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Production Function as a Set of Discrete Options: Neoclassical, Net Zero, and Climate Neutrality

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Abstract

This study examines the relationship between the neoclassical, net-zero, and climate neutrality perspectives, an area that has received limited attention in formal economic analysis. Adopting the concept of factor substitution, we model the production function as a set of discrete, substitutable options to explore the properties and interactions of these three perspectives. The findings demonstrate that each perspective yields a non-empty subset of solution options. Climate neutrality solutions are situated between neoclassical and net-zero solutions, exhibit discrete convexity, and are influenced by the level of GHG credit costs. Lower GHG credit costs tend to favour neoclassical solutions, while higher costs shift preference toward net-zero solutions. This highlights the importance of GHG credit pricing in guiding the transition to a low-emissions economy. Moreover, the framework enables the categorization of new climate mitigation options based on their effects, whether they are irrelevant, complementary, or disruptive. Overall, the proposed model provides an alternative formal approach that enhances the economic analysis of climate change mitigation strategies.

Keywords: neoclassical, net zero, climate neutrality, substitution, and production function.

1. Introduction

The rise in greenhouse gas (GHG) emissions since the Industrial Revolution has become one of the most pressing challenges for human civilization (Brohé, 2017; IPCC, 2021a). Climate change, defined as long-term shifts in temperatures and weather patterns, has had a wide-ranging impact on Earth's ecological system, including rising global temperatures, shifts in precipitation, extreme weather events, glacial melt, droughts, and forest fires, all of which pose risks to human well-being (IPCC, 2022a). These changes also affect various social and economic dimensions, increasing the likelihood of conflict, social unrest, and substantial global economic losses (Mach, et al., 2019; Beals, 2019).

In response, the discourse on climate change has generated not only a range of normative initiatives in global economic policy but also significant positive developments in economic practice. The two most prominent concepts are net zero and climate neutrality. Although these two terms are often used interchangeably in public narratives, conceptually these terms have different definitions, implications, and consequences (Jeudy-Hugo, Re, & Falduto, 2021; Rogelj, Geden, Cowie, & Reisinger, 2021). Net zero refers to minimizing GHG emissions as much as technologically and economically possible, bringing net emissions close to zero (Chen, Lim, Yeo, & Tseng, 2024). In contrast, carbon neutrality focuses on balancing the carbon emissions with carbon removal initiatives or carbon offsets (Chen, Lim, Yeo, & Tseng, 2024; Chen, et al., 2022). Despite the widespread use, the two key concepts are rarely explored through formal economic analysis. In particular, the relationship between the neoclassical, net zero, and climate neutrality perspectives warrant further exploration within a formal analytical framework.

This study aims to fills that gap by examining the relationship between neoclassical, net zero, and climate neutrality in an integrated manner. We adapt the idea of factor substitution (McFadden, 1962), treating the production function as a set of discrete substitutable options. This framework is built on the principle of substitution within the production process, the idea that the same output can be achieved through different combinations of inputs (Lachmann, 1947), where each factor input is viewed as a set of substitutable options. This formal approach aims to explain the properties and relationships between the neoclassical, net zero, and climate neutrality.

2. Production Function as Systematics of Options

A firm produces output using a combination of factor inputs such as equipment, raw materials, gasoline, natural gas, electricity, capital, labour, and land (McFadden, 1962; Wiese, 2021). This production process can be expressed as:

$$Q = f(X) = f(x_1, x_2, \dots, x_i, \dots, x_I) \tag{1}$$

In Equation (1), f represents the production function, which defines the technological relationship between the vector of factor inputs X and the resulting output Q (McFadden, 1962). This function maps all feasible input combinations to a corresponding level of output, forming a production set that is assumed to be non-empty, closed, and bounded (Wiese, 2021). In this study, we extend the classical idea of factor substitution in production analysis (McFadden, 1962). Traditional substitution theory emphasises that identical output can be achieved with different combinations of inputs, for example, trading capital for labour (Lachmann, 1947; Arrow, Chenery, Minhas, & Solow, 1961).

Here, however, we shift the focus from substitution between factor classes to substitution within a given factor class. Each factor input is treated as a set of mutually substitutable options. In other words, the “gasoline” input is not a single scalar but a menu that might include conventional gasoline, bio-gasoline, or synthetic gasoline. Likewise, an “electricity” input can be supplied by coal, gas-turbine, solar, or hydro plants, and an “air-conditioning” input might be met by a central chiller, a mini-split unit, or an evaporative cooler.

<u>Axiom</u>: Existence of a Non-Empty Set of Options (X_i)

$$\forall\ i\ \exists\ X_i\ = \{x_{i,1}, x_{i,2}, x_{i,3}, \dots, x_{i,j}, \dots\} \neq \emptyset$$

Each X_i represents a set of available options for the i^{th} factor input. Within this set:

- $x_i \in X_i$ denotes the selected input that is used in the production function.
- $x_{i,j} \in X_i$ denotes a potential option of input that could be selected but has not yet been.

All members of the set X_i are perfectly substitutable and produce the same output in Equation (1) or $f(x_1, \ldots, x_{i,1,}, \ldots, x_I) = f(x_1, \ldots, x_{i,2,}, \ldots, x_I) = \cdots$ for $\forall\, x_{i,1}, x_{i,2}, \ldots \in X_i$. There exists an amount $x_{i,j}$ of option j within factor input i is associated with a non-negative cost (p) and produces a non-negative greenhouse gas (GHG) emission (e).

$$\forall\, i,j \;\exists\, x_{i,j} \geq 0 : \; p_{i,j} \geq 0 \;\cap\; e_{i,j} \geq 0 \qquad (2)$$

For each factor input i, the firm selects one member from the set of options for factor input i, such that $x_i \in X_i$. The total production costs (P) can be calculated by summing the costs of all selected factor inputs (Marshall, 1890).

$$P = \sum_{i=1}^{I} p_i \qquad (3)$$

In this model, each firm is responsible for the GHG emissions from the input factors used. GHG emissions from the firm's output are assumed to be the responsibility of downstream firms or end consumers.

<u>Assumption 1</u>: Firm's Emissions Responsibilities

Each company/firm is responsible for the GHG emissions from the input factors used.

This assumption views the sale of production output as a transfer of ownership along with the corresponding rights and obligations. The total emissions (E) produced by a firm can be calculated as the accumulated emissions from all selected factor inputs (Nordhaus, 2010; Richter & Schiersch, 2017).

$$E = \sum_{i=1}^{I} e_i \qquad (4)$$

In this study, we assume the uniqueness of the set X_i meaning no two members have identical costs and emissions.

<u>Assumption 2</u>: Uniqueness in the Set X_i

$$\left(p_{i,j_1} = p_{i,j_2} \cap \; e_{i,j_1} = e_{i,j_2}\right) \rightarrow (j_1 = j_2)$$

When multiple options share identical cost and emission values, they can be represented by a single option without affecting the outcome. Assumption 2 ensures that each set X_i is efficient or has no trivial information.

An option can dominate or outperform other options (Leyton-Brown & Shoham, 2008) in terms of costs, in terms of emissions, or in terms of costs and emissions simultaneously. We define three types of weak dominance: weakly dominated in cost, weakly dominated in emissions, and weakly dominated.

<u>Definition 1</u>: Weakly Dominated in Cost $(\preccurlyeq_p)$

$\left(p_{i,j_1} \geq p_{i,j_2}\right) \leftrightarrow \left(x_{i,j_1} \preccurlyeq_p x_{i,j_2}\right)$, meaning that option x_{i,j_1} is weakly dominated in cost by option x_{i,j_2}.

Definition 2: Weakly Dominated in Emission ($\preccurlyeq_e$)

$(e_{i,j_1} \geq e_{i,j_2}) \leftrightarrow (x_{i,j_1} \preccurlyeq_e x_{i,j_2})$, which indicates that option x_{i,j_1} is weakly dominated in emission by option x_{i,j_2}.

Definition 3: Weakly Dominated ($\preccurlyeq$)

$(p_{i,j_1} \geq p_{i,j_2} \cap e_{i,j_1} \geq e_{i,j_2}) \leftrightarrow (x_{i,j_1} \preccurlyeq x_{i,j_2})$, implying that option x_{i,j_1} is weakly dominated by option x_{i,j_2}.

These definitions adopts the concept of "weakly dominated" ($\preccurlyeq$) which accommodates conditions $p_{i,j_1} = p_{i,j_2}$ or $e_{i,j_1} = e_{i,j_2}$, in contrast to "strictly dominated" ($\prec$) (Leyton-Brown & Shoham, 2008). An option may be considered dominated by another option based on any of the 3 definitions presented. Figure 1 illustrates examples of options that are weakly dominated in cost, weakly dominated in emission, and weakly dominated by another option.

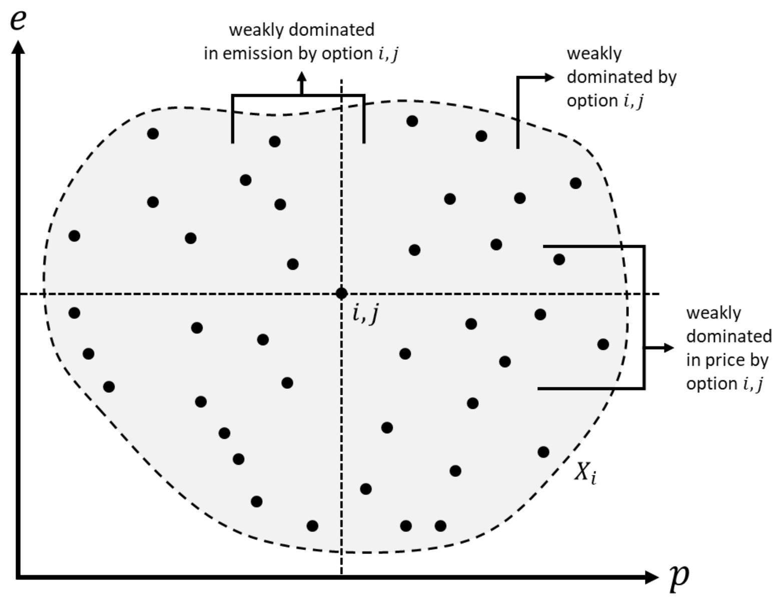

Figure 1. Illustration of options that are weakly dominated in cost, are weakly dominated in emission, and are weakly dominated by option i, j.

Building on the simple conceptual framework above, we now formalize three distinct types of production functions, each reflecting a different strategic perspective in the context of climate-related economic analysis: (1) the neoclassical production function, (2) the net zero production function, and (3) the climate neutrality production function.

3. Formalization: Subset of Discrete Options

3.1. Neoclassical: Minimum Cost

Alfred Marshall, one of the central figures who founded neoclassical economics (Veblen, 1919; Aspromourgos, 1986), introduced the concept of the "principle of substitution" in his book "The Principles of Economics" (1890). He observed that producers have flexibility in how they meet demand, and the principle of substitution emphasizes that producers will favour production factors that minimize total production cost. When a lower-cost alternative becomes available, producers are incentivized to substitute existing inputs with more cost-efficient ones to maximize profit (Marshall, 1890).

Marshall also introduced the concept of elasticity (Marshall, 1890), which later became an important element in neoclassical economics. This concept underpins a wide range of economic analyses, including price elasticity of supply, price elasticity of demand, income elasticity of demand, and cross-price elasticity of demand. Building on this foundation, John Hicks then developed a quantitative basis for the concept of substitution by introducing the concept of elasticity of substitution between factors of production (Hicks, 1932). The elasticity of substitution concept captures how producers may adopt new production methods that utilize less costly inputs to enhance profitability (Hicks, 1932). Minimum production cost orientation is a fundamental basis in the production function of neoclassical economics (Marshall, 1890; Hicks, 1932; McFadden, 1962). From this foundation, we define a subset of neoclassical options that yield minimum production costs:

<u>Definition 4</u>: Subset of Neoclassical Options $\left(X_i^G\right)$

$$\forall\, x_{i,j'} \in X_i \cap x_{i,j} \in X_i^G \subseteq X_i :\ x_{i,j'} \preccurlyeq_p x_{i,j}$$

Let j' be any option such that $j' \neq j$. A neoclassical option is one that has lower costs than all alternative options—in other words, all other options are weakly dominated in cost ($\preccurlyeq_p$) by the neoclassical options. Theorem 1 demonstrates that the subset of neoclassical options is not an empty set.

<u>Theorem 1</u>: Existence of A Non-Empty Subset of Neoclassical Options

$$X_i^G \neq \emptyset$$

Proof:

The axiom states $X_i \neq \emptyset$ which implies that the number of set members (n) is a positive integer, i.e., $n(X_i) \geq 1$. According to Equation (2), there exists a set of $x_{i,j}$ options that is associated with a non-negative cost (p), This allows us to define a corresponding mapping:

$$p_i\colon X_i \to \mathbb{P}_i$$

Where $\mathbb{P}_i = \{p_{i,1}, p_{i,2}, p_{i,3}, \dots\} \neq \emptyset$ is the set of cost value linked to the members of X_i. We can construct a partially ordered set of these costs:

$$\widehat{\mathbb{P}}_i = \{p_{i,1^*}, p_{i,2^*}, \dots, p_{i,j^*}, p_{i,j+1^*}, \dots\} \neq \emptyset$$

Where members of the cost set are ordered such that $p_{i,j^*} \leq p_{i,j+1^*}$ for all j (Simovici & Djeraba, 2008). Let K be the number of options with the minimum cost such that:

$$p_{i,1^*} = p_{i,2^*} = \cdots = p_{i,K^*}$$

with the following conditions:

i. $K \geq 1$ because $\widehat{\mathbb{P}}_i \neq \emptyset$; and
ii. $K \leq n(X_i)$ because subset $\widehat{\mathbb{P}}_i$ cannot contain more members than the set X_i.

Define a subset of substitutable minimum-cost options as:

$$\widehat{\mathbb{P}}_i^G = \{p_{i,1^*}, p_{i,2^*}, \dots, p_{i,K^*}\}$$

Thus, the full ordered set of cost options can be partitioned as:

$$\widehat{\mathbb{P}}_i = \widehat{\mathbb{P}}_i^G \cup \widehat{\mathbb{P}}_i^{\sim G} \neq \emptyset$$

Where $\widehat{\mathbb{P}}_i^{\sim G} = \{\, p_{i,K+1^*}, p_{i,K+2^*}, \dots\}$

- If $0 < K < n(X_i)$ then $\widehat{\mathbb{P}}_i^G \neq \emptyset$ and $\widehat{\mathbb{P}}_i^{\sim G} \neq \emptyset$.
- If $K = n(X_i)$ then $\widehat{\mathbb{P}}_i^G \neq \emptyset$ and $\widehat{\mathbb{P}}_i^{\sim G} = \emptyset$.

In all cases, since $K \geq 1$ and $K \leq n(X_i)$, it follows that $\widehat{\mathbb{P}}_i^G \neq \emptyset$, and therefore the corresponding subset of options $X_i^G \neq \emptyset$ (■).

Theorem 1 establishes the existence of a non-empty subset of neoclassical options. This implies that there is at least one option with minimum cost. In this study, the costs associated with different factor inputs are assumed to be independent, as seen in Equation (3). Consequently, selecting the minimum-cost option for any given input will result in lower total production costs compared to alternative options.

3.2. Net Zero: Minimum Emission

Despite their frequent interchangeable use, net zero and climate neutrality refer to conceptually distinct approaches, each with its own definitions, underlying assumptions, and implications. (Jeudy-Hugo, Re, & Falduto, 2021; Rogelj, Geden, Cowie, & Reisinger, 2021). Net zero means to reduce GHG emissions as much as possible, such that the residual emissions approaches zero (Chen, Lim, Yeo, & Tseng, 2024). According to the Science Based Targets initiative (SBTi), their Corporate Net-Zero Standard encourages firms to set a long-term target that eliminates all possible emissions by more than 90% allowing only a small fraction of unavoidable emissions to remain (SBTi, 2024). In the "Net Zero Guidelines" by the International Organization for Standardization (ISO), it is stated that GHG emissions reduction is prioritized, and carbon removals may only be applied after all feasible reduction measures have been implemented (ISO, 2022).

The scope and targets of net zero extends beyond just reducing carbon dioxide emissions but to also include all major greenhouse gases—such as methane, nitrous oxide, hydrofluorocarbons, and perfluorocarbons—each of which is converted into a standard unit of carbon dioxide equivalent (CO_2e) based on its global warming potential (United Nations, 2022; ISO, 2022; Chen et al., 2024; WBCSD & WRI, 2015; Corporate Finance Institute, 2024). Based on the definition and concept of net zero above, we define a subset of net zero options as follows:

Definition 5: Subset of Net Zero Options $\left(X_i^H\right)$

$$\forall\, x_{i,j'} \in X_i \cap x_{i,j} \in X_i^H \subseteq X_i\colon\; x_{i,j'} \preccurlyeq_e x_{i,j}$$

The subset X_i^H consists of net zero options, defined as those with lower emissions compared to other alternatives in the set X_i or in other words all other options are weakly dominated in emission ($\preccurlyeq_e$) by net zero options. Theorem 2 establishes the existence of a nonempty subset of net zero options.

Theorem 2: Existence of A Non-Empty Subset of Net Zero Options

$$X_i^H \neq \emptyset$$

Proof:

Because the axiom states that $X_i \neq \emptyset$, the number of members in set X_i satisfies $n(X_i) \geq 1$. According to Equation (2), there exists a set of $x_{i,j}$ options that is associated with a non-negative GHG emission (e), This allows us to define a corresponding emission mapping:

$$e_i: X_i \to \mathbb{E}_i$$

Where $\mathbb{E}_i = \{e_{i,1}, e_{i,2}, e_{i,3}, \dots\} \neq \emptyset$ is the set of emissions value linked to the members of X_i. Therefore, there is a partial order set:

$$\widehat{\mathbb{E}}_i = \{e_{i,1^\#}, e_{i,2^\#}, \dots, e_{i,j^\#}, e_{i,j+1^\#}, \dots\} \neq \emptyset$$

where $e_{i,j^\#} \leq e_{i,j+1^\#}$ for all j (Simovici & Djeraba, 2008). Let K be the number of substitutable options with the minimum emission such that:

$$e_{i,1^\#} = e_{i,2^\#} = \dots = e_{i,K^\#}$$

with the following conditions:

i. $K \geq 1$ because $\widehat{\mathbb{E}}_i \neq \emptyset$; and
ii. $K \leq n(X_i)$ because subset $\widehat{\mathbb{E}}_i$ cannot contain more members than the set X_i.

Define a subset of minimum-emission options as:

$$\widehat{\mathbb{E}}_i^H = \{e_{i,1^\#}, e_{i,2^\#}, \dots, e_{i,K^\#}\}$$

Thus, the full ordered set of emission options can be partitioned as:

$$\widehat{\mathbb{E}}_i = \widehat{\mathbb{E}}_i^H \cup \widehat{\mathbb{E}}_i^{\sim H} \neq \emptyset$$

Where $\widehat{\mathbb{E}}_i^{\sim H} = \{ e_{i,K+1^\#}, e_{i,K+2^\#}, \dots\}$

- If $0 < K < n(X_i)$ then $\widehat{\mathbb{E}}_i^H \neq \emptyset$ and $\widehat{\mathbb{E}}_i^{\sim H} \neq \emptyset$.
- if $K = n(X_i)$ then $\widehat{\mathbb{E}}_i^H \neq \emptyset$ and $\widehat{\mathbb{E}}_i^{\sim H} = \emptyset$.

In all cases, since $K \geq 1$ and $K \leq n(X_i)$, it follows that $\widehat{\mathbb{E}}_i^H \neq \emptyset$, and therefore the corresponding subset of options $X_i^H \neq \emptyset$ (■).

The characterization of minimum-emission options provides a foundation for understanding how environmental priorities shape the structure of feasible solutions. However, in many real-world policy and market contexts, decision-making does not rely on emissions alone. This opens the way for a broader formulation that accounts for both environmental outcomes and economic considerations.

3.3. Climate Neutral: Cost Optimization

3.3.1. Undominated Options

Emissions trading is a market-based mechanism for climate change mitigation that aims to control carbon and other greenhouse gas (GHG) emissions by providing economic incentives for emission reduction (Stavins, 2003). From an economic point of view, climate change arises because GHG producers do not bear the full external costs of their emissions. Therefore, it is essential to assigned costs to the emissions produced (IMF, 2008). Emissions trading has become an important tool in controlling GHG emissions at the local, national, and international levels (IPCC, 2021b), by disincentivizing emission-intensive activities and rewarding those that contribute to emission removal or reduction.

To internalize the cost of emissions, a GHG offset cost (ρ) is applied to each emission-producing option. Where ρ is constrained by $0 < \rho < \mathcal{L}$, and $\mathcal{L}$ is a large but finite upper bound. The total cost (φ) for option i, j can be calculated as follows:

$$\varphi_{i,j,\rho} = p_{i,j} + \rho \cdot e_{i,j} \tag{5}$$

Figure 2 illustrates the relationship between option cost (p), GHG credit costs (ρ), and total option cost (φ) within the cost–emissions diagram.

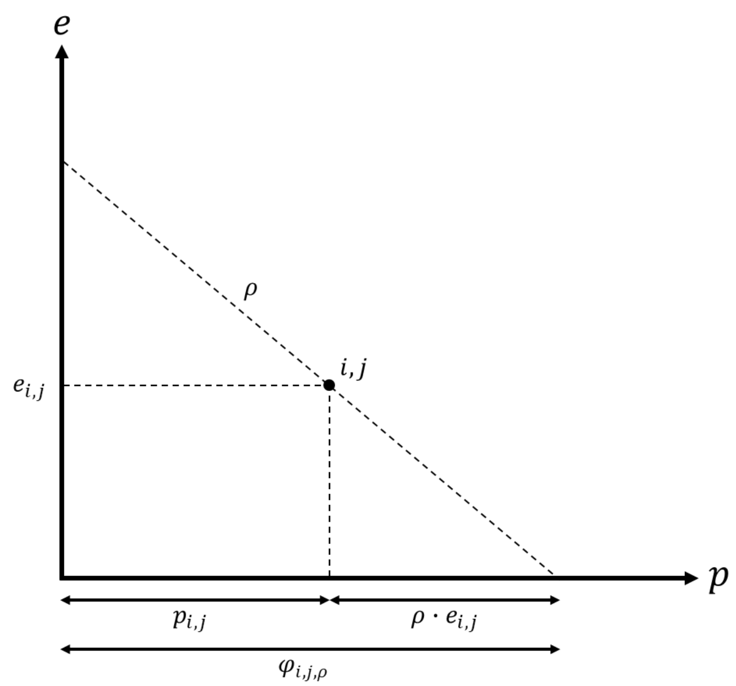

Figure 2. The relationship between option cost (p), GHG credit costs (ρ), and total option cost (φ).

Building on this visualization, Corollary 1 formalizes the result that a weakly dominated option always incurs a higher total cost—and conversely, any option with a consistently higher total cost for all $0 < \rho < \mathcal{L}$ is weakly dominated.

<u>**Corollary 1**</u>: Relationship Between Weakly Dominated and Total Cost

$$\forall\, 0 < \rho < \mathcal{L} : \left(x_{i,j} \preccurlyeq x_{i,j'}\right) \leftrightarrow \left(\varphi_{i,j,\rho} \geq \varphi_{i,j',\rho}\right)$$

Proof:

From Definition 3, an option $x_{i,j}$ is said to be weakly dominated by another option $x_{i,j'}$ if and only if:

$$\left(x_{i,j} \preccurlyeq x_{i,j'}\right) \leftrightarrow \left(p_{i,j} \geq p_{i,j'} \ \cap \ e_{i,j} \geq e_{i,j'}\right)$$

Now, referring to Equation (5), the total cost of an option is defined as:

$$\varphi_{i,j,\rho} = p_{i,j} + \rho \cdot e_{i,j}$$

Substituting the condition from Definition 3 into the total cost formula, we obtain:

$$\left(\varphi_{i,j,\rho} \geq \varphi_{i,j',\rho}\right) \ \equiv \ p_{i,j} + \rho \cdot e_{i,j} \geq p_{i,j'} + \rho \cdot e_{i,j'}$$

Thus,

$$\left(x_{i,j} \preccurlyeq x_{i,j'}\right) \leftrightarrow \left(\varphi_{i,j,\rho} \geq \varphi_{i,j',\rho}\right) \ (\blacksquare).$$

A weakly dominated option cannot be considered a valid solution, as it always incurs a higher total cost. Therefore, any selected option must belong to the subset of options that are not dominated by any other. This leads to the definition of the subset of undominated options, denoted $\left(X_i^U\right)$, as follows:

<u>Definition 6</u>: Subset of Undominated Options $\left(X_i^U\right)$

$$\forall \ x_{i,j'} \in X_i \cap x_{i,j} \in X_i^U \subseteq X_i: \ x_{i,j} \not\preccurlyeq \ x_{i,j'}$$

This definition states that each member of the subset of undominated options are not weakly dominated by any other option. Conversely, any option that is weakly dominated by another option cannot be included in the subset of undominated options.

This concept is closely related to what is commonly known in multi-objective optimization literature as the Pareto front or Pareto frontier, the set of non-dominated solutions where no objective (in this case, cost) can be improved without worsening another (such as emissions). Here, we adopt the term undominated options to emphasize its role as a foundational subset within the climate neutrality framework. The following corollary formalizes this exclusion principle.

<u>Corollary 2</u>: Options Not Belonging to the Undominated Subset

$$\left(x_{i,j} \preccurlyeq x_{i,j'}\right) \leftrightarrow \left(x_{i,j} \notin X_i^U\right)$$

Proof:

From Definition 6, an option $x_{i,j}$ is said to be an undominated option, if and only if:

$$x_{i,j} \not\preccurlyeq \ x_{i,j'} \leftrightarrow x_{i,j} \in X_i^U$$

From there we can imply that:

$$\left(x_{i,j} \preccurlyeq x_{i,j'}\right) \leftrightarrow \left(x_{i,j} \notin X_i^U\right) \ (\blacksquare).$$

Building on Corollary 2, which characterizes options excluded from the undominated subset, Corollary 3 identifies the unique option that lies at the intersection of the neoclassical and undominated subsets.

<u>Corollary 3</u>: Intersection between Neoclassical and Undominated Options

$$n\left(X_i^G \cap X_i^U\right) = 1$$

Proof:

From the proof of Theorem 1, we know that the set of minimum-cost options is:

$$\widehat{\mathbb{P}}_i^G = \{p_{i,1^*}, p_{i,2^*}, \dots, p_{i,K^*}\} \neq \emptyset,$$

with all members having interchangeable costs:

$$p_{i,1^*} = p_{i,2^*} = \cdots = p_{i,K^*}.$$

From Assumption 2 it follows that the associated emissions are all distinct:

$$e_{i,1^*} \neq e_{i,2^*} \neq \cdots \neq e_{i,K^*}.$$

These emission values form a partially ordered set (Simovici & Djeraba, 2008)

$$\widehat{\mathbb{E}}_i^G = \{e_{i,1^{*\#}}, e_{i,2^{*\#}}, \dots, e_{i,k^{*\#}}, \dots, e_{i,K^{*\#}}\} \neq \emptyset,$$

Ordered such that:

$$e_{i,k^{*\#}} < e_{i,k+1^{*\#}} \text{ and } x_{i,k^{*\#}} \in X_i^G$$

Now consider the following cases:

- If $k \neq 1$ then $e_{i,1^{*\#}} < e_{i,k^{*\#}}$, meaning $x_{i,k^{*\#}}$ is weakly dominated in emissions and therefore $x_{i,k^{*\#}} \notin X_i^U$.
- If $k = 1$ and $x_{i,j} \in X_i^G$, then $e_{i,1^{*\#}} < e_{i,j}$.
- If $k = 1$ and $x_{i,j} \notin X_i^G$, then $p_{i,1^{*\#}} < p_{i,j}$.

Which implies:

$$x_{i,1^{*\#}} \not\preccurlyeq x_{i,j}$$

Thus, $x_{i,1^{*\#}}$ is not weakly dominated and belongs to the undominated set:

$$x_{i,1^{*\#}} \in X_i^U$$

We therefore conclude that:

$$X_i^G \cap X_i^U = \{x_{i,1^{*\#}}\} = \{x_{i,j^{g\cap u}}\} \ (\blacksquare).$$

For simplicity, we will refer to $x_{i,1^{*\#}}$ as $x_{i,j^{g\cap u}}$ in the remainder of the study.

Extending the result of Corollary 3, which identified the unique intersection between the neoclassical and undominated subsets, the next corollary establishes the existence of a single option at the intersection of the net-zero and undominated subsets.

Corollary 4: Intersection between Net Zero and Undominated Options

$$n(X_i^H \cap X_i^U) = 1$$

Proof:

From the proof of Theorem 2, we know that the set of minimum-emission options is:

$$\widehat{\mathbb{E}}_i^H = \{e_{i,1^\#}, e_{i,2^\#}, \dots, e_{i,K^\#}\} \neq \emptyset,$$

with all members having interchangeable costs:

$$e_{i,1^\#} = e_{i,2^\#} = \cdots = e_{i,K^\#}.$$

From Assumption 2 it follows that the associated costs are all distinct:

$$p_{i,1^\#} \neq p_{i,2^\#} \neq \cdots \neq p_{i,K^\#}.$$

These emission values form a partially ordered set (Simovici & Djeraba, 2008)

$$\widehat{\mathbb{P}}_i^H = \{p_{i,1^{\#*}}, p_{i,2^{\#*}}, \dots, p_{i,k^{\#*}}, \dots, p_{i,K^{\#*}}\} \neq \emptyset,$$

Ordered such that:

$$p_{i,k^{\#*}} < p_{i,k+^{\#*}} \text{ and } x_{i,k^{\#*}} \in X_i^H.$$

Now consider the following cases:

- If $k \neq 1$ then $p_{i,1^{\#*}} < p_{i,k^{\#*}}$, meaning $x_{i,k^{\#*}}$is weakly dominated in cost and therefore $x_{i,k^{\#*}} \notin X_i^U$.
- If $k = 1$ and $x_{i,j} \in X_i^G$, then $p_{i,1^{\#*}} < p_{i,j}$.
- If $k = 1$ and $x_{i,j} \notin X_i^G$, then $e_{i,1^{\#*}} < e_{i,j}$.

Which implies:

$$x_{i,1^{\#*}} \not\preccurlyeq x_{i,j}$$

Thus, $x_{i,1^{\#*}}$ is not weakly dominated and belongs to the undominated set:

$$x_{i,1^{\#*}} \in X_i^U$$

We therefore conclude that:

$$X_i^H \cap X_i^U = \{x_{i,1^{\#*}}\} = \{x_{i,j^{h\cap u}}\} \text{ (■)}.$$

For simplicity, we will refer to $x_{i,1^{\#*}}$ as $x_{i,j^{h\cap u}}$ in the remainder of the study.

Drawing from the results of Corollaries 3 and 4, we can now formalize the key properties of the subset of undominated options.

Theorem 3: Properties of the Subset of Undominated Options

(a) $X_i^U \neq \emptyset$.
(b) If $n(X_i^U) = 1 \rightarrow X_i^U = \{x_{i,j^{g\cap h\cap u}}\}$.
(c) If $n(X_i^U) = 2 \rightarrow X_i^U = \{x_{i,j^{g\cap u}}, x_{i,j^{h\cap u}}\}$.
(d) If $n(X_i^U) \geq 3 \rightarrow X_i^U = \{x_{i,j^{g\cap u}}, X_i^{U\cap G'\cap H'}, x_{i,j^{h\cap u}}\}$ and $n\left(X_i^{U\cap G'\cap H'}\right) = n(X_i^U) - 2$.

Proof:

(a) If $X_i^U = \emptyset$; it contradicts Corollaries 3 and 4, which each guarantee a non-empty intersection with X_i^U. It also violates the principle that every finite partially ordered set has at least one minimal element/member (Johnsonbaugh, 2019) (■).

(b) From Corollary 3, if $n(X_i^U) = 1$ then $X_i^G \cap X_i^U = \{x_{i,j^{g\cap u}}\}$. From Corollary 4, if $n(X_i^U) = 1$ then $X_i^H \cap X_i^U = \{x_{i,j^{h\cap u}}\}$. It can be obtained that:

$$X_i^G \cap X_i^U = X_i^H \cap X_i^U = \{x_{i,j^{g\cap u}}\} = \{x_{i,j^{h\cap u}}\} = \{x_{i,j^{g\cap h\cap u}}\} (■).$$

(c) Assuming $n(X_i^U) \geq 2$ and suppose, for contradiction, that

$$x_{i,j^{g\cap u}} = x_{i,j^{h\cap u}}$$

So that:

$$X_i^U = \{x_{i,j^{g\cap h\cap u}}, X_i^W\}, \textit{ where } x_{i,j^W} \in X_i^W$$

and X_i^W represents the set of possible additional undominated options beyond the shared neoclassical-net-zero-undominated option. For x_{i,j^W} to belong to X_i^U it must satisfy Definition 3 and Assumption 2:

$$\left(\left(p_{i,j^w} < p_{i,j^{g\cap h\cap u}}\right) \cap \left(e_{i,j^w} > e_{i,j^{g\cap h\cap u}}\right)\right)$$

or

$$\left(\left(p_{i,j^w} > p_{i,j^{g\cap h\cap u}}\right) \cap \left(\left(e_{i,j^w}\right) < e_{i,j^{g\cap h\cap u}}\right)\right)$$

However:

- The first condition contradicts $x_{i,j^{g\cap h\cap u}} \in X_i^G$ (i.e., it has minimum cost), and
- The second condition contradicts $x_{i,j^{g\cap h\cap u}} \in X_i^H$. (i.e., it has minimum emissions).

Thus, $x_{i,j^W} \notin X_i^U$, implying $X_i^W = \emptyset$ and therefore $n(X_i^U) = 1$, which contradicts our initial assumption that $n(X_i^U) \geq 2$. Hence, for $n(X_i^U) \geq 2$ to hold, it must be that:

$$x_{i,j^{g\cap u}} \neq x_{i,j^{h\cap u}}$$

And if $n(X_i^U) = 2$, then:

$$X_i^U = \left\{x_{i,j^{g\cap u}}, x_{i,j^{h\cap u}}\right\}(\blacksquare).$$

(d) If $\left(X_i^U\right) \geq 3$, then the neoclassical–undominated option and the net-zero–undominated option must be distinct, i.e.,

$$x_{i,j^{g\cap u}} \neq x_{i,j^{h\cap u}}$$

In this case, the remaining undominated options must belong to the subset

$$X_i^{U\cap G'\cap H'}$$

which consists of options that are undominated but are neither neoclassical nor net-zero. Thus, we can express the undominated subset as:

$$X_i^U = \left\{x_{i,j^{g\cap u}}, X_i^{U\cap G'\cap H'}, x_{i,j^{h\cap u}}\right\}$$

with the number of such intermediate options given by:

$$n\left(X_i^{U\cap G'\cap H'}\right) = n\left(X_i^U\right) - 2\ (\blacksquare).$$

Based on the three theorems and four corollaries presented above, we can observe the relationship between the subset of neoclassical options $\left(X_i^G\right)$, the subset of net zero options $\left(X_i^H\right)$, and the subset of undominated options $\left(X_i^U\right)$, as illustrated in Figure 3.

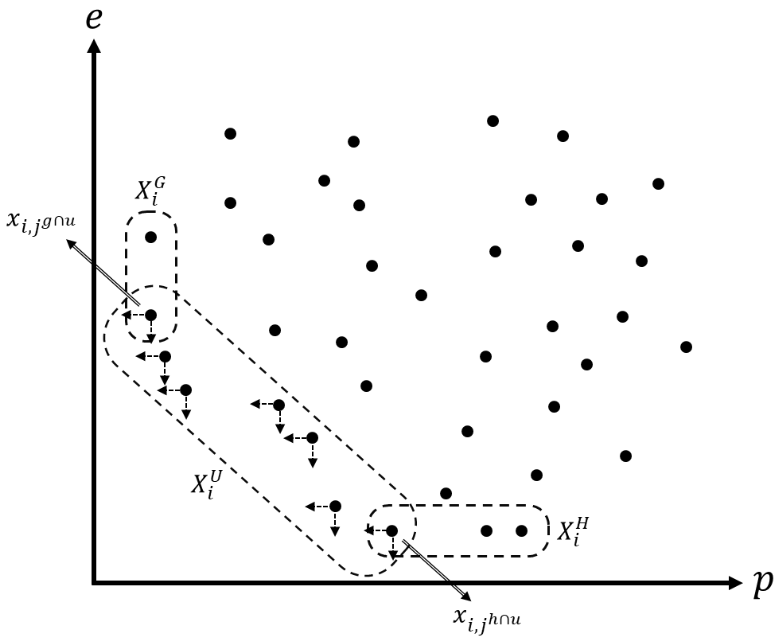

Figure 3. Relationship between subset of neoclassical options $\left(X_i^G\right)$, subset of net zero options $\left(X_i^H\right)$, and subset of undominated options $\left(X_i^U\right)$.

The undominated subset X_i^U consists of options that are not weakly dominated by any other options. This subset contains one unique member that overlaps with the subset of neoclassical options ($x_{i,j^{g\cap u}}$) and one unique member that overlaps with the subset of net zero options ($x_{i,j^{h\cap u}}$).

3.3.2. Properties of Climate Neutrality

Although the terms net zero, carbon neutrality, and climate neutrality are often used interchangeably in policy and public discourse, they differ significantly in scope, definition, and implementation (Jeudy-Hugo, Re, & Falduto, 2021; Rogelj, Geden, Cowie, & Reisinger, 2021). Carbon neutrality, though not central to this study, is frequently referenced in climate policy. It generally refers to balancing carbon dioxide emissions with equivalent removals or offsets. However, its usage is often ambiguous, sometimes referring strictly to CO_2, and at other times used more broadly to imply all greenhouse gas (GHG) emissions (IPCC, 2022b; Rogelj et al., 2021). To avoid this ambiguity, this study adopts the term climate neutrality (or GHG neutrality), which explicitly encompasses all types of GHGs. Climate neutrality refers to a state in which total anthropogenic GHG emissions are balanced by an equivalent number of removals from the atmosphere. This balance can be achieved not only through emission reductions and avoidance strategies but also using carbon offsets, making offsets a formally recognized component of the climate neutrality framework (UN Climate Change, 2021; IPCC, 2022b). The concept of climate neutrality is grounded in Article 4(1) of the Paris Agreement, which calls for "a balance between anthropogenic emissions by sources and removals by sinks of greenhouse gases in the second half of this century" (United Nations, 2015; European Parliament, 2022).

In contrast, the net zero approach emphasizes the reduction of emissions to the lowest possible level, with GHG removals permitted only to address unavoidable residual emissions (ISO, 2022; SBTi, 2024; Chen, Lim, Yeo, & Tseng, 2024). While both concepts aim at long-term climate stabilization, net zero places stricter limits on the role of offsets and prioritizes direct emission abatement as the primary pathway. While climate neutrality and net zero may converge conceptually at the global level, their implementation often varies across sub-global levels, such as for individuals, organizations, corporations, or countries (UN Climate Change, 2021; IPCC, 2022b). Achieving a perfect balance between GHG emissions and removals is particularly challenging at the micro level. Most firms lack the capacity to independently offset their emissions with equivalent removals, which makes it necessary to apply policy instruments that influence behaviour through economic incentives (Stavins, 2003).

Among these instruments, emissions trading is a prominent market-based mechanism for climate policy. In contrast to command-and-control regulations, which impose fixed rules or emission standards, emissions trading offers firms the flexibility to determine how best to meet policy targets (Callan & Thomas, 2013). This flexibility has made emissions trading a widely used approach for countries fulfilling their Paris Agreement commitments, and it is regarded as a key tool for climate change mitigation because it internalizes external emission costs by assigning them a market price (IPCC, 2007; ICAP, 2024). Such market mechanisms promote economic efficiency by enabling emission reductions to occur where marginal abatement costs are lowest (Coase, 1960; IPCC, 2001). Polluters are therefore incentivized to reduce emissions in the most cost-effective way and at the most strategic time (IPCC, 1996). Under this system, each emitter can select the optimal option based on the relationship between option cost, emissions level, and the price of GHG credits (ρ).

Building on this foundation, we define the subset of climate neutrality options as follows:

Definition 7: Subset of Climate Neutrality Options $\left(X_i^V\right)$

$$\forall\, x_{i,j'} \in X_i \cap x_{i,j} \in X_i^V \subseteq X_i \cap 0 < \rho < \mathcal{L} \colon \exists\, \rho \text{ where } \varphi_{i,j,\rho} \leq \varphi_{i,j',\rho}$$

In Definition 7, an option belongs to the subset of climate neutrality options if it has the lowest total cost for at least one value of the GHG credit cost (ρ) within the range $(0, \mathcal{L})$.

To examine climate neutrality further, we need to distinguish collectively dominated from weakly dominated. While weak dominance, as defined in Definition 3, refers to a situation where one option is dominated by a single other option. In this section, we try to broaden the scope of exploration by introducing the concept of collectively dominated, collective dominance extends this idea. It describes a case where an option is dominated not by one, but by a combination of two or more other options. This broader concept is formalized in the following Definition 8.

Definition 8: Collectively Dominated

> For all $p_{i,j-r} < p_{i,j} < p_{i,j+s}$ where r, s are positive integers and $p_{i,j} = \alpha \cdot p_{i,j-r} + (1-\alpha) \cdot p_{i,j+s}$ and $0 < \alpha < 1$, and $e_{i,j-r} > e_{i,j} > e_{i,j+1}$ where $e_{i,j} > \alpha \cdot e_{i,j-r} + (1-\alpha) \cdot e_{i,j+s}$ then $x_{i,j}$ is neither weakly dominated by $x_{i,j-r}$ nor $x_{i,j+s}$, but collectively dominated by $x_{i,j-r}$ and $x_{i,j+s}$ or $\left(x_{i,j} \preccurlyeq \left[x_{i,j-r}, x_{i,j+s}\right]\right)$.

Figure 4 illustrates this situation, where $x_{i,j}$ appears acceptable in isolation but fails to deliver a better trade-off than the convex combination of the two surrounding options. This inefficiency justifies excluding it from the optimal choice set.

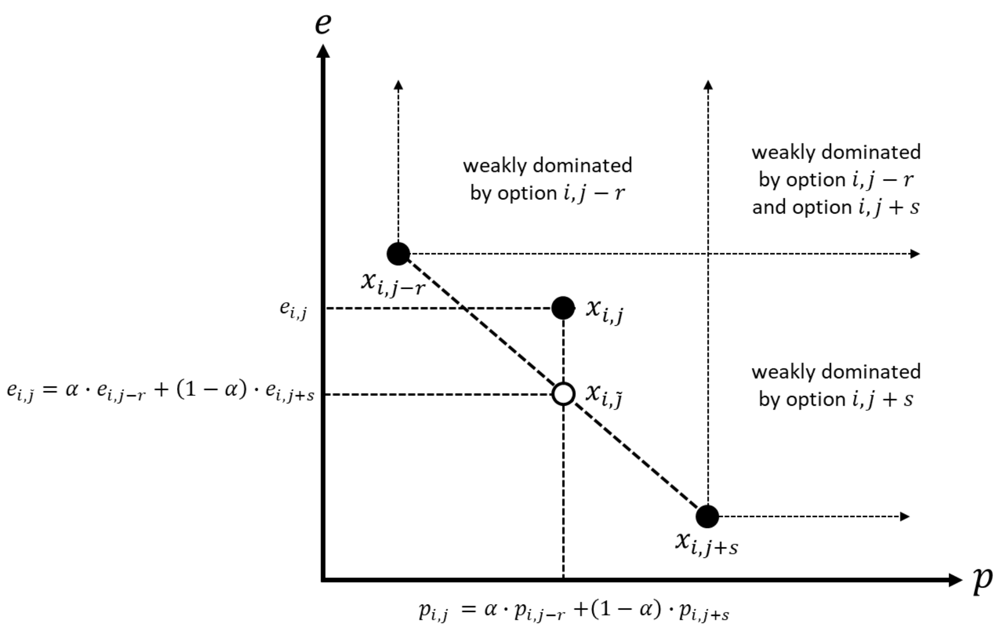

Figure 4. Illustration of an option that is not weakly dominated by any single option, but is collectively dominated by two other options.

The total cost (φ) of the collectively dominated options tends to be more expensive.

Corollary 5: Collectively Dominated Option is Always More Expensive

$$\forall\, 0 < \rho < \mathcal{L} : x_{i,j} \preccurlyeq \left[x_{i,j-r}, x_{i,j+s}\right] \rightarrow \left(\varphi_{i,j,\rho} > \varphi_{i,j-r,\rho}\right) \cup \left(\varphi_{i,j,\rho} > \varphi_{i,j+s,\rho}\right).$$

Proof:

From Equation 5, it is known that if $p_{i,j} = p_{i,\tilde{j}}$, $e_{i,j} > e_{i,\tilde{j}}$, and $\rho > 0$, then:

$$\varphi_{i,j,\rho} > \varphi_{i,\tilde{j},\rho}$$

Geometrically (see Figure 4), the GHG credit cost threshold ($\bar{\rho}$) can be defined as:

$$\bar{\rho} = \frac{p_{i,j+s} - p_{i,j-r}}{e_{i,j-r} - e_{i,j+s}} = \frac{p_{i,\tilde{j}} - p_{i,j-r}}{e_{i,j-r} - e_{i,\tilde{j}}} = \frac{p_{i,j+s} - p_{i,\tilde{j}}}{e_{i,\tilde{j}} - e_{i,j+s}}$$

- If $\bar{\rho} > \rho$, then $\frac{p_{i,\tilde{j}} - p_{i,j-r}}{e_{i,j-r} - e_{i,\tilde{j}}} > \rho \rightarrow p_{i,\tilde{j}} + \rho e_{i,\tilde{j}} > p_{i,j-r} + \rho e_{i,j-r} \rightarrow \varphi_{i,\tilde{j},\rho} > \varphi_{i,j-r,\rho}$. Since $\varphi_{i,j,\rho} > \varphi_{i,\tilde{j},\rho}$, then $\varphi_{i,j,\rho} > \varphi_{i,j-r,\rho}$

- If $\bar{\rho} < \rho$ then $\frac{p_{i,j+s} - p_{i,\tilde{j}}}{e_{i,\tilde{j}} - e_{i,j+s}} < \rho \Rightarrow p_{i,j+s} + \rho e_{i,j+s} < p_{i,\tilde{j}} + \rho e_{i,\tilde{j}} \rightarrow \varphi_{i,j+s,\rho} < \varphi_{i,\tilde{j},\rho}$ or $\varphi_{i,\tilde{j},\rho} > \varphi_{i,j+s,\rho}$. Since $\varphi_{i,j,\rho} > \varphi_{i,\tilde{j},\rho}$, then $\varphi_{i,j,\rho} > \varphi_{i,j+s,\rho}$

- If $\bar{\rho} = \rho$, then $\frac{p_{i,j+s} - p_{i,j-r}}{e_{i,j-r} - e_{i,j+s}} = \rho$ and $\frac{p_{i,\tilde{j}} - p_{i,j-r}}{e_{i,j-r} - e_{i,\tilde{j}}} = \rho \Rightarrow p_{i,j+s} + \rho e_{i,j+s} = p_{i,j-r} + \rho e_{i,j-r}$ and $p_{i,\tilde{j}} + \rho e_{i,\tilde{j}} = p_{i,j-r} + \rho e_{i,j-r} \Rightarrow \varphi_{i,\tilde{j},\rho} = \varphi_{i,j-r,\rho} = \varphi_{i,j+s,\rho}$

Given $\varphi_{i,j,\rho} > \varphi_{i,\tilde{j},\rho}$, it follows that:

$$\varphi_{i,j,\rho} > \varphi_{i,j-r,\rho} \text{ and } \varphi_{i,j,\rho} > \varphi_{i,j+s,\rho}$$

Since the only possible conditions are $\bar{\rho} > \rho$ or $\bar{\rho} < \rho$ or $\bar{\rho} = \rho$, the consequences are:

$$\varphi_{i,j,\rho} > \varphi_{i,j-r,\rho} \text{ or } \varphi_{i,j,\rho} > \varphi_{i,j+s,\rho} \ (\blacksquare).$$

From Definition 7, it is stated that for each member of the climate neutrality options set, for all j' and all $\rho \in \mathbb{R}^+$, there exist a ρ such that: $\varphi_{i,j,\rho} \leq \varphi_{i,j',\rho}$, or $\exists\, \rho$ such that: $\bigcap_{\forall j'}\left(\varphi_{i,j,\rho} \leq \varphi_{i,j',\rho}\right)$. This implies that for options that are not members of the climate neutrality option set, $\neg\exists\, \rho$ such that $\bigcap_{\forall j'}\left(\varphi_{i,j,\rho} \leq \varphi_{i,j',\rho}\right)$.

Corollary 6 follows by showing the implications for weakly dominated or collectively dominated options.

<u>Corollary 6</u>: Not A Member of The Subset of Climate Neutrality Options

$$\left(x_{i,j} \preccurlyeq x_{i,j'}\right) \cup \left(x_{i,j} \preccurlyeq \left[x_{i,j-r}, x_{i,j+s}\right]\right) \rightarrow \left(x_{i,j} \notin X_i^V\right)$$

Proof:

From Definition 7, we know the following equivalence:

$$\forall j' \colon \left(\exists\, \rho \colon \varphi_{i,j,\rho} \leq \varphi_{i,j',\rho}\right) \leftrightarrow \left(x_{i,j} \in X_i^V\right)$$

or

$$\left(\exists\, \rho \colon \bigcap_{\forall j'}\left(\varphi_{i,j,\rho} \leq \varphi_{i,j',\rho}\right)\right) \leftrightarrow \left(x_{i,j} \in X_i^V\right)$$

Equivalently,

$$\left(\neg\exists\ \rho: \bigcap_{\forall j'} \left(\varphi_{i,j,\rho} \leq \varphi_{i,j',\rho}\right)\right) \leftrightarrow \left(x_{i,j} \notin X_i^V\right)$$

From Corollary 1, we have:

$$\left(x_{i,j} \preccurlyeq x_{i,j'}\right) \leftrightarrow \left(\varphi_{i,j,\rho} \geq \varphi_{i,j',\rho}\right)$$

Since

$$\left(\varphi_{i,j,\rho} \geq \varphi_{i,j',\rho}\right) \rightarrow \left(\neg\exists\ \rho: \bigcap_{\forall j'} \left(\varphi_{i,j,\rho} \leq \varphi_{i,j',\rho}\right)\right)$$

It follows that:

$$\left(x_{i,j} \preccurlyeq x_{i,j'}\right) \rightarrow \left(x_{i,j} \notin X_i^V\right)$$

From Corollary 5, we know that

$$\left(x_{i,j} \preccurlyeq \left[x_{i,j-r}, x_{i,j+s}\right]\right) \rightarrow \left(\varphi_{i,j,\rho} > \varphi_{i,j-r,\rho}\right) \cup \left(\varphi_{i,j,\rho} > \varphi_{i,j+s,\rho}\right)$$

This implies:

$$\neg\exists\ \rho: \left(\varphi_{i,j,\rho} \leq \varphi_{i,j-r,\rho}\right) \cap \left(\varphi_{i,j,\rho} \leq \varphi_{i,j+s,\rho}\right) \rightarrow \left(x_{i,j} \notin X_i^V\right).$$

Therefore, based on both conditions:

- $\left(x_{i,j} \preccurlyeq x_{i,j'}\right) \rightarrow \left(x_{i,j} \notin X_i^V\right)$, and
- $\left(x_{i,j} \preccurlyeq \left[x_{i,j-r}, x_{i,j+s}\right]\right) \rightarrow \left(x_{i,j} \notin X_i^V\right)$,

through simplification of disjunctive antecedents (Alonso-Ovalle, 2004), we conclude:

$$\left(x_{i,j} \preccurlyeq x_{i,j'}\right) \cup \left(x_{i,j} \preccurlyeq \left[x_{i,j-r}, x_{i,j+s}\right]\right) \rightarrow \left(x_{i,j} \notin X_i^V\right)\ (\blacksquare).$$

This leads to Corollary 7, which states that the subset of climate neutrality options is itself a subset of the undominated options.

Corollary 7: The Climate Neutrality Subset is a Subset of the Undominated Options

$$X_i^V \subseteq X_i^U$$

Proof:

From Corollary 6, we know that:

$$\left(x_{i,j} \preccurlyeq x_{i,j'}\right) \cup \left(x_{i,j} \preccurlyeq \left[x_{i,j-r}, x_{i,j+s}\right]\right) \rightarrow \left(x_{i,j} \notin X_i^V\right)$$

By using simplification of disjunctive antecedents (Alonso-Ovalle, 2004), we can separate this into two implications:

$$\left(x_{i,j} \preccurlyeq x_{i,j'}\right) \rightarrow \left(x_{i,j} \notin X_i^V\right) \text{ and } \left(x_{i,j} \preccurlyeq \left[x_{i,j-r}, x_{i,j+s}\right]\right) \rightarrow \left(x_{i,j} \notin X_i^V\right).$$

Taking the contrapositive of the first implication (Johnsonbaugh, 2019), we obtain:

$$\left(x_{i,j} \in X_i^V\right) \rightarrow \left(x_{i,j} \nleq x_{i,j'}\right)$$

From the negation of Corollary 2, it is established that:

$$\left(x_{i,j} \nleq x_{i,j'}\right) \leftrightarrow \left(x_{i,j} \in X_i^U\right)$$

Therefore, it follows that:

$$\left(x_{i,j} \in X_i^V\right) \rightarrow \left(x_{i,j} \in X_i^U\right)$$

Thus, we conclude:

$$X_i^V \subseteq X_i^U \ (\blacksquare).$$

<u>**Theorem 4**</u>: The Properties of the Subset of Climate Neutrality Options

(a) $X_i^V \neq \emptyset$
(b) $n\left(X_i^G \cap X_i^V\right) = 1$
(c) $n\left(X_i^H \cap X_i^V\right) = 1$
(d) If $n\left(X_i^U\right) = 1 \rightarrow n\left(X_i^U\right) = n\left(X_i^V\right)$
(e) If $n\left(X_i^U\right) = 2 \rightarrow n\left(X_i^U\right) = n\left(X_i^V\right)$
(f) If $n\left(X_i^U\right) \geq 3 \rightarrow n\left(X_i^U\right) \geq n\left(X_i^V\right)$

Proof:

(a) From Equation (5), it is established that:

$$\varphi: \rho, \mathbb{P}_i, \mathbb{E}_i \rightarrow \Phi_i$$

We know the following:

- From the proof of Theorem 1, $\mathbb{P}_i \neq \emptyset$
- From the proof of Theorem 2, $\mathbb{E}_i \neq \emptyset$
- From the requirement of Equation 5, $0 < \rho < \mathcal{L}$, therefore, it follows that $\Phi_i \neq \emptyset$.

For all ρ, there exist a partially ordered set

$$\widehat{\Phi}_i = \left\{\varphi_{i,1^*}, \varphi_{i,2^*}, \dots, \varphi_{i,j^*}, \qquad \varphi_{i,j+1^*}, \dots\right\} \neq \emptyset$$

Such that $\varphi_{i,j^*} \leq \varphi_{i,j+1^*}$ for all j (Simovici & Djeraba, 2008). Let K be an integer such that:

$$\varphi_{i,1^*} = \varphi_{i,2^*} = \cdots = \varphi_{i,K^*}$$

with the following conditions:

(i) $K \geq 1$ because $\Phi_i \neq \emptyset$, and
(ii) $K \leq n(\Phi_i)$, because the number of subset members is less than or equal to the number of set members in Φ_i.

Define the subset of equal-valued minimum members as:

$$\widehat{\Phi}_i^V = \left\{\varphi_{i,1^*}, \varphi_{i,2^*}, \dots, \varphi_{i,K^*}\right\}$$

Thus, the full set can be written as:

$$\widehat{\Phi}_i = \{\widehat{\Phi}_i^V, \varphi_{i,K+1^*}, \varphi_{i,K+2^*}, \dots\} = \{\widehat{\Phi}_i^V, \widehat{\Phi}_i^{\sim V}\} \neq \emptyset$$

There are two cases:

- if $0 < K < n(\Phi_i)$, then both $\widehat{\Phi}_i^V \neq \emptyset$ and $\widehat{\Phi}_i^{\sim V} \neq \emptyset$
- if $K = n(\Phi_i)$, then $\widehat{\Phi}_i^V \neq \emptyset$ and $\widehat{\Phi}_i^{\sim V} = \emptyset$.

Since $K \geq 1$ and $K \leq n(\Phi_i)$, it follows that $\widehat{\Phi}_i^V \neq \emptyset$. Therefore, the set of climate neutrality options is non-empty:

$$X_i^V \neq \emptyset \ (\blacksquare).$$

(b) From Corollary 3, it is established that

$$n(X_i^G \cap X_i^U) = 1$$

This can also be expressed as:

$$X_i^G \cap X_i^U = \{x_{i,j^{g \cap u}}\} \text{ or } x_{i,j^{g \cap u}} \in X_i^G \text{ and } x_{i,j^{g \cap u}} \in X_i^U.$$

From Definition 4, if $x_{i,j^{g \cap u}} \in X_i^G$ then $x_{i,j'} \preccurlyeq_p x_{i,j^{g \cap u}}$ and from Definition 1 it is known that $p_{i,j^{g \cap u}} \leq p_{i,j'}$. From Equation (5), we know that:

$$\lim_{\rho \to 0^+} \varphi_{i,j,\rho} = p_{i,j}$$

Hence it follows that:

$$\lim_{\rho \to 0^+} \varphi_{i,j^{g \cap u},\rho} \leq \varphi_{i,j',\rho}$$

According to Definition 7, if there exist a ρ where:

$$\varphi_{i,j^{g \cap u},\rho} \leq \varphi_{i,j',\rho}$$

Then:

$$x_{i,j^{g \cap u}} \in X_i^V$$

From here we can conclude that:

$$X_i^G \cap X_i^U \cap X_i^V = \{x_{i,j^{g \cap u}}\}$$

or equivalently:

$$n(X_i^G \cap X_i^V) = 1 \ (\blacksquare).$$

(c) From Corollary 4 it is established that

$$n(X_i^H \cap X_i^U) = 1$$

This can also be stated as:

$$X_i^H \cap X_i^U = \left\{x_{i,j^{h\cap u}}\right\}$$

Which implies:

$$x_{i,j^{h\cap u}} \in X_i^H \; and \, x_{i,j^{h\cap u}} \in X_i^U$$

From Definition 4 it is known that if $x_{i,j^{h\cap u}} \in X_i^H$ then $x_{i,j'} \preccurlyeq_e x_{i,j^{h\cap u}}$, and from Definition 1 it is known that $e_{i,j^{h\cap u}} \leq e_{i,j'}$. From Equation (5), it is known that:

$$\lim_{\rho \to \mathcal{L}^-} \frac{\varphi_{i,j,\rho}}{\rho} = e_{i,j}$$

Thus, it follows that:

$$\lim_{\rho \to \mathcal{L}^-} \varphi_{i,j^{h\cap u},\rho} \leq \varphi_{i,j',\rho}$$

According to Definition 7, if there exist a ρ such that:

$$\varphi_{i,j^{h\cap u},\rho} \leq \varphi_{i,j',\rho}$$

Then:

$$x_{i,j^{h\cap u}} \in X_i^V$$

Therefore, we conclude:

$$X_i^H \cap X_i^U \cap X_i^V = \left\{x_{i,j^{h\cap u}}\right\}$$

or equivalently:

$$n\left(X_i^H \cap X_i^V\right) = 1 \; (\blacksquare).$$

(d) Theorem 3(b) states that if $n\left(X_i^U\right) = 1$ then $X_i^U = \left\{x_{i,j^{g\cap h\cap u}}\right\}$. From Corollary 7, it is known that:

$$X_i^V \subseteq X_i^U$$

And from Theorem 4(a), it is established that:

$$X_i^V \neq \emptyset$$

Given that X_i^U contains only one member, and $X_i^V \subseteq X_i^U$ with $X_i^V \neq \emptyset$, it must be the case that:

$$X_i^V = \left\{x_{i,j^{g\cap h\cap u}}\right\}$$

Therefore, this result implies:

$$X_i^V = \left\{x_{i,j^{g\cap h\cap u}}\right\} = X_i^U \; (\blacksquare).$$

(e) Theorem 3(c) states that if $n(X_i^U) = 2$ then $X_i^U = \{x_{i,j^{g \cap u}}, x_{i,j^{h \cap u}}\}$. Note that $x_{i,j^{g \cap u}} \neq x_{i,j^{h \cap u}}$, because if $x_{i,j^{g \cap u}} = x_{i,j^{h \cap u}}$, then $n(X_i^U) = 1$, which contradicts the condition $n(X_i^U) = 2$.

From Theorem 4(b), we have:

$$X_i^G \cap X_i^V = \{x_{i,j^{g \cap u}}\}$$

From Theorem 4(c), we have:

$$X_i^H \cap X_i^V = \{x_{i,j^{h \cap u}}\}$$

Given that $x_{i,j^{g \cap u}} \neq x_{i,j^{h \cap u}}$, it follows that:

$$X_i^V = \{x_{i,j^{g \cap u}}, x_{i,j^{h \cap u}}\}$$

Thus, we can conclude:

$$X_i^U = \{x_{i,j^{g \cap u}}, x_{i,j^{h \cap u}}\} = X_i^V \, (\blacksquare).$$

(f) From Corollary 7, it is established that:

$$X_i^V \subseteq X_i^U$$

Therefore,

$$n(X_i^U) \geq n(X_i^V) \, (\blacksquare).$$

The following section introduces the concept of discrete convexity, which extends the idea of convexity from continuous functions to discrete spaces (Yüceer, 2002).

<u>Definition 9</u>: Discrete Convex

Let $\mathbb{P}_i^W$ be a subspace of a discrete n-dimensional space. A function $z: \mathbb{P}_i^W \to \mathbb{E}_i^W$ is discretely convex if for all $p_{i,j-r}$, $p_{i,j}$, $p_{i,j+s} \in \mathbb{P}_i^W$ where r, s are positive integers and $p_{i,j-r} < p_{i,j} < p_{i,j+s}$ then $z(p_{i,j}) = z(\alpha \cdot p_{i,j-r} + (1+\alpha) \cdot p_{i,j+s}) \leq \alpha \cdot z(p_{i,j-r}) + (1+\alpha) \cdot z(p_{i,j+s})$.

<u>Theorem 5</u>: Discrete Convex in the Subset of Climate Neutrality Options

X_i^V is discrete convex

Proof:

From Definition 8 it is known that if $x_{i,j-r}, x_{i,j}, x_{i,j+s} \in X_i^V$ and $p_{i,j-r} < p_{i,j} < p_{i,j+s}$ and $e_{i,j} > \alpha \cdot e_{i,j-r} + (1-\alpha) \cdot e_{i,j+s}$ or $z(p_{i,j}) > \alpha \cdot z(p_{i,j-r}) + (1-\alpha) z \cdot (p_{i,j+s})$ then $(x_{i,j} \preccurlyeq [x_{i,j-r}, x_{i,j+s}])$, so that from Corollary 6 we obtain $x_{i,j} \notin X_i^V$.

In order not to contradict $x_{i,j} \in X_i^V$ then $z(p_{i,j}) \leq \alpha \cdot z(p_{i,j-r}) + (1+\alpha) \cdot z(p_{i,j+s})$, or in line with the definition of discrete convex in Definition 9 ($\blacksquare$).

4. Discussion

While the terms net zero and climate neutrality are often used interchangeably, they in fact refer to distinct concepts with different definitions, implications, and policy consequences (Jeudy-Hugo, Re, & Falduto, 2021; Rogelj, Geden, Cowie, & Reisinger, 2021; Chen, Lim, Yeo, & Tseng, 2024). This study formally explores these two strategic approaches within the economic analysis of climate change by adapting the concept of production factor substitution (McFadden, 1962), framing the production function as a set of discrete options. This discrete-option framework enables an integrated comparison of three perspectives: neoclassical, net zero, and climate neutrality. The neoclassical perspective, which emphasizes cost minimization (Marshall, 1890; Hicks, 1932; McFadden, 1962), yields a non-empty subset of optimal solutions, as established in Theorem 1. The net-zero perspective, which prioritizes minimizing direct GHG emissions (Chen, Lim, Yeo, & Tseng, 2024; SBTi, 2024), also results in a non-empty solution subset (Theorem 2). Lastly, the climate neutrality perspective, which balances emissions with carbon removal or offset initiatives, thereby influencing total costs, likewise produces a non-empty subset of solutions (Theorem 4(a)) (Chen, Lim, Yeo, & Tseng, 2024; Chen, et al., 2022; IMF, 2008).

As illustrated in Figure 5, the subset of climate neutrality solution options intersects with the subsets of neoclassical, net-zero, and undominated options. Specifically, the climate neutrality subset shares one solution with the neoclassical subset, namely, the neoclassical option with the lowest emissions (see Corollary 3 and Theorem 4(b)). It also intersects with one option from the net-zero subset, which is the net-zero option with the lowest cost (see Corollary 4 and Theorem 4(c)). Furthermore, the climate neutrality subset is entirely contained within the undominated options subset (see Corollary 7), which comprises options not dominated by any other individual option (see Corollary 2). A key distinguishing feature of the climate neutrality subset is that its elements are not only free from individual dominance but are also not collectively dominated by any combination of two other options (see Corollary 6). An additional noteworthy property of the climate neutrality solution set is that it satisfies discrete convexity, meaning it preserves a form of convex structure in a discrete space (see Theorem 5).

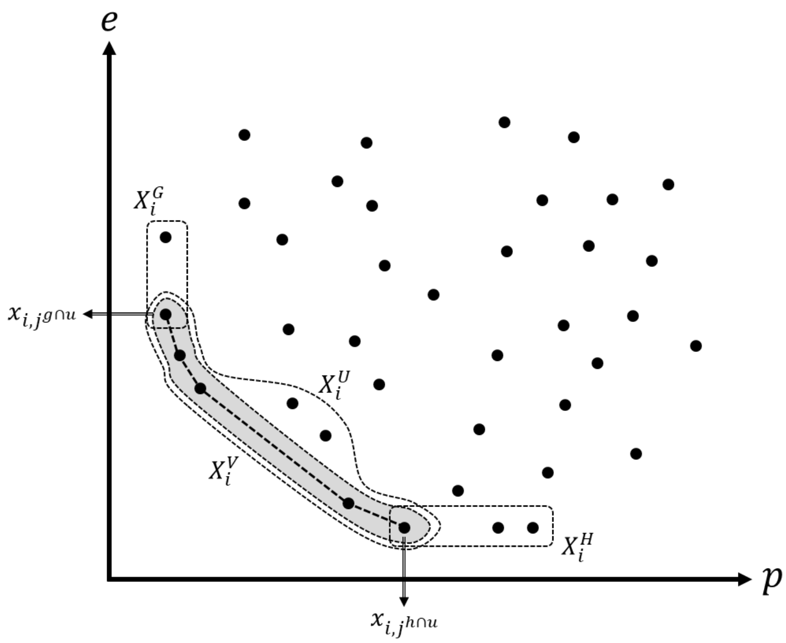

Figure 5. Illustration of the relationship between the subset of climate neutrality options and the subset of neoclassical, net-zero, and undominated options.

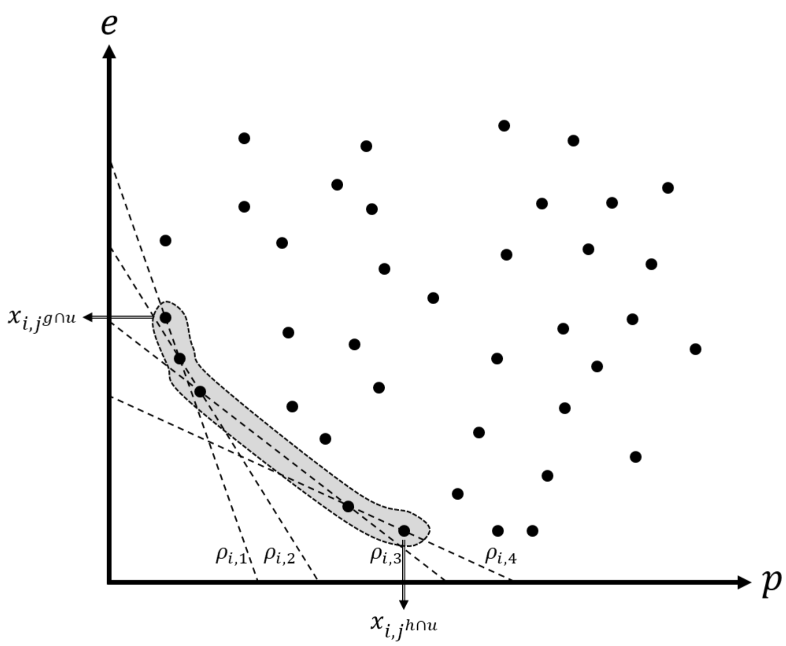

Figure 6. Effect of changes in GHG credit cost (ρ) on total option cost (φ). At a given GHG credit cost (ρ), one or two members of the subset of climate neutrality solution options have the smallest total option cost.

Each member of the climate neutrality solution subset represents the option with the lowest total cost (φ) at a given GHG credit cost (ρ), as defined in Definition 7. As illustrated in Figure 6, the option with the lowest total cost may shift from one member of the climate neutrality subset to another as ρ changes. At any given GHG credit cost (ρ) there may be one or two options within the climate neutrality subset that achieve the minimum total cost. The relationship between GHG credit cost (ρ) and the smallest total cost option ($\hat{x}$), where:

$$\hat{x}_{i,j} : \varphi_{i,j,\rho} = min_{\forall j'}\{\varphi_{i,j',\rho}\}$$

Maps from the domain set P (the set of possible GHG credit costs) to the codomain set $\hat{X}$ (the set of cost-minimizing options), i.e.,

$$P \rightarrow \hat{X}$$

However, this mapping does not satisfy the formal definition of a function (Johnsonbaugh, 2019; Bartle & Sherbert, 1999) because some values of $\rho \in P$ correspond to multiple elements in $\hat{X}$. As shown in Figure 7, the resulting relationship resembles a step function, but it fails to meet the criteria of a mathematical function due to these instances of non-uniqueness.

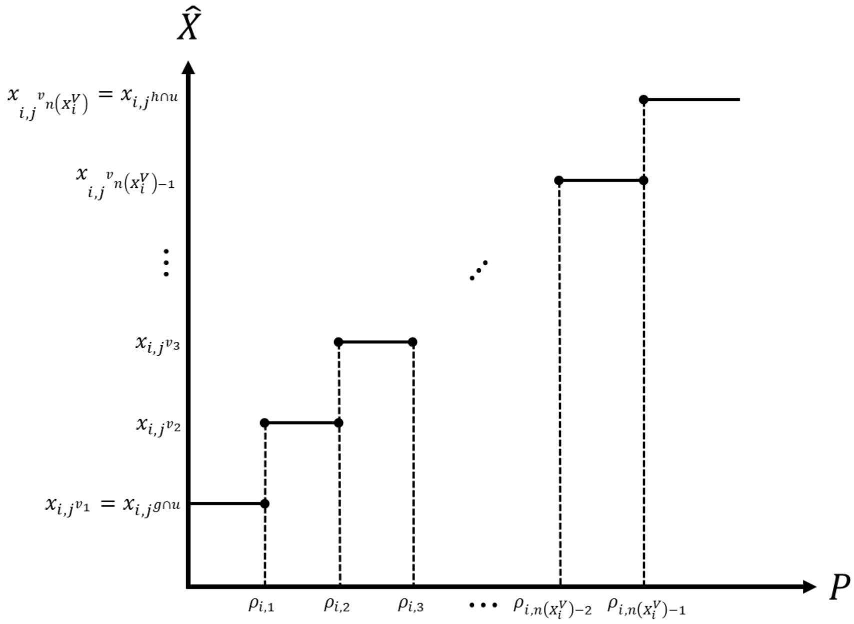

Figure 7. The relationship between GHG credit costs (ρ) and the option with the smallest total cost ($\hat{x}$).

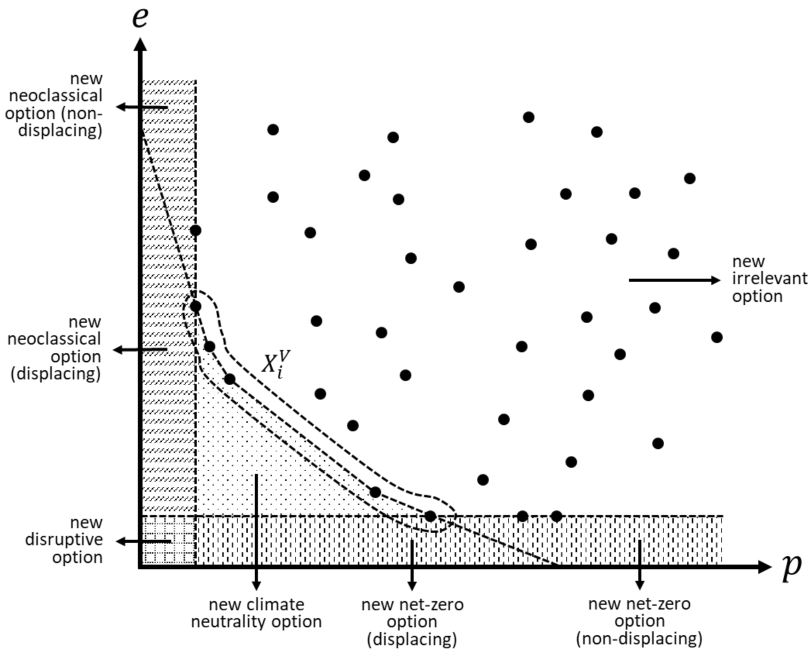

Figure 7. Categorization of new option types based on their impact on the configuration of a subset of climate neutrality options.

The emergence of a new option can have different impacts on the configuration of the climate neutrality solution subset. Broadly, there are seven distinct types of new options, categorized by their impact on this subset, as illustrated in Figure 7:

1. **New irrelevant option**: a new option that does not affect the existing subset of climate neutrality options.
2. **New neoclassical option (non-displacing):** a new neoclassical solution option that does not eliminate the previous neoclassical solution in the subset of climate neutrality options.

3. **New neoclassical option (displacing):** a new neoclassical solution option that eliminates the previous neoclassical option in the subset of climate neutrality options.
4. **New net-zero option (non-displacing):** a new net-zero solution option that does not eliminate the previous net-zero solution in the subset of climate neutrality options.
5. **New net-zero option (displacing):** a new net-zero solution option that eliminates the previous net-zero option in the subset of climate neutrality options.
6. **New climate neutrality option**: a new option that do not eliminate the previous neoclassical option and do not eliminate the previous net-zero option in the subset of climate neutrality options.
7. **New disruptive option**: a new option that eliminates all previous options in the subset of climate neutrality options.

The geometric position of the new option within the costs-emission space determines its effect that ranges from having no impact, to partially modifying the subset, to fully replacing all previous options. This analytical framework can inform broader discussions on climate strategy, particularly in relation to disruptive innovations in the climate transition (McDowall, 2018; Kivimaa, Laakso, Lonkila, & Kaljonen, 2021).

5. Conclusions

Viewing the production function as a set of discrete, substitutable options enables a structured examination of the relationship between neoclassical, net-zero, and climate neutrality perspectives. This formal model illustrates both the properties and interconnections among these three frameworks, each of which yields a non-empty set of solution options. The study formally demonstrates that a solution under the climate neutrality perspective lies between the neoclassical solution with the lowest emissions and the net-zero solution with the lowest total cost. The model not only establishes the structural relationship among the three perspectives but also highlights the unique properties of climate neutrality. Compared to neoclassical and net-zero, climate neutrality exhibits more complex characteristics, including discrete convexity, and the selection of the most optimal option is influenced by the GHG credit cost.

The relationship between GHG credit cost and the optimal solution resembles a step function, though it does not meet the formal definition of a function since some domain values correspond to multiple codomain values. In this framework, lower GHG credit costs tend to favour neoclassical solutions, while higher costs shift preference toward net-zero solutions. Thus, GHG credit cost plays a central role in shaping the transition to a low-emissions economy. Furthermore, this approach provides a basis for categorizing the impact of newly introduced climate mitigation options—classifying them as irrelevant, new neoclassical, new net-zero, new climate neutrality, or disruptive. Ultimately, conceptualizing the production function as a set of discrete, substitutable options offers a novel and rigorous lens to enrich the economic analysis of climate change.

References

Alonso-Ovalle, L. (2004). Simplification of Disjunctive Antecedents. In K. Moulton, & M. Wolf, *Proceedings of the North East Linguistic Society.* Amherst: GLSA.
Arrow, K. J., Chenery, H. B., Minhas, B. S., & Solow, R. M. (1961). Capital-Labor Substitution and Economic Efficiency. *Review of Economics and Statistics, 43*(3), 225-250. doi:10.2307/1927286

Aspromourgos, T. (1986). On The Origins of The Term 'Neoclassical'. *Cambridge Journal of Economics, 10*(3), 265-270.

Bartle, R. G., & Sherbert, D. R. (1999). *Introduction to Real Analysis* (3rd ed.). New York: John Wiley & Sons Inc.

Beals, R. K. (2019, November 20). *Global GDP will suffer at least a 3% hit by 2050 from unchecked climate change, say economists*. Retrieved from Market Watch.

Brohé, A. (2017). *The Handbook of Carbon Accounting.* New York: Routledge.

Callan, S. J., & Thomas, J. M. (2013). *Environmental Economics and Management: Theory, Policy, and Applications* (6th ed.). Mason: South-Western.

Chen, G., Lim, M. K., Yeo, W., & Tseng, M.-L. (2024). Net zero vs. carbon neutrality: supply chain management challenges and future research agenda. *International Journal of Logistics Research and Applications*, 1–36. doi:10.1080/13675567.2024.2359058

Chen, L., Msigwa, G., Yang, M., Osman, A. I., Fawzy, S., Rooney, D. W., & Yap, P.-S. (2022). Strategies to achieve a carbon neutral society: a review. *Environmental Chemistry Letters, 20*, 2277–2310. doi:10.1007/s10311-022-01435-8

Coase, R. (1960). The Problem of Social Cost. *Journal of Law and Economics, 3*, 1–44. doi:10.1086/466560

Corporate Finance Institute. (2024). *Carbon Accounting.* Vancouver: Corporate Finance Institute.

European Parliament. (2022). *EU regions in the transformation towards a climate-neutral future.* Strasbourg: European Parliament.

Hicks, J. R. (1932). *The Theory of Wages.* London: Macmillan.

ICAP. (2024). *Emissions Trading Worldwide: Status Report 2024.* Berlin: International Carbon Action Partnership.

IMF. (2008). *The Fiscal Implications of Climate Change.* Washington, DC: International Monetary Fund.

IPCC. (1996). *Climate Change 1995: Economic and Social Dimensions of Climate Change.* Cambridge and New York: Cambridge University Press.

IPCC. (2001). *Climate Change 2001: Mitigation.* Cambridge and New York: Cambridge University Press.

IPCC. (2007). *Climate Change 2007: Mitigation of Climate Change.* Cambridge and New York: Cambridge University Press.

IPCC. (2021a). *Climate Change 2021: The Physical Science Basis.* Cambridge: Cambridge University Press. doi:10.1017/9781009157896

IPCC. (2021b). *Summary For Policymakers, Climate Change 2001: Mitigation.* Cambridge and New York: Cambridge University Press.

IPCC. (2022a). *Climate Change 2022: Impacts, Adaptation and Vulnerability.* Cambridge: Cambridge University Press.

IPCC. (2022b). *Climate Change 2022: Mitigation of Climate Change.* Cambridge and New York: Cambridge University Press.

ISO. (2022). *IWA 42:2022 Net zero guidelines.* Geneva: International Organization for Standardization.

Jeudy-Hugo, S., Re, L. L., & Falduto, C. (2021). *Understanding Countries' Net-Zero Emissions Targets.* Paris: OECD and IEA.

Johnsonbaugh, R. (2019). *Discrete Mathematics* (8th ed.). New York: Pearson.

Kivimaa, P., Laakso, S., Lonkila, A., & Kaljonen, M. (2021). Moving beyond disruptive innovation: A review of disruption in sustainability transitions. *Environmental Innovation and Societal Transitions, 38*, 110-126. doi:10.1016/j.eist.2020.12.001

Lachmann, L. M. (1947). Complementarity and Substitution in the Theory of Capital. *Economica, 14*(54), 108-119. doi:10.2307/2549487

Leyton-Brown, K., & Shoham, Y. (2008). *Essentials of Game Theory: A Concise, Multidisciplinary Introduction.* San Rafael: Morgan and Claypool.

Mach, K. J., Kraan, C. M., Adger, W. N., Buhaug, H., Burke, M., Fearon, J. D., & Field, C. B. (2019). Climate as a risk factor for armed conflict. *Nature, 571*, 193–197. doi:10.1038/s41586-019-1300-6

Marshall, A. (1890). *Principles of Economics.* London: Macmillan.

McDowall, W. (2018). Disruptive innovation and energy transitions: Is Christensen's theory helpful? *Energy Research & Social Science, 37*, 243-246. doi:10.1016/j.erss.2017.10.049

McFadden, D. (1962). *Factor Substitution in The Economic Analysis of Production.* Minnesota: University of Minnesota.

Nordhaus, W. D. (2010). Economic aspects of global warming in a post-Copenhagen environment. *Proceedings of the National Academy of Sciences, 107*(26), 11721–11726.

Richter, P. M., & Schiersch, A. (2017). CO2 emission intensity and exporting: Evidence from firm-level data. *European Economic Review, 98*, 373-391. doi:10.1016/j.euroecorev.2017.07.011

Rogelj, J., Geden, O., Cowie, A., & Reisinger, A. (2021). Net-zero emissions targets are vague: three ways to fix. *Nature, 591*, 365-368. doi:10.1038/d41586-021-00662-3

SBTi. (2024, July 8). *The Corporate Net-Zero Standard*. Retrieved from Science Based Targets: https://sciencebasedtargets.org/net-zero

Simovici, D. A., & Djeraba, C. (2008). *Mathematical Tools for Data Mining: Set Theory, Partial Orders, Combinatorics.* London: Springer-Verlag.

Stavins, R. N. (2003). Experience with Market-Based Environmental Policy Instruments. In K.-G. Mäler, & J. R. Vincent, *Handbook of Environmental Economics* (Vol. I, pp. 355–435). Amsterdam: Elsevier Science. doi:10.1016/s1574-0099(03)01014-3

UN Climate Change. (2021). *Climate Neutral Now: Guidelines for Participation.* Bonn: UN Climate Change.

United Nations. (2015). *Paris Agreement.* Paris: United Nations.

United Nations. (2022). *Integrity Matters: Net-Zero Emissions Commitments of Non-State Entities.* New York: United Nations' High-Level Expert Group on the.

Veblen, T. (1919). *The Place of Science in Modern Civilisation and Other Essays.* New York: B. W. Huebsch.

WBCSD and WRI. (2015). *A Corporate Accounting and Reporting Standard: Revised Edition.* World Business Council for Sustainable Development (WBCSD) and World Resources Institute (WRI).

Wiese, H. (2021). *Advanced Microeconomics.* Leipzig: Springer Gabler.

Yüceer, Ü. (2002). Discrete convexity: convexity for functions defined on discrete spaces. *Discrete Applied Mathematics, 119*(3), 297-304. doi:10.1016/S0166-218X(01)00191-3