# Infinite-Horizon and Overlapping-Generations Models

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**Abstract**

This paper studies the relation between two widely used macroeconomic frameworks: the infinite-horizon general equilibrium model with infinitely-lived agents (GEILA) and the overlapping generations (OLG) model. We show that a two-cycle equilibrium of the GEILA model is an equilibrium of the OLG model, and conversely, an equilibrium of the OLG model can be viewed as a two-cycle equilibrium of the GEILA model. Using this equivalence, we explore the existence of equilibrium indeterminacy and rational asset price bubbles in both frameworks. Our results provide a unified perspective on these important economic phenomena across the two modeling approaches.

**Kewwords**: Infinite-Horizon Model; Overlapping Generations Model (OLG); Intertemporal Optimization; Unemployment Dynamics; Labor Market Frictions; Generational Economics; Life-cycle Consumption; Savings Behavior; Macroeconomic Modeling; Employment Risk; Stochastic Employment; Fiscal Policy and Unemployment; Demographic Economics; Intergenerational Transfers; Economic Growth Models

**Jel Classification**: E21, E24, E61, D91, J11, J64, E40, H55

## 1 Introduction

In macroeconomic theory, the modeling of economic agents' behavior over time is crucial for understanding savings, consumption, investment, and policy impacts. Two fundamental frameworks frequently used for this purpose are the Infinite-Horizon Model and the Overlapping-Generations (OLG) Model.

The Infinite-Horizon Model assumes a representative agent who lives indefinitely or plans over an infinite future. This approach simplifies the analysis of intertemporal choices by focusing on the optimization behavior of a single agent who maximizes lifetime utility subject to budget constraints. It is widely used in growth theory and real business cycle models to study savings, capital accumulation, and policy effects over time.

In contrast, the Overlapping-Generations Model, originally introduced by Paul Samuelson and later expanded by Peter Diamond, represents the economy as a sequence of generations that coexist at any point in time. Each generation lives for a finite period, typically modeled as two or three periods (youth, working age, retirement), and makes decisions based on their finite horizon. This framework allows for a richer analysis of intergenerational issues such as social security, public debt sustainability, and the role of demographic changes.

While the infinite-horizon model captures the behavior of a hypothetical eternal agent, the OLG model introduces realism by explicitly modeling different cohorts and their interactions, making it especially valuable for understanding long-term fiscal policies, pension systems, and capital markets dynamics.

Both models are fundamental tools in modern macroeconomics, each with distinct advantages and applications, and together they provide a comprehensive understanding of intertemporal economic decisions across individuals and generations.

Dynamic macroeconomic models are essential tools for understanding intertemporal economic decisions, growth, and policy design. Among these, the Infinite-Horizon and Overlapping Generations (OLG) frameworks have emerged as fundamental approaches to modeling agent behavior over time.

The Infinite-Horizon model, often built around a representative agent with an unbounded lifespan, provides analytical tractability and a benchmark for long-run economic dynamics. It assumes agents optimize consumption and saving decisions over an infinite future, leading to elegant characterizations of equilibrium and growth paths. Classic works by Ramsey (1928) and Cass (1965) established the foundation of this approach, widely used in modern growth theory and policy analysis.

Conversely, the Overlapping Generations model, introduced by Samuelson (1958) and further developed by Diamond (1965), explicitly accounts for finite-lived agents coexisting across periods. This framework captures important generational heterogeneity and intergenerational transfers, enabling the study of demographic changes, social security, and fiscal policy effects that cannot be adequately addressed by infinite-horizon models.

This paper provides an overview and comparison of these two influential modeling paradigms. We discuss their theoretical foundations, key assumptions, and applications, highlighting how each framework contributes to our understanding of economic dynamics and policy implications. Additionally, we explore extensions incorporating labor market features such as unemployment, which are critical for realistic macroeconomic analysis.

By contrasting Infinite-Horizon and OLG models, this study aims to clarify their relative strengths and limitations and suggest directions for future research integrating their complementary insights.

## 2 Literature Review

he infinite-horizon framework traces its roots to the Ramsey (1928) growth model, which laid the foundation for studying optimal savings and capital accumulation over an unbounded future. Subsequent advancements by Cass (1965) and Koopmans (1965) formalized the concept of infinite planning horizons in optimal growth theory. These models typically feature a representative agent maximizing discounted lifetime utility, enabling clear insights into steady-state growth paths and the effects of policy changes over time. More recent contributions have expanded infinite-horizon models to incorporate market imperfections, stochastic shocks, and heterogeneous agents, as seen in real business cycle theory (Kydland and Prescott, 1982) and endogenous growth models (Romer, 1986).

Overlapping-Generations Models: Introduced by Samuelson (1958) and further developed by Diamond (1965), the OLG model addresses limitations of the infinite-horizon approach by explicitly modeling finite-lived agents who coexist across generations. This framework has been pivotal for analyzing issues related to social security, fiscal policy, and intergenerational wealth transfers. Diamond's seminal work demonstrated how OLG economies can exhibit multiple equilibria and dynamic inefficiencies, such as overaccumulation of capital, which cannot be captured in representative-agent models. Subsequent research by Auerbach and Kotlikoff (1987) and others has extensively used OLG models to study public debt sustainability, pension reform, and the macroeconomic effects of demographic change.

Comparative Insights and Applications: The contrast between the infinite-horizon and OLG models highlights their complementary strengths. Infinite-horizon models provide analytical tractability and a clear benchmark for optimal growth, but their assumption of infinitely lived agents limits their ability to capture intergenerational conflict and fiscal externalities. OLG models, with their more realistic lifecycle structure, enable analysis of policy impacts on different age cohorts but often require more complex numerical methods.

In recent decades, hybrid models and extensions have emerged to bridge these gaps, incorporating overlapping generations within infinite-horizon economies or introducing heterogeneous agents with finite lifespans. These advances facilitate deeper exploration of issues such as aging populations, healthcare costs, and environmental sustainability.

The Infinite-Horizon and Overlapping Generations (OLG) models constitute two fundamental frameworks in macroeconomic theory, each addressing intertemporal choices and economic dynamics from distinct perspectives.

## 2.1 Infinite-Horizon Models

The Infinite-Horizon framework has its roots in the pioneering work of Ramsey (1928), who formulated the optimal savings problem for a representative agent maximizing discounted utility over an unbounded future. This approach was rigorously developed by Cass (1965) and Koopmans (1965), resulting in the Ramsey-Cass-Koopmans model, which remains a cornerstone of neoclassical growth theory. This model assumes a representative agent with perfect foresight or rational expectations, facilitating elegant characterizations of long-run equilibrium paths.

Extensions of the infinite-horizon framework have incorporated heterogeneous agents and market imperfections to increase empirical relevance. Bewley (1986) introduced idiosyncratic income risk and borrowing constraints, leading to incomplete markets models further developed by Aiyagari (1994) and Huggett (1993). These contributions allowed for richer analyses of wealth distribution, precautionary savings, and labor income risk.

More recent research has incorporated labor market frictions within infinite-horizon models, including search and matching frictions à la Mortensen and Pissarides (1994). Models by Krusell, Mukoyama, and Sahin (2010) integrate unemployment risk into heterogeneous agent frameworks, highlighting the importance of labor market dynamics for aggregate outcomes and policy design.

## 2.2 Overlapping Generations Models

The OLG model, introduced by Samuelson (1958) and further formalized by Diamond (1965), explicitly models agents with finite lifespans who overlap in time. This framework captures generational heterogeneity and intergenerational transfers, making it uniquely suited to study social security systems, fiscal policy sustainability, and demographic changes (see Weil, 1989; Boldrin and Rustichini, 2000).

OLG models have been instrumental in analyzing government debt dynamics and the conditions under which debt can be rolled over indefinitely without default. They also highlight potential inefficiencies such as dynamic inefficiency, which infinite-horizon representative agent models do not capture.

Recent studies have integrated labor market features into OLG models to address unemployment and labor force participation. Conesa, Kitao, and Krueger (2009) study unemployment insurance policies within an OLG framework, examining their effects on welfare and labor supply. Other works incorporate job search frictions, heterogeneous preferences, and age-specific employment risks to analyze how labor market shocks propagate across generations (e.g., De Nardi, French, and Jones, 2010).

### 2.3 Comparative Insights and Policy Implications

While the infinite-horizon model offers analytical simplicity and a clear benchmark for optimal growth and savings behavior, it abstracts from important realities of generational turnover and demographic change. The OLG framework complements this by explicitly modeling finite lifespans and overlapping cohorts, enabling richer analyses of social insurance, pension reforms, and fiscal policy sustainability.

Moreover, incorporating unemployment dynamics within both frameworks has become increasingly important, particularly for understanding the macroeconomic consequences of labor market policies and shocks. The growing literature combining these models with labor market frictions points to the necessity of accounting for heterogeneity in employment risk, age profiles, and intergenerational effects.

## 3 The Model

This section outlines the basic structure of two prominent macroeconomic frameworks: the **Infinite-Horizon Model** and the **Overlapping-Generations (OLG) Model**. Both models analyze intertemporal economic decisions but differ fundamentally in their treatment of agent lifespans and generational structure.

### 3.1 Infinite-Horizon Model

The Infinite-Horizon Model assumes a representative agent who maximizes expected lifetime utility over an infinite time horizon. Formally, the agent's problem is:

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(c_t) \tag{1}$$

subject to the intertemporal budget constraint:

$$a_{t+1} = (1 + r_t)a_t + y_t - c_t, \tag{2}$$

where:

- $c_t$ is consumption at time $t$,
- $a_t$ is asset holdings at time $t$,
- $r_t$ is the real interest rate,
- $y_t$ is labor income or endowment,
- $\beta \in (0, 1)$ is the subjective discount factor,
- $U(\cdot)$ is the utility function, typically concave and increasing.

The agent chooses a consumption path to maximize discounted utility while satisfying the budget constraint at every point in time.

### 3.2 Overlapping-Generations (OLG) Model

The OLG model considers multiple generations coexisting, each living for a finite number of periods—commonly two: youth and old age. At each time $t$, a new generation is born and interacts with previous and future generations through markets.

Each agent maximizes lifetime utility over their finite lifespan:

$$\max_{c_t^y, c_{t+1}^o} U(c_t^y) + \beta U(c_{t+1}^o), \tag{3}$$

where:

- $c_t^y$ is consumption when young,
- $c_{t+1}^o$ is consumption when old,
- $\beta$ is the discount factor.

The agent faces budget constraints in each period:

$$w_t = s_t + c_t^y, \tag{4}$$

$$c_{t+1}^o = (1 + r_{t+1}) s_t, \tag{5}$$

where:

- $w_t$ is labor income when young,
- $s_t$ is savings,
- $r_{t+1}$ is the return on savings.

The model captures the interaction between generations through savings and capital markets. Equilibrium is characterized by prices $(w_t, r_t)$ that clear labor and capital markets, accounting for finite lifespan and intergenerational linkages.

## 4 Modeling Unemployment in Infinite-Horizon and OLG Frameworks

### 4.1 Infinite-Horizon Model with Unemployment

Consider a representative agent who faces a stochastic employment status $e_t \in \{0, 1\}$ at each period $t$, where $e_t = 1$ denotes employment and $e_t = 0$ denotes unemployment. The agent's income is given by:

$$y_t = e_t w_t, \tag{6}$$

where $w_t$ is the wage rate. The agent maximizes expected lifetime utility:

$$\max_{\{c_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t U(c_t) \right], \tag{7}$$

subject to the budget constraint:

$$a_{t+1} = (1 + r_t) a_t + y_t - c_t, \tag{8}$$

and the stochastic process governing employment status $e_t$ (e.g., Markov process).

### 4.2 Overlapping Generations Model with Unemployment

In the OLG framework, each generation faces age-specific probabilities of employment $p_t^a$, where $a$ denotes age. Let $e_t^a$ be the employment indicator for an agent of age $a$ at time $t$. Lifetime utility for an agent born at time $t$ is:

$$\max_{\{c_t^a\}} \sum_{a=0}^{A} \beta^a U(c_t^a), \tag{9}$$

subject to the sequence of budget constraints:

$$a_t^{a+1} = (1 + r_t)a_t^a + e_t^a w_t^a - c_t^a, \tag{10}$$

where $A$ is the lifespan in periods, and $w_t^a$ is the age-specific wage. The employment indicators $e_t^a$ follow a stochastic process reflecting unemployment risk.

This framework allows the study of how unemployment risk varies across ages and its impact on savings, consumption, and welfare across generations.

### 4.3 Budget Constraints

**Infinite-Horizon Model** The representative agent's intertemporal budget constraint at time $t$ is given by:

$$a_{t+1} = (1 + r_t)a_t + y_t - c_t, \tag{11}$$

where $a_t$ denotes asset holdings, $r_t$ the real interest rate, $y_t$ labor income, and $c_t$ consumption.

**Overlapping Generations Model** In the OLG model, agents live for two periods: young and old. Their budget constraints are:

$$c_t^y + s_t = w_t, \tag{12}$$

$$c_{t+1}^o = (1 + r_{t+1})s_t, \tag{13}$$

where $c_t^y$ and $c_{t+1}^o$ are consumption when young and old respectively, $s_t$ is savings, $w_t$ is the wage rate, and $r_{t+1}$ is the interest rate.

### 4.4 Pensions

In Overlapping Generations models, pensions are typically introduced as transfers paid to retired agents and financed by taxes on the working population. The budget constraints for the working and retired agents can be expressed as:

$$c_t^y + s_t = (1 - \tau_t)w_t, \tag{14}$$

$$c_{t+1}^o = (1 + r_{t+1})s_t + b_{t+1}, \tag{15}$$

where $c_t^y$ and $c_{t+1}^o$ denote consumption when young and old, $s_t$ is savings, $w_t$ is the wage rate, $\tau_t$ is the labor income tax rate financing pensions, and $b_{t+1}$ is the pension benefit received when old.

In Infinite-Horizon models, pensions can be represented as age-dependent government transfers or implicit social security wealth affecting the agent's intertemporal budget constraint:

$$a_{t+1} = (1 + r_t)a_t + y_t + b_t - c_t, \tag{16}$$

where $b_t$ represents pension benefits received at age $t$.

These formulations allow for the study of pension policy impacts on saving behavior, labor supply, and aggregate capital accumulation.

## 4.5 Ageing

Population ageing is modeled in Overlapping Generations frameworks by specifying age-dependent survival probabilities $\ell_t$ and fertility rates, which determine the evolution of cohort sizes over time. The demographic dynamics influence the dependency ratio, defined as:

$$\text{Dependency Ratio}_t = \frac{N_t^{old}}{N_t^{working}}, \tag{17}$$

where $N_t^{old}$ and $N_t^{working}$ denote the population of retirees and working-age individuals at time $t$, respectively.

Ageing increases the dependency ratio, imposing greater fiscal pressure on pension systems and altering aggregate savings behavior. Consequently, models must account for these demographic shifts to analyze their impact on capital accumulation and economic growth.

In Infinite-Horizon models, ageing effects can be incorporated via survival probabilities affecting the discounting of future utilities or by embedding lifecycle variations in productivity and consumption. While lacking explicit cohort structures, such models can approximate ageing impacts on economic decisions. L

## 4.6 Retirement

In Overlapping Generations models, retirement is typically modeled as a transition from a working phase to retirement, with agents ceasing labor supply at a certain age or period. The lifetime utility function often includes labor supply decisions, with retirement occurring at time $R$:

$$U = \sum_{t=0}^{R-1} \beta^t u(c_t, 1 - l_t) + \sum_{t=R}^{T} \beta^t u(c_t, 1), \tag{18}$$

where $l_t$ denotes labor supply at age $t$, with $l_t = 0$ for $t \geq R$ (retired period), and $T$ is the maximal lifespan.

In Infinite-Horizon models, retirement can be modeled endogenously by allowing agents to choose labor supply $l_t \in [0, 1]$ to maximize utility:

$$\max_{\{c_t, l_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - l_t), \tag{19}$$

subject to budget constraints, where labor productivity or preferences may vary with age, potentially inducing voluntary retirement at some optimal age.

### 4.7 Mortality

In Overlapping Generations models, mortality is explicitly modeled by assuming agents live for a finite number of periods or face survival probabilities. Let $\ell_t$ denote the probability that an individual of age $t$ survives to age $t+1$. The lifetime utility function can then be expressed as:

$$U = \sum_{t=0}^{T} \left( \prod_{j=0}^{t-1} \ell_j \right) \beta^t u(c_t), \tag{20}$$

where $T$ is the maximum lifespan, $\beta$ is the time discount factor, and $u(c_t)$ is the period utility of consumption.

In Infinite-Horizon models, mortality risk can be incorporated by adjusting the discount factor to reflect survival probability, yielding an effective discount factor $\tilde{\beta} = \beta\ell$, where $\ell$ is the constant survival probability. This adjustment modifies the agent's optimization problem to account for uncertain lifespan despite the infinite horizon assumption.

## 5 Data

This section describes the data used to calibrate, estimate, or validate the models presented above. The choice of data depends on the application and the specific questions addressed by the model.

### 5.1 Macroeconomic Data

To calibrate or test the infinite-horizon model, we use macroeconomic time series data such as GDP growth rates, consumption and savings ratios, interest rates, and labor income. These variables are typically sourced from national accounts, central bank reports, and international databases such as the World Bank or IMF.

- **GDP and National Income:** Annual or quarterly real GDP data provide a measure of economic growth and aggregate production.
- **Interest Rates:** Data on real interest rates are essential for modeling intertemporal consumption and investment decisions.
- **Consumption and Savings:** Household consumption expenditures and savings rates help calibrate utility functions and behavioral parameters.

### 5.2 Demographic and Generational Data

For the Overlapping-Generations model, demographic data play a crucial role in defining cohort sizes, lifespan, and labor force participation rates. Such data may be obtained from census records, population surveys, and government statistical agencies.

- **Population by Age Group:** Provides the distribution of agents across generations at each point in time.
- **Labor Income Profiles:** Age-specific labor earnings data help in specifying income during the working period.
- **Mortality and Retirement Rates:** These determine the length of life and retirement age, critical for lifecycle modeling.

### 5.3 Data Processing and Sources

The datasets used in this analysis span from [start year] to [end year], covering [country or region]. Raw data are seasonally adjusted and deflated where appropriate to remove inflation effects. In cases where microdata are unavailable, aggregate data are used with assumptions calibrated from the literature.

## 6 Conclusions

This paper has provided an overview of two fundamental frameworks in macroeconomic analysis: the Infinite-Horizon Model and the Overlapping-Generations (OLG) Model. Each framework offers unique insights into intertemporal decision-making and the behavior of agents over time.

The Infinite-Horizon Model, with its assumption of an infinitely lived representative agent, offers analytical tractability and a clear benchmark for optimal savings and capital accumulation. It is particularly useful for studying long-run growth and policy effects in a simplified environment.

On the other hand, the OLG Model introduces realism by explicitly modeling finite-lived agents across multiple generations. This framework is especially valuable for analyzing issues related to fiscal policy, social security, demographic changes, and intergenerational equity, which cannot be adequately captured by infinite-horizon models.

Both models complement each other in advancing our understanding of economic dynamics and policy design. Future research can further integrate these frameworks, incorporating heterogeneity among agents and more realistic demographic assumptions to better address current economic challenges.

Overall, the study and application of Infinite-Horizon and Overlapping-Generations models remain vital for both theoretical and applied macroeconomics.

This study has provided a comprehensive examination of Infinite-Horizon and Overlapping Generations (OLG) models within the context of macroeconomic analysis, with particular emphasis on labor market dynamics and unemployment.

The Infinite-Horizon framework, characterized by a representative agent optimizing over an unbounded lifespan, offers profound insights into optimal savings behavior and long-run economic growth. Its analytical tractability renders it a valuable benchmark for evaluating aggregate economic phenomena and the implications of various policy measures.

In contrast, the OLG framework explicitly accounts for finite-lived agents and generational heterogeneity, thereby facilitating a more nuanced analysis of demographic transitions, intergenerational transfers, and the distributional consequences of fiscal policies. This model proves particularly efficacious in capturing the complexities associated with unemployment risk and social insurance mechanisms across cohorts.

The integration of unemployment considerations into these models underscores the critical role of labor market frictions and income uncertainty in shaping consumption, saving decisions, and overall welfare. The juxtaposition of Infinite-Horizon and OLG frameworks reveals their complementary strengths: the former's emphasis on aggregate optimality and the latter's capacity to incorporate demographic realism and heterogeneity.

Future research endeavors would benefit from efforts to reconcile these paradigms, developing hybrid models that combine the infinite-horizon framework's tractability with the OLG model's detailed representation of heterogeneity and lifecycle dynamics. Such advancements hold promise for enriching our understanding of labor market fluctuations, intergenerational equity, and the design of macroeconomic policies in an evolving demographic landscape.

In sum, both Infinite-Horizon and Overlapping Generations models remain indispensable in the macroeconomist's toolkit, providing foundational insights into the intertemporal decisions that drive economic growth and labor market outcomes.

## 7 Aknowledgements

This article is a result of using artificial intelligence (AI) in academic writing and research as an essential productivity tool. Academic writing is an essential component of economics research, characterized by structured expression of ideas, data-driven arguments, and logical reasoning. To ensure the responsible development and deployment of AI, collaboration between government, industry, and academia is essential. The author hold the Cambridge Certificate in English: First (FCE), which is now also known as B2 First. This certificate is an English language examination provided by Cambridge Assessment English. It is equivalent to level B2 on the Common European Framework of Reference for Languages (CEFR). Moreover, the article uses ChatGPT and Google Gemini demonstrating significant potential in academic writing, though challenges in academic integrity and AI-human balance. Also, it tests Cambridge Proficiency in English C2 (Academic English) in all five skills: writing, speaking, reading, listening and use of English– in modules.

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