Pro-Patent Policy in the Knowledge-Based Economy

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Abstract

In a knowledge-based economy, innovation plays a significant role in determining the level of economic growth and social welfare. Meanwhile, patent protection is a pivotal factor for research and development (R&D) incentives, and innovation performance depends on the degree of patent protection. Therefore, elucidating the mechanism for impact of patent protection on innovation; and hence economic growth is a crucial issue from the perspective of macroeconomic policy. Our research questions are twofold. (1) What conditions are necessary for patent protection to effectively promote innovation and economic growth? (2) Can strengthening patent protection enhance social welfare? This study addresses these problems using an expanding variety model of R&D-based endogenous growth. Our major findings are summarized as follows: If an economy satisfies conditions that the productivity in the final goods sector and labor force population are relatively large, while the patent duration elasticity of patent fee is relatively small, extending the patent duration fosters on the rate of innovation, the growth rate of gross domestic product (GDP) per capita, and the growth rate of livelihood-based public infrastructure. Moreover, strengthening patent protection by extending the duration of the patent right does not necessarily enhance social welfare. Furthermore, the patent duration that maximizes social welfare may be shorter than the patent duration that maximizes the growth rate of GDP per capita, the rate of innovation, or the growth rate of livelihood-based public infrastructure.

Keywords: Economic growth, Innovation, Patent duration, Patent fee, R&D, Social welfare

JEL Classification: H54, O32, O34, O38, O41

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1 Introduction

The position of knowledge-intensive industries is becoming increasingly important in achieving sustained economic growth. Therefore, to cope with the intensifying international competition, various measures have been implemented in each country with an emphasis on the protection of intellectual property rights (IPRs). In particular, developed countries have been actively developing laws for pharmaceuticals, biotechnology, and program copyrights, which are considered to be major future industrial sectors and have been inadequately protected by patent rights. Additionally, with the entry into force of the Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS), which is a founding element of the World Trade Organization (WTO), in January 1995, strengthening of IPR protection became an inevitable policy issue for developing countries as well.[1] Consequently, since January 1995, the strengthening of protection of IPRs has become a worldwide trend in both developed and developing countries.

How does strengthening of IPR protection affect the long-term macroeconomic performance in each country? The accumulation of findings of theoretical research on this question is still insufficient, and there is no consensus among researchers.[2] Under the circumstances of a maturing knowledge-based economy in many countries in North America, Europe, and East Asia, clarifying the impact of strengthening IPR protection from a long-term perspective is a crucial issue for predicting future macroeconomic trends. Based on this awareness of the problem, we focus on patent rights, which have received particular attention among IPRs, and address the following two research questions. First, what conditions are necessary for the strengthening of patent protection to function effectively as a policy instrument to promote innovation and economic growth? Second, does a policy to strengthen patent protection enhance social welfare? Our aim is to answer these questions based on a growth-theoretic framework in which takes into account new elements that have been overlooked in previous studies.

The strength of patent rights protection has traditionally been evaluated through two dimensions: duration and breadth. The duration of a patent rights denotes the period during which the patent rights remain in force, while the breadth of the patent rights refers to the scope of products that the patentee can prevent other firms from producing and selling.[3] Therefore, the government would be able to control the degree of patent protection through the duration and/or breadth of patent rights. Previous studies focusing on the aspect of the duration of patent rights include Segerstrom et al. (1990), Michel and Nyssen (1998), Iwaisako and Futagami (2003), Futagami and Iwaisako (2007), Chu (2010), Chen and Iyigun (2011),

[1]When the WTO was launched in 1995, a product of the Uruguay Round, one of its main pillars would be the Agreement on TRIPS. For most WTO members, TRIPS is the introduction of exogenous rules and standards (Archibugi and Filippetti, 2010). Thus, once the Uruguay Round was concluded and the new WTO's rules came into effect in 1995, countries began revising their national laws to come into conformity with TRIPS. According to Arza et al. (2023), because of TRIPS, most developing countries were forced to strengthen IPR protection in their national regulations and patent opportunities became more global.

[2]Regarding a literature review on IPRs and their macroeconomic effects, see Neves et al. (2021).

[3]More precisely, the concept of patent breadth can be categorized into leading and lagging breadth. Leading patent breadth defines the degree of superiority a non-patented product can have compared to the patented product while still being legally produced and sold. In contrast, lagging patent breadth indicates how much inferior a non-patented product must be to meet legal production and sales criteria (Iwaisako and Futagami, 2013). Additionally, note that leading patent breadth represents protection against future innovators, whereas lagging patent breadth represents protection against imitation (O'Donoghue and Zweimüller, 2004).

Sorek (2011), Noda (2012), Lin (2015), Lin and Shampine (2018), Noda and Kano (2021), and Iwaisako (2024).[4] On the other hand, previous studies focusing on the aspect of the breadth of patent rights include Li (2001), Goh and Olivier (2002), O'Donoghue and Zweimüller (2004), Chu et al. (2012), Iwaisako (2013), Iwaisako and Futagami (2013), Pan et al. (2018), Qian (2018), Chu et al. (2020), Tabata (2021), Yang (2021), and Iwaisako (2023).[5] In this study, considering an affinity with the structure of our model, we develop an argument focusing on the aspect of the duration of patent rights, one of the two above-mentioned aspects of patent protection. Our study has partial similarities with those of Michel and Nyssen (1998), and Iwaisako and Futagami (2003), Futagami and Iwaisako (2007), Iwaisako (2013), and Tabata (2021), and is particularly closely related to those of Iwaisako and Futagami (2003) and Futagami and Iwaisako (2007) in terms of an awarness of the problem.

We now mention an important point of discussion regarding the patent system that has been overlooked in the above-mentioned series of previous studies. Under the current patent system, patent holders are obliged to pay a certain amount of patent fees to the patent office in order to maintain their patent rights.[6] Despite the existence of such a rule, previous studies have not taken into account the patent fee, which is a necessary cost to maintain the patent right. Therefore, it appears that the conclusions derived in previous studies require some attention. However, it should not be overlooked that in practice, patent fee is incurred to maintain patent rights in accordance with the provisions of the law. For example, when the patent duration is extended as part of the strengthening of patent protection, the additional patent fee may be a significant burden for the patent holder. All models in the related literature lack analysis from this perspective. For this reason, we embed the patent fee into our model, and thereby fill this gap. Our modeling in a realistic setting that takes the role of patent fee into account can be regarded as an originality of this study.

Some previous studies have considered the role of services derived from public expenditure flow or public capital stock within the context of patent policy (e.g., Iwaisako, 2013; Tabata, 2021). Note that all of those literatures have focused on productive public spending or productive public capital that contributes to increasing private firms' productivity. Although the importance of private firms' productivity-enhancing public services (i.e., industrial-base public infrastructure) is unquestionable, households' utility-enhancing public services (i.e., livelihood-based public infrastructure) is also essential for people's lives, as emphasized in Arrow and Kurz (1970, Ch.4), Baier and Glomm (2001), Chen and Guo (2018), and Fang and Noda (2024). Therefore, it is an important issue to deepen our understanding of the relationship between the strengthening of patent protection and the development of livelihood-based public infrastructure. However, previous studies have not elucidated the impact of the strengthening of patent protection on the development of livelihood-based public infrastructure. For this reason, we incorporate livelihood-based public infrastructure into our model, applying the idea of Arrow and Kurz (1970, Ch.4), Baier and Glomm (2001), Chen and Guo (2018), and Fang and Noda (2024) that households receive benefits from public services pro-

[4] An early seminal work that examined the relationship between patent duration and social welfare based on the framework of a dynamic general equilibrium model is Judd (1985). However, it should be noted that the structure of Judd's (1985) model is an exogenous growth model, not an endogenous growth model.

[5] In Iwaisako and Futagami (2003), Futagami and Iwaisako (2007), and Iwaisako (2024), the issue of breadth of the patent rights is also discussed. However, the main focus of their analysis is on the duration of the patent rights.

[6] In Japan, for example, article 107 of the patent law contains this provision.

vided by the government.[7] To the best of our knowledge, no previous studies have analyzed the effects of strengthening of patent protection on livelihood-bese public infrastructure development, innovation, economic growth, and social welfare within the context of a unified model. In this sense, the present article proposes a novel theoretical framework for analyzing the impacts of patent protection on economic growth and social welfare. Thus, this point is another originality of our study.

We employ the research and development (R&D)-based endogenous growth model in which product innovation shows up as an expansion of the number of varieties of intermediate goods. This implies that we approach the above two research objectives using a type of so-called expanding variety models represented by Romer (1990), Grossman and Helpman (1991, Ch.3), Bénassy (1998), Arnold (2000), Matsuyama (2001), Barro and Sala-i-Martin (2004, Ch.6), Acemoglu (2009, Ch.13), and Bucci et al. (2021). Our model is based in particular on the model of Barro and Sala-i-Martin (2004, Ch.6). However, unlike the model of Barro and Sala-i-Martin (2004, Ch.6), which assumes an infinite patent duration, our analysis is conducted in the setting of a finite patent duration. Thus, our model can be interpreted as a modified version of Barro and Sala-i-Martin's (2004, Ch.6) model. Additionally, our study can also be positioned as a complement to the above series of previous studies. This is because those models share the common but never fully justified assumption that patents are infinitely lived, as pointed out by Michel and Nyssen (1998).

Our main findings can be summarized as follows: When the productivity in the final goods sector and the labor force population are relatively small, while the patent duration elasticity of patent fees is relatively high, extending the duration of patent rights negatively affects on the rate of innovation, the growth rate of gross domestic product (GDP) per capita, and the growth rate of livelihood-based public infrastructure. In contrast, if the productivity in the final goods sector and labor force population are relatively large, while the patent duration elasticity of patent fee is relatively small, then an extension of the patent duration fosters on the rate of innovation, the growth rate of GDP per capita, and the growth rate of livelihood-based public infrastructure. Moreover, our model implies that there is a negative relationship between the duration of the patent rights and social welfare when the extension of the patent duration had no effect on the growth rate of GDP per capita. However, the effect of patent duration on social welfare is unclear when the extension of patent duration has a positive effect on the growth rate of GDP per capita, or when the extension of patent duration has a negative effect on the growth rate of GDP per capita. Therefore, strengthening patent protection by extending the duration of the patent right does not necessarily improve social welfare. Furthermore, our model suggests that the patent duration that maximizes social welfare is shorter than the patent duration that maximizes the growth rate of GDP per capita, the rate of innovation, or the growth rate of livelihood-based public infrastructure.

The remainder of this article is organized as follows: Section 2 reviews earlier studies from perspectives of the duration of the patent right and the breadth of the patent right. Section 3 provides an overview of internationally comparable patent protection data. Section 4 describes basic structure of our closed economy model consisting of final goods sector, intermediate goods production/R&D sector, government, and households. Additionally, the relationships that hold among main variables in a steady-growth equilibrium path are derived. Section

[7] In our modeling, we follow Arrow and Kurz (1970, Ch.4) in considering services derived from public capital stocks rather than services derived from public expenditure flows as treated by Baier and Glomm (2001), Chen and Guo (2018), and Fang and Noda (2024).

5 explores the effects of an extension of duration of patent rights on economic growth and social welfare in a steady-growth equilibrium path. Section 6 summarizes the main findings and concludes. Appendix A explains the process of derivation the equation that expresses the relationship between the duration of patent rights and social welfare. Appendix B examines that effects of extension of duration of patent rights on the initial value of consumption per capita.

2 Related Literature

Economic research on patent rights has been an active effort in both microeconomics and macroeconomics, from theoretical and empirical approaches. In particular, industrial organization literature and economic growth literature have made important contributions to this field. In this section, instead of a general survey, we focus on the literature of theoretical approach based on economic growth models.[8] Specifically, we categorize and review selected previous studies in terms of duration and breadth of patent rights.

2.1 Previous Studies on Duration of Patent Rights

Segerstrom et al. (1990) developed a dynamic general equilibrium model of North-South trade in which R&D races between firms determine the rate of product innovation in the North. They considered each R&D race as an invention lottery in which the probability of winning the race is proportional to resources devoted to R&D by each firm. The duration of each R&D race is a deterministic decreasing function of the amount of aggregate resources devoted to R&D. Every time a new product of consumer goods is discovered, a new R&D race between firms in the North begins. The winner of R&D race earns dominant firm profits for an exogenously given patent period, after which perfect competition prevails. The results showed that when wages in the North and in the South are equal, an extension of the duration of patent rights increases the rate of product innovation in the North. This is because extending the duration of patent rights increases the rewards for winning R&D race. However, when Northern workers earn higher wages than Southern workers, extending the duration of patent rights decreases the rate of product innovation in the North. This is because the extension of patent duration itself increases the reward for innovative activity, it also causes wages in the North to rise sufficiently to offset this effect.

Michel and Nyssen (1998) examined the macroeconomic effects of the patent policy using an endogenous growth model with expansion of the variety of consumer products. Michel and Nyssen's (1998) model can be regarded as a generalization of Grossman and Helpman's (1991, Ch.3) expanding variety model. In Michel and Nyssen's (1998) model, it is assumed that patents not only represent a commercial protection for innovators but also entail a partial property right on information. Thus, extending duration of patent rights increases the profitability of a given R&D project but also decreases the knowledge spillovers that play a crucial role in the growth process. Moreover, they showed that economic growth is maximized by a finite duration of patent rights when the instantaneous diffusion of knowledge is low, whereas economic growth is maximized by an infinite duration of patent rights when the instantaneous diffusion of knowledge is high. Furthermore, in the former case, the optimal

[8] See Chu (2022, 2024) for a comprehensive survey of patent policy and economic growth.

duration of patent rights is also finite and shorter than the growth maximizing duration of patent rights. Michel and Nyssen's (1998) model implied that the design of the patent policy must take into account not only the usual arbitrage between private profitability and distortions due to the existence of monopolies, but also the diffusion of knowledge that directly influences the pace of growth.

Based on an endogenous growth model in which the variety of intermediate goods expands through firms' R&D activities, Iwaisako and Futagami (2003) investigated how patent policy affects economic growth and social welfare. In their model, the longer the duration of a patent right, the higher the rate of economic growth. Thus, the maximum rate of economic growth is achieved when the duration of the patent right is infinite. However, the duration that maximizes the social welfare is finite. The implication of their model that social welfare is maximized under a finite duration of patent right contrasts with those of seminal works by Judd (1985) and Gilbert and Shapiro (1990), which suggested that the optimal patent policy involves an infinite duration of the patent rights.

Futagami and Iwaisako (2007) investigated the dynamic properties of an expanding variety model with finite duration of patent right similar to Iwaisako and Futagami (2003). It should be noted, however, that the model in Iwaisako and Futagami (2003) is a continuous-time model setting, whereas the model in Futagami and Iwaisako (2007) is a discrete-time model setting for the purpose of analyzing the dynamic properties of the model. As a main result, in the model of Futagami and Iwaisako (2007), there exists a unique equilibrium growth path and this equilibrium path exhibits damped oscillations in contrast to the equilibrium path of an endogenous growth model with infinite duration of patent right. This suggests that finite duration of patent right may cause fluctuations, even in economies with simple production structures. Moreover, the model of Fitagami and iwaisako (2007) implies that the longer the duration of the patent right, the higher the growth rate of consumption per capita. Furthermore, the social-welfare maximizing duration of patent right is finite whether there are scale effects or not. They, like Iwaisako and Futagami (2003), presented a rationale for the adoption of a finite duration of patent right in the current patent policy based on the finding that the duration of a patent right that maximizes social welfare is finite.

To analyze whether changing the duration of patent rights is an effective policy instrument for promoting R&D, Chu (2010) developed an R&D-based growth model, which is a modified version of Romer's (1990) expanding variety model. Chu's (2010) awareness of the problem is why duration of patent rights have not been extended to increase R&D, despite insufficient investment in R&D in a market economy.[9] With the goal of finding an answer to this question, Chu (2010) calibrated the model to aggregate data for the U.S. economy to quantify the structural relationship among duration of patent rights, R&D, and consumption. The quantitative analysis revealed that under parameter values consistent with the empirical flow-profit depreciation rate of patents and other key features of the U.S. economy, extending the duration of patent rights beyond 20 years leads to a negligible increase in R&D, even though equilibrium R&D is underinvested. In contrast, shortening the patent term substantially reduces R&D and consumption. That is, there is an asymmetric effect between extending and shortening the duration of patent rights. Additionally, Chu (2010) identified and analytically derived a dynamic distortionary effect of duration of patent rights on investment in capital that has been neglected by previous studies, which focused mostly on the static distortionary effect of

[9]Regarding under-investment in R&D in the market economy, see, for example, findings of Jones and Williams (1998, 2000).

markup pricing. The dynamic distortion arises because when the duration of patent rigths is extended, the fraction of monopolistic industries rises and the resulting higher aggregate markup increases the wedge between the social marginal product of capital and the rental price. Consequently, the equilibrium capital-investment rate decreases and deviates from the social optimum.

Chen and Iyigun (2011) developed an economic growth model in which both the investment to develop a new technology and entry of imitators are determined endogenously. On the basis of the model of Chen and Iyigun (2011), how soon the new-technology machine is launched after the patent is granted is influenced by returns to scale in technology development and strategic delays. Additionally, strategic delays in technology development are most likely to occur when earlier dates of success enable imitators to enter an industry. Furthermore, they showed that an inverted U-shape relationship exists between the expected rate of innovation and duration of patent rights. It should be noted, however, that their model does not explicitly account for the utility-maximizing behavior of consumers and is essentially a partial equilibrium model.

Sorek (2011) explored how duration of patent affects economic growth based on an overlapping generations (OLG) model with an improvement in quality of consumer products. Following Chou and Shy (1993), Sorek (2011) considered the two extreme cases of duration of patent right: a one period patent and infinite patent length. In Sorek's (2011) quality-ladder OLG model, the inter-temporal elasticity of substitution (IES) between consumption in any different periods plays an important role in the effects of patent policy. Specifically, when (a) IES $= 1$, (b) small innovations with IES < 1, or (c) large innovations with IES > 1, R&D investment is higher under one period patent length than under infinite patent length and thus growth is faster. Additionally, they confirmed that if the IES is smaller than 1, then long patents are likely to crowd out R&D investment, as pointed by Chou and Shy (1993) in an expanding variety model (in which the IES plays no qualitative role).

Noda (2012) modified the expanding-variety semi-endogenous growth model of Jones (1995), in which the duration of the patent right was infinite, to a finite patent right duration setting. The quantitative analysis revealed two key findings： First, when duplication of research effort is very likely, the share of labor devoted to R&D in a decentralized economy will always be below the socially optimal level. Second, when duplication of research effort is unlikely to occur, the share of labor devoted to R&D in a decentralized economy may exceed the socially optimal level. This possibility arises especially in situations where the invention of new technologies is difficult. Additionally, examination of data for the Japanese economy suggested that the R&D investment in the Japanese economy was lower than the socially optimal level. Thus, from a macroeconomic perspective, the policy implication can be derived that more aggressive R&D efforts are essential in Japan.

Using an R&D-based endogenous growth model with an expanding variety of consumer goods, Lin (2015) explored more channels through which the optimal patent duration is determined. In Lin's (2015) model, goods are all horizontally differentiated and imperfectly substitutable, innovation of a new good gives birth to a new firm, which is granted by the government a patent with a finite term. Such a patented firm continues to be a monopolist, either until the patent it holds has expired or until it has been forced prematurely to lose monopoly power to newer product innovation. Lin (2015) used the term creative destruction of a patent's monopoly profits to represent the event of a newer product innovation coming to force any patented firms to lose monopoly power before their patents expire. Lin's (2015) model features an endogenous hazard rate at which patented firms' monopoly profits are creatively destructed by arrivals of newer varieties. Thus, a patent's effective life is endogenized and less than its legal life. Moreover, the model was calibrated to a global economy with a set of baseline parameter values, and the computation with algorithm known as Golden Section Search showed that the world's optimal patent length equals 18.35 years. Furthermore, the world needs a longer patent term to maximize social welfare when there is the creative-destruction hazard, the world needs a shorter patent duration when there is the prevalence of research congestion. Interestingly, if global welfare strongly values product variety, the optimal patent duration could theoretically exceed 1000 years.

To address the problem of long-run optimal duration of patent rights, Lin and Shampine (2018) developed a non-scale R&D-based growth model with variety expansion of durable intermediate goods. In the model of Lin and Shampine (2018), the long-run optimal patent duration is defined as that maximizes the economy's steady state scale-adjusted consumption along a balanced-growth path. Lin and Shampine (2018) assumed that all legally live patents confront two distinct idiosyncratic risks at any moment: one refers to an imitation hazard of imitative goods coming to underprice some patented goods, while the other pertains to creative destruction, creating an innovation hazard of innovative goods forcing a time-varying proportion of patented and non-patented goods to exit the market. This assumption gives rise to an autonomous system of mixed-type functional differential equations (FDEs) and its dynamics are driven by current, delayed and advanced states. Moreover, Lin and Shampine (2018) presented a relaxation algorithm to solve these FDEs by solving a sequence of standard boundary value problems for systems of ordinary differential equations. Furthermore, they applied this algorithm to simulate a calibrated U.S. economy's transitional dynamics by making discrete changes from the baseline 20 years patent length. The results showed that the long-run optimal duration of patent rights can be either infinite or close to 23.17 years. Specifically, the infinite duration of patent rights is optimal in the long run when a parameter set that features a relatively large knowledge spillover externality and a relatively small innovation hazard is used. However, the long-run optimal duration of patent rights becomes finite at 23.17 years when a parameter set that features a relatively small knowledge-spillover externality and a relatively large innovation hazard is used.

Noda and Kano (2021) extended Matsuyama's (1999) R&D-based growth model with endogenous fluctuations, and examined the relationship between pollution abatement and sustained economic growth. Specifically, the idea of the kindergarten rule of pollution abatement from the perspective of Brock and Taylor (2005) was embedded into a modified version of Matsuyama's (1999) model, assuming a productivity difference between old and new intermediate goods firms. Noda and Kano's (2021) model leads to the appearance of a no-innovation growth phase (called the Solow regime) and innovation-led growth phase (called the Romer regime) in the presence of pollution abatement. When period-2 cycles, alternating between Solow and Romer regimes emerge, the model implies that the zero net emission of pollution flow is compatible with continued growth of consumption. Quantitative considerations of the model suggested the case that period-2 cycles exist is plausible. Additionally, comparing the upper bounds of the average annual growth rate for the cases of 20 years, 25 years, and 30 years for duration of patent rights when period-2 cycles, its value is the highest for the case with a patent term of 20 years. Furthermore, the gap among the three upper bounds of the average annual growth rate widens as the productivity difference between old and new intermediate goods firms increases.

Based on the model extended by incorporating endogenous labor with linear disutility of labor into the expanding variety model of Romer (1990), Iwaisako (2024) examined the welfare-maximizing patent duration. The awareness of problem in Iwaisako (2024) is twofold. The first is how the optimal patent duration is determined, while the second is what factors determine the optimal patent duration. The results showed that the optimal patent duration depends on the parameter that determines the elasticity of substitution among intermediate goods, which also determines the price elasticity of demand. If the parameter is higher (i.e., the elasticity of substitution among the goods is higher and the price elasticity of demand is higher), the welfare-maximizing patent duration is shorter. Additionally, Iwaisako (2024) analyzed numerically the optimal mix problrm of patent duration and breadth, and obtained a similar result. That is, a higher elasticity of substitution (or the price elasticity of demand) is higher shortens the welfare-maximizing duration.

2.2 Previous Studies on Breadth of Patent Rights

To reconcile the policy implications of the the quality-ladder model of Grossman and Helpman (1991, Ch.4) with the widely-accepted empirical finding of substantial R&D under-investment, Li (2001) attempted to extend Grossman and Helpman's (1991, Ch.4) model in two ways. First, preferences were generalized by adopting the Dixit-Stiglitz-type constant elasticity of substitution utility function, allowing the elasticity of substitution between any two consumption goods to be greater than one. Second, interactions between industrial and patent policies were considered. Additionally, Li (2001) examined the patent breadth policy with an endogenous size of innovation. In Li's (2001) model, patent breadth defines the extent of quality improvement to which a product is protected from the infringement of its patent by lower-quality goods producers. A key question is whether R&D subsidies become more or less important when the government can set patent breadth. Answering this question requires the analysis of the optimal mix of industrial and patent policies. This is a novel feature of Li's (2001) model. Li (2001) demonstrated that the optimal policy always involves R&D subsidies for incremental as well as radical innovation (i.e., irrespective of the size of quality improvement). Therefore, when patent policy is available, the importance of R&D subsidies become more pronounced in achieving the social optimum.

Using an extended version of the combined expanding variety models in Romer (1990) and Grossman and Helpman (1991, Ch.3), Goh and Olivier (2002) addressed the issue of optimal patent protection in an economy with downstream and upstream sectors. Specifically, final goods and intermediate goods sectors correspond to downstream and upstream sectors, respectively. Goh and Olivier (2002) assumed that the duration of patent rights is infinite and focused on the protection granted by the breadth of patent rights. Moreover, following Gilbert and Shapiro (1990), they defined patent breadth as the ability of the patentee to raise the price for the single product that embodies the innovation. The key finding is that stronger patent protection in the downstream sector increases R&D incentives of firms in the upstream sector, but discourages innovation in the upstream sector because of a market size effect. Thus, higher patent protection in the upstream sector accelerates growth, while higher patent protection in the downstream sector slows growth. Furthermore, from the viewpoint of welfare analysis, they showed that if some innovation in the intermediate input sector is socially desirable, then welfare is strictly increasing in the patent breadth in the upstream industry, regardless of parameter values.

O'Donoghue and Zweimüller (2004) examined the role of patent policy within the context of a quality ladder model along the lines of Grossman and Helpman (1991) and Aghion and Howitt (1992). They interpreted the endogenous-growth patent policy as infinitely-lived patents that prevent all imitation (i.e., there is complete lagging patnet breadth), but allow any superior product to displace the innovator (i.e., there is no leading patent breadth). There are four lessons obtained from the analytical results. The first lesson is that whenever R&D firms face threats from future innovators, patents play a critical role in protecting firms from future innovators. The second lesson is that in addition to stimulating R&D investment, patent policies can be effective in influencing the direction of firms' inventive activity. Their model takes the form of imposing a patentability requirement to counter the tendency of firms to pursue suboptimally small innovations. Similarly, the patentability requirement and the breadth of prior patents can affect the characteristics of new products and the types of cost savings that firms pursue. The third lesson is that any consideration of government policies on R&D must carefully assess the impact of policy proposals on static efficiency, and to do so, the economy-wide impact of policies must be considered. According to their analysis, policies that affect all industries equally or only a few industries may have a small impact on static efficiency, while policies that have an asymmetric impact across industries may have a large impact on static efficiency. A fourth lesson is that the theoretical literature on R&D may miss an important issue if it assumes that R&D capacity is symmetric, and the magnitude of the distortion depends on the particular patent policy. However, this lesson does not apply only to the analysis of patent policy. When analyzing government policies toward R&D, we need to ask how they affect R&D distortions.

Chu et al. (2012) analyzed the effect of patent protection on innovation based on the framework of the R&D-based growth model with elastic labor supply. Specifically, they considered two innovation specifications: the knowledge-driven specification in Romer(1990) and the lab-equipment specification in Rivera-Batiz and Romer (1991). The results revealed that the effect of patent breadth on innovation is positive under the knowledge-driven specification in Romer (1990), whereas the effect becomes inverted-U under the lab-equipment specification in Rivera-Batiz and Romer (1991). In particular, the latter finding highlights an important but often neglected interaction between elastic labor supply and the innovation process through which strengthening patent protection may have an inverted-U effect on innovation as documented in recent empirical studies, such as Lerner (2009) and Qian (2007). Work of Chu et al. (2012) is related to the literature on patent policy and economic growth such as O'Donoghue and Zweimüller (2004), Furukawa (2007, 2010), and Chen and Iyigun (2011), which showed that strengthening patent protection via different patent instruments could generate a non-monotonic effect on innovation.

Iwaisako (2013) noted that while patent protection has generally been strengthened in both developing and developed countries since the TRIPS Agreement, the degree of patent protection remains relatively low in some developing countries. He then raised the question as to why some developing countries have relatively weak patent protection. His focus is on public services. Because public services exert an impact on the productivity of private firms, and these also have a significant impact on the effects of strengthening patent protection. Thus, by introducing idea of productive public spending of Barro (1990) into an expanding variety model of Rivera-Batiz and Romer (1991), Iwaisako (2013) constructed a framework where both R&D and public services are the engines of economic growth. Additionally, Iwaisako (2013) assumed that the duration of patent rights is fixed and infinite, and that government can control the degree of patent protection using only patent breadth. Within the context of this framework, Iwaisako (2013) considered the role of public services provided by government as the main reason for the differences in the strengthof patent protection found across countries, and explored the welfare-maximizing degree of patent protection. The result showed that when public services are small, the welfare-maximizing level of patent protection is weaker. More specifically, a country that can maintain only a lower level of public services prefers weaker patent protection as long as the ratio of public services to output is not too high. This finding may explain why the patent protection in some developing countries is relatively weak. Furthermore, this is also consistent with the empirical evidence he identified that there is a positive correlation between the ratio of government spending to GDP and an index of patent protection.

To investigate how strengthening patent protection affects economic growth, Iwaisako and Futagami (2013) incorporated physical capital accumulation into Grossman and Helpman's (1991, Ch.3) expanding variety model. Therefore, both innovation and capital accumulation are the driving forces of economic growth in Iwaisako and Futagami's (2013) model. Additionally, they assumed a situation in which the government could control the degree of patent protection only with respect to the breadth of patent rights. The results showed that stronger patent protection does indeed promote innovation. However, stronger patent protection may also impede capital accumulation. The reasons for this can be explained as follows. Strengthening patent protection allows firms producing intermediate goods to charge higher prices and decreases the amount of production. This leads to reduce the demand for capital and capital rents, inhibiting physical capital accumulation. Thus, if the negative effect on capital accumulation exceeds the positive effect on innovation as a result of stronger patent protection, then economic growth will decline. Furthermore, whether the negative effect on capital accumulation exceeds the positive effect on innovation depends on the productivity of R&D relative to the productivity of physical capital production. Specifically, when the relative productivity of R&D is sufficiently low, the negative effect on capital accumulation exceeds the positive effect on innovation, and stronger patent protection impedes economic growth.

Pan et al. (2018) pointed out that people care not only about the absolute level of consumption, but also about their status in society. Therefore, Pan et al. (2018) developed an endogenous growth model with status preferences to explore the impact of patent protection (breadth of patents) on innovation and social welfare. In their model, the marginal rate of substitution (MRS) between assets and consumption decreases with the quantity of assets and increases with the quantity of consumption. Patent protection promotes innovation by increasing the value of innovation, but when patent protection is strengthened due to the presence of status preference, individuals (households) choose a higher level of asset ownership to improve their social ranking. This puts pressure on contemporary consumption, reduces the attractiveness of capital accumulation for future consumption, and suppresses the rate of asset accumulation and innovation. This negative effect of patent protection is reflected by reducing the MRS between assets and consumption; Pan et al. (2018) define this as the substitution effect of patent protection on innovation. When the degree of patent protection is low (high), the MRS is large (small) and the positive (negative) effect of patent protection is dominant. As a result, a non-monotonic relationship exists between patent protection and innovation. Moreover, the stronger the status preference, the greater the substitution effect, so the degree of patent protection that maximizes the growth rate decreases with the strength of the status preference. Furthermore, Pan et al. (2018) conducted a numerical analysis and showed that the effect of patent protection on social welfare depends on status preferences. Specifically, when status preference is strong (i.e., the substitution effect of patent protection on innovation is large), patent protection reduces social welfare, whereas when the strength of status preference is weak, there is a non-monotonic relationship between patent protection and social welfare.

Based on a modified version of an expanding variety model of Rivera-Batiz and Romer (1991), Qian (2018) explored the effect of patent protection on innovation. Qian (2018) noted that patent breadth promotes innovation by increasing the benefits derived from it, but that patent breadth inhibits innovation by increasing the cost of innovation because it increases the price of intermediates used in the R&D sector. The results indicated that increasing patent breadth stimulates innovation when R&D is less intermediates-intensive than production, whereas it has a non-monotonic effect on innovation and economic growth when the former is more intermediates-intensive than the latter. The policy implication is that a one-size-fits-all type of patent protection is not always optimal. Work of Qian (2018) is closely related to the macroeconomic literature on patent breadth, such as Li (2001), Goh and Oliver (2002), Chu et al. (2012), and Pan et al. (2018), contributes to the literature by providing a novel channel that lead to a non0monotonic effect of patent protection.

Chu et al. (2020) investigated the effects of patent protection within the context of a R&D-based growth model with financial frictions. Their growth-theoretic analysis of patent policy features financial frictions in the form of credit constraints that potentially constrain R&D entrepreneurs. Specifically, because of moral hazard, R&D entrepreneurs may not be able to borrow as much as they would like for R&D investments. When these credit constraints are non-binding, strengthening patent protection by increasing patent breadth provides more monopolistic profits, which stimulates R&D and technological progress. This positive monopolistic profit effect captures the traditional view of patent protection. However, when the credit constraints are binding, the monopolistic distortion arising from patent protection leads to more severe financial frictions, which stifle R&D and slow down technological progress. This effect is referred to as the negative financial distortionary effect of patent protection. The results showed that whether stronger patent protection stimulates or stifles innovation depends on credit constraints faced by R&D entrepreneurs. When credit constraints are nonbinding (binding), strengthening patent protection stimulates (stifles) R&D. The overall effect of patent protection on innovation follows an inverted-U pattern. By relaxing the credit constraints, financial development stimulates innovation. Furthermore, patent protection is more likely to have a positive effect on innovation under a higher level of financial development. Chu et al. (2020) conducted cross-country panel regressions and confirmed that patent protection and financial development have a positive interaction effect on innovation.

Tabata (2021) analyzed the interactive effects of public investment policy and patent policy, based on a framework that incorporated productive public capital into the expanding variety model of Rivera-Batiz and Romer (1991) and introduced the concept of patent breadth in accordance with Goh and Olivier (2002). In Tabata's (2021) model, the engines of economic growth are private R&D and public capital accumulation. Tabata (2021) examined how the balanced-growth-maximizing public investment policy changes as patent protection becomes stronger. The results showed that as patent protection becomes stronger, the income tax rate to finance public investment should be lower and the expenditure share of new investment should be higher. Moreover, Tabata's (2021) model implies that the balanced-growth-maximizing public investment policy is equivalent to the welfare maximizing public investment policy along the balanced growth path. Furthermore, Tabata (2021) is closely related to the work of Iwaisako (2013), which examined the optimal strength of patent protection, and showed that as public services decrease, the optimal strength of patent protection decreases. Although Iwaisako (2013) condidered the effect of public services on the optimal design of patent policy, Tabata (2021) focused on the effect of patent protection on the government's design of public investment policy. In this sense, Tabata (2021) complements the analysis conducted by Iwaisako (2013).

To examine the growth and welfare effects of patent protection in a dynamic general equilibrium framework where innovation and capital accumulation are both driving forces of economic growth, Yang (2021) follows Iwasako and Futagami (2013) and incorporates the capital production sector in addition to the innovation production sector, using a Romer (1990)-type expanding variety model that incorporates the capital-producing sector as well as the innovation-producing sector. Additionally, the level of patent breadth, which determines the degree of the degree of firms' market power, is influenced by the patent authority's policy and reflects the strength of patent protection. The results indicated that the comparison between R&D productivity and capital productivity plays an important role in welfare analysis. Specifically, when the relative productivity of R&D compared to capital is high (low), social welfare takes an inverted-U shape for (is decreasing in) the strength of patent protection, and the welfare-maximizing degree of patent protection is no greater than (identical to) the growth-maximizing degree. Furthermore, the model was calibrated to the U.S. economy, and the results of numerical analysis supported these welfare implications.

Iwaisako (2023) analyzed the optimal mix problem of R&D subsidies and patent protection using R&D-based endogenous growth model with heterogeneous industries. Note that, for simplicity, Iwaisako (2023) assumed that patent duration is infinite and that the government determines the degree of patent protection by controlling only patent breadth. Regarding the optimal mix of R&D subsidies and patent protection, the following results are presented. First, the patent breadth should be set so that the markup is uniform across all industries so as not to distort the labor allocation across industries. Such uniform markups uniform markups distort the R&D allocation to be uniform across industries. Second, to improve R&D allocation, R&D subsidies should be higher (lower) for industries with higher (lower) R&D contribution to total factor productivity. However, it may not be possible for R&D subsidies to vary by industry. Therefore, Iwaisako (2023) derived the welfare-maximizing mix of a uniform R&D subsidy and patent protection across industries. Additionally, Iwaisako (2023) evaluated the growth and welfare losses due to this policy constraint of uniformity and demonstrated numerically that heterogeneity leads to significant growth and welfare losses.

3 Overview of Data on International Patent Protection

This section provides an overview of internationally comparable data on patent protection strength.[10] There are various internationally comparable data on the strength of the level of patent protection. However, most of them are integrated indices that score the strength of patent protection from a comprehensive perspective. In other words, in most cases, detailed data on the components of such an integrated index are not disclosed. As a preliminary for

[10]For a literature review on the measurement of international patent protection strength, see Chen et al. (2024).

model analysis of the role of patent duration in economic growth, we would like to investigate internationally comparable data on the level of patent protection strength in terms of the duration of patent rights. Therefore, we focus on data from Park (2008), which provides relevant indicators.

Here, we briefly mention the Park's (2008) international patent protection data, which is a standard measure of patent strength across countries (Chu et al., 2021). Park (2008) classified indicators related to patent laws into five categories (i.e., coverage, membership in international treaties, duration of protection, enforcement mechanisms, and restrictions) and assigned a score ranging from 0 to 1 to each category.[11] For each category, the closer the value is to 1, the higher the level of that category. The scores in the five categories are then summed to calculate an overall index that indicates the strength of patent protection. Thus, the maximum value of the overall index of patent protection strength in Park (2008) is 5. For detailed explanations of the data construction in Park (2008), see Ginarte and Park (1997). The data set of Park (2008) have been most recently updated through 2015 and is available on his website.

The data in Park (2008) cover 122 countries, including both developed and developing countries, although there are years with missing data for some countries. Figure 1 compares the overall index of strength of patent protection in 1960 and 2015, focusing on the 113 countries for which data exist for both 1960 and 2015.

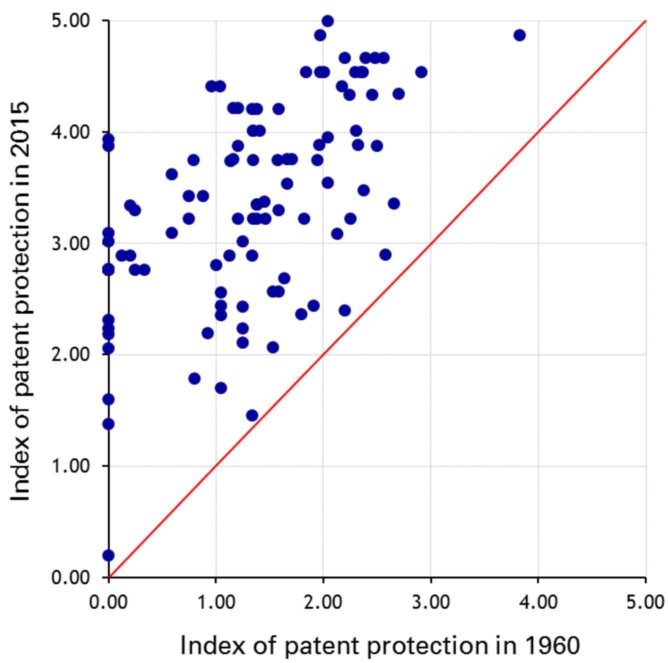

Figure 1: Comparison of the index of patent protection in 1960 and 2015

The scatter plot in Figure 1 shows that all points are located above the 45-degree line. This means that for all 113 countries, the level of patent protection in 2015 is higher than in 1960. It also shows that in 1960, the overall index of the strength of patent protection

[11] Coverage means the extent of inventions that are patentable. Restrictions correspond, for example, compulsory licensing in the event that a patented invention is not sufficiently exploited.

was in a relatively low range between 0 and 2, but by 2015, many countries have reached a level where the value exceeds 3. That is, the level of strength of patent protection in these countries increased rapidly in the period from 1960 to 2015. The facts observed from Figure 1 suggest that for more than half a century since 1960, there has been a global trend toward increasing levels of patent protection in both developing and developed countries. Park (2008) mentioned such trend reflects the adoption of stronger patent laws across countries particularly after TRIPS came into force, as well as the introduction of patent laws in countries which did not previously have patent systems. That is, TRIPS Agreement was a crucial milestone toward strengthening global patent protection because all WTO member countries had to align their IPR laws to the minimum standards set by TRIPS.

Next, we turn our attention to the index of patent protection strength in terms of the duration of patent rights. In Figure 2, graphs on the mean and standard deviation for the index of the duration of patent protection are plotted for 112 countries that data are continuously available from 1960 to 2015, respectively.

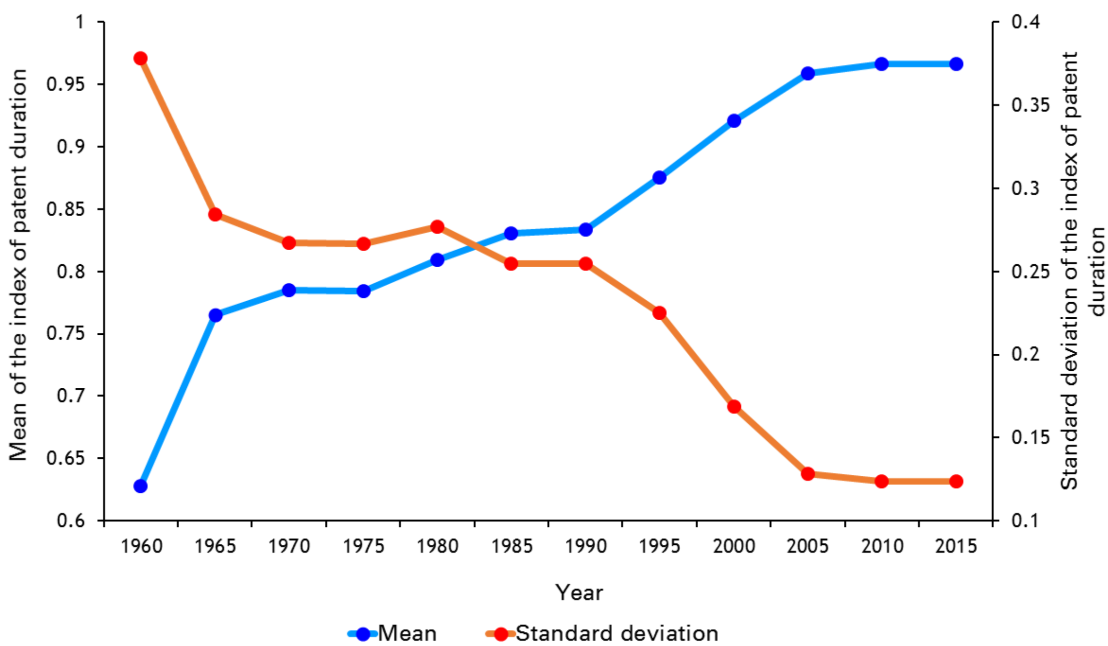

Figure 2: Changes in the mean and standard deviation of the index of the patent duration

The following characteristics of the two graphs can be seen from Figure 2. First, from the perspective of the entire period from 1960 to 2015, the mean value shows an upward trend.[12] In contrast, the standard deviation shows a downward trend. The upward trend of the mean value literally indicates an increase in the international average level of duration of patent protection over time, while the downward trend of the standard deviation suggests the harmonization across countries regarding the duration of patent protection.[13] Moreover, a

[12]Table 1 in Lerner (2002) indicates data that, for the 60 countries listed in the International Monetary Fund's International Financial Statistics with the largest GDP in 1997, the average duration of patent rights in these countries increased between 1850 and 1999.

[13]Using a two-country (innovative North and imitating South) model of product-cycle trade, Dinopoulos and

closer look reveals that both the mean and standard deviation indicate sharp changes during the 1960s and from the mid-1990s to the early 2000s. Specifically, the mean rose sharply during the 1960s and from the mid-1990s to the early 2000s. Conversely, the standard deviation fell sharply during those periods. In particular, the rapid changes since the mid-1990s are considered to be triggered by the TRIPS agreement of the WTO that came into effect in January 1995. Furthermore, both the mean and the standard deviation have changed very little since the early 2000s. However, it should be noted that the TRIPS was amended in January 2017. Therefore, changes may have occurred since 2017 reflecting the amendment of the TRIPS, although this cannot be confirmed from the current data in Park (2008).

Table 1 presents descriptive statistics on the index of the duration of patent protection from 1995 to 2015 for the 122 countries covered by Park (2008).

Table 1: Descriptive statistics on the index of the duration of patent protection

	1995	2000	2005	2010	2015
Mean	0.886	0.928	0.962	0.969	0.969
Standard deviation	0.218	0.163	0.123	0.119	0.119
Kurtosis	9.010	16.376	33.387	39.929	39.929
Skewness	−2.881	−3.584	−5.123	−5.737	−5.737
Number of observations	122	122	122	122	122

As already observed in the two graphs in Figure 2, Table 1 also confirms that the mean value increases over time, while the standard deviation decreases over time. Moreover, we find the feature of increasing the value of kurtosis over time. This suggests that the width of the distribution for the index of duration of patent protection is narrowing over time, changing to a more elongated bell-shaped distribution. Furthermore, we can identify that the skewness is negative and also becomes smaller over time. As described in Park (2008), this indicates that most of the countries have a score of the index of duration of patent protection that is above the mean. It should be noted, however, that all descriptive statistics for 2010 and 2015 are identical. That is, there is no change in mean, standard deviation, kurtosis, and skewness from 2010 to 2015. This is because the index of duration of patent protection in Park (2008) is the same for all countries when comparing the data of 2010 and 2015.

Table 2 shows the number of countries that the index for the duration of patent protection had a value of 0 or 1 in each year from 1995 to 2015 for the 122 countries covered in Table 1.

From Table 2, we can identify that the number of countries with a value of 0 was in a decreasing trend. In contrast, the number of countries with a value of 1 was in an increasing trend. More precisely, the number of countries with a value of 0 has remained constant since 2005, while the number of countries with a value of 1 has remained constant since 2010. In any case, it is worth noting that the percentage of countries with a value of 1, corresponding to

Kottaridi (2008) demonstrated that a move towards harmonization (i.e., a global patent regime with a common enforcement policy and equal-duration patents) based on stronger Southern IPR protection accelerates the long-run global rates of innovation and growth, and reduces the North-South wage gap. Additionally, Naghavi and Prarolo (2018) concluded that the harmonization of intellectual property culture leads to its globalization. Furthermore, Arza et al. (2023) described that in the last decades there has been a clear trend towards the harmonization of IPRs regimes around the world and this process was enforced globally in the context of the negotiations that led to the creation of the WTO.

Table 2: Number of countries that the index of the duration of patent protection has a value of 0 or 1

	1995	2000	2005	2010	2015
Number of countries with a value of 0	5	2	1	1	1
Number of countries with a value of 1	78	91	106	110	110
Number of observations	122	122	122	122	122

the maximum value of the index of the duration of patent protection, increased from 63.93% to 90.16% during the period from 1995 to 2015. Additionally, since 2005, there is only one country where the index of the duration of patent protection has a minimum value of 0. This is only 0.82% of the total. It is clear that behind such facts lie the influence of TRIPS, which came into effect in January 1995.

In summary, the analysis of internationally comparable data highlights a consistent global trend toward stronger patent protection, particularly after the implementation of the TRIPS Agreement. This upward trend provides a critical empirical foundation for theoretical examination. Building upon these findings, the next section introduces a theoretical framework to explore the implications of patent protection on economic growth and social welfare.

4 The Model

4.1 Final Goods Producers

All firms in the final goods sector are perfectly competitive, and each firm produces a homogeneous good (i.e., a single final good) using labor and a variety of non-durable intermediate goods. These goods are either consumed by households, converted into intermediate goods, or used for investment in households' utility-enhancing public capital (i.e., livelihood-based public infrastructure development). We assume that each intermediate good available at time t is numbered with a real number in the interval $[0, N(t)]$. Thus, $N(t)$ can be interpreted as the number of varieties of intermediate goods available at time t.[14] In other words, $N(t)$ represents the range of types of intermediate goods available at time t.

We follow Barro and Sala-i-Martin (2004, Ch.6) by writing the the production function for firm i in the final goods sector as

$$Y_i(t) = AL_i(t)^{1-\alpha} \int_0^{N(t)} x_{ij}(t)^{\alpha} dj, \tag{1}$$

where $j \in [0, N(t)]$ represents the index of intermediate goods types, $Y_i(t)$ is the output of final goods produced by firm i, $L_i(t)$ is the amount of labor input in firm i, and $x_{ij}(t)$ is the amount of input of type j's intermediate good in firm i. For the parameters, A represents a multifactor productivity index. $\alpha \in (0, 1)$ coincides with the inverse of the price of all types

[14] Barro and Sala-i-Martin (2004, Ch.6) described that N should be viewed as a tractable proxy for the technological complexity of the typical firm's production process or, alternatively, for the average degree of specialization of the factors of production employed by the typical firm.

of differentiated intermediate goods, which are within the duration of patent rights, as will be confirmed later. Additionally, $1-\alpha$ equals the labor elasticity of output.

We regard final goods as numéraire and normalize their prices to unity. If we express the wage rate as $w(t)$ and the price of intermediate good of type $j \in [0, N(t)]$ as $p_j(t)$, then the profit of firm i, $\Pi_i(t)$, is given by

$$\Pi_i(t) = AL_i(t)^{1-\alpha} \int_0^{N(t)} x_{ij}(t)^{\alpha} dj - \int_0^{N(t)} p_j(t) x_{ij}(t) dj - w(t) L_i(t). \tag{2}$$

Given $\{p_j(t)\}_{j=0}^{N}$, $w(t)$, and $N(t)$, the final-goods-producing firms choose the combination of inputs of the production factors that maximizes their profit. Let $(\widehat{L}_i,\ \{\widehat{x}_{ij}\}_{j=0}^{N})$ be the combination of labor input and a set of intermediate goods inputs that maximizes profit. Therefore, the following relation holds.

$$\left. \frac{\partial\, \Pi_i(L_i,\ \{\widehat{x}_{ij}\}_{j=0}^{N})}{\partial L_i} \right|_{L_i = \widehat{L}_i} = 0. \tag{3}$$

Consequently, Eqs. (2) and (3) imply that the quantity of labor demand of firm i in the subjective equilibrium is given by

$$\widehat{L}_i(t) = \frac{(1-\alpha) Y(t)}{w(t)}. \tag{4}$$

Note that the quantity demanded for intermediate good of type $j \in [0, N(t)]$ that maximizes profit of firm i in the final goods sector, $\widehat{x}_{ij}$, is the same as the quantity demanded that maximizes D_i, defined below.

$$D_i \equiv \int_0^{N(t)} \left[A\widehat{L}_i(t)^{1-\alpha} x_{ij}(t)^{\alpha} - p_j(t) x_{ij}(t) \right] dj. \tag{5}$$

Thus, when we apply calculus of variations to derive the quantity demanded for intermediate good of type $j \in [0, N(t)]$ by firm i, Eq. (5) yields

$$\widehat{x}_{ij}(t) = L_i(t) \left[\frac{\alpha A}{p_j(t)} \right]^{\frac{1}{1-\alpha}}. \tag{6}$$

Therefore, the price elasticity of demand for each intermediate good is equal to $1/(1-\alpha)$. In the following discussion, the hat symbol is omitted.

4.2 Intermediate Goods Producers and R&D

Suppose a situation in which R&D on the invention of a new intermediate good takes place in the private sector with the intention of earning profits in the market. Thus, product innovation, expressed in the form of an expansion of the variety of intermediate goods, arises as a result of R&D activities of profit-motivated private firms. Following Barro and Sala-i-Martin (2004, Ch.6), we consider the case that the producer of each intermediate good is also the inventor of that good. That is, producers of new intermediate goods also have the aspect of being innovators who have conducted R&D of those intermediate goods. Moreover, when a new product is invented, the government immediately grants a patent right to the inventor of the technical information regarding production of its new intermediate good (i.e., the firm that produces its new intermediate good). The duration of the patent rights for all inventions is defined by the finite length of $T \in (0, \infty)$, which is treated as a parameter. We express the lump-sum patent fee paid to the government by the firm granted the patent as F, and formulate $F = T^{\beta}/\beta$.[15] Here, the parameter β satisfies the condition $\beta > 2$. Note that the relation $\beta = \partial \log F / \partial \log T$ holds. Therefore, β represents the patent duration elasticity of patent fee. We suppose the case that patent fees include filing fees. However, as mentioned above, examination fees are ignored because our model considers the situation where all new inventions are granted patents unconditionally. Furthermore, for simplicity, we assume that for all intermediate goods, it costs one unit of the final good to produce one unit of them. In other words, the marginal and average costs are 1 when the final good is converted to the intermediate good.

In the following argument, we first explain the profit-maximizing behavior of firms as intermediate goods' producers. At each point in time, in the intermediate goods production sector, there are monopolistically competitive firms producing differentiated intermediate goods (with patent rights still in effect) and perfectly competitive firms producing homogeneous intermediate goods (with expired patent rights). Monopolistically competitive firms finance their production costs by issuing corporate stock. We focus on the profit-maximizing behavior of the firm that produces intermediate good of type j for which the duration of the patent right is valid (hereafter referred to as firm j). Because firm j is a monopolistically competitive firm, it chooses the price and output of the intermediate good of type j that maximizes the profit, considering the structure of the demand function for its product. Eq. (6) implies that the demand function for the intermediate good of type j, $x_j(p_j)$, is represented as

$$x_j(p_j) = \left[\frac{\alpha A}{p_j}\right]^{\frac{1}{1-\alpha}} \sum_i L_i. \tag{7}$$

Consequently, from Eq. (7), the profit function of firm j, $\pi_j(p_j)$, is given by

$$\begin{aligned} \pi_j(p_j) &= (p_j - 1)x_j(p_j) \\ &= (p_j - 1)\left[\frac{\alpha A}{p_j}\right]^{\frac{1}{1-\alpha}} \sum_i L_i. \end{aligned} \tag{8}$$

When we express the price of intermediate good of type j that maximize the profit of firm j by $\widehat{p}_j$, the following relation holds.

$$\left.\frac{d\,\pi_i(p_j)}{dp_j}\right|_{p_j=\widehat{p}_j} = 0. \tag{9}$$

Thus, Eqs. (8) and (9) imply that the price of an intermediate good of type j in a subjective equilibrium is expressed as

$$p_j(t) = \frac{1}{\alpha}. \tag{10}$$

[15]For instance, in the case of Japan, the patent fee for each year increases almost exponentially, albeit in steps, depending on the number of years that the patent right is maintained. For details, refer to the website of the Japanese Patent Office (`https://www.jpo.go.jp/e/index.html`). If we denote the total amount of patent fee paid by a patentee firm for maintaining a patent right from time 0 to time T as F and the number of years that the patent right is maintained as h, then we have $F = \int_0^T h^{\kappa} dh$, where $\kappa > 1$. Hence, by defining $\beta \equiv 1 + \kappa$, the relation $F = T^{\beta}/\beta$ holds.

From Eq. (10), we can identify that the price of an intermediate good of type j is equal to the marginal cost 1 multiplied by the markup $1/\alpha$. Moreover, substituion of Eq. (10) into Eq. (6) leads to

$$x_j(t) = \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} \sum_i L_i. \tag{11}$$

Furthermore, substituion of Eq. (10) into Eq. (7) yields

$$\pi_j \equiv \pi = \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} \Big(\frac{1-\alpha}{\alpha}\Big) \sum_i L_i. \tag{12}$$

Thus, from Eqs. (10)–(12), we find that in a subjective equilibrium, intermediate goods' price, output, and profit of all monopolistically competitive firms are equal.

Now, we examine the subjective equilibrium of the firm producing intermediate goods of type k as a perfectly competitive firm (hereafter referred to as firm k). Let p^c and x_k denote the price and quantity of intermediate good produced by firm k, respectively. Note that firm k is a price taker because it is a perfectly competitive firm. Given the market price of an intermediate good of type k, firm k determines the output of the intermediate good of type k that maximizes the profit, π_k, as follows:

$$\pi_k = (p^c - 1)x_k. \tag{13}$$

As a result, under the perfectly competitive market, the market price of an intermediate good of type k equals its marginal cost. That is, the following relation holds.

$$p^c = 1. \tag{14}$$

It is clear from Eqs. (13) and (14) that the profit of firm k in the subjective equilibrium is zero. Additionally, the output of intermediate goods of type k by firm k is represented as

$$x_k = \alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} \sum_i L_i. \tag{15}$$

Naturally, Eqs. (14) and (15) apply to all other perfectly competitive firms.

Note that monopolistically competitive firms earn positive profits because each firm can set the price of the intermediate good higher than its marginal cost over the duration of the patent rights. However, the duration of the patent right for all intermediate goods is set to a finite value, T. Thus, a firm that invents a new intermediate good at time t can earn a monopolistic positive profit until time $t + T$. After this patent expires, the production technology for its product becomes common knowledge. Consequently, after time $t+T$, other firms are free to enter the market for its intermediate good. As a result, the monopolistically competitive firm loses the ability to control the price of the intermediate good and becomes a perfectly competitive firm, with zero profit. Therefore, when the instantaneous interest rate at time ω is expressed as $r(\omega)$, the sum of the present discounted value of its profit stream subsequent to time t resulting from the invention of the new intermediate good is given by

$$\begin{aligned} V(t) &= \int_t^{t+T} \exp\Big[-\int_t^s r(\omega)d\omega\Big] \cdot \pi(s)ds \\ &= \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} \Big(\frac{1-\alpha}{\alpha}\Big) \sum_i L_i \int_t^{t+T} \exp\Big[-\int_t^s r(\omega)d\omega\Big]ds. \end{aligned} \tag{16}$$

Next, we turn to a description of firms' R&D activities. In our model, R&D efforts are essential for expanding the variety of intermediate goods, i.e., for realizing product innovation. What then is the relationship between investment in R&D and the creation of new products? For example, a situation in which ideas are depleted as public knowledge capital increases, making it difficult to create new products, implies that negative externalities are dominant.Conversely, positive externalities dominate in situations where an increase in public knowledge capital promotes the creation of new technologies. We can also consider a situation in which positive and negative externalities in the economy cancel each other out, in which case the increase in public knowledge capital has no effect on the invention of new products from the macroeconomic level perspective. For analytical convenience, we assume the case of offsetting positive and negative externalities, as in Barro and Sala-i-Martin (2004, Ch.6). Additionally, all firms have the same production technology and that one unit of new product is always invented when η units of resources are used in terms of the final good. Therefore, if the input of resources for R&D is expressed as Y_R, then the following relation holds.

$$\dot{N} = \frac{1}{\eta} Y_R, \tag{17}$$

where $\dot{N}(t) \equiv dN(t)/dt$, and we use the same notation for the derivative with respect to time for other variables. The formulation in Eq. (17) describes a situation in which firms with more R&D funds are able to create more new technologies.

Our model assumes a situation where free entry to the market is guaranteed. An entrepreneur who aims to be an innovator and has the funds, $\eta + T^\beta/\beta$, the sum of R&D costs, η, and the patent fee, T^β/β, will earn the sum of the the present discounted value of profit stream, V, shown in Eq. (16). There are three possible cases for the relationship between V and $\eta + T^\beta/\beta$, as shown in panels (a)–(c) of Figure 3.

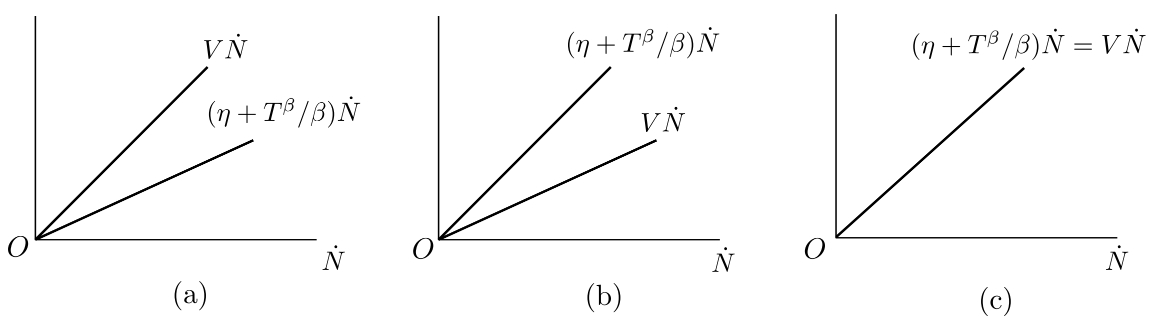

Figure 3: Comparison of sum of the present discounted value of profit stream by R&D and R&D cost

In panel (a), because the sum of the present discounted value of profit stream exceeds R&D costs, firms are willing to use infinitely more resources in R&D activities. However, such a situation is not possible in equilibrium. In the case of panel (b), the R&D cost exceeds the sum of the present discounted value of profit stream. Thus, the firm has no incentive to engage in R&D activities, and $\dot{N}(t) = 0$ in equilibrium in this case. That is, innovation does not occur. Hence, in an equilibrium where free entry into the market regarding R&D is guaranteed and the variety of intermediate goods increases over time (i.e., sustained innovation occurs), the relation $V(t) = \eta + T^\beta/\beta$ must hold. In other words, the only equilibrium in which the innovation rate is positive and finite resources are used for R&D activities is the case in panel (c). As a result, the following subjective equilibrium condition is derived.

$$\alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} \Big(\frac{1-\alpha}{\alpha}\Big) \sum_i L_i \int_t^{t+T} \exp\Big[-\int_t^s r(\omega)d\omega\Big] ds = \eta + \frac{T^\beta}{\beta}. \tag{18}$$

4.3 Government

In addition to patent-related work, the government also conducts development of livelihood-based public infrastructure that enhances households' utility such as welfare facilities, housing and living environment facilities, and educational and cultural facilities.[16] The gross investment $I(t)$ in livelihood-related infrastructure (i.e., household utility-enhancing public capital stock) is financed by the patent fee $\dot{N}(t)T^\beta/\beta$ collected from the R&D sector. Additionally, we assume that the government maintains a balanced budget at each point in time. Therefore, the following relation holds.

$$\frac{T^\beta}{\beta}\dot{N}(t) = I(t). \tag{19}$$

In Eq. (19), the left-hand and right-hand sides represent revenue and expenditure, respectively.

The net increase in the livelihood-based public infrastructure at a point in time equals gross investment less depreciation. Thus, the livelihood-based public infrastructure is accumulated as follows:

$$\dot{G}(t) = I(t) - \delta G(t). \tag{20}$$

In Eq. (20), $G(t)$ represents the level of livelihood-based public infrastructure and the parameter δ represents the depreciation rate of of livelihood-based public infrastructure.

4.4 Households

In our model economy, there are many infinite-lived households with perfect foresight, each household being a representative household (individual) with identical preference and work ability. All individuals are engaged in labor. The total population of the economy is expressed as L, which is an exogenous constant. We normalize the working hours of the representative household (or the work intensity per person) at each time point to 1. Thus, we identify the labor input and labor force population; hence the total labor input, $\sum_i L_i(t)$, is equal to the total population, L. That is, the relation $\sum_i L_i(t) = L$ holds. Because the total population, L, is a constant, it follows that growth rats of GDP, $Y(t)$, and GDP per capita, $y(t) \equiv Y(t)/L$, are equal. At each time point, the representative household receives wage, $w(t)$, by inelastically providing 1 unit of labor services to the firm in the final goods sector. Households can borrow or lend freely at the instantaneous, risk-free, interest rate, $r(t)$, and also receive income from their assets (i.e., capital income).

[16]As an illustrative example, in Japan, after the Great Hanshin-Awaji Earthquake in January 1995 and the Great East Japan Earthquake in March 2011, the government provided many temporary housing units to the victims.

When total consumption in the household sector is expressed as $C(t)$ and total assets held by the household sector as $Z(t)$, respectively, the following relation holds.

$$\dot{Z}(t) = w(t)L + r(t)Z(t) - C(t). \tag{21}$$

Note that in general equilibrium, total assets of household sector, $Z(t)$, are equal to the total market value of the R&D sector (i.e., firms producing intermediate goods), $V(t)N(t) = (\eta + T^{\beta}/\beta)N(t)$. Dividing both sides of Eq. (20) by L yields the following budget constraint for a representative household:

$$\dot{z}(t) = w(t) + r(t)z(t) - c(t), \tag{22}$$

where $z(t) \equiv Z(t)/L$ and $c(t) \equiv C(t)/L$.

The representative household maximizes the lifetime utility over an infinite horizon. Following Arrow and Kurz (1970, Ch.4), the representative household's lifetime utility evaluated at time 0, U_0, is given by

$$U_0 = \int_0^{\infty} e^{-\rho t}[\log c(t) + \varepsilon \log g(t)]dt, \tag{23}$$

where $g(t) \equiv G(t)/L$ represents the benefits that a household receives from services by livelihood-based public infrastructure.[17] Parameters $\varepsilon > 0$ and $\rho > 0$ represent the relative weight of the benefits obtained from livelihood-based public infrastructure and the rate of time preference, respectively. The term $u(t) \equiv \log c(t) + \varepsilon \log g(t)$ denotes the instantaneous utility at time t. Therefore, the lifetime utility at time 0, U_0, denotes the sum of the present discounted value of the stream of instantaneous utility, u, subsequent to time 0.

From Eqs. (22) and (23), given $\{r(t)\}_{t=0}^{\infty}$, $\{w(t)\}_{t=0}^{\infty}$, $\{g(t)\}_{t=0}^{\infty}$ and $z(0) \equiv z_0$, the household solves the optimization problem as follows:

$$\max \quad \int_0^{\infty} e^{-\rho t}[\log c(t) + \varepsilon \log g(t)]dt,$$

$$\begin{aligned} \text{s.t.} \quad \dot{z}(t) &= w(t) + r(t)z(t) - c(t), \\ z(0) &\equiv z_0 > 0. \end{aligned}$$

To solve this optimization problem, we set up the current value Hamiltonian, $\mathcal{H}(c, z, \lambda, t)$, as follows:

$$\mathcal{H}(c, z, \lambda, t) = \log c + \varepsilon \log g(t) + \lambda[w(t) + r(t)z - c]. \tag{24}$$

In Eq. (24), variable λ is called the co-state variable or the adjoint variable, which corresponds the marginal value of the state variable $z(t)$ (see, e.g., Kamien and Schwartz, 1991). In this case, the necessary conditions for maximizing the objective function are given by

$$\frac{\partial \mathcal{H}}{\partial c} \Longrightarrow \frac{1}{c(t)} = \lambda(t), \tag{25}$$

[17]Similar to Greiner et al. (2005, Ch.6), the term where the aggregate livelihood-based public infrastructure G is divided by the total population L can be interpreted as a kind of congestion phenomenon.

$$\dot{\lambda} = -\frac{\partial \mathcal{H}}{\partial z} + \rho\lambda \Longrightarrow \dot{\lambda}(t) = [\rho - r(t)]\lambda(t), \tag{26}$$

and

$$\lim_{t\to\infty} [e^{-\rho t}\lambda(t)z(t)] = 0, \tag{27}$$

where Eq. (26) is called the adjoint eqation and Eq. (27) is called the transversality condition.

Taking the natural logarithm of both sides of Eq. (25) and differentiating with respect to time yields

$$-\frac{\dot{c}(t)}{c(t)} = \frac{\dot{\lambda}(t)}{\lambda(t)}. \tag{28}$$

Moreover, Eq. (26) can be rewritten as follows.

$$\frac{\dot{\lambda}(t)}{\lambda(t)} = -r(t) + \rho. \tag{29}$$

Furthermore, Eqs. (28) and (29) lead to

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho. \tag{30}$$

Thus, the growth rate of consumption per capita equals the difference between the interest rate and the rate of time preference.

4.5 Growth Rates in Steady-Growth Equilibrium Path

Now, we consider a situation that the economy is on the steady-growth equilibrium path. In our model, the steady-growth equilibrium path is defined as the path where general equilibrium is satisfied and all variables grow at constant rates. In the following, we use the notation γ_v for the growth rate of variable v when expressing the growth rate of a variable in the steady-growth equilibrium path. Our main interest is how extension of the duration of patent rights (i.e., strengthening patent protection) affects the growth rate of key variables such as innovation and GDP per capita.

As confirmed by the data on international patent protection in Section 3, longer patent durations have been granted in both developed and developing countries throughout the postwar period. How does this global trend toward longer patent durations affect a country's macroeconomic performance? In the following discussion, we first examine the growth rate relationships among GDP per capita, consumption per capita, wage rate, variety of intermediate goods, and livelihood-based public infrastructure under the steady-growth equilibrium path. Next, we examine how the growth rate of the variety of intermediate goods (i.e., the rate of product innovation) and the growth rate of GDP per capita are affected by changes in the duration of the patent right, that is, the growth effect of the duration of the patent right. We also consider the relationship between the duration of a patent right and the level of social welfare.

Suppose that at time t, intermediate goods in the range $[0, N^c(t)]$ are produced by perfectly competitive firms, while intermediate goods in the range $(N^c(t), N(t)]$ are produced by monopolistically competitive firms. Let $N^m(t)$ be the variety of intermediate goods produced by a monopolistically competitive firm. Thus, noting that $N^c(t) = N(t-T)$, we find that the following relation holds.

$$\begin{aligned} N^m(t) &= N(t) - N^c(t) \\ &= N(t) - N(t-T). \end{aligned} \tag{31}$$

Considering Eqs. (1), (6), (10) and (14), we find that the total output of the final goods at time t can be represented as

$$Y(t) = \alpha^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} L[e^{-\gamma_N T} + \alpha^{\frac{\alpha}{1-\alpha}}(1 - e^{-\gamma_N T})]N(t) \tag{32}$$

The form of Eq. (32) indicates that our model belongs to a kind of AK-type endogenous growth model.[18] It is clear from Eq. (32) that the growth rate of GDP, γ_Y, and the innovation rate, γ_N, are equal. Moreover, Eqs. (19) and (20) yield

$$\gamma_G = \gamma_N \frac{T^\beta}{\beta} \frac{N(t)}{G(t)} - \delta. \tag{33}$$

Therefore, Eq. (33) implies that $N(t)/G(t)$ is a constant on the steady-growth equilibrium path and that the growth rate of livelihood-based public infrastructure, γ_G, and the innovation rate, γ_N, are equal.

Now, if the output of intermediate goods produced by any perfectly competitive firm is expressed by x^c and the output of intermediate goods produced by any monopolistically competitive firm by x^m, then the macroeconomy's resource constraint must be satisfied as follows:

$$Y(t) = C(t) + [N^c(t)x^c + N^m(t)x^m] + \eta \dot{N}(t) + I(t). \tag{34}$$

Specifically, Eq. (34) denotes the allocation of total value added in a country's macroeconomy (i.e., GDP or aggregate final goods) to households, firms producing intermediate goods, R&D firms, and government. From Eqs. (11), (15), and (31), the second term on the right-hand side of Eq. (34) can be rewritten as follows:

$$\begin{aligned} N^c(t)x^c + N^m(t)x^m &= N^c(t)\alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} L + N^m(t)\alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L \\ &= N(t)e^{-\gamma_N T}\alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} L + N(t)(1 - e^{-\gamma_N T})\alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L \\ &= N(t)\alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} L[e^{-\gamma_N T} + \alpha^{\frac{1}{1-\alpha}}(1 - e^{-\gamma_N T})]. \end{aligned} \tag{35}$$

Thus, substituting Eqs. (19), (32), and (35) into (34) and solving for $C(t)/N(t)$ yields

$$\begin{aligned} \frac{C(t)}{N(t)} &= \alpha^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} L[e^{-\gamma_N T} + \alpha^{\frac{\alpha}{1-\alpha}}(1 - e^{\gamma_N T})] \\ &\quad - \alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} L[e^{-\gamma_N T} + \alpha^{\frac{1}{1-\alpha}}(1 - e^{\gamma_N T})] \\ &\quad - \Big(\eta + \frac{T^\beta}{\beta}\Big)\gamma_N. \end{aligned} \tag{36}$$

[18]For details on the AK models, see Barro and Sala-i-Martin (2004, Ch.4), Acemoglu (2009, Ch.11), and Bénassy (2011, Ch.9).

Consequently, Eq. (36) shows that $C(t)/N(t)$ is a constant on the steady-growth equilibrium path and that the growth rate of consumption, γ_C, and the innovation rate, γ_N, are equal. Furthermore, Eq. (4) implies that the growth rate of GDP, γ_Y, and the growth rate of wage, γ_w, are equal. From Eqs. (4), (30), (32), (33), (36) and the assumption that the total population is a constant, we obtain

$$\gamma_y = \gamma_c = \gamma_w = \gamma_N = \gamma_G \equiv \gamma. \tag{37}$$

Hence, Eq. (37) implies that the steady-growth equilibrium path in our model is consistent with a balanced growth path.

5 Impacts of Extending Patent Duration on Economic Growth and Social Welfare

Patent duration is a fundamental design of the patent system, indicating the duration of the legal monopoly granted by patents (Zhang et al., 2024). How will product innovation and economic growth be affected when patent protection is strengthened in the form of extending patent duration? Our model allows us to examine such a question by means of comparative statics when T is changed as a policy parameter. Based on the above-mentioned awareness of problem, we conduct a theoretical analysis of the relationship between the duration of patent rights and the growth rate of GDP per capita. In our model, through understanding the effect of change in the duration of patent rights on the growth rate of GDP per capita, while at the same time, we can comprehend the impacts on the growth rate of consumption per capita, the growth rate of wage, the rate of innovation, and the growth rate of livelihood-based public infrastructure through the result presented in Eq. (37).

In the steady-growth equilibrium path, Eq. (30) implies that $r(t)$ must be a constant. Therefore, we express $r(t) = r$ and the time symbol t are omitted hereafter. When $r(t) = r$, Eq. (18) for the free entry condition with positive innovation can be rewritten as

$$\phi(1 - e^{-rT}) = \Big(\eta + \frac{T^\beta}{\beta}\Big) r, \tag{38}$$

where $\phi \equiv \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L[(1-\alpha)/\alpha]$. Thus, as shown in Figure 4, the value of r satisfying Eq. (38) is uniquely determined.

We express the level of r satisfying Eq. (38) by $\widehat{r}$. Note that in the steady-growth equilibrium path, there is no arbitrage opportunity. Recall that at each point in time, households can receive the interest rate $r(t)$ on funds lent to other households. Thus, in the steady-growth equilibrium path, the interest rate that households receive when they lend funds to other households must equal the rate of return on their investment in R&D firms (i.e., the intermediate goods producers). For this reason, $\widehat{r}$ represents the rate of return on R&D.

The value of the rate of return on R&D, $\widehat{r}$, determined by Eq. (38), varies with the duration of the patent right, T. Here, to conduct a comparative statics analysis on the change of $\widehat{r}$ when T increases, we denote $\widehat{r} = \widehat{r}(T)$ in the following discussion. Thus, Eqs. (30) and (37) imply that the growth rate of GDP per capita is also the function of the duration of the patent right. That is, in the sense that γ is a function of T, it is expressed as

$$\gamma(T) = \widehat{r}(T) - \rho. \tag{39}$$

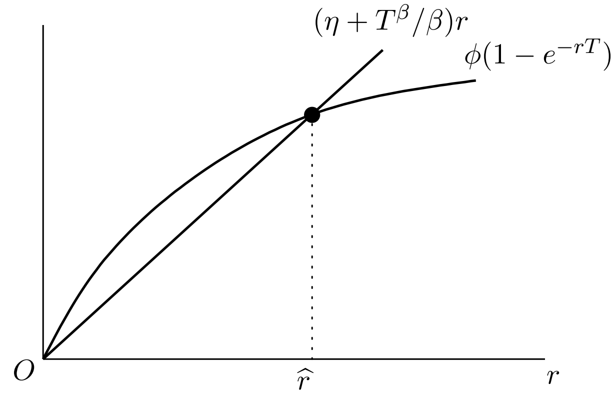

Figure 4: Rate of return on R&D in the steady-growth equilibrium path

Thus, based on Eqs. (38) and (39), the relationship between the duration of the patent right, T, and the growth rate of GDP per capita, γ, can be understood. Specifically, Eqs. (38) and (39) lead to

$$\frac{d\widehat{r}(T)}{dT} = -\frac{\widehat{r}(T)[T^{\beta-1} - \phi e^{-\widehat{r}(T)T}]}{\eta + \dfrac{T^\beta}{\beta} - \phi T e^{-\widehat{r}(T)T}}. \tag{40}$$

From Figure 4, we find that $\eta + T^\beta/\beta > \phi T e^{-\widehat{r}(T)T}$ holds when $r = \widehat{r}$. This relation must be satisfied for any $T > 0$. Therefore, the sign of the denominator in Eq. (40) is positive. However, the sign of the numerator in Eq. (40) is not uniquely determined, and there are three possibilities (i)–(iii) as follows:

$$\text{(i)} \quad \widehat{r}(T) > \frac{1}{T}\log\left(\frac{\phi}{T^{\beta-1}}\right) \Longleftrightarrow \frac{d\,\widehat{r}(T)}{dT} < 0 \Longleftrightarrow \frac{d\gamma(T)}{dT} < 0,$$

$$\text{(ii)} \quad \widehat{r}(T) = \frac{1}{T}\log\left(\frac{\phi}{T^{\beta-1}}\right) \Longleftrightarrow \frac{d\,\widehat{r}(T)}{dT} = 0 \Longleftrightarrow \frac{d\gamma(T)}{dT} = 0,$$

$$\text{(iii)} \quad \widehat{r}(T) < \frac{1}{T}\log\left(\frac{\phi}{T^{\beta-1}}\right) \Longleftrightarrow \frac{d\,\widehat{r}(T)}{dT} > 0 \Longleftrightarrow \frac{d\gamma(T)}{dT} > 0.$$

Panels (a)–(c) in Figure 5 correspond to cases (i)–(iii), respectively.

The curve in each panel of Figure 5 represents a graph of $(1/T)\cdot\log(\phi/T^{\beta-1})$. Given $T = \widetilde{T}$, the relations $d\gamma(\widetilde{T})/dT < 0$, $d\gamma(\widetilde{T})/dT = 0$, and $d\gamma(\widetilde{T})/dT > 0$ hold in panels (a), (b), and (c) in Figure 5, respectively. In other words, the effect of extending duration of patent rights on economic growth depends on the relative magnitude between the rate of return on R&D, $\widehat{r}(\widetilde{T})$, and the threshold value, $(1/\widetilde{T})\cdot\log(\phi/\widetilde{T}^{\beta-1})$. Specifically, panel (a) indicates that if the rate of return on R&D is relatively high compared with the threshold value, then the growth rate of GDP per capita declines as the duration of the patent rights is extended. A situation as shown in panel (a) occurs when ϕ is small, while β is large. This corresponds to the economy that the multifactor productivity index of firms producing the final goods and labor force population are small, while the patent duration elasticity of patent fee is large. In contrast, panel (c) indicates that if the rate of return on R&D is relatively low with the threshold value, then the growth rate of GDP per capita increases as the duration of

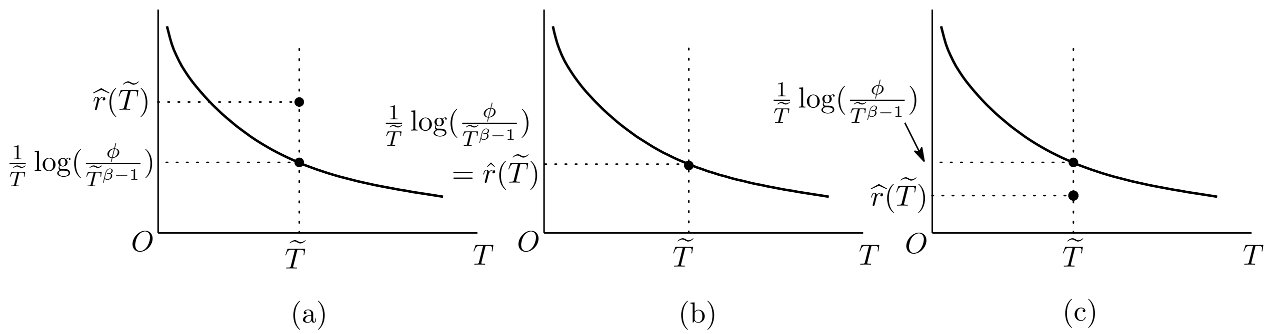

Figure 5: Changes in the growth rate of GDP per capita associated with an extension of the duration of the patent right

the patent rights is extended. A situation as shown in panel (c) occurs when ϕ is large, while β is small. This corresponds to the economy that the multifactor productivity index of firms producing the final goods and labor force population are large, while the patent duration elasticity of patent fee is small. Finally, panel (b) indicates that if the rate of return on R&D is equal to the threshold value, then the duration of the patent rights is uncorrelated with the GDP per capita growth rate. These results imply that strengthening patent protection in terms of extending the duration of patent rights does not necessarily increase the growth rate of GDP per capita.

Here, we focus on the case of panel (a) and give an intuitive interpretation of the result. When the rate of return on R&D is above a certain level, households will provide excessive funds to R&D firms. In other words, an excessive amount of resources is used for R&D activities. This results in a decrease in the amount of resources available for the production of intermediate goods. Consequently, the output of intermediate goods decreases. In this case, the extension of the duration of the patent rights would prolong the period of exclusive production of the differentiated intermediate goods. When the duration of the patent rights is extended, the overall amount of production of the intermediate goods is less than if the duration of the patent rights is not extended. Thus, the extension of the duration of the patent rights means the prolongation of the period during which the overall amount of production of the intermediate goods is reduced. It follows that a decrease in the output of intermediate goods leads to a decrease in the output of final goods because final goods producers use various intermediate goods as factors of production. As a consequence, economic growth performance declines.

As described in Section 2, the models of Iwaisako and Futagaki (2003) and Futagami and Iwaisako (2007) implied that a definite positive relationship exists between the duration of the patent rights and the growth rate of GDP per capita (or consumption per capita). These results may be attributable to the setup of their model that does not take into account the patent fee. In other words, the assumption that there are no patent fee is closely related to the background of derivation of such definite results. In practice, as shown in our analytical results in this section, unlike the results of Iwaisako and Futagaki (2003) and Futagami and Iwaisako (2007), the impacts of extending duration patent rights on the growth rate of GDP per capita is complex under a realistic setting that takes into account the patent fee.

Next, we investigate the effect of a change in the duration of a patent right on the level of social welfare. Following Grossman and Helpman (1991, Ch.3), we regard the lifetime utility of representative household as our measure of social welfare. From Eq. (36), the initial value of consumption per capita, c_0, can be expressed as a function of the duration of the patent right, T, as follows:

$$\begin{aligned} c_0(T) \;=\; & N_0\Big\{\alpha^{\frac{\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}\Big[e^{-\gamma(T)T}+\alpha^{\frac{\alpha}{1-\alpha}}(1-e^{-\gamma(T)T})\Big] \\ & -\alpha^{\frac{1}{1-\alpha}}A^{\frac{1}{1-\alpha}}\Big[e^{-\gamma(T)T}+\alpha^{\frac{1}{1-\alpha}}(1-e^{-\gamma(T)T})\Big]-\frac{1}{L}\Big(\eta+\frac{T^{\beta}}{\beta}\Big)\gamma(T)\Big\}, \end{aligned} \tag{41}$$

where N_0 is the initial level of variety of intermediate goods. Considering Eq. (41), the lifetime utility, U_0, can be represented as a function of the duration of the patent right, T, as follows:

$$U_0(T)=\frac{1}{\rho^2}[\rho\log c_0(T)+2\gamma(T)]+\frac{\varepsilon}{\rho}\log g_0, \tag{42}$$

where g_0 is the initial value of livelihood-based public infrastructure per capita. See Appendix A for details on the derivation of Eq. (42). Differentiating both sides of Eq. (42) with respect to the duration of the patent right yields

$$\frac{dU_0(T)}{dT}=\frac{1}{\rho^2}\Big[\rho\frac{d\log c_0(T)}{dT}+2\frac{d\gamma(T)}{dT}\Big]. \tag{43}$$

In Appendix B, we examine the direction of change in $\log c_0(T)$ with increasing T. The results are as follows: If the sign of $d\gamma(T)/dT$ is positive or $d\gamma(T)/dT$ is zero, then the sign of $d\log c_0(T)/dT$ is negative. Additionally, if the sign of $d\gamma(T)/dT$ is negative, then the sign of $d\log c_0(T)/dT$ can be both positive and negative. Thus, $dU_0(T)/dT<0$ when $d\gamma(T)/dT=0$. However, the sign of Eq. (43) is indeterminate when $d\gamma(T)/dT<0$ or $d\gamma(T)/dT>0$. In other words, if the sign of $d\gamma(T)/dT$ is positive or negative, then the impact of extending the duration of patent rights on social welfare is unclear. Therefore, we find that strengthening patent protection in terms of extending the duration of patent rights does not necessarily enhance social welfare.

The following intuitive interpretation of above results may be made. First, we mention the case in which the extension of the duration of the patent rights promotes GDP per capita growth. In this case, the level of household consumption will increase because the growth rate of wage increases concomitantly with an increase in the growth rate of GDP per capita. Furthermore, it should be noted that the extension of patent duration means a prolonged monopoly position for firms producing differentiated intermediate goods. This implies that differentiated intermediate goods will be sold to firms producing final goods at higher prices over a longer period of time, and prices of final goods will be correspondingly higher. As a consequence, household consumption will decline. That is, there is also the possibility that the extension of the duration of the patent right leads to reduce the level of household consumption. The net impact of extending the duration of the patent rights on household consumption depends on the magnitude of these conflicting effects. Thus, the relation that $d\log c_0(T)/dT<0$ holds when $d\gamma(T)/dT>0$ implies that the negative effect of the extending the duration of the patent right on household consumption definitely exceeds positive effect.

Next, we describe the case in which the extension of the duration of the patent rights causes a decline in GDP per capita growth. Household consumption is negatively affected by lower GDP per capita growth. Additionally, the extension of the duration of the patent right acts to raise the price of the final good. However, a decrease in household consumption reduces the quantity demanded for the final good, resulting in a decrease in the price of the final good. Thus, because of these conflicting effects on the price of the final good, it is unclear how household consumption changes as a result. If the effect of lowering prices exceeds the effect of raising prices, household consumption will ultimately increase; if the effect of raising prices exceeds the effect of lowering prices, the opposite will occur. The complex mechanisms described above obscure the effect of extending the duration of the patent rights on household consumption. Therefore, this is reflected in the result that the sign of $d \log c_0(T)/dT$ is indeterminate when $d\gamma(T)/dT < 0$.

Finally, we state the case in which the extension of the duration of the patent rights has no effect on the growth rate of GDP per capita. In this case, we need only focus on the increase in the price of the final good caused by the extension of the patent duration. An increase in the price of the final good leads to a decrease in household consumption. In sum, household consumption definitely decreases associated with the extension of the duration of the patent rights. Consequently, if $d\gamma(T)/dT = 0$, then the relation $d \log c_0(T)/dT < 0$ holds.

It should be noted that the implication of $dU_0(T)/dT < 0$ when $d\gamma(T)/dT = 0$. This relationship suggests that if there is a duration of patent rights that maximizes the growth rate of GDP per capita, which equals the rate of innovation or the growth rate of livelihood-based public infrastructure, then it is longer than the duration of patent rights that maximizes social welfare. In other words, the patent duration that maximizes social welfare may be shorter than the patent duration that maximizes the growth rate of GDP per capita, the rate of innovation, or the growth rate of livelihood-based public infrastructure.

6 Concluding Remarks

How does strengthening of patent protection affect economic growth and social welfare? In this study, we addressed this issue using an expanding variety model of R&D-based endogenous growth. In particular, this study focused on the duration of the patent rights, one of the two aspects of patent protection that can be viewed in terms of duration and breadth.

The present article contains three major contributions to the R&D-based endogenous growth literature. First, our model can be regarded as a modified version of Barro and Sala-i-Martin's (2004, Ch.6) expanding variety model. The model of Barro and Sala-i-Martin (2004, Ch.6) assumes that the duration of patent rights is infinite, but in this study we modify the setting of their model by embedding a finite duration of patent rights. Second, we developed our discussion under a realistic model that introduced a patent fee. Although patent fee cannot be ignored by patent holders in maintaining their patent rights from a legal perspective and are therefore an essential element in analyzing the effects of patent protection, the role of patent fee has not been considered in the existing relevant literature. This research effort fills this gap. Indeed, we identified that patent fee plays a crucial role in the resluts of our model analysis. Third, this study complements the work of Iwaisako (2013) and Tabata (2021) that analyzed the impact of patent protection on economic growth and social wilfare. Specifically, Iwaisako (2013) and Tabata (2021) examined the role of productive public services flow and productive public capital stock, respectively, within the context of a model focusing on patent breadth.

In contrast, we incoporated a livelihood-based infrastructure provided by the government (i.e., public capital stock that enhances household utility) into our model. To the best of our knowledge, no other study has analyzed the relationship between patent protection and livelihood-based infrastructure. Therefore, our study contributes to the development of this field in the sense that it presents new insights.

Our key findings regarding the impact of patent duration on the rate of innovation, the growth rate of GDP per capita, and the growth rate of livelihood-based public infrastructure can be summarized as follows: If the multifactor productivity index in the final goods sector and the labor force population are relatively small, while the patent duration elasticity of patent fee is relatively high, extending the duration of patent rights negatively affects on the rate of innovation, the growth rate of GDP per capita, and the growth rate of livelihood-based public infrastructure. Conversely, when the multifactor productivity index in the final goods sector and labor force population are relatively large, while the patent duration elasticity of patent fee is relatively small, an extension of the patent duration positively affects the rate of innovation, the growth rate of GDP per capita, and the growth rate of livelihood-based public infrastructure.

The following mechanism may exist behind these results. The extension of duration of patent rights means an increase in the maintenance cost of patent rights for R&D firms, which reduces the incentive of firms to engage in R&D activities. At the same time, because the extension of duration of the patent rights increases the revenue of the R&D firms, there is also an aspect that acts in the direction of increasing the incentive of the firms to engage in R&D activities. Therefore, if an increase in revenue is expected to exceed an increase in the maintenance cost through extension of duration of patent rights, then R&D firms will actively invest in R&D, thereby more innovation will be created and economic growth will be promoted. In short, the relative relationship between an increase in the revenue and an increase in the cost resulting from patent term extension determines the rate of return (in the net sense) on R&D. Moreover, the larger the multifactor productivity index in the final goods sector and labor force population, the larger the increase in revenue through R&D activities is expected to be, reflecting the greater demand for intermediate goods in the final goods sector. Furthermore, the smaller the patent duration elasticity of patent fee, the smaller an increase in maintenance cost due to the extension of duration of the patent rights. Therefore, when the multifactor productivity index in the final goods sector and labor force population large, while the patent duration elasticity of patent fee is small, extending the duration of the patent rights increases the rate of return on R&D; and thus stimulate the incentive for R&D activities, which will contribute to enhancing the creation of innovation and economic growth. In contrast, if the multifactor productivity index in the final goods sector and labor force population are small, while the patent duration elasticity of patent fee is large, the extension of duration of patent rights will cause a decline in innovation and economic growth because extending duration of patent rights leads to decrease the rate of return on R&D; and hence reduce the incentive for R&D activities.

The models of Iwaisako and Futagaki (2003) and Futagami and Iwaisako (2007) showed that the growth rate of GDP per capita or consumption per capita is an increasing function of the duration of patent rights. However, their models, unlike the model in this study, did not take patent fee into account. Thus, one of the reasons why conclusions such as those of Iwaisako and Futagaki (2003) and Futagami and Iwaisako (2007) were drawn may lie in the non-existence of the patent fee. If the patent fee is introduced into the model, the relationship between patent duration and economic growth should become more complex, as demonstrated in this study. In fact, empirical studies by Qian (2007) and Dosi et al. (2023) indicate that the effect of patent protection on innovation is non-monotonic and support our results based on theoretical analysis. Additionally, Thompson and Rushing (1999) found that strengthening patent protection will stimulate economic growth only for developed countries, while it has an insignificant correlation with economic growth for developing countries. A rigorous examination based on data analysis is needed to identify whether developed countries satisfy the implications of our model for the conditions under which stronger patent protection works effectively in promoting economic growth.

As for the impact of extension of patent duration on social welfare, a negative relationship was found between the duration of the patent rights and social welfare only when the extension of the patent duration had no effect on the growth rate of GDP per capita. In other cases, however, no clear relationship was found between the duration of the patent rights and social welfare. In other words, the effect of patent duration on social welfare is unclear when the extension of patent duration has a positive effect on the growth rate of GDP per capita, or when the extension of patent duration has a negative effect on the growth rate of GDP per capita. Thus, we can conclude that strengthening patent protection in terms of extending the duration of patent rights does not necessarily enhance social welfare. Additionally, the results of our model analysis suggested that if there exists a duration of patent rights that maximizes the growth rate of GDP per capita, which equals the rate of innovation or the growth rate of livelihood-based public infrastructure, then it is longer than the duration of patent rights that maximizes social welfare. In other words, the patent duration that maximizes social welfare is shorter than the patent duration that maximizes the growth rate of GDP per capita, the rate of innovation, or the growth rate of livelihood-based public infrastructure.

Appendices

A. Duration of Patent Rights and Social Welfare

In the welfare analysis in Section 3, the effect of the duration of the patent right on social welfare was examined based on Eq. (43). In this Appendix, we explain the derivation process of Eq. (43).

In the steady-growth equilibrium path, the relations $c(t) = c_0 e^{\gamma t}$ and $g(t) = g_0 e^{\gamma t}$ hold. Substituting these into Eq. (23) yields

$$U_0 = \int_0^\infty e^{-\rho t}[\log c_0 e^{\gamma t} + \varepsilon \log g_0 e^{\gamma t}]dt. \tag{A1}$$

Moreover, Eq. (A1) can be rewritten as

$$U_0 = \log c_0 \int_0^\infty e^{-\rho t}dt + \varepsilon \log g_0 \int_0^\infty e^{-\rho t}dt + 2\gamma \int_0^\infty e^{-\rho t}tdt. \tag{A2}$$

Note that the following relation holds for the third term on the right-hand side of Eq. (A2).

$$\int_0^\infty te^{-\rho t}dt = \Big[\frac{e^{-\rho t}}{-\rho}t\Big]_0^\infty - \int_0^\infty \frac{e^{-\rho t}}{-\rho}dt = \frac{1}{\rho^2}. \tag{A3}$$

Furthermore, recall that from the discussion in Section 3, it is confirmed that c_0 and γ depend on the duration of the patent rights, T, in the steady-growth equilibrium path. Thus, considering Eqs. (A2) and (A3), we can derive social welfare as a function of T in the form of Eq. (42).

B. Effects of Extension of Duration of Patent Rights on the Initial Value of Consumption Per Capita

In this appendix, we examine the direction of changes in $\log c_0$ with the extension of duration of patent rights, T. From Eq. (41), $\log c_0(T)$ is represented as

$$\log c_0(T) = \log N_0 + \log \theta(T), \tag{B1}$$

where

$$\begin{aligned}\theta(T) \equiv\ & \alpha^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} [e^{-\gamma(T)T} + \alpha^{\frac{\alpha}{1-\alpha}}(1 - e^{-\gamma(T)T})] \\ & -\alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} [e^{-\gamma(T)T} + \alpha^{\frac{1}{1-\alpha}}(1 - e^{-\gamma(T)T})] - \frac{1}{L}\Big(\eta + \frac{T^\beta}{\beta}\Big)\gamma(T) > 0.\end{aligned}$$

Differentiating both sides of Eq. (B1) with respect to the duration of patent rights yields

$$\frac{d \log c_0(T)}{dT} = \frac{1}{\theta(T)} \frac{d\theta(T)}{dT}. \tag{B2}$$

In Eq. (B2), $d\theta(T)/dT$ is given by

$$\frac{d\theta(T)}{dT} = \Psi e^{-\gamma(T)T}\Big[T\frac{d\gamma(T)}{dT} + \gamma(T)\Big] - \frac{1}{L}\bigg[T^{\beta-1}\gamma(T) + \Big(\eta + \frac{T^\beta}{\beta}\Big)\frac{d\gamma(T)}{dT}\bigg], \tag{B3}$$

where $\Psi \equiv A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} [(1 - \alpha^2) - (1 - \alpha)\alpha^{-\frac{\alpha}{1-\alpha}}] < 0$.

We find from Eq. (B3) that when the sign of $d\gamma(T)/dT$ is positive or $d\gamma(T)/dT$ is zero, the sign of $d\theta(T)/dT$ is negative. Therefore, from Eqs. (B2) and (B3), if $d\gamma(T)/dT \geq 0$, then the relation $d \log c_0(T)/dT < 0$ holds. Additionally, when the sign of $d\gamma(T)/dT$ is negative, the sign of $d\theta(T)/dT$ can be either positive or negative depending on the magnitude of the various parameters. Consequently, if $d\gamma(T)/dT < 0$, then the sign of $d \log c_0(T)/dT$ is indeterminate.

Declarations

Conflict of interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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