Public Investment Financed by Seigniorage, Money Supply Control and Inflation Dynamics in Sub-Saharan African Countries

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Abstract

In this study, we attempt to construct an overlapping generations model designed to theoretically analyze the macroeconomic situation of sub-Saharan African countries. Our aim is to examine the conditions necessary for the effective functioning of infrastructure development financed by seigniorage and monetary control policies in some sub-Saharan African countries with stagnant macroeconomic performance. We also consider the implications of our model in terms of inflation and population aging. As a result, when the government selects the monetary growth rate that maximizes the long-term growth rate of gross domestic product (GDP), the absolute value of the monetary growth rate elasticity of the private capital–public capital ratio must be equal to the reciprocal of the private capital elasticity of GDP, which is greater than 1. Thus, seigniorage per se is not the cause of economic stagnation in some sub-Saharan African countries. If maximizing social welfare is equivalent to maximizing the long-term growth rate of GDP in terms of selecting the public investment share, then the public investment share elasticity of the private capital–public capital ratio is zero. Moreover, when the initial value of the private capital–public capital ratio is sufficiently low (high) level, inflation (deflation) occurs during the transition process to a steady state. Furthermore, population aging does not necessarily constitute a bottleneck for economic growth in sub-Saharan African countries.

Keywords: Economic growth; Inflation; Infrastructure; Seigniorage; Sub-Saharan Africa

JEL Classification: E60; H54; O40

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1 Introduction

Since the seminal work of Sachs and Warner (1997), poverty and economic growth in sub-Saharan African countries have been the most important and high-profile subjects in the field of development economics (see, e.g., Bolarinwa and Simatele, 2023; Chen *et al.*, 2024; Labidi, 2024). When the macro economy of sub-Saharan African countries is analyzed, the first point to note is that, as highlighted by Combes (2015), the share of seigniorage revenue as a percentage of gross domestic product (GDP) is high compared with other countries and regions.[1] In this regard, as addressed by Adam *et al.* (2009), Sami *et al.* (2016), and Champ *et al.* (2022, pp.73-94), seigniorage is often discussed in relation to inflation. Furthermore, it is noteworthy that population aging has been rapidly progressing in sub-Saharan African countries in recent years (see, e.g., Ojagbemi *et al.*, 2020; Włodarcz *et al.*, 2020; Okoh *et al.*, 2024).[2]

In light of the above situation regarding sub-Saharan African countries, the following questions arise. Why are some sub-Saharan African countries achieving steady economic growth and others stagnating? This can also be viewed as a problem of what conditions are necessary for fiscal and monetary policies to function effectively in sub-Saharan African countries. Moreover, when infrastructure development is implemented using revenue from seigniorage, what relationships exist between economic development (in the sense of an increasing accumulation of the ratio of private capital to public capital, i.e., private capital–public capital ratio) and inflation? Furthermore, does the progress of population aging cause the stagnation of future economic growth in sub-Saharan African countries? Clearly, these are important outstanding problems.

Regarding economic research on sub-Saharan African countries, a large amount of literature exists on empirical analysis; by contrast, very little literature exists on theoretical analysis. Therefore, the purpose of this study is to theoretically approach the aforementioned issues from the perspective of a dynamic macroeconomic model. Specifically, to achieve this objective, we attempt to construct an overlapping generations (OLG) model with an uncertain lifetime that captures the essential characteristics of sub-Saharan African countries. To the best of our knowledge, no other theoretical study has highlighted the characteristics of sub-Saharan African countries in a dynamic macroeconomic model and addressed the issues of seigniorage-financed infrastructure development and monetary control policies, inflation, and population aging within the context of a unified framework.

Incidentally, numerous studies have been conducted on the macroeconomic effects of infrastructure development.[3] In this regard, in most previous theoretical studies, researchers

[1]For other empirical studies on international comparisons of seigniorage as a share of GDP or government expenditure, see Cukierman *et al.* (1992), Click (1998), Aisen and Veiga (2008), and Elkamel (2018).

[2]Generally, population aging in developing countries does not draw as much attention as it does in developed countries. However, population aging is also steadily progressing in developing countries (see, e.g., Shetty, 2012; Pillay and Maharaj, 2013; Bloom and Luca, 2016).

[3]Irmen and Kuehnel (2009) surveyed the major theoretical studies following Barro (1990) in the research field on the macroeconomic effects of productive government spending and public investment. In addition, Gramlich (1994) and Straub (2011) conducted a survey of empirical research in this field.

have considered taxes and government bonds as financial sources for infrastructure development. However, it should be noted that Combes (2015) found that seigniorage accounts for a non-negligible share of government resources in sub-Saharan African countries. In addition, according to Combes *et al.* (2015), the majority of such countries have limited ability to levy taxes and restrained access to international credit markets; therefore, the proportion of seigniorage revenue in the government budget increases. Furthermore, Romer (2019, pp.642-643) stated that wars, falls in export prices, tax evasion, and political stalemate frequently leave governments with large budget deficits. Therefore, considering investors' willingness to buy government bonds, they often do not have sufficient confidence that the government will honor its debts. Thus, the government's only choice is to resort to seigniorage. It follows that macroeconomic policies in seigniorage-dependent countries should be discussed within the context of a framework that incorporates seigniorage. Given this actual situation, when infrastructure development in sub-Saharan countries is analyzed, it is considered appropriate to focus on seigniorage as a financial source. In addition, for the sake of clarity and simplicity in our analysis, we consider only seigniorage revenue as a source of funding for infrastructure development.

Our research is related to the following literature. Assuming resource-rich low-income countries in the framework of a small open economy, Algozhina (2022) constructed a perfect foresight general equilibrium model of representative individuals with an infinite time horizon. Based on this model, Algozhina (2022) examined an optimal rule-based policy of accumulating public capital and its associated public investment path in the model. The resource sector is owned by foreigners and the government. To carry out public investment, the government collects revenue from the natural resource sector, which consists of royalties levied on the production quantity and dividend of resource profits, consumption tax, labor income taxes, and interest income from the sovereign wealth fund. As a result, based on calibration for African countries, Algozhina (2022) found that the front-loaded public investment path is optimal, given an initial one-period resource windfall, public investment inefficiency, and absorptive capacity constraints. However, Algozhina's (2022) model does not take into account seigniorage, which is an economic feature of sub-Saharan African countries.

Other previous studies relevant to our study include those by Crettez *et al.* (2002), Yakita (2008), Maebayashi (2013), and Yanagihara and Lu (2013). All of these studies are characterized by a framework based on OLG models of the type proposed by Diamond (1965). Specifically, Crettez *et al.* (2002) and Yanagihara and Lu (2013) developed an argument related to public policy that uses seigniorage. In particular, Crettez *et al.* (2002) focused on public services that affect household utility and examined the design of seigniorage in optimal monetary policy. In addition, Crettez *et al.* (2002) analyzed the Laffer curve of seigniorage and its relationship with optimal monetary policy. Yanagihara and Lu (2013) considered the role of public education financed by seigniorage in human capital accumulation, and confirmed that there exists an optimal money growth rate that maximizes the economic growth rate along the steady growth path. Yakita (2008) considered a situation in which government revenue from labor income taxes is allocated to new investments in infrastructure and the maintenance of infrastructure. In addition, Yakita (2008) examined the selection of income tax rates to maximize the GDP growth rate and the allocation problem in relation to government spending. Maebayashi (2013) addressed the issue of social security benefits and investment in infrastructure based on the model of Yakita (2008), although Maebayashi (2013) ignored infrastructure maintenance in an effort to simplify the analysis. Note that seigniorage was not considered in the models of Yakita (2008) and Maebayashi (2013).

The insights of Rioja (2008), who analyzed the role of infrastructure maintenance in economic growth, are also noteworthy, although Rioja's (2008) analysis was not based on the OLG model but rather on a continuous-time representative individual model with an infinite time horizon. Considering that many developing countries assign great importance to infrastructure maintenance, Rioja (2003) pointed out the lack of attention to this activity in conventional studies. In addition to Rioja's (2003) finding, our recognition of this issue is derived from the current situation, in which there has been an inadequate theoretical examination of infrastructure development relying on seigniorage, despite seigniorage revenue remaining an important fund-raising instrument for the governments of some developing countries, as mentioned above.

In our model, the cash-in-advance (CIA) constraint plays an important role.[4] More specifically, we assume that households are subject to the CIA constraint, as in the seigniorage-financed public education model of Yanagihara and Lu (2013), and incorporate fiat money into the OLG model of Yakita (2008). Thus, we extend the model to a setting that allows the consideration of seigniorage. However, to enable us to focus on the macroeconomic effects of seigniorage, we only examine revenue from seigniorage as a financial resource for investment in infrastructure and maintenance by the government, similar to Yanagihara and Lu (2013), which was considered to be the case where the government finances public education fully using seigniorage. Under such a model setting, we make it possible to analyze not only infrastructure development issues but also money supply control using a unified framework. In effect, both monetary control and infrastructure development are available as policy options rather than governments being forced to choose one or the other. In addition, a theoretical analysis of infrastructure development that separates investment in infrastructure and maintenance is meaningful for developing a more realistic argument; that is, when examining the macroeconomic policies of sub-Saharan African countries that are highly dependent on seigniorage, we believe that analyzing both investment in infrastructure and its maintenance is more realistic.

Our main results can be summarized as follows: If the government attempts to select the monetary growth rate that maximizes the long-term GDP growth rate through controlling the money supply, then at that level of the monetary growth rate, the mon-

[4] The fundamental premise of the CIA constraint, which is that transactions must be in cash that must be held before the transactions are undertaken, was first presented by Clower (1967).

etary growth rate elasticity of the private capital–public capital ratio must be equal to the reciprocal of the absolute value of the private capital–public capital ratio elasticity of GDP. Regarding the proportion of investment in infrastructure development, at the level of the public investment share (i.e., the ratio of infrastructure investment to government spending) that maximizes the long-term GDP growth rate, the public investment share elasticity of the private capital–public capital ratio must be greater than the private capital–public capital ratio elasticity of the monetary growth rate. Therefore, our model implies that a part of the reason behind sluggish macroeconomic performance in some sub-Saharan African countries may be that the economic conditions described above are not satisfied. Moreover, our model suggests that it is extremely difficult to simultaneously maximize both social welfare and the long-term GDP growth rate. Furthermore, we find that the use of seigniorage does not always cause inflation in sub-Saharan African countries. Regarding the relationship between population aging and the long-term GDP growth rate, population aging does not necessarily constitute a bottleneck for the long-term GDP growth rate in a seigniorage-dependent economy such as sub-Saharan African countries.

The remainder of this paper is organized as follows: In Section 2, we explain the basic structure of the model that takes into account the characteristics of sub-Saharan African countries. In addition, we consider the dynamic properties of the main economic variables. In Section 3, we examine the conditions necessary for the effective functioning of policies relating to infrastructure development and monetary control in some sub-Saharan African countries with stagnant macroeconomic performance from the viewpoints of growth-maximizing and welfare-maximizing policies. In Section 4, we analyze the relationship between the private capital–public capital ratio and inflation. In Section 5, we examine the influence of population aging on economic growth and social welfare. In Section 6, we describe the contributions of our study, summarize the major findings, and present our conclusions. We present the derivations of some equations in the Appendices.

2 The Model

2.1 Households

We consider a closed economy in which at the beginning of period $t = 1, 2, \ldots$, the total number of individuals is N_t. The cohort of individuals who were born at the beginning of period t is called generation t. The individuals are unisex and live for, at most, two periods: a young period and old period. For simplicity, we follow Yakita (2008) by assuming that individuals of generation t become pregnant with n_t children during period t (when young) and give birth at the beginning of period $t+1$ (when old). Moreover, we assume that individuals of generation t live for two periods with probability λ, but die immediately after being born at the beginning of period $t+1$ with probability $1-\lambda$. Therefore, we can also interpret λ as the life expectancy (expected lifespan) at the end of the young period.

Figure 1 depicts the changes in the population of each generation from generation $t-1$ to generation $t+1$.

Generation \ Period	$t-1$	t	$t+1$	$t+2$
$t-1$	Young (N_{t-1})	Old (λN_{t-1})		
t		Young (N_t)	Old (λN_t)	
$t+1$			Young (N_{t+1})	Old (λN_{t+1})

Figure 1: Changes in the population of each generation

As shown in Figure 1, we can confirm that the overlap between the old people of the generation $t-1$ and the young people of the generation t in period t, and the overlap between the old people of the generation t and the young people of the generation $t+1$ in period $t+1$. Regarding the total population in a country, specifically, the total population at the beginning of period t is $\lambda N_{t-1} + N_t$ and the population at the beginning of period $t+1$ is $\lambda N_t + N_{t+1}$. In this overlapping generations economy, individuals supply one unit of labor inelastically while young and receive a wage-based income. They do not work when old. Therefore, because people engage in labor only in their youth, we find from Figure 1 that the labor force population at the beginning of period $t-1$ is N_{t-1}, the labor force population at the beginning of period t is N_t, and the labor force population at the beginning of period $t+1$ is N_{t+1}.

Let us assume that the lifetime utility of an individual of generation t, u_t, depends on consumption when young, c_t; consumption when old, d_{t+1}; and the number of children, n_t. We normalize the time endowment of an individual in the working period to 1. The amount of time spent on prenatal training, which was called child-rearing time by Yakita (2008), is proportional to the number of children. More specifically, if child-rearing time per child is $\theta \in (0,1)$, an individual who gives birth to $n_t \geq 1$ children must allocate θn_t to child-rearing time. Therefore, the time available for labor by an individual is $1 - \theta n_t$.

For each generation, the number of people N_{t+1} of generation $t+1$ when young is equal to $n_t N_t$, which is the total number of births by generation t when old. The total population of a country at the beginning of period t is expressed as $\lambda N_{t-1} + N_t$. Moreover, our model implies that the percentage of older people in the total population (i.e., the rate of aging) is represented by $\lambda/(\lambda + n_{t-1})$.

The lifetime utility of an individual of generation t, u_t, is given by

$$u_t = \log c_t + \lambda\rho \log d_{t+1} + \varepsilon \log n_t, \tag{1}$$

where $\rho \in (0,1)$ represents a subjective discount factor and $\varepsilon > 0$ is the weight of the subutility resulting from the number of children (or the degree of preference for having children). We find that $\partial u_t/\partial n_t = \varepsilon/n_t$. This implies that when the number of children is 1, the marginal utility of having a child is equal to ε. The budget constraint of an individual of generation t in period t is given by

$$c_t + s_t + \frac{M_t}{P_t} = w_t(1 - \theta n_t), \tag{2}$$

where s_t denotes nonmonetary savings, M_t denotes fiat money holdings, P_t is the general price level, and w_t is the wage rate. In this study, money is defined as an asset that does not generate interest. The budget constraint of an individual of generation t in period $t+1$ is given by

$$d_{t+1} = \frac{R_{t+1}s_t}{\lambda} + \frac{M_t}{\lambda P_{t+1}}. \tag{3}$$

In Eq. (3), R_{t+1} is the real gross return on investment in private capital. [5] We assume that private capital depreciates fully at the end of each period.

We assume that part of the funds used for purchasing goods in period $t+1$ must be prepared in the form of money at the beginning of period t. More specifically, we follow Yanagihara and Lu (2013) by assuming that individuals of generation t are subject to the following CIA constraint:

$$M_t \geq \mu P_{t+1} d_{t+1}, \tag{4}$$

where the parameter $\mu \in (0,1)$ represents the level of the CIA constraint.[6] The closer μ is to 1, the tighter the constraint. In addition, we assume that μ satisfies the condition $\mu < \lambda$. Following Yanagihara and Lu (2013), the real rate of return on money that is held is less than the rate of return on private capital, that is, $R_{t+1} > P_t/P_{t+1}$. Consequently, Eq. (4) can be rewritten as

$$M_t = \mu P_{t+1} d_{t+1}. \tag{5}$$

An individual maximizes their lifetime utility as indicated in Eq. (1), subject to the conditions of Eqs. (2), (3), and (5). When an individual pursues utility-maximizing behavior, we find that

$$c_t = \frac{1}{1 + \lambda\rho + \varepsilon} w_t, \tag{6}$$

[5] Note that the probability of survival, λ, is included on the right-hand side of Eq. (3). When an individual is certain to live until the individual is old, that is, when $\lambda = 1$, Eq. (3) takes the same form as the budget constraint of old individuals in the models of Crettez *et al.* (2002) and Yanagihara and Lu (2013).

[6] Conventional monetary economic models include the money-in-the-utility-function (MIUF) model and CIA model. The MIUF model is characterized by the formulation of a utility function that incorporates a real money balance on the assumption that the possession of money affects individuals' utility. Blanchard and Fischer (1989, p. 192) pointed out two shortcomings of the MIUF model-based approach. The first is that the roles played by the actual transactions and money are often overlooked. The second is that it remains unclear what constraints should be placed on the objective function.

$$d_{t+1} = \frac{\lambda\rho}{(1+\lambda\rho+\varepsilon)\Big(\dfrac{\lambda-\mu}{R_{t+1}} + \dfrac{\mu P_{t+1}}{P_t}\Big)} w_t, \tag{7}$$

$$n_t = \frac{\varepsilon}{\theta(1+\lambda\rho+\varepsilon)} \equiv n. \tag{8}$$

Using Eqs. (6) and (7), the allocation of consumption between the periods when the individual is young and when the individual is old for an individual of generation t is expressed as

$$\frac{d_{t+1}}{c_t} = \frac{\lambda\rho}{\dfrac{\lambda-\mu}{R_{t+1}} + \dfrac{\mu P_{t+1}}{P_t}}. \tag{9}$$

Other things being equal, Eq. (8) implies that an increase in the probability of survival in old age serves to reduce the number of children that maximizes utility. Let χ be the percentage of older adults in the total population in equilibrium. Therefore,

$$\chi = \frac{\lambda\theta(1+\lambda\rho+\varepsilon)}{\varepsilon + \lambda\theta(1+\lambda\rho+\varepsilon)}. \tag{10}$$

Based on Eq. (10), the higher the probability of survival λ, the higher the proportion of the elderly population χ. Because λ and χ move in the same direction, the effect of population aging is understood through λ.

2.2 Firms

In this economy, a single homogeneous good is produced, which is regarded as the numéraire, and its price is normalized to unity. To produce homogeneous goods, perfectly competitive firms input private capital and labor. The production function for firm $i \in [1, F]$ has the following Cobb–Douglas form:

$$Y_{i,t} = K_{i,t}^{\alpha}(A_t L_{i,t})^{1-\alpha}. \tag{11}$$

In Eq. (11), $Y_{i,t}$ is the output of firm i, $K_{i,t}$ is the private capital input of firm i, $L_{i,t}$ is the labor input of firm i, and A_t is a measure of labor-augmenting technology. $\alpha \in (0,1)$ is a parameter.

Let Y_t be the aggregate output of homogeneous goods, K_t be the aggregate private capital, and L_t be the aggregate labor; that is, $Y_t \equiv \sum_{i=1}^{F} Y_{i,t}$, $K_t \equiv \sum_{i=1}^{F} K_{i,t}$, and $L_t \equiv \sum_{i=1}^{F} L_{i,t}$. Following Kalaitzidakis and Kalyvitis (2004) and Yakita (2008), A_t is defined as

$$A_t \equiv \frac{K_t^{\beta} G_t^{1-\beta}}{L_t}, \tag{12}$$

where G_t is the aggregate infrastructure (public capital) and $\beta \in (0,1)$ is a parameter. The formulation of A_t shows that the index measure of labor-augmenting technology increases in tandem with the levels of private and public capital. This relationship has a strong positive effect on the production activities of individual firms. However, the greater the total amount of labor, the smaller this benefit. For each firm, A_t is regarded as given.

The profit of firm i, $\Pi_{i,t}$, can be written as $\Pi_{i,t} = L_{i,t}[(1-\alpha)k_{i,t}^{\alpha}A_t^{1-\alpha} - w_t]$, where $k_{i,t} \equiv K_{i,t}/L_{i,t}$. Firm i, which takes R_t and w_t as given, maximizes its profit for a given $L_{i,t}$ by setting

$$R_t = \alpha k_{i,t}^{\alpha-1}A_t^{1-\alpha}. \tag{13}$$

In addition, in the market equilibrium, w_t equals the marginal product of labor corresponding to the value of $k_{i,t}$ that satisfies

$$w_t = (1-\alpha)k_{i,t}^{\alpha}A_t^{1-\alpha}. \tag{14}$$

The condition of Eq. (14) ensures that profit equals zero for any value of $L_{j,t}$.

As is clear from Eqs. (13) and (14), all firms select the same amount of private capital per unit of labor in equilibrium. Considering Eq. (11), GDP, given by $Y_t \equiv \sum_{i=1}^{F} Y_{i,t}$, is equal to

$$Y_t = K_t^{\Omega}G_t^{1-\Omega}, \tag{15}$$

where $\Omega \equiv \alpha + \beta(1-\alpha)$. Using $k_{i,t} = K_t/L_t$ and Eq. (12), Eq. (13) can be rewritten as

$$R_t = \alpha\Big(\frac{K_t}{G_t}\Big)^{\Omega-1}. \tag{16}$$

Therefore, from Eq. (16), the rental rate of private capital, R_t depends on the private capital–public capital ratio, K_t/G_t. Moreover, Eq. (14) can be rewritten as

$$w_t = (1-\alpha)\Big(\frac{K_t}{G_t}\Big)^{\Omega}\Big(\frac{G_t}{L_t}\Big). \tag{17}$$

From Eqs. (16) and (17), we find that the relationship $w_t/R_t = [(1-\alpha)/\alpha](K_t/L_t)$ holds. This means that the factor price ratio of labor and private capital is proportional to the private capital per worker. Moreover, s_t is proportional to w_t in the general equilibrium, as shown in Appendix A.

2.3 Government

We suppose that an integrated government includes not only the central government but also the central bank. Hereafter, we refer to this integrated government as simply the government. Let $\overline{M}_t$ be the total money supply. We assume that the government increases money at a rate of $\nu > 0$, the relationship between $\overline{M}_t$ and $\overline{M}_{t-1}$ is represented by

$$\overline{M}_t = (1+\nu)\overline{M}_{t-1}. \tag{18}$$

The total demand for money is written as $N_t M_t$. From Eq. (18), in the money market equilibrium, the following relationship holds:

$$\begin{aligned} N_t M_t &= \overline{M}_t \\ &= (1+\nu)\overline{M}_{t-1}. \end{aligned} \tag{19}$$

As mentioned in Section 1 , we assume that the government's only source of revenue is seigniorage. The government maintains a balanced budget and allocates a share of revenue from seigniorage to investment in infrastructure and maintenance of infrastructure. Let B_t be the revenue from seigniorage, E_t be the investment in infrastructure, and Z_t be the expenditure on infrastructure maintenance. When the percentage of government spending on infrastructure investment (i.e., the share of expenditure on infrastructure investment) is $\phi \in (0,1)$, the percentage of government spending on infrastructure maintenance (i.e., the share of expenditure on infrastructure maintenance) is $1-\phi$. Moreover, the relationships $E_t = \phi B_t$ and $Z_t = (1-\phi)B_t$ hold. Considering Eq. (18), seigniorage is expressed as $(\nu \overline{M}_{t-1})/P_t$. Therefore, the government's budget constraint in period t is given by

$$\begin{aligned} E_t + Z_t &= B_t \\ &= \frac{\nu}{P_t}\overline{M}_{t-1}. \end{aligned} \tag{20}$$

Infrastructure maintenance has the dual effects of decreasing the depreciation rate of infrastructure and increasing its durability. Infrastructure stock is accumulated based on

$$G_{t+1} = E_t + \big(1-\delta_{G,t}\big)G_t, \tag{21}$$

where $\delta_{G,t} \in (0,1)$ is the depreciation rate of infrastructure. In addition, we assume that a relationship exists between the depreciation rate of infrastructure and the share of expenditure on maintenance as follows:

$$\delta_{G,t} = 1 - \zeta\Big(\frac{Z_t}{B_t}\Big), \tag{22}$$

where $\zeta \in (0,1)$ is a parameter. Substituting Eq. (22) into Eq. (21) yields

$$G_{t+1} = E_t + \zeta\Big(\frac{Z_t}{B_t}\Big)G_t. \tag{23}$$

In Eq. (23), given a share of expenditure on maintenance, the higher ζ, the lower the infrastructure depreciation rate, which encourages the accumulation of infrastructure. However, if ζ is sufficiently small, infrastructure is not steadily accumulated. In this sense, ζ can be interpreted as an indicator of the efficiency of infrastructure maintenance (see Agénor, 2013, pp.41-42).

2.4 Dynamics

Now, we examine the dynamic properties of our model. As is clear from Eq. (15), GDP depends on both private capital and public capital. Therefore, we can gain an understanding of the behavior of GDP by investigating the behaviors of private and public capital during the transition process toward a steady state. The dynamic equation for public capital is expressed as

$$\frac{G_{t+1}}{G_t} = \phi\nu\frac{\alpha\mu}{\lambda-\mu}\left(\frac{K_t}{G_t}\right)^{\Omega} + \zeta(1-\phi), \tag{24}$$

and the dynamic equation for private capital is expressed as

$$\frac{K_{t+1}}{K_t} = \frac{1}{1+\lambda\rho}\left[\lambda\rho(1-\alpha) - (1+\nu)(1+\lambda\rho)\frac{\alpha\mu}{\lambda-\mu}\right]\left(\frac{K_t}{G_t}\right)^{-(1-\Omega)}. \tag{25}$$

See Appendix B for the derivations of Eqs. (24) and (25).

Regarding the private capital–public capital ratio, we define $x_t \equiv K_t/G_t$. Using Eqs. (24) and (25), we obtain

$$\begin{aligned} x_{t+1} &= \left[\frac{\lambda\rho(1-\alpha)(\lambda-\mu) - \alpha\mu(1+\nu)(1+\lambda\rho)}{(1+\lambda\rho)(\lambda-\mu)}\right]\frac{(\lambda-\mu)x_t^{\Omega}}{\phi\nu\alpha\mu x_t^{\Omega} + \zeta(1-\phi)(\lambda-\mu)} \\ &\equiv f(x_t). \end{aligned} \tag{26}$$

When all government revenue from seigniorage is allocated to investment in infrastructure, which means that $\phi = 1$, the private capital–public capital ratio, x_t, is constant over time. This implies that, in the case of $\phi = 1$, no transitional dynamics exist. However, because $0 < \phi < 1$ is assumed, the case in which there are no transitional dynamics is excluded.

Let $\widehat{x}$ be the private capital–public capital ratio when the relationship $x_t = x_{t+1}$ ($t = 0, 1, 2, \ldots$) holds. Figure 2 provides a graphical illustration of Eq. (26) and the 45-degree line.

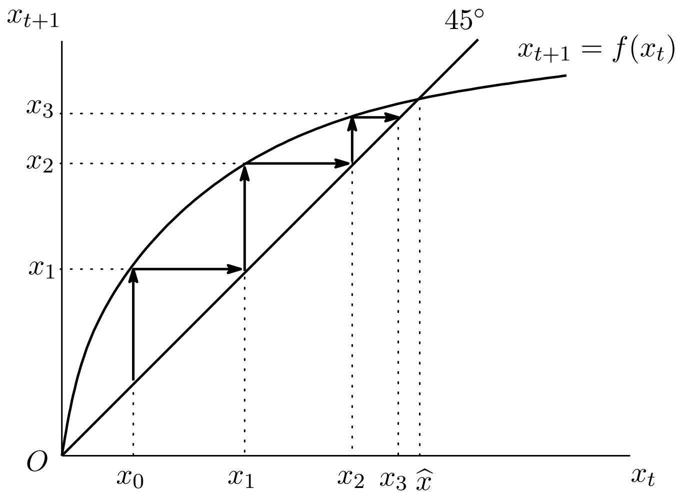

Figure 2: Behavior of the private capital–public capital ratio

It is evident from Figure 2 that when the initial value, x_0, of the private capital–public capital ratio is less than the steady-state value, $\widehat{x}$, this ratio increases monotonously over time and approaches its unique steady-state value, $\widehat{x}$. Conversely, if x_0 is greater than $\widehat{x}$, the private capital–public capital ratio decreases monotonously over time and converges to $\widehat{x}$. Therefore, the steady growth equilibrium of our model is globally stable.

Because the relationship $K_t = \widehat{x}G_t$ holds in the steady state, the growth rates of private capital and public capital become equal. Considering Eq. (24), the gross economic growth rate, Y_{t+1}/Y_t, is represented by

$$\frac{Y_{t+1}}{Y_t} = \Big(\frac{x_{t+1}}{x_t}\Big)^{\Omega}\Big(\frac{G_{t+1}}{G_t}\Big). \tag{27}$$

When $x_t = x_{t+1} = \widehat{x}$, Eq. (27) implies that the gross growth rate of GDP is equal to that of public capital. Consequently, the gross growth rates (hereafter, simply referred to as the growth rates) of GDP, private capital, and public capital become equal in the steady state; that is, the relationship $\gamma \equiv Y_{t+1}/Y_t = K_{t+1}/K_t = G_{t+1}/G_t \geq 1$ holds. Considering Eqs. (24) and (27),

$$\gamma = \phi\nu\frac{\alpha\mu}{\lambda-\mu}\widehat{x}^{\Omega} + (1-\phi)\zeta. \tag{28}$$

Note that the GDP growth rate shown in Eq. (28) is expressed as a weighted average, with the share of expenditure on infrastructure investment ϕ and the share of expenditure on infrastructure maintenance $1-\phi$ as the weights. In addition, GDP per worker is expressed as $y_t \equiv Y_t/N_t$ and the GDP growth rate per worker is expressed as $\widetilde{\gamma} \equiv (y_{t+1}/y_t)$. Therefore, considering Eq. (8), the relationship $\gamma = n\widetilde{\gamma}$ between the growth rate of GDP and the growth rate of GDP per worker holds. Thus, noting from Eq. (8) that n is a constant, maximizing the GDP growth rate has the same meaning as maximizing the GDP growth rate per worker in the steady state of the model.

The relationship between Eqs. (24) and (25) is shown in Figure 3.

Regarding the features of these growth rates, Figure 3 shows that although the growth rate of public capital increases monotonously with an increase in the private capital–public capital ratio, the growth rate of private capital declines monotonously. When the private capital–public capital ratio increases over time, the growth rate of public capital also increases. Therefore, the GDP growth rate increases consistently in accordance with Eq. (27). In addition, we can see from Eq. (27) that the percentage of public capital included in GDP, G_t/Y_t, decreases monotonously.

3 Policy Implications

3.1 Growth-maximizing policy

Generally, policymakers are focused on the promotion of economic growth. This highlights the importance of implications provided by the model from the perspective of growth-promoting policies. We first consider the monetary growth rate that maximizes the GDP growth rate in the steady state (hereafter, the steady economic growth rate) for a given share of expenditure on infrastructure investment in government spending. Subsequently, we examine the share of expenditure on infrastructure investment that maximizes the

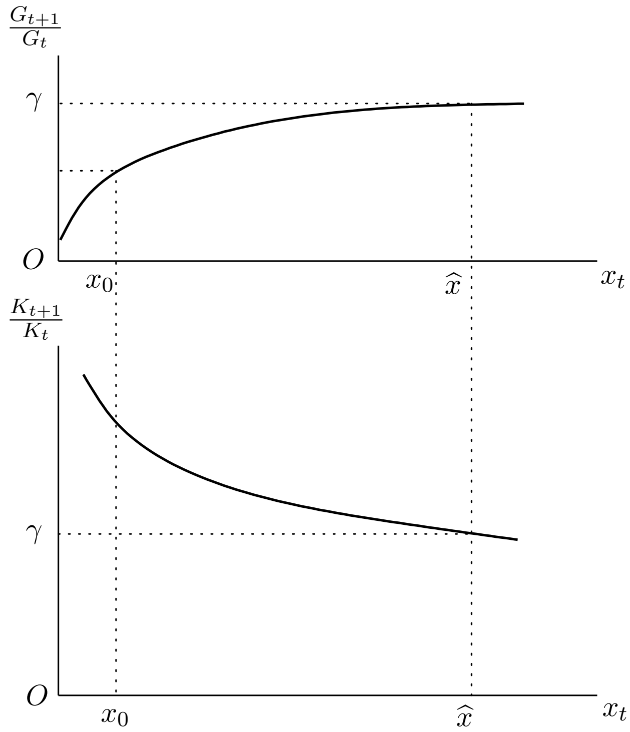

Figure 3: Growth rates of private capital and public capital

steady economic growth rate for a given monetary growth rate. We define $\widetilde{\nu}$ and $\widetilde{\phi}$ as the monetary growth rate and the share of expenditure on infrastructure investment, respectively, that maximize the steady economic growth rate. Clearly, the share of expenditure on infrastructure maintenance that maximizes the steady economic growth rate is $1 - \widetilde{\phi}$.

We investigate the monetary growth rate that maximizes the steady economic growth rate for a given share of expenditure on infrastructure investment. The derivative of Eq. (28) with respect to ν is

$$\frac{\partial \gamma}{\partial \nu} = \frac{\phi \alpha \mu}{\lambda - \mu} \widehat{x}^{\Omega} \Big[1 + \Omega \Big(\frac{\nu}{\widehat{x}} \frac{\partial \widehat{x}}{\partial \nu} \Big) \Big]. \tag{29}$$

When $\nu = \widetilde{\nu}$, the money supply increases in accordance with the rule that $\overline{M}_{t+1} = (1 + \widetilde{\nu})\overline{M}_t$, which is under the control of policymakers. Moreover, because the steady economic growth rate is maximized at $\nu = \widetilde{\nu}$, the relationship $(\partial \gamma / \partial \nu)|_{\nu = \widetilde{\nu}} = 0$ holds. Therefore, Eq. (29) implies that

$$\Big(\frac{\nu}{\widehat{x}} \frac{\partial \widehat{x}}{\partial \nu} \Big) \Big|_{\nu = \widetilde{\nu}} = -\frac{1}{\Omega} < 0. \tag{30}$$

From Eq. (15), note that Ω is the private capital elasticity of GDP. It follows that $1/\Omega$ is the reciprocal of the private capital elasticity of GDP. We call $(\nu/\widehat{x}) \cdot (\partial \widehat{x} / \partial \nu)$ the monetary growth rate elasticity of the private capital–public capital ratio. Given the share of expenditure on infrastructure investment, when the monetary growth rate that maximizes the steady economic growth rate is selected, the absolute value of the monetary growth rate elasticity of the private capital–public capital ratio at the level of the monetary growth rate that maximizes the steady economic growth rate must be equal to $-(1/\Omega)$, from Eq. (30). The condition $0 < \Omega < 1$ implies that the absolute value of the monetary growth rate elasticity of the private capital–public capital ratio at the level of the monetary growth rate that maximizes the steady economic growth is greater than 1 when the government selects the monetary growth rate that maximizes the steady economic growth rate. Furthermore, Eq. (30) implies that the relationship $(\partial \widehat{x}/\partial \nu)|_{\nu=\widetilde{\nu}} < 0$ holds. Hence, in the steady state, the private capital–public capital ratio decreases as the monetary growth rate increases when $\nu = \widetilde{\nu}$.

Regarding the case in which the share of expenditure on infrastructure investment maximizes the steady economic growth rate for a given monetary growth rate, Eq. (28) leads to

$$\frac{\partial \gamma}{\partial \phi} = \frac{\nu \alpha \mu}{\lambda - \mu} \widehat{x}^{\Omega} \Big[1 + \Omega \Big(\frac{\phi}{\widehat{x}} \frac{\partial \widehat{x}}{\partial \phi} \Big) \Big] - \zeta. \tag{31}$$

Note that $(\partial \gamma/\partial \phi)|_{\phi=\widetilde{\phi}} = 0$. Therefore, from Eq. (31), we obtain

$$\Big(\frac{\phi}{\widehat{x}} \frac{\partial \widehat{x}}{\partial \phi} \Big) \Big|_{\phi=\widetilde{\phi}} = -\frac{1}{\Omega} \Big[1 - \frac{\zeta(\lambda - \mu)}{\widehat{x}^{\Omega} \nu \alpha \mu} \Big]. \tag{32}$$

Moreover, Eqs. (30) and (32) imply that

$$\Big(\frac{\phi}{\widehat{x}} \frac{\partial \widehat{x}}{\partial \phi} \Big) \Big|_{\phi=\widetilde{\phi}} > \Big(\frac{\nu}{\widehat{x}} \frac{\partial \widehat{x}}{\partial \nu} \Big) \Big|_{\nu=\widetilde{\nu}} = -\frac{1}{\Omega}.$$

Therefore, the public investment share (i.e., the ratio of infrastructure investment to government expenditure) elasticity of the private capital–public capital ratio at the level of the public investment share that maximizes the steady economic growth rate is greater than the monetary growth rate elasticity of the private capital–public capital ratio at the level of the monetary growth rate that maximizes the steady economic growth rate.

Now, we define T as

$$T \equiv \frac{\zeta(\lambda - \mu)}{\widehat{x}^{\Omega} \widetilde{\nu} \alpha \mu},$$

and $\overline{x}$ as

$$\overline{x} = \Big\{ \frac{\zeta(\lambda - \mu)}{\nu \alpha \mu} \Big\}^{\frac{1}{\Omega}}.$$

Figure 4 shows the relationship between $\widehat{x}$ and T.

From Figure 4, we find that the sign of the public investment share elasticity of the private capital–public capital ratio is positive when the value of this ratio such as $\widehat{x}(\phi)$ is less than $\overline{x}$. Conversely, when the value of the ratio is greater than $\overline{x}$, the sign of the public investment share elasticity of the private capital–public capital ratio becomes negative. Thus, in an economy in which the private capital–public capital ratio corresponding to the expenditure share of public investment that maximizes the steady economic growth rate is sufficiently small (large), the public investment share elasticity of the private capital–public capital ratio has a positive (negative) value.

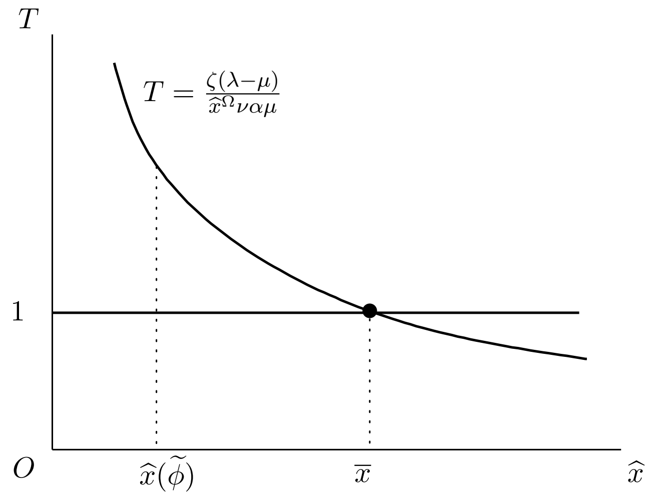

Figure 4: Growth-maximizing private capital–public capital ratio

3.2 Welfare-maximizing policy

We suppose that a benevolent government maximizes social welfare. The policy instruments of the government are designed to control monetary growth and the share of expenditure on infrastructure investment (or maintenance). The monetary growth rate and the share of expenditure on infrastructure investment that maximize social welfare are referred to as the optimal monetary growth rate and the optimal share of public investment (i.e., infrastructure investment as a percentage of government spending), respectively.

Social welfare, U, is given by

$$U = \sum_{t=0}^{\infty} \sigma^t \Big(\log c_t + \frac{\lambda\rho}{\sigma} \log d_t + \varepsilon \log n_t \Big), \tag{33}$$

where $\sigma \in (0,1)$ is the social discount factor.[7] We express the values of c_t, d_t, and n_t in the steady state as $\widehat{c}_t$, $\widehat{d}_t$, and $\widehat{n}$, respectively. In the steady state, Eq. (33) can be rewritten as

$$\begin{aligned} U &= \sum_{t=0}^{\infty} \Big(\sigma^t \log \widehat{c}_t \Big) + \sum_{t=0}^{\infty} \Big(\sigma^t \frac{\lambda\rho}{\sigma} \log \widehat{d}_t \Big) + \Big(\sum_{t=0}^{\infty} \sigma^t \varepsilon \log \widehat{n} \Big) \\ &= \frac{1}{1-\sigma} \log \Big[\frac{1-\alpha}{1+\lambda\rho} \widehat{x}^{\Omega} \Big(\frac{G_0}{N_0} \Big) \Big(\frac{\gamma}{\widehat{n}} \Big)^{\frac{\sigma}{1-\sigma}} \Big] \\ &\quad + \frac{\lambda\rho}{\sigma(1-\sigma)} \log \Big[\frac{\alpha}{\lambda-\mu} \widehat{x}^{\Omega} \Big(\frac{G_0}{N_0} \Big) \widehat{n} \Big(\frac{\gamma}{\widehat{n}} \Big)^{\frac{\sigma}{1-\sigma}} \Big] \\ &\quad + \frac{\varepsilon}{1-\sigma} \log \widehat{n}. \end{aligned} \tag{34}$$

See Appendix C for the derivation of Eq. (34). Differentiating Eq. (34) with respect to ν yields

$$\frac{\partial U}{\partial \nu} = \frac{1}{1-\sigma} \Big(1 + \frac{\lambda\rho}{\sigma} \Big) \Big[\Big(\frac{\Omega}{\widehat{x}} \frac{\partial \widehat{x}}{\partial \gamma} \Big) + \frac{\sigma}{1-\sigma} \Big(\frac{1}{\gamma} \frac{\partial \gamma}{\partial \nu} \Big) \Big]. \tag{35}$$

[7] See de la Croix and Michel (2002, pp.91-92) for the theoretical basis of the social welfare function in an OLG model.

The monetary growth rate that maximizes social welfare is represented by ν^*. Considering the relationship $(\partial U/\partial \nu)|_{\nu=\nu^*} = 0$, Eq. (35) implies that

$$\Big(\frac{\nu}{\widehat{x}}\frac{\partial \widehat{x}}{\partial \nu}\Big)\Big|_{\nu=\nu^*} = -\frac{\sigma}{\Omega(1-\sigma)}\Big(\frac{\nu}{\gamma}\frac{\partial \gamma}{\partial \nu}\Big)\Big|_{\nu=\nu^*}. \tag{36}$$

In Eq. (36), the term $(\nu/\widehat{x}) \cdot (\partial \widehat{x}/\partial \nu)$ represents the the monetary growth rate elasticity of the private capital–public capital ratio and the term $(\nu/\gamma) \cdot (\partial \gamma/\partial \nu)$ represents the monetary growth rate elasticity of the steady economic growth rate. Because $\sigma/[(1-\sigma)\Omega] > 0$, if the sign of the monetary growth rate elasticity of the private capital–public capital ratio is positive, the sign of the monetary growth rate elasticity of the steady economic growth rate is negative. Furthermore, from Eq. (36), we find that

$$\frac{\partial \gamma}{\partial \nu}\Big|_{\nu=\nu^*} \lesseqgtr 0 \Longrightarrow \Big(\frac{\nu}{\widehat{x}}\frac{\partial \widehat{x}}{\partial \nu}\Big)\Big|_{\nu=\nu^*} \gtreqless 0. \tag{37}$$

We can interpret Eq. (37) as follows: If the monetary growth rate that maximizes social welfare and the monetary growth rate that maximizes the steady economic growth rate are equal, at the level of the optimal monetary growth rate, the monetary growth rate elasticity of the private capital–public capital ratio that maximizes social welfare is zero. However, when this elasticity is not zero, the optimal monetary growth rate that maximizes social welfare is not equal to the monetary growth rate that maximizes the steady economic growth rate. Thus, maximizing the steady economic growth rate and maximizing social welfare can be achieved simultaneously when the condition that, at the level of the monetary growth rate that maximizes social welfare, the monetary growth rate elasticity of the private capital–public capital ratio is zero is satisfied.

Regarding the effect of the share of expenditure on infrastructure investment (i.e., public investment share) on social welfare, we obtain

$$\frac{\partial U}{\partial \phi} = \frac{1}{1-\sigma}\Big(1+\frac{\lambda\rho}{\sigma}\Big)\Big[\Big(\frac{\Omega}{\widehat{x}}\frac{\partial \widehat{x}}{\partial \phi}\Big) + \frac{\sigma}{1-\sigma}\Big(\frac{1}{\gamma}\frac{\partial \gamma}{\partial \phi}\Big)\Big]. \tag{38}$$

Let ϕ^* represent the optimal public investment share and let $1-\phi^*$ represent the optimal infrastructure maintenance share that maximizes social welfare. When $\partial U/\partial \phi = 0$, Eq. (38) leads to

$$\Big(\frac{\phi}{\widehat{x}}\frac{\partial \widehat{x}}{\partial \phi}\Big)\Big|_{\phi=\phi^*} = -\frac{\sigma}{\Omega(1-\sigma)}\Big(\frac{\phi}{\gamma}\frac{\partial \gamma}{\partial \phi}\Big)\Big|_{\phi=\phi^*}, \tag{39}$$

where the term $(\phi/\widehat{x}) \cdot (\partial \widehat{x}/\partial \phi)$ represents the public investment share elasticity of the private capital–public capital ratio and the term $(\phi/\gamma) \cdot (\partial \gamma/\partial \phi)$ represents the public investment share elasticity of the steady economic growth rate. Then, Eq. (39) implies that

$$\frac{\partial \gamma}{\partial \phi}\Big|_{\phi=\phi^*} \lesseqgtr 0 \Longrightarrow \Big(\frac{\phi}{\widehat{x}}\frac{\partial \widehat{x}}{\partial \phi}\Big)\Big|_{\phi=\phi^*} \gtreqless 0. \tag{40}$$

Based on Eq. (40), if the public investment share that maximizes social welfare and the public investment share that maximizes the steady economic growth rate are equal, then, at the level of the optimal public investment share, the public investment share elasticity of the private capital–public capital ratio is zero. However, if this elasticity is not zero, the public investment share that maximizes social welfare and the public investment share that maximizes the steady economic growth rate do not match. Hence, both maximizing the steady economic growth rate and maximizing the social welfare can be achieved when the condition under which, at the level of the public investment share that maximizes social welfare, the public investment share elasticity of the private capital–public capital ratio is zero is satisfied.

4 Inflation

What influence do changes in the monetary growth rate and the share of expenditure on infrastructure investment have on the inflation rate? Analyzing this issue is conducive to understanding whether infrastructure development and monetary control in a seigniorage-dependent economy such as Sub-Saharan African countries lead to an increase in the inflation rate. In our model, P_{t+1}/P_t can be written as

$$\frac{P_{t+1}}{P_t} = \Big[\frac{(1+\nu)(1+\lambda\rho)(\lambda-\mu)}{\lambda\rho(1-\alpha)(\lambda-\mu)-\alpha\mu(1+\nu)(1+\lambda\rho)}\Big] x_t^{1-\Omega}. \tag{41}$$

See Appendix D for the derivation of Eq. (41).

Figure 2 shows that, when the initial value of the private capital–public capital ratio, x_0, is sufficiently small, x_t increases over time. Therefore, Eq. (41) implies that if x_0 is sufficiently small, P_{t+1}/P_t increases over time. This means that a sustained increase in the general price level, that is, inflation occurs during the transition process to a steady state. However, as x_t approaches the steady state, the rate of increase of the private capital–public capital ratio gradually falls; that is, the inflation rate declines. Conversely, when the initial value x_0 is higher than the steady-state value $\widehat{x}$, x_t decreases over time, which results in deflation during the transition process to a steady state.

Now, we focus on the steady state and denote the inflation rate in the steady state by $\xi \equiv P_{t+1}/P_t$. For a given share of expenditure on infrastructure investment that maximizes the steady economic growth rate, when the monetary growth rate that maximizes the steady economic growth rate is selected,

$$\begin{aligned}\frac{\partial \xi}{\partial \nu}\Big|_{\nu=\widetilde{\nu}} &= \frac{(1+\lambda\rho)(\lambda-\mu)[\Psi+\alpha\mu(1+\widetilde{\nu})(1+\lambda\rho)]}{\Psi^2}(\widehat{x}|_{\nu=\widetilde{\nu}})^{1-\Omega} \\ &\quad + \frac{(1+\widetilde{\nu})(1+\lambda\rho)(\lambda-\mu)}{\Psi}(1-\Omega)(\widehat{x}|_{\nu=\widetilde{\nu}})^{-\Omega}\,\frac{\partial \widehat{x}}{\partial \nu}\Big|_{\nu=\widetilde{\nu}},\end{aligned} \tag{42}$$

where $\Psi \equiv \lambda\rho(1-\alpha)(\lambda-\mu)-\alpha\mu(1+\widetilde{\nu})(1+\lambda\rho)$. The sign of the first term on the right-hand side of Eq. (42) is positive, whereas the sign of the second term on the right-hand side is negative because $(\partial\widehat{x}/\partial\nu)|_{\nu=\widetilde{\nu}} < 0$ from Eq. (30). Therefore, we find that

$$\frac{\widetilde{\nu}[\Psi+\alpha\mu(1+\widetilde{\nu})(1+\lambda\rho)]}{\Psi(1+\widetilde{\nu})(1-\Omega)} \gtreqless -\Big(\frac{\nu}{\widehat{x}}\frac{\partial \widehat{x}}{\partial \nu}\Big)\Big|_{\nu=\widetilde{\nu}} \Longrightarrow \frac{\partial \xi}{\partial \nu}\Big|_{\nu=\widetilde{\nu}} \gtreqless 0. \tag{43}$$

Therefore, the direction of the change in the inflation rate caused by an increase in the monetary growth rate is dependent on the monetary growth rate elasticity of the private capital–public capital ratio. More specifically, Eq. (43) implies that if the absolute value of the monetary growth rate elasticity of the private capital–public capital ratio at the level of the monetary growth rate that maximizes the steady economic growth rate is sufficiently small (large), an increase in the monetary growth rate leads to an increase (decrease) in the inflation rate at the monetary growth rate that maximizes the steady economic growth rate. This suggests that an increase in the monetary growth rate in the steady state does not necessarily accelerate inflation.

Considering the effects of the share of expenditure on infrastructure investment on the change in the general price level, we obtain

$$\left.\frac{\partial \xi}{\partial \phi}\right|_{\phi=\widetilde{\phi}} = \frac{(1+\nu)(1+\lambda\rho)(\lambda-\mu)(1-\Omega)}{\Psi}\left(\widehat{x}|_{\phi=\widetilde{\phi}}\right)^{-\Omega}\left.\frac{\partial \widehat{x}}{\partial \phi}\right|_{\phi=\widetilde{\phi}}. \tag{44}$$

As a result, Eq. (44) implies that

$$\left.\frac{\partial \widehat{x}}{\partial \phi}\right|_{\phi=\widetilde{\phi}} \gtreqless 0 \Longrightarrow \left.\frac{\partial \xi}{\partial \phi}\right|_{\phi=\widetilde{\phi}} \gtreqless 0. \tag{45}$$

For example, we find from Eq. (45) that, at the public investment share that maximizes the steady economic growth rate, when the private capital–public capital ratio increases as the share of expenditure on infrastructure investment increases, the inflation rate increases in line with the increase in the share of expenditure on infrastructure investment.

Focusing on Eq. (9), inflation has an influence on the allocation of consumption between the period when individuals are young and that when they are old. Considering the relationships expressed by Eqs. (9), (16), (41), and $\gamma \equiv Y_{t+1}/Y_t = K_{t+1}/K_t = G_{t+1}/G_t$ in the steady state, we can confirm that the steady-state value of d_{t+1}/c_t in Eq. (9) depends on $\widehat{x}$. We recall from Eq. (30) that $(\partial \widehat{x}/\partial \nu)|_{\nu=\widetilde{\nu}} < 0$. Therefore, when the monetary growth rate that maximizes the steady economic growth rate is selected, an increase in the monetary growth rate leads to a decrease in the private capital–public capital ratio, which results in an increase in the ratio of consumption when old to consumption when young.

5 Population Aging

As mentioned in Section 1, the impact of population aging on macroeconomic performance has often been the subject of debate in Sub-Saharan African countries. Yakita (2008) considered the case in which the government does not change its policy, among various other factors, and analyzed the relationship between population aging and economic growth. More specifically, Yakita (2008) focused on the steady state of the model and treated the expenditure share of public investment as a constant parameter. The results of analysis by Yakita (2008) indicated an increase in the economic growth rate as the population aged.

However, Yakita (2008) did not closely examine the relationship between population aging and social welfare.

Following Yakita (2008), in this section, we assume the case in which the government does not change its policy regarding the monetary growth rate and the expenditure share of public investment, and proceed to investigate the impact of population aging on the GDP growth rate and social welfare in a steady state. If the monetary growth rate and the expenditure share of public investment are expediently regarded as constant parameters, $(\partial\gamma/\partial\lambda)$ yields

$$\frac{\partial\gamma}{\partial\lambda}\Big|_{\nu,\phi:\text{fixed}} = \phi\nu\frac{\alpha\mu}{\lambda-\mu}\widehat{x}^{\Omega}\Big[\Big(\frac{\lambda}{\widehat{x}}\frac{\partial\widehat{x}}{\partial\lambda}\Big)\Big|_{\nu,\phi:\text{fixed}} - \frac{1}{\lambda-\mu}\Big]. \tag{46}$$

Therefore, from Eq. (46), we obtain

$$\frac{\partial\gamma}{\partial\lambda}\Big|_{\nu,\phi:\text{fixed}} \gtreqless 0 \Longleftrightarrow \Big(\frac{\lambda}{\widehat{x}}\frac{\partial\widehat{x}}{\partial\lambda}\Big)\Big|_{\nu,\phi:\text{fixed}} \gtreqless \frac{\lambda}{\Omega(\lambda-\mu)}. \tag{47}$$

As we can see from Eq. (47), the sign of $(\partial\gamma/\partial\lambda)|_{\nu,\phi:\text{fixed}}$ becomes less certain in the sense that there are three possibilities. Specifically, when the life expectancy elasticity of the private capital–public capital ratio is sufficiently small (large), the steady economic growth rate increases (decreases) as aging progresses. For example, for the comparative indicator $\lambda/[\Omega(\lambda-\mu)]$ consisting of the probability of survival (λ), the private capital elasticity of GDP (Ω), and the level of the CIA constraint (μ), the case of $\Omega = 0.5$, $\lambda = 0.8$, and $\mu = 0.02$ leads to $\lambda/[\Omega(\lambda-\mu)] \cong 2.05$. This means that if the life expectancy elasticity of the private capital–public capital ratio is higher (lower) than 2.05, then the long-term GDPgrowth rate is regarded as increasing (decreasing) as aging progresses.

In the model of Yakita (2008), the private capital–public capital ratio increases because of the progress of aging. However, when the model used in our study considers the monetary growth rate and the expenditure share of public investment as constant parameters, the direction of the changes in the private capital–public capital ratio caused by the progress of aging is unclear. As a result, three possible relationships between population aging and the steady economic growth rate arise, as shown in Eq. (47). The implications of the model used in our study can be summarized as follows: When the life expectancy elasticity of the private capital–public capital ratio is $\lambda/[\Omega(\lambda-\mu)]$ or higher, the progress of population aging does not have a negative effect on the steady economic growth rate. Therefore, our model suggests that , in sub-Saharan African countries where the life expectancy elasticity of the private capital–public capital ratio is sufficiently high, population aging is not necessarily an impediment to the steady economic growth rate.

Next, we analyze the impact of population aging on social welfare. Differentiating both sides of Eq. (34) with respect to λ yields

$$\begin{aligned}
\frac{\partial U}{\partial \lambda} &= \frac{1}{1-\sigma}\Bigg\{ -\frac{\rho}{1+\lambda\rho} + \frac{\Omega}{\lambda}\Big(\frac{\lambda}{\widehat{x}}\frac{\partial \widehat{x}}{\partial \lambda}\Big) + \frac{\sigma}{\lambda(1-\sigma)}\Big[\Big(\frac{\lambda}{\gamma}\frac{\partial \gamma}{\partial \lambda}\Big) + \frac{\lambda\rho}{1+\lambda\rho+\varepsilon}\Big]\Bigg\} \\
&\quad + \frac{\rho}{\sigma(1-\sigma)}\Bigg\{ \log\Big[\frac{\alpha}{\lambda-\mu}\widehat{x}^{\Omega}\Big(\frac{G_0}{N_0}\Big)\widehat{n}\Big(\frac{\gamma}{\widehat{n}}\Big)^{\frac{\sigma}{1-\sigma}}\Big] - \frac{\lambda}{\lambda-\mu} + \Omega\Big(\frac{\lambda}{\widehat{x}}\frac{\partial \widehat{x}}{\partial \lambda}\Big) \\
&\quad + \frac{\sigma}{1-\sigma}\Big(\frac{\lambda}{\gamma}\frac{\partial \gamma}{\partial \lambda}\Big) + \frac{2\sigma-1}{1-\sigma}\Big(\frac{\lambda\rho}{1+\lambda\rho+\varepsilon}\Big)\Bigg\} \\
&\quad - \frac{\rho}{1-\sigma}\Big(\frac{\varepsilon}{1+\lambda\rho+\varepsilon}\Big). \qquad (48)
\end{aligned}$$

The first term on the right-hand side of Eq. (48) represents the change in the partial utility of consumption in the young phase caused by the extension of life expectancy. The second term is the change in the subutility of consumption in the old phase caused by the extension of life expectancy. The third term indicates the change in the subutility of having a child caused by the extension of life expectancy. First, the sign of the third term is negative to aid our understanding. Therefore, the progress of aging causes a decline in the number of children in a steady state and negatively affects social welfare. However, the signs of the first and second terms remain unclear. Whether the sign of $\partial U/\partial \lambda$ in Eq. (48) becomes positive, negative, or zero cannot be observed.

Now, we describe the fact that the first and second terms on the right-hand side of Eq. (48) include $\partial \widehat{x}/\partial \lambda$ and $\partial \gamma/\partial \lambda$. Although their signs are unclear, the following theoretical analysis is possible. If $\partial \widehat{x}/\partial \lambda > 0$, then the private capital–public capital ratio increases as the population ages, which has a positive effect on social welfare. If $\partial \gamma/\partial \lambda > 0$, the steady economic growth rate increases as the population ages, which has a positive effect on social welfare; that is, an increase in the private capital–public capital ratio and an increase in the steady economic growth rate caused by population aging raises the level of social welfare.

6 Conclusion

In the previous sections, based on the OLG model with an uncertain lifetime that takes into account the essential characteristics of sub-Saharan African countries, we theoretically analyzed infrastructure development financed by seigniorage and monetary control policies. Specifically, we examined the preconditions that must be satisfied when attempting to maximize the long-term GDP growth rate and social welfare. In addition, we considered the relationship between the private capital–public capital ratio and inflation (or deflation). Furthermore, we explored how population aging affected the long-term GDP growth rate and social welfare in countries where government expenditure is reliant on seigniorage, such as sub-Saharan African countries.

We recall that in previous economic studies on sub-Saharan African countries, very little research was conducted using a theoretical approach. Therefore, a major contribution of our study is the construction of a tractable model that reflects the essential characteristics of sub-Saharan African countries in a simple form and the effort of conducting a theoretical analysis in the context of a unified framework not found in previous studies on the aforementioned issues. Particularly, this can be seen as a contribution to the development of theoretical research in the field of development economics on sub-Saharan African countries. In addition, another important contribution of our study is that it has expanded horizons in terms of theoretical research on the determinants of long-term GDP growth and social welfare by incorporating into the model elements that were overlooked in previous studies, such as that by Algozhina (2022) with African countries in mind, and has led to new findings. Furthermore, our modeling and theoretical analysis can be regarded as an attempt to complement the previous studies of Crettez *et al.* (2002), Yakita (2008), Maebayashi (2013), Yanagihara and Lu (2013), and Algozhina (2022). Therefore, this can also be considered as our theoretical contribution.

The major findings of our study can be summarized as follows: When the government attempts to select the monetary growth rate that maximizes the long-term GDP growth rate through controlling the money supply, then at that level of the monetary growth rate, the monetary growth rate elasticity of the private capital–public capital ratio must be equal to the reciprocal of the absolute value of the private capital–public capital ratio elasticity of GDP, which is greater than 1. Regarding the proportion of investment in infrastructure development or maintenance in government expenditure, at the level of the public investment share (i.e., the ratio of infrastructure investment to government spending) that maximizes the long-term GDP growth rate, the public investment share elasticity of the private capital–public capital ratio must be greater than the private capital–public capital ratio elasticity of the monetary growth rate, which equals the reciprocal of the absolute value of the private capital elasticity of GDP and its value is greater than 1. Therefore, when economic policies aimed at maximizing the long-term GDP growth rate are implemented, whether such policies are effective depends on the reciprocal of the absolute value of the private capital elasticity of GDP. Thus, based on the implications of our model, the stagnant macroeconomic performance of some sub-Saharan African countries may be partly explained by the situation in which the economic conditions described above have not been satisfied. In addition, the lackluster macroeconomic performance of some countries in sub-Saharan Africa that are relatively highly dependent on seigniorage is not necessarily caused by seigniorage per se.

The monetary growth rate that maximizes social welfare and the monetary growth rate that maximizes long-term GDP growth are the same only when the monetary growth rate elasticity of the private capital–public capital ratio is zero at the level of the monetary growth rate that maximizes social welfare. Moreover, if the public investment share that maximizes social welfare and the public investment share that maximizes the long-term GDP growth rate are the same, then the public investment share elasticity of the private capital–public capital ratio must be zero at the level of the public investment share that maximizes social welfare. These implications of our model suggest that it is extremely difficult to simultaneously maximize both social welfare and the long-term GDP growth rate. Therefore, realistically, depending on the situation in each country in Sub-Saharan Africa, it is likely to prioritize either maximizing social welfare or maximizing the long-term GDP growth rate.

Regarding the pattern of changes in the general price level in response to changes in the private capital–public capital ratio, in an economy in which the private capital–public capital ratio starts from a sufficiently low level, inflation occurs during the transition to a steady state. Conversely, in an economy that starts with a sufficiently high private capital–public capital ratio, deflation occurs during the transition to a steady state; that is, whether inflation or deflation occurs depends on the initial value of the private capital–public capital ratio. Moreover, the direction of the change in the inflation rate caused by an increase in the monetary growth rate is dependent on the monetary growth rate elasticity of the private capital–public capital ratio in the steady state. Specifically, when the absolute value of the monetary growth rate elasticity of the private capital–public capital ratio at the monetary growth rate that maximizes the long-term GDP growth rate is sufficiently small (large), an increase in the monetary growth rate leads to an increase (decrease) in the inflation rate at the monetary growth rate that maximizes the steady economic growth rate. This suggests that an increase in the monetary growth rate in the steady state does not necessarily accelerate inflation. Therefore, the use of seigniorage in sub-Saharan African countries does not necessarily cause inflation from the viewpoint of the model setting used in our study

Whether population aging increases or reduces the long-term GDP growth rate depends on the magnitude of the life expectancy elasticity of the private capital–public capital ratio. For example, when the life expectancy elasticity of the private capital–public capital ratio is sufficiently small (large), the long-term GDP growth rate increases (decreases) as aging progresses. Moreover, our model suggests that, in sub-Saharan African countries where the life expectancy elasticity of the private capital–public capital ratio is sufficiently high, population aging is not necessarily an impediment to the long-term GDP growth rate. Thus, based on our model, the aging of the population does not necessarily become a bottleneck for long-term GDP growth in sub-Saharan African countries. Furthermore, the impact of population aging on social welfare could be positive or negative.

Traditionally, political instability and corruption have been viewed as the main causes of economic stagnation in some sub-Saharan African countries (e.g., Narayan *et al.*, 2011; Araral *et al.*, 2019; Adegboye *et al.*, 2020). However, the implications of our model suggest that, even if the problems of political instability and corruption are resolved, some sub-Saharan African countries with stagnant macroeconomic performance will not improve unless the preconditions to maximize the long-term GDP growth rate described above are satisfied.

Appendix A: Relationship Between Nonmonetary Savings and Wages

We prove that a proportional relationship exists between the nonmonetary savings of households and the wage income that households receive in the general equilibrium. We recall that the depreciation rate of private capital is assumed to be 1. Consequently, in the equilibrium private capital market, aggregate private capital at the beginning of period $t+1$ is equal to aggregate nonmonetary savings in period t; that is,

$$K_{t+1} = N_t s_t. \tag{A1}$$

Moreover, in the labor market equilibrium,

$$L_t = (1 - \theta n_t) N_t. \tag{A2}$$

Considering the CIA constraint in Eq. (5), Eq. (19) can be rewritten as

$$\overline{M}_{t-1} = N_{t-1} \mu P_t d_t, \tag{A3}$$

and Eq. (3) can be rewritten as

$$d_t = \frac{1}{\lambda - \mu} R_t s_{t-1}. \tag{A4}$$

Substituting Eq. (A4) into Eq. (A3) yields

$$\overline{M}_{t-1} = N_{t-1} \frac{\mu}{\lambda - \mu} P_t R_t s_{t-1}. \tag{A5}$$

From Eqs. (16), (21), (A1), and (A5), we obtain

$$m_t = (1+\nu) \frac{\alpha\mu}{\lambda - \mu} \Big(\frac{K_t}{G_t}\Big)^{\Omega - 1} \Big(\frac{K_t}{N_t}\Big), \tag{A6}$$

where $m_t \equiv M_t / P_t$. Using Eq. (A2), Eq. (A6) can be rewritten as

$$m_t = (1+\nu) \frac{\alpha\mu}{\lambda - \mu} \Big(\frac{K_t}{G_t}\Big)^{\Omega - 1} (1 - \theta n_t) \Big(\frac{K_t}{L_t}\Big). \tag{A7}$$

In addition, Eqs. (A7) and (8) lead to

$$m_t = (1+\nu) \frac{\alpha\mu}{\lambda - \mu} \Big(\frac{K_t}{G_t}\Big)^{\Omega - 1} \Big(\frac{K_t}{L_t}\Big) \frac{1 + \lambda\rho}{1 + \lambda\rho + \varepsilon}. \tag{A8}$$

Furthermore, Eqs. (2), (6), and (8) yield

$$s_t + m_t = \frac{\lambda\rho}{1 + \lambda\rho + \varepsilon} w_t. \tag{A9}$$

Combining Eqs. (17), (A8), and (A9), we obtain

$$s_t = \frac{1}{(1-\alpha)(1 + \lambda\rho + \varepsilon)} \Big[\lambda\rho(1-\alpha) - (1+\nu)(1+\lambda\rho)\frac{\alpha\mu}{\lambda - \mu}\Big] w_t. \tag{A10}$$

From Eq. (A10), we can confirm that nonmonetary savings change at the same rate as wages.

Appendix B: Derivation of Equations (24) and (25)

First, we derive the dynamic equation for public capital. Dividing both sides of Eq. (23) by G_t implies that

$$\frac{G_{t+1}}{G_t} = \frac{E_t}{G_t} + \zeta\Big(\frac{Z_t}{B_t}\Big). \tag{B1}$$

Considering the relationships $E_t = \phi B_t$, $Z_t = (1-\phi)B_t$, and Eq. (20), Eq. (B1) can be rewritten as

$$\frac{G_{t+1}}{G_t} = \frac{\phi}{G_t}\frac{\nu}{P_t}\overline{M}_{t-1} + \zeta(1-\phi). \tag{B2}$$

Substituting Eqs. (16), (A1), and (A5) into Eq. (B2) yields Eq. (24).

Next, we derive the dynamic equation for private capital. Eqs. (A1) and (A10) imply that

$$K_{t+1} = N_t\frac{1}{1+\lambda\rho+\varepsilon}\Big[\lambda\rho(1-\alpha) - (1+\nu)(1+\lambda\rho)\frac{\alpha\mu}{\lambda-\mu}\Big]\Big(\frac{K_t}{G_t}\Big)^{\Omega}\Big(\frac{G_t}{L_t}\Big). \tag{B3}$$

Substituting Eq. (A2) into Eq. (B3) yields

$$K_{t+1} = \frac{1}{1+\lambda\rho}\Big[\lambda\rho(1-\alpha) - (1+\nu)(1+\lambda\rho)\frac{\alpha\mu}{\lambda-\mu}\Big]\Big(\frac{K_t}{G_t}\Big)^{\Omega}G_t. \tag{B4}$$

Furthermore, Eq. (B4) can be rewritten to obtain Eq. (25).

Appendix C: Derivation of Equation (34)

Using Eqs. (6), (8), and (17),

$$\widehat{c}_t = \frac{1}{1+\lambda\rho}(1-\alpha)\widehat{x}^{\Omega}\Big(\frac{G_t}{N_t}\Big). \tag{C1}$$

Because the relationships $G_t = \gamma^{\,t} G_0$ and $N_t = \widehat{n}^{\,t} N_0$ hold in the steady state, Eq. (C1) can be rewritten as

$$\widehat{c}_t = \frac{1-\alpha}{1+\lambda\rho}\widehat{x}^{\Omega}\Big(\frac{\gamma}{\widehat{n}}\Big)^{t}\Big(\frac{G_0}{N_0}\Big). \tag{C2}$$

From Eqs. (16), (A1), and (A4), we obtain

$$\widehat{d}_t = \frac{\alpha}{\lambda-\mu}\widehat{x}^{\Omega}\Big(\frac{\gamma}{\widehat{n}}\Big)^{t}\Big(\frac{G_0}{N_0}\Big)\widehat{n}. \tag{C3}$$

Substituting Eqs. (8), (C2), and (C3) into Eq. (33) yields Eq. (34).

Appendix D: Derivation of Equation (41)

From Eqs. (5), (A4), and (A9), we obtain

$$\frac{\lambda\rho}{1+\lambda\rho\varepsilon}w_t - s_t = \frac{\mu}{\lambda-\mu}\Big(\frac{P_{t+1}}{P_t}\Big)R_{t+1}s_t. \tag{D1}$$

Using Eqs. (16), (17), and (A1), Eq. (D1) can be rewritten as

$$\frac{\lambda\rho}{1+\lambda\rho+\varepsilon}(1-\alpha)x_t^{\Omega}\frac{G_t}{L_t} - \frac{K_{t+1}}{N_t} = \frac{\mu}{\lambda-\mu}\Big(\frac{P_{t+1}}{P_t}\Big)\alpha x_t^{\Omega-1}\frac{K_{t+1}}{N_t}. \tag{D2}$$

Substituting Eqs. (A2) and (25) into Eq. (D2) yields Eq. (41).

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Data Availability

No data was used for the research in this article.

Declarations

Conflict of interest

The authors have no relevant financial or non-financial interests to disclose.

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