The Relationship between the Modified Golden Rule in the Ramsey Model and the Equilibrium Solutions in the Overlapping Generations Model

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Abstract

This paper demonstrates that the modified golden-rule level of capital stock derived in the Ramsey model can also be obtained within the overlapping generations model. While the standard overlapping generations model typically addresses a two-period optimization problem and involves dynamic inefficiency, the introduction of a bequest motive allows the equilibrium to coincide with that of the Ramsey model under certain conditions. This indicates that, despite differing motivations for leaving wealth, the solutions of the two models can converge.

Keywords：Overlapping Generations Model, Dynamics of Capital Stock, Ramsey Model

JEL Classifications：E0

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1. Introduction

This paper clarifies that the modified golden-rule level of capital stock in the Ramsey model can also be derived within the overlapping generations (OLG) model. Whereas the OLG model typically considers only a two-period optimization problem for individuals, the Ramsey model is based on intertemporal optimization. This difference is reflected in the optimality of the equilibrium solutions in the steady state. In the Ramsey model, an equilibrium satisfying the modified golden rule is obtained, while in the OLG model the equilibrium is often dynamically inefficient, leaving room for Pareto improvements. However, by introducing a bequest motive, it can be shown that under certain parameter conditions the equilibrium in the OLG model coincides with that of the Ramsey model. This indicates that, whether wealth is transferred for the sake of children's utility or for one's own utility, the two models can converge to the same solution. Moreover, it is confirmed that similar results can be obtained even in the OLG model without bequests.

The structure of this paper is as follows. Section 2 presents the setup of the Ramsey model; Section 3 analyzes its equilibrium solution. Section 4 introduces the OLG model and derives its equilibrium solution, and Section 5 concludes the paper.

2. Model

The population size is normalized to one, and population growth is assumed to be zero. The economy consists of households and firms. We begin by presenting the model setup necessary to derive the level of capital accumulation in the Ramsey model. This model is known as Ramsey (1928), Cass (1965) and Koopmans (1965) model.

2.1 Households

Utility function is assumed as follows.

$$u_t = \alpha ln c_{1t} + (1-\alpha) ln c_{2t+1} + \varepsilon u_{t+1}, 0 < \alpha < 1, 0 < \varepsilon < 1. \qquad (1)$$

c_{1t} : consumption in young period, c_{2t+1}: consumption in old period, ε: level of altruism

In the household sector, each individual lives for two periods: youth and old age. The individual's utility includes not only their own consumption but also the utility of their children, with the degree of altruism denoted by ε. This framework can be interpreted as a dynastic model, in which it is as if a single individual continues to live infinitely across periods. Furthermore, in any period t, both the young and the old generations coexist. While this coexistence of generations may resemble an overlapping generations (OLG) model, it differs from the typical OLG framework in that a single individual chooses both youth and old-age consumption.

Budget Constraint is given by the follows,

$$K_{t+1} = w_t + r_t K_t + (1-\delta)K_t - c_{1t} - c_{2t}, 0 < \delta < 1. \tag{2}$$

K_t:capital stock, w_t: wage rate, r_t: interest rate, δ: depreciation rate

Labor is supplied inelastically during youth, generating labor income. In addition, individuals earn capital income, which is allocated among consumption in youth, consumption in old age, and savings, the latter becoming the capital stock in the following period.

The optimal allocations are as follows. Equation (3) represents the intertemporal allocation of consumption, which is referred to as the Euler equation for consumption. Equation (4) describes the intra-temporal allocation of consumption, namely the allocation between consumption in youth and consumption in old age.

$$\frac{c_{1t+1}}{c_{1t}} = \varepsilon(r_{t+1} + 1 - \delta) \tag{3}$$

$$\frac{c_{1t}}{c_{2t}} = \frac{\alpha\varepsilon}{1-\alpha} \tag{4}$$

2.2 Firms

Production function is assumed as the following Cobb=Douglas production function.

$$Y_t = K_t^{\theta} L_t^{1-\theta}, 0 < \theta < 1. \tag{5}$$

Y_t: final goods, L_t: labor input

In a competitive market, factor prices are equal to marginal products under the profit-maximization condition, from which the following equations can be derived.

$$r_t = \theta K_t^{\theta-1} \tag{6}$$

$$w_t = (1-\theta)K_t^{\theta} \tag{7}$$

3. Equilibrium

We derive the equilibrium of this model economy. Defining $\Delta c_{1t} = c_{1t+1} - c_{1t}$ and $\Delta K_t = K_{t+1} - K_t$, and then the following equations can be obtained because of (3) and (4).

$$\Delta c_{1t} = \varepsilon(r_{t+1} + 1 - \delta) - 1 \tag{8}$$

$$\Delta K_t = K_t^{\theta} - \delta K_t - \frac{1-\alpha+\alpha\varepsilon}{\alpha\varepsilon} c_{1t} \tag{9}$$

Once c_{1t} is determined, c_{2t} can be obtained from equation (4). By illustrating equations (8) and (9), we obtain the following figure.

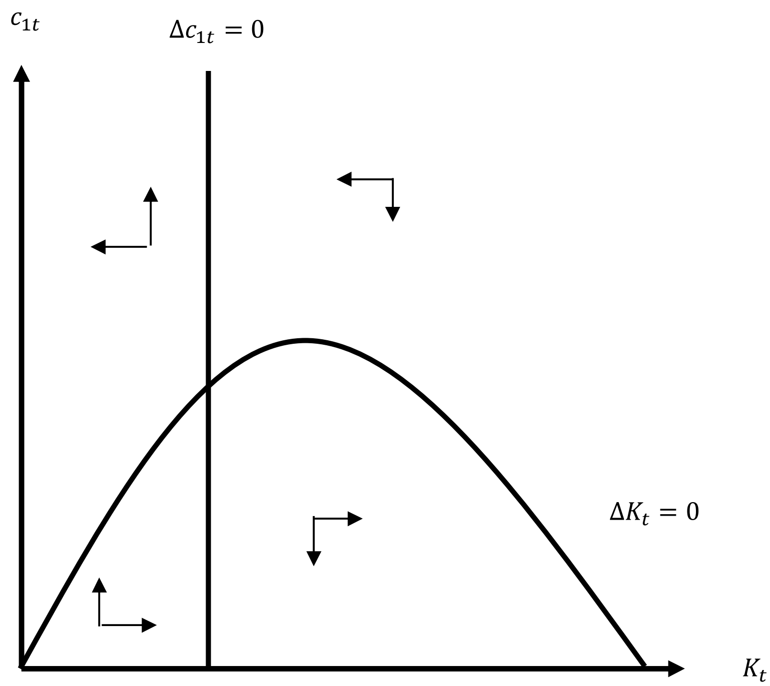

Figure 1: Dynamics of K_t and c_{1t}

This is the typical phase diagram derived in the Ramsey model, representing the steady-state equilibrium with saddle path.

4. Bequest-Consumption Model

Next, we consider the overlapping generations model. Here, the population size is normalized to one, and population growth is assumed to be zero. This model is based on Samuelson (1958) and Diamond (1965).

4.1 Households

Utility function is assumed as follows:

$$u_t = \bar{\alpha} ln c_{1t} + \bar{\beta} ln c_{2t+1} + \left(1 - \bar{\alpha} - \bar{\beta}\right) ln b_{t+1}, 0 < \bar{\alpha}, 0 < \bar{\beta}, \bar{\alpha} + \bar{\beta} < 1 . \quad (10)$$

b_{t+1}: bequest

The main difference from the standard two-period overlapping generations (OLG) model is the inclusion of bequests. In this context, bequests are treated within a bequest-consumption framework, where utility is derived directly from the bequest itself. It should be noted that this is not a model in which wealth is transferred out of concern for the utility of the child generation. The OLG model, by construction, differs from the Ramsey or dynastic models in that it essentially considers only the utility of the individual. If the utility of children were incorporated into equation (10), the framework would, in essence, involve intergenerational allocation, thereby becoming equivalent in nature to Ramsey or dynastic models. This would alter the essential characteristics of the OLG model. To avoid this, the setup is specified as in equation (10).

Budget constraint in young period is shown as follows:

$$b_t + w_t = c_{1t} + s_t. \tag{11}$$

Individuals receive b_t as a bequest from the parent generation and earn w_t through the inelastic supply of one unit of labor. This income is then allocated between consumption in youth and savings s_t for consumption in old age.

Budget constraint in old period is shown as follows:

$$(1 + r_{t+1})s_t = c_{2t+1} + b_{t+1}. \tag{12}$$

In old age, individuals receive both the principal and interest from their savings, which are allocated to consumption c_{2t+1} and bequests.

From equations (11) and (12), the lifetime budget constraint can be expressed as follows.

$$c_{1t} + \frac{c_{2t+1}}{1 + r_{t+1}} + \frac{b_{t+1}}{1 + r_{t+1}} = w_t + b_t \tag{13}$$

The allocation that maximizes the utility function (10) subject to the budget constraint (13) is given as follows.

$$c_{1t} = \bar{\alpha}(b_t + w_t) \tag{14}$$

$$c_{2t+1} = \bar{\beta}(1 + r_{t+1})(b_t + w_t) \tag{15}$$

$$b_{t+1} = \left(1 - \bar{\alpha} - \bar{\beta}\right)(1 + r_{t+1})(b_t + w_t) \tag{16}$$

The firm's production function is given by equation (5), and the factor prices, namely the wage rate and the interest rate, are represented by equations (6) and (7), respectively. In what follows, we assume that capital fully depreciates within one period, i.e., $\delta = 1$.

4.2 Equilibrium

The capital accumulation equation is given by $K_{t+1} = s_t$. It should be noted that the population size is normalized to one.

$$K_{t+1} = \left(b_t + w_t - \bar{\alpha}(b_t + w_t)\right) \tag{17}$$

Here, taking equations (6) and (7) into account, equations (16) and (17) can be expressed as follows.

$$K_{t+1} = (1 - \bar{\alpha})\left((1 - \theta)K_t^{\theta} + b_t\right) \tag{18}$$

$$b_{t+1} = \left(1 - \bar{\alpha} - \bar{\beta}\right)\theta K_{t+1}^{\theta - 1}\left((1 - \theta)K_t^{\theta} + b_t\right) \tag{19}$$

From equations (18) and (19), the following expression is obtained.

$$b_{t+1} = \left(1-\bar{\alpha}-\bar{\beta}\right)\theta K_{t+1}^{\theta-1}\frac{K_{t+1}}{1-\bar{\alpha}} \rightarrow b_t = \frac{\left(1-\bar{\alpha}-\bar{\beta}\right)\theta}{1-\bar{\alpha}}K_t^{\theta} \tag{20}$$

From equations (18) and (20), the following dynamic equation of capital can be derived.

$$K_{t+1} = (1-\bar{\alpha})\left((1-\theta)+\frac{(1-\bar{\alpha}-\bar{\beta})\theta}{1-\bar{\alpha}}\right)K_t^{\theta} \tag{21}$$

The equilibrium solution is characterized by the dynamic equation of capital (21). A graphical representation of equation (21) is shown as follows.

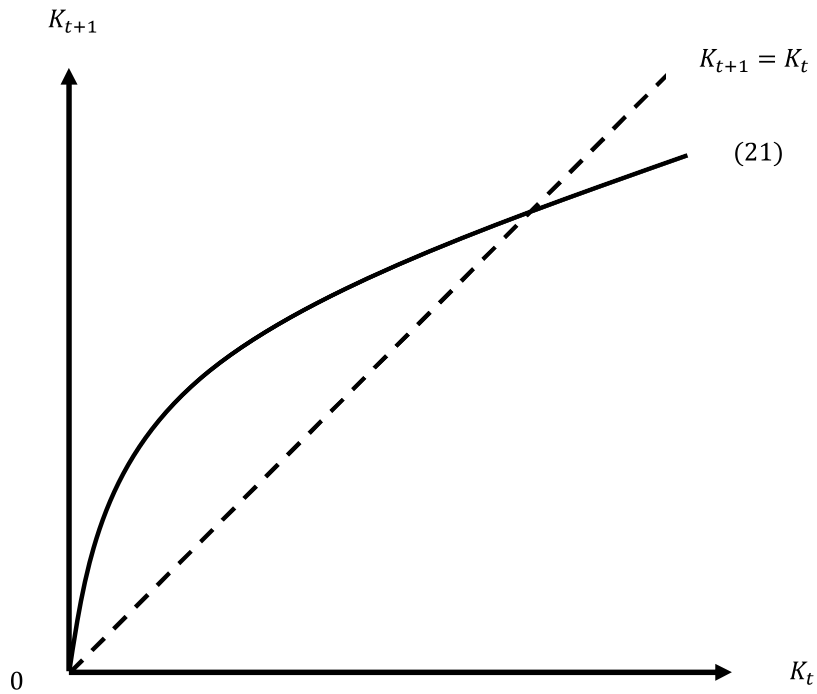

Figure 2. Dynamics of Capital Stock

The steady-state capital stock is given as follows:

$$K = \left((1-\bar{\alpha})\left((1-\theta)+\frac{(1-\bar{\alpha}-\bar{\beta})\theta}{1-\bar{\alpha}}\right)\right)^{\frac{1}{1-\theta}}. \tag{22}$$

We now derive the steady-state capital stock in the dynasty model. This can be obtained from equation (8) by setting $\Delta c_{1t} = 0$, as follows.

$$K = (\varepsilon\theta)^{\frac{1}{1-\theta}} \tag{23}$$

From equations (22) and (23), the steady-state levels of capital stock in the dynasty model and the bequest-consumption model are derived. Equation (22) represents the modified golden-rule level of capital stock, and the parameter conditions under which the capital stock given by equation (23) coincides with the modified golden-rule level are as follows.

$$\varepsilon = \frac{1-\bar{\alpha}}{\theta}\left((1-\theta) + \frac{(1-\bar{\alpha}-\bar{\beta})\theta}{1-\bar{\alpha}}\right) \tag{24}$$

In the overlapping generations model, dynamic inefficiency generally arises, and the fact that the equilibrium itself allows for potential Pareto improvements is regarded as a major issue. However, by incorporating bequests in the form of a bequest-consumption model and specifying the preference parameter for bequests, it becomes possible to obtain the equilibrium solution derived in the Ramsey model—namely, the equilibrium at the modified golden-rule level. This demonstrates that even without resorting to the Ramsey framework, the modified golden-rule level of capital stock can be derived within the overlapping generations model.

5. Conclusion

This paper has shown that the modified golden-rule level of capital stock in the Ramsey model can also be derived within the overlapping generations (OLG) framework. Whereas the OLG model generally considers only a two-period individual optimization, the Ramsey model is based on intertemporal optimization. This difference is reflected in the optimality of the equilibrium in the steady state. In the Ramsey model, the equilibrium achieves the modified golden rule, while in the standard OLG model the equilibrium is typically dynamically inefficient, leaving scope for Pareto improvements. However, by introducing a bequest-consumption model, it is found that under certain parameter conditions the equilibrium coincides with that of the Ramsey model. In other words, whether wealth is transferred for the sake of children's utility or for one's own utility—i.e., even when the motives for leaving bequests differ—the solutions of the two models can coincide.

Needless to say, similar results can also be obtained in an overlapping generations model without bequests. Given certain parameters, the equilibrium of no bequest model is equivalent with the case of Ramsey model in the steady state.

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