Separate needs for the leisure-consumption choice

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Abstract

This paper explores the labour supply and consumer demand equations derived from a utility function created by *adding* two S-shaped utilities.

An S-shaped cardinal utility for a commodity represents the individual's experience of fulfilment of a need – deprivation (increasing marginal utility (MU)), subsistence (a point of inflection), sufficiency (diminishing MU), and either satiation at finite consumption with the possibility of surfeit, or satiation at infinite consumption.

The utilities of commodities fulfilling the same need are weakly separable (multiplicative) and those of two commodities fulfilling different needs are strongly separable (additive).

Functional forms are derived from a utility function created by *adding* two *normal* distribution functions with satiation at infinity, the parameters of which have meaningful psychological interpretations. The indifference map, demand and Engels curve diagrams are explored.

The main outcomes:

- Concave- (dysfunctional poverty) and convex-to-the-origin indifference curves are separated by a straight-line indifference curve, (an absolute poverty line).
- Budget movements on the indifference curve map reveal: corner solutions and disequilibrium associated with dysfunctional poverty; optimisation occurs elsewhere, even with deprivation in one or other need.
- The derived functional forms are functions of only the real wage rate and endowments of unearned consumption.
- The derived functional form diagrams display: involuntary unemployment, disjointed curves, sticky wages and prices, wage- and price-elasticity associated with deprivation in a need, inferior normal and inferior-Giffen responses, and envelope curves.
- The slope of the straight-line indifference curve, (defined by the relative-intensities-of-need), and its intercept on the endowment axis, play significant and dramatic roles in both Engels diagrams.

(250 words)

Keywords: *S-shaped cardinal utility includes increasing marginal utility expressing deprivation; additive utilities represent separate needs; dysfunctional poverty causes involuntary unemployment and disequilibrium; absolute poverty line; sticky wages.*

JEL classification: D11, J22.

1 Introduction

The marginal utility revolution, founded by [1] Jevons (1871), [2] Menger (1871) and [3] Walras (1874), has provided the corner stone of neoclassical microeconomics. Notwithstanding the fact that, around the year 1900, Pareto dispensed with cardinal utility on the grounds that it cannot be measured, in favour of ordinal utility, the process of humanising the initial robot-like utility-maximising, *homo economicus* with perfect information began. Alternative objectives, such as minimising maximum regret and satisficing, have been introduced, while, in more recent decades, the individual's methods for dealing with imperfect, or the absence of, information have been modelled. Despite these contributions by behavioural economists, neoclassical microeconomics remains remarkably barren of basic psychology. It is devoid of any acknowledgement that people could experience deprivation to which they react differently, or that an individual has several different fundamental human needs, (despite [4] Maslow's (1943) 'Hierarchy of Needs' hypothesis being published more than 80 years ago). This paper aims to extend basic microeconomic utility theory into the realms of deprivation and separate needs.

The Law of Demand, claiming that demand curves are always negative, based on the assumption of diminishing marginal utility as personally experienced by economists, has rarely been challenged ([5] van Praag (1968), [6] Miller (1988)) in a century and a half. Van Praag proved that an S-shaped utility for a commodity, bounded below and above, ($0 \leq u \leq 1$), could be used to make interpersonal welfare comparisons, thus, partially solving the non-measurability problem of utility.

This paper builds on the theory developed in 'The Ubiquitous Giffen'[1], based on van Praag's seminal work, by recognising that a bounded *leaning-S-shaped* utility for a commodity (good, service or event) captures the individual's potential experiences associated with the fulfilment of a need. These cover deprivation (increasing marginal utility (MU)), subsistence (a point of inflection), sufficiency (diminishing MU), and satiation, either at finite consumption with the possibility of surfeit, or at infinite consumption. The separability rule is applied: utilities of commodities fulfilling the same need are weakly separable (multiplicative) and those of two commodities fulfilling different needs are strongly separable (additive). The rule fails to distinguish between multiplied and added utilities specifying only diminishing MU.

This paper derives labour supply and consumption demand equations from leaning-S-shaped bounded cardinal utilities for commodities fulfilling different needs and examines the individual's reactions to changes in the real wage rate and endowments of unearned consumption.

The method adopted involves three media. Firstly, the mathematical functional forms for labour supply and consumption demand are derived in Appendix A from a utility function created by the addition of two normal distribution functions, with satiation at infinite consumption. Equations for the boundaries between superior and inferior, and between inferior normal and inferior-Giffen, experiences were derived. Secondly, the equations were used to create the indifference curve map and the four derived functional form (DFF) diagrams. Thirdly, these very non-linear diagrams were carefully scrutinised. Their

[1] Author's name and other details to be supplied. The author has already given permission for some material in 'The Ubiquitous Giffen' to be replicated in this paper.

shapes and patterns yielded a wealth of visual information about human behaviour, on which to base a commentary.

In section 2, the paper briefly summarises the propositions, leading to the creation of the utility function. The distinctive features of the indifference curve map for leisure and consumption (Figure 2) are noted. Section 3 introduces the budget constraint and the consumption and labour equations.

In section 4, Table 1 summarises the relationships between the unfamiliar features observed on the indifference Map and their reflected outcomes in the DFF diagrams (Figures 3a – 3d). Table 2 compares the four DFF diagrams in detail. Figure 4 presents the four diagrams together as one diagram, sharing the four axes: labour, real wage rates, consumption and endowments. The reservation wage diagram reveals it as a U-shaped function of endowments. Section 5 concludes by summarising the assumptions and outcomes.

In the mathematical Appendix A, the utility function is created, from which the functional forms are derived, using the familiar leisure-consumption notation, together with equations for the boundaries and envelope curves between superior and inferior responses, and the boundary between inferior normal and inferior-Giffen ones.

Appendix B comprises a set of four tables, detailing the shapes of the curves in each DFF diagram, which follow axes and can be disjointed.

2 A utility function and an indifference curve map

On a graph of utility vs consumption, the representation of diminishing marginal utility (MU) is that of an arc hanging in mid-air without parameters or reference points.

Van Praag's (1968) S-shaped utility function for a commodity (good, service or event), was based on a lognormal distribution function with satiation at infinity. It is *bounded above and below* ($0 \leq u \leq 1$). He showed that it could be used for interpersonal welfare comparisons, thus partially solving the non-measurability problem of utility. He also used it to create a multiplicative utility function. Van Praag's work has been developed and successfully applied by The Leyden School ([7] Van Herwaarden and Kapteyn, 1981; [8] Hagenaars, 1986; [9] van Praag and Kapteyn, 1994).

The *shape* of an S-shaped utility for a single commodity in Figure 1[2] can be recognised as an expression of the different stages experienced by the individual as her needs are being fulfilled – deprivation (increasing marginal utility (MU)), subsistence (a point of inflection), sufficiency (diminishing MU), and either satiation at finite consumption with the possibility of a surfeit, or satiation at infinite consumption.

The *separability rule* concerning human needs states:

> The utilities of commodities fulfilling the same need are weakly separable (multiplicative) and those of two commodities fulfilling different needs are strongly separable (additive).

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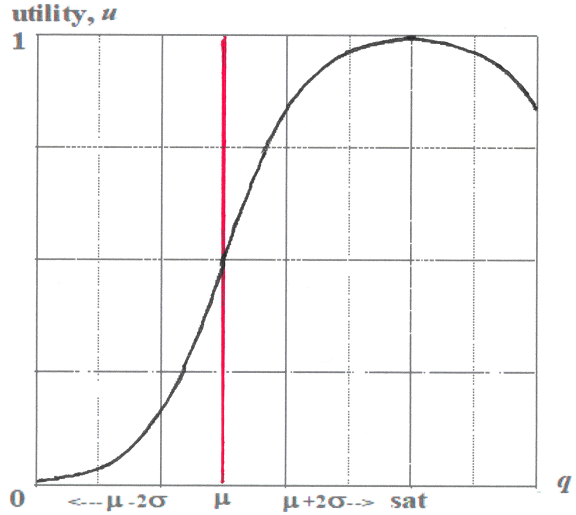
[2] Figures 1-4 were created using [10] Seppo Mustonen's program SURVO (1992).



KEY

u	utility
q	consumption
μ	subsistence consumption
sat	satiation consumption
σ	intensity-of-need for consumption

FIGURE 1 LEANING-S-SHAPED U-FN

Whereas van Praag's work is based on the n-variable, multiplicative form of the lognormal distribution function (n-Mult.LN-DF), the normal distribution function is more tractable for additive utilities (2-Add.N-DF). Both incorporate satiation at infinity, with 'near satiation' occurring at much lower, finite levels of consumption.

Two leaning-S-shaped, bounded cardinal utilities with satiation at infinity, representing the needs for leisure (unwaged time), q_0, and for consumption, q, with subsistence parameters, γ_0 and γ, and intensity-of-need parameters, σ_0 and σ, respectively, were added together to form a utility function.

The '2-Add.N-DF' **utility function** is given as:

$$u(q_0, q) = ½\, F_0(q_0) + ½\, F(q)$$

$$u(q_0, q) = \frac{1}{2}\int_{-\infty}^{q_0} \frac{\exp\left[-(R_0 - \gamma_0)^2/2\sigma_0^2\right]}{\sigma_0.\sqrt{2\pi}}\, dR_0 + \frac{1}{2}\int_{-\infty}^{q} \frac{\exp\left[-(R - \gamma)^2/2\sigma^2\right]}{\sigma.\sqrt{2\pi}}\, dR \qquad \text{(A1)}$$

It would be almost impossible to use equation (A1) to create an indifference curve map. Fortunately, as [11] Johnson and Kotz (1970; 244) state 'The shape of this [logistic] distribution is quite similar to that of the normal density function'.

$$P(t) = \frac{e^{t}}{[1 + e^{t}]^2} = \frac{e^{-t}}{[1 + e^{-t}]^2}.$$

This was used to create the indifference curve map in Figure 2, adjusted for location and scale.

$$q = \gamma - \{\log\,[\,(0.5 * bracket) / (u * bracket - 0.5) - 1]\,\} / (1.82/\sigma), \qquad \text{(A2)}$$

where u is utility and $bracket = (1 + \exp(-(1.82/\sigma_0) * (q_0 - \gamma_0)))$.

Equation (A2) was used to create Figure 2, the indifference curve map.

The following unfamiliar features can be observed on the indifference curve map.

The straight-line indifference curve, BA

The map is divided by a straight-line indifference curve, BA, through point E, (at co-ordinates, γ_0, γ), with a negative slope, σ/σ_0, creating an intercept on the right-hand vertical axis at q = A. B is the intercept of BA on the q-axis. At consumption q = B, utility is very close to satiation utility.

The ratio σ/σ_0 is defined by the consumer's *relative intensities-of-need*, in this case of leisure over consumption. The smaller the value of σ_0, the greater the slope of the straight-line indifference curve (measured at corner A). If $\sigma/\sigma_0 > 1$, then leisure is valued more highly than consumption. The greater the intensity-of-need, the more highly valued leisure becomes, compared with consumption.

The straight-line indifference curve BA separates the concave-to-the-origin indifference curves (defining 'dysfunctional poverty') close to the origin in the rhomboid B0TA, from the convex-to-the-origin indifference curves. The rhomboid B0TA represents nontangential choices, which can lead to corner solutions on the right- or left-hand q axes,

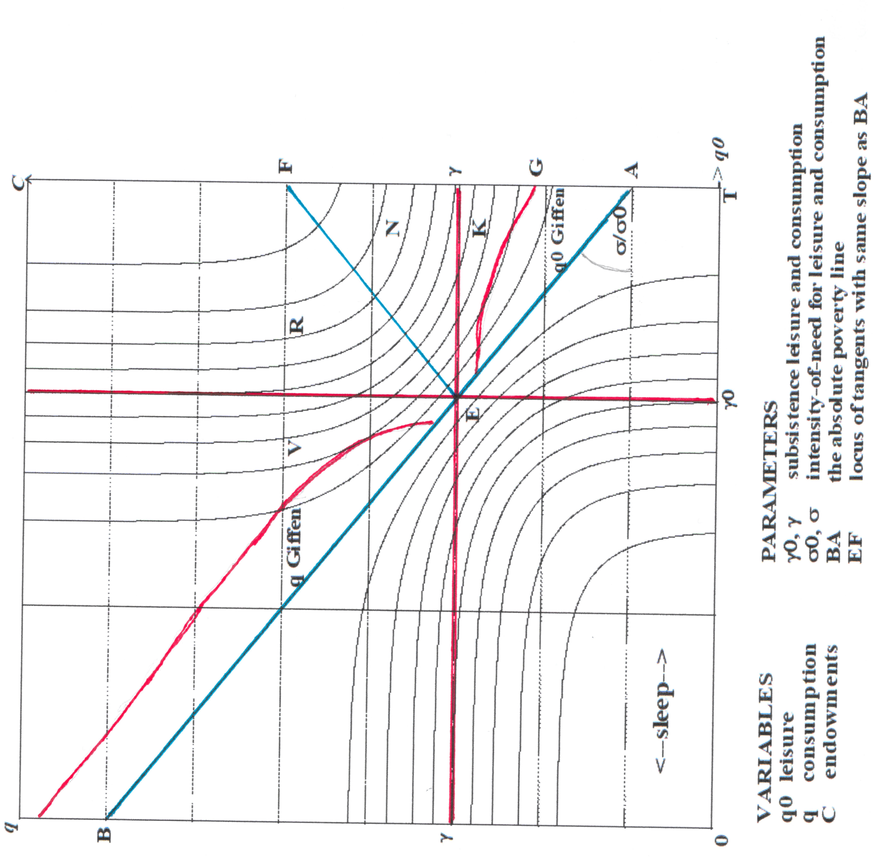


FIGURE 2 INDIFFERENCE CURVE MAP FOR LEISURE-CONSUMPTION CHOICE

representing deprivation in consumption and leisure respectively. BA can be identified as an Absolute Poverty Line.

Four quadrants

The map is further divided into four quadrants by the two subsistence parameters, γ_0 and γ. The left-hand and lower quadrants represent a border of deprivation beside the leisure and consumption axes respectively.

The locus EF

The line EF is the locus of points where the slopes of the indifference curves are parallel to BA, for $q_0 > \gamma_0$ and $q > \gamma$, creating an intercept on the right-hand vertical axis at q = F. For this model, at consumption q = F, the individual's utility is fairly close to satiation utility.

The convex-to-the-origin indifference curves

The convex-to-the-origin indifference curves, bounded below by BA, are divided into four areas by the two subsistence parameters, and EF.

It is shown in equation (A5) in Appendix A that, in the top right-hand quadrant of Figure 2, both commodities are experienced as superior goods, (additivity and positive diminishing marginal utilities always yield superior characteristics). With additive utilities, the two goods are net substitutes for each other. EF divides the top right-hand quadrant of Figure 2 into high-waged superior experiences for leisure in area R, and low-waged ones ($w/p < \sigma/\sigma_0$) in area N.

An individual experiences leisure as an inferior good in the triangular area marked as K in Figure 2 and consumption as an inferior good on area V. The experience of a commodity as inferior is associated with it being abundant enough for sufficiency, while combined with deprivation of a commodity fulfilling another need.

In area V, which is that part of the left-hand border where the indifference curves are convex-to-the-origin, the individual is deprived of leisure, (with increasing MU), and here leisure is termed an 'ultra-superior' good.

Thus, the subsistence parameters provide the boundaries between a commodity responding as inferior or superior and it responding as superior or ultra-superior.

Optimisation when deprived in one or other need may be regarded as 'functional poverty', as in in areas K and V.

Boundary between inferior normal and inferior-Giffen experience

Equation (A7a), derived in Appendix A, defines the boundary between leisure being experienced as inferior normal and its being inferior-Giffen in area K, but it must be solved numerically. The boundary has been drawn in area V for consumption, and in area K for leisure, where it creates an intercept on the right-hand vertical axis at G. The boundaries meet the straight-line indifference curve at small distances from E.

The Giffen experience is associated with more being consumed of an abundant but cheaper commodity in response to a rise in its price, enabled by relinquishing some of another more expensive good in which the individual is already extremely deprived ([11] Miller, 2025).

Endowments of unearned consumption

Individuals receive endowments during their lifetimes, from family, local communities, education, via unearned income and via state services and benefits. Each individual receives a maximum endowment of leisure, q_0 = T, on the horizontal axis, where a right-hand vertical axis measures the individual's endowment of unearned consumption, *C*.

An individual will respond very differently to changes in wage rates depending on his/her level of endowment, with markers A, G, γ and F playing significant roles, especially the 'survival endowment' at *C* = A.

3 Income equations and derived functional forms

Let *w* be the wage rate and *p* be the price of consumption.

Full income: $M = T.w + C.p$, where $M \geq 0$.

Survival income: $(\gamma_0.w + \gamma.p)$

Supernumerary income: $Z = (T.w + C.p) - (\gamma_0.w + \gamma.p)$

$= (T - \gamma_0).w + (C - \gamma).p$.

The linear budget constraint, $M = q_0.w + q.p$ is expressed as:

$$q = (M - q_0.w)/p. \qquad \text{(A3)}$$

In Appendix A, the utility function is maximised subject to the budget constraint, using the Lagrange multiplier method, and the functional forms are derived.

The equation for consumption demand is:

$$q = \gamma + \frac{\left((T-\gamma_0).x + (C-\gamma)\right) - \frac{x}{b}.\sqrt{\left[\left((T-\gamma_0).x+(C-\gamma)\right)^2 - \left(1-\left(\frac{x}{b}\right)^2\right).\left(\sigma^2.2.ln\left(\frac{x}{b}\right)\right)\right]}}{\left(1-\left(\frac{x}{b}\right)^2\right)} \qquad \text{(A9c)}$$

where $x = w/p$ and $b = \sigma/\sigma_0$. This equation must be constrained so that $q \geq 0 + C$ and $0 \leq lab \leq T$.

The labour supply equation, where $lab = T - q_0$, is given by:

$$lab = (T - \gamma_0) - \frac{\left((T-\gamma_0) + (C-\gamma)/x\right) - \frac{b}{x}.\sqrt{\left[\left((T-\gamma_0) + \left(\frac{C-\gamma}{x}\right)\right)^2 - \left(1-\left(\frac{b}{x}\right)^2\right).\left(\sigma_0^2.2.ln\left(\frac{b}{x}\right)\right)\right]}}{\left(1-\left(\frac{b}{x}\right)^2\right)} \qquad \text{(A10)}$$

Equation (A10) must also be constrained such that $0 \leq lab \leq T$.

These two equations for the derived functional forms confirm that consumption demand, *q*, and labour supply, *lab* = (T – q_0), are functions of only two independent variables, the real wage rate, *x* = *w/p*, and the endowment of unearned consumption, *C*.

The equations were used to create four diagrams – labour supply, consumption demand and their associated Engels curves.

4 The indifference curve map and the four derived functional form (DFF) diagrams

In section 3 above, the utility function was maximised subject to a budget constraint creating the DFF equations. In this section, the series of maximisation points created by the budget line on the indifference Map are related to the corresponding curves on the four DFF diagrams, particularly where the straight-line indifference curve is involved. These are summarised in Tables B1 – B4 in Appendix B, mapping areas K, N, R and V on the Map to their manifestation as sections K, N, R and V on the DFF diagrams.

The indifference Map is the key to understanding the four derived diagrams. Each diagram comprises a set of curves which are the outcomes from tracing the effects of changes in *w/p* or *p/w* and *C* on labour and consumption. Each starts with the budget line at the origin (*lab* = 0, *q* = 0) of the Map, before increases in *w/p* rotates it clockwise, or increases in *p/w* rotates it anticlockwise, with the origin as a pivot, or it moves up the *C* axis. The budget line movements and their optimisation (or other maximisation) points are then related to the curves on the four derived diagrams, each of which has three or four distinctive curve patterns.

Each DFF diagram is divided into four quadrants, by its own subsistence parameter and either by the 'survival endowment', *C* = A, or by an 'equilibrium wage rate', (or 'equilibrium price'), equal to the slope of BA, $w/p = \sigma/\sigma_0$, (the relative-intensities-of-need).

The movements of the budget line over the Map create unfamiliar phenomena, (such as corner solutions and disequilibria in the dysfunctional poverty area, sticky wages and prices, and responses to deprivation). These are matched by their corresponding manifestations (curves being constrained to follow axes, disjointed curves, high wage- and price-elasticities) on the four DFF diagrams. These are summarised in Table 1 below.

A disequilibrium can cause an apparent instability in behaviour; that is, large reactions can occur in response to small changes in real wage rates. If wages were to waver slightly around σ/σ_0, then behaviour could appear to oscillate markedly.

Table 2 then compares how the unfamiliar features from the indifference Map manifest as outcomes in each of the four DFF diagrams.

TABLE 1 Unfamiliar features observed on the indifference curve map are reflected in outcomes on the derived functional form diagrams

MAP AREA FIGURE 2	UNFAMILIAR FEATURES			DERIVED DIAGRAMS FIGURES 3a, 3b, 3c and 3d
CONCAVE TO ORIGIN		**REAL WAGE, *w/p*** **PRICE, *p/w***	**ENDOW-MENTS, *C***	**LS**, labour supply. **LE**, labour Engels. **CD**, consumption demand, and **CE**, consumption Engels.
DYSFUN-CTIONAL POVERTY	**Corner solutions**	$w/p < \sigma/\sigma_0$	$0 \leq C < A$	LS, CD: Curves move along wage/price axis LS: **Involuntary unemployment**. LE curves move along *C* axis, *lab* = 0 CE: *q* is a corner solution, $q = 0 + C$.
	Disequilibrium: choice **jumps** from one axis to the other.	$w/p = \sigma/\sigma_0$	$0 \leq C < A$	LS: **disjointed curve** from *lab* = 0 to *lab* = T as *w/p* increases. CD: **disjointed curve** from $q \leq B$ to $q = 0 + C$, *as p/w* increases.
	Disequilibrium: Equal utility on axes.	$w/p = \sigma/\sigma_0$	$0 \leq C < A$	LE, CE: Curves move along **parallel paths** to *C* = A. and exit as a straight line to *C* = F.
BA	Slope = $-\sigma/\sigma_0$, relative intensities-of-need. Absolute poverty line	$w/p = \sigma/\sigma_0$	*C* = A	LS, CD: **Sticky wages/prices**. LE, CE: **Survival endowment**
CONVEX				
V	**Deprivation** of q_0. q_0 is ultra-superior ***q* is inferior normal & inferior-Giffen**	$w/p > \sigma/\sigma_0$ $p/w < \sigma_0/\sigma$	$0 \leq C < A$ *C* - elastic	LS, LE: *lab* falls – reduces deprivation LS, CD: v hi-waged/low-priced elastic CD, **U-shaped curves**: **envelope**. CE: **U-shaped curves**: **envelope**.
R	*q* and q_0 are superior	$w/p > \sigma/\sigma_0$	$0 \leq C$ *C* - elastic	LS, CD: very wage / price inelastic LE: *lab* = *lab*(*w/p*, –*C*). CE: *q* = q(*p/w*, +*C*).
EF		$w/p = \sigma/\sigma_0$	A < *C* < F	*lab* declines} *q* increases} in straight lines to *C* = F.
N	q_0 and *q* are superior	$w/p < \sigma/\sigma_0$	$A < C \leq F$ C - elastic	LS, CD: wage / price inelastic LE: *lab* = *lab*(*w/p*, –*C*). CE: *q* = *q*(*p/w*, +*C*).
K	**Deprivation** of *q*. *q* is ultra-superior **q_0 is inferior normal & inferior-Giffen**	$p/w > \sigma_0/\sigma$ $w/p < \sigma/\sigma_0$	$A < C \leq \gamma$ *C* - elastic	CD, CE: *q* = *q*(-*p/w*, +*C*) LS, CD: v. low-waged/hi-priced elastic LS: **U-shaped curves**: **envelope**. LE: **envelope**.
	U-shaped reservation wage	*w/p* = f(*C* | *lab* = 0)		LS: the intercepts on the *w/p* axis form **a U-shaped curve**. Minimum *w/p*, (*w/p**), occurs at $C = \gamma$.
OVERALL			$C \geq 0$	LE: **Maximum labour falls. Polarisation of labour falls.** CE: *C* provides **a consumption floor**.

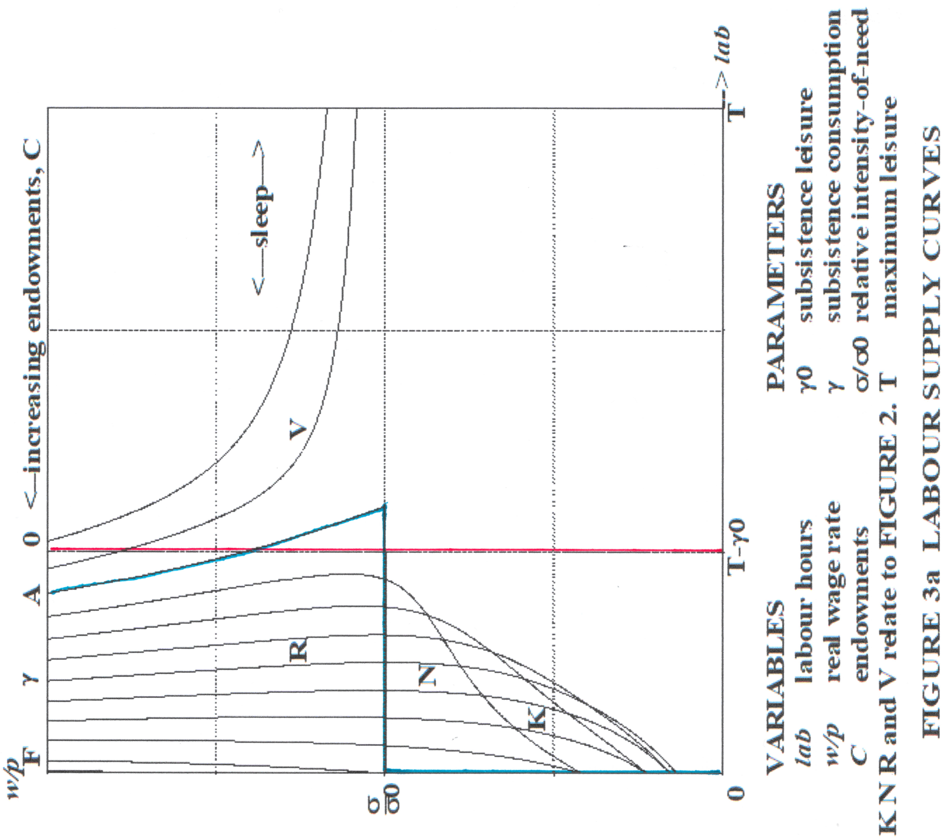
				Polarisation of consumption falls.

TABLE 2 Unfamiliar features to note on the four derived functional form diagrams as *w/p* and *C* increase from zero

FEATURES	**Figure 3a LS**	**Figure 3b LE**	**Figure 3c CD**	**Figure 3d CE**
$0 \leq C < A$. **Dysfunctional poverty** **Corner solutions** on Indifference curve Map.	*lab* = 0, LS moves up the *w/p* axis for all $0 \leq w/p \leq \sigma/\sigma_0$. **Involuntary unemployment**	*lab* = 0, LE moves along the *C* axis for all $0 \leq w/p \leq \sigma/\sigma_0$.	*q* = 0, CD moves down the *p/w* axis, for $p/w > \sigma_0/\sigma$.	*q* = 0 + C. CE moves along the 45° line, for $p/w > \sigma_0/\sigma$.
Disequilibrium	$w/p = \sigma/\sigma_0$. **Disjointed LS curves**, from *lab* = 0 to *lab* = T	$w/p = \sigma/\sigma_0$. Parallel pathways lab = 0 and lab = T to *C* = A.	$p/w = \sigma_0/\sigma$. **Disjointed CD curves**, from *q* = 0 + *C* to $(B – A) \leq q \leq B$.	$p/w = \sigma_0/\sigma$. Parallel pathways from *q* = 0 to *q* = 0 + C = A, and *q* = (B–A) to *q* = B.
More corner solutions	LSs move up *w/p* axis at *lab* = T to $\sigma/\sigma_0 < w/p \leq B/T$.		CDs slope negatively from $(B–A) \leq q$ to *q* = B	
OPTIMISATION Functional poverty. Deprivation of leisure, section V	$w/p > \sigma/\sigma_0$, labour falls. **very wage elastic.** Thence LS falls, inelastic, thro' R.	$w/p > \sigma/\sigma_0$, labour falls steeply, very *C* elastic.	$p/w < \sigma_0/\sigma$. Consumption falls steeply, very price elastic U-shaped curves over V and R.	$p/w < \sigma_0/\sigma$. very *C* elastic U-shaped curves within section V
Envelope curves…			… between inferior section V and superior R. At *p/w* = 0, *q* = B (section R)	… between Giffen-deprived and 'normal' deprived, within section V.
C = A, $w/p = \sigma/\sigma_0$, $p/w = \sigma/\sigma_0$. **BA**	**Sticky wages** from *lab* = 0 to *lab* > $(T – \gamma_0)$.	**Survival endowment** from *lab* = 0 to *lab* = T.	**Sticky prices** from *q* < 0 + A to $q > \gamma$.	**Survival endowment** from *q* = 0 + *C* = A to *q* = B.
$A < C \leq F$. $w/p = - \sigma/\sigma_0$, $p/w = - \sigma/\sigma_0$. **EF**		$w/p = \sigma/\sigma_0$. *lab* decreases in a straight line from *lab* = $(T – \gamma_0)$ to *lab* = F.		$p/w = \sigma_0/\sigma$. *q* increases in a straight line from $q = \gamma$ to *q* = 0 + *C* = F.
$A < C \leq \gamma$. **Functional poverty**. **Deprivation of consumption,** section K	$w/p < \sigma/\sigma_0$, **very wage elastic.**	very *C* elastic. Boundary between K and N extends from *lab* = $(T – \gamma_0)$ to *lab* = 0, for $w/p^* \leq w/p \leq \sigma/\sigma_0$.	$p/w > \sigma_0/\sigma$, very price elastic	$p/w > \sigma_0/\sigma$, very C-elastic.
Envelope curves…	… between inferior section K and superior section N.	… between Giffen-deprived and 'normal' deprived within section K.		
F < C		*lab* = 0. **Voluntary unemployment.**		*q* = 0 + C.
	Intercepts on the *w/p* axis form a **U-shaped reservation wage** curve with minimum at $C = \gamma$	Section R extends down behind K and N as w/p increases.	As $p/w \rightarrow \infty$, the CD curves tend to *q* = 0 + *C*.	*q* = 0 + *C* provides a **consumption floor**.

*w/p** is the minimum level of *w/p* as indicated in the LS diagram.
The lowest non-zero value for *w/p*, when C = γ, is given by equation (A14) in Appendix A.

Figure 3a Labour supply diagram



The LS diagram is divided into four quadrants by the subsistence parameter (T – γ_0) and the equilibrium wage rate, *w/p* = σ/σ_0.

The least familiar features are:

- the very elastic LS curves in section V, for very low endowments, $0 \leq C < A$, and high wage rates, *w/p* > σ/σ_0, associated with deprivation of leisure. These have started at the origin, climbed the *w/p* axis with *lab* = 0 (involuntary unemployment) to *w/p* = σ/σ_0, where facing disequilibrium, they have jumped to *lab* = T creating a disjointed LS curve, then moved up the *q* axis for *lab* = T, before optimising in sections V and R.

- the LS curve for the 'survival endowment', *C* = A, also starts at the origin and climbs the *w/p* axis to the equilibrium wage rate, *w/p* = σ/σ_0, where *lab* can take any of a range of values (determined by demand for labour), before exiting as a negative sloping curve briefly through section V to R.

- the very elastic LS curves in section K, for low endowments, $A < C \leq \gamma$, and low wage rates, *w/p* < σ/σ_0, associated with deprivation of consumption. These start at the origin, climb up the *w/p* axis to a point *w/p* < σ/σ_0, where they create a series of decreasing intercepts on the *w/p* axis as *C* increases, with elastic, positive-sloping LS curves before passing briefly through section N to section R with inelastic, negative-sloping LS curves.

- The last unfamiliar feature is the envelope curve below the U-shaped LS curves, for $A \leq C \leq \gamma$, reflecting the boundary between leisure responding as inferior in section K and as superior in section N. It stretches from the point where the 'survival endowment' LS curve at C = A exits from the equilibrium price to an intercept on the *w/p* axis at *w/p* > 0, which is the minimum level of the real wage rate (equation A14) which occurs when C = γ.

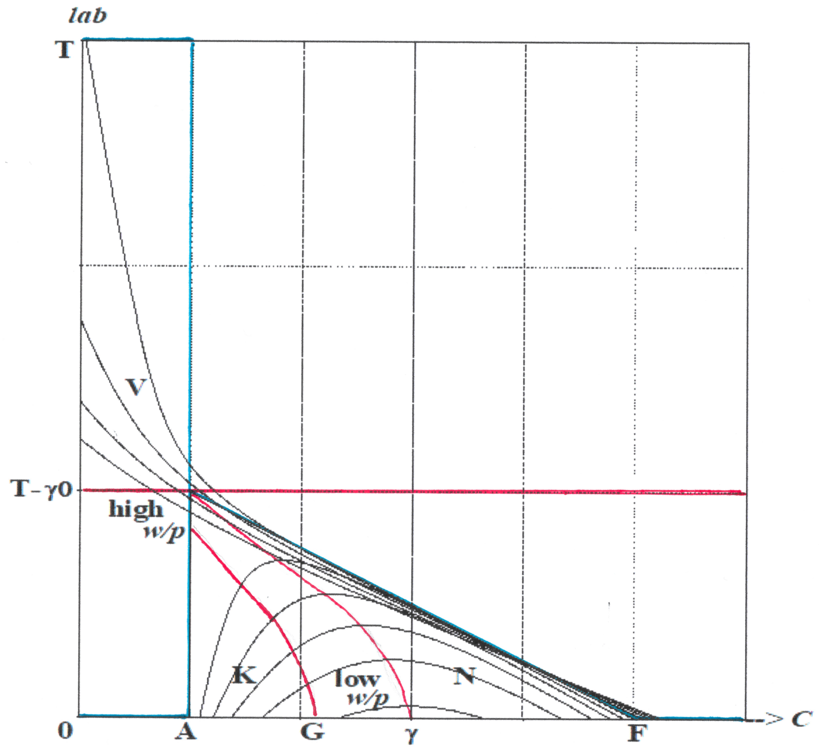
The curves in sections N and R, for *C* > γ, follow a more familiar pattern, with intercepts increasing as *C* increases.

Figure 3b Labour Engels diagram

The LE diagram is divided into four quadrants by the subsistence parameter (T – γ_0) and the survival endowment, *C* = A. *Higher* values of labour, *lab* > T – γ_0, represent deprivation of leisure.

Figure 3b is difficult to interpret on account of three apparent anomalies. Firstly, the space on the LE diagram on which to represent section R is extremely limited. Secondly, there is no clear boundary between sections K and N, and thirdly the anticipated envelope curve, between leisure being experienced as inferior-Giffen and inferior normal, does not seem to have materialised.

Yet, the LE curve(s) for w/p = σ/σ_0 in Figure 3b gives the most dramatic economic narrative of all four DFF diagrams. With low endowments, $0 \leq C < A$ the individual faces stark choices, either deprived of leisure on high wages, or in involuntary unemployment.



VARIABLES		PARAMETERS	
lab	labour hours	$\gamma 0$	subsistence leisure
C	endowments	γ	subsistence consumption
w/p	real wage rate	A	survival endowment
K N R and V relate to FIGURE 2.		T	maximum leisure

FIGURE 3b LABOUR ENGELS CURVES

When *w/p* = σ/σ_0, and *C* = 0, the budget line is parallel to BA on the Map and presents equal utility at *lab* = 0 and *lab* = T. Disequilibrium prevails as endowments increase to *C* = A. This is represented on Figure 3b as two (disjointed) LS curves running parallel to each other at *lab* = 0 and *lab* = T for 0 ≤ C < A. At *C* = A, a transformation occurs, where the individual appears to face the whole range of labour possibilities, but, as *C* increases, optimisation leads in a negative-sloping straight line from *lab* = T – γ_0 to *lab* = 0 at *C* = F. *Lab* = 0, (voluntary unemployment), for all C > F. When facing *w/p* > σ/σ_0 and *w/p* < σ/σ_0, the individual can experience similar, but not as dramatic, changes when endowments change.

The LE diagram illustrates the drama facing an individual if two events were to coincide. With 'survival endowments', *C* = A, and wage rate, w/p = σ/σ_0, s/he could experience passing from either full deprivation at lab = T or from involuntary unemployment at *lab* = 0 to 'full employment' at *lab* = (T – γ_0) – or vice versa. When this latter occurs at society level, it can be very dramatic, when large numbers of people face a drop in the real value of their state benefits and suddenly become unemployed and their health deteriorates due to the deprivation associated with involuntary unemployment.

What of **section R**, which is not labelled on the LE diagram? The space available for LE curves associated with area R initially appears as a very slim space lying below lab = (T – γ_0) and around the negative straight-line LE curve for *C* = A, reflecting the boundary between sections R and N, (EF on the Map). However, section R grows downwards as *w/p* increases, (the LE curves creating intercepts on the *lab* axis, each one lower than the previous one, from *C* = 0, and behind sections K and N), with negative-sloping, *C*-elastic LE curves, reaching *lab* = 0 at *C* > F. When *w/p* = ∞, *lab* = 0, for all *C* ≥ 0, and the LE curve is co-incidental with the *C* axis.

The **boundary between sections K and N** is found (by solving equation (A5) numerically) to comprise the locus through the turning points of the LE arcs for *w/p** ≤ *w/p* < σ/σ_0 and A < *C* < F, ending at C = γ. This was added to the LE diagram.

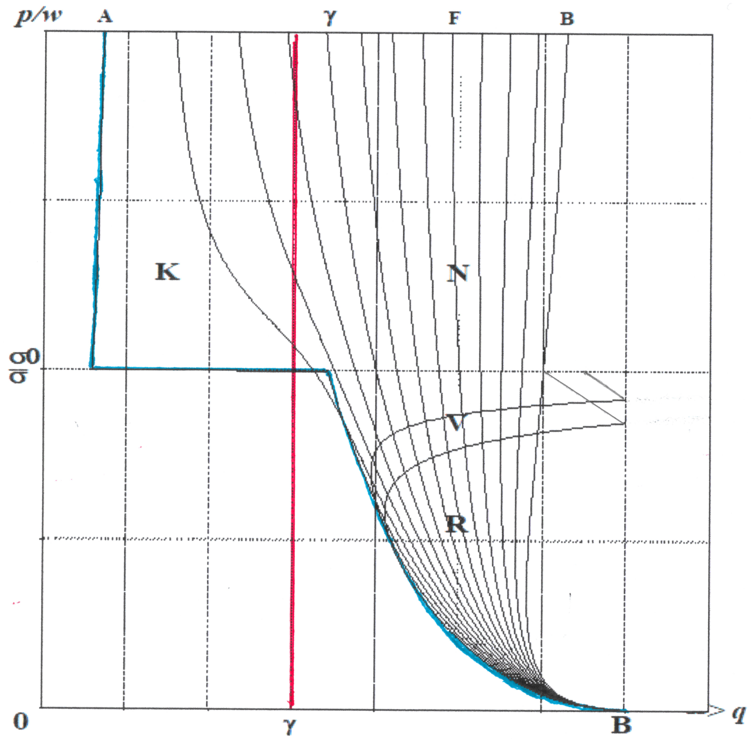
Similarly, the '**envelope**' on the boundary between leisure responding as inferior normal and as inferior-Giffen in area K was found (by solving equation (A7a) numerically). This also was added to the LE diagram as the arc from (*lab* < (T – γ_0), *C* = A) to a point G roughly halfway between A and γ on the *C* axis. It separates the more elastic from the less elastic part of each LE curve in section K.

Figure 3c Consumption demand diagram

The CD diagram is divided into four quadrants by the consumption subsistence parameter, γ, and the equilibrium price, *p/w* = σ_0/σ.

The process of creating Figure 3c from the indifference Map is the reverse of that followed for the labour supply diagram, Figure 3a. However:

- the focus is now on consumption.
- At *p/w* = 0, the budget line lies along the *q* axis on the right-hand side of the Map, and as *p/w* increases, it rotates anticlockwise on the same pivot (*lab* = 0, *q* = 0), and optimisation occurs in the reverse order of that for labour supply.



VARIABLES		PARAMETERS	
q	consumption	γ	subsistence consumption
p/w	price of consumption	$\sigma 0/\sigma$	relative intensity-of-need

K N R and V relate to FIGURE 2

A γ F and B indicate different levels of endowments, C

FIGURE 3c CONSUMPTION DEMAND CURVES

- A complication arises in disequilibrium because, whereas labour always jumps between the extremes of *lab* = 0 and *lab* = T, consumption often jumps between points *q* > 0 and *q* < B.
- Consumption must be modified such that it comprises both its level as indicated on the indifference map, together with the added endowment element, for instance, *q* = 0 + *C* for all *C* on the *C* axis.

When *p/w* = 0, the individual can consume his/her fill, but on the CD diagram this is limited to *q* = B, since, for this utility function, at *q* = B, s/he is very close to satiation level of utility. As *p/w* increases, consumption is experienced as superior and price-inelastic through section R. This occurs for any level of endowment, *C*.

The least familiar features are:

- for *C* = 0, and low prices, *p/w* < σ_0/σ, the CD curve starts at *q* = B, and declines through section R as price increases, declining briefly through section V, before increasing through the Giffen-deprived part of section V to *q* = B, creating a U-shaped curve. On the Map, for *C* = 0, the budget line, now in the dysfunctional poverty area, begins to maximise utility along the *q* axis to *q* = (B – A), now parallel with BA, and so is in disequilibrium and jumps to *q* = 0. This is represented on the CD diagram, as a short straight line from *q* = B to *q* = (B – A) where it is then disjointed between *q* = (B – A) and *q* = 0. It then maximises utility along the *p/w* axis at *q* = 0 for further price increases.

- The two curves through section V provide the beginning of an envelope curve below the two U-shaped CD curves, reflecting the boundary between consumption responding to price rises as superior in section R and inferior in section V.

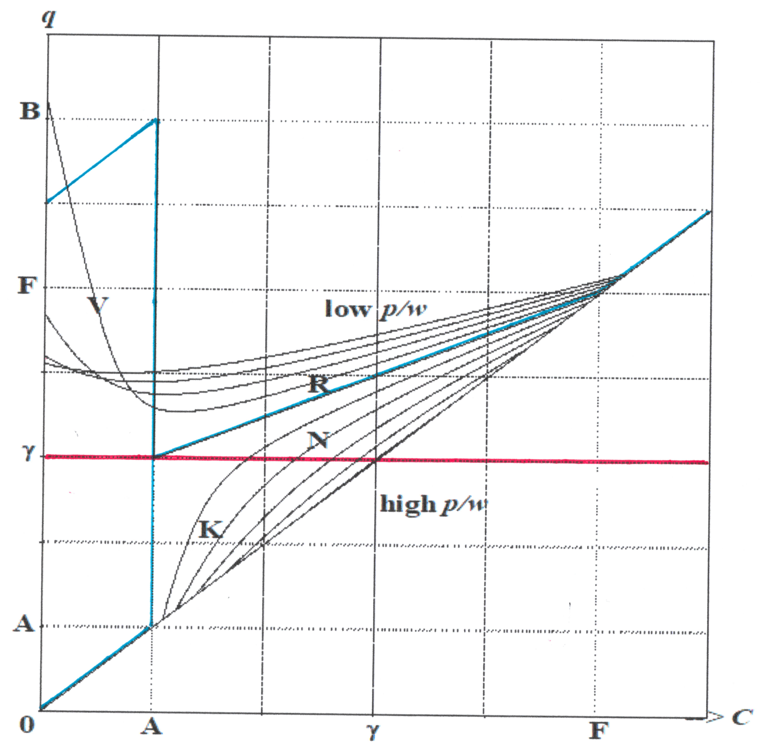
- The CD curve for the 'survival endowment', *C* = A, also starts at *q* = B and decreases as p/w increases to the equilibrium price, *p/w* = σ_0/σ, where *q* can take any of a range of values (determined by supply of consumption), before exiting as a slightly-positive-sloping curve for C = A.

- As with the LS curves, the most price elastic curves are associated with deprivation of leisure in section V and deprivation of consumption in section K.

Figure 3d   Consumption Engels diagram

The price of consumption, *p/w*, is the inverse of the real wage rate, *w/p*. When *p/w* = ∞, *w/p* = 0 and when *p/w* = σ_0/σ, *w/p* = σ/σ_0. Both versions of the diagram were created initially. The two versions were very similar for *w/p* > σ/σ_0 and *p/w* < σ_0/σ, although with different scales, but they differ slightly for *w/p* < σ/σ_0 and *p/w* > σ_0/σ, where lower wage rates provide a fuller picture compared with higher prices, which would have required a value of *p/w* = ∞. Thus, the CE diagram was created using increments of *w/p*, rather than *p/w*, but it is interpreted in terms of *p/w*.

The process of creating Figure 3d from the indifference Map is the same as for the labour Engels diagram, Figure 3b, but it now explores the effects of varying *C* for a given level of *p/w* on the Map and relates these to the CE diagram. So, the focus is on consumption,



VARIABLES
- q consumption
- C endowments
- p/w price of consumption

PARAMETERS
- γ subsistence consumption
- A survival endowment

K N R and V relate to FIGURE 2.

FIGURE 3d CONSUMPTION ENGELS CURVES

which must be modified such that it comprises both its level as indicated on the indifference map, together with the added endowment element, for instance, $q = 0 + C$ for all $C \geq 0$. Each CE curve represents a different price level, *p/w*.

The process starts on the Map with the budget line at $C = 0$, sloping at a given price level to which the budget line remains parallel as it moves up the *C* axis.

The least familiar features are:

- The individual again faces stark choices with low endowments, $0 \leq C < A$, either consuming his/her inadequate endowments or facing high consumption at low prices (and long work hours) in section V.

- The envelope curve below the U-shaped curves in section V reflects the boundary between consumption responding as inferior-normal and inferior-Giffen.

- With $p/w = \sigma/\sigma_0$, and $C = 0$, the budget line is parallel to BA on the Map and presents equal utility at both $q = 0$ and $q = (B – A)$. This disequilibrium prevails as endowments increase to $C = A$. This is represented on Figure 3d as two (disjointed) CE curves running parallel to each other from $q = 0$ to $q = A$ and from $q = (B – A)$ to $q = B$ for $0 \leq C < A$. At $C = A$, a transformation occurs, where the individual appears to face the whole range of consumption possibilities, but, as *C* increases, optimisation leads in a positive-sloping straight line from $q = \gamma$ to $q = F$ at $C = F$.

- As in the LE diagram, the combination of survival endowment and equilibrium price is a transformative event, whether facing low or high prices. Those facing high prices can start to optimise in section K.

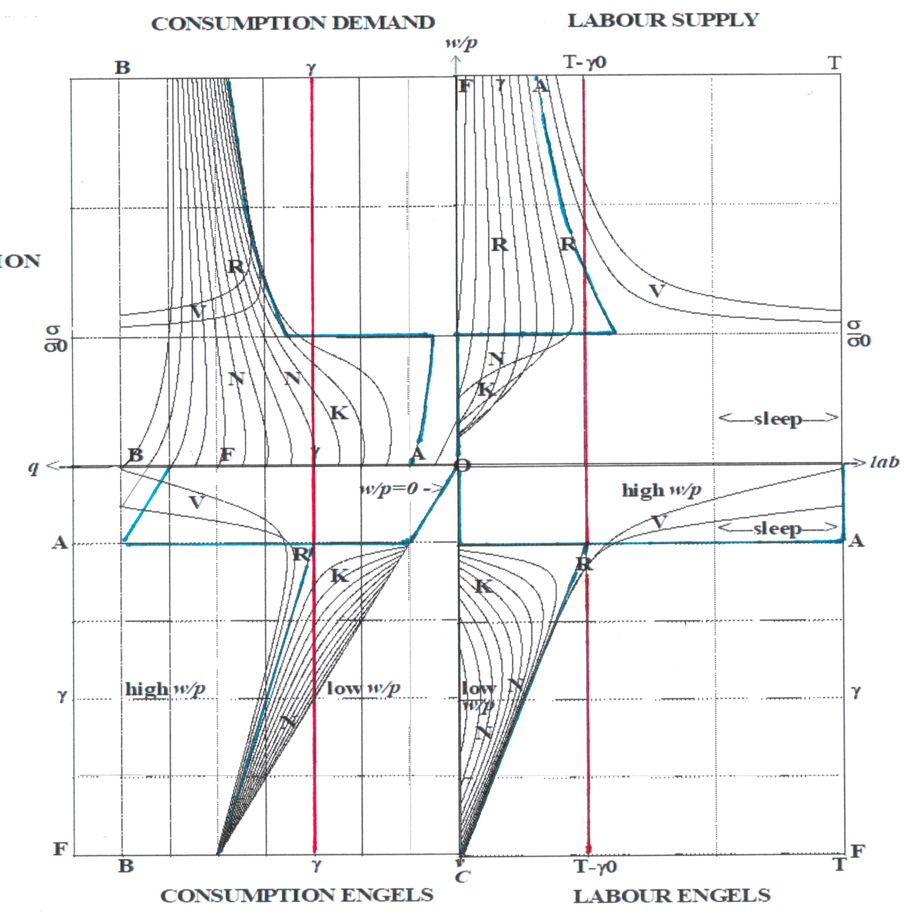
- Low endowments are very divisive to a society, but the divisions are reduced as *C* increases.

- Endowments provide a floor, below which the individual's consumption cannot fall. An endowment greater than survival, $C \geq A$, would provide a base for a working society, but a society can only really be healthy when no-one is deprived, and all receive $C \geq \gamma$.

Figure 4 The 4-axes diagram

Figure 4 presents all four DFF diagrams together, relating the two dependent variables, q_0 and q, to the two independent variables, *w/p* and *C*, sharing the four axes. It provides a visual impression of how each of the four diagrams is related to the others and more patterns can be discerned.

FIGURE 4. DERIVED FUNCTIONAL FORMS FOR THE LEISURE-CONSUMPTION CHOICE



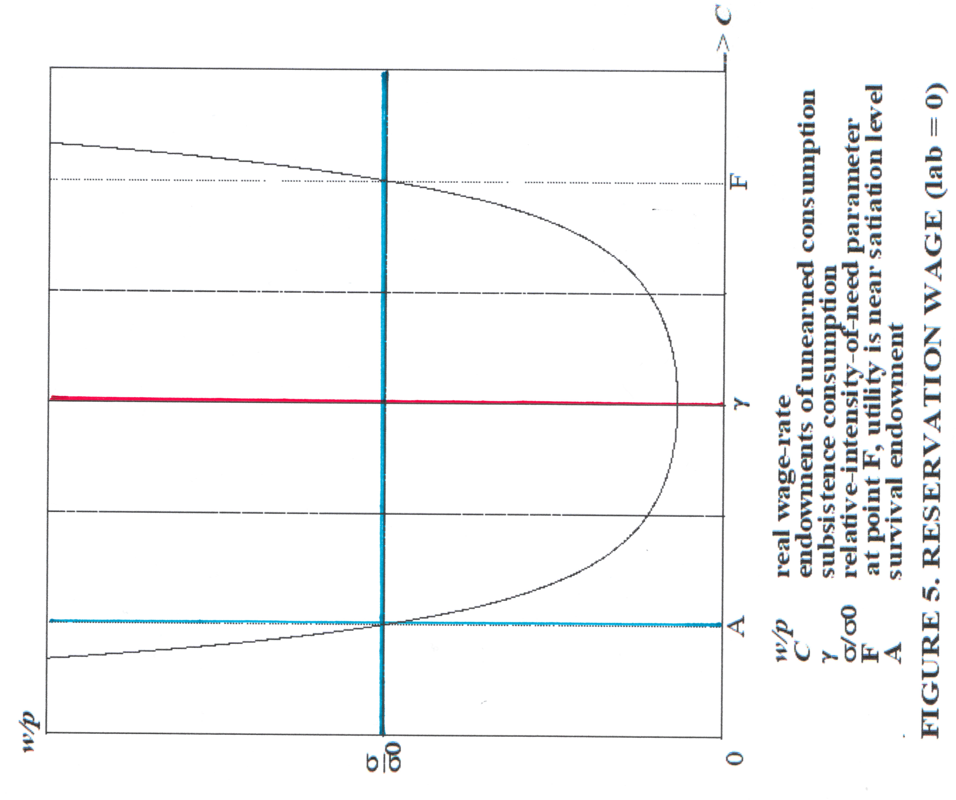
KEY:

VARIABLES:
$q0$ leisure
lab labour
q consumption
w/p real wage rate
C unearned endowments

PARAMETERS:
$\gamma 0$ subsistence leisure
γ subsistence consumption
$\sigma 0$ leisure intensity-of-need
σ consumption intensity-of-need

T is maximum time endowment
A is survival endowment
F - utility is close to satiation level
K, N, R and V relate to indifference map

Figure 5 The reservation wage.



Re-arranging the labour equation (A10) gives the relationship between the two independent variables such that *w/p* is a function of *C*. This could provide a map of contours for different values of *lab*. The contour for *lab* = 0 gives the reservation wage, (below which it is not worth the individual working). This can also be identified from the points on the *w/p* axis in Figure 3a from which each new non-zero LS curve starts for *C* > A. Careful examination reveals that this initially decreases and then increases again as endowments increase. For $0 \leq C < A$, the relationship between *w/p* and *C* is more difficult to discern from the four derived diagrams. However, it is confirmed in equation (A13) in Appendix A that the reservation wage is a U-shaped function of endowments.

[Figure 5 near here.]

Figure 5 illustrates the U-shaped curve that emerges. The minimum wage rate, as derived in equation (A14) in Appendix A, occurs when $C = \gamma$,

$$\frac{w}{p} = \frac{\sigma}{\sigma_0}.\sqrt{exp\left[-\left(\frac{T-\gamma_0}{\sigma_0}\right)^2\right]}. \qquad \text{(A14)}$$

5 Conclusion

Adding two bounded leaning-S-shaped utility functions for single commodities extends the indifference curve map to include a straight-line curve, dividing concave-to-the-origin indifference curves, representing ‘dysfunctional poverty’, from convex-to-the-origin indifference curves which include areas of deprivation in one or other need. The Giffen experience manifests when more is consumed of an abundant cheaper good in response to a rise in its price, enabled by relinquishing some of a more expensive good in which the individual is already extremely deprived.

Defined by the individual’s subsistence and relative intensities-of-need parameters, the straight-line indifference curve plays two vital roles. Its slope determines the equilibrium wage rate, (or equilibrium price), while its intercept on the endowments axis defines the ‘survival endowment’. The co-incidence of the budget line with the straight-line indifference curve, and thus the co-incidence of survival endowment with equilibrium wage/price, could transform society and the labour market.

This extension of marginal utility theory reveals that poverty is a combination of low endowments of unearned consumption and low pay. The outcomes expected for endowments below the survival level are very polarising depending on whether the individual faces high or low wage rates. With a high wage, an individual can face disequilibrium, and then optimise, leading to deprivation of leisure, while, when facing a low wage, s/he is in dysfunctional poverty, experiencing involuntary unemployment.

Mainstream marginal microeconomics has been a story about comfortably off individuals, by comfortably off individuals (experiencing diminishing marginal utility), for other comfortably off individuals. It assumes that the poorest members of society are merely the least comfortably off, who could be expected to behave in the same way as the other, more comfortably off, people. However, representing deprivation by increasing marginal utility, combined with the introduction of different needs, reveals that deprived individuals respond very differently compared with the comfortably off neighbours. Wage rates and prices are most elastic when an individual is deprived of either leisure or consumption.

One of the most important outcomes of this work could be to change society's perception of deprived people, presenting them in a more sympathetic light. Rather than being misrepresented, and labelled 'lazy scoundrels', society will recognise that their deprivation and involuntary unemployment undermines their physical and mental health rendering them malnourished and financially insecure, and not well enough to work for pay. Few of us would insist that someone who had not eaten for three days should labour for a ten-hour shift on a construction site. And a slap-up meal beforehand would not solve his problem. Why would we treat other deprived people differently? The solution is to invest in the population, ensuring that each receives at least a 'survival endowment', so that they, too, will be able to fulfil their potential and contribute to society.

The addition of bounded leaning-S-shaped utilities for leisure and consumption, meeting different human needs, adds a new dimension to cardinal utility theory, offering an exciting new range of potential theoretical developments and empirical analyses.

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APPENDIX A

NOTATION for leisure, consumption, and labour:

VARIABLES:

u = utility, $0 \le u \le 1$
q_0 = leisure, $0 \le q_0 \le T$
q = consumption
w = wage rate, $w \ge 0$.
p = price of consumption, $p \ge 0$.
$x = w/p$, real wage rate
C = endowment of unearned consumption

$lab = (T - q_0)$, is labour, hours worked for pay.

PARAMETERS:

γ_0 = subsistence leisure
γ = subsistence consumption
σ_0 = intensity-of-need for leisure
σ = intensity-of-need for consumption
b = σ/σ_0, relative intensities-of-need
T = maximum endowment of leisure in a given time period

The utility function for a commodity (good, service or event) is an S-shaped function, bounded below and above, ($0 \le u \le 1$). The functional form chosen for this paper is the distribution function of the normal distribution (N-DF).

The '2.Add.N-DF' **utility function** is given as:

$$u(q_0, q) = \tfrac{1}{2} F_0(q_0) + \tfrac{1}{2} F(q)$$

$$u(q_0, q) = \frac{1}{2}\int_{-\infty}^{q_0} \frac{\exp\left[-(R_0-\gamma_0)^2/2\sigma_0^2\right]}{\sigma_0.\sqrt{2\pi}}\, dR_0 + \frac{1}{2}\int_{-\infty}^{q} \frac{\exp\left[-(R-\gamma)^2/2\sigma^2\right]}{\sigma.\sqrt{2\pi}}\, dR \qquad (1)$$

It would be almost impossible to use equation (1) to create an indifference curve map. The logistic distribution Equation (A2), based on the logistic distribution, adjusted for location and scale, was used to draw the indifference curve map.

$$P(t) = \frac{e^{t}}{[1 + e^{t}]^2} = \frac{e^{-t}}{[1 + e^{-t}]^2}.$$

$$q = \gamma - \{\log [\, (0.5 * bracket) / (u * bracket - 0.5) - 1] \} / (1.82/\sigma), \qquad (A2)$$

where u is utility, and $bracket = (1 + \exp(-(1.82/\sigma_0) * (q_0 - \gamma_0)))$.

Full income: $M = T.w + C.p$, where $M \ge 0$.
Survival income is: $(\gamma_0.w + \gamma.p)$
Supernumerary income: $Z = M$ – survival income.
$= (T - \gamma_0).w + (C - \gamma).p$.

The linear budget constraint, $M = q_0.w + q.p$ is expressed as:

$$q = (M - q_0.w)/p. \qquad (A3)$$

Maximising $u(q_0, q)$ subject to the budget constraint M, and using the Lagrange multiplier method leads to:

$$\frac{du(q_0,q)}{dq_0} = \frac{\frac{1}{2}.exp[-(q_0 - \gamma_0)^2 /2\sigma_0^2]}{\sigma_0.\sqrt{2\pi}} - \lambda.w = 0. \qquad \text{(A1a)}$$

$$\frac{du(q_0,q)}{dq} = \frac{\frac{1}{2}.exp[-(q - \gamma)^2/2\sigma^2]}{\sigma.\sqrt{2\pi}} - \lambda.p = 0. \qquad \text{(A1b)}$$

$$\frac{exp[-(q_0 - \gamma_0)^2 /2\sigma_0^2]}{exp[-(q- \gamma)^2 /2\sigma^2 \;]} = \frac{\sigma_0.w}{\sigma.p}. \qquad \text{(A1c)}$$

Thus, this yields the **optimality condition**:

$$\left(\frac{q- \gamma}{\sigma}\right)^2 - \left(\frac{q_0- \gamma_0}{\sigma_0}\right)^2 = ln\left(\frac{\sigma_0.w}{\sigma.p}\right)^2 \qquad \text{(A4)}$$

Boundary between superior and inferior responses

By expressing equation (A4) in terms of q_0, and differentiating with respect to q, dq_0/dq can be found.

$$q_0 = \gamma_0 + \sigma_0.\left[\left(\frac{q-\gamma}{\sigma}\right)^2 - 2.ln\left(\frac{\sigma_0.w}{\sigma.p}\right)\right]^{\frac{1}{2}} \qquad \text{(A4)}$$

$$\frac{dq_0}{dq} = \frac{1}{2}.\left[\left(\frac{q-\gamma}{\sigma}\right)^2 - 2.ln\left(\frac{\sigma_0.w}{\sigma.p}\right)\right]^{\frac{-1}{2}}.\left(\frac{2q- 2\gamma}{\sigma}\right).\left(\frac{\sigma_0}{\sigma}\right) = 0.$$

$$\frac{dq_0}{dq} = \frac{\left(\frac{q-\gamma}{\sigma}\right).\left(\frac{\sigma_0}{\sigma}\right)}{\sqrt{\left[\left(\frac{q-\gamma}{\sigma}\right)^2 - 2.ln\left(\frac{\sigma_0.w}{\sigma.p}\right)\right]}} = 0. \qquad \text{(A5)}$$

By setting $dq_0/dq = 0$, as in equation (A5), the **locus for the threshold between q_0 being superior and its being inferior** on the indifference curve map, is found to be coincidental with $q = \gamma$, for $q_0 > \gamma_0$. This is the boundary between areas K and N. Similarly, $q_0 = \gamma_0$, for $q > \gamma$ is the boundary between q being inferior in area V and its being superior in area R.

Boundary between inferior normal and inferior-Giffen responses

The optimality condition of equation (A4) gives the locus of points describing the *price ratio-consumption locus* for a given income, M, on the indifference curve map, for any two commodities, q_0 and q, fulfilling different needs, and is not dependent on endowments.

To obtain the **locus of points for the threshold between q_0 being inferior normal and its being inferior-Giffen**, the following procedure is adopted.

Re-arranging the budget equation (A3) in terms of w/p, where $M = (Tw + C.p)$, gives

$w/p = (C - q)/(q_0 - T)$

The price ratio, *w/p*, is substituted into the optimality condition, equation (A4), eliminating prices from equation (A6),

$$\left(\frac{q-\gamma}{\sigma}\right)^2 - \left(\frac{q_0-\gamma_0}{\sigma_0}\right)^2 = ln\left(\frac{\sigma_0.w}{\sigma.p}\right)^2 \qquad \text{(A4)}$$

$$exp\left[\left(\frac{(q-\gamma)}{\sigma}\right)^2 - \left(\frac{(q_0-\gamma_0)}{\sigma_0}\right)^2\right] = \left[\left(\frac{(C-q)}{(q_0-T)}\right).\left(\frac{\sigma_0}{\sigma}\right)\right]^2 . \qquad \text{(A6)}$$

To simplify the notation, let $\left(\frac{q-\gamma}{\sigma}\right)^2 - \left(\frac{q_0-\gamma_0}{\sigma_0}\right)^2 = S$

Re-arranging equation (A6) in terms of *C* gives:

$$C = q + (q_0 - T).\left(\frac{\sigma}{\sigma_0}\right).\sqrt{[exp(S)]}.$$

Differentiating *C* with respect to q_0 and q yields the following:

$$\frac{dC}{dq_0} = \left(\frac{\sigma}{\sigma_0}\right).\sqrt{[exp(S)]} + (q_0 - T).\left(\frac{\sigma}{\sigma_0}\right).\frac{1}{2}.[exp(S)]^{-\frac{1}{2}}.\exp(S).(-2)(q_0-\gamma_0)/\sigma_0^2.$$

$$\frac{dC}{dq_0} = [1 - (q_0 - T).(q_0 - \gamma_0)/\sigma_0^2].\left(\frac{\sigma}{\sigma_0}\right).\sqrt{[exp(S)]}.$$

$$\frac{dC}{dq} = 1 + (q_0 - T).\left(\frac{\sigma}{\sigma_0}\right).\frac{1}{2}.[exp(S)]^{-1/2}exp(S).(+2)(q-\gamma)/\sigma^2.$$

$$\frac{dC}{dq} = \left[1 + (q_0 - T).(q-\gamma)/(\sigma^2).\left(\frac{\sigma}{\sigma_0}\right).\sqrt{[exp(S)]}\right].$$

Using implicit differentiation, dq_0/dq is obtained and set equal to zero, eliminating *C*. The Giffen boundary is independent of endowments, thus T can also be eliminated by setting T = 0. The equation is multiplied through by –1, resulting in equation (A7).

$$\frac{dq_0}{dq} = -\frac{dC}{dq} \Big/ \frac{dC}{dq_0}$$

$$\frac{dq_0}{dq} = -\frac{1+q_0.(q-\gamma)/(\sigma^2).\left(\frac{\sigma}{\sigma_0}\right).\sqrt{[exp(S)]}}{\left[[1-q_0.(q_0-\gamma_0)/\sigma_0^2].\left(\frac{\sigma}{\sigma_0}\right).\sqrt{[exp(S)]}\right]} = 0.$$

$$\frac{dq_0}{dq} = +\frac{1+q_0.(q-\gamma)/(\sigma^2).\left(\frac{\sigma}{\sigma_0}\right).\sqrt{[exp(S)]}}{\left[[1-q_0.(q_0-\gamma_0)/\sigma_0^2].\left(\frac{\sigma}{\sigma_0}\right).\sqrt{[exp(S)]}\right]} = 0. \qquad \text{(A7)}$$

Thus, the locus for the threshold between q_0 being inferior normal and inferior-Giffen is given by the numerator of equation (A7).

$$\sqrt{\left[exp\left(\left(\frac{q-\gamma}{\sigma}\right)^2-\left(\frac{q_0-\gamma_0}{\sigma_0}\right)^2\right)\right]}\cdot\left(\frac{q_0}{\sigma_0}\right)\cdot\left(\frac{q-\gamma}{\sigma}\right)+1=0. \quad \text{(A7a)}$$

This locus does not go through E, (γ_0, γ), but cuts the straight-line indifference curve BA at a point which can be found from the positive solution to the quadratic equation in q_0 within the denominator of equation (A7) as follows:

$$\sqrt{\left[exp\left(\left(\frac{q-\gamma}{\sigma}\right)^2-\left(\frac{q_0-\gamma_0}{\sigma_0}\right)^2\right)\right]}\cdot\left(\frac{\sigma}{\sigma_0}\right).[1-q_0.(q_0-\gamma_0)/\sigma_0^2]=0$$

$[1 - q_0.(q_0 - \gamma_0)/\sigma_0^2] = 0.$

$q_0^2 - \gamma_0.q_0 - \sigma_0^2) = 0.$

$q_0 = [\gamma_0 \pm \sqrt{\{ \gamma_0^2 + 4\sigma_0^2\}}]/ 2.$ (A7b)

Equation (A7a) must be solved numerically to find the solutions for q for given values of $q_0 > [\gamma_0 + \sqrt{\{\gamma_0^2 + 4.\sigma_0^2\}}]/2$. Equations (A7a and A7b) can be adapted for q. These loci have been drawn on the indifference curve map in Figure 2.

Derivation of the demand equation

Substituting for $q = (M - q_0.w)/p$, from the budget constraint,
and for $M = Z + \gamma_0.w + \gamma.p$ from the supernumerary income equation,
into equation (A4), yields an '**implicit demand equation**' (A8):

$$\left(\frac{q-\gamma}{\sigma}\right)^2 - \left(\frac{q_0-\gamma_0}{\sigma_0}\right)^2 = ln\left(\frac{\sigma_0.w}{\sigma.p}\right)^2 \quad \text{(A4)}$$

$$\left[\frac{\frac{Z}{p}-(q_0-\gamma_0).x}{\sigma}\right]^2 = \left[\frac{(q_0-\gamma_0)}{\sigma_0}\right]^2 + ln\left[\frac{x}{b}\right]^2 \quad \text{(A8)}$$

$$\frac{\sigma_0^2.\left(\frac{Z}{p}\right)^2-2.\sigma_0^2(q_0-\gamma_0).x.\left(\frac{Z}{p}\right)+\sigma_0^2.(q_0-\gamma_0)^2.x^2-\sigma^2.(q_0-\gamma_0)^2-2.\sigma_0^2.\sigma^2.ln\left(\frac{x}{b}\right)}{\sigma_0^2.\sigma^2} = 0.$$

$$\frac{(\sigma_0^2.x^2-\sigma^2).(q_0-\gamma_0)^2 - 2.\sigma_0^2.\left(\frac{Z}{p}\right).x.(q_0-\gamma_0) + \sigma_0^2.\left(\frac{Z}{p}\right)^2-2.\sigma_0^2.\sigma^2.ln\left(\frac{x}{b}\right)}{\sigma_0^2.\sigma^2} = 0.$$

which is a quadratic equation in ($q_0 - \gamma_0$), which is solved using the *negative* square root, yielding **demand equation** (A9a) for the first commodity:

Let the root be $(q_0-\gamma_0) = \frac{-b-\sqrt{b^2-4.a.c}}{2.a}$;

$$a = (\sigma_0^2.x^2 - \sigma^2)\;; \qquad b = -2.\sigma_0^2.x.Z/p\;; \qquad c = \sigma_0^2.\left(\left(\frac{Z}{p}\right)^2 - \sigma^2.2.ln\left(\frac{x}{b}\right)\right).$$

$$(q_0 - \gamma_0) = \frac{2.\sigma_0^2.\left(\frac{Z}{p}\right).x - \sqrt{\left[4.\sigma_0^4\left(\frac{Z}{p}\right)^2.x^2 - 4.(\sigma_0^2.x^2 - \sigma^2).\sigma_0^2.\left(\left(\frac{Z}{p}\right)^2 - .\sigma^2.2.ln\left(\frac{x}{b}\right)\right)\right]}}{2.(\sigma_0^2.x^2 - \sigma^2)}$$

$$(q_0 - \gamma_0) = \frac{\sigma_0^2.\left(\frac{Z}{p}\right).x - \sqrt{\left[.\sigma_0^2.\sigma^2.\left(\frac{Z}{p}\right)^2 + (\sigma_0^2.x^2 - \sigma^2).\left(\sigma_0^2.\sigma^2.2.ln\left(\frac{x}{b}\right)\right)\right]}}{(\sigma_0^2.x^2 - \sigma^2)}$$

Dividing numerator and denominator by σ_0^2:

$$(q_0 - \gamma_0) = \frac{\left(\frac{Z}{p}\right).x - \sqrt{\left[b^2.\left(\frac{Z}{p}\right)^2 + (x^2 - b^2).\left(\sigma_0^2.\left(\frac{\sigma^2}{\sigma_0^2}\right).2.ln\left(\frac{x}{b}\right)\right)\right]}}{(x^2 - b^2)}$$

$$q_0 = \gamma_0 + \frac{\left(\frac{Z}{p}\right).x - b.\sqrt{\left[\left(\frac{Z}{p}\right)^2 + (x^2 - b^2).\left(\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right)\right]}}{(x^2 - b^2)} \tag{A9a}$$

Expanding $Z/p = (T - \gamma_0).w/p + (C - \gamma)$, and dividing numerator and denominator by x^2, gives an alternative version of the leisure demand equation:

$$q_0 = \gamma_0 + \frac{((T-\gamma_0) + (C-\gamma)/x) - \frac{b}{x}.\sqrt{\left[((T-\gamma_0)+(C-\gamma)/x)^2 + \left(1-\left(\frac{b}{x}\right)^2\right).\left(\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right)\right]}}{\left(1 - \left(\frac{b}{x}\right)^2\right)} \tag{A9b}$$

The corresponding equation (A9c) for consumption demand must be constrained so that $q \geq 0 + C$ and $0 \leq lab \leq T$.

$$q = \gamma + \frac{((T-\gamma_0).x + (C-\gamma)) - \frac{x}{b}.\sqrt{\left[((T-\gamma_0).x+(C-\gamma))^2 + \left(1-\left(\frac{x}{b}\right)^2\right).\left(\sigma^2.2.ln\left(\frac{b}{x}\right)\right)\right]}}{\left(1 - \left(\frac{x}{b}\right)^2\right)} \tag{A9c}$$

These confirm that demand for leisure and consumption can be expressed as a function of only two variables, *x* and *C*, where *x* = *w/p*.

The labour supply equation, where $lab = T - q_0$, is given by:

$$lab = (T - \gamma_0) - \frac{((T-\gamma_0) + (C-\gamma)/x) - \frac{b}{x}.\sqrt{\left[((T-\gamma_0).+(C-\gamma)/x)^2 + \left(1-\left(\frac{b}{x}\right)^2\right).\left(\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right)\right]}}{\left(1 - \left(\frac{b}{x}\right)^2\right)} \tag{A10}$$

Equation (A10) must also be constrained such that $0 \leq lab \leq T$.

Envelope curve on the demand curves reflects the boundary between superior and inferior responses.

By differentiating q_0 in demand equation (A9b), with respect to C, and setting the partial derivative equal to zero, one obtains:

$$\frac{dq_0}{dC} = \frac{\frac{1}{x} - \frac{2}{x^2}((T-\gamma_0).x + (C-\gamma)) . \frac{b}{2x}.\left[((T-\gamma_0) + (C-\gamma)/x)^2 + \left(1-\left(\frac{b}{x}\right)^2\right).\left(\sigma_0^2 .2.ln\left(\frac{x}{b}\right)\right)\right]^{-1/2}}{\left(1-\left(\frac{b}{x}\right)^2\right)} = 0.$$

Re-arranging this and squaring both sides, to express it in terms of C, gives:

$$\left[((T-\gamma_0) + (C-\gamma)/x)^2 + \left(1-\left(\frac{b}{x}\right)^2\right)\left(\sigma_0^2 .2.ln\left(\frac{x}{b}\right)\right)\right] = ((T-\gamma_0) + (C-\gamma)/x)^2 \left(\frac{b}{x}\right)^2.$$

Simplifying, by substituting $Z/(p.x) = ((T-\gamma_0) + (C-\gamma)/x)$, gives

$$\left[\left(\frac{Z}{p.x}\right)^2 + \left(1-\left(\frac{b}{x}\right)^2\right)\left(\sigma_0^2 .2.ln\left(\frac{x}{b}\right)\right)\right] = \left(\frac{Z}{p.x}\right)^2 . \left(\frac{b}{x}\right)^2$$

$$\left(\frac{Z}{p.x}\right)^2 - \left(\frac{Z}{p.x}\right)^2 . \left(\frac{b}{x}\right)^2 = \left[-\left(1-\left(\frac{b}{x}\right)^2\right)\left(\sigma_0^2 .2.ln\left(\frac{x}{b}\right)\right)\right].$$

$$\left(\frac{Z}{p.x}\right)^2 . \left(1-\left(\frac{b}{x}\right)^2\right) = \left[-\left(1-\left(\frac{b}{x}\right)^2\right)\left(\sigma_0^2 .2.ln\left(\frac{x}{b}\right)\right)\right].$$

$$\left(\frac{Z}{p.x}\right)^2 = \left[-\left(\sigma_0^2 .2.ln\left(\frac{x}{b}\right)\right)\right].$$

$$Z^2 = -\left[p^2 .x^2 .\left(\sigma_0^2 .2.ln\left(\frac{x}{b}\right)\right)\right]$$

$Z = \sigma_{0.}w\sqrt{\left[-2.ln\left(\frac{x}{b}\right)\right]}$, if (x/b) < 1; that is, x < b, or $p_1/p_2 < \sigma_2/\sigma_1$.

$$\left(\frac{C-\gamma}{x}\right) = -(T-\gamma_0) + \sigma_0.\sqrt{\left[+2.ln\left(\frac{b}{x}\right)\right]} \qquad \text{if (b/x) > 1, if x < b.} \qquad \text{(A11)}$$

Substituting for $(C-\gamma)/x$ from equation (A11) into equation (A9b) gives the envelope curve on the demand equations, for $x \leq$ b, that is, for $p_1/p_2 \leq \sigma_2/\sigma_1$.

$$q_0 = \gamma_0 + \frac{((T-\gamma_0) + (C-\gamma)/x) - \frac{b}{x}.\sqrt{\left[((T-\gamma_0).+(C-\gamma)/x)^2 + \left(1-\left(\frac{b}{x}\right)^2\right).\left(\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right)\right]}}{\left(1-\left(\frac{b}{x}\right)^2\right)} \qquad \text{(A9b)}$$

$$q_0 = \gamma_0 + \frac{\sqrt{\sigma_0^2.2.\ln\left(\frac{b}{x}\right)} - \frac{b}{x}.\sqrt{\left[\sigma_0^2.2.ln\left(\frac{b}{x}\right) + \left(1-\left(\frac{b}{x}\right)^2\right).\left(\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right)\right]}}{\left(1-\left(\frac{b}{x}\right)^2\right)} .$$

$$q_0 = \gamma_0 + \frac{\sqrt{\sigma_0^2.2.\ln\left(\frac{b}{x}\right)} - \frac{b}{x}.\sqrt{\left[\sigma_0^2.2.ln\left(\frac{b}{x}\right) - \left(1-\left(\frac{b}{x}\right)^2\right).\left(\sigma_0^2.2.ln\left(\frac{b}{x}\right)\right)\right]}}{\left(1-\left(\frac{b}{x}\right)^2\right)} .$$

$$q_0 = \gamma_0 + \frac{\sqrt{\sigma_0^2.2.\ln\left(\frac{b}{x}\right)} - \frac{b}{x}.\sqrt{\left[\ + \left(\frac{b}{x}\right)^2.\left(\sigma_0^2.2.ln\left(\frac{b}{x}\right)\right)\right]}}{\left(1-\left(\frac{b}{x}\right)^2\right)} .$$

$$q_0 = \gamma_0 + \frac{\sqrt{\sigma_0^2.2.\ln\left(\frac{b}{x}\right)} - \left(\frac{b}{x}\right)^2\sqrt{\left[.\left(\sigma_0^2.2.ln\left(\frac{b}{x}\right)\right)\right]}}{\left(1-\left(\frac{b}{x}\right)^2\right)} .$$

$$q_0 = \gamma_0 + \frac{\sqrt{\sigma_0^2.2.\ln\left(\frac{b}{x}\right)} - \left(\frac{b}{x}\right)^2\sqrt{\left[.\left(\sigma_0^2.2.ln\left(\frac{b}{x}\right)\right)\right]}}{\left(1-\left(\frac{b}{x}\right)^2\right)} .$$

$$q_0 = \gamma_0 + \sqrt{\left[\sigma_0^2.2.ln\left(\frac{b}{x}\right)\right]}\ ,\ for\ \frac{w}{p} < \frac{\sigma}{\sigma_0},\ \text{for } x < b \qquad \text{(A12)}$$

Reservation wage

The **reservation wage**, $x = w/p$, is a function of endowments of unearned consumption, C. It can be obtained by setting *lab* = 0 in equation (A10) and rearranging it in terms of x, as follows:

$$lab = (T - \gamma_0) - \frac{\left((T-\gamma_0) + (C-\gamma)/x\right) - \frac{b}{x}.\sqrt{\left[\left((T-\gamma_0).+(C-\gamma)/x\right)^2 + \left(1-\left(\frac{b}{x}\right)^2\right).\left(\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right)\right]}}{\left(1-\left(\frac{b}{x}\right)^2\right)} \qquad \text{(A10)}$$

Let *lab* = 0 in equation (A10) and re-arrange.

$$-(T-\gamma_0).\left(1-\left(\frac{b}{x}\right)^2\right) + \left[(T-\gamma_0) + \left(\frac{C-\gamma}{x}\right)\right] = \left(\frac{b}{x}\right)\sqrt{\left[\left((T-\gamma_0) + (C-\gamma)/x\right)^2 + \left(1-\left(\frac{b}{x}\right)^2\right).\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right]} .$$

Square both sides of the equation and collect terms:

$$\left[(T-\gamma_0).\left(\frac{b}{x}\right)^2 + \left(\frac{C-\gamma}{x}\right)\right]^2 = \left(\frac{b}{x}\right)^2.\left[\left((T-\gamma_0) + (C-\gamma)/x\right)^2 + \left(1-\left(\frac{b}{x}\right)^2\right).\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right] .$$

$$\left[(T-\gamma_0)^2.\left(\frac{b}{x}\right)^4 + \left(\frac{C-\gamma}{x}\right)^2 + 2(T-\gamma_0).\left(\frac{b}{x}\right)^2.\left(\frac{C-\gamma}{x}\right)\right]^1 = \left(\frac{b}{x}\right)^2.\left[\left((T-\gamma_0)^2 + \left(\frac{C-\gamma}{x}\right)^2 + 2.(T-\gamma_0)(C-\gamma)/x\right)^1 + \left(1-\left(\frac{b}{x}\right)^2\right).\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right] .$$

$$\left[-(T-\gamma_0)^2.\left(\frac{b}{x}\right)^2.\left(1-\left(\frac{b}{x}\right)^2\right)+\left(\frac{C-\gamma}{x}\right)^2.\left(1-\left(\frac{b}{x}\right)^2\right)\right]^1=\left(\frac{b}{x}\right)^2.\left[\left(1-\left(\frac{b}{x}\right)^2\right).\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right].$$

Divide through by (1 – (b/x)^2)

$$-(T-\gamma_0)^2.\left(\frac{b}{x}\right)^2+\left(\frac{C-\gamma}{x}\right)^2=\left(\frac{b}{x}\right)^2.\sigma_0^2.2.ln\left(\frac{x}{b}\right).$$

$$-(T-\gamma_0)^2+\left(\frac{x}{b}\right)^2.\left(\frac{C-\gamma}{x}\right)^2=\sigma_0^2.2.ln\left(\frac{x}{b}\right).$$

$$-(T-\gamma_0)^2+\left(\frac{C-\gamma}{b}\right)^2=\sigma_0^2.2.ln\left(\frac{x}{b}\right).$$

$$-\left(\frac{T-\gamma_0}{\sigma_0}\right)^2+\left(\frac{C-\gamma}{\sigma}\right)^2=\ln\left(\frac{x}{b}\right)^2.$$

Rearrange this in terms of x:

$$x=b.\sqrt{exp\left[\left(\frac{C-\gamma}{\sigma}\right)^2-\left(\frac{T-\gamma_0}{\sigma_0}\right)^2\right]}. \quad \text{(A13)}$$

This quadratic in C can be solved using the negative root, yielding an expression for the U-shaped reservation wage, x, which is symmetric about $C = \gamma$ for the 2.Add.N-DF model.

When $C = \gamma$, $$x=b.\sqrt{exp\left[-\left(\frac{T-\gamma_0}{\sigma_0}\right)^2\right]}. \quad \text{(A14)}$$

APPENDIX B

The four Tables in Appendix B relate the unfamiliar features observed on the indifference Map to their manifestation as curves in each of the four derived functional form diagrams.

TABLE B1 The relationship between the Map and the LS diagram

	Endowments	Wage rate	Map	LS diagram
1	$0 \leq C < A$		**Dysfunctional poverty** **Corner solutions**	
		$w/p < \sigma/\sigma_0$	*lab* = 0.	LS curves move up *w/p* axis from $w/p = 0$ to $w/p = \sigma/\sigma_0$. *lab* = 0. **Involuntary unemployment**
		$w/p = \sigma/\sigma_0$	Budget line parallel to BA. **Disequilibrium**	**Disjointed LS curves**; labour jumps from *lab* = 0 to *lab* = T at $q = (B – A)/T$
		$w/p > \sigma/\sigma_0$	Corner solutions. Greater utility at *lab* = T, moving up *q* axis to B.	LS curve continues to move up *w/p* axis at *lab* = T, to $w/p = (B – A + C)/T = B/T$.
			OPTIMISATION	
	$0 \leq C < A$	$w/p > \sigma/\sigma_0$	**Functional poverty.** **Deprivation of leisure**, Area V	Negative-sloping LS curve in section V. Labour falls steeply. **Very wage elastic.**
			Area R	Labour falls less steeply. Wage inelastic
2	$C = A$	$w/p < \sigma/\sigma_0$	Corner solution; *lab* = 0.	LS curve moves up *w/p* axis from $w/p = 0$ to $w/p = \sigma/\sigma_0$. *lab* = 0.
		$w/p = \sigma/\sigma_0$	Co-incidental with **BA**. An infinity of options between *lab* = 0 & *lab* = T?	**Sticky wages** as LS varies in the face of persistent demand for labour, over range *lab* = 0 to *lab* > $(T – \gamma_0)$.
		$w/p > \sigma/\sigma_0$	Optimisation. Area V	Labour falls through V very briefly.
			Area R	Labour falls slightly. Wage inelastic.
3	$A < C < \gamma$	$w/p < \sigma/\sigma_0$	Corner solution; *lab* = 0.	LS curve moves up *w/p* axis from $w/p = 0$ to $w/p < \sigma/\sigma_0$; *lab* = 0.
			Functional poverty **Deprivation of Consumption,** Area K.	LS curves create a set of decreasing Intercepts on the *w/p* axis. Steep, positive-sloping LS curves. **Labour increases**. **Very wage elastic**.
			Boundary between leisure responding as inferior in area K and superior in area N.	U-shaped LS curves. **Envelope curve** starts at point (*w/p**, *lab* = 0) and ends at point ($w/p = \sigma/\sigma_0$, *lab* > $(T – \gamma_0)$).
			Area N, leisure superior	**Labour increases** through N briefly.
		$w/p \geq \sigma/\sigma_0$	Area R, leisure superior	Labour falls slightly through R. Very wage inelastic
4	$C \geq \gamma$	$w/p < \sigma/\sigma_0$	Corner solutions in areas N and R. *lab* = 0.	Each LS curve moves up the *w/p* axis, creating a series of intercepts, starting at lowest non-zero *w/p*, *w/p**. Intercepts increase as *C* increases.
			Area N	Positive-sloping LS curves. **Labour increases**. Wage elastic.
		$w/p \geq \sigma/\sigma_0$	Area R	Labour barely changes through R. Very wage inelastic

When $C = \gamma$, $$\left(\frac{w}{p}\right)^* = \frac{\sigma}{\sigma_0}.\sqrt{exp\left[-\left(\frac{T-\gamma_0}{\sigma_0}\right)^2\right]}. \quad \text{(A14)}$$

TABLE B2 The relationship between the Map and the LE diagram

	Wage rate	Endow- ments	Map	LE diagram
			Dysfunctional poverty. Corner solutions	
1	$0 \le w/p < \sigma/\sigma_0$	$0 \le C < A$	Budget lies over the *lab* axis, *lab* = 0. Corner solution. As budget shifts up the *C* axis, no optimisation is possible	LE curves move along the *C* axis, *lab* = 0. **Involuntary unemployment.**
	$0 \le w/p \le w/p^*$	A < C < F	Budget lies over *lab* axis, *lab* = 0. Corner solution. As budget shifts up the *C* axis, no optimisation is possible.	LE curves move along the *C* axis, *lab* = 0, **involuntary unemployment**.
	$w/p = w/p^*$	A < *C* < F	**OPTIMISATION** is registered	LE curves move along the *C* axis, registering optimisation at $C = \gamma$.
	$w/p^* < w/p < \sigma/\sigma_0$	A < *C* < F	**Functional poverty, deprivation of consumption** in area K. Optimisation starts in area K just before $C = \gamma$ and ends in area N, just after $C = \gamma$, with wider bases as *C* increases.	LE curves comprise a series of arcs, Becoming increasingly wider, higher and more asymmetric as *w/p* increases. In section K, arcs are **C-elastic**.
			Boundary between leisure responding as Giffen-deprived and 'normal'-deprived, in area K.	'**Envelope**' from point (*C* = A, *lab* < (T – γ_0)) to point (*C* = G, *lab* = 0) – equation (A7a) solved numerically.
			Boundary between areas K and N.	**Curved boundary** = locus of maximum points of the LE arcs, from point (*C* = A, *lab* = (T – γ0)) to point ($C = \gamma$, *lab* = 0) – equation (A5) solved numerically.
		$C \ge F$	Constraint: if *lab* < 0, then set *lab* = 0. Budget follows *C* axis.	LE curve moves along the *C* axis. *lab* = 0. Voluntary unemployment.
2	$w/p = \sigma/\sigma_0$	$0 \le C < A$	Budget line parallel to BA. **Disequilibrium**: equal utility at *lab* = 0 and *lab* = T	**Parallel paths along the *C* axes** at *lab* = 0, (**involuntary unemployment)**, and *lab* = T.
		C = A	Budget co-incidental with **BA**. An infinity of options between *lab* = 0 and *lab* = T.	**Drama at survival endowment**, ***C* = A**.
		$A < C \le F$	**Optimisation**. Budget line traces locus EF, separating areas N and R.	LE curve is a **negative straight line** from point (*C* = A, *lab* = (T – γ_0)) to point (*C* = F, *lab* = 0).
				Pointed LE curve is the apex of the series of LE arcs for $w/p^* < w/p < \sigma/\sigma_0$ and A < *C* < F.
		C > F	**Constraint:** if $lab \le 0$, then set *lab* = 0. Budget follows *C* axis.	LE curve moves along *C* axis, for all *C* > F. *lab* = 0. Voluntary unemployment.
3	$w/p > \sigma/\sigma_0$	$C \ge 0$	Corner solutions. At *C* = 0, the budget lies over origin and point (*lab* = T, *q* = (B – A)), and moves up the *q* axis to *q* = B as *C* increases.	LE curves move along the *C* axis, at *lab* = T.
			Optimisation Functional poverty, deprived of leisure in area V and on through area R	Each LE curve falls steeply through section V. **Very *C*-elastic**.

			to the C axis, where *lab* = 0 for all *C* > F.	It falls less steeply in an almost straight line through section R, from *C* = A to *C* > F, then along the *C* axis.
			Subsequent budgets, for higher *w/p*, lie over (lab = T, *q* = B) and (lab = 0, C < 0), the latter offering higher utility.	Subsequent LE curves create intercepts on the *lab* axis (*C* < 0) and then *lab* falls steeply through V and less steeply through section R. The intercept of each successive LE curve is lower than the last, and each curve meets the *C* axis slightly further along than the previous one; *lab* = 0.
	w/p = ∞	*C* ≥ 0		The LE curve is co-incidental with the *C* axis, *lab* = 0.

TABLE B3 The relationship between the Map and the CD diagram

	Endow-ment	price	Map	CD diagram
1	$0 \leq C < A$		**Dysfunctional poverty** **Corner solutions**	
		$p/w < \sigma_0/\sigma$	Budget line lies over *q* axis at *lab* = 0. $q = \infty$	Free consumption limited to *q* = B.
			Budget rotates anticlockwise. **Optimisation** is possible through area R & consumption falls as p/w Increases.	Negative CD curve passes through section R as *p/w* increases.
			Consumption increases again to *q* = B through area **V** – **deprivation of leisure, functional poverty**.	**Very price elastic** CD curves increase through V to *q* = B again, creating a series of **U-shaped curves** bounded by an **envelope curve**, reflecting the superior-inferior boundary between sections R and V.
			Budget line moves down *q*-axis, (**dysfunctional poverty**, **corner solution)**, until it is parallel to BA at *q* = (B – A).	CD curves decrease in straight lines from *q* = B to *q* = (B – A) as *p/w* increases
		$p/w = \sigma_0/\sigma$	**Disequilibrium** – budget choice **jumps** as *p/w* increases further, from *q* = (B – A) to *q* = 0 + *C*, with the latter offering greater utility.	**Disjointed CD curves** jump from *q* = (B – A) to *q* = 0 + *C*.
		$p/w > \sigma_0/\sigma$	Consumption stays at *q* = 0 + *C* as *p/w* increases.	For *C* = 0, CD curve moves up *p/w* axis, for $p/w > \sigma_0/\sigma$. For 0 < *C* < A, CD curves move upwards at $0 < q \leq C$, and *q* = *C* at $p/w = \infty$.
2	*C* = A	$p/w < \sigma_0/\sigma$	Budget line starts in same way as above.	Negative-sloping CD curve passes through section R, as *p/w* increases.
		$p/w = \sigma_0/\sigma$	Budget line **co-incidental with BA**. Apparent range of options between *q* = B & *q* = 0 + *C* = A?	CD curve meets $p/w = \sigma_0/\sigma$ at a point $q > \gamma$. Actual range is much shorter, from $q > \gamma$ to *q* = 0 + *C* = A. **Sticky prices**.
		$p/w > \sigma_0/\sigma$	Budget line continues to rotate anticlockwise, on point *C* = A.	For *C* = A, CD curve moves up the price axis at *C* < A almost vertically. *q* = 0 + *C* = A at $p/w = \infty$.
3	A < *C*	$p/w < \sigma_0/\sigma$	Budget line starts in same way, optimising in area R.	Negative-sloping CD curve passes through section R.
	$A < C < \gamma$	$p/w > \sigma_0/\sigma$	Optimisation continues in area K, **deprived of consumption – functional poverty**.	CD curves, still falling through section K, become **price elastic.**
	$C \geq \gamma$	$p/w > \sigma_0/\sigma$	Optimisation continues through area N.	Negative CD curves are very price inelastic through section N. Increased endowments create gaps between subsequent CD curves, which become narrower as *C* increases.

TABLE B4 The relationship between the Map and the CE diagram

	Price	Endow-ment	Map	CE diagram
1	$0 \leq p/w < T/B$	*C* < 0	**Dysfunctional poverty.**	

			Corner solutions The budget line moves up the C axis from $C < 0$ with the value $q > 0$ offering greater utility than $q = 0$. At q = B, the budget **optimises** through area V, **deprived of leisure**.	These CE curves move up the q axis and create as a series of decreasing intercepts on the q axis as p/w decreases, before q declines through section V.
	p/w = T/B	$C = 0$	The budget line lies over the origin & point (q = B, lab = T). p/w = T/B. As p/w increases, optimisation traces a path through V, **deprivation of leisure, functional poverty.**	The CE curve starts at q = B and falls steeply through the Giffen-deprived part of section V, then briefly and less steeply through the 'normal deprived' part of V. ***C*-elastic**.
	T/B ≤ $p/w < \sigma_0/\sigma$	0 < C < A	For example, w/p = T/(B – A/2) The budget line moves up the C axis from $C = 0$ to C = A/2. q moves from q = (B – A/2) *to* q = B which also offers greater utility than $q = 0$. Optimisation occurs in area V.	The CE curve moves up a 45° line from q = (B – A/2) at $C = 0$ to q = B at C = A/2. As C increases further, the CE curve falls steeply through section V.
	0 ≤ $p/w < \sigma_0/\sigma$	0 ≤ C < A		The CE curves create the **Giffen envelope curve** in section V.
		A ≤ C ≤ F	The budget line optimises through area R.	The CE curve increases as C increases in R.
		C > F	The budget line is constrained and moves up the q axis at lab = 0.	The CE curve meets and joins the 45° straight line, $q = 0 + C$, *after* C = F, ie. C > F.
2	$p/w = \sigma_0/\sigma$	0 ≤ C < A	**Dysfunctional poverty**. Budget line lies parallel to BA. **Disequilibrium**: $q = 0 + C$ and q = (B – A + C) offer equal utility as C increases.	CE curves move along **parallel straight 45° paths,** from $q = 0$ to $q = 0 + C$ = A and from q = (B – A) to q = (B – A + C) = B.
		C = A	Budget line **co-incidental with BA**. Range of options covers: 0 + C = A ≤ q ≤ B.	**Survival endowment**. CE curve for C = A is vertical line at C = A. Actual range is much smaller: 0 + C = A ≤ $q \leq \gamma$.
		A < C ≤ F	Budget line traces the locus, **EF**, from $q = 0 + C$ = A to q = F.	CE curve exits the survival endowment at $q = \gamma$ in positive-sloping straight line to q = F.
		C > F	Budget line constrained to a corner solution and traces the q axis at lab = 0.	The CE curve for C = A meets and joins the 45° line at C = F.
3	$p/w > \sigma_0/\sigma$	0 ≤ C ≤ A	**Dysfunctional poverty**. Corner solution, $q = 0 + C$, offers greater utility.	The CE curve, $q = 0 + C$, is the 45° straight line through the origin and through q = A at C = A.
		C > A	**Optimisation** can start in area K **deprived of consumption, functional poverty** – then continue through area N.	CE curves increase steeply through section K, (***C* elastic**), and then less steeply through N, each successive curve becoming less bowed, all meeting and joining the 45° straight line, $q = 0 + C$, *before* C = F.
	$p/w = \infty$	$C \geq 0$	Budget lies over lab axis. Corner solutions, $q = 0 + C$, as C increases.	The CE curve, $q = 0 + C$, is the 45° straight line through the origin and passing through q = F at C = F.