An intergenerational welfare analysis in a small open economy model between social security systems

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An Intergenerational Welfare Analysis in a Small Open Economy Model Between Social Security Systems

By Elvis Jurado*

This thesis investigates the long-term macroeconomic and welfare impacts of transitioning from a Pay-As-You-Go to a fully funded pension system, specifically within the Ecuadorian economic context. The study is motivated by financial and demographic challenges that threaten the sustainability of the current pension structure. Understanding the effects of such a transition is essential for informed implementation. The research has two primary objectives: first, to simulate this reform under various economic shocks, particularly changes in oil income and interest rates given that variability in oil revenues directly affects the economy as oil is Ecuadors main source of income; and second, to evaluate how the timing of the changes influences welfare outcomes across generations. The analysis is based on a transition from a Pay-As-You-Go system to a Fully Funded system, allowing for a more flexible response to demographic and fiscal pressures. To achieve this, a calibrated Overlapping Generations model is employed, integrated with a Small Open Economy framework and tailored to Ecuadorian data. This model allows for simulation of the pension reform under different macroeconomic conditions and transition scenarios. Findings suggest that while a fully funded system may increase welfare in the new steady-state equilibrium relative to a PAYG system reflecting the right timing for replacing the social security system under a general equilibrium model positive economic shocks can produce large welfare gains. However, welfare outcomes during the transition period remain highly sensitive to shocks, which in some scenarios can cause net losses for certain generations. The impact varies depending on the type of shock and the timing of reform implementation. These results highlight the importance of timing and economic context when designing pension policy. A poorly timed reform could reduce expected benefits, even if long-term outcomes appear favorable.

Keywords: *Overlapping Generations, Welfare, Small Open Economy, Demography, Ageing*

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I. Introduction

The sustainability and efficiency of social security have been discussed in research for decades due to variables, such as demographic transition and fertility, that directly affect an important part of the economy.

One by-product of demographic transition is ageing, which presents itself as a rather complicated issue that The Organization for Economic Co-operation and Development (OECD) countries must constantly face. Needless to say, the viability of social security is simply worrying in the long run, considering that the rate of both, population growth and fertility in the twentieth century, have actually seen a decrease. With this in mind, younger individuals born in later generations are increasingly unable to sustain older ones. We must, therefore, acknowledge that this gap keeps widening as time goes by.

Another point worth considering is that over time, health-care expenditures are expected to increase while future per capita Gross Domestic Product (GDP) growth to decline. When different methods of pension systems are reviewed, the one that is the most commonly used in Latin American countries is the Pay-As-You-Go system, which makes it impossible to cope with the problem of demographic transition; for that reason, countries must find a way to escape this challenging cycle.

Experts have come to the conclusion that the given demographic change will cause profound long-lasting economic impacts, both globally and within individual countries. It is vital to consider the optimal timing for changing a countrys pension system; to ensure that societal welfare is maintained; and also bearing in mind that changing a pension system does require numerous reforms, which could potentially become significant barriers.

In general, most Latin American countries have small open economies that have indeed been affected by the demographic transition; this directly influences household consumption, capital, aggregate output, and economic growth in both the short and long run. Due to these phenomena, it is imperative to find a path that points us towards the right time to adjust our social security system, one that is managed more efficiently and ensures only a minimal loss of welfare in the economy.

This study is motivated to find the right moment to change from the current social security system, Pay-As-You-Go, to a new social security system, Fully Funded. To achieve this goal, two primary objectives have to be completed: first, to simulate this reform under various economic shocks, particularly changes in oil income and interest rates, and second, to evaluate how the timing of the reform influences welfare outcomes across generations. Given the central role that oil plays in Ecuadors economy, special attention is paid to how fluctuations in oil prices or extraction levels may affect the performance and long-term sustainability of the proposed pension system.

II. Methods

The first step of this new model is to select and adapt the models that must be consistent with our Ecuadorian economy. Additionally, the new model will integrate parts of these models to create a combined framework.

A. Overlapping Generations Model

One important model to take into consideration in this research is the Overlapping Generations Model (OLG) which was developed by Allais (1947), Samuelson (1958), and Diamond (1965). This fundamental model, well-known in the field of macroeconomics, operates as follows: at any given time, individuals from different generations are alive and maybe trading with one another. Each generation interacts with other generations at different periods of its life. The OLG model is extensively studied because it allows for the analysis of the aggregate implications of life-cycle saving. These savings become capital stock as individuals need to finance their consumption during retirement. A key result of the OLG model is that the competitive equilibrium may not be Pareto optimal, as individual savings may be over-accumulated. The simplest OLG model is the two-period version, where individuals live for only two periods.

In this model, individuals interact in the market at different stages of their life cycles; a young person interacts with an older person, and later, as the young person ages, the interaction shifts to mostly younger individuals. Typically, this economy is composed of two cohorts or generations, often referred to as the young and the old.

In the economy, various subjects interact, including individuals and firms. In this case, individuals live for only two periods: they are born at time t, and consume C_t in period t, and C_{t+1} in the period t+1 with a utility function of:

$$u(c_t) + (1+\theta)^{-1}u(c_{t+1}),$$

where,

$$\theta \geq 0, u'(\cdot) > 0, u''(\cdot) < 0.$$

It is important to note that individuals work only in the first period of their lives, supplying inelastically one unit of labor and earning a real wage of w_t. They only consume a portion of their salary in the first period and save the remainder to consume during the second period of retirement. The number of individuals born at time t and working in period t is N_t. Also, the population grows at a rate n, so $N_t = N_0(1+n)^t$. On the other hand, firms act competitively and use constant returns technology, represented by $Y = F(K, N)$. The goal of each firm is to maximize profits, taking the wage rate, w_t, and the rental rate on capital r_t as given. Next, we will examine the optimization problems of individuals and firms and derive the market equilibrium.

In the case of individuals, the maximization problem at time t is:

$$\max\ u(c_t) + (1+\theta)^{-1}u(c_{t+1})$$

subject to

$$c_t + s_t = w_t,$$

$$c_{t+1} = (1 + r_{t+1})s_t$$

where w_t is the wage received in period t, and $r_t + 1$ is the interest rate on savings from period t to period t+1. In the final period, individuals consume all their resources, encompassing both interest and principal.

The first-order condition for the maximization problem is expressed as:

$$u'(c_t) - (1+\theta)^{-1}(1+r_{t+1})u'(c_{t+1})$$

B. Small Open Economy

The world economy comprises many small economies that interact with each other; with each transaction having a negligible impact on the global economy. This model assumes that each Small Open Economy shares identical preferences, technology, and market structures. Most macroeconomic interactions in a small open economy are related to the inter-temporal trade, which involves the exchange of resources across time. Inter-temporal trade is measured by the current account of the balance of payments. An adaptation of Irving Fishers (1930) model will be used for the case of a small open economy that consumes a single good over two periods: the young and the old.

In this model, an individual i maximizes lifetime utility U_1^i, which depends on consumption levels in both periods c^i.

$$U_y^i = u(c_y^i) + \beta u(c_o^i), \quad 0 < \beta < 1.$$

Where β is the subjective discount factor or time-preference factor that measures the individual's impatience to consume. As usual, the assumptions for the utility function $u(c^i)$ are: $u'(c^i) > 0$ strictly increasing in consumption, and $u''(c^i) < 0$ strictly concave.

Let y^i denote the individual's output and r the real interest rate in the world capital market on date 1. The lifetime budget constraint for consumption is:

$$c_y^i + \frac{c_o^i}{1+r} = y_y^i + \frac{y_o^i}{1+r}$$

This constraint restricts the present value of consumption spending to be equal to the present value of output. Since output is perishable, it cannot be stored for later consumption. The first-order condition for the previous problem is:

$$u'(c_y^i) = (1+r)\beta u'(c_o^i),$$

Which is known as the intertemporal Euler equation.

C. Social Security Systems

It is well-known that Social Security System affects both capital accumulation and the welfare of an economy. Due to this, social security programs were introduced to ensure a minimum level of income in retirement, as individuals might not save enough for their old age. Additionally, any program that impacts peoples income will have repercussions on savings and capital accumulation.

Individuals make a social security contribution while they are young and receive payments in their old age from the social security system. Let d_t denote the contribution of a young person at time t and b_t denote the benefit received by an old person in period t. There are two fundamentally different methods to run a social security system:

Fully Funded System: In this system, the contributions of the young at time t are invested and returned with interest at time t+1 to the then-old. For this case, $b_t = (1 + r_t)d_{t-1}$ where r_t is the rate of return on social security contributions.

Pay-As-You-Go System: This system acts like an unfunded scheme where current contributions made by the young are directly transferred to the current old. In this case, $d_t = (1 + n)d_t$ and the rate of return on the contributions is n.

III. The Model

The main objective of this model is to analyze and research the long-run macroeconomics and welfare levels in two different pension reforms using a model calibrated to Ecuadorian historical data. Additionally, the model combines the Overlapping Generations (OLG) and Small Open Economy (SOE) frameworks, where the main economic assumption that is taken into the model is that each generation lives for only 30 years and the range of analysis will span eight generations.

A. Demographics

In each period t, a new generation is born; the duration of a generation is 30 years. Individuals grow at a rate η_t per period. The population growth in this model is given by the following equation:

$$N_{t+1} = N_t(1 + \eta_{t+1}) \tag{1}$$

B. Technology

A representative firm uses a linear Cobb-Douglas production function that uses only labor as input and labor that augments technological growth to produce output. The function is represented by:

$$Y_t = A_t L_t \tag{2}$$

Where A_t is the labor augmenting technology factor and L_t is the labor input such as hours of work. The labor technology factor is determined by:

$$A_{t+1} = A_t(1 + g_{t+1}) \tag{3}$$

Where g is the growth rate. The next maximization linear problem gives the general equilibrium of this representative firm:

$$max\left\{A_t L_t - w_t L_t\right\}, \tag{4}$$

In this equilibrium, capital is not part of the maximization problem. The process of obtaining the equation is also in the appendix. Here w_t is the wage rate and the equilibrium of the linear problem yields the result:

$$w_t = A_t \tag{5}$$

C. Households problem

Individuals derive utility from the consumption during both their youth and old age; furthermore, the population does not have a bequest motive. Individuals seek to maximize their utility. As a consequence, each individual born at time t in different generations faces the following problem:

$$maxU_t = \frac{(C_t^y)^{1-\gamma} - 1}{1-\gamma} + \beta E_t \frac{(C_{t+1}^o)^{1-\gamma} - 1}{1-\gamma} \tag{6}$$

Subject to a set of budget constraints:

$$C_t^y + a_{t+1} = w_t - \tau_t \tag{7}$$
$$C_{t+1}^o = (1 + r_{t+1})a_{t+1} + T_{t+1} \tag{8}$$

In the previous equations β is the discount factor, C_t^y is the consumption of the youth, C_{t+1}^o is the consumption of the old ones. Also, a_{t+1} is the assets of each individual, r_{t+1} is the interest rate, T_{t+1} is the lump-sum transfer receives when individuals are old and comes from the social security payroll tax rate τ_t paid during their youth. It is also clear that hours of work and consumption cannot be negative; therefore, they must satisfy the following conditions:

$$\begin{aligned} a_{t+1} &\in [0, 1] \\ C_t^y &\geq 0 \\ C_{t+1}^o &\geq 0 \end{aligned}$$

The maximization problem of the households gives the following equation of Euler:

$$(C_t^y)^{-\gamma} = E_t \beta (1 + r_{t+1})(C_{t+1}^o)^{-\gamma} \tag{9}$$

D. Lump sum transfers

Since agents live in only one generation at a time, all the resources are consumed within the same period of life, with no bequest to the next generation. As a result, the government uses oil income to distribute lump sum transfers to agents alive in their old age at no cost. With this in mind, the government transfers the income in the following form:

$$N_t\tau_t + QP_t^{oil} + IT = N_{t-1}T_t \tag{10}$$

$$\tau_t = \tau w_t \tag{11}$$

Where QP_t^{oil} represents oil income at time t; also, N_t, N_{t-1} refer to the population sizes of the young and old, respectively. Additionally, τ_t is given by a payroll tax τ on the salaries w_t in each period t. It is important to clarify that the transfers depend on the type of social security system applied in the economy.

E. Social Security

An agent who works throughout his life must retire at some point t, and receive pension benefits T_{t+1}, calculated as a fraction of his social security payroll and the oil income.

In the Pay-As-You-Go system the equation that illustrates the transfers to agents is given by:

$$T_{t+1} = \frac{N_{t+1}\tau_{t+1} + QP_{t+1}^{oil} + IT}{N_t} \tag{12}$$

The transfers T_{t+1} in this Social security system depend on social security payroll the entire population $N_{t+1}\tau_{t+1}$ and the oil income QP_{t+1}^{oil} divided by the number of agents N_t. On the other hand, the Fully-Funded system is calculated as follows:

$$T_{t+1} = (1 + r_{t+1})\left(\tau_t + \frac{QP_{t+1}^{oil} + IT}{N_t}\right) \tag{13}$$

In this system, transfers under the Fully Funded system T_{t+1} depend on the social security payroll and oil income per capita, adjusted to the future by an interest rate r_{t+1}.

F. Equilibrium

An equilibrium for this economy, given the demographic growth structure consists of sequences over different generations of two social security tax rates τ_t, lump-sum transfers T_{t+1}, households allocations $[C_t^y, C_{t+1}^o, a_{t+1}]$, the factor for the firm A_t and factor prices w_t, r_{t+1} given by:

1) Given the two payroll tax rates, lump-sum transfers, and factor prices, households can solve their optimization problem.

2) Given factor prices, the representative firms optimization problem can be solved.

3) The equilibrium is defined as the point where all markets clear:

   - The labor market equilibrium (5) is:

$$A_t = w_t$$

   - The household equilibrium (9) is:

$$(C_t^y)^{-\gamma} = E_t \beta (1 + r_{t+1})(C_{t+1}^o)^{-\gamma}$$

   - Combining the transfers as the Pay-as-You-Go system (12) and the Euler equation for consumption (9) leads to the following expression:

(14)

$$(1 - \tau - \hat{a}_{t+1})^{-\gamma} = E_t \beta (1 + r_{t+1})^{1-\gamma} (\hat{a}_{t+1} + \frac{(1 + \eta_{t+1})(1 + g_{t+1})}{1 + r_{t+1}} (\tau + Z_{t+1}))^{-\gamma}$$

   - Similarly, combining the transfers under the Fully-Funded system (13) and the Euler equation for consumption (9) results in the following:

$$(1 - \tau - \hat{a}_{t+1})^{-\gamma} = E_t \beta (1 + r_{t+1})^{1-\gamma} (\hat{a}_{t+1} + (\tau + Z_t))^{-\gamma} \quad (15)$$

   - Where:

$$Z_t \equiv \frac{QP_t^{oil} + IT}{N_t A_t} \quad (16)$$

$$a_{t+1} = \hat{a}_{t+1} A_t \quad (17)$$

   The preceding equations are re-expressions of the oil income and the augmenting technology.

4) Using a numerical solution of the algorithm "By-section", where we aim to find the zero of a one-dimensional function that represents the transition between the Pay-as-You-Go and Fully-Funded systems. The equation is given by:

(18)

$$(1 - \tau - \hat{a}_{t+1})^{-\gamma} = (1 + \pi)^2 E_t \beta (1 + r_{t+1})^{1-\gamma} (\hat{a}_{t+1} + \frac{(1 + \eta_{t+1})(1 + g_{t+1})}{1 + r_{t+1}} (\tau + Z_{t+1}))^{-\gamma}$$
$$+ \pi E_t \beta (1 + r_{t+1})^{1-\gamma} (\hat{a}_{t+1} + (\tau + Z_t))^{-\gamma} + (1 - \pi) \pi E_t \beta (1 + r_{t+1})^{1-\gamma} (\hat{a}_{t+1})^{-\gamma}$$

5) The variables r_{t+1}, η_{t+1}, g_{t+1} and Z_{t+1} are subject to different random shocks, as given by:

$$r_{t+1} = (1-\theta_r)r^* + \theta_r r_t + \sigma_r \epsilon^r_{t+1} \tag{19}$$

$$\eta_{t+1} = (1-\theta_\eta)\eta^* + \theta_\eta \eta_t + \sigma_\eta \epsilon^\eta_{t+1} \tag{20}$$

$$g_{t+1} = (1-\theta_g)g^* + \theta_g g_t + \sigma_g \epsilon^g_{t+1} \tag{21}$$

$$Z_{t+1} = (1-\theta_Z)Z^* + \theta_Z Z_t + \sigma_Z \epsilon^Z_{t+1} \tag{22}$$

6) We must examine how the welfare analysis is derived, considering that we assumed our economy is governed by a central planner who discounts the utility of each generation at a rate R. Additionally, we must assume that the utility of both current and future generations is the primary concern of the planner, who seeks a social welfare function that represents the sum of the utilities of all generations over a specific period. In a Benthamite fashion, he weights utility by the size of each generation for this reason, I will employ the method outlined in Blanchard,O.& Fisher,S. (1993). Which yields the Benthamite Equation:

$$U = (1+\theta)^{-1}u(c_{2o}) + \sum_{t=0}^{T-1}(1+R)^{-t-1}[u(c_{1t}) + (1+\theta)^{-1}u(c_{2t+1})] \tag{23}$$

G. Calibration

The model requires the calibration of several parameters that will help define the economy and enable a numerical solution.

Table 1—: Parameters.

Parameters Requiring Calibration										
γ	β	τ	η	ϕ_r	ϕ_g	ϕ_Z	σ_r	σ_g	σ_Z	π

The parameter γ measures the degree to which an agent dislikes risk relation to their current wealth level. An individual whose gamma value is high means the individual is more risk-averse. The value for γ is determined from the lectures, where a common value is near 2.

The parameter β characterizes the impatience of agents, reflecting their preference for current consumption over future consumption. As a result, future benefits are valued less than the present ones. In standard economic models, the average value of β is typically set at 0.97.

I set $\tau = 0.2$ to approximate the social security tax rate in Ecuador, as the combined employee and employer tax is, on average, close to this value.

The rate of growth in the number of agents in a period t is defined as η. To calibrate η, I reference the average annual population growth of Ecuador (2000-2023) from World Health Organization data, which is approximately 1.1% with a projected increase of 24% by 2050.

The interest rate is denoted by r. To calibrate r, I estimate an Ar(1) model using interest rate data (2000-2023) from the Central Bank of Ecuador, as follows:

$$r_{t+1} = \phi_r r_t + \epsilon^r_{t+1} \tag{24}$$

The parameter ϕ_r allows me to fit the interest rate, and the estimated value based on the data is $\phi_r = 0.7905$, with a standard deviation $\sigma_r = 0.2119$.

The rate of growth in labor technology is defined by g. To calibrate g, I estimate an Ar(1) model using the inflation data series (2003-2023) from the Central Bank of Ecuador, which is given by:

$$g_{t+1} = \phi_g g_t + \epsilon^g_{t+1} \tag{25}$$

The result is the value of the parameter ϕ_g that allows me to fit the g parameter. Using the data, I estimate $\phi_g = 0.7521$ with a standard deviation of $\sigma_g = 0.0854$.

Oil income is defined by Z in equation (16). To calibrate Z, I estimate an Ar(1) model using the oil income data series (1995-2023) from the Central Bank of Ecuador, given by:

$$Z_{t+1} = \phi_Z Z_t + \epsilon^Z_{t+1} \tag{26}$$

The result is the value of the parameter ϕ_Z that allows me to fit the Z parameter. Using data, I estimate $\phi_Z = 0.8104$ with a standard deviation of $\sigma_Z = 0.1353$.

The incidence probability is challenging to determine, as the exact timing of system changes is uncertain. In this case, an intermediate probability of $\pi = 0.5$ is selected, which is equivalent to the probability of tossing a fair coin.

To summarize the parameter values in this section.

Table 2—: Parameters Calibrated.

Parameters Calibration										
γ	β	τ	η	ϕ_r	ϕ_g	ϕ_Z	σ_r	σ_g	σ_Z	π
0.978	0.97	0.2	1.1%	1	0.397	1	0.011	0.154	0.06	0.5

Note: Most parameter calibrations used Ar(1) model.

Source: Central Bank of Ecuador.

H. Numerical Simulation

The competitive equilibrium is defined in the previous section, where different methods of transfers are considered, depending on the social security tax selected. When reassembling some growth variables, the set of equations that describe the solution are affected in the following ways:

- The social security transfers.
- The consumption of households.
- The transfers from governments.
- The welfare equation.

Social Security Transfers

Pension benefits that show the change between the Pay-as-You-Go and Fully-Funded systems paths are given in equation (18):

$$(1-\tau-\hat{a}_{t+1})^{-\gamma} = (1+\pi)^2 E_t\beta(1+r_{t+1})^{1-\gamma}(\hat{a}_{t+1} + \frac{(1+\eta_{t+1})(1+g_{t+1})}{1+r_{t+1}}(\tau+Z_{t+1}))^{-\gamma}$$
$$+\pi E_t\beta(1+r_{t+1})^{1-\gamma}(\hat{a}_{t+1}+(\tau+Z_t))^{-\gamma} + (1-\pi)\pi E_t\beta(1+r_{t+1})^{1-\gamma}(\hat{a}_{t+1})^{-\gamma}$$

This equation represents the probability of changing the social security system and will allow us to find the variables $\hat{a}_{t+1}, r, g, Z, n$.

On the other hand, pension benefits that represent the new social security system under Fully-Funded scenario are given in equation (15):

$$(1-\tau-\hat{a}_{t+1})^{-\gamma} = E_t\beta(1+r_{t+1})^{1-\gamma}(\hat{a}_{t+1}+(\tau+Z_t))^{-\gamma}$$

This equation represents the new system and allows us to find the updated values for the variables $\hat{a}_{t+1}, r, g, Z$, and n under the new scenario. Both pension benefit equations are solved in the appendix.

Optimal Consumption of Households

As we know, the equilibrium of consumption is given by the Euler equation (9):

$$(C_t^y)^{-\gamma} = E_t\beta(1 + r_{t+1})(C_{t+1}^o)^{-\gamma}$$

This equation shows the equality between current and future consumption. The Euler equation is solved in the appendix. Although equilibrium between both consumptions is achieved, the equations to calculate each one are necessary for the results. The process to obtain these equations is detailed in the appendix. The current and future consumptions are given by equations (7) and (8):

$$\begin{aligned} C_t^y &= A_t(1-\tau) - a_{t+1} \\ C_{t+1}^o &= (1 + r_{t+1})a_{t+1} + T_{t+1} \end{aligned}$$

The Transfers

The transfers problem is solved in the appendix. The re-expression of transfers as functions of endogenous variables is derived from the two social systems. The Fully-Funded transfer is given by:

$$T_{t+1} = (1 + r_{t+1})(\tau + Z_t)A_t \tag{27}$$

The Pay-as-You-Go transfer is given by:

$$T_{t+1} = (1 + n_{t+1})(1 + g_{t+1})(\tau + Z_t)A_t \tag{28}$$

The Welfare Equation

The welfare equation in our model, which comes from the Command Optimum in the Lecture of Macroeconomics book is given by:

$$U = \left\{ \frac{(C_t^y)^{1-\gamma} - 1}{1-\gamma} + \beta\frac{(C_{t+1}^o)^{1-\gamma} - 1}{1-\gamma} \right\} N_t \tag{29}$$

This equation measures the welfare level of different generations and will indicate which scenario is better at the societal level. Additionally, it will help us determine the optimal time to transition between systems.

I. Numerical Algorithm

The model that combines the OLG and SOE models is characterized by equations (6)-(8), (17)-(22), and (27)-(29). To solve the model I apply the following process:

1) Select the endogenous variables (r, n, g, z, $\hat{a}_{t+1}$, a_{t+1}, C_t^y, C_{t+1}^o, T_{t+1}, U_t, U).

2) The equilibrium is defined by equations (6)-(8), (17)-(22), and (27)-(29)

3) This system of eleven equations in eleven endogenous variables is solved using using MATLAB.

4) Iterate 1000 times and analyze various scenarios that depend on the temporal evolution of systems (2 or 6) and exogenous variables such as oil price, interest rate, and technology growth rate.

5) Select the optimal welfare value from the average welfare obtained across different iterations and generations in the code.

IV. Results

I aim to evaluate different scenarios that directly affect the agents consumption behavior. Based on these scenarios, the resulting levels of societal welfare will be assessed in order to identify the most favorable outcomes. I examine six scenarios in which the timing of transition between social security systems varies. Specifically, the system change occurs at either T=2 or T=6. For each transition period, the economy is subjected to different exogenous shocks: an increase in the oil income, a decrease in the interest rate, and a combination of both shocks. The figures below illustrate the outcomes of these scenarios:

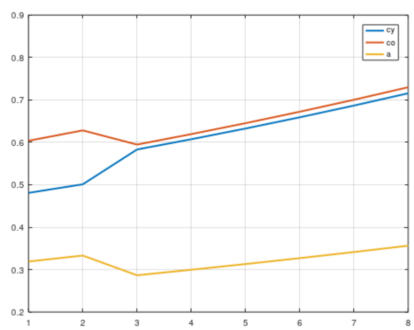

Figure 1. : Increase of Z - Jump in T=2.

With an increase in the price of oil and a change in the social security system at T=2, the consumption of young agents increases from 0.48 to 0.58. This occurs because higher disposable income enables them to shift some future consumption to the present.

In contrast, the consumption of elderly agents decreases from 0.63 to 0.59, as they receive lower benefits following the system change. At T=3, the consumption paths of both groups converge and stabilize, indicating a smoothing effect over time.

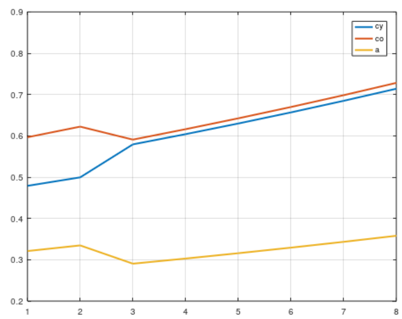

Figure 2. : Decrease of R - Jump in T=2.

In this scenario, a system change at T=2 coincides with a decrease in the interest rate. As a result, the consumption of young people increases from 0.50 to 0.57, due to a lower incentive to save for the future. Meanwhile, the consumption of the elderly decreases from 0.63 to 0.59, since the reduced interest rate yields lower returns on their savings. From T = 3, consumption levels of both groups smooth out and converge.

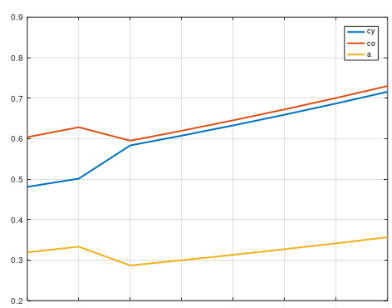

Figure 3. : Increase of Z -Decrease of R - Jump in T=2.

This scenario combines an increase in oil income and a decrease in the interest rate, with a system change occurring at T=2. The consumption of young people increases from 0.50 to 0.58, encouraged by the dual effect of higher income and the lower returns to saving, which incentivize present consumption. On the other hand, the consumption of elderly agents declines from 0.64 to 0.60, as both their savings and redistributed resources are negatively affected. From T=3, the consumption levels of both groups begin to smooth out and nearly overlap.

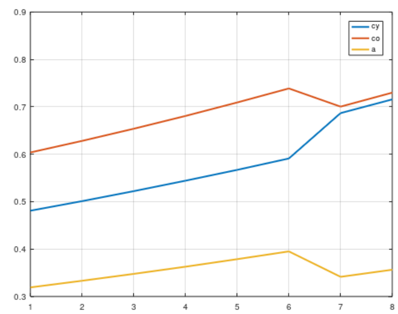

Figure 4. : Increase of Z - Jump in T=6.

In this scenario, the oil income increases while the system change is delayed until T=6. The consumption of young people increases, and it shows that their consumption increases from 0.59 to 0.78, as the additional income leads them to exchange future consumption for present consumption. In contrast, the consumption of elderly agents decreases from 0.74 to 0.7, since they receive fewer resources following the redistribution that comes with the new system. From T=7, both groupsconsumption paths stabilize permanently.

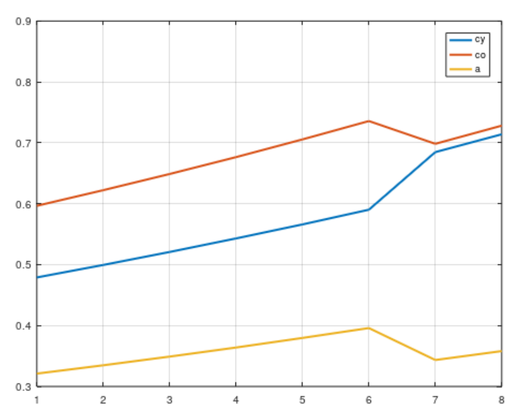

Figure 5. : Decrease of R - Jump in T=6.

Here, a decrease in the interest rate is combined with a system change at T=6. The consumption of young agents increases from 0.59 to 0.68, driven by the disincentive to save money, which encourages present consumption. Meanwhile, the consumption of elderly people drops from 0.74 to 0.70, as their savings generate lower returns. From T=7, consumption levels settle out permanently.

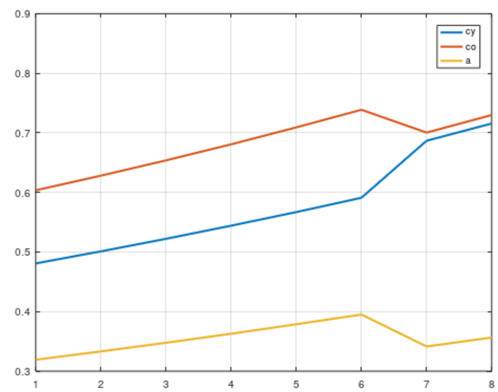

Figure 6. : Increase of Z -Decrease of R - Jump in T=6.

In the final scenario, both shocks are introduced at T=6, alongside the system change. The consumption of youth agents increases from 0.59 to 0.68 due to the combined effects of increased oil income and lower incentives to save. In turn, elderly consumption falls from 0.74 to 0.70, as the redistribution under the new system and lower returns on savings reduce their capacity to consume. From T=7, both groups experience stable, smoothed consumption paths.

A. *Welfare analysis*

In the previous scenarios, I used the Benthamite equation to measure welfare levels, following the method outlined by Blanchard,O.& Fisher,S. (1993). As mentioned earlier, six possible scenarios are analyzed.

Table 3—: Welfare Values.

Scenarios	T=2	T=6
Increase P(z)	-10,261	-10,479
Decrease r	-10,362	-10,543
Increase P(z) and Decrease r	-10,234	-10,439

Different scenarios can reflect various shocks that may affect the welfare, especially when we take into account the most important shocks in the Ecuadorian economy. This section evaluates welfare impacts under three shock scenarios over two time periods (T = 2 and T = 6), where we consider changing the social security system between Pay-As-You-Go and a Fully funded model. The Benthamite equation is used to quantify aggregate utility. Negative values reflect welfare losses. The scenarios examine: (1) an oil price increase, (2) an interest rate decrease, and (3) both shocks occurring simultaneously.

The first scenario reflects a shock caused by an oil price increase. Under this shock, the change between systems could be implemented at either T=2 or T=6. The welfare values in each period are -10.261 and -10.479, respectively.

The second scenario reflects a decrease in the interest rate. If the interest rate decreases, we could change our social security system at time T=2 or T=6, with the welfare values of -10.362 and -10.543 in each period.

The third scenario reflects the combination of both shocks. If both shocks occur in the economy, the change could be made at either time, with welfare values of -10.234 at time T=2 and -10.439 at time T=6.

If we compare the scenarios at times T=2 and T=6, the third scenario, representing the combination of both shocks, results in the lowest welfare values. Based on an analysis of times and scenarios, the third scenario with a change in the social security system at time T=2 shows the lowest welfare value of -10.234 among all the possible combinations.

V. Conclusion

This study has examined the long-run macroeconomic and welfare effects of introducing a fully funded pension system in an economy that will replace the older scheme, where the Pay-As-You-Go (PAYG) system is the primary mechanism, replicating key features of the Ecuadorian economy. It was found that the introduction of a fully funded pension system results in welfare gains for agents born into the new long-run equilibrium, compared to a scenario in which the PAYG system remains in place. However, the extent of these welfare improvements varies depending on the economic shocks affecting the economy. This analysis was conducted using an Overlapping Generations (OLG) model within a Small Open Economy framework calibrated to Ecuadorian data.

This study considered the effects of three macroeconomic shocks:

- An oil price increase.
- An interest rate decrease.
- Both occurring simultaneously.

The increase in household welfare at the long-run equilibrium depends on whether and how these shocks occur. A rise in oil prices directly boosts national income, enabling agents to increase current consumption. A decrease in interest rates raises consumption by discouraging savings and increasing disposable income.

Compared to the unfunded system, this study aimed to identify the optimal timing for transitioning to a funded system based on welfare outcomes. The most favorable scenario occurs when both shocks take place at time T=2, indicating that this timing leads to the highest welfare gains.

It is important to highlight that the models used in this thesis capture welfare changes only in response to the specific shocks applied within the Ecuadorian context, such as fluctuations in oil prices, given the countrys dependence on this income source. However, the model does not account for potential changes in other factors, such as how pension system reforms might affect households retirement decisions.

Moreover, the current model framework does not consider the effects of a pension reform on aggregate variables such as capital accumulation or commodity markets. While these aspects could be integrated into the OLG model, with relatively minor modifications, doing so would increase the computational complexity and cost of solving the model numerically.

In addition, this study does not account for transitional dynamics between long-run equilibria. Analyzing such transitional shocks is important, as agents living through the transition may be worse off even if the reform benefits future generations. As supported by both this research and the broader literature, it cannot be assumed that the introduction of a fully funded pension system would be Pareto optimal.

From a political economy perspective, future research could explore whether it is possible to improve overall welfare by transitioning to a fully funded system, using a model calibrated more precisely to Ecuadorian data and potentially accounting for transitional dynamics and broader macroeconomic variables.

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HouseholdsOptimization Problem
The representative consumer maximizes their life-cycle utility, given by:

$$maxU_t = \frac{(C_t^y)^{1-\gamma} - 1}{1-\gamma} + \beta E_t \frac{(C_{t+1}^o)^{1-\gamma} - 1}{1-\gamma} \tag{A1}$$

subject to the following budget constraints:

$$C_t^y + a_{t+1} = w_t - \tau_t \tag{A2}$$

$$C_{t+1}^o = (1 + r_{t+1})a_{t+1} + T_{t+1} \tag{A3}$$

The consumers problem is solved using the Lagrangian:

$$L = \frac{(C_t^y)^{1-\gamma} - 1}{1-\gamma} + \beta E_t \frac{(C_{t+1}^o)^{1-\gamma} - 1}{1-\gamma} + \lambda_1(w_t - \tau_t - C_t^y - a_{t+1}) + \lambda_2((1 + r_{t+1})a_{t+1} + T_{t+1} - C_{t+1}^o)$$

The first order conditions are:

$$\frac{\partial L}{\partial C_t^y} = (C_t^y)^{-\gamma} - \lambda_1 = 0 \tag{A4}$$

$$\frac{\partial L}{\partial C_{t+1}^o} = \beta(C_{t+1}^o)^{-\gamma} - \lambda_2 = 0 \tag{A5}$$

$$\frac{\partial L}{\partial a_{t+1}} = -\lambda_1 + \lambda_2(1 + r_{t+1}) = 0 \tag{A6}$$

Taking λ_1 and λ_2 from (A4) and (A5) we obtain:

$$\lambda_1 = (C_t^y)^{-\gamma} \tag{A7}$$

$$\lambda_2 = \beta(C_{t+1}^o)^{-\gamma} \tag{A8}$$

By combining (A7) and (A8) in in the first-order condition (A6), the intertemporal optimality condition is obtained:

$$(C_t^y)^{-\gamma} = E_t\beta(1 + r_{t+1})(C_{t+1}^o)^{-\gamma} \tag{A9}$$

Firms Optimization Problem

The following linear production function problem defines the Firms Optimization Problem:

(A10) $$Y_t = A_t L_t - w_t L_t$$

The first-order condition for maximum is:

(A11) $$\frac{\partial Y_t}{\partial L_t} = A_t - w_t = 0$$

Here, the equilibrium is presented as:

(A12) $$A_t = w_t$$

Social Security Transfers
The transfer mechanisms under both social security systems may yield different outcomes. The Fully-Funded system represents transfers through the following equation:

(A13) $$T_{t+1} = (1 + r_{t+1})(\tau_t + \frac{QP^{oil}_{t+1} + IT}{N_t})$$

Using equations (A9) and A(13), the pension benefits under the Fully-Funded system are derived by substituting C^y_t (A2) and C^o_{t+1} (A3) in (A9):

(A14) $$(w_t - \tau_t - a_{t+1})^{-\gamma} = E_t\beta(1 + r_{t+1})((1 + r_{t+1})a_{t+1} + T_{t+1})^{-\gamma}$$

By replacing into (A13), the equations (11),(17) and (A12) we obtain:

(A15)
$$(A_t - \tau A_t - \hat{a}_{t+1}A_t)^{-\gamma} = E_t\beta(1 + r_{t+1})((1 + r_{t+1})\hat{a}_{t+1}A_t + (1 + r_{t+1})(\tau A_t + \frac{QP^{oil}_{t+1} + IT}{N_t}))^{-\gamma}$$

By factoring out A_t and transferring it to the other side of the equation, we obtain:

(A16)
$$(1 - \tau - \hat{a}_{t+1})^{-\gamma} = E_t\beta(1 + r_{t+1})((1 + r_{t+1})\hat{a}_{t+1} + (1 + r_{t+1})(\tau + \frac{QP^{oil}_{t+1} + IT}{N_t A_t}))^{-\gamma}$$

By factoring out the common term $(1 + r_{t+1})$ on the right-hand side and substituting equation (16), we obtain:

(A17) $$(1-\tau-\hat{a}_{t+1})^{-\gamma} = E_t\beta(1+r_{t+1})^{1-\gamma}(\hat{a}_{t+1}+(\tau+Z_t))^{-\gamma}$$

The transfer under both social security systems may yield different outcomes. The Pay-as-You-Go system represents the transfers with the following equation:

(A18) $$T_{t+1} = \frac{N_{t+1}\tau_{t+1} + QP^{oil}_{t+1} + IT}{N_t}$$

Using equations (A9) and A(18), the pension benefits under the Pay-as-You-Go system are derived by substituting C^y_t (A2) and C^o_{t+1} (A3) into equation (A9):

(A19) $$(w_t-\tau_t-a_{t+1})^{-\gamma} = E_t\beta(1+r_{t+1})((1+r_{t+1})a_{t+1}+T_{t+1})^{-\gamma}$$

By replacing into (A12), the equations (11),(17) and (A18) we obtain:

(A20)
$$(A_t-\tau A_t-\hat{a}_{t+1}A_t)^{-\gamma} = E_t\beta(1+r_{t+1})((1+r_{t+1})\hat{a}_{t+1}A_t + \frac{N_{t+1}\tau A_{t+1} + QP^{oil}_{t+1} + IT}{N_t})^{-\gamma}$$

By factoring out A_t and isolating on the one side of the equation, we obtain:

(A21) $$(1-\tau-\hat{a}_{t+1})^{-\gamma} = E_t\beta(1+r_{t+1})((1+r_{t+1})\hat{a}_{t+1} + \frac{N_{t+1}}{N_t}\frac{A_{t+1}}{A_t}\tau + \frac{QP^{oil}_{t+1}+IT}{N_tA_t})^{-\gamma}$$

By multiplying and dividing the final term by $N_{t+1}A_{t+1}$ and factoring out the term $\frac{N_{t+1}A_{t+1}}{N_tAt}$, we obtain:

(A22)
$$(1-\tau-\hat{a}_{t+1})^{-\gamma} = E_t\beta(1+r_{t+1})((1+r_{t+1})\hat{a}_{t+1} + \frac{N_{t+1}}{N_t}\frac{A_{t+1}}{A_t}(\tau + \frac{QP^{oil}_{t+1}+IT}{N_{t+1}A_{t+1}}))^{-\gamma}$$

By replacing the population growth rate $\frac{N_{t+1}}{N_t} = (1+\eta_{t+1})$, the technology growth rate $\frac{A_{t+1}}{A_t} = (1+g_{t+1})$ and equation (16), also, and by factoring out $(1+r_{t+1})$, we obtain:

(A23) $$(1-\tau-\hat{a}_{t+1})^{-\gamma} = E_t\beta(1+r_{t+1})^{1-\gamma}(\hat{a}_{t+1} + \frac{(1+\eta_{t+1})(1+g_{t+1})}{1+r_{t+1}}(\tau+Z_{t+1}))^{-\gamma}$$

The transfer in equation (A13) can be re-expressed by substituting equation (11) and multiplying, and dividing by A_t in the last term:

(A24) $$T_{t+1} = (1+r_{t+1})(\tau A_t + \frac{QP^{oil}_{t+1} + IT}{N_t A_t} A_t)$$

By factoring out A_t and applying equation (16), we obtain:

(A25) $$T_{t+1} = (1+r_{t+1})(\tau + Z_t)A_t$$

The transfer in equation (A18) can be re-expressed by substituting equation (11), and by multiplying, and dividing the first term by A_t and the second term $N_{t+1}A_{t+1}A_t$. We then obtain:

(A26) $$T_{t+1} = \frac{N_{t+1}}{N_t}\frac{A_{t+1}}{A_t}\tau A_t + \frac{QP^{oil}_{t+1} + IT}{N_{t+1}A_{t+1}}\frac{N_{t+1}}{N_t}\frac{A_{t+1}}{A_t}A_t$$

By substituting the population growth rate $\frac{N_{t+1}}{N_t} = (1+\eta_{t+1})$, the technological growth rate $\frac{A_{t+1}}{A_t} = (1+g_{t+1})$ and equation (16), and by factoring out the common terms $(1+\eta_{t+1})$, $(1+g_{t+1})$ and A_t, we obtain:

(A27) $$T_{t+1} = (1+\eta_{t+1})(1+g_{t+1})A_t(\tau + Z_{t+1})$$