A two-sector model of optimal growth in which labour is employed only in the industry of consumption goods: A complete characterization of equilibrium paths

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A Two-Sector Model of Optimal Growth in which Labour is Employed only in the Industry of Consumption Goods: A Complete Characterization of Equilibrium Paths*

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Abstract

In this paper, I develop a frictionless two-sector optimal growth model with endogenous labour supply in which the non-reproducible factor can be employed only in the production of consumption goods. From a theoretical point of view, I show that in this setting the optimal allocation of labour is constant over time, whereas its actual utilization relies on the availability of a convex technology. In parallel, a convex technology in the sector of investment goods is instead sufficient for the determinacy of meaningful equilibrium paths. Furthermore, calibrating the model to the US economy, I show that such a two-sector economy smooths the complementarity between the marginal propensities to consume and to save, and it also able to replicate the countercyclical (procyclical) pattern of the relative price of capital goods (real wages).

Keywords: Capital accumulation; Investment; Consumption; Two-sector growth model; Labour supply; Relative price of capital goods; Real Wage.

JEL Classification: C61; E21; E22; O11.

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1 Introduction

The large majority of contributions dealing with the two-sector optimal growth model pioneered by Meade (1961) and Huzawa (1961) – in which there is an explicit distinction between the production technologies of consumption and investment goods – are presented and developed by considering a fixed labour supply often normalized to one (cf. Pelgrin and Venditti, 2022; Senouci, 2011, 2012; Herrendorf and Velentinyi, 2006). As far as I can see, this way of treating labour provision follows directly from the customary theoretical approach used to address the optimal program within two-sector economies which relies on the determination of a sequence of decentralized allocations – or momentary equilibria – where profit-maximizing firms pin down the capital-labour ratio in the two industries by equating the values of the marginal productivities of the employed factors across the existing sectors (cf. Solow, 1961; Gabisch, 1973). Within that setting, for a given value of the wage rate, should also be given the labour supply of price-taking workers-consumers. In parallel, for the same value of the wage, firms should decide how many of these workers-consumers have to be employed in each industry, a figure that cannot be higher than the already mentioned labour supply.

According to the widely accepted Neoclassical theory, however, actual labour provision should follow – in each period – from balancing at the margin the utility of the additional consumption allowed by the active participation of workers-consumers to the production process, to the disutility arising from the corresponding reduction of leisure whose opportunity cost is indeed given by the value of the prevailing wage (cf. Rolf and Colombino, 2015). Therefore, as far as a general competitive equilibrium is concerned, whenever the optimal program of the two-sector economy implies adjustments of the capital intensities that alter the productivity of labour, the profit-maximizing level of the wage will be modified and variations in labour supply in principle cannot be ruled out unless assuming an inelastic labour supply schedule (cf. Hahn, 1965).

Given that the non-reproducible factor can be alternatively directed in the production of different kind of goods, the endogenization of labour supply in a two-sector model of optimal growth raises some intriguing and unexplored issues, especially when the relevant economic decisions are taken at a centralized level by a unique perfectly informed optimizing agent. On the one hand, for a certain amount of forgone leisure in the current period, providing labour to the production of consumption goods immediately boosts the utility of the optimizing agent as it happens in the traditional one-sector model (cf. Cass, 1965). On the other hand, similarly to what usually happens for saving decisions, offering labour to the production of investment goods provides a lagged contribution to the utility of the optimizing agent because such a choice enhances the production of commodities that – in a later period – can be eventually exploited to produce welfare-enhancing consumption goods. To the best of my knowledge, this intertemporal trade-off remained largely unexplored in the literature of two-sector models. In addition, on a more analytical ground, whenever the optimal growth rate of the two types of labour inputs cannot be univocally conveyed in terms of per capita consumption and capital, the decision about the amount of labour to provide in each industry may alter the bi-dimensionality of the dynamic system in which the two-sector problem of optimal growth is usually framed (cf. Drabicki and Takayama, 1975; Galor, 1992).

In this paper, I develop a frictionless two-sector optimal growth model with endogenous labour supply in which all the relevant decisions are efficiently taken at a centralized level by an infinitely lived agent. Moreover, along the lines of the OLG model set forth by Hashimoto and Sakuragawa (1999), I consider the situation in which capital – on account of its *malleability* – is employed in the production processes of both sectors whereas the non-reproducible factor is instead constrained by imperfect mobility and it can be employed only in the industry of consumption goods (cf. Cardi et al. 2020).[1] In other words, I take into consideration the case in which the choice to offer labour services has an instantaneous cost in terms of forgone leisure – or some disutility from offering labour – that has to be optimally counterbalanced by the increased utility that such a decision raises in terms of the additional consumption goods that can be produced and consumed by employing those services in the relative industry.

From a theoretical point of view, I show that relying on logarithmic preferences over consumption, separable CRRA disutility with respect to labour provision and Cobb-Douglas technologies, the two-sector model presented in this paper leads to the constancy of the non-reproducible factor allocated in the sector of consumption goods whose actual employment in that industry is also associated to the availability of a specific convex production technology. At the same time, a likewise convex technology in the investment good sector is instead sufficient for the determinacy of the equilibrium path followed by the production of consumption goods and the total stock of capital. Furthermore, parametrizing and calibrating the model to the US economy, I show that the two-sector economy under scrutiny relaxes the strict complementarity between the marginal propensity to consume and to save of the optimizing agent that holds in the one-sector model, and it also able to replicate the observed countercyclical pattern of the relative price of capital goods as well as the procyclical pattern followed by some measures of the real wage rate commonly used to quantify the purchasing power of employed workers (cf. Cass, 1965; Collins and Williamson, 2001; Restuccia and Urrutia, 2001; Fisher, 2006; Justiniano et al. 2011; Hergovich and Merz, 2018; Lian et al. 2020; Sallivan, 1997).

The rest of the paper proceeds as follows. Section 2 illustrates the model economy. Section 3 explores its quantitative implications. Finally, Section 4 concludes.

2 Theoretical framework

Measuring time (t) on a continuous scale, so that $t \in \mathbb{R}_+$, I consider a model economy in which there are two distinct sectors, one that produces consumption goods that – net of labour

[1] A two-sector economy in which capital is non-shiftable across industries is analysed by Ryder (1969).

provision – contribute to the utility of the unique optimizing agent, and another one in which are produced investment goods that boost the overall capital accumulation in the society (cf. Srinivasan, 1964). Consequently, in each instant, the total stock of capital available in the economy – say $K(t)$ – is broken down between the stock of capital allocated in the sector of consumption goods and the stock of capital allocated in the sector of investment goods, respectively denoted by $K_C(t)$ and $K_I(t)$. Therefore, full employment of the available capital stock will always imply that

$$K(t) = K_C(t) + K_I(t) \tag{1}$$

Following Hashimoto and Sakuragawa (1999), I take into consideration a situation in which labour can be employed only in the industry that produces consumption goods, whereas investment goods are produced only by means of capital. Consequently, the productive technologies available in the two industries are assumed to be described by the following Cobb-Douglas expressions:

$$Y_I(t) = S_I(K_I(t))^{\alpha_I} \tag{2}$$

$$Y_C(t) = S_C(K_C(t))^{\alpha_C}(L_C(t))^{1-\alpha_C} \tag{3}$$

where $Y_I(t)$ ($Y_C(t)$) is the real output of the sector the produces investment (consumption) goods i, $S_I > 0$ ($S_C > 0$) is an industry-specific measure of productivity, $\alpha_I > 0$ ($\alpha_C \in 0, 1]$) is the value of the elasticity of output of the industry of investment (consumption) goods with respect to the stock of capital employed in that sector, whereas $L_C(t)$ is the amount of labour employed in the sector of consumption goods that in this case corresponds to total employment.

Since labour is completely omitted from the expression in eq. (2) and it enters only the one in eq. (3), the model economy described in this paper depicts a situation in which there is full automation of production in the industry that supplies investment goods – machines that produce machines – and the immobility of the labour factor from the sector of consumption goods, an industry that often supplies non-tradable goods (cf. Growiec, 2022; Cardi et al. 2020). Aiming at analysing the potential role of non-convexities in the production technologies, for the moment, I do not fix any upper bound for the elasticity of output respect to the stock of the employed in the sector of investment goods whereas I make the hypothesis of constant returns to scale with respect to capital and labour – or, alternatively, linearity in the employed capital – in the sector of consumption goods. It is worth noticing, however, that the case in which the technology prevailing in the sector of consumption goods becomes linear in the capital input, i.e., the situation in which α_C is equal to 1, corresponds to the case of full automation in both industries (cf. Guerrazzi, 2025).

In addition, under the assumption that the employed capital equally depreciates in both industries, the capital accumulation law can be written as

$$\dot{K}(t) = Y_I(t) - \delta K(t) \tag{4}$$

where $\delta > 0$ is the common value of the depreciation rate of capital.

According to the expression in eq. (4), $Y_I(t)$ is the output of capital goods available for net investment to add to the stock of malleable machines available in the economy (cf. Meade, 1961).

2.1 The centralized problem

In the model economy described above, the decision to consume a certain amount of resources straightforwardly implies to allocate a certain share of capital and some labour services in the sector of consumption goods by leaving the remaining fraction of capital to the sector of investment goods (cf. Cai, 2006; Guerrazzi, 2025). On the one hand, the expression in eq. (3) implies that in each instant the quantity of capital to employ in the industry of consumption goods – for each unit of Y_C to produce and consume – depends on the amount of supplied labour and it is given by the following expression:

$$K_C(t) = \left(\frac{Y_C(t)}{S_C(L_C(t))^{1-\alpha_C}}\right)^{\frac{1}{\alpha_C}} \tag{5}$$

On the other hand, eq. (5) together with the hypothesis of full employment of the reproducible factor conveyed by eq. (1) imply that the residual quantity of capital to employ in the sector of investment goods can be written as

$$K_I(t) = K(t) - \left(\frac{Y_C(t)}{S_C(L_C(t))^{1-\alpha_C}}\right)^{\frac{1}{\alpha_C}} \tag{6}$$

Consequently, given the expressions in (2), (4) and (6), a benevolent and well-informed social planner that lives forever endowed with logarithmic preferences over consumption and separate CRRA disutility with respect to labour provision will allocate the non-reproducible factor and the available capital in the two sectors by solving the following problem:

$$\begin{gathered} \max_{\mathcal{C}_S \in \mathcal{A}_0(\overline{K})} \int_{t=0}^{\infty} \exp(-\rho t)\left(\ln Y_C(t) - \tfrac{1}{1+\gamma}(L_C(t))^{1+\gamma}\right) dt \\ \text{s.to} \\ \dot{K}(t) = S_I\left(K(t) - \left(\tfrac{Y_C(t)}{S_C(L_C(t))^{1-\alpha_C}}\right)^{\frac{1}{\alpha_C}}\right)^{\alpha_I} - \delta K(t) \\ K(0) = \overline{K} \end{gathered} \tag{7}$$

where $\mathcal{C}_S \equiv \begin{pmatrix} Y_C(\cdot) & L_C(t) \end{pmatrix}$ is the set of control functions, $\mathcal{A}_0(\overline{K})$ is the set of admissible control strategies, $\rho > 0$ is the discount rate, $\gamma >$ is the inverse of the elasticity of labour supply, whereas $\overline{K} > 0$ is the initial value of the total capital stock (cf. Benhabib and Farmer, 1996).

In order to have economically meaningful trajectories, the set of all the admissible control strategies $\mathcal{C}_S$ starting from the initial couple $\left\{0, \overline{K}\right\}$ is defined as

$$\mathcal{A}_0\left(\overline{K}\right) := \left\{\mathcal{C}_S \in \mathbb{L}^1_{\mathbf{loc}}\left(\mathbb{R}_+; \mathbb{R}^2_+\right) : K\left(t\right) \in \mathbb{R}_+ \quad \forall t \in \mathbb{R}_+\right\} \tag{8}$$

According to the definition in (8), $\mathcal{C}_S$ has to belong to the set of locally integrable (or summable) functions such that the level of consumption goods which are produced and consumed, the corresponding amount of labour employed in the relative industry and the total stock of capital are positive all over the relevant time horizon.

The first-order conditions (FOCs) for the social planner problem in (7) are given by

$$1 - \frac{\alpha_I S_I q\left(t\right)\left(\frac{Y_C(t)}{S_C(L_C(t))^{1-\alpha_C}}\right)^{\frac{1}{\alpha_C}}}{\alpha_C\left(K\left(t\right) - \frac{Y_C(t)}{S_C(L_C(t))^{1-\alpha_C}}\right)^{1-\alpha_I}} = 0 \tag{9}$$

$$\frac{\alpha_I\left(1-\alpha_C\right) S_I q\left(t\right)\left(\frac{Y_C(t)}{S_C(L_C(t))^{1-\alpha_C}}\right)^{\frac{1}{\alpha_C}}}{\alpha_C\left(K\left(t\right) - \left(\frac{Y_C(t)}{S_C(L_C(t))^{1-\alpha_C}}\right)^{\frac{1}{\alpha_C}}\right)^{1-\alpha_I}\left(L_C\left(t\right)\right)^{1+\gamma}} - 1 = 0 \tag{10}$$

$$\dot{q}\left(t\right) = q\left(t\right)\left(\rho + \delta - \frac{\alpha_I S_I}{\left(K\left(t\right) - \left(\frac{Y_C(t)}{S_C(L_C(t))^{1-\alpha_C}}\right)^{\frac{1}{\alpha_C}}\right)^{1-\alpha_I}}\right) \tag{11}$$

$$\lim_{t\to\infty} \exp\left(-\rho t\right) q\left(t\right) K\left(t\right) = 0 \tag{12}$$

where $q\left(t\right)$ is the costate variable associated to the capital accumulation constraint.

Eq.s (9) and (10) are, respectively, the FOCs with respect to $Y_C\left(t\right)$ and $L_C\left(t\right)$ that have to hold in each instant of time. Moreover, the intertemporal relationship in eq. (11) conveys the optimal trajectory of the costate variable. Furthermore, the right-end point condition on the shadow-value of capital in (12) is the required transversality condition.

The expressions in eq.s (9) and (10) straightforwardly imply that in each instant the optimal amount of labour to employ in the sector of consumption goods is given by the following expression:

$$L_C\left(t\right) = L^*_C \equiv \left(1-\alpha_C\right)^{\frac{1}{1+\gamma}} \tag{13}$$

Eq. (13) reveals that as long as the production technology prevailing in the sector of consumption goods is convex, i.e., as long as $\alpha_C \in (0,1)$, the optimal amount of labour to supply and employ in that sector is positive and constant over time, so that it will always hold that $\dot{L}_C(t)/L_C(t) = 0$, for all t.[2] Consequently, whenever $\alpha_C \in (0,1)$, in the industry of consumption goods the non-reproducible factor becomes a fixed factor along the optimal trajectory. By contrast, when the technology prevailing in the sector of consumption goods is linear in the employed capital, i.e., whenever $\alpha_C = 1$, the optimal amount of labour implied by eq. (13) becomes equal to zero by confirming that in this case there is actually full automation of production in both industries (cf. Guerrazzi, 2025).[3]

Moreover, differentiating eq. (9) with respect to time and using its implied expression for $q(t)$ allows us to find the Euler equation for the production of consumption goods. Straightforward algebra together with the expression in eq. (13) reveal that such a Euler equation can be written as

$$\frac{\dot{Y}_C(t)}{Y_C(t)} = \frac{\alpha_C\left((1-\alpha_I)K(t)\frac{\dot{K}(t)}{K(t)} - \left(\frac{Y_C(t)}{S_C(1-\alpha_C)^{\frac{1-\alpha_C}{1+\gamma}}}\right)^{\frac{1}{\alpha_C}}\frac{\dot{q}(t)}{q(t)}\right)}{\alpha_I\left(\frac{Y_C(t)}{S_C(1-\alpha_C)^{\frac{1-\alpha_C}{1+\gamma}}}\right)^{\frac{1}{\alpha_C}} + (1-\alpha_I)K(t)} \tag{14}$$

Eq. (14) shows that the optimal growth rate of the output of consumption goods is given by a non-linear combination between the growth rate of capital – which can be easily retrieved from eq.s (2), (4) and (6) – and the optimal growth rate of the costate variable implied by eq. (11). Obviously, eq. (14) together with the optimal dynamics of the stock of capital and the result in eq. (13) provide the dynamic system that describes the motion of the model economy by preserving the (bi)dimensionality of the traditional setting in which two-product two-factor economies are usually framed (cf. Drabicki and Takayama, 1975; Galor, 1992).

2.2 First-best pricing

The theoretical setting developed above describes the outcomes of a centralized economy in which all the relevant decisions are taken by a unique optimizing agent by considering the prevailing quantities with no regard for any price signal. However, it is well known that in a decentralized setting in which there is a market for each good and for each factor and the two existing goods are then produced by atomistic profit-maximizing firms that take market prices as given, the value of the marginal product of capital must be equalized across industries in order to verify the condition for the allocative efficiency of capital (cf. Huzawa, 1961; Senouci, 2011).[4] In a similar manner, profit maximization requires that the value of the marginal productivity

[2]This result recalls recent findings on the relatively small values of the wage elasticities of labour supply observed in many countries (cf. Bargain et al. 2014).

[3]Since the utility arising from consumption goods satisfies the Inada conditions, no matter the employment of labour, the optimizing agent will always choose to produce this kind of goods (cf. Drabicki and Takayama, 1975; Senouci, 2011).

[4]Obviously, the consistency of this kind of behaviour at the decentralized level requires decreasing returns in the sector of investment goods, i.e., it must hold that $\alpha_C \in (0,1)$.

of labour in the sector of consumption goods must be equal to the wage rate that elicits the matching labour supply.

Consequently, in a decentralized competitive setting with continuous market clearing, the ratio between the marginal productivities of capital employed in the two industries must return the implicit path of the relative price of the two goods and – at the same time – the marginal productivity of labour prevailing in the sector of consumption goods has to be equal to the real wage rate that implement the centralized allocations described by the solution of the problem in (7). Formally speaking, according to the expressions eq.s (2), (3), (5), (6) and (13), the relative price of investment goods and the real wage that implement first-best allocations should be, respectively, equal to

$$\frac{p_I(t)}{p_C(t)} = \frac{\alpha_C S_C (1-\alpha_C)^{\frac{1-\alpha_C}{1+\gamma}} \left(K(t) - \left(\frac{Y_C(t)}{S_C} \right)^{\frac{1}{\alpha_C}} \right)^{1-\alpha_I}}{\alpha_I S_I \left(\frac{Y_C(t)}{S_C} \right)^{\frac{1-\alpha_C}{\alpha_C}}} \tag{15}$$

$$\frac{w(t)}{p_C(t)} = (1-\alpha_C)^{\frac{1-\alpha_C}{1+\gamma}} Y_C(t) \tag{16}$$

where $p_I(t)$ ($p_C(t)$) is the price of investment (consumption) goods, whereas $w(t)$ is the nominal wage rate.

On the one hand, eq. (15) shows that the relative price of investment goods is a decreasing function of the total factor productivity (TFP) in that sector relative to the TFP in the consumption good sector whereas its path depends on the allocation of capital in the two industries over time (cf. Ferreira et al. 2014). On the other hand, eq. (16) reveals that the path followed by the real wage rate is given by a constant share of the output produced in the sector of consumption goods whose magnitude is driven by the labour – which is fixed – and the capital allocated in that industry.

2.3 Steady state and golden rule

Since the optimal amount of labour employed in the sector of consumption goods has to be constant over time at the level implied by the expression in eq. (13), in the model economy described above efficient steady-state allocations are simply defined as the set of pairs $\mathcal{S} := \{Y_C^*, K^*\} \in \mathbb{R}^2_{++}$ such that $\dot{Y}_C(Y_C^*, K^*) = \dot{K}(Y_C^*, K^*) = 0$ whose point values will be also associated to the equilibrium level of the production of investment goods ($Y_I^* > 0$), the corresponding value of the amounts of capital allocated, respectively, in the sector of consumption goods ($K_C^* > 0$) and in the sector of investment goods ($K_I^* > 0$), and the first-best prices (p_I^*/p_C^* and w^*/p_C^*). In case of asymptotic stability, some elements of $\mathcal{S}$ will be also characterized by the fact that $\lim_{t\to\infty} Y_C(t) = Y_C^* \wedge \lim_{t\to\infty} K(t) = K^*$, a feature that may lead to the convergence of all the remaining endogenous and time-varying variables.

The long-run value of each endogenous variable as a function of the model's parameters can be easily retrieved as follows. First, setting $\dot{q}(t) = 0$ in eq. (11) by considering the result in eq. (6) pins down the steady-state level of the stock of capital allocated in the sector of investment goods, namely

$$K_I^* = \left(\frac{\alpha_I S_I}{\rho + \delta}\right)^{\frac{1}{1-\alpha_I}} \tag{17}$$

Second, eq. (17) together with the expression in eq. (2) allows us to obtain the steady-state level of the output in the sector of investment goods, that is

$$Y_I^* = S_I \left(\frac{\alpha_I S_I}{\rho + \delta}\right)^{\frac{\alpha_I}{1-\alpha_I}} \tag{18}$$

Third, setting $\dot{K}(t) = 0$ in eq. (4) and using the result in eq. (18) determines the steady-state value of the total stock of capital, namely

$$K^* = \frac{S_I}{\delta} \left(\frac{\alpha_I S_I}{\rho + \delta}\right)^{\frac{\alpha_I}{1-\alpha_I}} \tag{19}$$

Forth, consistently with eq. (1), subtracting the expression in eq. (17) from the expression in eq. (19) allows us to find the steady-state level of the stock of capital allocated in the sector of consumption goods, that is

$$K_C^* = \frac{S_I (\rho + \delta (1 - \alpha_I))}{\delta (\rho + \delta)} \left(\frac{\alpha_I S_I}{\rho + \delta}\right)^{\frac{\alpha_I}{1-\alpha_I}} \tag{20}$$

Eq. (20) straightforwardly reveals that the equilibrium stock of capital allocated in the production of consumption goods is positive whenever α_I is lower that $1 + \rho/\delta$, a condition that – in principle – does not rule out the possibility to have a non-convex technology in the sector of investment goods. Moreover, eq. (20) together with the expressions in eq. (3) and (13) pins down the steady-state level of the output in the sector of consumption goods, that is

$$Y_C^* = S_C (1 - \alpha_C)^{\frac{1-\alpha_C}{1+\gamma}} \left(\frac{S_I (\rho + \delta (1 - \alpha_I))}{\delta (\rho + \delta)}\right)^{\alpha_C} \left(\frac{\alpha_I S_I}{\rho + \delta}\right)^{\frac{\alpha_C \alpha_I}{1-\alpha_I}} \tag{21}$$

Eq.s (19) and (20) straightforwardly imply that $(\rho + \delta (1 - \alpha_I)) / (\rho + \delta)$ $(\alpha_I \delta / (\rho + \delta))$ is the equilibrium share of capital allocated in the sector of consumption (investment) goods. In this respect, it is worth noticing that its magnitude – as well as the ones of K_I^* and K_C^* – is completely unrelated to the conditions of production prevailing in the sector of consumption goods which matter only for the determination of Y_C^* (cf. Senouci, 2011).

Fifth, eq.s (15), (16), (17), (20) and (21) allows us to derive the equilibrium values of the Pareto optimal relative price of investment goods and the real wage, in detail

$$\frac{p_I^*}{p_C^*} = \frac{\alpha_C S_C \left(1-\alpha_C\right)^{\frac{2(1-\alpha_C)}{1+\gamma}}}{\left(\rho+\delta\right)\left(\frac{S_I(\rho+\delta(1-\alpha_I))}{\delta(\rho+\delta)}\right)^{1-\alpha_C}\left(\frac{\alpha_I S_I}{\rho+\delta}\right)^{\frac{\alpha_I(1-\alpha_C)}{1-\alpha_I}}} \tag{22}$$

$$\frac{w^*}{p_C^*} = S_C\left(1-\alpha_C\right)^{\frac{2(1-\alpha_C)}{1+\gamma}}\left(\frac{S_I\left(\rho+\delta\left(1-\alpha_I\right)\right)}{\delta\left(\rho+\delta\right)}\right)^{\alpha_C}\left(\frac{\alpha_I S_I}{\rho+\delta}\right)^{\frac{\alpha_C\alpha_I}{1-\alpha_I}} \tag{23}$$

Furthermore, setting $\dot{K} = 0$ in eq. (4) by taking into account the results in eq.s (2) and (6), allows us to find the golden-rule level of the stock of capital that in the present context is given by the equilibrium level of K that maximizes the production and the consumption of Y_C. Straightforward algebra reveals that such a critical level of the capital stock is given by the following expression:

$$K_{\text{GR}} = \frac{S_I}{\delta}\left(\frac{\alpha_I S_I}{\delta}\right)^{\frac{\alpha_I}{1-\alpha_I}} \tag{24}$$

As illustrated in the diagram of Figure 1, the expressions in eq.s (24) and (19) imply that as long as the optimizing agent that solves the problem in (7) discounts future utility and disutility streams at a positive rate, the golden-rule level of the capital stock is higher than its long-run value and it falls short of its maximum level ($K_{\max} \equiv \left(S_I/\delta\right)^{1/(1-\alpha_I)}$) that would prevail if the equilibrium value of the production of consumption goods were set to zero. Once again, it is worth noticing that K_{GR} is not related to the production technology prevailing in the sector of consumption goods.

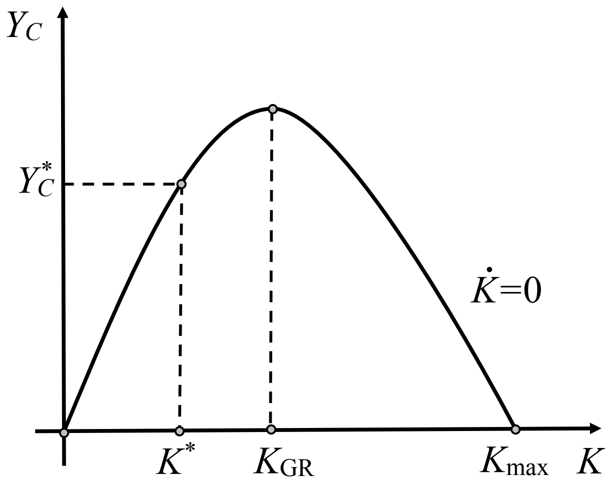

Figure 1: Golden rule

2.4 Local dynamics

Given the expressions in eq.s (2), (4), (6), (13), (14), (19) and (21), the local dynamics of Y_C and K around the unique element of $\mathcal{S}$ is given by the following 2×2 linear system:

$$\begin{pmatrix} \dot{Y}_C(t) \\ \dot{K}(t) \end{pmatrix} = \begin{bmatrix} 0 & -\frac{\alpha_C(1-\alpha_I)(\rho+\delta)\delta Y_C^*}{(\rho+\delta-\alpha_I(\rho+\delta(1-\alpha_I)))K^*} \\ -\frac{\rho+\delta}{\alpha_C S_C}\left(\frac{(\rho+\delta(1-\alpha_I))K^*}{(\rho+\delta)L_C^*}\right)^{1-\alpha_C} & \rho \end{bmatrix} \begin{pmatrix} Y_C(t) - Y_C^* \\ K(t) - K^* \end{pmatrix} \tag{25}$$

The trace of the Jacobian matrix in (25) – say $\boldsymbol{J} \in \mathbb{R}^{2\times 2}$ – is equal to $\rho > 0$, whereas its determinant is equal to

$$-\frac{(\rho+\delta)(1-\alpha_I)(\rho+\delta(1-\alpha_I))\delta}{\alpha_I\delta + (1-\alpha_I)(\rho+\delta(1-\alpha_I))} \tag{26}$$

For given values of ρ and δ, the expression in eq. (26) consists in the ratio between two parabolas whose independent variable is α_I. On the one hand, the parabola at the numerator has two distinct roots, namely $\overline{\alpha}_I = 1$ and $\overline{\overline{\alpha}}_I = 1 + \rho/\delta$ and it is positive for external values. On the other hand, for values of the discount rate lower than three-times the depreciation rate of capital, a range of quite acceptable parameter values, the parabola at the numerator has a negative discriminant so that it is always positive.

Consequently, recalling the result in eq. (20), as long as the technology prevailing in the sector of investment goods is convex, i.e., as long as $\alpha_I \in (0, 1)$, the expression in eq. (26) is negative and the equilibrium stock of capital allocated in the sector of consumption goods takes a meaningful positive value.[5] In this case, no matter the shape of the technology prevailing in the industry of consumption goods – which may even be the sector that produces the largest share of output – $\boldsymbol{J}$ will have two real eigenvalues with opposite sign, say $\lambda_1 < 0$ and $\lambda_2 > 0$. Therefore, as it would happen in a companion one-sector model with logarithmic preferences and Cobb-Douglas technology with decreasing returns to scale, the unique element of S will be a saddle point, and the slope of the saddle path will be positive (cf. Cass, 1965).

In other words, given the value of $\overline{K}$, there will be a unique value of $Y_C(0)$ that places the system in (25) on the stable branch of the saddle path whereas all the others will tend to diverge by violating the resource constraint implied by eq.s (2), (4) and (6), or the transversality condition in (12). Moreover, as long as $\alpha_I \in (0, 1)$, whenever the selected value of $\overline{K}$ undershoots – or overshoots – its steady-state reference, then the same must hold even for $Y_C(0)$. Interestingly, in the model economy under scrutiny, the convexity of the technology prevailing in the sector of investment goods is responsible for the determinacy of the equilibrium path followed by Y_C and K, whereas – recalling the expression in eq. (13) – the convexity of the

[5] For the sake of completeness, it is worth noticing that whenever α_I falls in the interval $\left(\overline{\alpha}_I, \overline{\overline{\alpha}}_I\right)$ the equilibrium stock of capital allocated in the sector of consumption goods is positive, but the stationary solution is an unstable source.

technology available in the sector of consumption goods is responsible for the positivity of the labour input employed in that industry.

The analogy with the textbook Ramsey model discussed earlier can be further expanded. Notably, the two elements on the main diagonal of $\boldsymbol{J}$ – namely, 0 and ρ – match exactly those found in a corresponding one-sector model with logarithmic preferences and Cobb-Douglas technology. The remaining two elements of $\boldsymbol{J}$, however, may not necessarily align with their counterparts in the one-sector model, as their actual values are influenced by the chosen calibration. It is well understood that, in the one-sector model, consumption directly reduces capital accumulation on a one-to-one basis. As a result, the first element of the second row of $\boldsymbol{J}$ – denoted as $j_{2,1}$ – is always equal to -1, since any additional unit of Y_C consumed by the optimizing agent leads to an equivalent reduction of saving and investment (cf. Cass, 1965). Given the expressions in eq.s (13), (19) (20) and (25), in the theoretical framework under scrutiny this holds when the parameters of the model fulfil a particular relationship which is described by the following equality:

$$\frac{K_C^*}{L_C^*} = \left(\frac{\alpha_C S_C}{\rho + \delta}\right)^{\frac{1}{1-\alpha_C}} \tag{27}$$

The equality in eq. (27) is the one that prevails when the marginal propensity to consume of the optimizing agent is strictly complementary to its marginal propensity to save and – recalling the result in eq. (17) – it corresponds to the case in which the equilibrium capital intensity in the sector of consumption goods (K_C^*/L_C^*) is just determined in the same way as the equilibrium level of capitalization in the sector of investment goods (K_I^*), i.e., by equating the marginal productivity of capital in the sector of consumption goods to the opportunity cost of capital which is given by the sum between the discount and the depreciation rates.

Suppose now that $\mathbb{V}(\lambda_1) \in \mathbb{R}^{2\times 1}$ is the column-eigenvector associated to the convergent eigenvalue. Thereafter, the dynamics of the output in the sector of consumption goods and the dynamics of the capital stock are conveyed by

$$\begin{pmatrix} Y_C(t) \\ K(t) \end{pmatrix} = \begin{pmatrix} Y_C^* \\ K^* \end{pmatrix} + \begin{pmatrix} \frac{v_{1,1}(\lambda_1)}{v_{2,1}(\lambda_1)} \\ 1 \end{pmatrix} \left(\overline{K} - K^*\right) \exp(\lambda_1 t) \tag{28}$$

where $v_i(\lambda_1) \in \mathbb{R}$ is the i-th element of $\mathbb{V}(\lambda_1)$.

Relying on a suitable discretization and a tailored calibration, the expression in (28) will be used to examine the numerical properties of the two-sector model economy under scrutiny.

3 Numerical properties

To evaluate its numerical properties, the theoretical framework developed above is calibrated on an annual basis by taking as reference the US economy. Specifically, following Christiano and Fisher (2003), the values of the sector-specific output elasticities with respect to capital

– α_C and α_I – are based on the estimates provided by Hornstein and Praschnick (1997). The common depreciation rate of capital (δ) is set according to Kydland and Prescott (1982), while – consistently with the requirement for a negative determinant of $\boldsymbol{J}$ – the discount rate (ρ) follows the value suggested by Itskhoki and Moll (2019). The inverse elasticity of labour supply (γ) is calibrated in line with Benhabib and Farmer (1996). Additionally, after normalizing S_I to 1, the value of S_C is calibrated to match the average ratio between the share of consumption and investment expenditures over GDP, a figure that according to historical data amounts to 4.566.[6] The values of each parameter together with their own concise description is summarized in Table 1.

Parameter	Description	Value
S_C	Productivity in the sector of consumption goods	2.253
α_C	Elasticity of capital in the sector of consumption goods	0.450
S_I	Productivity in the sector of investment goods	1.000
α_I	Elasticity of capital in the sector of investment goods	0.260
δ	Capital depreciation rate	0.100
ρ	Discount rate	0.030
γ	Inverse elasticity of labour supply	1.000

Table 1: Calibration

The figures in Table 1 imply that in the steady-state equilibrium the share of capital employed in the sector of consumption goods (K_C^*/K^*) amounts to 80%, so that the remaining 20% is allocated in the production of investment goods. Unsurprisingly, in the calibrated two-sector economy the consumption goods industry is more capital-intensive than the investment goods one.

In addition, under the baseline calibration the equality in eq. (27) is not verified. Specifically, the adopted parameter values imply that the first element on the second row of $\boldsymbol{J}$ in (25) is less than one in absolute value. To be precise, $j_{2,1} = -0.5061$. This means that according to the figures in Table 1 the decision to produce and consume an additional unit of Y_C restrains capital accumulation but less than one-to-one as it happens in the conventional one-sector model of optimal growth (cf. Cass, 1965). In other words, given the values of the involved parameters, in the two-sector model developed above the marginal productivity of the stock of capital employed in the sector of consumption goods is so high that the production and the consumption of an additional unit of Y_C is possible by slowing capital accumulation by a lower amount.[7] Therefore, in the calibrated model, the sum between the marginal propensity

[6] US macroeconomic data on national account can be retrieved at https://fred.stlouisfed.org.

[7] Obviously, this follows from the fact that K_C^*/L_C^* is lower than the amount appointed by eq. (27).

to consume and the marginal propensity to save – and invest – is larger than 1 (cf. Senouci, 2011).

On a visual perspective, the parameters values in Table 1 lead to the trajectories of Y_C and K illustrated in the diagram of Figure 2.[8]

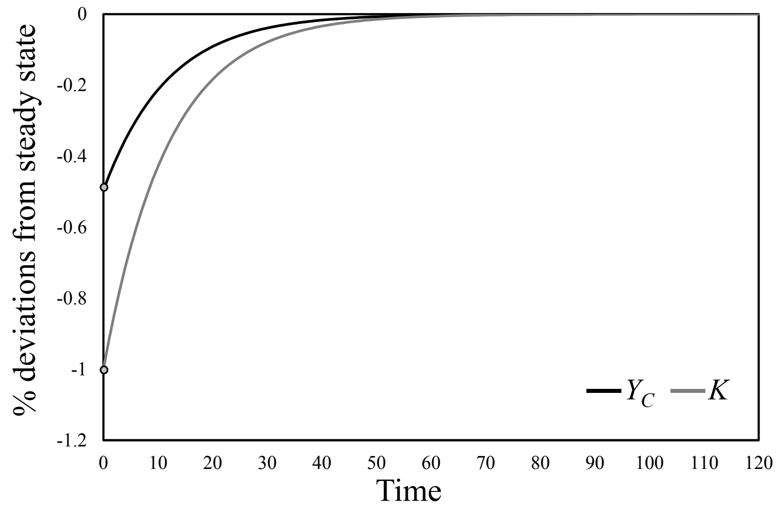

Figure 2: Saddle-path dynamics

The plot in Figure 2 shows that when the stock of capital undershoots its steady-state value by 1% the production of consumption goods does the same but to a lower extent (about 0.5%). Obviously, the smaller deviations of Y_C from Y_C^* during the adjustment process are due to the risk-aversion implied by the logarithmic preferences over consumption of the optimizing agent that solves the intertemporal problem in (7). In the present context, recalling the result in eq. (13), along the optimal path consumption smoothing is mainly realized through the constancy of the labour input that – together with capital – allows to produce the goods that contribute to the utility of the maximizing agent. Thereafter, as it happens in the standard one-sector model, Y_C and K monotonically converge towards their long-run references along the upward-sloped saddle path mentioned in Section 2.

Another interesting aspect of the model economy under scrutiny is certainly the way in which capital is optimally allocated over time in the two sectors. The adjustments of K_C and K_I implied by eq.s (5) and (6) obtained when K initially undershoots its steady-state value by 1% as it does in Figure 2 are tracked in Figure 3.

[8]The time step of the simulations is set to 1. MAT LAB codes are available from the author upon reasonable request.

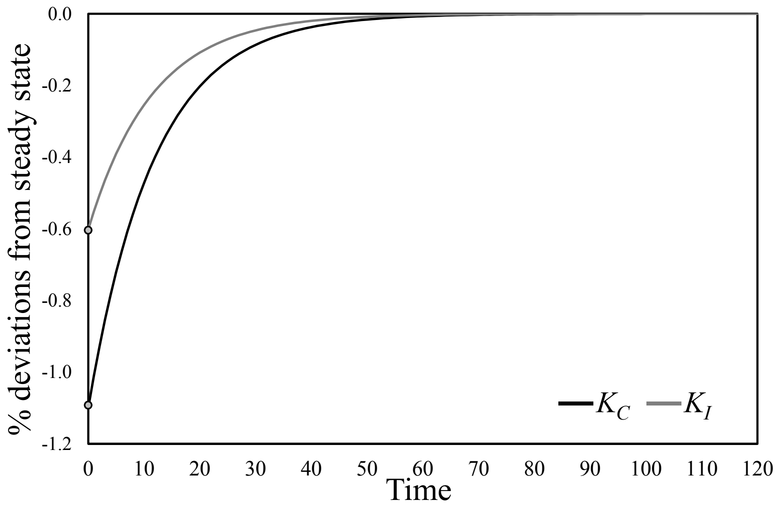

Figure 3: Capital adjustments in the two sectors

The plot in Figure 3 shows that the amount of capital allocated in the sector of consumption (investment) goods undershoots its long-run reference to a higher (lower) extent with respect to the aggregate stock of capital and then they converge towards their respective equilibrium value without undergoing any factor reversal. In other words, along the optimal path, the implied share of capital allocated in sector of consumption goods is always above the one allocated in the sector of investment goods. Furthermore, recalling the expressions in eq.s (15) and (16), the allocation of capital in the two sectors is also the determinant of the relative prices that in a decentralized environment implement the centralized solution of the problem in (7). Specifically, the competitive paths of the ratios p_I/p_C and w/p_C implied by the adjustments of Y_C and K plotted in Figure 2 are illustrated in the two panels of Figure 4.

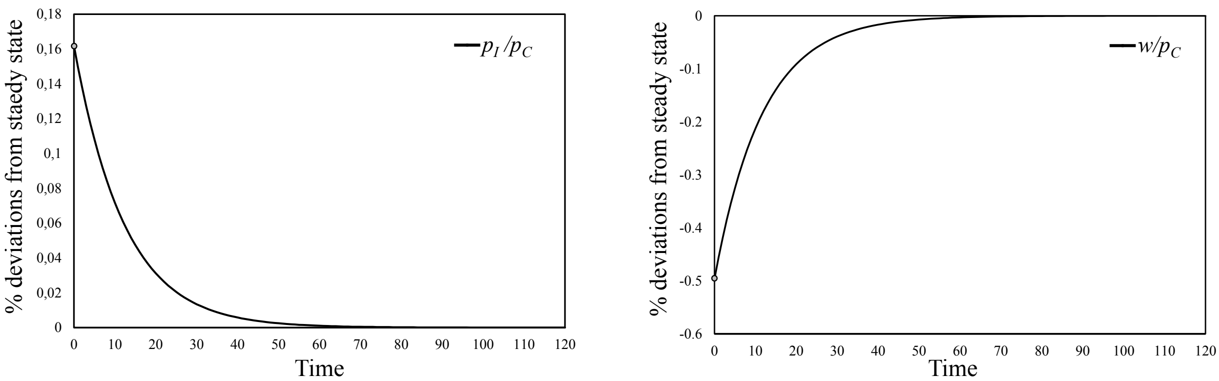

Figure 4: The implied paths of the relative price of investment goods and the real wage

The plot on the left-hand-side of Figure 4 reveals the price of investment goods measured in units of the price of consumption goods which is consistent with the solution of the central planner problem follows a decreasing path towards its steady-state value. Since the quantity adjustments plotted in Figures 2 and 3 imply that the production of the two goods – Y_C and Y_I – follows an increasing pattern towards the respective long-run values, this means that whenever the supply functions of the two goods are upward sloped the growth rate of the price of investment goods has to fall short of the growth rate of the price of consumption goods all over the adjustment process towards the stationary solution.[9] Moreover, the plot on the right-hand side shows that the profit-maximizing level of the real wage follows an increasing path towards its steady-state reference which is driven by the accumulation of capital employed in the sector of consumption goods.

From an empirical point of view, the dynamic behaviour of p_I/p_C shown in the left-hand-side of Figure 4 is consistent with the countercyclical pattern displayed by actual data on the relative price of tradable capital goods stressed by many authors and often attributed to technical progress and/or market integration (cf. Collins and Williamson, 2001; Restuccia and Urrutia, 2001; Fisher, 2006; Justiniano et al. 2011; Hergovich and Merz, 2018; Lian et al. 2020). In our two-sector economy, such a decreasing path is obtained through the optimal allocation of labour and capital in the two industries even without any technical progress (cf. Guerrazzi, 2025). On other side, the increasing pattern of the real wage plotted in right-hand-side is likewise consistent with the procyclical pattern displayed by the value of hourly compensation – which includes not only wage and salary compensation, but also all the other entries of labour remuneration such as fringe benefits and contributions for social insurance programs – deflated with the consumer price index for urban consumers. According to actual data, such a measure of the real wage is also the one that closely follows the observed path of labour productivity (cf. Sallivan, 1997).[10]

4 Concluding remarks

In this paper, along the lines of Hashimoto and Sakuragawa (1999), I developed a frictionless two-sector optimal growth model with endogenous labour supply in which the non-reproducible factor can be employed only in the industry that produces consumption goods by boosting households' utility. From a theoretical perspective, I showed that when all the relevant decisions are taken at a centralized level by an infinitely lived optimizing agent this setting leads to the constancy of the labour input allocated in the sector of consumption goods whose actual employment in that industry is also related to the availability of a convex technology. In parallel, a likewise convex technology in the investment good sector turned out to be instead sufficient

[9]Denoting with $r(t)$ the rental rate paid by firms in a decentralized environment for each unit of employed capital, the supply functions of each goods would be, respectively, given by $p_C(t) = (1/S_C)(r(t)/\alpha_C)^{\alpha_C}(w(t)/(1-\alpha_C))^{1-\alpha_C}$ and $p_I(t) = r(t)(Y_I(t))^{1/(1-\alpha_I)}/\alpha_I S_I^{1/\alpha_I}$.

[10]Further evidence on the behaviour of the relative price of capital goods and the one of real wage rates is given in Appendix.

for the determinacy of economically meaningful equilibrium paths. Furthermore, parametrizing and calibrating the model to the US economy, I showed that such a two-sector economy smooths the usual complementarity between the marginal propensities to consume and to save and it also able to replicate the countercyclical pattern of relative price of tradable capital goods and the procyclical path followed by some empirical proxies of the real wage rate (cf. Sallivan, 1997; Collins and Williamson, 2001; Restuccia and Urrutia, 2001; Fisher, 2006; Justiniano et al. 2011; Hergovich and Merz, 2018; Lian et al. 2020).

The analysis carried out in this paper can be further developed in many directions. Obviously, it would be interesting to derive a symmetric version of the two-sector model of optimal growth in which labour can be employed only in the sector of investment goods by considering full automation in the production of consumption goods. This unexplored theoretical framework could be a mandatory prologue for the subsequent derivation a two-sector model of optimal growth in which labour supply is fully endogenous, i.e., the situation in which labour becomes malleable as capital and even the non-reproducible factor can be employed in both industries. With the exclusion of settings characterized by very peculiar assumptions about the preferences of the optimizing agent and/or about the factor elasticities displayed by the technologies available in the two industries, such a possibility has been explored only incidentally in the literature on two-sector models of optimal growth (cf. Benhabib and Farmer, 1996; Benhabib and Nishimura, 1998; Harrison and Weder, 2002; Zhang, 2005; Dufourt et al. 2015). In addition to addressing the intertemporal trade-off between offering labour in the sector of consumption goods and in the one of investment goods, the latter extension should also deliver some interesting insights about the determinants of industry wage differentials (cf. Reilly and Zanchi, 2003; Plasman et al. 2007). For the mentioned reasons, these could be intriguing avenues for additional deepening.

Appendix: The observed paths of the relative price of capital goods and real earnings

An empirical appraisal of the path followed by the relative price of capital goods and the real wage rate in the US can be grasped by looking at, respectively, the ratio between the estimated values of the producer price index for capital equipment (PPICPE) and price index for personal consumption expenditure (PCEPI), and the values of the median usual weekly real earnings for workers of 16 years and over (LEU0252881600A). The pattern of these two time series together with their linear trends are illustrated in the two panels of Figure A1.

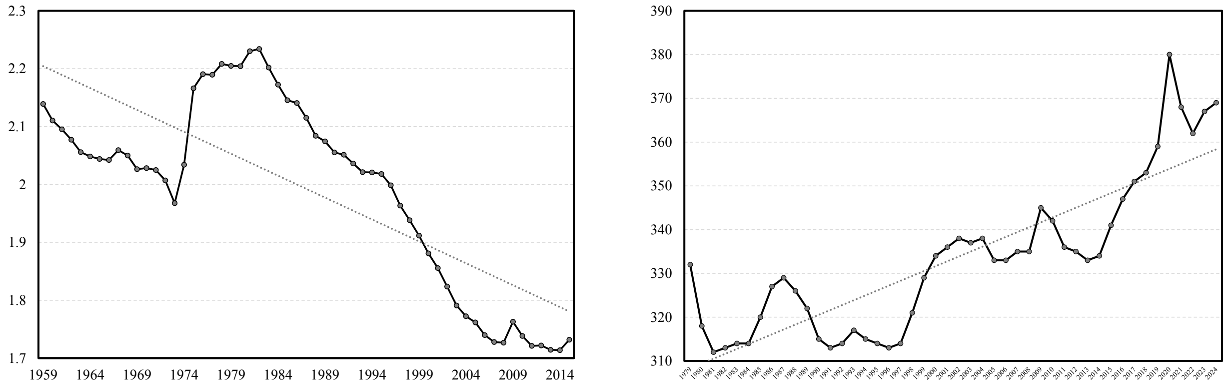

Figure A1: The relative price of capital goods and real earnings in the US

Consistently with the calibrated version of the two-sector model presented in the main text, the two plots in Figure A1 show that the actual relative price of capital goods followed a decreasing path, whereas the adopted measure for the real wage rate followed an upward sloped path. As we can see, in real-world data, the main deviations from the respective monotonic trends occurred in correspondence of recession episodes. Those divergences may signal that during this type of events the values of the productivity parameters in eq.s (2) and (3), namely, S_C and S_I, can hardly be taken as time-invariant (cf. Ferreira et al. 2014).

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