Prefix-Based Collection Auction: A Mechanism against Market Power and Collusion

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Abstract

We introduce a new auction mechanism for selling multiple identical items to a single winner — the Prefix-Based Collection Auction. In this format, the winner is restricted to a contiguous prefix of their bids, and the payment rule follows a strategy-proof (DSIC) second-price design. The prefix structure provides strong protection against both market power and bidder collusion, while preserving intuitive and fair bidding behavior. The mechanism is robust, simple to implement, and has potential applications in art-collection markets, digital advertising, and other settings where the value of ownership depends on controlling an entire collection.

1 Introduction

In many auction environments, a single participant may seek to acquire multiple identical items, forming what we call a collection. Standard mechanisms such as the Vickrey auction [1], Clarke's mechanism [2], or Groves' framework [3], while strategy-proof, do not adequately address the risks of market power or collusion. Dominant bidders may manipulate outcomes, and groups of participants may coordinate their bids to suppress prices [4].

To address these concerns, we introduce the Prefix-Based Collection Auction. In this mechanism, a single bidder may win only a contiguous prefix of their submitted bids, and the winner pays a second price determined by the best competing prefix offered by others. This simple payment rule preserves the desirable DSIC property of Vickrey-style auctions while embedding additional robustness through the prefix structure.

The prefix restriction plays a key strategic role. Unlike sequential or single-parameter auctions, where collusion is relatively easy to coordinate, a multi-parameter prefix auction makes collusion substantially harder: conspirators must agree not on a single number, but on an entire sequence of bids. Experimental evidence confirms that collusion is more feasible in sequential than in simultaneous multi-object auctions [5]. By contrast, prefix auctions increase the dimensionality of coordination and thereby reduce the scope for stable collusive arrangements.

Moreover, even if collusion arises, the structured nature of prefix bids generates patterns that are more easily detectable with modern machine learning methods. Recent laboratory studies show how algorithmic collusion can be detected through pattern recognition in bid data [6]. Thus, prefix auctions not only discourage collusion ex ante, but also provide tools for ex post monitoring and enforcement.

We argue that this format is particularly relevant in applications such as digital advertising markets. Advertisers often wish to acquire all ad slots on a page to guarantee exclusivity. Our mechanism accommodates such demands by allowing bidders to compete over entire prefixes of slots, while ensuring that the payment reflects true competition.

In contrast to multi-winner auction formats such as the Vickrey–Clarke–Groves mechanism (VCG), combinatorial auctions, or direct fractional auctions (DFA) [7], the prefix-based design guarantees that only one bidder wins, receiving the full prefix of items. This "collectible" property distinguishes it sharply from fractional or divisible ownership models, and underlines its relevance in markets where exclusivity over the collection is essential.

Finally, for clarity, we distinguish two components of the mechanism: the Winner Determination Rule (WDR), which specifies allocation, and the Winner Payment Rule (WPR), which specifies pricing. Unlike the traditional notion of the Winner Determination Problem (WDP), which emphasizes computational difficulty, the concept of WDR highlights that allocation is not merely a problem to be solved, but a normative rule that shapes the overall properties of the mechanism.

2 Winner Determination Rule (WDR)

2.1 Setting and Definitions

The seller offers K identical indivisible items, where K is the total number of units available for sale. Each bidder i submits a bid sequence (prefix): $b_i(1)$, $b_i(2)$, …, $b_i(K)$. The sequence must be non-decreasing: $b_i(1) \le b_i(2) \le \cdots \le b_i(K)$.

In general, the winner determination problem in multi-unit auctions is formulated as the search for a set of winning bidders whose allocations maximize total value. In our prefix framework, the same principle applies: formally, one seeks an allocation of the K items across bidders. However, as shown in Appendix A (Singleton Theorem), any such allocation reduces to a singleton: the optimal solution always assigns all K items to a single bidder.

Accordingly, the Winner Determination Rule can be stated as:

$$i^* = \arg \max_{i=1,\ldots,N} \sum_{k=1}^{K} b_i(k). \tag{1}$$

This structural property simplifies strategic behavior and provides protection against collusion and price manipulation.

3 Winner Payment Rule (WPR) and Truthfulness

3.1 Payment Rule

Our goal is to define a payment rule such that truthful bidding is a dominant strategy for each participant, meaning it maximizes their utility regardless of others' actions [1, 3]. Let the winner i^* be determined by equation (1), receiving a prefix of length K. The payment rule is:

$$P(i^*) = \max_{i \neq i^*} S_i(K), \tag{2}$$

where $S_i(K) = \sum_{j=1}^{K} b_{i,j}$ is the prefix sum of bidder i. This is a prefix version of the Vickrey second-price rule: the winner pays not their own submitted prefix value, but the highest alternative prefix value offered by competitors for the same collection of K items.

3.2 Truthfulness

Under this payment rule, truthful reporting of valuations is a dominant strategy. If bidder i underbids, they risk losing even when their true valuation would allow a positive utility. If they overbid, they may win but be forced to pay more than their valuation. Thus, bidding truthfully maximizes expected payoff, as in the classic Vickrey auction [1].

More formally, the utility of the winning bidder i^* with true valuation vector $v = (v_1, \ldots, v_K)$ is

$$u = \begin{cases} \sum_{j=1}^{K} v_j - P, & \text{if the bidder wins,} \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Here P is defined by (2).

3.3 Weak DSIC Property of Unit Bids

It is important to note that while the mechanism as a whole is DSIC, the individual unit bids $b_{i,j}$ are not strictly dominant strategies in isolation. A bidder could, in principle, shade one component of the sequence without directly changing the outcome, provided the prefix sum remains unchanged. However, such deviations yield no advantage, since the payment depends only on the highest competing prefix, not on the bidder's own report. For this reason, we describe the bids $b_{i,j}$ as weakly DSIC: misreporting them does not strictly harm the bidder in every case, but also never produces a gain. Thus, truthful bidding is weakly dominant for each unit, and strictly dominant for the sequence as a whole.

4 Collusion Resistance and Strategic Robustness

A central motivation for designing the Prefix-Based Collection Auction is its robustness against bidder collusion. In traditional Vickrey auctions, especially when multiple objects are sold, collusion is comparatively easy to sustain: bidders can coordinate to suppress the second price, lowering the payment of the winner. In sequential multi-object auctions, items are sold one after another. Collusion is straightforward: bidders can simply agree to alternate victories, e.g., "I will take the first item, you take the second." This form of tacit coordination has been well documented in both experimental and empirical studies [5].

By contrast, in simultaneous multi-object auctions, all items are sold at once, which makes collusion more difficult. Coordination now requires bidders to agree on strategies across multiple objects simultaneously. Still, even in this format, groups of bidders can sometimes manipulate aggregate outcomes by aligning their bids across objects.

The Prefix-Based Collection Auction introduces an additional layer of complexity. Instead of bidding separately on individual items, each bidder must submit a monotone prefix of bids. To collude successfully, participants must not only agree on who gets which items, but also align entire bid sequences in a way that sustains the collusive outcome. This transforms collusion into a multi-parameter coordination problem, which is substantially more fragile than single-price agreements. Thus, prefix auctions make collusion harder to initiate and harder to maintain: deviations from the collusive plan are easier, punishments are harder to enforce, and the credibility of the agreement is weaker compared to sequential formats.

Even if collusion arises, the structured nature of prefix bids creates recognizable patterns. For example, conspirators may generate unnaturally synchronized or stepwise bid sequences. Recent laboratory evidence [6] shows that algorithmic collusion leaves detectable statistical traces in bidding data. Because prefix auctions constrain bids to monotone sequences, anomalies become even more conspicuous, making them easier to identify using modern machine learning tools.

Taken together, these properties show that prefix auctions combine two layers of collusion resistance: (i) strategic resistance ex ante — collusion is harder to arrange because of the multi-parameter prefix format; and (ii) detectability ex post — if collusion still occurs, structured bid sequences make detection significantly easier. In this sense, the prefix structure and the second-price rule complement each other: while the latter guarantees truthful bidding (DSIC), the former ensures robustness against collusion, making the Prefix-Based Collection Auction simultaneously incentive-compatible and strategically resilient.

5 Conclusion

The Prefix-Based Collection Auction introduces a simple yet powerful modification of the classical Vickrey framework. By restricting allocations to monotone prefixes and applying a second-price payment rule, the mechanism preserves strategy-proofness while adding robustness against collusion. Relative to sequential auctions, the prefix structure eliminates the possibility of straightforward alternation agreements ("I win the first item, you win the second"). Relative to simultaneous multi-object auctions, it further constrains coordination by requiring bidders to submit entire monotone sequences, transforming collusion into a multi-parameter coordination problem that is difficult to sustain. Moreover, the structure of prefix bids generates regular patterns, making any collusive behavior more easily detectable with modern data-driven methods. From a mechanism design perspective, the auction remains modular and transparent. The Winner Determination Rule (WDR) specifies allocation by maximizing prefix sums, and the Winner Payment Rule (WPR) applies a direct extension of the Vickrey second-price principle. This division underscores the conceptual clarity of the format while avoiding the computational complications often associated with multi-unit mechanisms. Taken together, these properties position the Prefix-Based Collection Auction as a middle ground between classical second-price auctions and more elaborate multi-object formats. It retains the desirable DSIC foundation of the former, while introducing structural safeguards that strengthen resistance to market power and collusion. As such, it offers both theoretical insight and practical promise for applications in markets where collections, rather than individual units, are the relevant object of competition [7].

Appendix A. Theorem about Singleton Solutions

Notations and Inequalities

Consider N non-decreasing sequences of the same length K:

$$(b_i(1), b_i(2), \ldots, b_i(K)), \quad i = 1, \ldots, N. \tag{A1}$$

Assume $b_i(1) \leq b_i(2) \leq \cdots \leq b_i(K)$ for each i. Define the prefix sum $S_i(k) = \sum_{j=1}^{k} b_i(j)$. Define

$$F^* = \max_{i=1,\ldots,N} S_i(K), \quad \text{so that } S_i(K) \leq F^*,\ i = 1, \ldots, N. \tag{A2}$$

Allocations

Definition: A K-allocation $\alpha(K) = (k(i_1), k(i_2), \ldots, k(i_m))$ is a set of prefixes such that their lengths $k(i) \neq 0$ together partition K. That is,

$$\sum_{i=1}^{N} k(i) = K. \tag{A3}$$

The value of allocation $\alpha(K)$ is

$$F(\alpha(K)) = \sum_{i=1}^{N} S_i(k(i)). \tag{A4}$$

Theorem (Singleton Solutions)

If $b_i(1), b_i(2), \ldots, b_i(K)$ are non-decreasing sequences, then for any K-allocation $\alpha(K)$ the following holds:

$$F(\alpha(K)) = \sum_{i=1}^{N} S_i(k(i)) \leq \max_{i=1,\ldots,N} S_i(K) = F^*. \tag{A5}$$

Proof

For non-decreasing sequences, the averages of their prefixes do not decrease:

$$\frac{S_i(k)}{k} \leq \frac{S_i(k+1)}{k+1}. \tag{A6}$$

Substituting K in place of $k+1$ gives

$$S_i(k(i)) \leq \frac{k(i)}{K}\, S_i(K). \tag{A7}$$

Using (A7) and (A2) yields

$$S_i(k(i)) \leq \frac{k(i)}{K} F^*. \tag{A8}$$

Substituting (A8) into (A4), and using (A3), we obtain

$$\sum_{i=1}^{N} S_i(k(i)) \leq \sum_{i=1}^{N} \frac{k(i)}{K} F^* = \frac{1}{K} \Big(\sum_{i=1}^{N} k(i) \Big) F^* = 1 \cdot F^*.$$

End of proof. □

From the proven inequality $F(\alpha(K)) \leq F^*$ it follows that the best allocation can always be the set with only one element (the singleton), namely

$$i^* = \arg \max_{i=1,\ldots,N} S_i(K).$$

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