_d-48(p_d)^2+32(p_d)^3}{48}, \end{aligned}$$

in which D_1 is the mass of consumers who purchase at $t = 1$ given price p_d, and $D_{2u}(\hat{p}_{2u})$ is the mass of consumers from unrecognized consumers who purchase at price $\hat{p}_{2u}$ at $t = 2$, and the third term is the seller's profit from recognized consumers at $t = 2$. Thus, the seller's unique optimal price choice is $\hat{p}_d = \frac{2-\sqrt{3}}{4}$, leading to the overall profit $\widehat{\Pi}^d(\hat{p}_d) = \frac{4+\sqrt{3}}{16}$, which is strictly higher than the total profit $\frac{1}{4}$ when the prices satisfy $p_d > \hat{p}_{2u}$.

Thus, we conclude that the equilibrium outcome under the disclosure mode is given by $\hat{p}_d =$

$\frac{2-\sqrt{3}}{4}$ and $\hat{p}_{2u} = \frac{1}{2}$, with the seller's total profit $\widehat{\Pi}^d = \widehat{\Pi}^d(\hat{p}_d) = \frac{4+\sqrt{3}}{16}$. □

Proof of Proposition 5. To characterize the SPE of the game with anonymous option under two-dimensional private information, we differentiate two cases, $p_d \leq p_a < p_{2u}$ and $p_d \leq p_{2u} \leq p_a$, and analyze the optimal prices in each case in sequence.

Case 1: $p_d \leq p_a < p_{2u}$ in which p_{2u} is the anticipated second-period price. No consumers purchase their first units at $t = 2$. For p_{2u} to be sequentially optimal, it maximizes the seller's profit from the unrecognized segment at $t = 2$,

$$p_{2u} = \arg\max_{p} \ \pi_{2u}(p) = D_{2u}(p)p,$$

where the second-period demand from the unrecognized segment is

$$D_{2u}(p) = \begin{cases} 1-p & \text{if} \quad p \geq p_a - p_d + p_{2u} \\ 1-p-\frac{1}{2}(p_a-p_d+p_{2u}-p)^2 & \text{if} \quad p_{2u} \leq p \leq p_a - p_d + p_{2u} \\ 1-p_{2u}-\frac{1}{2}(p_a-p_d)^2+(p_{2u}-p)(1-p_a+p_d) & \text{if} \quad p_a \leq p < p_{2u} \end{cases}.$$

The derivatives of $\pi_{2u}(p)$ with respect to p are given by

$$\frac{d\pi_{2u}(p)}{dp}\Big|_{p_{2u}\leq p\leq p_a-p_d+p_{2u}} = 1-2p-\frac{1}{2}\Big((p_a-p_d+p_{2u}-p)^2-2(p_a-p_d+p_{2u}-p)p\Big),$$
$$\frac{d\pi_{2u}(p)}{dp}\Big|_{p_a\leq p<p_{2u}} = 1-p_{2u}-\frac{1}{2}(p_a-p_d)^2+(1-p_a+p_d)(p_{2u}-2p).$$

For p_{2u} satisfying $p_d \leq p_a \leq p_{2u}$ to be optimal at $t = 2$, it is necessary that

$$\frac{d\pi_{2u}(p))}{dp}\Big|_{p\geq p_{2u}} \leq 0 \quad \text{and} \quad \frac{d\pi_{2u}(p))}{dp}\Big|_{p<p_{2u}} \geq 0$$

hold at $p = p_{2u}$, leading to

$$p_{2u} = \frac{1-\frac{1}{2}(p_a-p_d)^2}{2-p_a+p_d}. \tag{31}$$

The seller's profit when setting $p_d \le p_a$ can then be written as follows:

$$\widehat{\Pi}^1(p_a, p_d) = \underbrace{\frac{1}{2}(2p_{2u} - 2p_d)(p_a - p_d)}_{D_{1D}} p_d + \underbrace{\Big(1 - p_a - \frac{1}{2}(p_a - p_d)^2 - (p_{2u} - p_a)(p_a - p_d)\Big)}_{D_{1A}} p_a$$
$$+ \underbrace{\Big(1 - p_{2u} - \frac{1}{2}(p_a - p_d)^2\Big)}_{D_{2u}(p_{2u})} p_{2u} + \underbrace{\int_0^{p_a - p_d} \int_{p_d + K}^{p_a - p_d + p_{2u} - K} v \, \mathrm{d}v \mathrm{d}K}_{= \frac{1}{2}(p_a - p_d)(p_{2u} - p_d)(p_a + p_{2u})},$$

in which D_{1D} is the mass of consumers choosing option D, D_{1A} is the mass of consumers choosing option A at $t = 1$, $D_{2u}(p_{2u})$ is the mass of consumers from the unrecognized segment who purchase at the uniform price p_{2u} at $t = 2$, and the last term is the seller's second-period profit from the recognized segment. Solving the maximization problem subject to (31) and $p_d \le p_a$ gives $\hat{p}_d^1 = 0.1881$ and $\hat{p}_a^1 = 0.5251$. Observing these prices and rationally anticipating $\hat{p}_{2u}^1 = 0.5672$, consumers with $v_i \in [0.1881 + K_i, 0.9042 - K_i]$ and $K_i \le 0.337$ choose option D, others with $v_i \ge \hat{p}_a^1 = 0.5251$ choose option A, and those with $v_i < \hat{p}_a^1 = 0.5251$ choose option N. At $t = 2$ the optimal uniform price is indeed $\hat{p}_{2u}^1 = 0.5672$. The total seller profit in this case is $\widehat{\Pi}^1(\hat{p}_a^1, \hat{p}_d^1) = 0.5192$.

Case 2: $p_d \le p_{2u} \le p_a$ in which p_{2u} is the anticipated second-period price. A consumer chooses option D if $p_d + K_i \le v_i \le p_a - p_d + p_{2u} - K_i$ and $K_i \le p_{2u} - p_d$. The remaining consumers choose option A if $v_i \ge p_a$ and choose option N otherwise. Consumers with $v_i \in [p_{2u}, p_a]$ and $K_i \ge p_{2u} - p_d$ wait until the second period to purchase their first units.

Given p_a, p_d and the anticipated p_{2u}, the seller chooses p at $t = 2$ to maximize his profit from the unrecognized segment, and p_{2u} is consistent with the seller's optimal price choice at $t = 2$:

$$p_{2u} = \arg\max_p \ \pi_{2u}(p) = D_{2u}(p)p$$
$$= \begin{cases} \Big(1 - p_a - \frac{1}{2}(p_{2u} - p_d)^2 + (p_a - p)(1 - p_{2u} + p_d)\Big)p & \text{if} \quad p_{2u} \le p \le p_a \\ \Big(1 - p - \frac{1}{2}(p_{2u} - p_d)^2 - (p_a - p_{2u})(p_{2u} - p_d) - \frac{1}{2}(p_{2u} - p_d + p - p_d)(p_{2u} - p)\Big)p & \text{if} \quad p < p_{2u} \end{cases}.$$

The derivative of $\pi_{2u}(p)$ with respective to p is

$$\frac{d\pi_{2u}(p)}{dp}\Big|_{p \ge p_{2u}} = 1 - p_a - \frac{1}{2}(p_{2u} - p_d)^2 + (p_a - 2p)(1 - p_{2u} + p_d),$$
$$\frac{d\pi_{2u}(p)}{dp}\Big|_{p \ge p_{2u}} = 1 - 2p - \frac{1}{2}(p_{2u} - p_d)^2 - (p_a - p_{2u})(p_{2u} - p_d) - \frac{1}{2}((p_{2u})^2 - 2p_{2u}p_d + 4p_d p - 3p^2).$$

For the uniform price p_{2u} to be optimal at $t = 2$, the following inequalities must hold at $p = p_{2u}$

$$\frac{d\pi_{2u}(p)}{dp}\,|_{p\geq p_{2u}}\leq 0 \quad \text{and} \quad \frac{d\pi_{2u}(p)}{dp}\,|_{p\leq p_{2u}}\geq 0,$$

leading to

$$1 - p_a - \frac{1}{2}(p_{2u} - p_d)^2 + (p_a - 2p_{2u})(1 - p_{2u} + p_d) = 0. \tag{32}$$

The seller's profit in this case can then be written as:

$$\begin{aligned}\widehat{\Pi}^2(p_a, p_d) = &\underbrace{\frac{1}{2}(2p_a - 2p_d)(p_{2u} - p_d)}_{D_{1D}} p_d + \underbrace{\Big(1 - p_a - \frac{1}{2}(p_{2u} - p_d)^2\Big)}_{D_{1A}} p_a \\ &+ \underbrace{\Big(1 - p_{2u} - \frac{1}{2}(p_{2u} - p_d)^2 - (p_a - p_{2u})(p_{2u} - p_d)\Big)}_{D_{2u}(p_{2u})} p_{2u} + \underbrace{\int_0^{p_{2u}-p_d}\int_{p_d+K}^{p_a-p_d+p_{2u}-K} v\,dv\,dK}_{=\frac{1}{2}(p_a-p_d)(p_{2u}-p_d)(p_a+p_{2u})}.\end{aligned}$$

Maximizing $\widehat{\Pi}^2(p_a, p_d)$ with respect to p_a and p_d, subject to $p_d \leq p_{2u} \leq p_a$ and (32) gives $\hat{p}_d^2 = 0.2077$ and $\hat{p}_a^2 = 0.5706$. Given these prices and rationally anticipating $\hat{p}_{2u}^2 = 0.5706$, the consumers with $v_i \in [K_i + 0.2077, 0.9335 - K_i]$ and $K_i \leq 0.3629$ choose option D, other consumers with $v_i \geq 0.5706$ choose option A, and consumers with $v_i < 0.5706$ do not purchase at $t = 1$. At $t = 2$, consumers in the recognized segment pay $p_{2i} = v_i$, and consumers in the unrecognized segment pay the uniform price $\hat{p}_{2u}^2 = 0.5706$. This leads to the total profit of $\widehat{\Pi}^2(\hat{p}_a^2, \hat{p}_d^2) = 0.5173$.

Since $\widehat{\Pi}^1(\hat{p}_a^1, \hat{p}_d^1) = 0.5192 > \widehat{\Pi}^2(\hat{p}_a^2, \hat{p}_d^2) = 0.5173$, it is optimal for the seller to choose $p_d \leq p_a \leq p_{2u}$ in equilibrium. Thus, the seller's optimal price choices are given by $\hat{p}_a^A = \hat{p}_a^1 = 0.5251$, $\hat{p}_d^A = \hat{p}_d^2 = 0.1881$, $\hat{p}_{2u}^A = \hat{p}_{2u}^1 = 0.5672$.

Finally, we compare the equilibrium outcome with anonymous option to that without anonymous option. Since $\widehat{\Pi}^A = \widehat{\Pi}^1(\hat{p}_a^1, \hat{p}_d^1) = 0.5192 > \widehat{\Pi}^{nd} = 0.5$, the anonymous option strictly increases the seller's profit.

Consumers are on average worse off with the anonymous option because

$$\begin{aligned}\widehat{CS}^A =& \int_0^{\hat{p}_a^A-\hat{p}_d^A}\int_{\hat{p}_d^A+K}^{\hat{p}_a^A-\hat{p}_d^A+\hat{p}_{2u}^A-K}(v-\hat{p}_d^A-K)\mathrm{d}v\mathrm{d}K+\frac{1}{2}\Big((1-\hat{p}_{2u}^A)^2+(1-\hat{p}_a^A)^2\Big)(1-\hat{p}_a^A+\hat{p}_d^A)\\ &+\int_0^{\hat{p}_a^A-\hat{p}_d^A}\int_{\hat{p}_a^A-\hat{p}_d^A+\hat{p}_{2u}^A-K}^{1}(v-\hat{p}_a^A+v-\hat{p}_{2u}^A)\mathrm{d}v\mathrm{d}K\\ =&0.0305+0.1369+0.0541=0.2215<\widehat{CS}^{nd}=\frac{1}{4}.\end{aligned}$$

Total welfare is also lower with the anonymous option:

$$\widehat{W}^A=\widehat{\Pi}^A+\widehat{CS}^A=0.7407<\widehat{W}^{nd}=\widehat{\Pi}^{nd}+\widehat{CS}^{nd}=\frac{3}{4}.$$

□

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