Transaction Process, Seigniorage Channel, and Monetary Effectiveness in Flexible Price Economy

Huang, Guangming

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Guangming Huang[1][2]

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Abstract

Embedding seigniorage and transaction process into the RBC model, this paper proposes a new monetary economy, seigniorage channeled monetary economy, briefly SCME, in which monetary shocks can affect the real variables effectively and persistently in flexible price conditions. The mechanism of the effectiveness is the resource occupation effect of money issuance, in other words, money is a public goods and new money issuance is a form of taxation. The preliminary applications of SCME have clearly explained some notable puzzles or hotly debated issues in empirical studies, such as the price puzzle, the missing liquidity effect, the best inflation rate, the negative movement of hours under a positive technology shock, and the Friedman rule. In addition, we obtained interactive pricing, origin of money market interest rate, the best money market interest rate, and the best tax rate (in other words, the best government debt level) in this paper, and there is no forward guidance puzzle in SCME. Because resource allocation in the unique equilibrium of SCME is Pareto optimal, which is starkly different from the existing theories' sub-optimal result for the monetary and fiscal economy, a profound consequence of SCME is that it is a proof of the Invisible Hand Conjecture of Adam Smith in the economy with tax and money.

Keywords

Effectiveness of Monetary Shock, Seigniorage, Transaction Side of Economy, Interactive Pricing, Nonneutrality of Inflation, Liquidity Effect, Price Puzzle, Forward Guidance Puzzle, Monetary Transmission Mechanism, Money Market Interest Rate, Friedman Rule, Reactive Monetary Policy, Neoclassical Macroeconomics, New Keynesian Economics, Invisible Hand Conjecture

E13, E3, E4, E52, E6

1. Introduction

Constructing a monetary economic model and using it to clearly explain the complex monetary and macroeconomic phenomena in the real world economy is a long-standing pursuit of economists. In the literature, there are two main approaches to embedding money into and searching for the effective role of monetary shocks in the intertemporal optimization general equilibrium context. The first, from the Neoclassical school, was launched by Sidrauski (1967), Brock (1974), and Fischer (1979), among

[1] PhD in economics. Independent economic researcher. Email: huangguangming@263.net. Mobile Phone: +86-13121302871. I acknowledge Prof. Harald Uhlig, Prof. Carl E. Walsh, Prof. Qiang Chen, Prof. Weiping Zhang, Dr. Jian Huo, Dr. Jia Song, Mr. Joseph Klunder, and many others for helping during the process of this study. All errors are solely mine. This paper is posted on http://ideas.repec.org; http://www.ssrn.com .

[2] A used name of this paper is: Seigniorage Channel and Monetary Effectiveness in Flexible Price Economy.

other writers who adopted the MIU treatment, as well as by Lucas (1982), Svensson (1985), and Lucas and Stokey (1987), among other researchers who adopted the CIA treatment. Unfortunately, after finding that monetary shocks cannot impact the real economy effectively based on an RBC economy with the CIA treatment, Cooley and Hansen (1989) concluded that "if money does have a major effect on the cyclical properties of the real economy, it must be through channels that we have not explored here" (p735). A similar result could be obtained in the flexible price quantitative DSGE economy with the money-in-utility treatment[3]. Subsequently, the search for the effective role of monetary shocks to the real economy in the neoclassical flexible price condition became dormant. [4]

The failure of the first approach, to some extent, triggered the flourishing of the other: dynamic new Keynesian macroeconomics. Yun (1996), Woodford (2003), Christiano, Eichenbaum, and Evans (2005), and many other researchers of the New Keynesian School, who sought to obtain the monetary effectiveness result in the context of rational expectation intertemporal optimization, suggested constructing a monetary economy with a sticky price mechanism. Although this school does obtain the monetary effectiveness result and is currently the dominant workhorse of monetary economics, one of the main problems with the sticky price theory is that when a part of the firms set the product price in the model economy, the product price as a whole is not derived from the first order. In other words, the pricing arrangement of the new Keynesian School is ad hoc. In addition, Del Negro, Giannoni, Paterson (2023) raised the forward guidance puzzle about this school of studies, and Chari, Kehoe, and McGrattan (2009) doubted the studies of this school as well.

Furthermore, there are four dubious treatments in both the above two schools of monetary economics:

1. Unilaterally pricing. It is common sense that product pricing is an interactive action between the demand side and the supply side. However, in existing neoclassical monetary economics, the product price is decided unilaterally by the demand side, i.e., by the household. In new Keynesian macroeconomics, it is the supply side, i.e, the firms, that have the chance of pricing the product unilaterally.

2. The government lump-sum transfer mechanism adopted in the existing neoclassical and new Keynesian monetary economics makes money issuance a resource income rather than an expense of the household, which is starkly inconsistent with the fact that money issuance is seigniorage, a kind of tax, to households.

3. Because equality between the interest rate of money and the interest rate of a bond is a necessary part of the existing neoclassical and new Keynesian literature, money has to be treated as an interest-bearing asset to the household; see Chapter 2 of Woodford (2003) for reference. Unfortunately, from the time when gold was a medium of exchange, we know money was a liquidity instrument; it does not generate any interest to the household. This fact has not changed today, and we see that central banks generally do not pay interest on the base money, and commercial banks generally do not pay interest on the demand deposit. Some central banks and commercial banks do pay some interest

[3] See Chapter 2 of Walsh (2017) for reference.

[4] The New Monetarism (see Williamson and Wright (2010) and the reference mentioned there) is another line of study regarding money in a flexible price condition. Though the point of this line of study is much different from that of this paper, a common characteristic shared by the New Monetarism and the present paper is the emphasis on the transaction side of the economy, which is absent in the mainstream dynamic general equilibrium monetary theories; this will become clear in section 3.

to the base money and demand deposit, respectively, but the interest rates of these debts are much lower than the respective interest rates of the assets in their respective balance sheets.
4. Both the existing neoclassical and new Keynesian monetary economics admit and accept the non-Pareto optimal result of the equilibrium of an economy with money and tax. This is inconsistent with the Invisible Hand Conjecture of Adam Smith, which believed the resource allocation in a market economy was Pareto optimal. Intuition also tells us that the economy with money and tax, which is created by nature, should be a perfect object; that is, the resource allocation in its equilibrium should be Pareto optimal, at least in a benchmark case.

The above problems show that the journey of modeling monetary economy is still under way. This paper provides a new trial to fix all the above problems by developing a new economy named the seigniorage channeled monetary economy, briefly SCME. It insists on the neoclassical tradition, that is, maintaining the flexibility of prices, proposes a new way, that is, the seigniorage channel, to introduce money into an RBC economy, and obtains the result that monetary shocks can sharply affect the real variables. This is a brand new conclusion in flexible price quantitative DSGE monetary economics. The readers will find that SCME is an interactive pricing economy with the transaction process; it discards the equality between the interest rate of money and the interest rate of bonds, it nests the RBC economy as a special case, and the resource allocation in the unique equilibrium of this economy is Pareto optimal. The above mentioned four dubious treatments in the existing theories will be clearly discussed when we have developed the seigniorage channeled monetary economy and more problems within the existing monetary economics will be found and discussed in this paper.

In Section 2, we first discuss the dubious design in the budget of existing neoclassical monetary economics, that is, government lump-sum transfers, and find that it is a cause of the monetary ineffectiveness result of this theory. Then, we consider the case when simply discarding the lump-sum transfers in the traditional budget, unfortunately, the ineffectiveness result still emerges. However, this new budget is the one adopted in the SCME models of this paper.
Section 3 studies the seigniorage channeled monetary economy with an exogenous monetary aggregate rule, where the effectiveness of monetary shocks under flexible price conditions is obtained. In subsection 3.1, a modified version of the RBC economy developed by King, Plosser, and Rebelo (1988) is provided, and this is the basis of the SCME in this paper. The modification here is letting households consume both private goods and public goods and allowing the latter be produced from tax. A shocking characteristic of this RBC economy with taxation is that the resource allocation in its unique equilibrium could be a Pareto optimum, a characteristic that will be maintained in SCME. A by-product here is that we can obtain the best tax rate of the economy, where the best government debt level can be obtained accrodingly.
The incorporation of the transaction process is the most essential reform of SCME. The readers will see in Section 3 that the involvement of transaction equations in the model economy has two fundamental consequences. 1. It makes the economy complete and integrated, which means that the major processes in the real world economy; that is, transaction, production, and resource allocation, are all realized in the model economy. In contrast, it will be shown in this section that the transaction part is lacking and impossible in existing neoclassical and new Keynesian monetary economics because the cash-in-advance treatment and the money-in-utility treatment are both incompatible with the equation of exchange, a result of the involving of transaction process. The effectiveness of monetary

shock in flexible price condition is the result of the combination of the equation of exchange and the seigniorage channel in the budget. The equation of exchange will be derived from the first orders. 2. In the meantime, the involvement of transaction equations makes product pricing an interactive behavior between the supply side and the demand side of the economy, which, as will be shown, is again impossible in existing dynamic general equilibrium monetary economics. The involvement of the interactive pricing in the decision process makes obtaining the Pareto optimal equilibrium of an economy much easier than the traditional way. Traditionally, because of the price-taking setting, there is no interactive pricing to coordinate the product and factor markets, and the Pareto optimum is obtained through the benevolent social planner treatment, and the equivalence between the optimum and the equilibrium is obtained through the two welfare theorems; see Debreu (1959) for reference. Note that the traditional price-taker assumption has already been discarded in the existing neoclassical and new Keynesian monetary economics, as discussed above on the unilaterily pricing of the present theories. Acutally, in any representative agent economy, agent can decide the price because she knows that she is representative, which means she knows all other agents will act the same as her.

Two other issues about SCME are discussed in detail in Section 3. The first is the mechanism of monetary effectiveness. In SCME, the monetary shock takes effect by influencing resource allocation through the seigniorage channel. The second issue is the steady-state analysis, including the optimal rate of inflation and the cost of inflation, actually, we obtain an inflation arch here, which show the optimal inflation rate. In addition, the optimal monetary aggregate growth rate will be obtained.

Section 4 studies the interest rate version of SCME. In the literature, the nominal interest rate is involved by introducing the bond into the economy. The quantity of the bond eventually degenerates to zero due to the representative household design, which implies that one cannot lend to himself or herself in equilibrium. In contrast, a money market interest rate is involved in the economy in a new way. Specifically, we find that a money market interest rate, which equals the expected growth rate of money, is inherent in the seigniorage equation. The interest rate version of SCME is subsequently obtained, and we will obtain the best money market interest rate, which is different from the Friedman rule. In addition, three more issues, which could be regarded as applications of SCME, are discussed: a) the absence of the liquidity effect in the monetary aggregate rule economy; b) the time lag in monetary policy implementation, which is found to be the cause of the hump in the impulse-response curves; and c) the price puzzle in the interest rate rule economy.

The last subsection of Section 4 studies the reactive interest rate rule, which could also be regarded as an application of SCME. An approach for choosing a reactive monetary policy is developed, and we find the respective parameters are much smaller than those suggested by Taylor (1998). In addition, the negative movement of the hours under a positive technology shock is obtained in the reactive interest rate rule case. The simulation of a reactive interest rate rule economy is provided in detail, which reproduces many of the findings in the empirical studies in the literature, and the forward guidance puzzle does not appear in this economy

Section 5 concludes this paper by summarizing the findings and initiatives of this paper. The matters that need attention and directions for further studies are mentioned as well. The findings of this paper show that it is a promising way to replicate the real-world economy by involving credit in the simple SCME of this paper, a work conducted in the companion paper, Huang (2025).

2. The Household Budget Constraint: from Government Lump Sum Transfers to Seigniorage Channel

The government lump-sum transfer in the household budget of the existing neoclassical monetary economics is a cause that prevent the effectiveness of monetary shocks. The well-accepted budget constraint of both MIU and CIA is

$$C_t + \mathrm{X}_t + \frac{M_t}{P_t} = Y_t + \Xi_t + \frac{M_{t-1}}{P_t} \qquad \text{E2.1}$$

where C, X, M, P, Y, and Ξ are consumption, investment, monetary aggregate, product price, output, and government lump-sum transfers.[5] The government transfer mechanism discussed here is the requirement extensively adopted in the literature that, in equilibrium,

$$\Xi_t = \frac{M_t}{P_t} - \frac{M_{t-1}}{P_t} \qquad \text{E2.2}$$

With E2.2, the budget constraint, E2.1, becomes

$$C_t + X_t = Y_t \qquad \text{E2.3}$$

E2.3 means that under the government lump-sum transfer mechanism E2.2, the real resource allocation is not directly connected with money. With the traditional neoclassical production function, which also lacks a position for money, it is obvious that money does not exist in either the resource allocation or the resource production process in this economy. Consequently, the channel for a monetary shock to affect real variables is substantively eliminated.

The above comment can be verified mathematically. Generally, we can only obtain the solution of the dynamic model numerically; however, we can understand the basic idea analytically and clearly with simple cases. Here, we use the one-factor production CIA economy as an example; the MIU economy can be analyzed similarly. Assume that labor, N, is the only production factor. That is, the production function is $Y_t = N_t{}^{1-\alpha}$, where $0 \le \alpha < 1$ is the parameter and $0 < N_t \le 1$. The population of the economy is assumed to be constant. Additionally, assume that the monetary aggregate policy is $M_t = Z_t^M \Theta^t M$. Z^M is the exogenous monetary supply shock, Θ is the exogenous gross growth rate of the monetary aggregate, and M is the exogenously given quantity of money. Let the present-period utility function be $U_t = \mathrm{U}(C_t, J_t)$, where $J_t = 1 - N_t$ is the leisure enjoyed by the household. The permanent utility from period t is $UU_t = \sum_{i=0}^{\infty} \beta^i E_t U_{t+i}$. β is the subjective discount rate, and E is the expectation operator. For simplicity, assume that $\mathrm{U}(C_t, 1 - N_t) = \Phi(C_t) + \Psi(N_t)$. With the undetermined coefficient method, we can obtain the relation between output and the monetary shock of this CIA economy as

$$\widehat{Y_t} = -\frac{1-\rho^M}{\Phi_{CC}C - (1-\rho^M) + (2-\frac{\beta}{\Theta}\rho^M)\frac{\Psi_{NN}N^{1+\alpha} - \alpha(1-\alpha)\frac{\Phi_C}{2-\frac{\beta}{\Theta}}}{(1-\alpha)^2}} \widehat{Z_t^M} \qquad \text{E2.4}$$

where a variable with a ^ denotes the percentage deviation of the variable from its steady state and a variable without a time indicator represents its steady-state value. Additionally, assume that $\widehat{Z_t^M} = \rho^M \widehat{Z_{t-1}^M} + \varepsilon_t^M$, where $0 \le \rho^M < 1$, and $\varepsilon^M{}_t \sim N(0, \sigma_M{}^2)$ is a white noise process.

[5] The complete form of the budget in the literature includes bonds, B, and the gross interest rate on bonds, R^B, which is

$$C_t + \mathrm{X}_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = Y_t + \Xi_t + \frac{M_{t-1}}{P_t} + \frac{R_{t-1}^B B_{t-1}}{P_t}$$

However, because B_t is set to be zero in equilibrium in the literature, we can discuss the monetary issue of MIU and CIA with the simplified version of the budget, E2.1. Because this budget is also used in New Keynesian models, the discussion here is applicable to New Keynesian macroeconomics.

When $\Phi_C>0$, $\Phi_{CC}<0$, $\Psi_{NN}<0$, which are held in the literature, together with the well-accepted values of the parameters, the denominator on the right-hand side of E2.4 is negative. Let the value of the coefficient of $\widehat{Z_t^M}$ in E2.4 be Ⓩ. We can see that the value of Ⓩ ranges from approximately 0.2 to 0 as ρ^M spans from 0 to 1. Because ρ^M is close to 1 in the literature, Ⓩ is a small positive number that is much closer to 0, from which we obtain the weak relation between output and the monetary shock. The numerical solution of the more complicated MIU and CIA economies provides similar results.

There is still another crucial problem with the government transfer arrangement. From E2.1, when $M_t>M_{t-1}$, which is generally held, the money issuance activity becomes income for the household. This is starkly inconsistent with the fact that seigniorage is an expense of the household. As mentioned in the introduction section, this treatment is also necessary to obtain the equivalence between the bond rate R^B and the money market interest rate, R, which is an indispensable part of the existing neoclassical monetary economics and dynamic new Keynesian economics. To make thing clear, some explanations are needed here: E2.1 do put $M_t - M_{t-1}$ at the expenditure side of the budget, so it seems that a rise in $M_t - M_{t-1}$ will force the household to reduce consumption or investment. However, this is not the case. The problem of E2.1 is that, according to E2.2, the increase in $M_t - M_{t-1}$ will be offset by the increase in the government lump-sum transfer in the income side of the budget, that is to say, a rise in $M_t - M_{t-1}$ will not lead to a reduction of household consumption or investment, and it implies an increase in government lump-sum transfer, which is an income of the household. We can even add government expenditure on the left side of E2.1; under this case, the government lump-sum transfer on the right side will increase in the same amount to make the two sides of E2.1 equal. The source of government lump-sum transfer is mysterious and dubious because the government is a resource consumer and cannot produce any new resource by itself.

A reasonable treatment is discarding Ξ in E2.1, so we have the following budget,

$$C_t + \mathrm{X}_t + \frac{M_t}{P_t} = Y_t + \frac{M_{t-1}}{P_t} \qquad \text{E2.5}$$

Note the term $\frac{M_t}{P_t} - \frac{M_{t-1}}{P_t}$, which is the seigniorage, is embedded in E2.5. However, the ineffectiveness result is retained under this economy because adjusting the price sufficiently is an efficient way for a household to encounter the monetary shock in this flexible price economy. In subsection 3.3, we will see that the deep reason for the ineffectiveness result of this type of economy is its absence of the transaction process.

3. Seigniorage Channeled Monetary Economy

3.1 A Pareto Optimal Tax Economy

Before diving into the study, let us prepare the economy on which the SCME is based. It is the RBC economy in King, Plosser, and Rebelo (1988) with the following modifications:

a. Let the household consume private goods, C, and public goods, G. Then, the period utility function takes the following form:

$$U_t = \mathrm{U}(C_t, G_t, J_t) \qquad \text{E3.1}$$

b. Assume that the public goods are produced from tax by the government sector and that, simply,

$$G_t = T_t \qquad \text{E3.2}$$

c. With the flat-rate income tax rate τ, it is not difficult to obtain

$$T_t = \tau Y_t \qquad \text{E3.3}$$

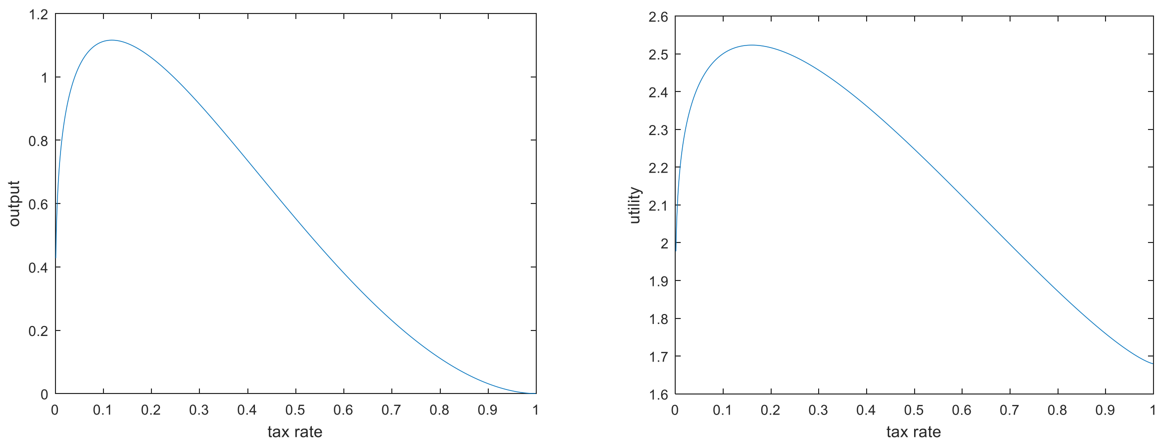
Consequently, the budget constraint of the household in this modified version of the RBC economy is

$$C_t + X_t + T_t = Y_t \qquad \text{E3.4}$$

With the above modifications, the only intertemporal optimizer in this economy, the household, is subject to two constraints: the budget constraint E3.4 and the public goods constraint E3.2. When the production and utility functions meet the requirements in Stokey, Lucas, and Prescott (1989), we can obtain a unique equilibrium for this version of the RBC economy.

An outstanding characteristic of this modified economy is that when public goods are transformed from the tax by the government employees as in E3.2 and enter the utility of the household as in E3.1, tax is consumed by households in the form of public goods, and it no longer distorts the economy. No resource is wasted, and the resource allocation in the unique equilibrium of this RBC economy with tax is Pareto optimal. In contrast, in the traditional treatment, taxation introduces distortion because households cannot consume the portion of output that constituting government spending.

A by-product here is that we can obtain the best tax rate, which is shown in the following Figure 1:



(a) (b)

Figure 1

In the economy of Figure 1, the period t utility function we adopt is $U_t = \frac{(C_t{}^{\chi} G_t{}^{1-\chi})^{1-\eta}}{1-\eta} + \xi(1 - N_t)$, the production function is $Y_t = K_{t-1}^{\alpha} N_t{}^{1-\alpha}$, and the capital reproduction function is $K_t = (1 - \delta)K_{t-1} + X_t$, where α=0.36, β=0.97, δ=0.025, χ=0.8, η=0.5 are adopted, the value of ξ is adopted to ensure the steady state value of N=1/3 when τ is 0.15, and the value of χ is close to the relative value between C and G in the steady state, which can be obtained from the steady state condition of the model.

With this modified version of the RBC economy, we begin the study of SCME.

3.2 The Model

In this economy, money is a public goods and new money issuance is a form of taxation. The representative household, which owns the firm and makes intertemporal consumption and investment decisions, weighs consumption, public goods, and leisure in its utility. The government in this economy includes both monetary and fiscal authorities. The government is not a utility maximizer; it simply implements monetary and fiscal policies according to the respective rules and provides public goods, such as national defense and monetary services, to the household. The arrangement of this economy is stated in parts A, B, and C below, the unique Pareto optimal equilibrium is obtained in part D, and a comment on the MIU and CIA economies is provided in part E.

A. Monetary Issues and Transactions

a. *Monetary Policy*. We adopt a simple exogenous monetary aggregate rule in this section:

$$\mathrm{M_t} = Z_t^M \Theta^t M \qquad \text{E3.5}$$

where Θ is the constant gross growth rate of money and M is the amount of the monetary aggregate, which is exogenously given. The log form of the monetary shock Z^M is a stationary first-order autoregressive process:

$$\ln \mathrm{Z_t^M} = (1-\rho^{\mathrm{M}}) \ln \mathrm{Z^M} + \rho^{\mathrm{M}} \ln \mathrm{Z_{t-1}^M} \qquad \text{E3.6}$$

where $0<\rho^M<1$. The steady state of Z^M is set to unity. The white noise shock $\varepsilon^M{}_t \sim N(0, \sigma_M{}^2)$ is added to the log-linear form of E3.6, so we have $\widehat{Z_t^M} = \rho^M \widehat{Z_{t-1}^M} + \varepsilon_t^M$. At the beginning of each period, ε^M is realized.

As will be clear from the transaction process we will discuss soon, the firm holding M_{t-1} at the end of period t-1, the newly issued money in period **t** is $M_t - M_{t-1,}$ and M_t is the stock of money used in mediating the transactions in period **t**.

b. *Renting*. The firm rents labor and capital from the household, and the household receives money from the firm, which will be used to purchase products later in the same period. Therefore, we have

$$W_t^K K_{t-1} = P_t^M M_t^K \qquad \text{E3.7}$$

$$W_t^N N_t = P_t^M M_t^N \qquad \text{E3.8}$$

where K and N are capital and working hours, and W^K and W^N are their respective rent rates. As the numeraire, the price of money, $P^M{}_{t,}$ is constant and normalized to unity, so we have $P^M{}_t \equiv 1$. Note that we will not show P^M or $P^M{}_t$ again in the rest of the paper. M^K and M^N are the money used in the rental of capital and labor hours, respectively, and we have

$$M_t^K + M_t^N = M_t \qquad \text{E3.9}$$

which means that information is complete, and the quantity of money is known to everyone in the economy.

c. *Purchasing*. After production, the household purchases products with money. Thus, we have

$$M_t^K = P_t Y_t^K \qquad \text{E3.10}$$

$$M_t^N = P_t Y_t^N \qquad \text{E3.11}$$

where Y^K and Y^N are products purchased with M^K and M^N, respectively, and P is the price of product Y.

E3.7-E3.11 show that all transactions are mediated by M_t , and after the transactions, M_t is held by the firm at the end of period t.

B. Supply Side of the Economy

d. *Production*. The production function in this paper is the standard constant returns to scale Cobb–Douglass function:

$$Y_t = Z_t^T K_{t-1}^{\alpha} (A_t N_t)^{1-\alpha} \qquad \text{E3.12}$$

where α is the share of capital in production, and the growth rate of technology is exogenously given. That is, A_t/A_{t-1} is set to be a constant Γ.

Similar to the monetary shock, the log form of the technology shock, Z^T, is a stationary first-order autoregressive process:

$$\ln Z_t^T = (1-\rho^T)\ln Z^T + \rho^T \ln Z_{t-1}^T \qquad \text{E3.13}$$

where $0<\rho^T<1$. The steady state of Z^T is set to unity. The white noise process $\varepsilon^T_t \sim N(0, \sigma_T^2)$ is added to the log-linear form of E3.13, so we have $\widehat{Z_t^T} = \rho^T \widehat{Z_{t-1}^T} + \varepsilon_t^T$. At the beginning of each period, ε^T is realized. ε^T and ε^M are independent of one another in this paper.

e. *Firm Profit Maximization*. The firm maximizes its profit, D, with

$$\mathrm{D}_t = Y_t - Y_t^K - Y_t^N \qquad \text{E3.14}$$

From the maximization process of the firm, we have

$$W_t^K K_{t-1} = \alpha P_t Y_t \qquad \text{E3.15}$$

$$W_t^N N_t = (1-\alpha) P_t Y_t \qquad \text{E3.16}$$

In addition, we can obtain $Y_t^K = \alpha Y_t, Y_t^N = (1-\alpha)Y_t, and\ \mathrm{D}_t = 0$ from E3.7, E3.8, E3.10, E3.11, E3.15, and E3.16.

f. *Equation of Exchange*. From E3.7-E3.11 and E3.15-E3.16, we obtain the equation of exchange:

$$M_t = P_t Y_t \qquad \text{E3.17}$$

C. Demand Side of the Economy

g. *Budget Constraint with Seigniorage and Taxation*. Under the representative agency arrangement of the economy, the household is the owner of the firm, correspondingly, the seigniorage is paid by the household with its income. The income of the household, that is, $Y_t^K + Y_t^N + D_t$, equals Y_t, which is obvious from parts A and B of this subsection.

As to the seigniorage, we have,

$$S_t = \frac{M_t - M_{t-1}}{\mathrm{P}_t} \qquad \text{E3.18}$$

Concerning the tax, T, the simple flat-rate income tax is adopted in this paper, and we have

$$T_t = \tau Y_t \qquad \text{E3.19}$$

where **τ** is the tax rate.[6]

Consequently, we obtain the total tax, TT, of this economy as

$$TT_t = T_t + S_t \qquad \text{E3.20}$$

And we obtain the representative household's budget constraint as

$$C_t + X_t + TT_t = Y_t \qquad \text{E3.21}$$

with the left-hand side representing the household's resource allocation and the right-hand side representing his or her income.

h. *Public Goods Production*. In this economy, the household consumes public goods, and the public goods are produced by the government sector with taxes. In this paper, we simply let

[6] Here, we assume that the fiscal authority directly collects products, as a tax, from the household. It is not difficult to let the household pay the tax in monetary form and the fiscal authority purchase the equivalent quantity of products from the firm.

$$G_t = TT_t. \qquad \text{E3.22}$$

i. *Capital Accumulation*. Capital is held by the household, and its accumulaton is canonical:

$$K_t = (1-\delta)K_{t-1} + X_t \qquad \text{E3.23}$$

where δ is the depreciation rate of capital.

j. *Utility Function*. Assume the periodical utility of the household to be

$$U_t = \mathrm{U}(\frac{C_t}{A_t}, \frac{G_t}{A_t}, J_t) \qquad \text{E3.24}$$

where J_t=1-N_t is the leisure in period **t**. That is, the maximum number of hours is normalized to unity. E3.24 means that the household enjoys private goods, C_t, public goods, G_t, and leisure. The modification of C_t and G_t with the growth factor A_t means that the household cares about growth-adjusted consumption and public goods. This treatment helps obtain the detrended form of the economy easily. The gross growth rate of the population in this paper is assumed to be unity.

The permanent utility of households starting from period t, UU_t, is

$$UU_t = maxE_t \sum_{i=0}^{\infty} \beta^i U_{t+i} \qquad \text{E3.25}$$

where β is the subjective utility discount rate of the household and E is the expectation operator.

D. The Unique Pareto Optimal Equilibrium

E3.5-E3.25 consist of the basic arrangement of the SCME. In this economy, the household maximizes its permanent utility, E3.25, subject to the budget constraint, E3.21, and the public goods constraint, E3.22. In addition, the equation of exchange is already known by the household at the moment of decision, and it represents the household's third constraint. Let $\dot{Y}_t = \frac{Y_t}{A_t}, \dot{C}_t = \frac{C_t}{A_t}, \dot{X}_t = \frac{X_t}{A_t}, \dot{G}_t = \frac{G_t}{A_t}, \dot{S}_t = \frac{S_t}{A_t}, \dot{T}_t = \frac{T_t}{A_t}, \dot{TT}_t = \frac{TT_t}{A_t}, \dot{K_{t-1}} = \frac{K_{t-1}}{A_t}, \dot{M}_t = \frac{M_t}{\Theta^t}, \dot{P}_t = \frac{P_t}{\frac{\Theta^t}{A_t}}$, where a letter with a ·above it stands for its detrended form, we can obtain the detrended form of the utility function, the constraints of the household, the production function, and monetary policy as follows:

$$UU_t = maxE_t \sum_{i=0}^{\infty} \beta^i \mathrm{U}(\dot{C_{t+\iota}}, \dot{G_{t+\iota}}, J_t) \qquad \text{E3.26}$$

$$\dot{C}_t + \Gamma\dot{K}_t - (1-\delta)\dot{K_{t-1}} + \tau\dot{Y}_t + \frac{\theta\dot{M}_t - \dot{M_{t-1}}}{\theta\dot{P}_t} = \dot{Y}_t \qquad \text{E3.27}$$

$$\dot{G}_t = \tau\dot{Y}_t + \frac{\theta\dot{M}_t - \dot{M_{t-1}}}{\theta\dot{P}_t} \qquad \text{E3.28}$$

$$\dot{M}_t = \dot{P}_t\dot{Y}_t \qquad \text{E3.29}$$

$$\dot{Y}_t = Z_t^T \dot{K_{t-1}}^{\alpha} N_t^{1-\alpha} \qquad \text{E3.30}$$

$$\dot{M}_t = Z_t^M M \qquad \text{E3.31}$$

E3.26-E3.31 is a recursive dynamic optimization problem with two endogenous state variables, $\dot{K_{t-1}}$ and $\dot{M_{t-1}}$, and two exogenous shocks, Z^T_t and Z^M_t, and we can obtain the Bellman equation subject to the budget constraint, E3.27, public goods constraint, E3.28, and equation of exchange constraint, E3.29. Although with a big difference in economic sense, this economy is mathematically the same as the RBC economy of Stokey, Lucas, and Prescott (1989) and the fiscal economy of subsection 3.1, accordingly, the unique equilibrium of this economy can be obtained when the utility function E3.26 meet the required conditions of Stokey, Lucas, and Prescott (1989).

Note that because the firm's optimal behavior is embedded in the exchange equation, the optimal choice of the firm is simultaneously and uniquely determined when the household's problem is uniquely solved.

Obviously, the following two characteristics differentiate this unique equilibrium of SCME from those in the literature:

1. Because tax and seigniorage are consumed by households in the form of public goods, resource allocation in the unique equilibrium of SCME is Pareto optimal. Note that although there is cost, that is, seigniorage, in operating the monetary system, this cost is not a distortion or a deadweight loss. This cost is used to produce the monetary service, which is ultimately consumed by the household. No resource is wasted in this economy. When cost of maintaining the monetary system turns to be less than the amount of seigniorage, to obtain the same quantity of public goods, there can be an equivalent deduction of the tax, T. In other words, the improvement of the monetary regime leads to a Pareto improvement of the economy.
2. Because of the involvement of the transaction equations, prices (product price, wage rate, and rent rate) become an endogenous component of the optimization process, and we obtain the equilibrium prices in the optimization process directly. In contrast, in the literature, the connection between the Pareto optimum and the competitive equilibrium with a set of prices relies on the two welfare theorems; see part IV of Stokey, Lucas, and Prescott (1989) and references mentioned there.

With the above two characteristics, the existence of the unique Pareto optimal equilibrium of SCME is actually a proof, in an economy with money and tax, of the Invisible Hand Conjecture of Adam Smith, that is, the resource allocation in the market economy is Pareto optimal.

Another interesting aspect of SCME is that it embeds the RBC economy as a special case. With $\Theta=1$, which means that seigniorage degenerates to zero, and the monetary shock neglected, we obtain the modified version of the RBC economy described in subsection 3.1, which can be further simplified to the RBC economy of King, Plosser, and Rebelo (1988).

E. Comparison with MIU and CIA economies

Now let us compare SCME with the existing MIU and CIA economies to deepen our understanding of each of these economies. Two aspects are worth mentioning:

a. The involvement of the transaction equations, E3.7-E3.11, makes SCME a complete and integrated system that consists of all processes of the real-world economy: transaction, production, consumption, and investment. In contrast, the transaction part is lacking in existing dynamic general equilibrium monetary economics. Furthermore, it is actually impossible to embed the transaction part into existing theories. The reason is that in the CIA economy, when the goods are divided into cash goods, the transaction of which needs to be mediated by money, and credit goods, the transaction of which does not need to be mediated by money, the cash-in-advance constraint, $M_t \leq P_tC_t$, is directly inconsistent with the equation of exchange, $M_t = P_tY_t$, which is the outcome of the transaction process and firm behavior, except the case of $C_t = Y_t$, which is obviously unreasonable. In the MIU economy, the magnitude of the real balance, M_t/P_t, in the utility seems too large in scale because we can obtain $M_t/P_t = Y_t$ from the equation of exchange, which is obtained from the transaction equations and the behavior of the firm, and a real balance at the magnitude of Y_t is too large compared with the magnitude of C_t in the utility. Factually, from E3.18 and the equation of exchange E3.17, we have

$S_t = \frac{M_t - M_{t-1}}{M_t} Y_t$, which shows S_t is much less than Y_t. In steady state, $S = \frac{\Theta - 1}{\Theta} Y$, which means S≈0.015Y when Θ is about 1.015 in quarterly term as in the economic history of the US. Note that from E3.20 and E3.22, S actually enters the utility as a part of G in SCME.

b. Concerning the pricing mechanism, the involvement of the transaction equations in the SCME makes the decision of product pricing an interactive activity between the firm and the household. Both the decision of the firm and the household are based on the transaction process. In contrast, as mentioned in the introduction section, the product price is solely decided by the demand side in the existing neoclassical monetary economics and solely decided by the supply side in New Keynesian economics. In addition, from the incompatibility between the cash-in-advance treatment/the money-in-utility treatment and the equation of exchange we obtained above, interactive pricing is correspondingly impossible in the existing theories.

3.3 Mechanism of Monetary Effectiveness in SCME

Before discussing the mechanism of monetary effectiveness in SCME, let us simulate the model economy of the above subsection, briefly Model 3.2. Here, we need a concrete form of the utility function and parameter values. To ensure robustness, well-accepted functional forms and parameter values are adopted in this paper. In particular, we assume the following utility function[7]:

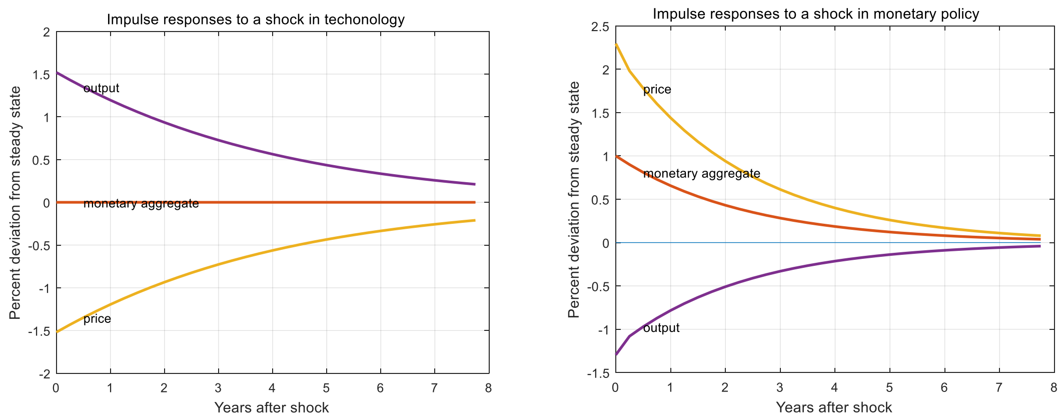
$$U_t = \frac{(\dot{C_t}^{\chi} \dot{G_t}^{1-\chi})^{1-\eta}}{1-\eta} + \xi \frac{(1-N_t)^{1-\eta N}}{1-\eta N} \qquad \text{E3.32}$$

where χ, η, and ηN are the respective coefficients, and ξ is the balance parameter, which will help in obtaining a reasonable steady-state value for hours in the simulation. Concerning the value of the parameters, let α=0.36, β=0.97, δ=0.025, Θ=1.015, Γ=1.0075, τ=0.17, η=0.5, ηN=0.5, and χ=0.8. The subjective discount rate, β, which is 0.97, is adopted to ensure that the steady-state C/Y, X/Y, T/Y, and S/Y ratios are close to those in the everyday economy, which are 0.64, 0.18, 0.17, and 0.015, respectively, in this model. The value of ξ is set to ensure that the steady-state value of hours be 1/3. Regarding the parameters in the shocks, let ρ^T=0.9, ρ^M=0.9, σ^T=0.7%, and σ^M=0.7%, which are extensively adopted in the literature.

The whole system of the simulated model is provided in Appendix A. By log-linearizing the model around its steady state. Figure 2 provides the impulse response of output and price of this SCME under a one-percent positive technology shock and monetary shock.[8] Panel (a) shows that the responses of output and price are approximately plus and minus 1.5 percent, respectively, in the first period of a technology shock, and Panel (b) shows that the responses are approximately minus 1.3 percent in output and plus 2.3 percent in price in the first period of the monetary shock, and the monetary effectiveness is persistent. Note that the amount of the movements of output and price is consistent with the log-linear form of the equation of exchange E3.29, $\hat{M} = \hat{P} + \hat{Y}$. During period t, $\hat{M} = 0$ in the technology shock case and $\hat{M} = 1$ in the monetary shock case, which is shown in Panels (a) and (b) of Figure 2, respectively. The negative output effect following a positive monetary aggregate shock is consistent with the findings of Eichenbaum (1992) and Leeper and Gordon (1992). More simulation results of this economy will be provided later in this subsection.

[7] Similar results can be obtained using logarithmic utility.

[8] The toolkit of Uhlig (1999) is used in simulation in this paper.



(a) (b)

Figure 2

The strong monetary effectiveness and persistence in the SCME originate from the cooperation of the seigniorage equation E3.18 and the equation of exchange E3.17, with which we obtain

$$S_t = \frac{M_t - M_{t-1}}{M_t} Y_t \qquad \text{E3.33}$$

E3.33 means M_t is essentially embedded into the budget E3.21, which in turn explicitly shows that a monetary shock can lead to resource reallocation.
In particular, from the detrended form of E3.33, we can obtain the following log-linear form:

$$\widehat{SOY_t} = \frac{1}{\Theta - 1}(\widehat{\dot{M}_t} - \widehat{\dot{M}_{t-1}}) \qquad \text{E3.34}$$

where $SOY_t = \frac{\dot{S}_t}{\dot{Y}_t}$. E3.34 means that a one-percent positive movement in $\dot{M}$ in period t, which is triggered by a one-percent monetary shock Z^M, see E3.31, will lead to a 66.67 percent change in $\frac{\dot{S}_t}{\dot{Y}_t}$ when Θ=1.015. From the detrended from of the budget constraint E3.21, we can obtain

$$\frac{\dot{C}}{\dot{Y}}\widehat{COY_t} + \frac{\dot{X}}{\dot{Y}}\widehat{XOY_t} + \frac{\dot{S}}{\dot{Y}}\widehat{SOY_t} = 0 \qquad \text{E3.35}$$

where $COY_t = \frac{\dot{C}_t}{\dot{Y}_t}, XOY_t = \frac{\dot{X}_t}{\dot{Y}_t}$. With $\frac{\dot{C}}{\dot{Y}} = 0.64, \frac{\dot{X}}{\dot{Y}} = 0.18$, and $\frac{\dot{S}}{\dot{Y}} = 0.015$ in the above-simulated economy, a 66.67 percent movement in SOY will lead to an approximately 1 percent change in the last term on the left-hand side of E3.35, which means that a one-percent monetary shock will trigger an approximately minus 1 percent movement in both COY and XOY. This implies a significant impact of a monetary shock on the real output, consumption, and investment variables.

The above analysis is elucidated when we use a simple case to obtain an analytical solution. In particular, let us study the case of a one-factor production function with no growth, as we did in reviewing the MIU and CIA economies in section 2. Here, let capital be the only factor, and we have $Y_t = K_{t-1}^{\alpha}$, where α=0.36. In addition, we neglect taxes and public goods, which means that the period utility function is $U_t = \mathrm{U}(C_t)$, and we have the budget constraint as $C_t + K_t + S_t = Y_t$, where δ=1 is assumed. There are no changes in the seigniorage or monetary policy functions. That is, E3.18 and E3.5

are maintained. The equation of exchange, in this case, is $M_t = \alpha P_t Y_t$. In this simple SCME, the representative household is subject to the budget constraint and the equation of exchange constraint.
When there is a one-percent positive movement in the monetary shock, there will be a 66.7 percent increase in $\widehat{S_t}$ from the following log-linear form of the seigniorage equation when Θ=1.015:

$$(1-\frac{1}{\Theta})\widehat{S_t} = (1-\frac{1}{\Theta})\widehat{Y_t} - \frac{1}{\Theta}(\widehat{M_{t-1}} - \widehat{M_t}) \qquad \text{E3.36}$$

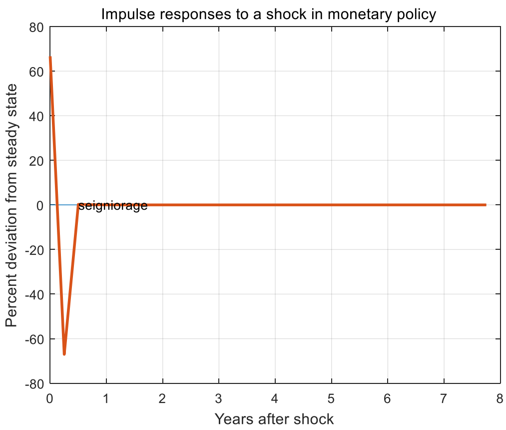
Note that we have $\widehat{Y_t} = 0$ from the production function. In addition, from the budget constraint, we have

$$C\widehat{C_t} + \mathrm{K}\widehat{K_t} + S\widehat{S_t} = Y\widehat{Y_t} \qquad \text{E3.37}$$

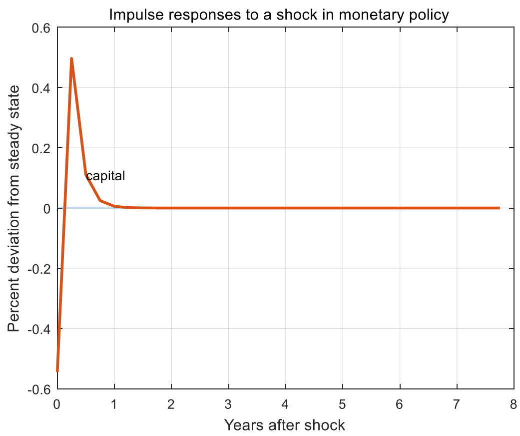
which means that if the household does not change its capital investment, that is, it keeps $\widehat{K_t} = 0$, then there will be a -1.51 percent decrease in $\widehat{C_t}$. According to the utility function, a decrease in consumption means reduced utility. However, from E3.37, it is possible to improve utility if the household decreases capital investment. That is, let $\widehat{K_t} < 0$. The result depends on the comparison between two opposite effects induced by the decrease in capital investment: the improvement in consumption in period t and the possible worsening of consumption in the following periods; the latter is possible because the decline in capital investment in this period will lead to reduced production in the next period. We can obtain the exact result with the undetermined coefficient method. Although we can obtain the analytical form of the recursive equilibrium laws between each variable and the endogenous state variables and the exogenous monetary shock by pencil and paper, it remains very complex even in this simple case. The strategy we adopt here is to provide the movement equation for capital investment solved by a personal computer directly below, where $U_t = \frac{{C_t}^{1-\eta}}{1-\eta}$, η=0.5, and ρ^M=0 are adopted:

$$\widehat{K_t} = 0.22 * \widehat{K_{t-1}} + 0.62 * \widehat{M_{t-1}} - 0.54 * \widehat{Z_t^M} \qquad \text{E3.38}$$

E3.38 means that given a one-percent positive movement in Z^M_t, the household's optimal choice is to decrease capital investment by 0.54 percent from its steady-state level. This decrease in $\widehat{K_t}$ implies, from E3.37, a 1.24 percent decrease in $\widehat{C_t}$, which will lead to greater utility compared with the 1.51 percent decrease in $\widehat{C_t}$ in the benchmark case with $\widehat{K_t} = 0$. The decreased $\widehat{K_t}$ leads to a 0.19 percent decrease in output in the next period, which is evident from the production function. Figure 3 below shows the variations in the main variables in this economy. The economy quickly returns to its steady state because the monetary shock and the seigniorage effect disappear quickly in the case of ρ^M=0. An interesting point here is that, contrary to that we expected above, capital investment increases rather than decreases because of the decrease in seigniorage in period t+1.
Compared with the technology shock, which influences the economy through resource production, this simple case shows that monetary shock takes effect through resource reallocation.



(a)



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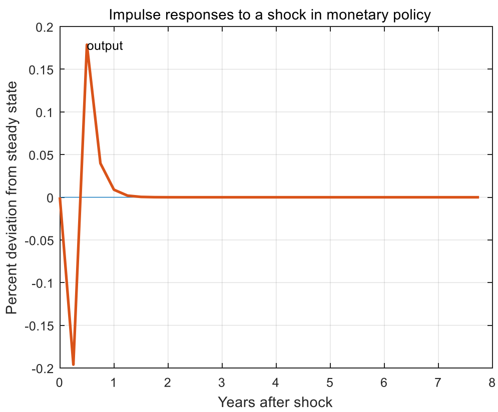

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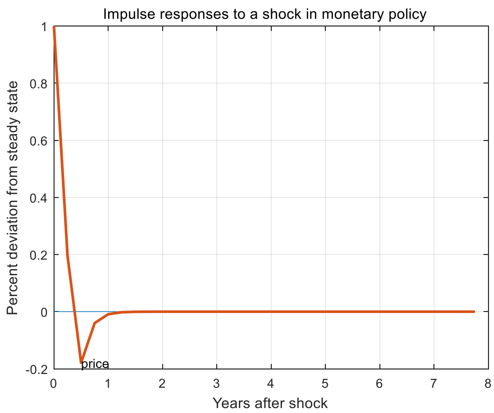

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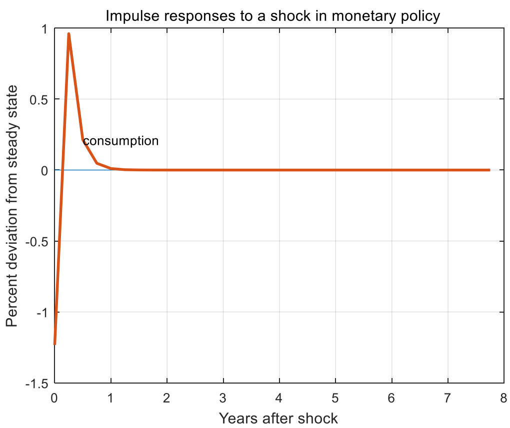

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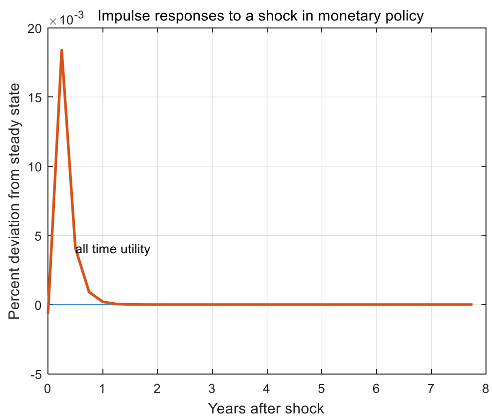


(f)

Figure 3

In the above simple case, we can obtain the precise values of the impact of the monetary shock on all variables by hand. For models such as Model 3.2, the mechanism is similar, but we can only obtain the result numerically. In Model 3.2, as shown in Figure 4 below, a one-percent positive monetary shock leads to a 66.7 percent increase in seigniorage in period **t**. It is the best response for a household in period t to increase consumption by 1.7 percent, decrease investment by 18 percent, which implies a capital decrease of approximately 0.5 percent, and decrease hours worked by approximately 2.2

percent. As a result, the household's permanent utility in period t increases by approximately 0.1 percent. In the resource reallocation process, the total tax increases by approximately 4 percent in the first period. The movements of output and price are shown in Panel (b) of Figure 2.

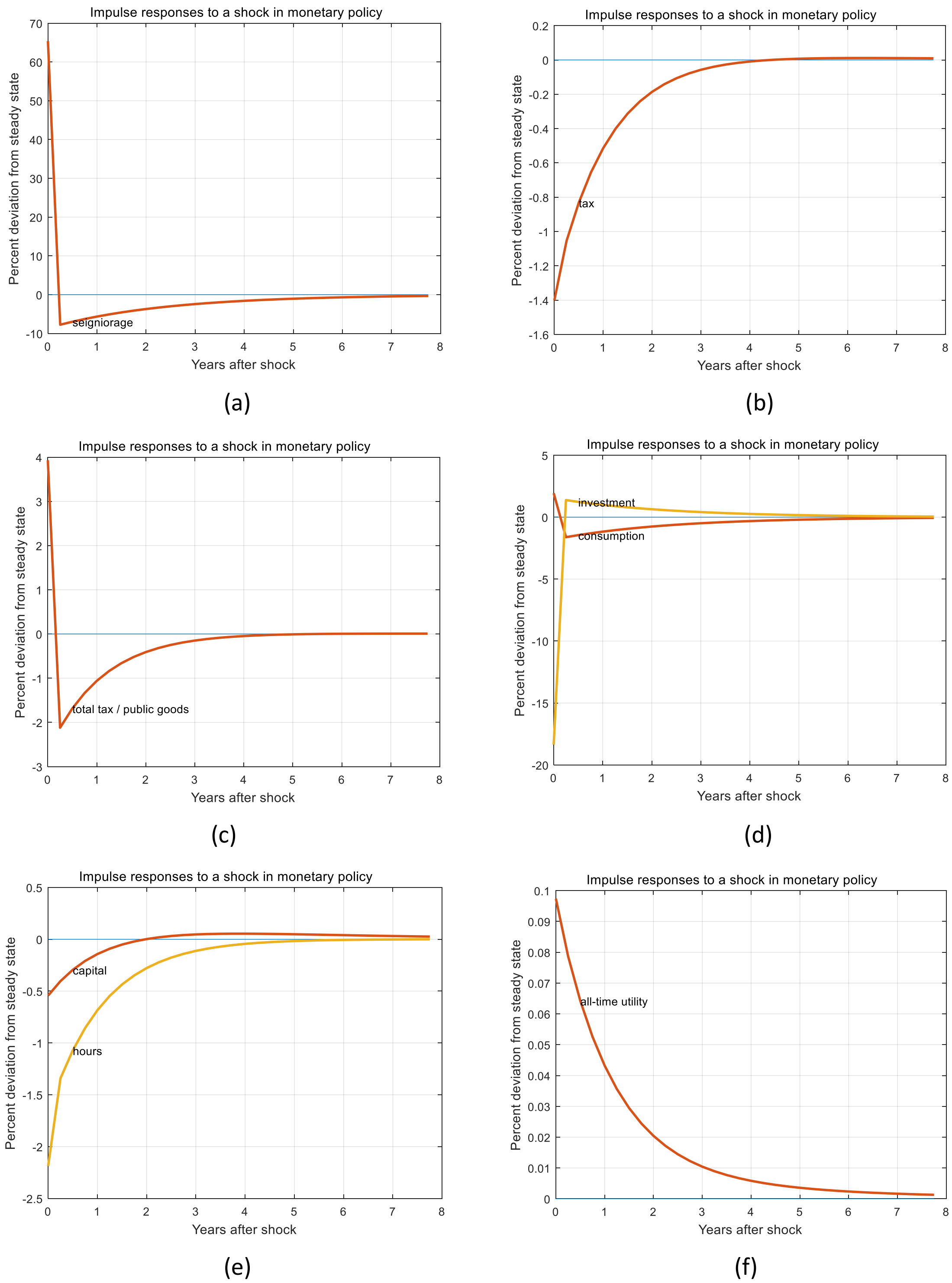


Figure 4

The importance and indispensability of the transaction part can be further understood when we go back to the economy with seigniorage we discusseded at the end of section 2. The huge difference between that economy, where monetary shock is ineffective, and SCME, where monetary shock is effective, shows that the transaction side, from which we obtained the equation of exchange, is a crucial part of the economy.

3.4 Steady-state Analysis: Nonneutrality of Growth Rate of Money and Inflation

Above, we studied the impact of a monetary shock on the economy. Now, let us consider the impact of money in the steady state, which has been another long-standing topic in monetary economics. Compared with the neutrality and superneutrality results in some of the studies of traditional neoclassical monetary economics, in SCME, the change in the monetary aggregate in a steady state is neutral. Nevertheless, the change in the growth rate of money and the inflation rate is nonneutral.

The neutrality of the change in the monetary aggregate is evident because it is the growth rate of money, rather than the stock of the monetary aggregate, that appears in the steady state of the economy. In other words, in steady state, the change in the stock of monetary aggregate will be entirely absorbed by the change in the price level.

The appearance of Θ in the steady state leads to the non-neutrality result of the growth rate of money in the SCME. In addition, we can obtain the non-neutrality of inflation with the following treatment: Dividing the equation of exchange of period t by that of period t-1, $\frac{M_t}{M_{t-1}} = \frac{P_t}{P_{t-1}}\frac{Y_t}{Y_{t-1}}$, we obtain, in steady state,

$$\Theta = \Pi\Gamma \qquad \text{E3.39}$$

where Π and Γ are the steady-state value of gross inflation and the gross economic growth rate, respectively. When replacing Θ in the steady-state equations of the household with E3.39, the non-neutrality of inflation appears.

Although it is difficult to obtain the analytical solution of the relation between output and inflation in economies such as Model 3.2, we can obtain the idea of quantifying the non-neutrality of the money growth rate and inflation with the simple economy described in Subsection 3.3 above. From its steady state, we obtain

$$\mathrm{Y} = (\frac{\Theta}{\beta^2\alpha})^{\frac{\alpha}{\alpha-1}} = (\frac{\Pi\Gamma}{\beta^2\alpha})^{\frac{\alpha}{\alpha-1}} \qquad \text{E3.40}$$

E3.40 is the output-money growth rate curve, briefly YΘC, and output-inflation curve, brief YIC, of the simple economy. Regarding the optimal rate of inflation, when the quantity of output is used as the criterion, from E3.40, we have

$$\frac{dY}{d\Pi} = \frac{\alpha}{\alpha-1}(\frac{\Pi\Gamma}{\beta^2\alpha})^{\frac{1}{\alpha-1}}\frac{\Gamma}{\beta^2\alpha} \qquad \text{E3.41}$$

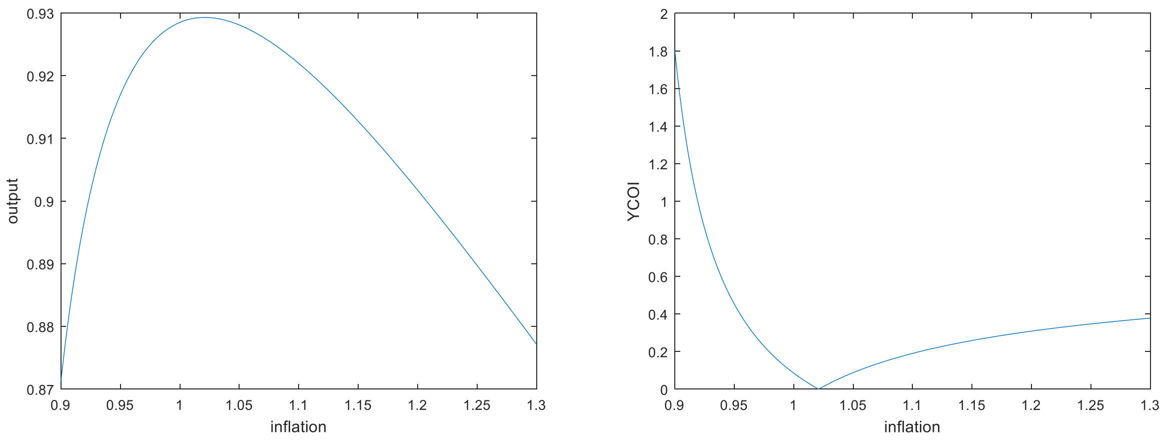
In addition, we can obtain the output cost of inflation, briefly YCOI, from $\left|\frac{\mathrm{d}\ln Y}{\mathrm{d}\ln\Pi}\right| = \left|\frac{\Pi}{Y}\frac{dY}{d\Pi}\right|$.

The mechanism of the non-neutrality of inflation is the same as that in the monetary shock case discussed in the above subsection: the rise of inflation, triggered by the increase in the growth rate of the monetary aggregate, leads to an increase in the seigniorage-output ratio and triggers the resource reallocation process that leads to utility maximization.

Because the growth rate of economy, which is decided by the technology side of the economy, is known in the steady state, another interesting conclusion here is that, from E3.39, we obtain the optimal rate of monetary aggregate growth rate when the optimal rate of inflation is obtained.

Panels (a) and (b) of Figure 5 below show the YIC and YCOI of Model 3.2, respectively, when gross inflation moves from 0.9 to 1.3. The comparison of the optimal rate of inflation and cost of inflation with those in existing empirical studies will be provided in the next section when the interest rate is involved.

Similarly, we can obtain the relation between each other variable and inflation. For example, the relation between utility and inflation, and the utility cost of inflation.



(a) (b)

Figure 5

4. SCME with Interest Rate Rule

Since it is the interest rate, rather than the monetary aggregate, that central banks use in monetary policy implementation, it is necessary to involve the interest rate in the economy. As mentioned in the introduction section, the canonical approach of applying the nominal interest rate in the literature is to introduce a pseudo bond in the budget constraint, which is accompanied by an interest rate. However, it is unacceptable for the quantity of bonds to be zero in equilibrium, which is inconsistent with reality, and the monetary authority does not directly control the bond rate in the real world economy. Therefore, this traditional way of treating nominal interest rates is dubious. Here, we find a new way to introduce the money market interest rate into the economy and study the SCME with the new interest rate.

4.1 Origin of Money Market Interest Rate, Missing of Liquidity Effect, and the Humps

Let the monetary authority collect the seigniorage with a gross money market interest rate, R_t, which simultaneously means the seigniorage on issuing M_t will be collected in the next period. Similar to E3.18, we have

$$E_t S_{t+1} = \frac{R_t M_t - M_t}{E_t P_{t+1}} \qquad \text{E4.1}$$

This new way of seigniorage collecting should not change the amount of seigniorage collected as in the old way; that is,

$$E_t S_{t+1} = \frac{E_t M_{t+1} - M_t}{E_t P_{t+1}} \qquad \text{E4.2}$$

Correspondingly, we have $R_t M_t = E_t M_{t+1}$, from which we obtain

$$R_t = \frac{E_t M_{t+1}}{M_t} \quad \text{E4.3}$$

Strikingly, E4.3 means that the gross interest rate of money equals its expected gross growth rate. [9]

This money market interest rate is strange at first glance. It may seem that the monetary authority can arbitrarily control the interest rate through money issuance. However, this is generally not true.
The explanation is as follows: although money issuance is the everyday work of monetary authority, money is issued according to a specific rule, that is, monetary policy, which could be regarded as a restriction on the behavior of the central bank. In addition, as shown in section 3, there is an optimal rate of inflation, which means that keeping the inflation rate at an acceptable level is an essential requirement of the household. From the equation of exchange and E4.3, we have

$$\Pi_t^e = \frac{R_t}{\Gamma_t^e} \quad \text{E4.4}$$

E4.4 means that the expected inflation rate, Π^e_t, which equals $E_t P_{t+1}/P_t$, is closely connected with R_t and the expected output growth rate, Γ^e_t, which equals $E_t Y_{t+1}/Y_t$. Because the economy's growth rate is generally stable, E4.4 shows that the relationship between inflation and the interest rate is close and direct. Monetary authorities must consider this close and direct relation when issuing money. Accordingly, the money market interest rate, E4.3, is a reasonable object. However, supposing that these two conditions, that is, the monetary policy rule and an acceptable inflation rate of the household, are not heeded by the monetary authority, the money market interest rate may become irrational. Unfortunately, we do observe repeated periods of the excessive inflation in economic history, some of which are indeed triggered by mistakes in conducting monetary policy.

Before studying the interest rate economy, let us take a detour to explain the liquidity effect puzzle with this new money market interest rate. When E4.3 is introduced into Model 3.2, Panel (a) of Figure 6 shows that a positive monetary aggregate shock will decrease the interest rate in the model economy. This is consistent with the liquidity effect, which means that an increased growth rate of money leads to a decrease in the interest rate but different from the empirical findings in Eichenbaum (1992) and Leeper and Gordon (1992), where the money market interest rate increases under a positive monetary aggregate shock. However, when the monetary transmission process is considered, the money market interest rate increases in the early stage(s) under a positive monetary shock. Specifically, let the following actual monetary aggregate, M^a of E4.5, be the quantity of money that mediates the transactions, which means **n** periods are needed to implement the monetary policy:

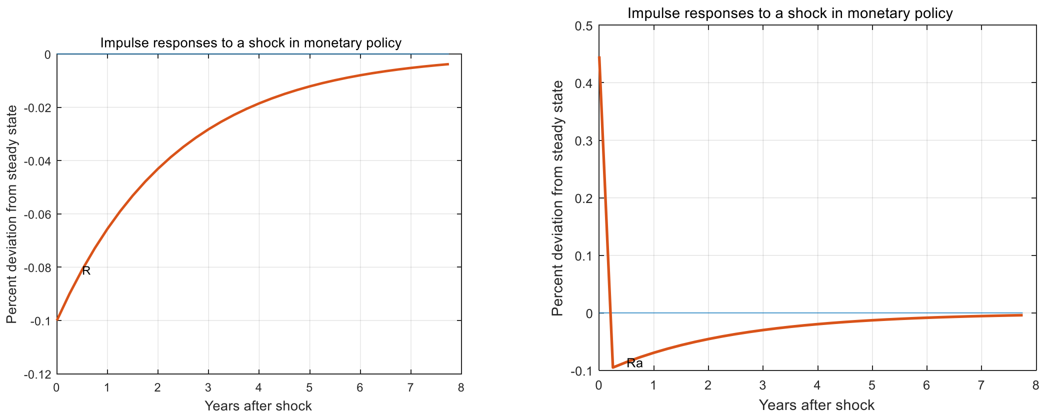
$$M_t^a = \frac{M_t + M_{t-1} + \cdots + M_{t-n+1}}{n} \quad \text{E4.5}$$

[9] Note that we can get the nominal rate of return of capital R^K of period t in Model 3.2 as, $R_t^K = \frac{E_t P_{t+1}(1-\delta)K_t + \alpha E_t P_{t+1} E_t Y_{t+1}}{P_t K_t}$. The numerator in the right side of the equation is the expected capital value of the next period, which consists of two parts: the expected value of depreciated capital and the expected rent income of capital, the denominator is the capital value of the present period. Similarly, we can get the real rate of return of capital, r^K, as, $r_t^K = \frac{(1-\delta)K_t + \alpha E_t Y_{t+1}}{K_t}$. With both the nominal and real rate of capital, we get the Fisher relation, $R_t^K = \Pi_t^e r_t^K$, where $\Pi^e_t = E_t P_{t+1}/P_t$ is the expected inflation rate. Note that it is the rate of return on capital, rather than a money market interest rate, that is adopted in the Fisher equation.

Correspondingly, we can obtain the actual money market interest rate of period t, R^a_t, by the weighted average, that is, $R_t^a = \frac{R_t M_t + R_{t-1} M_{t-1} + \cdots + R_{t-n+1} M_{t-n+1}}{M_t + M_{t-1} + \cdots + M_{t-n+1}}$, which can be simplified to

$$R_t^a = \frac{E_t M_{t+1}^a}{M_t^a} \qquad \text{E4.6}$$

Panel (b) of Figure 6 shows that the actual money market interest rate of E4.6, in the case of n=2, does increase in the first period of a positive monetary aggregate shock in Model 3.2.

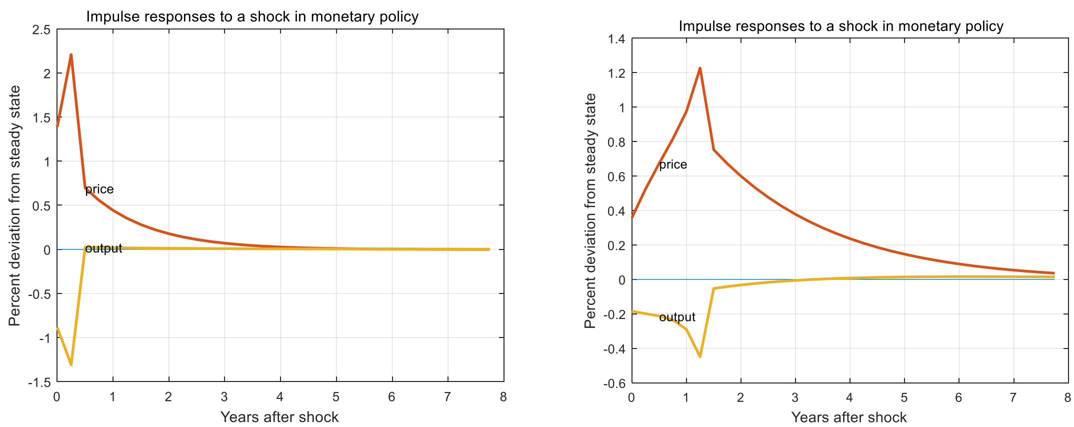


(a) (b)

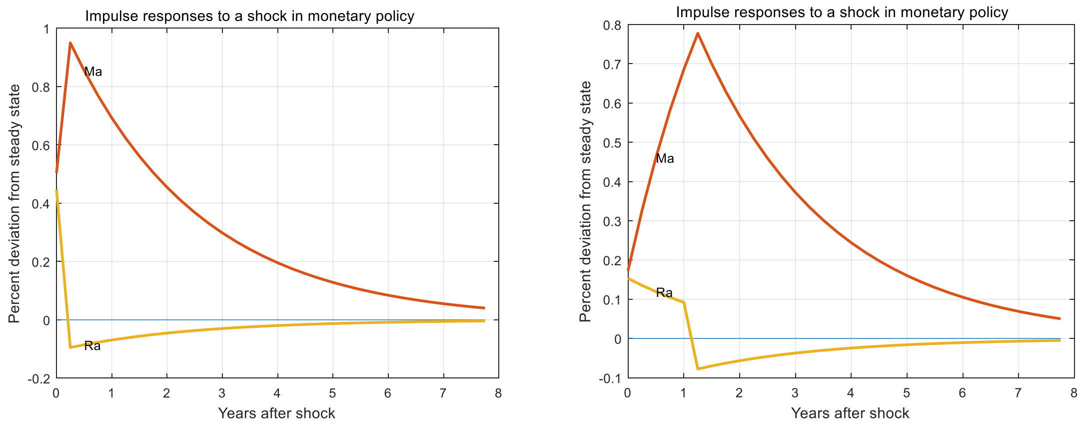
Figure 6

In addition, this treatment of monetary policy transmission leads to a hump in the response of the nominal and real variables under a monetary shock. The cases of n=2 and n=6 with Model 3.2 are shown below in Figure 7, where the chosen variables are price, output, M^a, and R^a. Note that the movement of price and output of the case of n=1 is shown in Panel (b) of Figure 2.

n=2 n=6



(a) (b)



(c) (d)

Figure 7

4.2 SCME with Exogenous Interest Rate Rule and Price Puzzle

Equipped with the money market interest rate, we can study the SCME with the interest rate rule. To obtain such an economy, we need the interest rate version of the seigniorage equation and equation of exchange.

First, from E3.18 and E3.17, we can obtain a new expression for the seigniorage at period t as,

$$S_t = (1 - \frac{1}{\frac{M_t}{M_{t-1}}})Y_t \qquad \text{E4.7}$$

When $M_t = E_{t-1}M_t$, that is, the realized monetary aggregate in period t equals the expected one, which means that there is no operational error in the monetary aggregate issuing process, we obtain the interest rate version of the seigniorage equation from E4.7 and E4.3 as,

$$S_t = (1 - \frac{1}{R_{t-1}})Y_t \qquad \text{E4.8}$$

It is possible to introduce a new shock, Z^{MM}, to introduce operational error into the monetary aggregate issuing process. However, my preliminary study shows that this error is not crucial, and we will not discuss this case in this paper. The assumption of $M_t = E_{t-1}M_t$ is retained in this paper. With E4.8, we can obtain the detrended form of the budget constraint and public goods constraint of the interest rate SCME, respectively, as

$$\dot{C}_t + \Gamma \dot{K}_t = \left(\frac{1}{R_{t-1}} - \tau\right)\dot{Y}_t + (1-\delta)\dot{K}_{t-1} \qquad \text{E4.9}$$

$$\dot{G}_t = (\tau + 1 - \frac{1}{R_{t-1}})\dot{Y}_t \qquad \text{E4.10}$$

Second, we have already obtained the interest rate version of the equation of exchange, that is, E4.4, which can be expressed as

$$R_t = \Pi_t^e \Gamma_t^e \qquad \text{E4.11}$$

In addition, we need an interest rate rule. Here, we adopt the following simple exogenous interest rate rule:

$$R_t = Z_t^M R \qquad \text{E4.12}$$

which means that the money market interest rate is set with two elements, a constant rate R and the monetary shock Z^M. Note that the value of R is the same as Θ in Model 3.2 because, according to E4.3, these two parameters are identical in the steady state.

All other things are the same as in Model 3.2, and we obtain the interest rate SCME. Note that because of the intimate relation between monetary aggregate and money market interest rate, that is, E4.3, and the adoption of the interest rate version of the equation of exchange in this kind of economy, that is, E4.11, it is convenient to deal with inflation rather than price level in an economy with an interest rate rule. The impulse response of actual inflation of this interest rate rule economy following a monetary shock is shown in Figure 9 below, where the utility function and parameter values are the same as those in Model 3.2; the only change is that Θ in Model 3.2 is replaced by R. The price puzzle, which means the actual inflation increases following a contractionary interest rate shock, appears in this interest rate rule SCME, which is close to the findings in Sims (1992) and Eichenbaum (1992). The price puzzle can be easily explained in SCME since we can obtain $\Pi_t = \frac{R_{t-1}}{\Gamma_t}$, which shows the intimate relation between actual inflation and the interest rate. The entire system of this exogenous interest rate rule economy is provided in Appendix B.

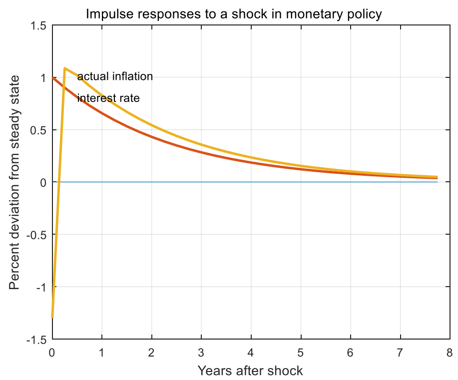


Figure 8

From the equivalence between Θ and R, the YIC and YCOI of this interest rate rule case are the same as those of Model 3.2, which are shown in Figure 5. Note the optimal inflation rate here is inconsistent with the the rule of Friedman (1969), which advocated a zero nominal interest rate. Regarding the cost of inflation, in the literature, as Gillman (1995) noted, the welfare cost of inflation for the United States ranges from 0.85 percent to 3 percent of real GNP per percent increase in the nominal interest rate above zero. As shown in Panel (b) of Figure 5, the cost of inflation of SCME is a little smaller than the results in the literature.

4.3 SCME with Reactive Interest Rate Rule

Since the monetary shock effectively triggers the movement of real variables, it is a natural idea to manipulate it to influence the economy, especially to counter the fluctuation in the economy. In this subsection, the reactive interest rate rule is embedded into SCME, and an approach for selecting monetary policy is developed. The negative movement of hours under a positive technology shock occurs in this flexible price economy with the reactive interest rate rule. The simulation and relative results of a 3-period monetary policy transmission SCME are provided in detail, reproducing many of the empirical findings in the literature and showing that SCME is promising in replicating the real-world economy. There is no forward guidance puzzle in this economy.

A. Choice of Reactive Interest Rate Rule

Assume, instead of E4.12, that monetary policy is reactive as follows:

$$R_t = Z_t^M R \Phi_\Pi^{\frac{\Pi_t^e}{\Pi}-1} \qquad \text{E4.13}$$

where $\frac{\Pi_t^e}{\Pi} - 1$ is the inflation gap, $\Phi_\Pi(\cdot)$ is a monotone function that responds to the expected inflation gap, and Φ_Π is a parameter. E4.13 means that the money market interest rate will be adjusted according to the expected inflation gap. Note that the monetary authority, in this new circumstance, still runs according to the rule. Namely, it is still not a utility optimizer.

Compared with Model 4.2, there is no change in the transaction equations, the firm's behavior, or the exchange equation in this new economy. However, the behavior of the household needs some modification because E4.13 means that the household can impact the money market interest rate through Π^e_t. Correspondingly, the endogenous monetary policy needs to be treated as a new constraint in the household's decision.

Equipped with the reactive monetary policy, it is time to implement the SCME concretely. The main problem encountered here is how to decide the value of the new parameter Φ_Π, that is, the choice of the policy rule. Note that Φ_Π is the only parameter in the reactive policy economy that is different from that in Model 4.2. When applying the Taylor rule (1998), that is, when setting $\ln \Phi_\Pi = 1.5$, the simulated economy fluctuates violently, with the standard deviation of output being more than 4%, and some of the statistical relationships disappear. For example, the money market interest rate decreases in response to a positive monetary shock, and there is no price puzzle. Therefore, we need to find a way to locate the value of Φ_Π in E4.13, or $\ln \Phi_\Pi$ when we consider the log-linear form.

Here, we begin with the following two principles in locating the value of $\ln \Phi_\Pi$:

Principle 1: Model 4.2 should be the benchmark for the reactive interest rate economy with rule E4.13, briefly Model 4.3A.

Principle 2: Model 4.3A should be close to the benchmark Model 4.2. In particular, the steady-state values of Model 4.3A should be the same or close to those of Model 4.2.

When comparing the difference between the two economies carefully, especially the steady state values, we find that the difference between them is not unmanageable, and the essential differences lie mainly in the following three aspects:

Difference 1: The policy rule, in log-linear form

Model 4.2 $$\widehat{R_t} = \widehat{Z_t^M} \qquad \text{E4.14}$$

Model 4.3A $$\widehat{R_t} = \widehat{Z_t^M} + \ln \Phi_\Pi \widehat{\Pi_t^e} \qquad \text{E4.15}$$

Difference 2: The steady-state Y/K ratio[10]

$$\frac{Y}{K}^{Model4.2} = \frac{\textcircled{1}+\textcircled{3}+\textcircled{6}}{\textcircled{2}-(\textcircled{4}-\textcircled{5})} \qquad \text{E4.16}$$

$$\frac{Y}{K}^{Model4.3A} = \frac{\textcircled{1}+\textcircled{3}+\textcircled{6}\textcircled{|}}{\textcircled{2}-(\textcircled{4}-\textcircled{5})\textcircled{|}} \qquad \text{E4.17}$$

where $\textcircled{|} = \frac{\ln \Phi_{\Pi}}{\ln \Phi_{\Pi}-1}$

Difference 3: The steady-state hours[11]

$$N^{Model4.2} = \frac{(\frac{(\textcircled{7}+\textcircled{8})\textcircled{9}}{\xi})^{\frac{1}{\eta}}}{1+(\frac{(\textcircled{7}+\textcircled{8})\textcircled{9}}{\xi})^{\frac{1}{\eta}}} \qquad \text{E4.18}$$

$$N^{Model4.3A} = \frac{(\frac{(\textcircled{7}+\textcircled{8}\textcircled{|})\textcircled{9}}{\xi})^{\frac{1}{\eta}}}{1+(\frac{(\textcircled{7}+\textcircled{8}\textcircled{|})\textcircled{9}}{\xi})^{\frac{1}{\eta}}} \qquad \text{E4.19}$$

Difference 1 implies that the value of $\ln \Phi_{\Pi}$ needs to be small to make the reactive policy rule close to the benchmark rule.

From Difference 2, we have the following: a. $\frac{Y}{K}^{Model4.3A}$ is close to $\frac{Y}{K}^{Model4.2}$, and b. The value of $\frac{Y}{K}^{Model4.2}$ is almost stable when the value of $\ln \text{ʊ}$ is small, which is shown in Panel (a) of Figure 9. Note that the value of the same parameter in Model 4.3A is set equal to that in Model 4.2 in the comparison.

E4.19 is depicted in Panel (b) of Figure 9, which shows that the steady-state hours of Model 4.3A are sensitive to the value of $\ln \Phi_{\Pi}$, and the value should be small to make the steady-state value of hours of Model 4.3A close to 1/3, the steady-state value in Model 4.2.

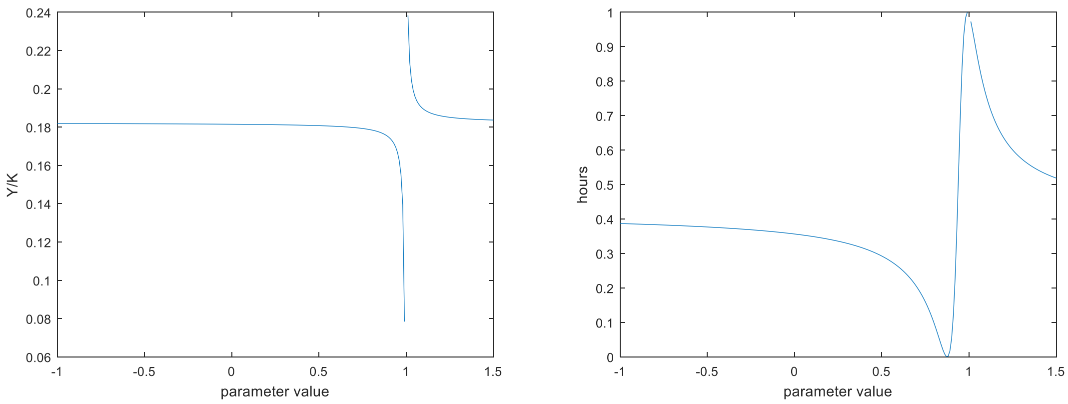
---

10 $\textcircled{1} = \chi(1-\frac{\beta}{\Gamma}(1-\delta))$; $\textcircled{2} = \frac{\beta}{\Gamma}(\frac{1}{R}-\tau)\alpha$; $\textcircled{3} = \frac{\beta}{\Gamma}(1-\chi)(\Gamma-(1-\delta))\alpha$;

$\textcircled{4} = (1-\beta)\beta\chi\frac{\alpha}{R\Gamma}$; $\textcircled{5} = (1-\beta)\beta(1-\chi)\frac{\frac{1}{R}-\tau}{\tau+1-\frac{1}{R}}\frac{\alpha}{R\Gamma}$; $\textcircled{6} = (1-\beta)\beta(1-\chi)\frac{\Gamma-(1-\delta)}{\tau+1-\frac{1}{R}}\frac{\alpha}{R\Gamma}$

11 $\textcircled{7} = \left(\frac{1}{R}-\tau\right)\chi + (\tau+1-\frac{1}{R})(1-\chi)\frac{\left(\frac{1}{R}-\tau\right)\frac{Y}{K}-(\Gamma-(1-\delta))}{(\tau+1-\frac{1}{R})\frac{Y}{K}}$; $\textcircled{8} = \frac{\beta}{\Gamma}(\chi-(1-\chi)\frac{\left(\frac{1}{R}-\tau\right)\frac{Y}{K}-(\Gamma-(1-\delta))}{(\tau+1-\frac{1}{R})\frac{Y}{K}})$;

$\textcircled{9} = \frac{\textcircled{10}}{\left(\frac{1}{R}-\tau\right)-\frac{(\Gamma-(1-\delta))}{Y/K}}\left(\frac{1}{Y/K}^{\frac{\alpha}{1-\alpha}}\right)^{1-\eta}(1-\alpha)$; $\textcircled{10} = \left(\left(\left(\frac{1}{R}-\tau\right)-\frac{(\Gamma-(1-\delta))}{Y/K}\right)^{\chi}\left(\tau+1-\frac{1}{R}\right)^{1-\chi}\right)^{(1-\eta)}$



(a) (b)

Figure 9

With the above analysis and experiments on Model 4.3A, we can conclude that the value of $\ln \Phi_\Pi$ should be small under the settings of the model.

B. A Simulated Reactive Interest Rate Rule SCME

Based on the study above, we provide in detail the simulation and relative results of an SCME with the following reactive interest rate policy:

$$R_t = Z_t^M R \Phi_\Pi^{\frac{\Pi_t^e}{\Pi}-1} \Phi_Y^{\frac{Y_t}{A_t Y}-1} \qquad \text{E4.20}$$

E4.20 means that the money market interest rate responds to both the expected inflation gap and output gap, where Φ_Π and Φ_Y are the respective parameters. The log-linear form of E4.20 is

$$\widehat{R_t} = \widehat{Z_t^M} + \ln \Phi_\Pi \widehat{\Pi_t^e} + \ln \Phi_Y \widehat{Y_t} \qquad \text{E4.21}$$

In particular, the rule used in the simulation below is

$$\widehat{R_t} = \widehat{Z_t^M} + 0.2\widehat{\Pi_t^e} + 0.2\widehat{Y_t} \qquad \text{E4.22}$$

that is, $\ln \Phi_\Pi = 0.2$ and $\ln \Phi_Y = 0.2$. The value of $\ln \Phi_Y$ can be obtained in a similar way as we study the value of $\ln \Phi_\Pi$.

In addition, to make the model economy closer to the real-world economy, we consider the case of 3-period monetary policy transmission, and from E4.5, E4.6, and E4.3, we have:

$$M_t^a = \frac{M_t + M_{t-1} + M_{t-2}}{3} \qquad \text{E4.23}$$

$$R_t^a = \frac{R_{t-2}(R_t R_{t-1} + R_{t-1} + 1)}{R_{t-1} R_{t-2} + R_{t-2} + 1} \qquad \text{E4.24}$$

Note that in this case, R^a in E4.24 is the interest rate entering into the seigniorage equation E4.8, and the interest rate version of the exchange equation becomes

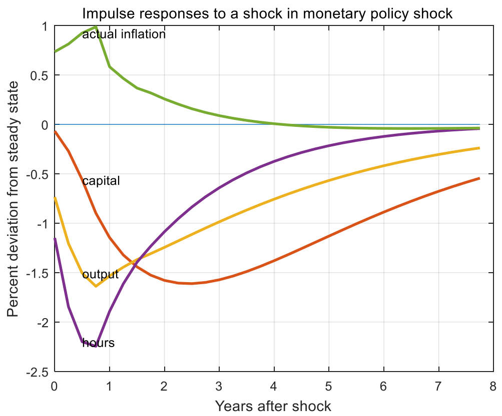
$$R_t^a = \Pi_t^e \Gamma_t^e \qquad \text{E4.25}$$

The simulated economy is called Model 4.3B, and the entire system is provided in Appendix C. With the commonly used parameter values of this paper, the steady-state values of C/Y, X/Y, T/Y, S/Y, and Y/K of this model economy are 0.6359, 0.1793, 0.17, 0.0148, and 0.1813, respectively, and we have R=1.015, Γ=1.0075, Π=R/Γ=1.0074, and N=1/3.

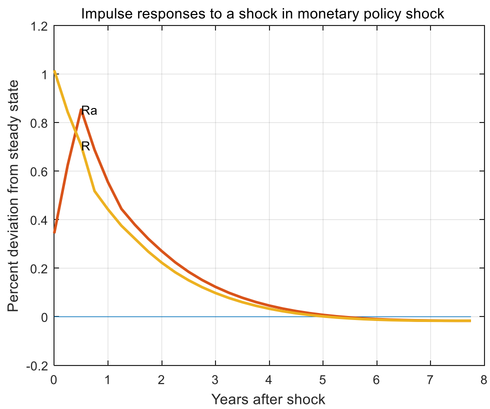
Now, let's compare the experimental results of Model 4.3B with the empirical results in the literature. Ramey (2016) summarized some of the main results from the literature on the impact of a monetary shock on output, which spans from -0.6% to -5% under a 100 basis point fund rate peak. The timing of the trough spans from 8 months to 8 quarters. In addition, the majority of the studies reported a 4%-10% 1 year - 5 years ahead forecast error variance of output explained by the monetary shock. Romer and Romer (2004) and Coibion (2012) are exceptions who reported major and moderate parts of the variance coming from the monetary shock, respectively.

Figure 10 provides the impulse-response curves of the variables in Model 4.3B under a one-percent positive interest rate shock (except Panel (f), which is the technology shock case), where monetary effectiveness and its persistence, price puzzle, and the humps are all obtained. Panel (a) shows that output decreases by approximately 1.6 percent in a trough in this model economy when the peak of R^a is approximately +0.85 percent, as shown in Panel (b). With the 3-period implementation process, the timing of the trough is at about the third quarter, which could be adjusted if we change the transmission periods. Panel (b) shows that the actual money market interest rate, R^a, increases in early periods, which imitates the step-by-step raising of the funds rate by the Board of the Federal Reserve in a contractionary interest rate policy operation. As shown in Panel (e), the permanent utility of the household decreased slightly. An interesting aspect of this economy is that hours decrease, as Panel (f) shows, under a positive technology shock, which is consistent with the empirical studies of Gali (1999) and Basu, Fernald and Kimball (2006), among others.

Note there is no forward guidance puzzle in this model economy.



(a)



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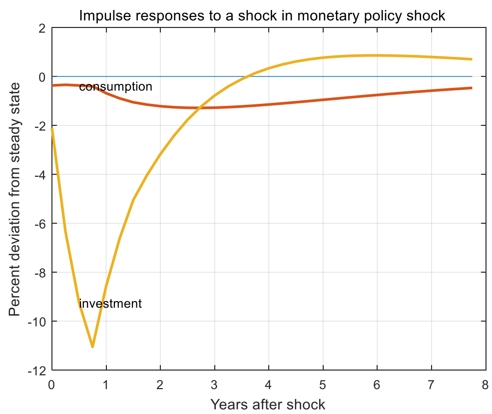

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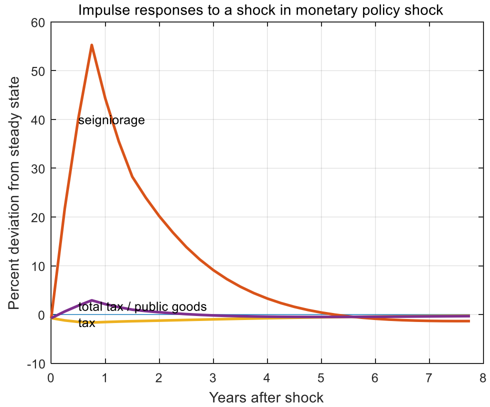

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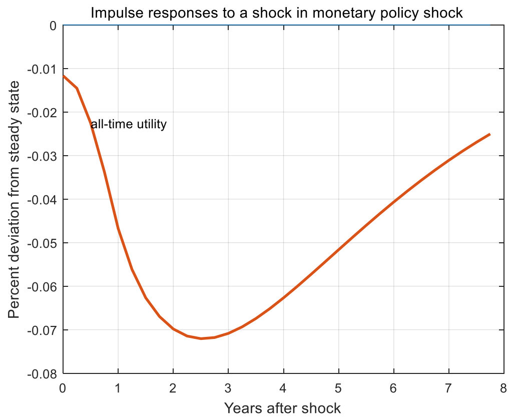
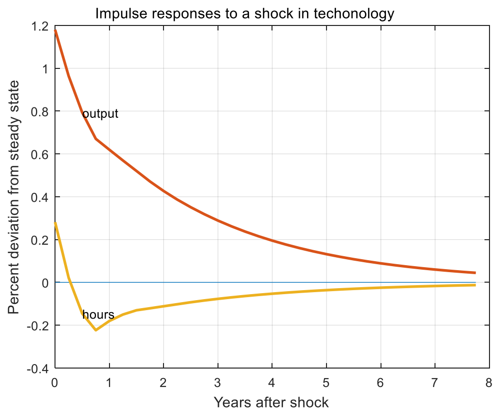


(e)



(f)

Figure 10

Table 1 provides the standard deviation and cross-correlation of the main variables in this economy. The standard output variation is 1.69%. By the way, we provide the standard deviation of the respective variables of the benchmark economy, that is, the three-period monetary transmission version of Model 4.2, briefly Model 4.2L3. The results show the reactive interest rate rule economy is much more stable than the exogenous interest rate rule economy.

Regarding variance decomposition, 76% of the 8-quarter-ahead forecast error variance for output is explained by the monetary shock in Model 4.3B, which is reported in Table 2.

Table 1: Standard Deviation and Cross-correlation of Model4.3B

	SD%		cross-correlation of output with:										
	Model4.3B	Model4.2L3	-5	-4	-3	-2	-1	0	1	2	3	4	5
Ra	0.76	0.72	0.26	0.18	0.04	-0.17	-0.40	-0.59	-0.64	-0.56	-0.43	-0.29	-0.19
output	1.69	3.84	0.00	0.15	0.35	0.58	0.81	1	0.81	0.58	0.35	0.15	0.00
actual inflation	1.35	2.84	0.29	0.27	0.19	0.06	-0.09	-0.60	-0.61	-0.53	-0.41	-0.29	-0.20
consumption	0.93	1.83	0.65	0.72	0.72	0.67	0.59	0.52	0.29	0.09	-0.08	-0.23	-0.33
investment	10.45	11.66	-0.28	-0.14	0.10	0.38	0.66	0.85	0.75	0.58	0.41	0.27	0.15
capital	1.15	1.04	0.62	0.72	0.79	0.78	0.69	0.52	0.28	0.06	-0.11	-0.24	-0.33
hours	2.10	5.20	-0.21	-0.08	0.10	0.34	0.59	0.80	0.77	0.64	0.46	0.29	0.15
seigniorage	49.50	50.05	0.18	0.05	-0.16	-0.39	-0.57	-0.62	-0.55	-0.42	-0.29	-0.18	-0.10
tax	1.69	3.84	0.00	0.15	0.35	0.58	0.81	1	0.81	0.58	0.35	0.15	0.00
total tax	3.24	6.83	0.22	0.13	-0.03	-0.20	-0.31	-0.27	-0.28	-0.24	-0.19	-0.15	-0.12
public goods	3.24	6.83	0.22	0.13	-0.03	-0.20	-0.31	-0.27	-0.28	-0.24	-0.19	-0.15	-0.12

Table 2: Percentage Variance Due to Technology Shock and Monetary Policy Shock								
	1 quarter ahead		4 quarters ahead		8 quarters ahead		20 quarters ahead	
	tech-shock	mp-shock	tech-shock	mp-shock	tech-shock	mp-shock	tech-shock	mp-shock
Ra	0.0800	0.9200	0.0905	0.9095	0.1044	0.8956	0.1231	0.8769
output	0.7204	0.2796	0.3288	0.6712	0.2351	0.7649	0.1790	0.8210
actual inflation	0.7204	0.2796	0.3693	0.6307	0.3445	0.6555	0.3548	0.6452
consumption	0.4516	0.5484	0.5862	0.4138	0.2541	0.7459	0.0895	0.9105
investment	0.8084	0.1916	0.0910	0.9090	0.0579	0.9421	0.0553	0.9447
capital	0.8084	0.1916	0.1225	0.8775	0.0360	0.9640	0.0156	0.9844
hours	0.0566	0.9434	0.0102	0.9898	0.0097	0.9903	0.0103	0.9897
seigniorage	0.7204	0.2796	0.1017	0.8983	0.1178	0.8822	0.1428	0.8572
tax	0.7204	0.2796	0.3288	0.6712	0.2351	0.7649	0.1790	0.8210
total tax	0.7204	0.2796	0.4696	0.5304	0.5014	0.4986	0.5581	0.4419
public goods	0.7204	0.2796	0.4696	0.5304	0.5014	0.4986	0.5581	0.4419

5. Conclusion

SCME, which is based on the RBC model, is a macroeconomic platform in which monetary shocks can effectively impact real variables in the flexible price condition, and the effectiveness is persistent and hump-shaped. With SCME, we obtain many new understandings of the operations of the real-world economy, which include the following:

(i) The mechanism for monetary effectiveness and persistence is resource reallocation triggered by the coopetation of the seigniorage channel and the equation of exchange.

(ii) In the steady state, the quantity of monetary aggregate is neutral, but the growth rate of money and inflation are nonneutral to the real economy. The YIC and YCOI are derived, and the optimal rate of inflation is obtained. With the optimal rate of inflation, we obtain the optimal rate of money market interest rate, which is inconsistent with the Friedman rule.

(iii) The price puzzle, that is, the increase in price levels under contractive interest rate policy shock, emerges in the SCME.

(iv) There is no forward guidance puzzle in SCME.

(v) The humps in the impulse response of the real and nominal variables are caused by the monetary transmission process, which also causes the absence of the liquidity effect.

(vi) A quantitative method for the choice of reactive monetary policy is developed, in which we found the parameter value is different from that suggested by Taylor (1998).

(vii) Decrease in hours under positive technology shocks is found when the interest rate rule is reactive.

(viii) The optimal tax rate, that is, the best government debt level is obtained.

(ix) The pricing is interactive in SCME.

(x) Money and taxes are not sources of distortion for the economy, and the resource allocation of the unique equilibrium of SCME is Pareto optimal.

The following four innovations are the pillars of the findings in this paper:

(1) Differentiating between taxes and public goods, that is, E3.1-E3.2. This modifies the utility and budget of the economy and makes the unique equilibrium of the economy with taxation a Pareto optimum. It provides the basis for studying SCME.

(2) The budget constraint with the seigniorage channel, that is, E3.21. The seigniorage equation provides a position for money in the economic system. Money and monetary policy can now substantially contribute to the economic rebalancing process under flexible prices.
(3) The integration of the transaction side into the economy, that is, E3.7-E3.11. In addition to helping handle the complex money and tax affairs in the economy, which clarify the operations of the complicated model economy, the inclusion of the transaction side makes the model economy a whole, which includes all the processes as those in the real-world economy. Above all, the transaction equations establish the interactive pricing mechanism and help obtain the equation of exchange, the latter is crucial for the effectiveness result.
(4) The way money market interest rate is defined, that is, E4.3. The close relationship between the monetary aggregate and interest rate rule economy is found.

The main purpose of this paper is to put forward the SCME platform, and as a by-product of this study, we found the failure of the neoclassical monetary economy and the new Keynesian economy came from their neglection of the transaction side of the economy and the government lump-sum transfer setting.
Many issues, such as the velocity of money, the Taylor rule, and the optimal monetary policy, need to be scrutinized in depth in SCME. Furthermore, to replicate the real-world economy, credit, asset prices, and foreign exchange are needed to be embedded into SCME.

Appendix

As mentioned in the text, although with a big difference in economic sense, SCME in this paper is mathematically the same as the RBC economy of Stokey, Lucas, with Prescott (1989). After obtaining the existence and uniqueness of the equilibrium with the method provided in Stokey, Lucas, with Prescott (1989), in the Appendix, we provide the log-linearized form of the whole model economy around its steady state with the LaGrange method of Uhlig (1999). All variables are in detrended form in the Appendix.

A. Detrended Form of Model 3.2

$$UU_t = maxE_t \sum_{i=0}^{\infty} \beta^i \left(\frac{\left(C_{t+i}{}^{\chi} G_{t+i}{}^{1-\chi}\right)^{1-\eta}}{1-\eta} + \xi \frac{(1-N_{t+i})^{1-\eta N}}{1-\eta N}\right) \quad \text{A.A.1}$$

$$Y_t = Z_t^T K_{t-1}^{\alpha} N_t{}^{1-\alpha} \quad \text{A.A.2}$$

$$\mathrm{M_t} = Z_t^M \overline{M} \quad \text{A.A.3}$$

$$W_t^K K_{t-1} = M_t^K \quad \text{A.A.4}$$

$$W_t^N N_t = M_t^N \quad \text{A.A.5}$$

$$M_t^K + M_t^N = M_t \quad \text{A.A.6}$$

$$M_t^K = P_t Y_t^K \quad \text{A.A.7}$$

$$M_t^N = P_t Y_t^N \quad \text{A.A.8}$$

$$U_t^F = max(Y_t - Y_t^K - Y_t^N) \quad \text{A.A.9}$$

$$\alpha P_t Y_t = W_t^K K_{t-1} \quad \text{A.A.10}$$

$$(1-\alpha) P_t Y_t = W_t^N N_t \quad \text{A.A.11}$$

$$M_t = P_t Y_t \quad \text{A.A.12}$$

$$S_t = \left(1 - \frac{M_{t-1}}{\Theta M_t}\right) Y_t \quad \text{A.A.13}$$

$$T_t = \tau Y_t \quad \text{A.A.14}$$

$$TT_t = T_t + S_t \quad \text{A.A.15}$$

$$C_t + X_t + TT_t = Y_t \quad \text{A.A.16}$$

$$\Gamma K_t = (1-\delta) K_{t-1} + X_t \quad \text{A.A.17}$$

$$G_t = TT_t \quad \text{A.A.18}$$

$$\Lambda_t^C = \chi \frac{(C_t^{\chi} G_t^{1-\chi})^{1-\eta}}{C_t} \quad \text{A.A.19}$$

$$\Lambda_t^G = (1-\chi) \frac{(C_t^{\chi} G_t^{1-\chi})^{1-\eta}}{G_t} \quad \text{A.A.20}$$

$$\Lambda_t^C - \frac{\beta}{\Gamma} E_t \Lambda_{t+1}^C \left(\left(\frac{M_t}{\Theta E_t M_{t+1}} - \tau\right) \alpha \frac{E_t Y_{t+1}}{K_t} + (1-\delta)\right) - \frac{\beta}{\Gamma} E_t \Lambda_{t+1}^G \left(\tau + 1 - \frac{M_t}{\Theta E_t M_{t+1}}\right) \alpha \frac{E_t Y_{t+1}}{K_t} -$$

$$\frac{\beta}{\Gamma} E_t \Lambda_{t+1}^{\mathrm{E}} E_t P_{t+1} \alpha \frac{E_t Y_{t+1}}{K_t} = 0 \quad \text{A.A.21}$$

$$\frac{\xi}{(1-N_t)^{\eta N}} = \Lambda_t^C \left(\frac{M_{t-1}}{\Theta M_t} - \tau\right) (1-\alpha) \frac{Y_t}{N_t} + \Lambda_t^G \left(\tau + 1 - \frac{M_{t-1}}{\Theta M_t}\right) (1-\alpha) \frac{Y_t}{N_t} + \Lambda_t^{\mathrm{E}} P_t (1-\alpha) \frac{Y_t}{N_t}$$

A.A.22

$$\Lambda_t^{\mathrm{E}} + \Lambda_t^C \frac{M_{t-1}}{\Theta M_t^2} Y_t - \beta E_t \Lambda_{t+1}^C \frac{1}{\Theta E_t M_{t+1}} E_t Y_{t+1} - \Lambda_t^G \frac{M_{t-1}}{\Theta M_t^2} Y_t + \beta E_t \Lambda_{t+1}^G \frac{1}{\Theta E_t M_{t+1}} E_t Y_{t+1} = 0 \quad \text{A.A.23}$$

$$\ln Z_t^M = (1-\rho^M) \ln Z^M + \rho^M \ln Z_{t-1}^M \quad \text{A.A.24}$$

$$\ln Z_t^T = (1-\rho^T)\ln Z^T + \rho^T \ln Z_{t-1}^T \qquad \text{A.A.25}$$

Λ^C, Λ^G, and Λ^E are the Lagrange multipliers of the budget constraint, A.A.16, the public goods constraint, A.A.18, and the equation of exchange constraint, A.A.12, respectively. A.A.19-A.A.23 are first orders on C_t, G_t, K_t, N_t, and M_t, respectively.

B. Detrended Form of Model 4.2

A.A.1 A.B.1

A.A.2 A.B.2

$$\mathrm{R_t} = Z_t^M \bar{R} \qquad \text{A.B.3}$$

A.A.4 A.B.4

A.A.5 A.B.5

A.A.6 A.B.6

A.A.7 A.B.7

A.A.8 A.B.8

A.A.9 A.B.9

A.A.10 A.B.10

A.A.11 A.B.11

A.A.12 A.B.12

$$R_t = \frac{E_t M_{t+1}}{M_t} \qquad \text{A.B.13}$$

$$R_t = \Pi_t^e \Gamma_t^e \qquad \text{A.B.14}$$

$$S_t = (1 - \frac{1}{R_{t-1}})Y_t \qquad \text{A.B.15}$$

A.A.14 A.B.16

A.A.15 A.B.17

A.A.16 A.B.18

A.A.17 A.B.19

A.A.18 A.B.20

A.A.19 A.B.21

A.A.20 A.B.22

$$\Lambda_t^C - \frac{\beta}{\Gamma} E_t \Lambda_{t+1}^C \left(\left(\frac{1}{R_t} - \tau \right) \alpha \frac{E_t Y_{t+1}}{K_t} + (1-\delta) \right) - \frac{\beta}{\Gamma} E_t \Lambda_{t+1}^G \left(\tau + 1 - \frac{1}{R_t} \right) \alpha \frac{E_t Y_{t+1}}{K_t} - \Lambda_t^{\mathrm{E}} \Pi_t^e \frac{E_t Y_{t+1}}{Y_t} \frac{\alpha}{K_t} +$$

$$\beta E_t \Lambda_{t+1}^{\mathrm{E}} E_t \Pi_{t+1}^e \frac{E_t Y_{t+2}}{E_t Y_{t+1}} \frac{\alpha}{K_t} = 0 \qquad \text{A.B.23}$$

$$\frac{\xi}{(1-N_t)^{\eta N}} = \Lambda_t^C \left(\frac{1}{R_{t-1}} - \tau \right) (1-\alpha) \frac{Y_t}{N_t} + \Lambda_t^G \left(\tau + 1 - \frac{1}{R_{t-1}} \right) (1-\alpha) \frac{Y_t}{N_t} - \Lambda_t^{\mathrm{E}} \Pi_t^e \Gamma \frac{E_t Y_{t+1}}{Y_t} \frac{1-\alpha}{N_t}$$

A.B.24

$$\Lambda_t^{\mathrm{E}} + \beta E_t \Lambda_{t+1}^C \frac{E_t Y_{t+1}}{R_t^2} - \beta E_t \Lambda_{t+1}^G \frac{E_t Y_{t+1}}{R_t^2} = 0 \qquad \text{A.B.25}$$

A.A.24 A.B.26

A.A.25 A.B.27

Λ^E is the Lagrange multiplier of the constraint on the interest rate form of the equation of exchange, A.B.14. A.B.25 is the first order on R_t.

C. Detrended Form of Model 4.3B

A.A.1 A.C.1

A.A.2 A.C.2

$$R_t = Z_t^M \bar{R} \Phi^{\frac{\Pi_t^e}{\Pi}-1} \Phi^{\frac{Y_t}{Y}-1} \qquad \text{A.C.3}$$

A.A.4 A.C.4

A.A.5 A.C.5

A.A.6 A.C.6

A.A.7 A.C.7

A.A.8 A.C.8

A.A.9 A.C.9

A.A.10 A.C.10

A.A.11 A.C.11

A.A.12 A.C.12

A.B.13 A.C.13

$$M_t^a = \frac{M_t + M_{t-1} + M_{t-2}}{3} \qquad \text{A.C.14}$$

$$R_t^a = \frac{R_{t-2}(R_t R_{t-1} + R_{t-1} + 1)}{R_{t-1} R_{t-2} + R_{t-2} + 1} \qquad \text{A.C.15}$$

$$R_t^a = \Pi_t^e \Gamma_t^e \qquad \text{A.C.16}$$

$$S_t = (1 - \frac{1}{R_{t-1}^a}) Y_t \qquad \text{A.C.17}$$

A.A.14 A.C.18

A.A.15 A.C.19

A.A.16 A.C.20

A.A.17 A.C.21

A.A.18 A.C.22

A.A.19 A.C.23

A.A.20 A.C.24

$$\Lambda_t^C - \frac{\beta}{\Gamma} E_t \Lambda_{t+1}^C \left(\left(\frac{1}{R_t^a} - \tau \right) \alpha \frac{E_t Y_{t+1}}{K_t} + (1 - \delta) \right) - \frac{\beta}{\Gamma} E_t \Lambda_{t+1}^G \left(\tau + 1 - \frac{1}{R_t^a} \right) \alpha \frac{E_t Y_{t+1}}{K_t} - \Lambda_t^{\mathrm{E}} \Pi_t^e \frac{E_t Y_{t+1}}{Y_t} \frac{\alpha}{K_t} +$$

$$\beta E_t \Lambda_{t+1}^{\mathrm{E}} E_t \Pi_{t+1}^e \frac{E_t Y_{t+2}}{E_t Y_{t+1}} \frac{\alpha}{K_t} - \frac{1}{\Gamma} \Lambda_t^{\mathrm{P}} R_t \ln \Phi \frac{E_t Y_{t+1}}{Y_t} \frac{\alpha}{K_t} + \frac{\beta}{\Gamma} E_t \Lambda_{t+1}^{\mathrm{P}} E_t R_{t+1} \ln \Phi \frac{E_t Y_{t+2}}{E_t Y_{t+1}} \frac{\alpha}{K_t} = 0$$

A.C.25

$$\frac{\xi}{(1-N_t)^{\eta N}} = \Lambda_t^C \left(\frac{1}{R_{t-1}^a} - \tau \right) (1 - \alpha) \frac{Y_t}{N_t} + \Lambda_t^G \left(\tau + 1 - \frac{1}{R_{t-1}^a} \right) (1 - \alpha) \frac{Y_t}{N_t} - \Lambda_t^{\mathrm{E}} \Pi_t^e \Gamma \frac{E_t Y_{t+1}}{Y_t} \frac{1-\alpha}{N_t} -$$

$$\Lambda_t^{\mathrm{P}} R_t \ln \Phi \frac{E_t Y_{t+1}}{Y_t} \frac{(1-\alpha)}{N_t} \qquad \text{A.C.26}$$

$$\Lambda_t^{\mathrm{E}}\widetilde{R_t^a} + \beta E_t\Lambda_{t+1}^{\mathrm{E}}\widetilde{E_tR_{t+1}^a} + \beta^2E_t\Lambda_{t+2}^{\mathrm{E}}\widetilde{E_tR_{t+2}^a} + \Lambda_t^{\mathrm{P}} + \beta E_t\Lambda_{t+1}^{C}\frac{\widetilde{R_t^a}}{(R_t^a)^2}E_tY_{t+1} + \beta^2E_t\Lambda_{t+2}^{C}\frac{\widetilde{E_tR_{t+1}^a}}{(E_tR_{t+1}^a)^2}E_tY_{t+2} + \beta^3E_t\Lambda_{t+3}^{C}\frac{\widetilde{E_tR_{t+2}^a}}{(E_tR_{t+2}^a)^2}E_tY_{t+3} - \beta E_t\Lambda_{t+1}^{G}\frac{\widetilde{R_t^a}}{(R_t^a)^2}E_tY_{t+1} - \beta^2E_t\Lambda_{t+2}^{G}\frac{\widetilde{E_tR_{t+1}^a}}{(E_tR_{t+1}^a)^2}E_tY_{t+2} - \beta^3E_t\Lambda_{t+3}^{G}\frac{\widetilde{E_tR_{t+2}^a}}{(E_tR_{t+2}^a)^2}E_tY_{t+3} = 0 \quad \text{A.C.27}$$

$$\Lambda_t^{\mathrm{E}}\frac{\Gamma E_tY_{t+1}}{Y_t} + \Lambda_t^{\mathrm{P}}R_t\ln \mho\frac{1}{\Pi} = 0 \quad \text{A.C.28}$$

A.A.24 A.C.29

A.A.25 A.C.30

Λ^P is the Lagrange multiplier of the monetary policy constraint, A.C.3. A.C.28 is first order on $\Pi^e{}_t$. A term with a ~ symbol above it stands for its derivative on R_t.

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