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Concepts from Mathematical Finance for Assessing and Achieving Intergenerationally Equitable Climate Mitigation: Implied CO2-Price, Carbon Interest Rate, Fair Share of GDP, and the Extension of an Integrated Assessment Model with a Climate Transformation Fund

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Implied CO_2-Price, Carbon Interest Rate, Fair Share of GDP, and the Extension of an Integrated Assessment Model with a Climate Transformation Fund

Concepts from Mathematical Finance for Assessing and Achieving Intergenerationally Equitable Climate Mitigation

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Abstract

This paper applies concepts from mathematical finance to the analysis of climate change mitigation costs and policy design. We define three metrics: an *implied CO_2-price* based on the total discounted cost of abatement and damages; a *carbon interest rate*, representing the internal rate of return of abatement actions; and a *fair share of GDP* to support effort-based climate funding. These metrics provide ex post evaluations of optimal emission pathways in integrated assessment models (IAMs), offering a descriptive framework for understanding the cost structure of climate policy.

In addition, we extend an integrated assessment model by incorporating a *climate transformation fund*, funded by a fixed GDP share, a CO_2-price, or a mix) and that finances climate-related costs over time. This extension improves intergenerational equity.

We consider the general case of a stochastic model.

Our numerical experiments on a classical (deterministic) DICE model show that the implied CO_2-price is a factor of 10 larger than the classical social cost of carbon (a marginal price) and that the implied share of GDP is roughly 3 %. However, the model exhibits substantial intergenerational inequality.

Introducing a climate transformation fund, our numerical result shows that roughly 2.4 % of the GDP is sufficient to cover all climate mitigation costs (including abatement and damage cost), equally distributing the burden among all generations. This intergenerational equitable climate change mitigation results in only a modest reduction of the GDP or consumption (significantly smaller than the funding rate).

Analysing a stochastic extension of a DICE model, we see that the presence of a climate transformation fund significantly reduces the convexity and volatility of the cost structure.

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1. Introduction

Integrated assessment models try to find an optimal emission path to mitigate climate change.

A classical example is the DICE model [19], which obtains the optimal emission path by maximizing the total (discounted) social welfare. This objective function may lead to intergenerational inequality, [10]. This is due to future damage cost changes being balanced to current abatement cost changes. The temporal distribution of the burden of the total cost is an outcome, it is not explicitly optimized for equity. For example, in the classical DICE model, there is no additional mechanism balancing the temporal distribution of the burden in any measure. Hence, the corresponding optimal emission pathways are optimal with respect to an aggregated utility, but do not consider intergenerational equity.

We use concepts from mathematical finance, like the par-swap-rate and internal rate of return, and introduce an implied par CO_2-price, an implied par CO_2-rate (implied rate of carbon) and an implied fair share of GDP covering the total cost that give an intuitive representation of the cost associated with climate change mitigation.

These are descriptive metrics, derived ex post from the cost function without actually interacting with the economic model.

We then augment the integrated assessment model with a model for a climate transformation fund with different funding methods: a fraction of the GDP, a CO_2 price, or a mixture.

We consider the general case of a stochastic model.

Our numerical experiments are performed on the DICE model [18, 19] and extensions thereof. The DICE model is frequently used to demonstrate the impacts of modifications [3, 28, 14, 12, 17].

We analyse optimal emission pathways maximising the aggregated social welfare and non-optimal pathways under the classical DICE model and the under the constraint of equi-distribution of burden.

A posteriori analysis on the classical DICE 2016 gives an implied CO_2–price of 500 \$/tCO_2, where 350 \$/tCO_2 are to be attributed to future emissions. The cost correspond to 3.8 % of the GDP.

Introducing a climate transformation fund, our numerical result shows that roughly 2.4 % of the GDP is sufficient to cover all climate mitigation cost (including abatement and damage cost), where the burden will be equally distributed among all generation. This

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intergenerational equitable climate change mitigation comes at a very small reduction of the total discounted utility.

Analysing a stochastic extension of a DICE model, we see that the presence of a climate transformation fund significantly reduces the convexity of the model, i.e., the risk.

As the numbers are derived from a DICE 2016 model, they may underestimate actual mitigation costs. However, the methods presented are general and can also be applied to other integrated assessment models.

Remark This paper combines and extends ideas that were previously published in the working papers [8] (implied CO_2-price) and [7] (fair share of GDP and climate transformation fund).

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2. Concepts to Express Cost of Climate Change Mitigation

Integrated assessment models (IAMs), as the DICE model, determine the optimal emission path by maximizing the total social welfare. The total annualized cost $t \mapsto C(t)$ associated with climate change is an output of the model.

In a classical DICE model [19] we would have $C(t) = C_{\mathrm{A}}(t) + C_{\mathrm{D}}(t)$, where C_{A} are the costs associated with abatement and C_{D} are the costs associated with damage.[1]

In this section, we will derive different concepts to express the total cost and the temporal relation of abatement and damage cost. The concepts share superficial similarities to the *social cost of carbon*, but, as we will illustrate, give a much fairer assessment of the true social cost of carbon.

The concepts we present are inspired by classic financial products, such as interest rate swaps.

Stochastic Model, Equivalent Martingale Measure We consider the general case of a stochastic model, where the respective quantities are random variables over a filtered probability space $(\Omega, \mathbb{Q}, \{\mathcal{F}_t\})$, were $\mathbb{Q} = \mathbb{Q}^N$ is an equivalent martingale measure associated with a numeraire N.

We will not make use of this more general setup until the very end, however, since the quantities that we consider are derived from valuation formulas, the only difference between the deterministic and the stochastic case is the addition of an expectation operator $\mathrm{E}^{\mathbb{Q}^N}$.

For a classical deterministic DICE model it is sufficient to just ignore the expectation $\mathrm{E}^{\mathbb{Q}^N}$.

2.1. Short Recapitulation of the Social Cost of Carbon

Before we introduce alternative concepts, let us recall the definition of the social cost of carbon.

The *Social Cost of Carbon* (SCC) [19, 21] derived from an integrated assessment model is defined as

$$\frac{\partial V(0)}{\partial E(t)} \Big/ \frac{\partial V(0)}{\partial Z(t)},$$

[1] Here and in the following, we adopt the notation from [11].

where $V(0)$ is the total discounted welfare, $E(t)$ is the time t emissions measured in tCO_2 and $Z(t)$ is the time t consumption measured in Dollar.[2]

Model extensions updating IAMs [12, 17, 2, 24] show a wide range of potential SCC, while also econometrics-based SCC estimates for single impacts such as mortality [1] or energy consumption [23], but also general SCC [22] emphasize the need to improve the precision of the estimated SCC,

The SCC is sometimes considered a good choice for a CO_2-price. However, since partial derivatives are involved, we immediately see that the SCC is associated with a *change emanating from the equilibrium state*. It is a marginal price. It is unclear whether this price will cover the cost along the climate-change mitigation paths. For this purpose we will now defined the implied CO_2-price and our analysis in Section 5.1 will show that the difference of the two is significant.

2.2. An Implied CO_2-Price that covers the Cost of a Transition to Net Zero

Let $C(t)$ be the time t (annualized) total cost. Let $E(t)$ denote the time-t (annualized) emissions. Let $N(t)$ denote the value process of a risk-free account that serves as numéraire. We value the financial contract that exchanges the costs $C(t)$ with a (forward) price $K \cdot N(t)$ paid for the emissions $E(t)$. The value of this financial contract is:

$$V_{\mathrm{Gap},\mathrm{CO}_2}(K;0) \ := \ \mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T (C(t) - K \cdot N(t) \cdot E(t)) \, \frac{N(0)}{N(t)} \, \mathrm{d}t \right) . \qquad (1)$$

Here, the factor $\frac{N(0)}{N(t)}$ is the discount factor[3].

The assumption that the CO_2-price is given by a constant K times $N(t)$ constitutes the assumption that the time-value adjusted CO_2-price is constant.

The time horizon T is chosen such that, approximately, $E(t) = 0, \ C(t)/N(t) = 0$ for $t \geq T$, such that we distribute *all* costs to *all* contributing emissions. See Section A.1.

[2]Note that we use the letter Z for the consumption, while [19] uses the letter C for consumption. We made the change as we will denote the total cost (damage and abatement costs) by $C(t)$. Consumption will not play a role in this paper - except to state this formula here once.

[3]In a model with constant deterministic continuously compounding risk-free interest rate r we have that $N(t) = \exp(r \cdot t)$ describes the value process of a bank account with initial value $N(0) = 1$.

The value $V_{\mathrm{Gap,CO_2}}(K;0)$ can be interpreted as the *gap* that is left by a uniform (time-value adjusted) CO_2-price K. For $K = 0$ we have that $V_{\mathrm{Gap,CO_2}}(0;0)$ gives the accumulated cost. Based on (1) we define the value $\bar{p} := K_{\mathrm{par}}$ (corresponding to a par-rate) that puts the gap to zero:

Definition (Implied Cost-of-Carbon): Let $\bar{p}$ be such that $V_{\mathrm{Gap,CO_2}}(\bar{p};0) = 0$, i.e.,

$$\bar{p} \ := \ \frac{\mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T \frac{C(t)}{N(t)}\,\mathrm{d}t\right)}{\mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T E(t)\,\mathrm{d}t\right)}. \tag{2}$$

We call $\bar{p}$ the *implied cost-of-carbon* or *implied CO_2-price*.

The price $\bar{p}$ can be interpreted as an *implied CO_2-price*. It is implied by the cost $C(t)$ related to climate change and its mitigation. It can be interpreted as the fair price resulting from a strict application of a *polluter pays principle*.

2.2.1. Separating Costs from Historical Emissions

The implied price $\bar{p}$ carries the cost of future damages that are resulting from historical emissions. To separate these costs, we may run the model with the (extreme) abatement policy given by perfect (100%) abatement of emissions from the first year on, i.e., $\mu \equiv 1$. The damage cost observed in this model will be the one that have to be attributed to historical emissions.

Definition: Let $C_{\mathrm{D}}^{\circ}(t)$ denote the damage cost observed in a model with full abatement from the first year. We then define the *implied CO_2-price of the future total cost* as

$$\tilde{p} \ := \ \frac{\mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T \frac{C(t)-C_{\mathrm{D}}^{\circ}(t)}{N(t)}\,\mathrm{d}t\right)}{\mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T E(t)\,\mathrm{d}t\right)}. \tag{3}$$

In addition we define the *implied CO_2-price of the future damage cost* as

$$\tilde{p}|_{\mathrm{D}} \ := \ \frac{\mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T \frac{C_{\mathrm{D}}(t)-C_{\mathrm{D}}^{\circ}(t)}{N(t)}\,\mathrm{d}t\right)}{\mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T E(t)\,\mathrm{d}t\right)}. \tag{4}$$

The value $\tilde{p}$ can be interpreted as a fair (polluter pays) CO_2 price that covers the future cost of abatement and damages, excluding those damages resulting from historical emissions.

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2.2.2. Comparison to the Social Cost of Carbon

The *implied CO_2-price* introduced in this section covers the total cost. In contrast to this, the social cost of carbon is a marginal price that does not necessarily cover the cost. To compare the two, we may calculate the par-rate price that is associated with $SCC(t)$. If $SCC(t)$ is used as a CO_2 price, then the total value paid for CO_2 emissions is

$$V_{\mathrm{SCC}}(0) \ := \ \mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T (SCC(t) \cdot E(t)) \frac{N(0)}{N(t)} \, \mathrm{d}t \right).$$

The corresponding par-rate price is given K solving

$$\mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T (SCC(t) \cdot E(t) - K \cdot N(t) \cdot E(t)) \frac{N(0)}{N(t)} \, \mathrm{d}t \right) \ \stackrel{!}{=} \ 0. \tag{5}$$

The solution of (5) is given by $K = p_{\mathrm{SCC}}$ with

$$p_{\mathrm{SCC}} \ = \ \frac{\mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T (SCC(t) \cdot E(t)) \frac{1}{N(t)} \, \mathrm{d}t \right)}{\mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T \ E(t) \, \mathrm{d}t \right)}. \tag{6}$$

Here p_{SCC} represents a fair constant price for all times, while the social cost of carbon varies over time.

We report the values $\bar{p}$ and $\tilde{p}$ and p_{SCC} for a classical (calibrated) DICE model in Section 5.1.

2.3. An Implied Interest Rate of Carbon

The following concept is similar to the classical social cost of carbon as it considers marginal cost/cost changes. It will be helpful later to investigate the temporal cost structure. It is inspired by an internal rate of return.

Let μ denote the model's abatement policy, e.g., the one associated with the optimal emission path. Let $C_{\mathrm{A}}(t)$ denote the corresponding time-t abatement cost and $C_{\mathrm{D}}(t)$ the corresponding time-t damage cost. We assume that a local change $\mathrm{d}\mu(T)$ in the abatement factor $\mu(T)$ affects the abatement cost C_{A} in T only, as it is the case for the DICE model.

A local change $\mathrm{d}\mu(T)$ in the abatement factor generates a change to the time-T abatement cost $\frac{\mathrm{d}C_{\mathrm{A}}(T)}{\mathrm{d}\mu(T)}$, a change in the emissions, and hence a change in the future damage cost $\frac{\mathrm{d}C_{\mathrm{D}}(s)}{\mathrm{d}\mu(T)}$ in $s \geq T$. Hence, an increase of time-T abatement cost is associated with an decrease of time s, $s \geq T$, damage cost. Based on this we define

Definition (Interest Rate Term Structure of Carbon, Carbon Interest Rate): For any fixed time $T \geq 0$, let $r^{\mathrm{SCC}}(T)$ be the solution of the following equation

$$\frac{\mathrm{d}C_{\mathrm{A}}(T)}{\mathrm{d}\mu(T)} + \sum_{t_k \geq T} \frac{\mathrm{d}C_{\mathrm{D}}(t_k)}{\mathrm{d}\mu(T)} \exp(-r^{\mathrm{SCC}}(T) \cdot (t_k - T)) \ = \ 0. \tag{7}$$

We call $T \mapsto r^{\mathrm{SCC}}(T)$ *implied interest rate term structure of carbon* or, for short, the *carbon interest rate*.

For the interpretation of $r^{\mathrm{SCC}}(T)$ consider the case where the abatement factor $\mu(T)$ is *increased* by some $\mathrm{d}\mu(T)$. This increase will require abatement cost of $\mathrm{d}C_{\mathrm{A}}(T)$ (an investment), but will lead to an decrease of emissions $\mathrm{d}E(s)$ for $s \geq T$, and thus to an decrease of damage cost $\mathrm{d}C_{\mathrm{D}}(s)$ for $s \geq T$ (a return). This situation can be considered as an investment: one invests the abatement cost $\mathrm{d}C_{\mathrm{A}}(T)$ and earns the saved damage cost $\mathrm{d}C_{\mathrm{D}}(s)$. The rate $r^{\mathrm{SCC}}(T)$ is the *internal rate of return* (IRR) of this investment.[4]

The rate r^{SCC} provides important insights on the temporal distribution of the cost structure. If for example r^{Market} denotes a market rate, then $r^{\mathrm{SCC}}(t) > r^{\mathrm{Market}}$ implies that funding abatement with a market loan will be profitable for the society.

In the case of a stochastic model, Equation (7) defines a path-wise quantity (which is not even $\mathcal{F}_T$ measurable). In the stochastic case, the more meaning full definition of a term

[4]Likewise we can interpret a reduction of the abatement factor $\mu(T)$ by $\mathrm{d}\mu(T)$ as a loan: resulting in borrowing the saved abatement cost and repaying with the increased damage cost.

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structure curve would be to define the rate as a spread over the risk neutral rate, like for a Bond spread or (implied) default intensity:

Definition (Carbon Interest Rate Spread): For any fixed time $T \geq 0$, let $\lambda^{\mathrm{SCC}}(T)$ be the solution of the following equation

$$\mathrm{E}^{\mathbb{Q}^N}\left(\frac{\mathrm{d}C_{\mathrm{A}}(T)}{\mathrm{d}\mu(T)} + \sum_{t_k \geq T} \frac{\mathrm{d}C_{\mathrm{D}}(t_k)}{\mathrm{d}\mu(T)} \frac{N(T)}{N(t_k)} \exp(-\lambda^{\mathrm{SCC}}(T) \cdot (t_k - T))\right) \; = \; 0.$$

We call $T \mapsto \lambda^{\mathrm{SCC}}(T)$ the *carbon interest rate spread*.

However, in a stochastic model, it may be still informative to determine the random variable $T \mapsto r^{\mathrm{SCC}}(T)$ by solving Equation (7) path-wise, as its volatility indicates the variations, i,e., the risk in the cost structure.

2.3.1. The Carbon Interest Rate in the Classical DICE Model

For the DICE model we may derive the rate r^{SCC} in terms of other model components, if the abatement policy μ is optimal.

Differentiating the model's objective function with respect to $\mu(t_j)$ we find from its optimality $\mathrm{d}V(0)/\mathrm{d}\mu(t_j) \ = \ 0$ that

$$\begin{aligned} 0 \ &= \ \int_0^T \frac{\mathrm{d}U(t)}{\mathrm{d}\mu(t_j)} \frac{N(0)}{N(t)} \mathrm{d}t \ = \ \int_{t_j}^T \frac{\mathrm{d}U(t)}{\mathrm{d}\mu(t_j)} \frac{N(0)}{N(t)} \mathrm{d}t \\ &= \ \int_{t_j}^T \sum_{t_k \geq t_j} \frac{\mathrm{d}U(t)}{\mathrm{d}C(t_k)} \frac{\mathrm{d}C(t_k)}{\mathrm{d}\mu(t_j)} \frac{N(0)}{N(t)} \mathrm{d}t \ = \ \sum_{t_k \geq t_j} \frac{\mathrm{d}C(t_k)}{\mathrm{d}\mu(t_j)} \left(\int_{t_j}^T \frac{\mathrm{d}U(t)}{\mathrm{d}C(t_k)} \frac{N(0)}{N(t)} \mathrm{d}t \right) . \end{aligned}$$

That is

$$\sum_{t_k \geq t_j} \frac{\mathrm{d}C(t_k)}{\mathrm{d}\mu(t_j)} \cdot \exp\left(-r^{\mathrm{SCC}}(t_j)(t_k - t_j)\right) \ = \ \sum_{t_k \geq t_j} \frac{\mathrm{d}C(t_k)}{\mathrm{d}\mu(t_j)} \cdot \left(\frac{R(t_j, t_k)}{R(t_j, t_j)} \right) , \qquad (8)$$

with

$$R(t_j, t_k) \ = \ \int_{t_j}^T \frac{\mathrm{d}U(t)}{\mathrm{d}C(t_k)} \frac{N(0)}{N(t)} \mathrm{d}t.$$

Here U denotes the model's utility function. The rate r^{SCC} is the sum of the discount rate and a weighted average of the utility rate.[5]

It is important to note that this relation holds only for the optimal abatement policy. For non-optimal emission pathways, the rate r^{SCC} may differ significantly and provide important insights into intergenerational inequality.

We will report the *implied interest rate term structure of carbon* for different parameterizations of the classical DICE model in Section 5.2.

[5] Note that $r^{\mathrm{SCC}}(t_j) \ \neq \ -\frac{1}{t_k - t_j} \log\left(\frac{R(t_j,t_k)}{R(t_j,t_j)}\right)$.

2.4. An Implied Fair Share of GDP

In Section 2.2 the cost were assigned to emissions (a polluter-pays principle). An alternative is to consider cost on a per GDP basis, which follows more an effort sharing principle.

Let $C(t)$ be the time t (annualized) total cost. Let $GDP(t)$ denote the time-t (annualized) GDP (that has not yet been reduced by the cost $C(t)$).[6] We value the financial contract that exchanges the costs with a fraction $K \cdot GDP(t)$ of the GDP. The value of this financial contract is:

$$V_{\text{Gap,GDP}}(K;0) \ := \ \mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T (C(t) - K \cdot GDP(t)) \, \frac{N(0)}{N(t)} \, \mathrm{d}t \right). \tag{9}$$

Based on this we can define the value $\bar{q}$ (corresponding to a par-rate) that puts the gap to zero:

Definition (Implied Cost-of-Carbon as Fraction of the GDP (Implied Share of GDP)): Let $\bar{q}$ be the such that $V_{\text{Gap,GDP}}(\bar{q};0) = 0$, i.e.,

$$\bar{q} \ := \ \frac{\mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T \frac{C(t)}{N(t)} \, \mathrm{d}t \right)}{\mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T \frac{GDP(t)}{N(t)} \, \mathrm{d}t \right)}. \tag{10}$$

We call $\bar{q}(T)$ the *implied cost-of-carbon as fraction of the GDP (up to time T)* or, for short, *implied share of GDP*.

The fraction $\bar{q}$ can be interpreted as the fair share of GDP required to mitigate climate change costs.

Note that $\bar{q}$ is calculated a posterior from a model that already determined the optimal emission path by deducing the cost from the GDP. To determine the actual optimal emission path corresponding to an effort sharing that results in an equal share of the GDP, we need to modify the model, see Section 3.

[6]In, for example, the DICE model, the $GDP(t)$ is reduced by the cost $C(t)$, which results in $GDP_{\text{net}}(t) := GDP(t) - C(t)$.

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2.4.1. Separating Costs from Historical Emissions

As above, we may separate the cost associated with historical emissions and define

$$\tilde{q} \ := \ \frac{\mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T \frac{C(t)-C_{\mathrm{D}}^{\circ}(t)}{N(t)} \, \mathrm{d}t\right)}{\mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T \frac{GDP(t)}{N(t)} \, \mathrm{d}t\right)}. \tag{11}$$

2.4.2. Comparison to the Social Cost of Carbon

As in Section 2.2.2, we can calculate an implied share of GDP that matches the cost covered if the $SCC(t)$ is used as a CO_2 price.

The corresponding par-rate K is given by solving

$$\mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T (SCC(t) \cdot E(t) - K \cdot GDP(t)) \frac{N(0)}{N(t)} \, \mathrm{d}t\right) \stackrel{!}{=} 0. \tag{12}$$

The solution of (12) is given by $K = q_{\mathrm{SCC}}$ with

$$q_{\mathrm{SCC}} \ = \ \frac{\mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T (SCC(t) \cdot E(t)) \frac{1}{N(t)} \, \mathrm{d}t\right)}{\mathrm{E}^{\mathbb{Q}^N}\left(\int_0^T \frac{GDP(t)}{N(t)} \, \mathrm{d}t\right)}. \tag{13}$$

Here q_{SCC} represents a constant share of GDP over time, while the social cost of carbon varies.

We report the values $\bar{q}$ and $\tilde{q}$ and q_{SCC} for a classical (calibrated) DICE model in Section 5.1.

2.5. Summary

In this section we introduced some a posteriori expressions of the cost structure of an IAM. Expressing the total cost as an equally distributed rate does not mean that the cost is equally distributed: while $C(t)$ and $\bar{p} \cdot N(t) \cdot E(t)$ have the same discounted value, their temporal distribution is different. This difference can be interpreted as an intergenerational inequality in the burden of cost..

Modifying the model so that costs are distributed evenly over time will result in a different optimal emission path and, consequently, different total cost. To achieve this different cost distribution we modify the model and introduce a *climate transformation fund*.

3. An Integrated Assessment Model with Climate Transformation Fund

To address the issue of intergenerational inequality, we aim for a more balanced cost structure, where every generation carries to some extent the same burden. In other words, we aim to constrain the determination of the optimal emission pathways to those that comply with intergenerational equality. To achieve this, we introduce a climate transformation fund.

3.1. Model for the Climate Transformation Fund

We extend an integrated assessment model by incorporating a dedicated fund that is used to cover cost associated with climate change. Utilizing different funding models, this allows us to investigate some quantities for interest, e.g.,

- *the fair share of the GDP to mitigate climate change costs*, and,
- *the fair CO_2 price to mitigate climate change costs.*

In our numerical experiment we apply our model extension to the standard DICE model, but our assumptions does not rely on very specific properties of the model.

Let $t \mapsto F(t)$ denote the amount of cash in the fund at time t measured in trillion USD. The fund operates according to the following rules:

- Initially, the fund is empty, i.e., $F(0) = 0$
- The fund accrues interest at a specified rate r_F. This rate can be chosen to match the model's time-preference rate ρ, but this is not a strict requirement.
- Each year, a fixed amount $f(t)$ is allocated to the fund, increasing its balance while correspondingly reducing the GDP.
- The fund is used to cover damage and abatement costs. If total costs exceed the fund's balance, the fund is depleted ($F(t) = 0$), and the excess is covered by the GDP, i,.e., the excess costs are directly borne by the current generation, reducing their consumption and/or investment.

We then consider several different models for the financing $f(t)$ of the fund. These are

- **None**: For $f \equiv 0$ we recover the unmodified IAM, i.e., in our case, the classical DICE model.

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- **Fixed Share of the GDP**: In this model $f(t) = q \cdot GDP(t)$, where q is a constant. In other words, a constant fraction of the GDP is allocated to the fund.
- **CO_2-Price**: In this model, $f(t) = p \cdot E(t) \cdot N(t)$, where p is a constant, $N(t)$ is the accrual factor and $E(t)$ is the emission.
- **Mixed**: The model uses a mixed/hybrid funding with a CO_2-price p and a GDP fraction q, where, $f(t) = q \cdot GDP(t) + p \cdot E(t) \cdot N(t)$,

Given that setup, we investigate:

Implied CO_2-Price : *Is there a minimal price $p = p^*$ such that all climate costs (damage and abatement) will be covered from the fund?*

Fair Share of GDP : *Is there a minimal fraction $q = q^*$ such that all climate costs (damage and abatement) will be covered from the fund?*

Here, climate costs refer to climate change mitigation and adaptation costs, in terms of the model. For the DICE model this is the sum of abatement cost and damage cost.

3.2. Impact of the Climate Fund on Abatement Policy and Cost Allocation

The presence of the fund will usually alter the IAM's optimal emission path, even if the fund's accrual rate agrees with the model's time preference rate. In the DICE model, the optimal emission path is given by maximizing the *total* welfare. The distribution of the cost is implicit; it is an outcome. This results in the effect that costs are deferred to the future: small abatement costs are preferred in the present, allowing for large damage costs in the future. This cost structure represents an intergenerational inequality, see [10]. The fund facilitates effort sharing. For example, in the "fair share of GDP" model every generation takes the same share of GDP to mitigate abatement and damage cost.

Because the money from the fund cannot be consumed or invested (it can only be saved or used to compensate for abatement and damage costs), the fund changes the set of feasible choices and, consequently, the optimal emission path. Put differently, the determination of the optimal emission path is constrained to the space of all intergenerationally equitable solutions, where the funding model f defines the level of intergenerational equality.

Because the IAM's optimal emission path depends on the funding model f, the optimal values p^*, q^* usually have to be calculated numerically. We propose a candidate value for the funding model and then determine the abatement strategy that maximizes the total welfare (resulting in the optimal abatement policy conditional to the funding model). We then check if the fund was sufficient to cover the total costs.

3.3. Results on the Funding Models

3.3.1. Non-Existence of a CO_2-Price Covering the Total Cost

At least in the classical DICE model, a CO_2-price that finances the emission trading fund such that it covers the total cost along the optimal emission path does not exist.

In other words: we may calculate the implied CO_2-price $\bar{p}$ for a model with climate transformation fund funded by the price p (i.e., $f(t) = p \cdot E(t) \cdot N(t)$), and observe that, for the DICE model, $\bar{p} = \bar{p}(p) > p$ for all p.

As the funding is deduced from the GDP, the funding of the emission trading fund impacts the social welfare. As the funding is proportional to the emission, this represents an additional incentive to reduce the emissions. Reducing the emissions will reduce the damage cost, but also the amount available in the fund, as the funding is proportional to the emission. However, the reduction in cost is less than the reduction in the fund. This is perhaps most evident by considering the limit case: with a 100 % abatement, the emissions will be zero, hence, no funding will occur, but damages from historical emissions and abatement cost will still constitute a significant amount of cost.

However, for the implied CO_2-price $\tilde{p}|_{\mathrm{D}}$ covering the damage cost associated with future emissions, we find that there exists a model funding $f(t) = p \cdot E(t) \cdot N(t)$ such that $\tilde{p}|_{\mathrm{D}}(p) = p$. In this model, the funding covers the the damage cost associated with future emissions, while damage cost associated with historical emissions and abatement cost are still directly deduced from the GDP. See Section 5.3.1.

3.3.2. Existence of a Share of the GDP Covering the Total Cost

Under comparably mild assumptions we may prove that a solution q^* exists:

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Existence of an Equilibrium Share of the GDP Let $t \mapsto C(t)$ denote the cost structure of the optimal emission path for the model with $f \equiv 0$. Let $\bar{C}_0 := \sum_{i=0}^{n} C(t_i)/(1+r_F)^i$ denote the total discounted cost, discounted at the funding rate r_F. Let $GDP(0)$ denote the initial value of the GDP. Assume that $\frac{\partial C}{\partial GDP(0)} > 0$ and $\bar{C}_0 < GDP(0)$. Then there exists a funding f such that the fund covers the total cost of the optimal emission path.

Proof: From the assumption $\frac{\partial C}{\partial GDP(0)} > 0$ we have that a reduction of the initial GDP does not increase the total cost. From $\bar{C}_0 < GDP(0)$ we can take the amount $\bar{C}_0$ form the initial GDP and use it as an initial funding $F(0) = \bar{C}_0$. As this fund accrues at the rate r it will cover the total cost $\bar{C}_0$.

The assumption that $\frac{\partial C}{\partial GDP(0)} > 0$ is a reasonable one, as both, abatement cost and damage cost, should increase with industry activity, and industry activity should increases with initial GDP. For the DICE model we may explicitly verify this assumption.

3.3.3. A Mixed Funding Model

Using the mixed funding model

$$f(t) \ := \ q \cdot GDP(t) \ + \ p \cdot E(t) \cdot N(t)$$

we may determine for a given CO_2-price p the equilibrium fraction of the GDP $q = q^*$ such that the climate transformation fund covers all cost.

It is clear that even for high CO_2 prices p, a significant fraction q is still required to cover the abatement cost and the damage cost associated with historical emissions.

The specific function $p \mapsto q^*(p)$ is not clear a-priori, since the presence of the emission relative funding component p will alter the optimal emission path by incentifying a higher abatement policy, which increases the total cost, but speeds up the mitigation process.

3.4. The Cost of Intergenerational Equality

The climate transformation fund that covers all the costs of climate transformation provides a means to ensure equitable intergenerational climate mitigation.

By comparing the optimal emission paths of the classical model and the model with fund, one may, to some extent, assess the cost of intergenerational equality.

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We may, for example, compare the total cost (C), the consumption (Z), the GDP or the discounted utility (U).

The total utility is expected to decrease in a model with intergenerational equitable climate mitigation; otherwise the original emission paths would not maximize the utility. One might expect that the GDP and consumption decrease as well.

It is not necessarily the case that the total cost (C) rises in a model with intergenerational equitable climate mitigation. The opposite is true. This is due the climate transformation fund incentifying a faster climate change mitigation (impacting the GDP, but easing the total cost).

This compensating effect may result in a GDP reduction that is in total lower than the funding rate q^*, which may be a bit surprising at first.

Our numerical results using the classical DICE model show that the negative effect on the utility is comparably small, see Section 5.3.4.

3.5. Abatement Model Realism

For the DICE model, our numerical results show that the presence of a climate transformation fund strongly incentifies a much faster abatement. Within the model, the abatement rate instantaneously jumps from its initial value of $3\ \%$ to a value above $50\ \%$. This behaviour is also already present in the classical DICE model, and shows that the starting value is just sub-optimal.

A fast achievement of higher abatement values appears to be unrealistic. To account for this, one may constrain the slope of the abatement policy $t \mapsto \mu(t)$. While this leads to more realistic abatement policies, qualitatively our numerical results for the DICE model remain the same.

4. Stochastic Model, Risk and Convexity

In [10, 11] a stochastic DICE model with stochastic discount rates and stochastic abatement policy was introduced. Here, the stochastic abatement model mimics the adaptation of the abatement speed to the interest rate level.

In [10] it was shown that such a DICE model exhibits *convexity* such that the expected cost in derived from the stochastic model are significantly higher than the cost derived from the deterministic model where interest rates agree with the expected level.

As our numerical results show that the presence of the climate transformation fund significantly reduces the dependency of the carbon interest rate (hence, the cost structure) to the level of the discount rate, it is natural to investigate the effect of the climate transformation fund to convexity.

4.1. Stochastic Interest Rate and Stochastic Abatement Model

As in [10] we consider a classical Hull-White model for the short rate (discount rate), [16, 5].

$$\begin{aligned} \mathrm{d}\rho(t) &= (\theta(t) - a(t)\rho(t))\,\mathrm{d}t + \sigma(t)\,\mathrm{d}W(t), \quad \rho(t_0) := r_0, \\ \mathrm{d}N(t) &= \rho(t)\,\mathrm{d}t, \end{aligned} \tag{14}$$

where an exact time-discretization scheme [6] was used in the numerical experiments.

For the abatement policy we consider the three-parameter model

$$\begin{aligned} \mu(t,\omega) &= \min\left(a_0 + (a_1 + a_2 \cdot \rho(t,\omega)) \cdot t, 1.0\right), \\ \mu(0,\omega) &= \mu_0, \end{aligned} \tag{15}$$

where $\mu_0 = 0.03$ is fixed and a_0, a_1, a_2 are free parameters. Here a_0 allows to correct the non-optimal initial value, a_1 determines the linear abatement speed and a_2 the sensitivity of the abatement speed to the interest rate level.

The model captures the most important aspects by still remaining numerically tractable. In a more general setup we would consider the optimal stochastic policy adapted to the filtration associated with our stochastic model.

The stochastic abatement policy itself could be interpreted as re-calibration of a non-stochastic IAM, adapting to the change in the interest rate level.

Within this model, all quantities (abatement cost, damage cost, consumption, GDP, etc.) become stochastic as a result of the stochastic abatement policy.

The numerical calculation can become involved. For example, to determine the now stochastic carbon interest rate we use a stochastic algorithmic differentiation [9] for the partial derivative of the abatement and damage cost.

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5. Numerical Results

In this section we summarize numerical results obtained from a classical DICE model and its extension with a climate transformation fund.

We consider a model with time horizon $T = 500$ years, a time discretization step size $\Delta t = 1.0$ year, a time preference rate (discount rate) of 1.5 % (annual linear compounding) and an elasticity of the marginal utility of consumption of $\eta = 1.45$.

Due to the long time horizons, the choice of the discount rate and of the elasticity of the marginal utility of consumption have a strong influence on the resulting optimal pathways [13, 15, 20, 25, 26]. However, the qualitative aspects of our results do not depend significantly on these choices. For example, in the analysis of the carbon interest rate, a change of the discount rate will just result in a parallel shift of all quantities.

5.1. Implied CO_2-Price and Implied Share of GDP for the DICE Model

We first summarize in Table 1 the implied values defined in Section 2 for the optimal emission path of the classical DICE model.[7]

We see a significant mismatch between p_{SCC}, the SCC par-rate which we had defined in Section 2, and $\bar{p}$, the implied CO_2 -price. That $p_{\mathrm{SCC}} << \bar{p}$ implies that taking the classical Social Cost of Carbon as a CO_2-price does not cover the total cost of climate change mitigation. The difference is significant. The true cost of climate mitigation are a factor of 10 higher. The society covers the remaining cost, but without any relation to the emissions.

The implied fair share of GDP is around 3 %. To put this number into some comparison, the U.S. national defence spending is between 4% and 5% of the U.S. GDP (1990-2012), [27].

We will see in Section 5.3.2 that DICE 2016 would suggest that intergenerational equitable climate mitigation would have been possible at less that 3 % of the GDP.

[7]The results of this section can be re-created in the numerical experiment

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ClimateModelExperimentCO2Pricing
```

available at [4].

Implied CO_2-Price	
Implied CO_2 price $\bar{p}$ (total cost):	$\approx 500\ \$/\mathrm{tCO}_2$
Implied CO_2 price $\tilde{p}$ (without cost of historical emissions):	$\approx 350\ \$/\mathrm{tCO}_2$
Social Cost of Carbon Par-Rate p_{SCC}:	$\approx 45\ \$/\mathrm{tCO}_2$

Implied Fair Share of GDP	
Implied Fair Share of GDP $\bar{q}$ (total cost):	$\approx 3.20\ \%$
Implied Fair Share of GDP $\bar{q}$ (without cost of historical emissions):	$\approx 2.40\ \%$

Table 1: Implied values for the classical DICE model (DICE 2016) with discount rate 1.5%.

5.1.1. Temporal Distribution of the SCC

The social cost of carbon $SCC(t)$ has a fairly strong time-dependence. Figure 1 shows the difference of the discounted social cost of carbon $SCC(t)/N(t)$ and the par rate p_{SCC}. That $SCC(0)$ is lower than p_{SCC} implies that the present generation is paying less (compared to its contribution measured in terms of emissions). The difference

$$p_{\mathrm{SCC}} - \frac{SCC(t)}{N(t)}$$

can be interpreted as a measure of inter-generational inequality. For a discussion on inter-generational inequality, see [10].

5.2. Implied Carbon Interest Rate (according to DICE)

While the implied CO_2-price and a the implied share of the GDP give a 1-figure assessment of the cost, the implied interest rate term structure of carbon gives a deeper insight in the temporal relation of abatement cost to damage cost.

5.2.1. Implied Carbon Interest Rate for Optimal Emission Paths

We calculate the carbon interest rate $t \mapsto r^{\mathrm{SCC}}(t)$ in a classical DICE model and compare it to the discount rate (the pure time preference rate) r, and the effective rate of the

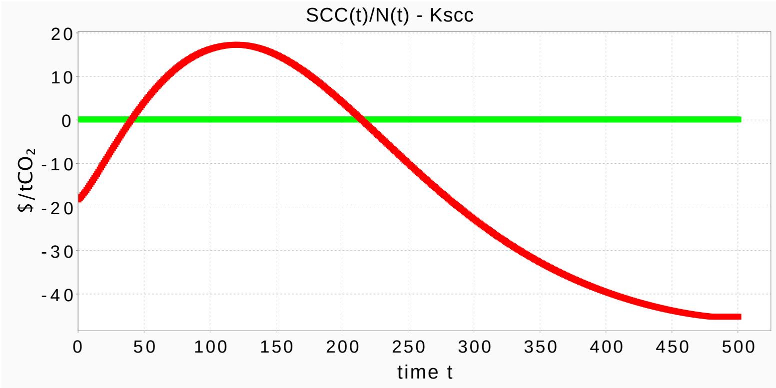

Figure 1: **Deviation of the SCC from the corresponding Par-Value.** The difference of the (discounted) social cost of carbon $t \mapsto SCC(t)$ from the corresponding (constant) forward price p_{SCC}. Generations living in years 40 to 220 pay more, while others pay less.

discounted utility, i.e., the relative utility gain by a reduction of the cost, to be precise

$$-\frac{1}{\Delta t} \log \left(\frac{\partial U(t+\Delta t)}{\partial C(t+\Delta t)} \Big/ \frac{\partial U(t)}{\partial C(t)} \right) . \tag{16}$$

Figure 2 shows the result for the optimal abatement policy. The carbon interest rate, which can be considered as an internal rate of return, is much higher than the model's discount rate (pure time preference rate) and slightly lower than the effective rate of the discounted utility. What is however more striking is that the curve is declining. This means that it is more profitable for current generation to increase the abatement (hence, invest abatement cost) than for future generations. This is in accordance with the declining growth rate utility.

5.2.2. Implied Carbon Interest Rate for Non-Optimal Emission Paths

The situation changes dramatically if we consider a non-optimal abatement policy.

We consider a simplified one-parametric abatement policy (see [11]) with

$$\mu(t) \ = \ \min \left(\mu(0) + \frac{1-\mu(0)}{T^{\mu=1}} \, t, 1.0 \right) ,$$

where the parameter $T^{\mu=1}$ specifies the time at which the policy achieves 100 % abatement.

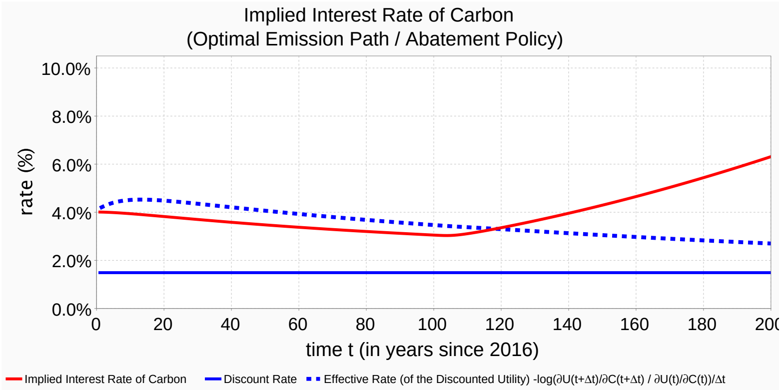

Figure 2: **Implied Carbon Rate.** The internal rate of return associated with carbon abatement (carbon interest rate, red) for a *calibrated* DICE model, i.e., the optimal abatement policy (green). The model's discount rate is 1.5% (blue). For the optimal emission path the carbon interest rate is close to the model's utility rate (dotted blue). The rising of the rate after year ≈ 120 coincides with the abatement becoming close to 100 %.

We use this abatement policy with $T^{\mu=1} = 150$, noting that the slow increase of the abatement policy rather reflects the actual real situation. Figure 4 compares the the optimal and this non-optimal abatement policy.

With this abatement policy we find the implied carbon interest rate as depicted in Figure 3, which we should compare to Figure 2.

Now, even from an utility point of view, it is profitable to perform more abatement, where early generations see a rate of return of 10 % or more to perform faster abatement.

Table 2 gives some key figures for the two models: The difference in the consumption appears small, the difference in the implied share of the GDP mild. The cost are higher, but emissions are much higher too, such that the implied CO_2-prices declines. However, the intergenerational inequality has worsen significantly on the model with non-optimal emission path (Figure 3).

We will see that this situation is changed by the introduction of a climate transformation fund.

5.2.3. The Carbon Interest Rate for the Current Generation

We now investigate the short end of the carbon interest rate term structure, i.e., the value of $r^{\mathrm{SCC}}(1)$, the interest rate associated with an increase in abatement in time $t = 1$.

Property	**Optimal Abatement**	**Non-Optimal Abatement**	**Difference**
Utility:	495359	495283	-0.02 %
Consumption (10^{12} USD):	36835	36583	-0.68 %
Cost (10^{12} USD):	1637	1910	+16.74 %
Emission (GtCO2):	3379	4568	+35.17 %
Implied CO_2 Price $\bar{p}$ (\$/tCO_2):	484	418	-13.64 %
Par Social Cost of Carbon p_{SCC} (\$/tCO_2):	46	48	+5.25 %
Implied CO_2 Price (Abatement) $\bar{p}\vert_{\mathrm{A}}$ (\$/tCO_2):	47	24	-50.17 %
Implied CO_2 Price (Damage) $\bar{p}\vert_{\mathrm{D}}$ (\$/tCO_2):	437	395	-9.67 %
Implied Fair Share of GDP $\bar{q}$:	3.2 %	3.7 %	+16.36 %

Table 2: Comparison of the model with optimal emission path and the model with the one-parametric non-optimal policy with $T^{\mu=1} = 150$.

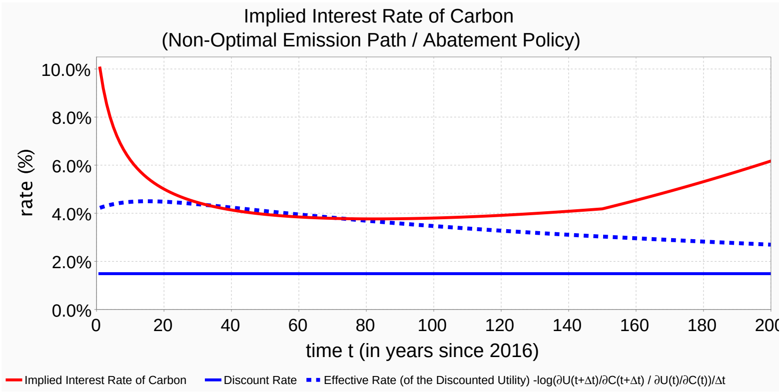

Figure 3: **Implied Carbon Rate.** The internal rate of return associated with carbon abatement (carbon interest rate, red) for a DICE model with a discount rate of 1.5% (blue) and fixed (non-optimal) abatement policy. The rate can become much larger that the model's utility rate (dotted blue).

We numerically calculate its dependency on different *non-optimal* abatement policies, specifically on the parameter $T^{\mu=1}$ that determines the slope of the abatement policy.[8] Figure 5 shows the dependency of $r^{\mathrm{SCC}}(1)$ on the model parameter $T = T^{\mu=1}$: the faster the abatement is performed, the lower is the internal rate of return associated with carbon abatement.

[8]The results of this section can be found in the numerical experiment

`ClimateModelExperimentSCCRate`

available at [4].

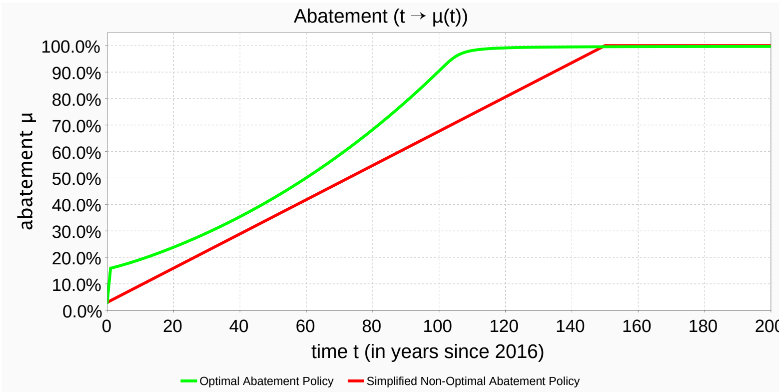

Figure 4: Comparison of the optimal abatement policy (green) and the one-parametric non-optimal policy with $T^{\mu=1} = 150$ (red).

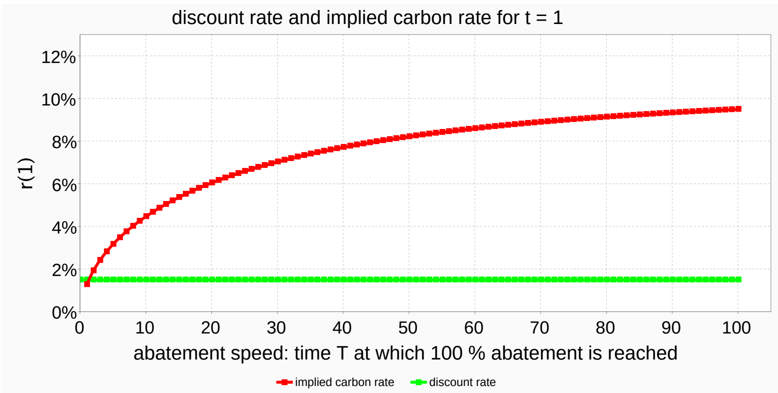

Figure 5: **Implied Carbon Rate for $t = 1$ as Function of Abatement Speed.** The internal rate of return associated with carbon abatement in $t = 1$ as a function of the time $T = T^{\mu=1}$ at which the model achieves 100 % abatement (red), in a DICE model with a discount rate of 1.5% (green).

5.3. IAM with a Climate Transformation Fund

We use a classical DICE model extended with the *climate transformation fund*.

All other parameters remains the same, e.g., the interest rate (time-preference) rate is $1.5\ \%$ per annum and agrees with the fund's accrual rate. For non-funding $f \equiv 0$ we recover the previously used DICE model.

5.3.1. An Equilibrium CO_2-Price covering the Future Damage Cost

As mentioned in Section 3.3.1 we cannot expect that a funding model $f(t) = p \cdot E(t) \cdot N(t)$ allows for an utility maximizing emission path that covers all cost.

To analyse this for the DICE model we create different models for given funding CO_2-prices p and then calculate the implied CO_2-price.

Figure 6 shows the total cost and total emissions as a function of p and it becomes obvious that the emissions fall faster than cost, such that the emission funded fund cannot cover the total cost.

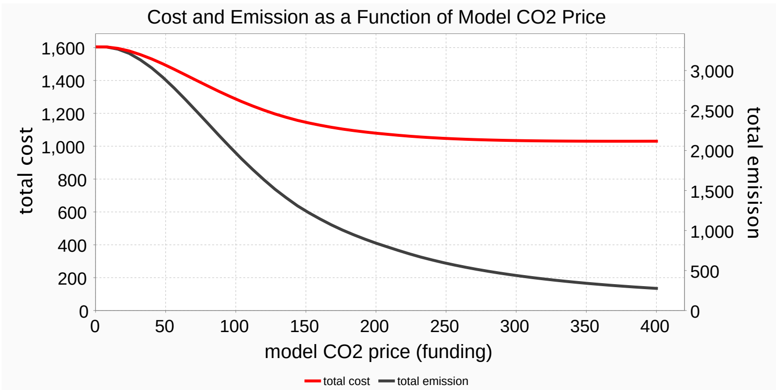

Figure 6: The total discounted cost and total emission for different funding CO_2 prices p.

In Figure 7 we depict the implied CO_2-prices $\bar{p}$, $\tilde{p}$ and $\tilde{p}|_{\mathrm{D}}$ as a function of the model's funding parameter p. While there is indeed no p such that $p = \bar{p}(p)$, it is possible to find an equilibrium price $p = p^*$ such that $p^* = \tilde{p}|_{\mathrm{D}}(p^*)$. Thus, in a DICE model, a CO_2-price can cover the future damage cost, but abatement cost and historical damage cost cannot be covered. We find that this equilibrium price is $p^* \approx 250\ \$/\mathrm{tCO}_2$.

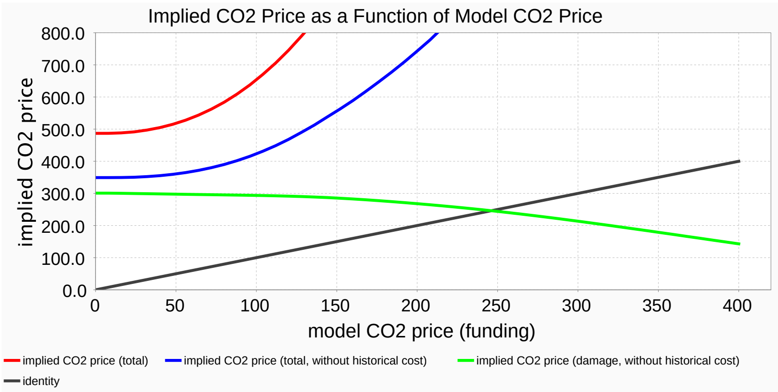

Figure 7: The implied CO_2-prices $\bar{p}$ (red), $\tilde{p}$ (blue) and $\tilde{p}|_{\mathrm{D}}$ (green) for different funding CO_2 prices p (DICE model with climate transformation fund).

5.3.2. Fair Share of GDP to Mitigate Climate Change Cost (according to DICE)

We use $f(t) = q \cdot GDP(t)$ to add a constant fraction of the GDP to the fund. Based on this, the model determines the optimal emission path for the given q. We then determine q^* as the minimum value q such that the fund remains positive for all times (i.e. all costs are covered by the fund).

It turns out that a funding rate $q^* = 2.4\ \%$ of the GDP is sufficient to cover the total cost.

Under the presence of the climate transformation fund, the optimal emission path prefers a much higher abatement (Figure 10), resulting in lower emissions (Figure 11). The fund will result in lower total cost (Figure 12).

Note that the fund results in an even distribution of cost per GDP among all generations. Here Figure 8 shows how the cost that are covered by the fund compare to the funds funding, with a breakdown to abatement and damage cost in Figure 9.

This intergenerational equitable climate change mitigation comes at surprisingly small cost, Table 3 gives a comparison of the most important quantities. While the utility is slightly negatively impacted, the total discounted consumption is even improved. This is due to a small temporal shift of the consumption: while current generations reduce their consumption slightly future generation have an over-compensating gain in consumption. For the unconstrained model, i.e., the model without fund, this situation is not optimal as current generations have a higher utility gain than future generations. That said, the effect is still small (see Figure 16).

The implied CO_2-price is significantly increasing, which is due to the emission reduction being much larger than the cost reduction.

However, a remarkable effect is seen in the comparison of the implied fair share of the GDP: the classical DICE models shows a value of 3.2 % to cover the total cost. The model with climate transformation fund shows a value of 2.4 %, which of course agrees with the fund rate to cover the total cost. This value is significantly lower. The effect is due to the cost reduction, which is much larger than the impact on the GDP (compare Figure 12 and Figure 15)

Property	**Classical Model**	$(q = 2.4\%)$ **Model with Fund**	**Difference**
Utility:	495359	493667	-0.34 %
Consumption (10^{12} USD):	36835	37265	+1.17 %
Cost (10^{12} USD):	1637	1212	-25.95 %
Emission (GtCO2):	3379	1602	-52.59 %
Implied CO_2 Price $\bar{p}$ (\$/tCO_2):	484	756	+56.17 %
Par Social Cost of Carbon p_{SCC} (\$/tCO_2):	46	0	-99.16 %
Implied CO_2 Price (Abatement) $\bar{p}\|_{\mathrm{A}}$ (\$/tCO_2):	47	208	+337.29 %
Implied CO_2 Price (Damage) $\bar{p}\|_{\mathrm{D}}$ (\$/tCO_2):	437	549	+25.63 %
Implied Fair Share of GDP $\bar{q}$:	3.2 %	2.4 %	-26.25 %

Table 3: Comparison of the classical DICE model and the same model extended with a climate transformation fund, funded by 2.4 % of the GDP (the minimum rate covering all climate cost, including damage cost associated with historical emissions).

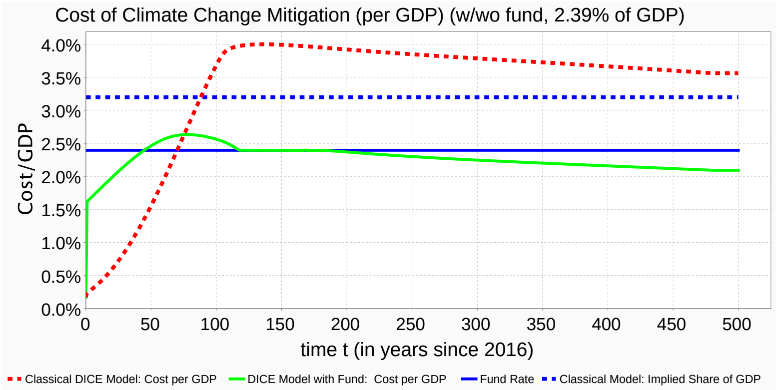

Figure 8: Cost per GDP in the DICE with Emission Fund 2.4 % of GDP (optimal emission path).

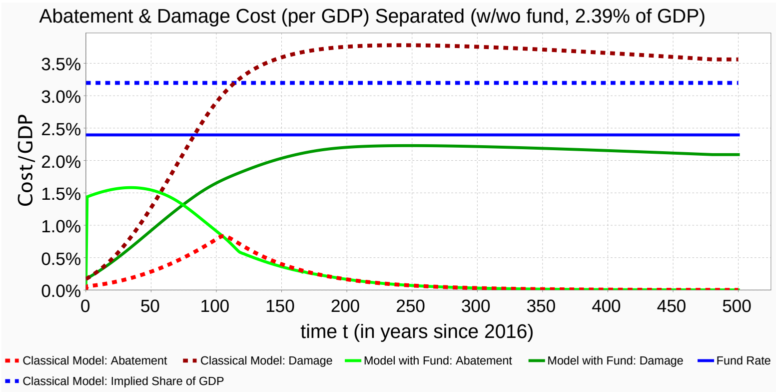

Figure 9: Cost per GDP as in Figure 8 separated for Abatement and Damage Cost in the Model with Emission Fund 2.4 % of GDP and the Classical Model (each with its optimal emission path).

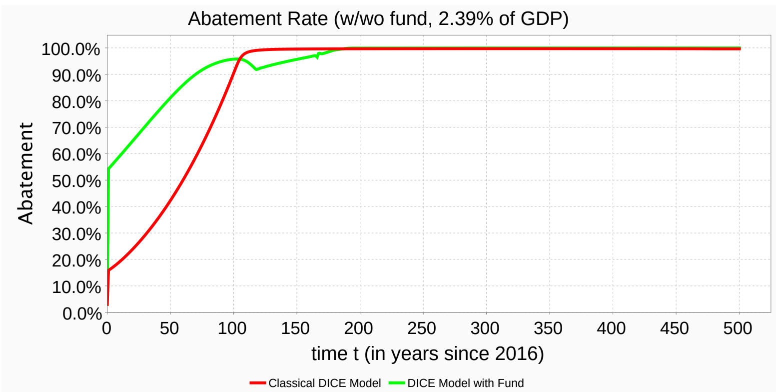

Figure 10: Abatement in the calibrated DICE with Emission Fund 2.4 % of GDP

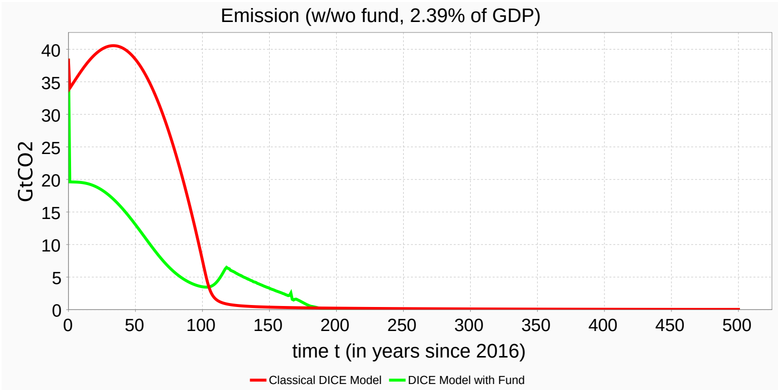

Figure 11: Emission in the calibrated DICE with Emission Fund 2.4 % of GDP

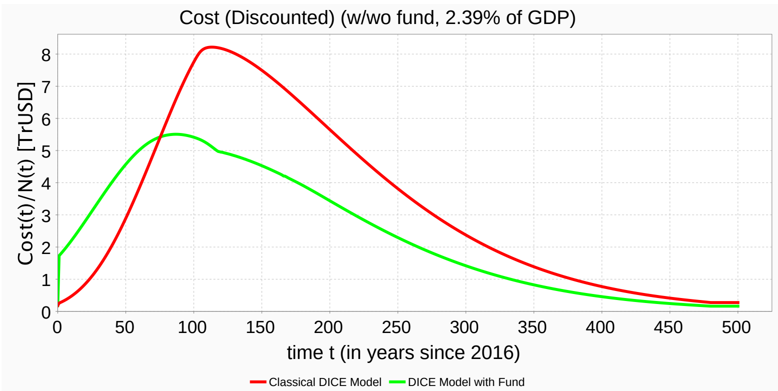

Figure 12: Cost in the calibrated DICE with Emission Fund 2.4 % of GDP

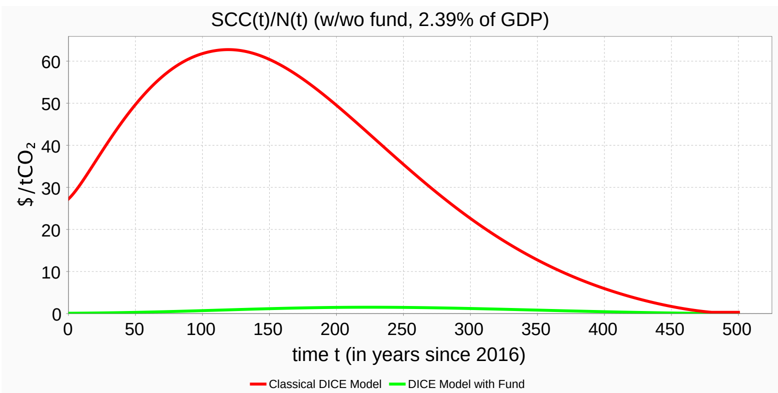

Figure 13: Discounted social cost of carbon

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5.3.3. Implied Carbon Interest Rate in the Model with Fund

We investigate the implied carbon interest rate $t \mapsto r^{\mathrm{SCC}}(t)$ in the model with climate transformation fund, the results are depicted in Figure 14.

The carbon interest rate is much lower, but still above the model's discount rate. It remains roughly constant for a significant time, then rises as 100 % abatement is approached. This can be seen as another indication that the climate transformation fund improves intergenerational equity.

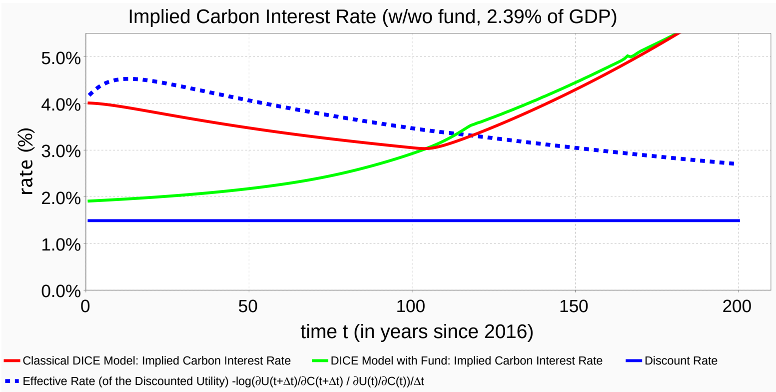

Figure 14: The implied carbon interest rate relating abatement cost to avoided damage cost: In the classical model it is more profitable to perform more abatement for current generations than for later generations (red curve). In the model fund the situation is changes, the implied interest rate of carbon remains constant for a significant time, then rises as 100 % abatement is approached (green).

5.3.4. The Impact of Intergenerational Equity on the GDP and the Consumption

The presence of the fund F results in different optimal emission paths, depending on the funding rate q. For the limit case of $q = 0$ we recover the optimal emission path of the classical DICE model.

The emission fund will levelize the burden of costs across generations, levelize in a "per GDP sense".

In other words, if we compare the model equipped with the fund F with the classical model, the question arises how the model with the fund redistributes certain quantities. We compare the output (GDP) and the consumption of the optimal emission path in the respective models.

Figure 15 shows the relative difference of the GDP. The funding puts a strain on the

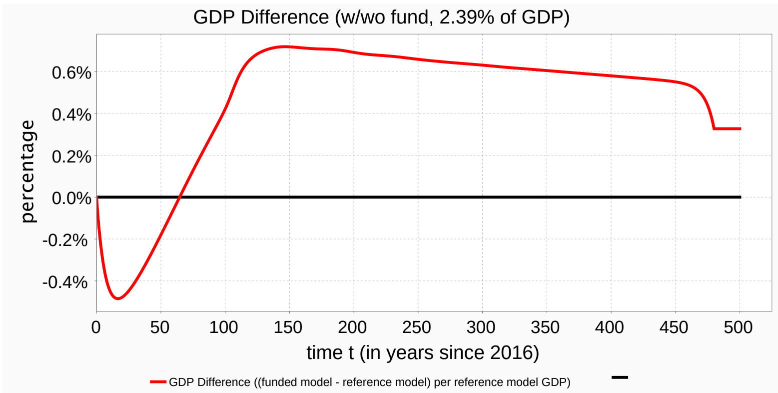

Figure 15: The difference of the GDP: funded model relative to the classical model. The plot shows the function $t \mapsto \frac{GDP^q(t) - GDP^0(t)}{GDP^0(t)}$, where $GDP^q(t)$ denotes the GDP in the model with funding rate q.

GDP and the GDP is lowered by 0.4% at first, but soon profits from the reduces damage costs.

The comparable low impact on the GDP can be explained by an adjustment of the savings rate, which is (as in the classical DICE model) optimized. Figure 16 shows the relative difference of the consumption, where we see that the initial supply to the fund corresponds with a reduced consumption.

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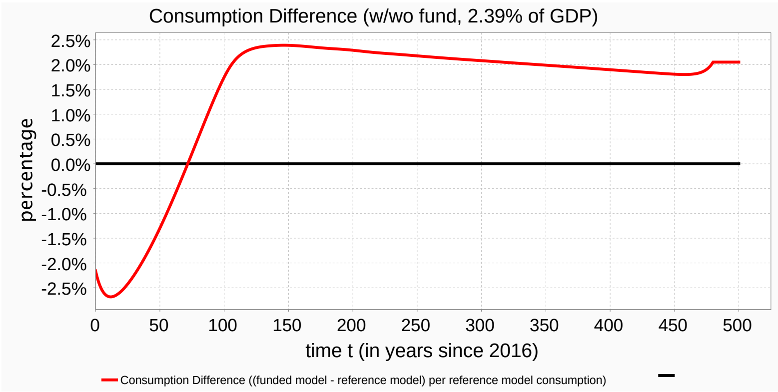

Figure 16: The difference of the Consumption: funded model relative to the classical model.

5.3.5. Role of the Discount Rate

The discount rate is a much debated exogenous parameter of the DICE model, [13, 15]. In the classical DICE model, it has a strong impact on the optimal abatement strategy.

For low to moderate discount rates we observe that adding a fund as a model extension to the DICE model significantly reduces the sensitivity of the optimal abatement strategy to the discount rate. While it is still qualitatively similar: higher interest rates lead to slower abatement, the presence of the fund will favor early abatement and the differences in the optimal abatement strategies are reduced, Figure 17. Note that for interest rates below 2.5%, the time at which the 85% abatement level is reached is almost independent of the discount rate in the model with fund. For low interest rates, the optimal funding rate q^* to cover all future costs is almost independent of the discount rate.

Figure 18 shows the corresponding emission path for different levels of the discount rate. Figure 19 shows the effective cost per GDP.

Table 4 gives an overview of numerical experiments with different discount rate levels.

For discount rates below 2.5%, the abatement strategy and the optimal funding rate depend only weakly on the discount rate. For discount rates higher than 2.5% the discount rate has again a strong impact on the optimal abatement policy, similar to the classical model. This behavior is reasonable, as for high discount rates, deferring abatement is favourable, as future damages are devalued by increasing the discount rate. Such an emission path creates more nominal damages in the future, hence more nominal costs. Thus, the required funding rate rises.

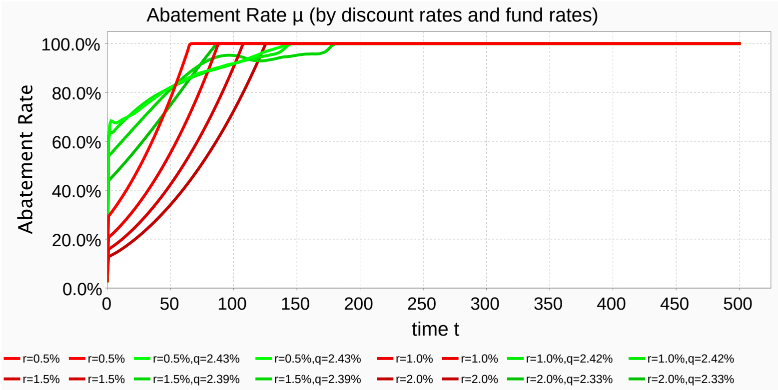

Figure 17: Optimal abatement policy for different discount rate levels, low to moderate discount rates.

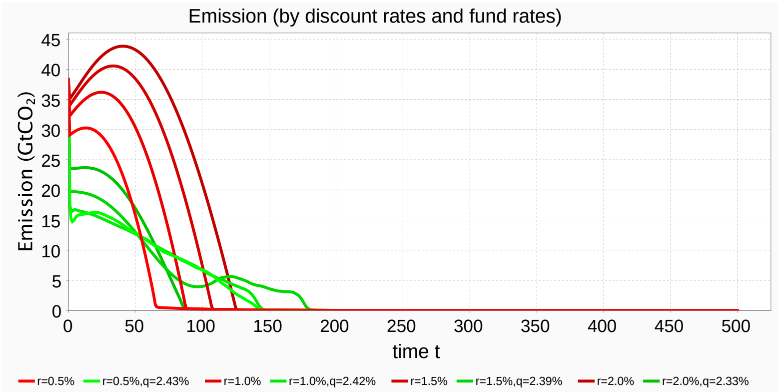

Figure 18: Emissions on the optimal abatement policy for different discount rate levels, low to moderate discount rates.

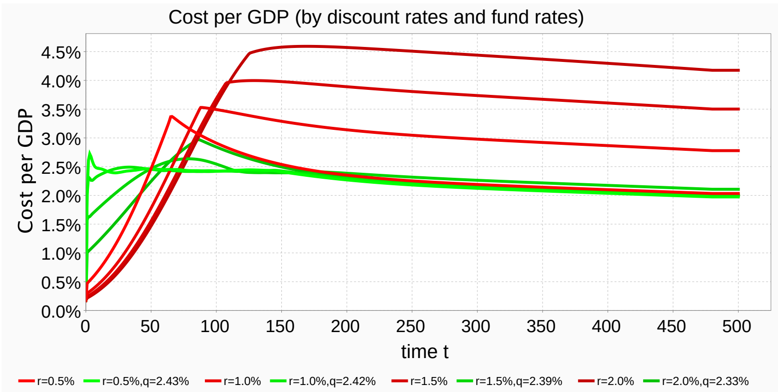

Figure 19: Cost per GDP for different discount rate levels, low to moderate discount rates.

Discount Rate	Funding Rate that Covers all Costs
0.5%	2.43%
1.0%	2.42%
1.5%	2.39%
2.0%	2.33%
2.5%	2.23%
3.0%	2.98%
3.5%	3.72%
4.0%	4.64%

Table 4: Funding rate required to cover abatement and damage costs, depending on the chosen discount rate.

The rise in the funding rate is remarkable, too. A high funding rate is clearly not desirable and illustrates that the optimal emission paths that are associated with higher discount rates could present economic and political challenges.

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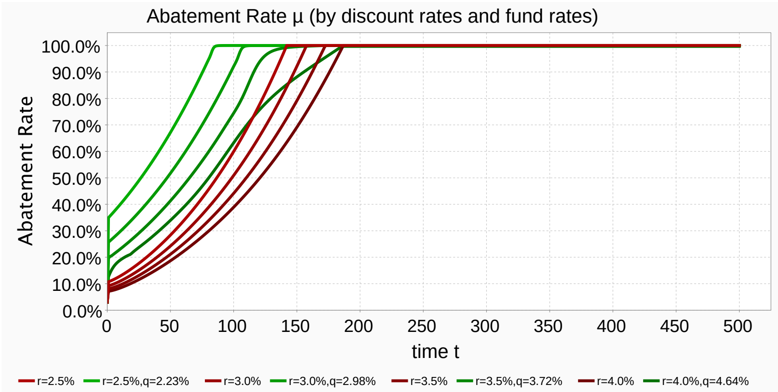

Figure 20: Optimal abatement policy for different discount rate levels, higher discount rates.

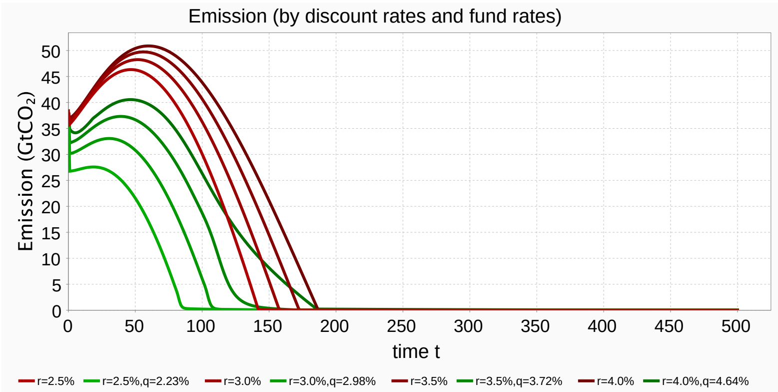

Figure 21: Emissions on the optimal abatement policy for different discount rate levels, higher discount rates.

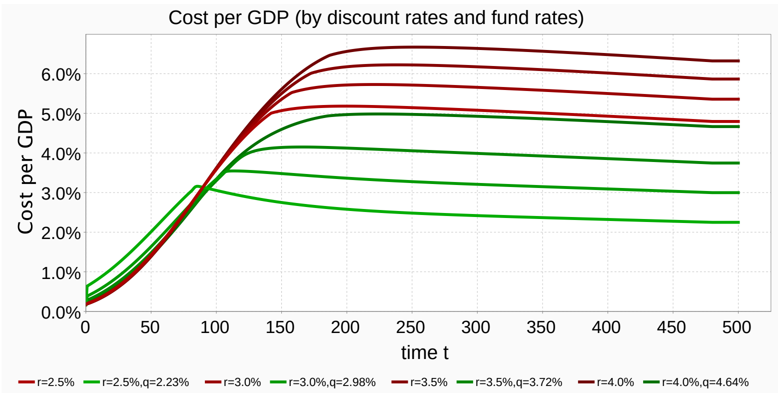

Figure 22: Cost per GDP for different discount rate levels, higher discount rates.

5.3.6. Effect of a Climate Transformation Fund on Risk and Convexity

We finally investigate the effect of the presence of a climate transformation fund in a stochastic IAM. We consider the DICE model with stochastic interest rate (time-preference rate) and stochastic abatement policy, as in Section 4.

Within this model, we determine the random variable $r^{\mathrm{SCC}}(T)$ by solving (7) path-wise.

Figure 23, 24 and 25 show the corresponding result for a model without climate transformation fund, a model with climate transformation fund funded by $q = 1\%$ of the GDP, and one funded by $q = 2\%$ of the GDP, respectively.

The climate transformation fund significantly reduces the volatility of $r^{\mathrm{SCC}}(T)$, such that present abatement cost are more in balance with future damage cost on a path-wise basis. Note that the fund with funding by 1% of GDP is - as we know - not sufficient to cover the total cost at the expected interest rate level. Hence, at least for many simulation paths, we expect that abatement cost are avoided in favor of future damage cost. This explains that even in Figure 24 we see an albeit later, rise of the carbon interest rate paths. Increasing the funding rate to 2% strongly delays this effect, Figure 25.

For the expectation, a positive drift is to be expected from the convexity of the exponential function in (7). However, the drift of the carbon interest rate is much higher than what is to be expected from this effect(compare for example the drift of the discount rate).

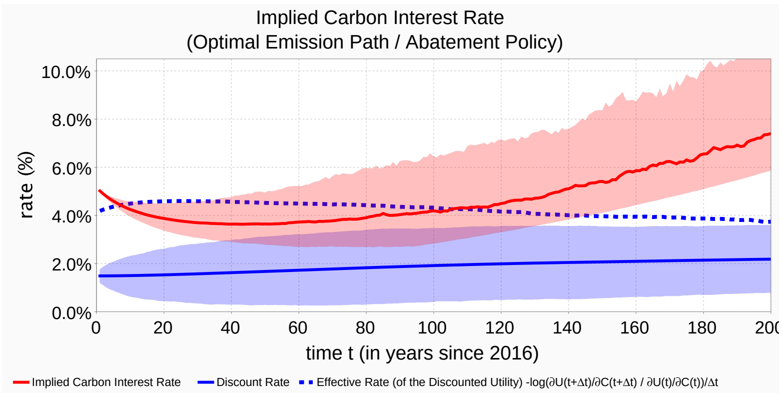

Figure 23: The (stochastic) implied carbon rate r^{SCC} (solving Equation 7 path-wise), expectation (red line) and 5 % to 95 % quantile (red area), for the optimal abatement parameters of the stochastic DICE model with stochastic interest rate (blue) and *without climate transformation fund*.

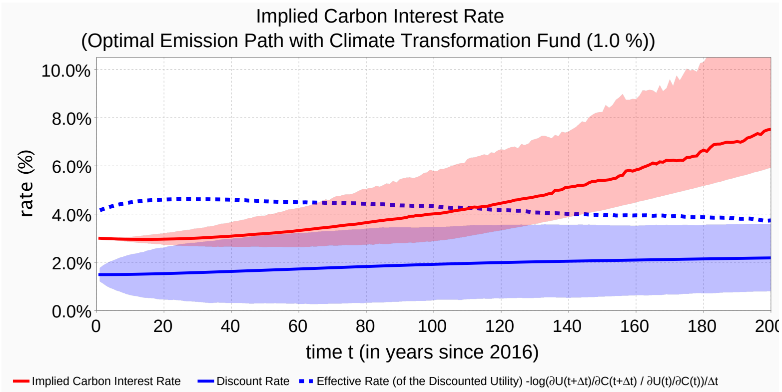

Figure 24: The (stochastic) implied carbon rate r^{SCC} (solving Equation 7 path-wise), expectation (red line) and 5 % to 95 % quantile (red area), for the optimal abatement parameters of the stochastic DICE model with stochastic interest rate (blue) and *with climate transformation fund, funded by 1 % of the GDP*.

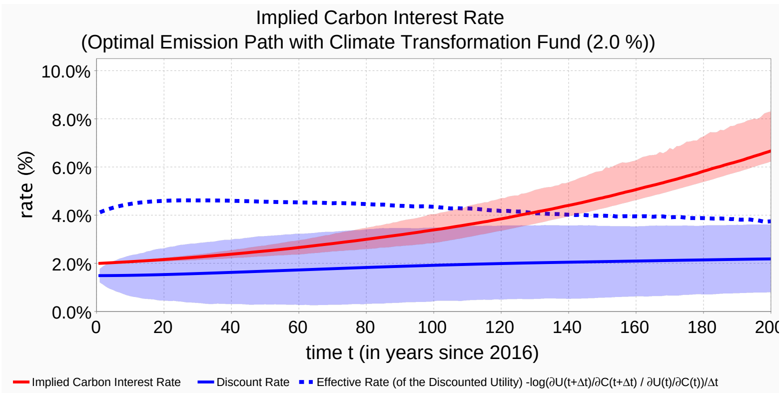

Figure 25: The (stochastic) implied carbon rate r^{SCC} (solving Equation 7 path-wise), expectation (red line) and 5 % to 95 % quantile (red area), for the optimal abatement parameters of the stochastic DICE model with stochastic interest rate (blue) and *with climate transformation fund, funded by 2 % of the GDP*.

6. Conclusion

We introduced some concepts to assess the cost of climate change mitigation. The concepts are inspired by concepts from mathematical finance, like the par-swap rate or the internal rate of return.

We introduced an *implied* CO_2-prices and an *implied* share of the GDP covering the cost of climate change mitigation.

To assess the temporal structure of the cost we introduced an internal rate of return relating the abatement cost to the associated damage cost saving: the *carbon interest rate*. For the unconstrained optimal emission path this rate agrees with an weighted average of the models utility rate, but for other paths it differs and provides additional insights by revealing a difference in the cost structure.

We observe that the classical *social cost of carbon* falls significantly short in covering the total cost (of carbon). The implied CO_2-price is by a factor of 10 higher.

The implied share of the GDP suggest that an intergenerational equitable climate change mitigation would be possible at a decent fraction of the GDP, which then motivated our subsequent model extension.

A fundamental issue with most reported values of the classical social cost of carbon, but also for our implied values, is that they are usually derived from the model's optimal emission paths. This neglects that we are likely following a non-optimal paths.

For (slightly) non-optimal emission paths the aggregated values (like social cost of carbon or implied CO_2-price) differ mildly, but the temporal cost structure changes significantly, introducing much stronger intergenerational inequality as can be revealed by the implied carbon interest rate.

To model an intergenerational equitable climate change mitigation, we extend the model with a *climate transformation fund* that has a constant funding and covers the abatement and damage cost. The presence of the fund may not only improve the distribution of the cost, i.e., the intergenerational equity, but also may reduce the total cost significantly, with a very small impact on the major driver of the model's objective function, the consumption.

With low to moderate discount rates, the DICE 2016 model suggests that a fund supplied with 2.4% of the GDP would be sufficient to cover all costs from climate change, that is, abatement and damage costs. To put this into comparison, the U.S. national defence spending is between 4% and 5% of the U.S. GDP (1990-2012), [27].

In the presence of the fund, the optimal emission path (with the otherwise unmodified objective function to maximize social welfare) results in much stronger emission reductions, which effectively reduces the (undiscounted) costs associated with climate change.

The fund rate is almost independent of the specific discount rate as long as the discount rate stays below 2.5%. This shows that the optimal funding rate is rather an endogenous quantity generated by the mode.

The fund has the advantage of distributing climate change costs equally (in terms of percentage of the GDP) across all generations, while the classical model may exhibit intergenerational inequality [10]. Our extended model gives the minimum funding rate required, and for that, the optimal emission pathways in the presence of the fund.

We are aware, that the absolute figures obtained from the DICE model should be taken with caution, due to the simplicity of the model. The results presented here should be interpreted on a more qualitative basis, illustrating basic concepts and relations.

Finally, we like to emphasize that utility-maximizing emission paths may exhibit significant intergenerational inequality and that the addition of a climate transformation fund establishes a certain level of equity among generations and improves resilience against external changes, e.g., the time preference rate. With the presence of a climate transformation fund, society sees faster abatement as optimal. The effort required is smaller than expected (e.g., the reduction of consumption or GDP), as it is smaller than the funding rate.

We suggest augmenting current policies and more general integrated assessment models with funding capacities ensuring or improving intergenerational equity.

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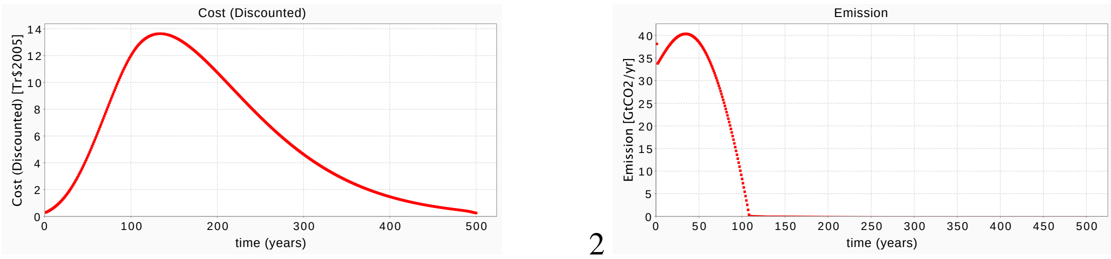
A. Appendix

A.1. Choice of the Time-Horizon

The definition of $\bar{p}$ is sensitive to the choice of the time horizon T as long as $E(t) > 0,\ C(t) > 0$. In the classical DICE model $E(t) = 0$ is never fulfilled, but $E(t)$ becomes negligibly small after $T = 500$. Similarly, the discounted cost $\frac{C(t)}{N(t)}$ decline significantly fast, see Figure 26.

If T is significantly large, $\bar{p}$ converges. Its limit is the price obtained by distributing all (future) costs to all (future) emitters. Figure 27 given the price $\bar{p}$ as a function of the chosen time-horizon $T \mapsto \bar{p}(T)$.



2

(a) **Cost**, $t \mapsto C(t)$. (b) **Emission**, $t \mapsto E(t)$.

Figure 26: **Cost and Emission.** The cost and emission in the calibrated model as a function of time showing that there is a sufficiently fast decay.

A.2. Implementation

We use an extended version of the DICE models. The model is implemented on a general value type that allows to model all quantities as random variables and allows use algorithmic differentiation to obtain various partial derivative.[9]

The implementation of the model can be found in the class

`SIAModelWithNonlinearFunding`

at [4]. The modelling of the climate transformation fund can be provided though the implementations of the interface

[9] The ability to consider stochastic quantities was not use in the present an.

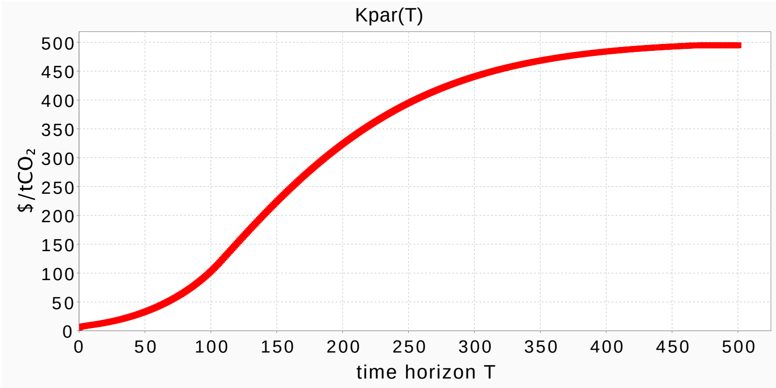


Figure 27: **Dependency on the Time-Horizon.** The implied CO_2-price as a function of the chosen time-horizon T. The price convergences as emission and cost go to zero. To read this figure: Requiring that the paying for the emissions of the next 150 years cover the cost of these 150 years would require a CO_2 price of $\bar{p}(150) \approx 225$ \$/tCO_2.

`ClimateTransformationFundModel.`

A.3. List of Symbols

Symbol	Description
$\bar{q}$	(a posteriori) implied fraction of the (accumulated) GDP that corresponds to the (accumulated) cost.
$\tilde{q}$	(a posteriori) implied fraction of the (accumulated) GDP that corresponds to the (accumulated) cost that can be attributed to additional emissions.
$\bar{q}^*$	smallest equilibrium fraction of the GDP required to fund a climate transformation that covers the (accumulated) cost.
$\bar{p}$	(a posteriori) time value adjusted CO_2-price, i.e., a fraction of the (accumulated) $E(t) \cdot N(t)$, that corresponds to the (accumulated) cost.
$\tilde{p}$	(a posteriori) time value adjusted CO_2-price, i.e., a fraction of the (accumulated) $E(t) \cdot N(t)$, that corresponds to the (accumulated) *future* cost.
$\tilde{p}\vert_{\mathrm{D}}$	(a posteriori) time value adjusted CO_2-price, i.e., a fraction of the (accumulated) $E(t) \cdot N(t)$, that corresponds to the (accumulated) *future damage* cost.
$\bar{p}^*$	smallest equilibrium time value adjusted CO_2-price, i.e., a fraction of the (accumulated) $E(t) \cdot N(t)$, required to fund a climate transformation that covers the (accumulated) cost.
$r^{\mathrm{SCC}}(t)$	carbon interest rate in time t: the internal rate of return by investing into higher abatement at time t.

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Notes

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