Understanding Credit Demand: Exploring the Impact of Leverage and Return on Borrowing

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Understanding Credit Demand:
Exploring the Impact of Leverage and Return on Borrowing[1]

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Abstract

This study presents a conceptual framework for analyzing the relationship between firm returns and credit demand. By examining the conditional distributions of return on borrowing and leverage, the framework investigates how changes in interest rates and the maximum leverage provision impact credit demand. The study emphasizes the role of the mean and standard deviation of these distributions and introduces the concept of elasticity to quantify the responsiveness of credit demand. The findings provide valuable insights for policymakers in understanding the dynamics of credit demand and formulating effective economic policies. The framework offers a useful tool to assess the implications of interest rate and leverage provision changes on credit demand and support sustainable economic growth in the credit market.

Keywords: credit demand, interest rate policy, distribution of leverage, economic policy.

JEL Code: E41, E43, E44, E51, E52

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1. Introduction

The COVID-19 pandemic originated as a health crisis but subsequently transformed into an economic crisis as governments implemented interventions aimed at mitigating the virus's spread, including mobility restrictions and social activity limitations. These measures have had a profound impact on global supply chains, leading to a significant contraction in the demand for goods and services. Consequently, businesses at the microeconomic level have experienced diminished returns and increased risks. It is important to note that the relationship between business returns and risks is idiosyncratic, as each firm or group of firms within an economic sector demonstrates a unique response to specific circumstances. Various iterations of capital asset pricing models have indicated a positive association between expected returns and idiosyncratic risk (Barberis & Huang, 2001; Goyal & Santa-Clara, 2003), which subsequently influences investment decisions (Panousi & Papanikolaou, 2012). During periods of recession, credit demand typically declines, reflecting reduced interest in pursuing new investments (Bernanke et al., 1991; Hall, 1993).

To counter the economic ramifications of the COVID-19 pandemic, financial authorities worldwide have implemented diverse policy packages aimed at boosting lending. These measures include providing liquidity facilities, reducing mandatory deposits at central banks, and lowering interest rates (Didier et al., 2021). Despite these policy interventions, many countries have witnessed a decrease in credit realization, indicating challenges in anticipating the pandemic's impact (Çolak & Öztekin, 2021). Stiglitz and Weiss (1981) elucidated the concept of credit rationing, where lenders limit or are unwilling to increase the supply of credit, resulting in excess demand in the credit market. However, during severe economic crises like the COVID-19 pandemic, the opposite phenomenon occurs, with a significant decline in credit demand (Danilowska, 2021; Temesvary & Wei, 2021; Wu et al., 2020). While macroeconomic policies play a crucial role in maintaining economic stability during challenging times, they alone are insufficient to address micro-level idiosyncratic problems. Economic prosperity emerges from the interaction between macro and micro levels, with macroeconomic policies providing opportunities while firms generate tangible wealth at the micro level (Porter, 2003). Effective recovery of the economy necessitates a series of macro policies that address real challenges at the micro level.

The credit market exhibits a dynamic and multifaceted structure. Monetary and financial authorities determine interest rates and other credit policies by considering dynamic macroeconomic variables such as economic output, inflation, unemployment, exchange rates, economic growth, and balance of payments (Hahn & Friedman, 1990). Subsequently, the banking industry supplies credit based on the regulations and policies set by these authorities. On the demand side, firms make decisions considering interest rates, credit policies, and microeconomic variables encompassing aspects of profitability (such as sales growth, revenue, net return, expected profit, return on assets) and business costs (such as cost of goods sold, operational costs, current ratio, leverage) (Hillier et al., 2019). However, the conceptual analysis of credit demand and supply is predominantly driven by macroeconomic approaches and exploring microeconomic variables can contribute to enriching macroeconomic analyses, particularly in understanding the dynamics of the credit market.

The dynamic of credit market during recessions or economic crises has been explored in various economic studies. During economic contractions, the financial structure of the business sector weakens, and firms experience a decline in performance, leading to simultaneous weakening of credit supply and demand (Wojnilower, 1985). Bernanke et al. (1991) and Hall (1993) examined the reduction in credit demand during recessions and found that declining demand for goods and services prompts firms to delay expansion and investment plans, resulting in a credit crunch during recessionary or subnormal conditions.

In the microeconomic literature, recessionary or economic crisis conditions are often associated with a decline in firm performance. Numerous empirical studies have explored the effects of macro-level economic contractions on micro-level firm performance, including the increased likelihood of financial distress for highly leveraged firms during the Great Depression in the 1930s (Graham et al., 2011), the continued use of debt financing by U.S. firms during the 1987-1988 crisis following the stock market crash (Bernanke et al., 1990), the impact of the 1997-1998 Asian financial crisis on the profitability of financial firms in Indonesia (Sufian & Habibullah, 2010), the influence of the 2008 financial crisis on the performance of firms in Gulf Cooperation Council countries (Zeitun & Saleh, 2015), and the effects of the COVID-19 pandemic on global firm performance (Hu & Zhang, 2021). Declining sales and increasing costs lead to reduced profitability or even losses for many businesses and industries (Richardson et al., 1998)resulting in decreased credit demand during recessionary periods. Integrating these micro perspectives into macroeconomic analyses can provide a more comprehensive understanding of credit demand dynamics.

Our objective to contribute to the existing literature on credit demand analysis by proposing a conceptual framework that establishes a connection between firm-level financial ratios and macro-level policies regarding interest rates and maximum leverage. Through deductive reasoning using basic model leverage-related factors associated with loan acquisition capacity (Jarrow, 2013) and interest rate factors influencing borrowing costs and profitability (Stiglitz & Weiss, 1981; Romer, 2012), this framework elucidates that aggregate credit demand can be determined by considering the distribution of leverage and return on borrowing. Return on borrowing serves as an indicator of credit productivity, reflecting the firm's ability to generate added value from its activities. A higher return on borrowing ratio signifies a more productive firm in terms of generating returns. Comparable to other profitability ratios, return on borrowing enables benchmarking a firm against the market as a whole and other firms within the same industry.

Another influential factor is leverage, which holds significant importance for firms when seeking credit. Firms may seek and utilize credit facilities if their leverage level falls below the maximum permissible leverage provision. The maximum leverage provision plays a crucial role in ensuring that firms possess the capacity to repay their credit obligations and that banks can recover the lent credit from borrowers. In conjunction with interest rates, the maximum leverage provision is employed to establish policies governing the maximum credit limit. These policies aim to influence credit demand in the market and maintain economic stability, particularly in situations where the credit market experiences disruptions or when managing economic expansion or contraction.

Moreover, this framework is expected to have practical implications for economic policies, particularly in estimating the elasticity of credit demand in response to changes in interest rates or maximum leverage provisions. Corporate credit can be important measure for policy makers, both from monetary policy and financial stability perspectives.

2. Micro Foundation of Credit Demand

This study adopts the microeconomic framework introduced by Stiglitz and Weiss (1981) to analyse the impact of interest rates on credit. Specifically, If firm i borrows funds of B_i with an interest rate of r_i , then $B_i.r_i$ is the interest cost of borrowing, where the firm is denoted as "default" when the return R_i (which is the difference between revenue and costs) and collateral C_i are insufficient to repay the agreed-upon borrowing amount. This relationship can be mathematically expressed as Equation (1).

$$R_i + C_i \leq B_i(1 + r_i). \quad (1)$$

The net return or profit (π) of firm i can be represented as a function of the return R_i and the interest rate r_i. In this context, the firm faces two possible outcomes: it must either repay the agreed-upon borrowing amount if the return is sufficient, or alternatively, release the collateral if the return falls short.

$$\pi_i(R_i, r_i) = max\{R_i - B_i(1 + r_i); -C_i\}. \quad (2)$$

A firm experiences loss $(\pi_i < 0)$ when the return (R_i) fails to cover the borrowing amount and its associated interest cost. At the break-even point $(\pi_i = 0)$, the return (R_i) is sufficient to meet the obligation of repaying the borrowing amount and its interest cost. In contrast, the firm achieves a positive net return $(\pi_i > 0)$ when the return (R_i) surpasses the agreed-upon borrowing amount along with its interest. The relationship between the net return $\pi_i(R_i, r_i)$ and the return (R_i) is visually represented in Figure 1, where the net return is illustrated by the red line as a function of the return.

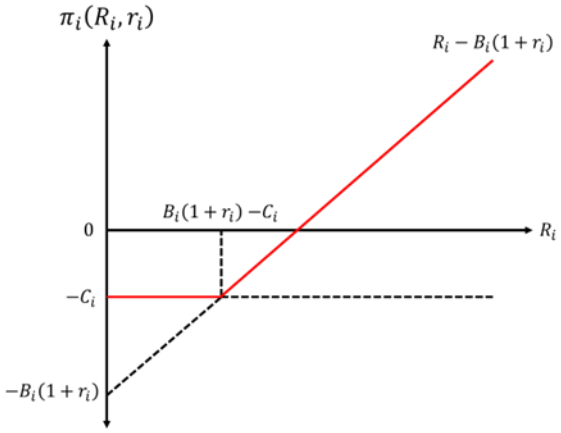


FIGURE 1. THE RELATIONSHIP BETWEEN A FIRM'S NET RETURN (PROFIT) AND ITS RETURN (STIGLITZ & WEISS, 1981)

The net return (π_i) follows a probability density function (pdf) denoted as f_i, which is dependent on the return R_i and the risk θ_i. By incorporating these variables, the expected net return (Π_i) can be computed using the equation proposed by Stiglitz and Weiss (1981). The convexity of the firm's profit function (as represented in Figure 1) implies that an increase in the risk parameter θ_i leads to a higher expected profit (Π_i), as established by Stiglitz and Weiss (1981)and Stiglitz and Rothschild (1970).

$$\Pi_i(r_i, \hat{\theta}_i) \equiv \int_0^\infty \pi_i(R_i, r_i) \cdot f_i\left(R_i, \hat{\theta}_i\right) dR_i. \tag{3}$$

When making borrowing decisions, firms should assess their capacity to repay credit obligations. As firms repay their debts using profits generated from their business activities, they expect their anticipated returns to be adequate to cover the agreed-upon borrowed amount along with the associated interest costs, as emphasized by Stiglitz and Weiss (1981). Consequently, it is crucial for a company to determine the portion of its obligations that can be met using its generated returns. The interest rate at which the return precisely matches the total cost of borrowing is referred to as the critical interest rate. A firm will seek credit if it secures a credit interest rate lower than its critical interest rate.

THEOREM 1: *Borrowing is feasible for the firm only if the interest rate is lower than a critical interest rate (threshold) determined by a specific risk level.*

Equations (2) and (3) facilitate the computation of the net return and the expected net return, which can be expressed as follows:

$$\Pi_i(r_i, \hat{\theta}_i) = \int_0^{\infty} \pi_i(R_i, r_i) \cdot f_i\left(R_i, \hat{\theta}_i\right) dR_i$$

$$= \int_0^{B_i(1+r_i)-C_i} -C_i \cdot dF_i(R_i, \hat{\theta}_i) + \int_{B_i(1+r_i)-C_i}^{\infty} R_i - B_i(1+r_i) \cdot dF_i(R_i, \hat{\theta}_i)$$

Under the assumption that the collateral value (C_i), borrowing amount (B_i), and return value (R_i) remain unaffected by changes in interest rates (r_i), the expected net return can be expressed as follows:

$$\frac{d\Pi_i(r_i, \hat{\theta}_i)}{dr_i} = -B_i \int_{B_i(1+r_i)-C_i}^{\infty} \cdot dF_i\left(R_i, \hat{\theta}_i\right) < 0. \quad (4)$$

Equation (4) reveals that the expected net return diminishes as interest rates rise. At a certain level of risk $\hat{\theta}_i$, there exists a critical interest rate $\hat{r}_i$. This critical interest rate signifies a point where the expected returns are just sufficient to repay the borrowing amount and interest costs, resulting in a break-even condition, as discussed by Stiglitz and Weiss (1981).

$$\Pi_i(\hat{r}_i, \hat{\theta}_i) \equiv 0. \quad (5)$$

Firm i will only borrow funds when the expected profit is positive, indicating that the projected profit is likely to exceed the obligation to repay the borrowed amount and the associated interest costs, expressed as $\Pi_i(r_i, \hat{\theta}_i) > \Pi_i(\hat{r}_i, \hat{\theta}_i) = 0$. By utilizing Equations (4) and (5), it can be concluded that in order to achieve a positive expected profit, the credit interest rate must be lower than the critical interest rate, or $\Pi_i(r_i, \hat{\theta}_i) > \Pi_i(\hat{r}_i, \hat{\theta}_i)$, if and only if $r_i < \hat{r}_i$ (Q.E.D.).

The provision of credit from banks to firms is governed by a legal agreement that stipulates the repayment of credit along with interest expenses. Prior to accessing credit, firms must satisfy certain requirements and covenants established by the banks. One important measure used to assess a firm's debt level is the leverage ratio, which quantifies the amount of debt taken on or given to the firm (Hillier et al., 2019). The leverage ratio of a firm is determined by its performance under specific circumstances and at a particular point in time. Typically, there is a maximum limit imposed on a firm's leverage ratio (Jarrow, 2013), ensuring that the firm remains capable of repaying its debts. This maximum leverage provision is also implemented by banks as a covenant, safeguarding the repayment of loans extended to firms. To qualify for credit, a firm's leverage ratio must be lower than the maximum leverage provision.

In addition to the factors mentioned earlier, a firm's decision to seek credit also considers two important considerations: (i) the anticipated profit and (ii) the associated borrowing costs (Stiglitz & Weiss, 1981). Loans present an opportunity for firms to generate future net returns, but they also entail contractual obligations to repay the borrowed amount in the future (Hillier et al., 2019). In this study, we estimated the expected profit by considering the return on assets (ROA) ratio and the cost of borrowing, which is determined by the leverage ratio multiplied by the interest rate. Following a martingale condition (Williams, 1991), firms apply for credit when the expected profits outweigh the anticipated costs of borrowing.

THEOREM 2: *Under a martingale condition, the firm will be able to borrow funds if its leverage ratio is lower than the maximum allowed and if the return on assets surpasses the product of the leverage ratio and the change in interest rate.*

Before seeking credit, it is essential for a firm to assess its leverage, which represents the extent of credit used in its business operations. The focus is on the debt-to-value ratio rather than the absolute debt amount (Bernanke et al., 1988). In this study, the leverage of a firm is measured by the debt-to-asset ratio $\lambda_{i,t}$, which is calculated as the ratio of the firm's debt $(B_{i,t})$ to its total assets value $(A_{i,t})$. The debt is determined as the difference between the firm's assets $(A_{i,t})$ and its equity $(W_{i,t})$. If $A_{i,t} > 0$ and $B_{i,t} \geq 0$, then $\lambda_{i,t} \geq 0$.

$$\lambda_{i,t} \equiv \frac{B_{i,t}}{A_{i,t}} = \frac{A_{i,t}-W_{i,t}}{A_{i,t}}. \quad (6)$$

Apart from the maximum credit limit set by the bank, a firm also has an internal limit on the total borrowing in relation to its total assets. This implies that the firm is eligible to seek credit only if the borrowing amount is lower than the value of its total assets, which can be utilized to repay the borrowed funds(Jarrow, 2013). The establishment of a maximum credit limit ensures that the firm retains its capacity to repay both the borrowing amount and the associated interest costs.

The maximum credit limit of the firm at a specific time, denoted as $(B^*_{i,t})$, is determined by the portion of assets that can be used to repay credit or $A_{i,t} \cdot (1 - m_{i,t})$, where $m_{i,t}$ represents the fraction of assets that cannot be used for credit repayment (Jarrow, 2013). Therefore, $B_{i,t} < A_{i,t} \cdot (1 - m_{i,t}) = B^*_{i,t}$. Since the maximum credit limit is directly related to the borrowing amount and assets, we can substitute with the firm's leverage ratio to obtain $\lambda_{i,t} < \lambda_{i,t}^{*}$. Thus, we can conclude that the firm is eligible for borrowing at a specific time if its leverage ratio is lower than the maximum leverage provision (Q.E.D.).

Firms seek bank credit with the expectation of generating profits, a goal that varies across industries and individual firms. To evaluate and compare a firm's profitability, we employ the widely recognized metric known as Return on Assets (ROA), denoted as α_i. ROA serves as a quantitative measure of the net return or profit (π_i) achieved by a firm relative to its total asset value (A_i). Consequently, we derive the ensuing equation:

$$\alpha_i \equiv \frac{\pi_i}{A_i} = \frac{max\{R_i - B_i(1+r_i); -C_i\}}{A_i}. \tag{7}$$

In cases where a firm fails to repay its loans, the ROA can be expressed $\alpha_i = \frac{-C_i}{A_i} < 0$. Conversely, when a firm is not in default, the ROA is defined as $\alpha_i = \frac{R_i - B_i(1+r_i)}{A_i}$. It is important to note that credit demand arises exclusively from firms that are not in default.

To investigate the impact of interest rate changes on firm profitability, we examine the differential equation presented as Equation (7). Considering that $A_{i,t} > 0$ and $B_{i,t} \geq 0$, we can establish the following relationship: an increase in interest rates leads to a decrease in the return on assets (ROA), indicating a decline in firm profitability.

$$\frac{d\alpha_i}{dr_i} = -\frac{B_i}{A_i} \leq 0. \tag{8}$$

To identify the presence of a critical ROA $(\hat{\alpha})$, where the expected profit becomes zero, it is necessary to examine the relationship between changes in expected profit and ROA. Both Equations (4) and (8) demonstrate that the expected return and ROA are affected by the interest rates at which a firm borrows debt. Hence, it can be argued that an increase in a firm's ROA (α_i) results in an increase in its expected profit (Π_i).

$$\frac{d\Pi_i}{d\alpha_i} = \frac{\left(\frac{d\Pi_i}{dr_i}\right)}{\left(\frac{d\alpha_i}{dr_i}\right)} = \frac{d\Pi_i}{dr_i} \cdot \frac{dr_i}{d\alpha_i} \geq 0. \qquad (9)$$

Given that firms aim to avoid future losses, it is reasonable to expect that the future expected profit will at least remain unchanged compared to the current profit. This ensures the firm's ability to sustain its business operations and meet its obligations. Under a martingale condition, where the expected value of the next period $(t+1)$ equals its current value at the current time (t) (Williams, 1991), Equation (3) can be expressed in the following manner:

$$\Pi_{i,t+1} = E\left[\pi_{i,t+1}\right] = \pi_{i,t.} \qquad (10)$$

The ROA serves as a measure of a firm's profitability in relation to its total assets. In certain cases, there exists a specific ROA ratio that results in an expected profit of zero. This ROA is referred to as the critical ROA $(\hat{\alpha}_{i,t})$. By examining Equations (2), (5), (6), (9), and (10), we can establish the following relationship:

$$\Pi_{i,t+1}\left(\hat{r}_{i,t+1}, \hat{\theta}_{i,t+1}\right) = \pi_{i,t}\left(R_{i,t}, \hat{r}_{i,t}\right) = \hat{\alpha}_{i,t} A_{i,t} = 0. \qquad (11)$$

When we assume that $A_{i,t} > 0$, the critical ROA of the firm is equal to zero.

$$\hat{\alpha}_{i,t} = 0. \qquad (12)$$

By analyzing Equations (9) and (12), we can determine the conditions necessary for achieving a positive expected net return. Specifically, the requirement is that the ROA exceeds the critical ROA, represented by $\Pi_i\left(r_i, \hat{\theta}_i\right) > \Pi_i\left(\hat{r}_i, \hat{\theta}_i\right)$, if and only if $\alpha_{i,t} > \hat{\alpha}_{i,t} = 0$. In other words, for a positive expected net return, the firm's ROA must surpass the critical ROA of zero.

To assess the firms' tendency towards applying for credit, it is essential to examine the interplay between ROA and borrowing costs, which are influenced by leverage ratios and the prevailing interest rates. By leveraging Equations (6) and (7) under the assumption of a martingale condition for λ, we can derive the following equation, enabling the calculation of the change in ROA:

$$\Delta\alpha_{i,t+1} = -E[\lambda_{i,t+1}]\Delta r_{i,t+1} = -\lambda_{i,t}\Delta r_{i,t+1.} \quad (13)$$

In a particular study, it is assumed that interest rates are explicitly included in the loan contract (Stiglitz & Weiss, 1981). The effective interest rate (r) for the subsequent period $(t+1)$ is determined by referencing the agreed interest rate $(\bar{r})$ outlined in the contract between the firm and the bank during the preceding period (t). This assumption provides a basis for analyzing the impact of interest rate dynamics on the firm's financial considerations.

$$r_{i,t+1} \equiv \bar{r}_{i,t.} \quad (14)$$

The equation mentioned above illustrates that the change in the effective interest rate for the next period is equal to the change in the agreed interest rate at the current time, denoted as $\Delta r_{i,t+1} = \Delta\bar{r}_{i,t}$. This relationship highlights the correspondence between the adjustments in the interest rate agreed upon between the firm and the bank and the subsequent change in the effective interest rate applicable to the firm's borrowing in the following period.

To determine if the firm is eligible to apply for credit, the condition is that the expected net return for the next period should be positive $(\Pi_{i,t+1} > 0)$. Based on Equations (12), (13), and (14), we can deduce that the condition $\Pi_{i,t+1} > 0$ is fulfilled only when $E[\alpha_{i,t+1}] = \alpha_{i,t} + \Delta\alpha_{i,t+1} = \alpha_{i,t} - \lambda_{i,t}\Delta r_{i,t+1} = \alpha_{i,t} - \lambda_{i,t}\Delta\bar{r}_{i,t} > E[\hat{\alpha}_{i,t+1}] = \hat{\alpha}_{i,t} = 0$. Therefore, the firm can apply for credit if $\alpha_{i,t} > \lambda_{i,t}\Delta\bar{r}_{i,t}$. This condition ensures that the firm's current profitability exceeds the product of its leverage ratio and the change in the agreed interest rate (Q.E.D.).

To practically and simply implement Theorem 2, the authors suggest using the return on borrowing as a metric. This ratio compares the firm's profit to the borrowing amount and reflects the firm's efficiency in utilizing credit to generate profits. This condition ensures that the firm's profitability from utilizing credit is expected to be higher than the impact of changes in the interest rate, making borrowing a viable option. Additionally, the return on borrowing can help anticipate the impact of interest rate changes on the firm's profitability.

COROLLARY 1: *The Firm is considered eligible for borrowing at specific time if the return on borrowing exceeds the change in the interest rate.*

By utilizing Equations (6) and (7), we can establish the relationship between the return on borrowing (β) and the net return or profit (π) relative to total borrowings (B):

$$\beta_{i,t} \equiv \frac{\pi_{i,t}}{B_{i,t}} = \frac{\alpha_{i,t} \cdot A_{i,t}}{\lambda_{i,t} \cdot A_{i,t}} = \frac{\alpha_{i,t}}{\lambda_{i,t}}. \tag{15}$$

Based on Theorem 2 and Corollary 1, firm i at time t is eligible to borrow only if two conditions are met; (i) the firm's leverage ratio must be lower than the maximum leverage provision ($\lambda_{i,t} < \lambda^*_{i,t}$) and (ii) the firm's return on borrowing must be greater than the change in interest rate ($\beta_{i,t} > \Delta\bar{r}_{i,t} = \bar{r}_{i,t} - \bar{r}_{i,t-1}$). The simplified visualization is presented in Figure 2 as a quadrant.

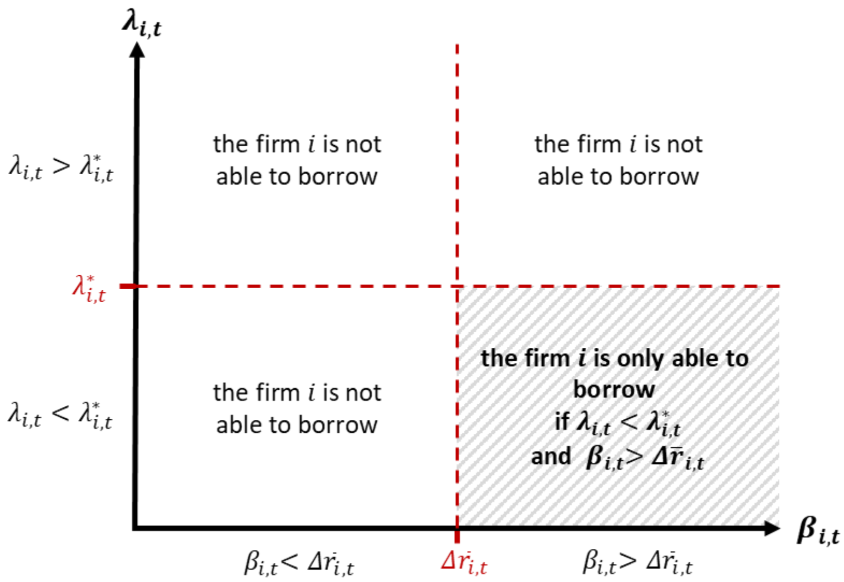


FIGURE 2. ILLUSTRATION OF THEOREM 2 AND COROLLARY 1.

3. Aggregate Credit Demand

According to Theorem 2, a firm is eligible to apply for credit if its leverage ratio is lower than the maximum leverage provision set by the lender. This maximum eligible credit is determined by the remaining borrowing capacity, which is the difference between the maximum leverage provision and the firm's current leverage ratio, multiplied by the total assets of the firm. However, it is important to note that the firm has the flexibility to choose whether to fully utilize, partially utilize, or not utilize its available borrowing limit. Therefore, the firm's credit demand at a given time is also influenced by its intention to borrow, if the borrowing remains within the maximum leverage limit.

Aggregate credit demand in the market refers to the combined credit demand of all firms at a particular point in time. For credit demand to exist, a firm must have available borrowing capacity and anticipate a positive net return. To determine the overall credit demand, we can analyze the distribution of leverage ratios and return on borrowing for all firms in the market. By examining these distributions, we can assess the extent of credit demand in the market and understand the overall borrowing needs of firms.

THEOREM 3: *Credit demand at a specific time can be estimated by considering the multivariate distribution of leverage ratios and return on borrowing in the market.*

From Corollary 1, it is established that for firm i at time t to be eligible for borrowing if if $\lambda_{i,t} < \lambda^*_{i,t}$ and $\beta_{i,t} > \Delta\bar{r}_{i,t} = \bar{r}_{i,t} - \bar{r}_{i,t-1}$. Furthermore, the credit demand (ψ) is influenced by the firm's intention to borrow $\omega_{i,t} \epsilon [0,1]$ and its maximum eligible credit limit. The maximum eligible credit limit of the firm is determined by its borrowing capacity based on the remaining borrowing space and its total assets. In other words, credit demand can be understood as a function of the firm's intention to borrow and its maximum eligible credit limit, where credit demand is positively influenced by a higher intention to borrow and a larger credit limit. This implies that firms with a stronger intention to borrow and a larger borrowing capacity are more likely to have higher credit demand.

$$\psi_{i,t} = \omega_{i,t} \cdot max\{\lambda^*_{i,t} - \lambda_{i,t}; 0\} \cdot A_{i,t}. \tag{16}$$

Although some firms meet the eligibility criteria for borrowing, indicated by have $\lambda_{i,t} < \lambda^*_{i,t}$ and $\beta_{i,t} > \Delta\bar{r}_{i,t}$, not all of them apply for credit. This is because their intention to borrow, represented by $\omega_{i,t} = 0$, meaning they have no interest in obtaining additional funds. On the other hand, there are a few firms that are highly aggressive in maximizing their borrowing and utilize their available borrowing space fully, resulting in $\omega_{i,t} = 1$.

At the macro level, the overall level of credit intensity in the market, denoted as Ω, captures the collective borrowing behavior of firms. It represents the extent to which firms are actively seeking credit. The value of Ω reflects the combination of firms with varying levels of intention to borrow, ranging from zero to one. A higher value of Ω suggests a greater overall demand for credit in the market, indicating a higher credit intensity among firms.

$$\Omega_t = E[\omega_{i,t}]. \tag{17}$$

At a specific time t the market consists of a set of firms denoted as I, where $I = \{1, 2, \cdots, i, \cdots, \mathbb{i}\}$. Each firm i in the market is characterized by its leverage value $(\lambda_{i,t})$ and return on borrowing value $(\beta_{i,t})$. This relationship is established at the given time t, and it is represented by the probability density function (pdf) of the distribution of leverage and return on borrowing, denoted as $g(\lambda_{i,t}, \beta_{i,t})$. The pdf captures the statistical distribution of the leverage and return on borrowing across the firms in the market, providing insights into the collective behavior of these variables among different firms.

To simplify the analysis, we make the following assumptions: (i) the interest rate for all firms at time t is the same, denoted as $\bar{r}_{i,t} = \bar{r}_t$, (ii) the maximum leverage provision for all firms at time t is identical, represented as $\lambda^*_{i,t} = \lambda^*_t$, and (iii) the firm-level credit intensity (ω) and total assets (A) are independent of each other and other variables. These assumptions allow us to focus on the common interest rate and leverage provisions across firms, while considering the independence of credit intensity and total assets from other factors. This simplification enables a more straightforward analysis of the relationship between variables and their impact on credit dynamics.

$$\Psi_t(\Delta\bar{r}_t) = \left(\sum_{\forall i} \psi_{i,t}\right) \cdot \int_{\Delta\bar{r}_t}^{\infty} \int_0^{\lambda^*_t} g(\lambda_t, \beta_t)\, d\lambda_t d\beta_t \qquad (18a)$$

$$= \left(\sum_{\forall i} \omega_{i,t} \cdot max\{\lambda^*_{i,t} - \lambda_{i,t}; 0\} \cdot A_{i,t}\right) \cdot \int_{\Delta\bar{r}_t}^{\infty} \int_0^{\lambda^*_t} g(\lambda_t, \beta_t)\, d\lambda_t d\beta_t$$

$$= \Omega_t \cdot \left(\sum_{\forall i} max\{\lambda^*_{i,t} - \lambda_{i,t}; 0\} \cdot A_{i,t}\right) \cdot \int_{\Delta\bar{r}_t}^{\infty} \int_0^{\lambda^*_t} g(\lambda_t, \beta_t)\, d\lambda_t d\beta_t$$

$$= \zeta_t \cdot \int_{\Delta\bar{r}_t}^{\infty} \int_0^{\lambda^*_t} g(\lambda_t, \beta_t)\, d\lambda_t d\beta_t$$

Where:

$$\zeta_t = \Omega_t \cdot \left(\sum_{\forall i} max\{\lambda^*_{i,t} - \lambda_{i,t}; 0\} \cdot A_{i,t}\right) > 0. \qquad (18b)$$

Based on Equation (18), we can approximate credit demand by considering the multivariate distribution of leverage and return on borrowing. This indicates that the credit demand at a certain time can be estimated by analysing the joint distribution of leverage and return on borrowing across firms in the market. This conclusion follows from the previous analysis and serves as a confirmation (Q.E.D.).

The interest rate prevailing in the market has a significant impact on the borrowing costs incurred by firms. Lower borrowing costs tend to attract credit demand from firms across a range of profitability levels, whereas higher interest rates may attract credit demand primarily from firms with higher profitability levels. This observation demonstrates the existence of the law of demand in the credit market, where lower borrowing costs stimulate greater credit demand, while higher borrowing costs may limit credit demand to more profitable firms.

COROLLARY 2: *The function of credit demand in the interest rate is monotonically decreasing. Therefore, if the interest rate increases, the credit demand decreases.*

Corollary 2 can be proven using Equation (18a) as follows:

$$\Psi_t(\Delta\bar{r}_t) = \zeta_t \cdot \int_{\Delta\bar{r}_t}^{\infty}\int_0^{\lambda_t^*} g(\lambda_t, \beta_t)\; d\lambda_t d\beta_t \qquad (19)$$

$$= \zeta_t \cdot \int_{\Delta\bar{r}_t}^{\infty}\left(\int_0^{\lambda_t^*} g(\lambda_t, \beta_t) d\lambda_t\right) d\beta_t$$

$$= \zeta_t \cdot \int_{\Delta\bar{r}_t}^{\infty} G_{\mathbb{L}_t<\lambda_t^*}(\beta_t)\; d\beta_t$$

Based on the concept of limits and derivatives of Equation (19), the following relationship can be observed:

$$\frac{d\Psi_t}{d\mathrm{r}_t} = \lim_{\varepsilon\to 0}\frac{\Psi_t(\bar{r}_t+\varepsilon) - \Psi_t(\bar{r}_t)}{\varepsilon} \qquad (20)$$

$$= \lim_{\varepsilon\to 0}\frac{=\zeta_t\cdot\int_{\Delta\bar{r}_t+\varepsilon}^{\infty} G_{\mathbb{L}_t<\lambda_t^*}(\beta_t)\; d\beta_t - =\zeta_t\cdot\int_{\Delta\bar{r}_t}^{\infty} G_{\mathbb{L}_t<\lambda_t^*}(\beta_t)\; d\beta_t}{\varepsilon} = -\zeta_t \cdot G_{\mathbb{L}_t<\lambda_t^*}(\Delta\bar{r}_t)$$

From Equation (18b), $\zeta_t > 0$, and in the continuous random variable $\mathbb{B}_t$ for $\forall\beta_t$, $G_{\mathbb{L}_t<\lambda_t^*}(\beta_t) \geq 0$ (Hogg et al., 2012) including when $\beta_t = \Delta\bar{r}_t$. Equation (20) implies that $\frac{d\Psi_t}{dr_t} \leq 0$, which means that if the interest rate increases, the credit demand decreases. This result aligns with the law of demand in economics, which states that there is an inverse relationship between price (in this case, the interest rate) and quantity demanded (credit demand) (Q.E.D.).

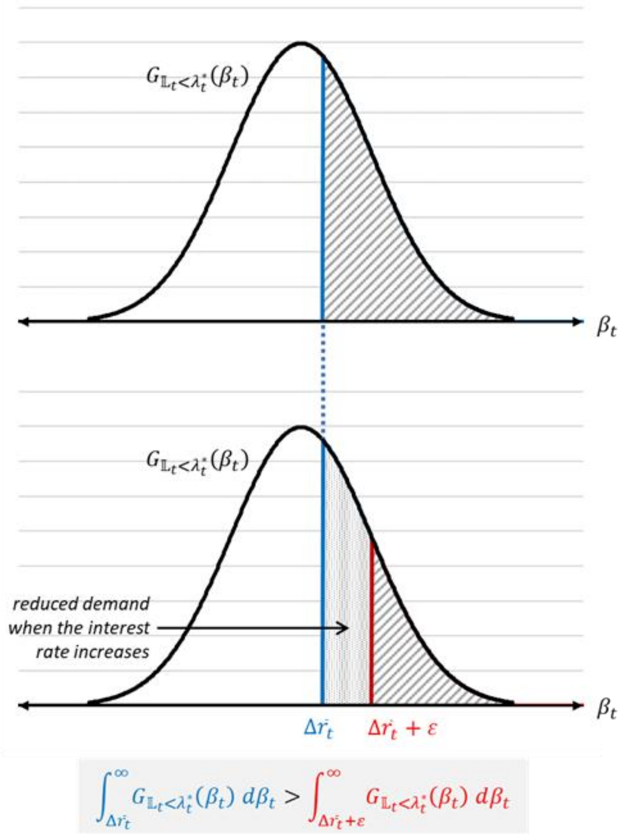


FIGURE 3. ILLUSTRATION OF COROLLARY 2

COROLLARY 3: *There is a maximum leverage provision for borrowers in a market, and it further asserts that the credit demand is influenced by changes in the maximum leverage provision. Specifically, if the maximum leverage provision increases, it allows borrowers to access a higher level of debt relative to their assets, resulting in an increase in credit demand.*

Corollary 3 can be established from Equation (18a) as follows:

$$\Psi_t(\Delta\bar{r}_t) = \zeta_t \cdot \int_{\Delta\bar{r}_t}^{\infty} \int_0^{\lambda_t^*} g(\lambda_t, \beta_t)\, d\lambda_t d\beta_t \qquad (21)$$

$$= \zeta_t \cdot \int_0^{\lambda_t^*} \left(\int_{\Delta\bar{r}}^{\infty} g(\lambda_t, \beta_t)\ d\beta_t \right) d\lambda_t$$

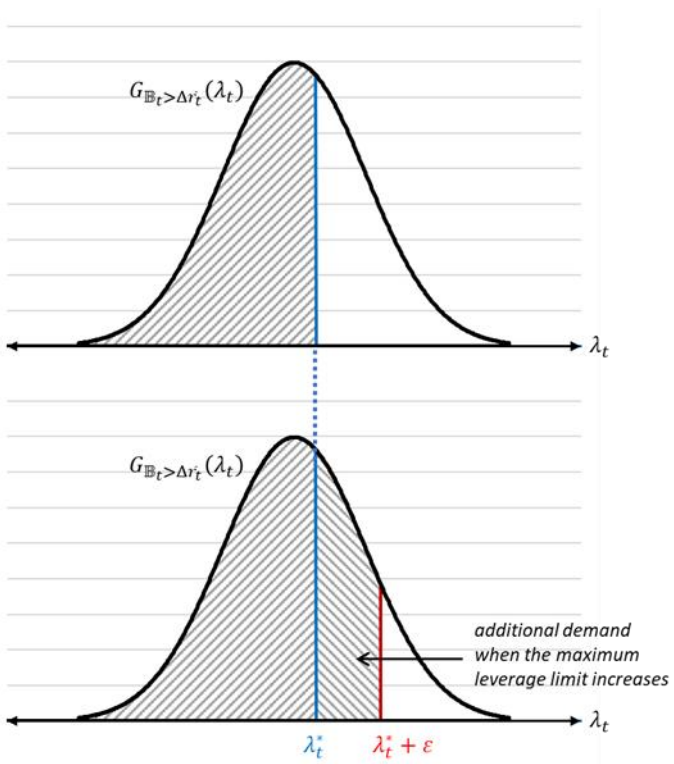
$$= \zeta_t \cdot \int_0^{\lambda_t^*} G_{\mathbb{B}_t > \Delta \bar{r}_t}(\lambda_t)\, d\lambda_t$$

Based on the concept of limits and derivatives of Equation (21), we can make the following observations:

$$\frac{d\Psi_t}{d\lambda_t^*} = \lim_{\varepsilon \to 0} \frac{\Psi_t(\lambda_t^* + \varepsilon) - \Psi_t(\lambda_t^*)}{\varepsilon} \tag{22}$$

$$= \lim_{\varepsilon \to 0} \frac{\zeta_t \cdot \int_0^{\lambda_t^* + \varepsilon} G_{\mathbb{B}_t > \Delta \bar{r}_t}(\lambda_t) d\lambda_t \; - \zeta_t \cdot \int_0^{\lambda_t^*} G_{\mathbb{B}_t > \Delta \bar{r}_t}(\lambda_t) d\lambda_t}{\varepsilon} = \zeta_t \,.\, G_{\mathbb{B}_t > \Delta \bar{r}_t}(\lambda_t^*)$$

Based on Equation (18b), we know that $\zeta_t > 0$. In the continuous random variable $\mathbb{L}_t$for $\forall \lambda_t$, the probability density function $G_{\mathbb{B}_t > \Delta \bar{r}_t}(\lambda_t) \geq 0$ (Hogg et al., 2012), including when λ_t = λ_t^. Therefore, from Equation (22), we can conclude that (dΨ_t)/(dλ_t^) ≥ 0. This implies that if the maximum leverage provision increases, the credit demand will increase. (Q.E.D.).



$$\int_0^{\lambda_t^*} G_{\mathbb{B}_t > \Delta \bar{r}_t}(\lambda_t) d\lambda_t < \int_0^{\lambda_t^* + \varepsilon} G_{\mathbb{B}_t > \Delta \bar{r}_t}(\lambda_t) d\lambda_t$$

FIGURE 4. ILLUSTRATION OF COROLLARY 3

4. The Role of the Conditional Distribution of Return on Borrowing

The performance of a firm is inherently influenced by the dynamic nature of changing economic conditions over time. Notably, individual firms exhibit diverse performance levels in response to fluctuations in economic circumstances, whereby certain firms may experience advantages under specific conditions, while others may suffer losses. Evaluating firm performance often involves considering various metrics, such as profitability ratios, which provide insights into the firm's ability to generate profits relative to the total borrowed capital employed in its business operations. This research focuses specifically on the examination of the return on borrowing ratio, which quantifies a firm's profitability in relation to the aggregate borrowing it has acquired for its operational needs. It is worth noting that, at any given time and within specific economic conditions, there exists a contingent distribution of returns on borrowing encompassing all firms in the market. This conditional distribution, denoted as $G_{\mathbb{L}_t<\lambda_t^*}(\beta_t)$ captures the dynamic nature of the return on borrowing metric, reflecting the varied profitability levels observed among firms in response to changing economic circumstances.

The conditional distribution of the return on borrowing in the market ($G_{\mathbb{L}_t<\lambda_t^*}(\beta_t)$) exhibits certain statistical characteristics that provide insights into the behaviour of firms within the market. Specifically, this distribution possesses a mean value ($\mu_{\beta,t}$) and a standard deviation value ($\sigma_{\beta,t}$). The mean value ($\mu_{\beta,t}$) represents the average return on borrowing achieved by all firms that possess leverage values lower than the maximum leverage provision set in the market. It serves as an indicator of the typical level of profitability attained by these firms. On the other hand, the standard deviation value ($\sigma_{\beta,t}$) measures the extent to which the return on borrowing varies around the mean value among firms with leverage values below the maximum leverage provision. It provides a measure of the dispersion or spread of the return on borrowing metric, indicating the degree of variability observed in the profitability levels of these firms. In this section, we explore into the dynamic impact of the mean and standard deviation of the conditional distribution of return on borrowing, shedding light on the changing patterns and trends in the profitability performance of firms within the market.

COROLLARY 4: *If the mean of the conditional distribution of return on borrowing increases, the credit demand* ($\boldsymbol{\Psi}_t$) *will also increase.*

Under ceteris paribus conditions, $\mu_{\beta',t} = \mu_{\beta,t} + \varepsilon$ exists if and only if $\mathbb{B}'_t = \mathbb{B}_t + \varepsilon$ due to the condition of $\mu_{\beta',t} = E[\mathbb{B}'_t] = E[\mathbb{B}_t + \varepsilon] = E[\mathbb{B}_t] + \varepsilon = \mu_{\beta,t} + \varepsilon$. In the linear transformation of $\mathbb{B}'_t = \mathbb{B}_t + \varepsilon$, we know that $\beta'_t = \beta_t + \varepsilon$ and $G'_{\mathbb{L}_t<\lambda^*_t}(\beta'_t) = G_{\mathbb{L}_t<\lambda^*_t}(\beta_t)$.

Based on the concept of limits and derivatives represented in Equation (19), we derive the following conditions:

$$\frac{d\Psi_t}{d\mu_{\beta,t}} = \lim_{\varepsilon\to 0} \frac{\Psi_t(\mu_{\beta,t}+\varepsilon) - \Psi_t(\mu_{\beta,t})}{\varepsilon} \tag{23}$$

$$= \lim_{\varepsilon\to 0} \frac{\zeta_t \cdot \int_{\Delta\bar{r}_t}^{\infty} G'_{\mathbb{L}_t<\lambda^*_t}(\beta'_t)\, d\beta'_t - \zeta_t \cdot \int_{\Delta\bar{r}_t}^{\infty} G_{\mathbb{L}_t<\lambda^*_t}(\beta_t)\, d\beta_t}{\varepsilon}$$

$$= \zeta_t \cdot \lim_{\varepsilon\to 0} \frac{\int_{\Delta\bar{r}_t-\varepsilon}^{\Delta\bar{r}_t} G_{\mathbb{L}_t<\lambda^*_t}(\beta_t)\, d\beta_t}{\varepsilon} = \zeta_t \cdot G_{\mathbb{L}_t<\lambda^*_t}(\Delta\bar{r}_t)$$

From Equation (18b), we can conclude that $\zeta_t > 0$, and for the continuous random variable $\mathbb{B}_t$ for $\forall \beta_t$, it holds true that $G_{\mathbb{L}_t<\lambda^*_t}(\beta_t) \geq 0$ (Hogg, et al., 2012), even when $\beta_t = \Delta\bar{r}_t$. As a result, based on Equation (23), we can infer that $\frac{d\Psi_t}{d\mu_{\beta,t}} \geq 0$. This means that an increase in the mean value of the conditional distribution of return on borrowing leads to an increase in credit demand. (Q.E.D.).

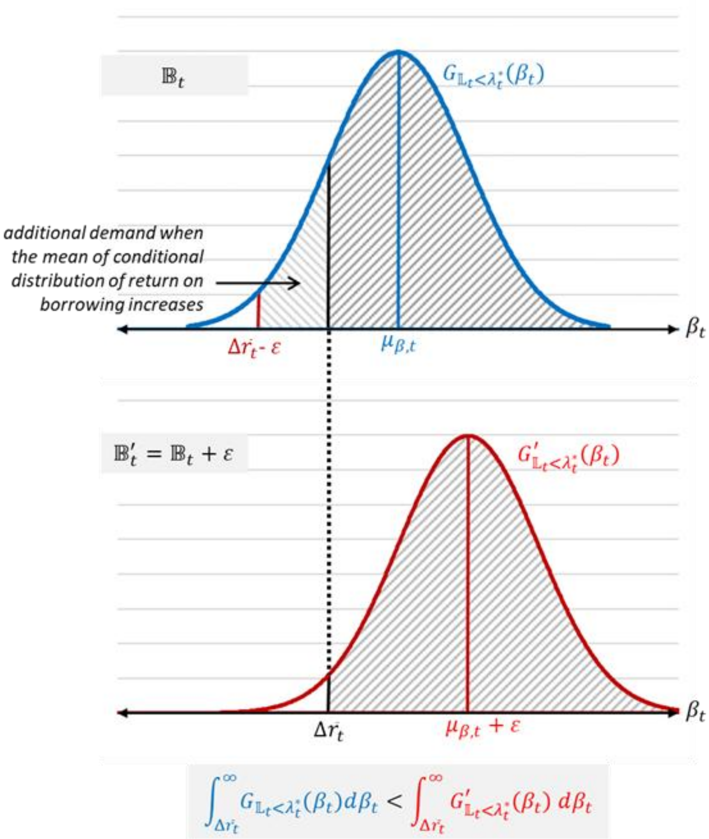


FIGURE 5. ILLUSTRATION OF COROLLARY 4

COROLLARY 5: *An increase in the standard deviation of the conditional distribution of return on borrowing will conditionally affect the credit demand, depending on the relationship between the change in the interest rates and the mean value of the distribution. If the change in the interest rate is greater than the mean the credit demand will increase.*

The condition of $\sigma'_{\beta',t} = (1+\varepsilon)\sigma_{\beta,t}$, where $\mu'_{\beta\prime,t} = \mu_{\beta,t}$ and $(1+\varepsilon) > 1$, exists if and only if $\mathbb{B}'_t - \mu_{\beta,t} = (1+\varepsilon)\cdot(\mathbb{B}_t - \mu_{\beta,t})$.

Given that $var(\mathbb{B}'_t) = E\left[\left(\mathbb{B}'_t - \mu_{\beta,t}\right)^2\right] = E\left[\left((1+\varepsilon)\left(\mathbb{B}_t - \mu_{\beta,t}\right)\right)^2\right] = (1+\varepsilon)^2 \cdot var(\mathbb{B}_t)$, then $\sigma'_{\beta',t} = \sqrt{var(\mathbb{B}'_t)} = \sqrt{(1+\varepsilon)^2 \cdot var(\mathbb{B}_t)} = (1+\varepsilon)\sigma_{\beta,t}$.

The concept of a standard score can be applied to the following relationship:

$$\int_{\Delta\bar{r}_t}^{\infty} G'_{\mathbb{L}_t<\lambda_t^*}(\beta'_t)\, d\beta'_t = \int_{\Delta\bar{r}_t+\vartheta_t}^{\infty} G_{\mathbb{L}_t<\lambda_t^*}(\beta_t)\, d\beta_t. \tag{24}$$

When considering the congruence of the conditional distributions of the random variables $\mathbb{B}'_t$ and $\mathbb{B}_t$, we observe that $\mathbb{B}'_t$' is a linear transformation of $\mathbb{B}_t$. When the standard scores of both variables are equal, represented as $z' = z$, the following relationship holds: $\frac{\Delta\bar{r}_t-\mu_{\beta,t}}{(1+\varepsilon)\sigma_{\beta,t}} = \frac{(\Delta\bar{r}_t+\vartheta_t)-\mu_{\beta,t}}{\sigma_{\beta,t}}$. Consequently, we can calculate the value of $\vartheta_t = \left(\frac{\Delta\bar{r}_t-\mu_{\beta,t}}{(1+\varepsilon)}\right) - \left(\frac{\Delta\bar{r}_t-\mu_{\beta,t}}{1}\right)$. Since $\varepsilon > 1$, we can derive the following result:

$$\vartheta_t = \begin{array}{l} if\ \Delta\bar{r}_t < \mu_{\beta,t} \ \rightarrow\ \vartheta_t > 0 \\ if\ \Delta\bar{r}_t = \mu_{\beta,t} \ \rightarrow\ \vartheta_t = 0 \\ if\ \Delta\bar{r}_t > \mu_{\beta,t} \ \rightarrow\ \vartheta_t < 0 \end{array} \tag{25}$$

From Equations (18) and (24), applying the concept of limits and derivatives, we can derive Equation (26) as follows:

$$\frac{d\Psi_t}{d\sigma_{\beta,t}} = \lim_{\varepsilon\to 0^+} \frac{\Psi_t\big((1+\varepsilon)\sigma_{\beta,t}\big) - \Psi_t(\sigma_{\beta,t})}{\varepsilon} \tag{26}$$

$$= \lim_{\varepsilon\to 0^+} \frac{\zeta_t \int_{\Delta\bar{r}_t}^{\infty} G'_{\mathbb{L}_t<\lambda_t^*}(\beta'_t)\, d\beta'_t - \zeta_t \int_{\Delta\bar{r}_t}^{\infty} G_{\mathbb{L}_t<\lambda_t^*}(\beta_t)\, d\beta_t}{\varepsilon}$$

$$= \zeta_t \lim_{\varepsilon\to 0^+} \frac{\int_{\Delta\bar{r}_t+\vartheta_t}^{\infty} G_{\mathbb{L}_t<\lambda_t^*}(\beta_t)\, d\beta_t - \int_{\Delta\bar{r}_t}^{\infty} G_{\mathbb{L}_t<\lambda_t^*}(\beta_t)\, d\beta_t}{\varepsilon}$$

From Equations (18b), (25), and (26), we can formulate the following relationship:

$$\frac{d\Psi_t}{d\sigma_{\beta,t}} = \begin{array}{l} if\ \Delta\bar{r}_t < \mu_{\beta,t} \ \rightarrow\ \frac{d\Psi_t}{d\sigma_{\beta,t}} < 0 \\ if\ \Delta\bar{r}_t = \mu_{\beta,t} \ \rightarrow\ \frac{d\Psi_t}{d\sigma_{\beta,t}} = 0 \\ if\ \Delta\bar{r}_t > \mu_{\beta,t} \ \rightarrow\ \frac{d\Psi_t}{d\sigma_{\beta,t}} > 0 \end{array} \tag{27}$$

From Equation (27), we can conclude that an increase in the standard deviation of the conditional distribution of return on borrowing will conditionally affect credit demand based on the relationship between $\Delta\bar{r}_t$ and $\mu_{\beta,t}$. Specifically, if $\Delta\bar{r}_t > \mu_{\beta,t}$ credit demand will increase. (Q.E.D.).

Corollaries 4 and 5 illustrate the impact of variations in the mean and standard deviation on credit demand, respectively, considering the dynamic nature of the conditional distribution of return on borrowing. However, to fully understand the effects of changes in other statistical moments, such as skewness and kurtosis, the Cornish-Fisher expansion can be employed. The Cornish-Fisher expansion is an asymptotic expansion technique used to estimate quantiles of a probability distribution based on the values of statistical moments, allowing for the exploration of their influence on credit demand (Cornish & Fisher, 1938).

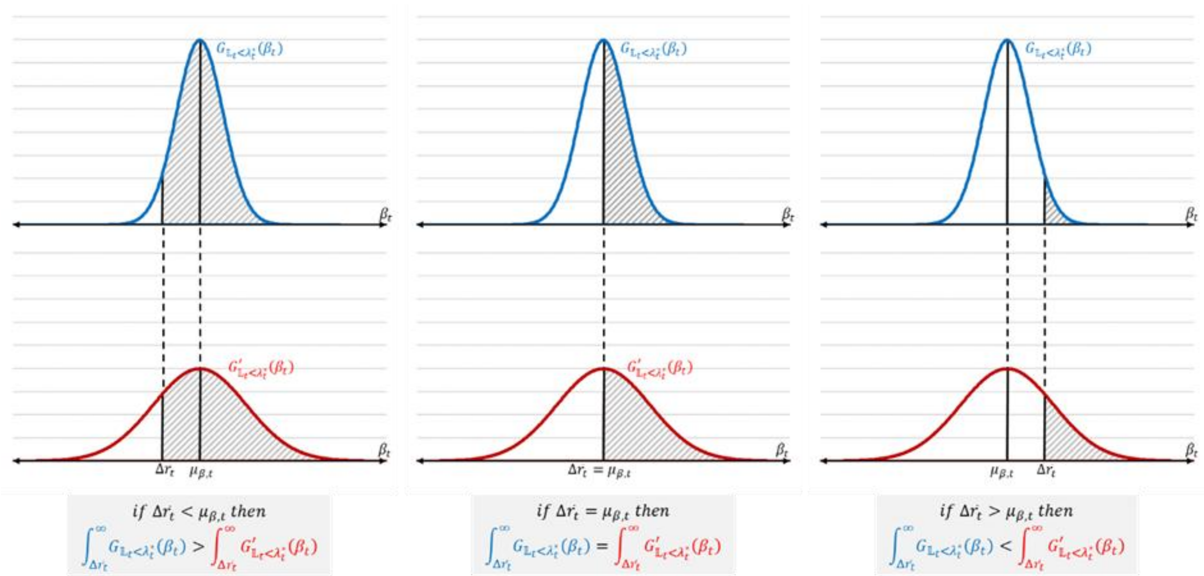


FIGURE 6. ILLUSTRATION OF COROLLARY 5

5. Elasticity in the Interest Rate Policy and Maximum Leverage Provision

The preceding discussions in the realm of positive economics aimed to explain the conceptual connection between firm-level returns and macro-level credit demand. In this section, the established conceptual framework is operationalized in the normative economic domain as a supportive tool for devising economic policies. Specifically, this section explores into the estimation of interest rate elasticity using the conditional distribution of return on borrowing and investigates the elasticity of the maximum leverage provision contingent upon the conditional distribution of leverage. Accurately estimating the impact of monetary and financial policies on credit demand assumes a crucial role in shaping economic policies (Lukas, 2017).

COROLLARY 6: *The elasticity of credit demand in response to changes in the interest rate can be determined by examining the cumulative conditional distribution function of return on borrowing.*

By analyzing the relationship between the interest rate and the corresponding probabilities of different levels of return on borrowing, we can assess the sensitivity of credit demand to changes in the interest rate. The elasticity of credit demand provides insights into the magnitude of the response of credit demand to variations in the interest rate, thus informing policymakers and stakeholders about the potential impact of interest rate changes on overall credit demand in the economy.

$$\eta_{\Psi_t,\Delta\bar{r}_t} = \left(\frac{1-\mathbb{G}_{\mathbb{L}_t<\lambda_t^*}(\Delta\bar{r}_t)}{1-\mathbb{G}_{\mathbb{L}_t<\lambda_t^*}(0)} - 1\right)\cdot\frac{\bar{r}_{t-1}}{\Delta\bar{r}_t}$$

The elasticity (η) of changes in quantity (Q) with respect to price (P) is determined by the following equation, as proposed by Marshall (2013) and Moore (1922):

$$\eta \equiv \frac{\Delta Q}{\Delta P}\cdot\frac{P}{Q}. \tag{28}$$

From Equations (19) and (28), the elasticity (η) of the credit demand (Ψ) can be obtained using the following equation consequent to the changes in interest rate $(\Delta\bar{r}_t)$:

$$\eta_{\Psi_t,\Delta\bar{r}_t} = \frac{\Psi_t(\Delta\bar{r}_t)-\Psi_t(0)}{\Delta\bar{r}_t}\cdot\frac{\bar{r}_{t-1}}{\Psi_t(0)} \tag{29}$$

$$= \frac{\left(\zeta_t \cdot \int_{\Delta \bar{r}_t}^{\infty} G_{\mathbb{L}_t < \lambda_t^*}(\beta_t)\, d\beta_t - \zeta_t \cdot \int_0^{\infty} G_{\mathbb{L}_t < \lambda_t^*}(\beta_t)\, d\beta_t\right)}{\Delta \bar{r}_t} \cdot \frac{\bar{r}_{t-1}}{\zeta_t \cdot \int_0^{\infty} G_{\mathbb{L}_t < \lambda_t^*}(\beta_t)\, d\beta_t}$$

$$= \left(\frac{\int_{\Delta \bar{r}_t}^{\infty} G_{\mathbb{L}_t < \lambda_t^*}(\beta_t)\, d\beta_t}{\int_0^{\infty} G_{\mathbb{L}_t < \lambda_t^*}(\beta_t)\, d\beta_t} - 1\right) \cdot \frac{\bar{r}_{t-1}}{\Delta \bar{r}_t}$$

$$= \left(\frac{1 - \mathbb{G}_{\mathbb{L}_t < \lambda_t^*}(\Delta \bar{r}_t)}{1 - \mathbb{G}_{\mathbb{L}_t < \lambda_t^*}(0)} - 1\right) \cdot \frac{\bar{r}_{t-1}}{\Delta \bar{r}_t}$$

Equation (29) asserts that the elasticity of credit demand, in response to changes in interest rate policy, can be estimated by utilizing the conditional cumulative distribution function of return on borrowing. (Q.E.D.). This implies that by examining the relationship between changes in interest rates and the corresponding changes in credit demand, we can quantify the sensitivity of credit demand to interest rate variations. The conditional cumulative distribution function provides a useful tool for analysing this relationship and estimating the elasticity of credit demand with respect to changes in interest rates.

COROLLARY 7: *Elasticity of the credit demand as a result of the changes in the maximum leverage provision can be determined using the conditional cumulative distribution function of leverage.*

Equations (21) and (28) provide a basis for deriving the elasticity (η) of credit demand (Ψ) in response to changes in the maximum leverage provision $(\Delta\lambda_t^*)$. By analyzing the relationship between changes in the maximum leverage provision and the corresponding changes in credit demand, we can determine the sensitivity of credit demand to variations in the maximum leverage provision. The elasticity of credit demand can be calculated using the following equation, which incorporates the derivatives of the credit demand function with respect to the maximum leverage provision:

$$\eta_{\Psi_t,\Delta\lambda_t^*} = \frac{\Psi_t(\lambda_{t-1}^*+\Delta\lambda_t^*)-\Psi_t(\lambda_{t-1}^*)}{\Delta\lambda_t^*}\cdot\frac{\lambda_{t-1}^*}{\Psi_t(\lambda_{t-1}^*)} \tag{30}$$

$$= \frac{\left(\zeta_t\cdot\int_0^{\lambda_{t-1}^*+\Delta\lambda_t^*} G_{\mathbb{B}_t>\Delta\bar{r}_t}(\lambda_t)d\lambda_t - \zeta_t\cdot\int_0^{\lambda_{t-1}^*} G_{\mathbb{B}_t>\Delta\bar{r}_t}(\lambda_t)d\lambda_t\right)}{\Delta\lambda_t^*}\cdot\frac{\lambda_{t-1}^*}{\zeta_t\cdot\int_0^{\lambda_{t-1}^*} G_{\mathbb{B}_t>\Delta\bar{r}_t}(\lambda_t)d\lambda_t}$$

$$= \frac{\left(\int_0^{\lambda_{t-1}^*+\Delta\lambda_t^*} G_{\mathbb{B}_t>\Delta\bar{r}_t}(\lambda_t)d\lambda_t - \int_0^{\lambda_{t-1}^*} G_{\mathbb{B}_t>\Delta\bar{r}_t}(\lambda_t)d\lambda_t\right)}{\int_0^{\lambda_{t-1}^*} G_{\mathbb{B}_t>\Delta\bar{r}_t}(\lambda_t)d\lambda_t}\cdot\frac{\lambda_{t-1}^*}{\Delta\lambda_t^*}$$

$$= \left(\frac{\mathbb{G}_{\mathbb{B}_t>\Delta\bar{r}_t}(\lambda_{t-1}^*+\Delta\lambda_t^*)}{\mathbb{G}_{\mathbb{B}_t>\Delta\bar{r}_t}(\lambda_{t-1}^*)}-1\right)\cdot\frac{\lambda_{t-1}^*}{\Delta\lambda_t^*}$$

Equation (30) demonstrates that the elasticity of credit demand in response to changes in the maximum leverage provision can be determined by utilizing the conditional cumulative distribution function of leverage. By analyzing the relationship between changes in the maximum leverage provision and the resulting changes in credit demand, we can calculate the elasticity of credit demand. The conditional cumulative distribution function of leverage provides valuable information on the probability distribution of leverage values and allows us to assess the impact of changes in the maximum leverage provision on credit demand. This finding supports the assumption that the elasticity of credit demand can be derived using the conditional cumulative distribution function of leverage (Q.E.D.).

6. Conclusion

In conclusion, this analysis has presented a conceptual framework that explores the relationship between firm returns and credit demand at both the micro and macro levels. The conditional distribution of return on borrowing and leverage has been utilized to understand the dynamics of credit demand in response to changes in interest rates and the maximum leverage provision. By examining the mean and standard deviation of these conditional distributions, we have shown that changes in these variables can have a significant impact on credit demand. Additionally, the concept of elasticity has been introduced to quantify the responsiveness of credit demand to changes in interest rates and the maximum leverage provision.

This framework has implications for formulating economic policies and understanding the effects of monetary and financial policies on credit demand. Estimating the elasticity of interest rates and the maximum leverage provision allows policymakers to assess the impact of these variables on credit demand and make informed decisions to achieve desired economic outcomes. By analyzing the conditional distributions of return on borrowing and leverage, policymakers can gain insights into the sensitivity of credit demand to changes in these variables and tailor their policies accordingly. This approach highlights the importance of considering both micro-level firm dynamics and macro-level credit demand in designing effective economic policies.

Further research can explore the effects of additional statistical moments, such as skewness and kurtosis, on credit demand using methods like the Cornish–Fisher expansion. Understanding the impact of these moments on credit demand can provide a more comprehensive picture of the relationship between firm returns and credit market dynamics. Additionally, incorporating other factors such as macroeconomic indicators and regulatory frameworks into the analysis can enhance our understanding of credit demand fluctuations and inform policy interventions. By continuously refining our understanding of the relationship between firm performance and credit demand, we can better address the challenges and opportunities within the credit market and foster sustainable economic growth.

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