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Can Criminal Symbiosis Explain the Persistence of Violence in Brazil?

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Abstract

This paper develops the concept of criminal symbiosis to explain the persistence of violence in Brazil. Using official historical series, we document a systematic co-movement between serious crimes and minor offenses, suggesting a process of mutual reinforcement. When minor crimes increase, policing costs rise and the expected punishment for severe offenses declines, enabling escalation into homicide and organized crime. Conversely, targeted repression of minor infractions helps restore social norms and generates measurable deterrent effects. We formalize this mechanism through a dynamic system linking offender stocks, institutional responses, and intergenerational transmission of crime. The framework provides clear testable implications and supports integrated crime-prevention strategies that combine enforcement, rehabilitation, and community resilience.

Keywords: Criminal symbiosis; violence persistence; social spillovers; enforcement policy; dynamic model; Brazil.

JEL Codes: K42; D85; I38; C61.

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1 Introduction

Crime constitutes one of the most persistent challenges in contemporary Brazil, ranging from minor offenses to highly complex and violent crimes. According to national indicators, the country consistently ranks among the highest in homicide rates per 100,000 inhabitants, positioning it as a critical case study for the economics of crime and public security. In this context, understanding the interactions between minor offenses and serious crimes is crucial for designing effective public policies.

The classical literature offers distinct interpretations of this phenomenon. The *Broken Windows Theory* (Kelling and Wilson, 1982), formulated within criminology, argues that tolerance of minor offenses signals social disorder and fuels the escalation of violence, creating a cumulative process of public order deterioration. In parallel, the economic approach initiated by Becker (1968) and expanded by Ehrlich (1973) conceptualized crime as a rational decision based on cost-benefit calculations: individuals commit offenses when expected returns outweigh the utility of lawful behavior, given probabilities of punishment and social costs. This tradition consolidated the field of *economics of crime*, providing tools to analyze incentives, bounded rationality, and enforcement mechanisms.

Both approaches, however, present limitations when addressing the persistence of violence in countries such as Brazil. The *Broken Windows Theory* emphasizes a unidirectional causality (minor infractions \rightarrow serious crimes), whereas economic models often focus on atomized decision-making, underestimating network effects, social externalities, and the endogeneity across crime categories. At this intersection, we introduce the concept of *criminal symbiosis*, defined as a dynamic and bidirectional relationship between minor offenses and serious crimes.

Our central hypothesis is that, under certain institutional conditions, minor offenses not only precede but are also reinforced by more severe crimes. Serious crimes, when visible and unpunished, may normalize or legitimize less severe unlawful conduct, transmitting social signals of tolerance. Conversely, the proliferation of seemingly trivial minor offenses reduces the expected cost of criminal behavior and fosters an environment conducive to violent crime. Thus, *criminal symbiosis* should be understood as a structure of mutual incentives rather than a linear causality.

The literature on routine activities and crime opportunities provides an additional framework. According to Felson (2006), offenses arise from the convergence of three elements: a motivated offender, a suitable target, and the absence of capable guardians. Within this framework, social interaction networks play a crucial role, as crimes do not occur in isolation but in contexts that either facilitate or inhibit their occurrence. Symbiotic relationships emerge when different actors derive illicit benefits from mutual cooperation: corruption that sustains drug trafficking, collusion between law enforcement and criminal organizations, or everyday arrangements such as commercial establishments indirectly profiting from illegal activities (e.g., motels and short-term prostitution).

From an economic perspective, such interactions generate negative externalities and market failures. Each criminal act imposes social costs that are not internalized by the offender, creating inefficiencies and distortions perpetuated by weak institutions. *Criminal symbiosis* amplifies these effects by establishing feedback mechanisms: minor offenses strengthen the logistical and cultural base sustaining serious crimes, while the latter, by their magnitude, lower the marginal cost of routine infractions. This analysis thus connects to broader debates on public goods (security), information asymmetry (perceptions of impunity), and incentive theory.

These mechanisms help explain the resilience of violence in Brazil. As highlighted by Williams and Godson (2021) and Rodriguez (2022), entrenched criminality cannot be addressed solely through police repression. Effective policies must encompass institutional strengthening, reduction of corruption within judicial and security systems, and the creation of legal opportunities in socially vulnerable contexts. The approach advanced in this article contributes to this debate by integrating the economics of crime with

critical criminology in a coherent analytical framework.

By emphasizing the interdependence between minor offenses and serious crimes, this study aims to: (i) systematize the concept of *criminal symbiosis* in light of economic theory; (ii) propose a formal dynamic model that captures feedback between crime categories; and (iii) outline public policy implications capable of disrupting the symbiotic links that perpetuate violence. We argue that security strategies that ignore the symbiotic dimension tend to be inefficient, as they address only one side of the criminal equation while leaving the structural foundations of violence reproduction untouched.

2 Preliminary Empirical Motivation

Before presenting the formal model, it is useful to examine two historical series that illustrate the interaction between youth offenders, minor offenses, and serious crimes. These observations are descriptive rather than formal statistical tests, yet they provide support for the plausibility of the *criminal symbiosis* hypothesis.

The analysis draws on official aggregate indicators: (i) custodial socio-educational measures for youth offenders (1996–2022), and (ii) homicide and suicide rates among individuals aged 15–29 (1980–2022). The interpretation of these data is conducted in light of both the *criminal symbiosis* hypothesis and the *Broken Windows Theory*.

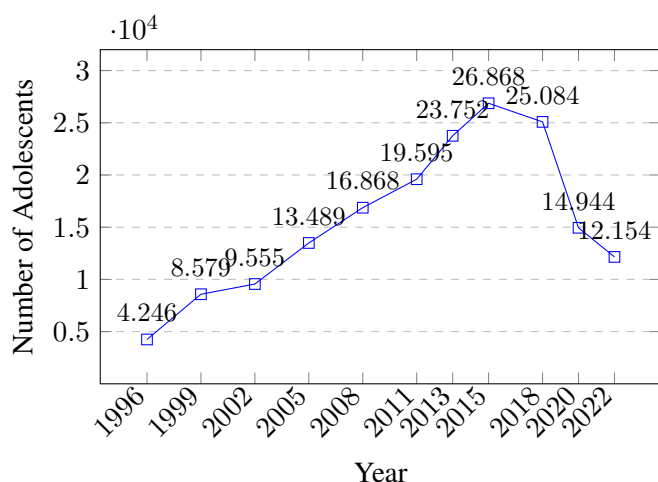


Figure 1: Adolescents under custodial socio-educational measures (Brazil, 1996–2022). Source: Data-SUS)

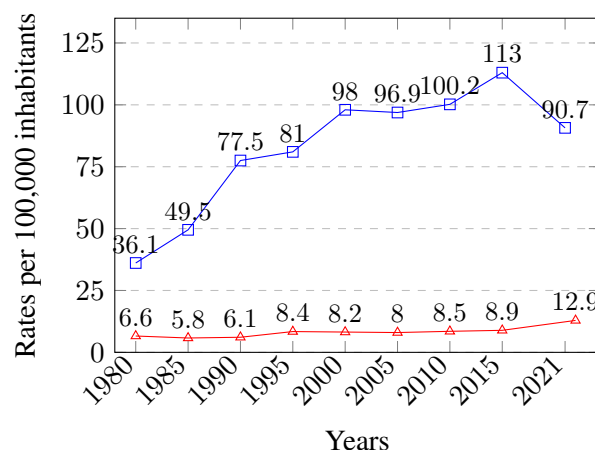


Figure 2: Homicide and Suicide Rates – Youth aged 15–29: Source: Data-SUS

2.1 Evolution of Socio-Educational Measures (1996–2022)

Figure 1 shows a sharp upward trajectory in the number of adolescents subject to custodial socio-educational measures, rising from approximately 4,000 in 1996 to over 26,000 in 2015. Following this peak, a marked decline is observed, reaching around 12,000 adolescents in 2022, which represents a reduction of more than 50%.

The initial increase may reflect the expansion of detention infrastructure and the tightening of institutional responses. The steep decline after 2015, in turn, may be associated with both legal and contextual changes, such as Recommendation No. 62 of the National Council of Justice during the pandemic, which

reduced juvenile incarceration. From the perspective of criminal symbiosis, the decline in custodial measures may indirectly affect serious crimes, since adolescents are often instrumentalized in minor offenses.

2.2 Youth Homicide and Suicide Rates (1980–2022)

Figure 2 reveals distinct dynamics. The homicide rate rose from 36.1 per 100,000 youth in 1980 to 113 in 2015, representing an increase of more than 200%. After this peak, a decline of nearly 20% was recorded up to 2021, although the levels remain alarming. The suicide rate, on the other hand, increased from 6.6 in 1980 to 12.9 in 2022, nearly doubling over four decades, though in a gradual and persistent manner.

These results indicate a dual vulnerability: on one hand, youth exposed to interpersonal violence; on the other, the worsening of mental health fragilities. The recent decline in homicides is not mirrored by suicides, which continue to rise. Thus, the concept of social disorder should encompass both crime and self-inflicted violence, thereby broadening the scope of the *Broken Windows Theory*.

2.3 Propagation Mechanism and Stability Conditions

We define a latent violence potential V_t , which determines the conversion of psychosocial stress into externalized or internalized harm. Let:

$$V_t = \alpha + \beta U_t - \delta C_t + \omega J_{t-1} + \xi_t, \quad \beta, \delta, \omega > 0,$$

where U_t denotes socioeconomic tension, C_t is collective protection, and J_{t-1} channels behavioral imitation.

Observable outcomes arise through two distinct activation surfaces:

$$\begin{aligned} H_t &= \max\{0, V_t - \tau_H\}, \\ S_t &= \max\{0, \tau_S - (V_t - \tau_S)\}, \end{aligned} \quad \tau_H > \tau_S > 0.$$

Thus, homicide increases monotonically with V_t , while self-harm emerges when V_t remains positive but below τ_H .

The dynamic structure is given by:

$$\mathbf{X}_t = \begin{pmatrix} H_t \\ J_t \\ S_t \end{pmatrix} = \mathbf{A} \begin{pmatrix} H_{t-1} \\ J_{t-1} \\ S_{t-1} \end{pmatrix} + \mathbf{B} \begin{pmatrix} U_t \\ C_t \\ E_t \end{pmatrix} + \boldsymbol{\varepsilon}_t,$$

with \mathbf{A} derived from behavioral thresholds and enforcement elasticity captured by E_t .

Proposition 1. *A reduction in youth imitation ω produces a diagonal-dominant \mathbf{A} , implying $\rho(\mathbf{A}) < 1$ and ensuring monotonic convergence to equilibrium.*

Proof. Diagonal dominance reduces $\|\mathbf{A}\|_\infty$, bounding $\rho(\mathbf{A})$ by Gershgorin's disks. Therefore, sufficiently strong enforcement E_t dampens the propagation channel $J_{t-1} \rightarrow H_t$, eliminating explosive paths. \square

Proposition 2. *Weakening of collective protection ($\delta \downarrow$) increases V_t , shifting the system towards ($H_t > 0, S_t > 0$) outcomes simultaneously, consistent with co-movement in the empirical figures.*

Remark 1. *The relative magnitudes of τ_H and τ_S determine whether violence is expressed externally (crime) or internally (self-harm), providing a unified interpretation of the observable indicators.*

3 Theoretical Framework

The literature on routine activities and opportunities argues that contexts of weak surveillance, combined with high potential returns, significantly increase the likelihood of criminal behavior (Felson, 2006). Empirical evidence reinforces this view by showing spatial co-location and temporal correlation between urban disorder and violence, suggesting that small signs of disorganization may trigger cumulative processes of criminality (Groff and McCord, 2012).

From the perspective of the economics of crime, inaugurated by (Becker, 1968), criminal conduct is understood as the outcome of a rational choice, in which the agent weighs the expected costs and benefits of offending. This decision depends on the potential gain, the probability of detection, the severity of sanctions, and the attractiveness of legal alternatives. Crime therefore emerges when illicit incentives outweigh the expected costs imposed by the justice system.

The notion of *criminal symbiosis* proposed in this article seeks to integrate these two approaches, emphasizing the dynamic interdependence between minor and more serious crimes. Serious crimes, when not effectively punished, tend to erode the social stigma associated with everyday infractions, thereby normalizing lower-level illicit behaviors. Conversely, the proliferation of minor offenses creates a permissive environment that reduces the expected cost of offending and expands opportunities for more severe crimes. In economic terms, this constitutes a feedback process in which both the stock of disorder and the social perception of impunity operate as negative externalities that affect the equilibrium of the criminal system.

This process, however, may be counterbalanced by a deterrent effect: policies of targeted policing against seemingly minor offenses and swift judicial responses tend to increase the expected cost of crime at all levels, reinforcing institutional credibility and constraining the escalation of violence (Funk and Kugler, 2003). *Criminal symbiosis*, therefore, should be understood not only as a social phenomenon, but also as an endogenous incentive structure in which the persistence of violence stems both from individual rationality and from the symbiotic interaction between categories of crime and weakened institutions.

4 Dynamic Theoretical Model of Criminal Symbiosis

4.1 Intuition and Objectives

The objective of this model is to formalize the hypothesis of *criminal symbiosis*: tolerance toward minor offenses reduces the expected cost of crime, fostering learning and social legitimation that, over time, increase the incidence of more serious crimes. To this end, we structure a discrete-time dynamic system composed of six interdependent blocks.

The first block refers to crimes, divided into $C_H(t)$ (serious crimes) and $C_L(t)$ (minor crimes). The second block concerns offender populations, represented by $A(t)$ (adult offenders) and $Y(t)$ (youth offenders). The third block corresponds to the judicial system, whose aggregate capacity and efficiency are denoted by $J(t)$. Finally, the fourth block encompasses the prison system, represented by $P(t)$, which measures the stock of incarcerated individuals.

The controllable public policy instruments in the model include targeted policing against serious crimes (u_H) and minor crimes (u_L), the effective conviction rate (τ), and the rehabilitation rate (r). The proposed formulation thus seeks to provide a theoretical foundation for understanding how the interaction between crimes of different severity, institutional structures, and public policy choices can help explain the persistence of violence in contexts such as Brazil.

4.2 Model Formulation

The dynamics of criminal symbiosis are formalized by a discrete-time system of equations that describe the interaction among the blocks defined above. The incidence of serious crimes, $C_H(t)$, depends on the participation of adult offenders $A(t)$ and on the learning derived from minor offenses committed by youths $Y(t)$, weighted by the efficiency of the judicial system $J(t)$ and the level of policing targeted at serious crimes u_H .

Similarly, minor crimes, $C_L(t)$, reflect both the behavior of youth offenders and the intensity of policing targeted at minor offenses u_L . The stock of adult offenders evolves through the transition of youths into adulthood, the entry of new agents due to failed rehabilitation, and exits resulting from imprisonment or conviction, regulated by τ and r . The youth offender population follows a comparable trajectory, determined by early criminal socialization and exits due to police or judicial repression.

The judicial system $J(t)$ directly affects the probability of conviction, while the prison system $P(t)$ accumulates the flow of convicted individuals, being reduced by the rehabilitation process. Thus, the model aims to capture the feedback loop between crimes of different severity and the role of control and rehabilitation policies, constituting a dynamic framework to analyze the persistence and diffusion of criminality.

4.3 Model Formulation and Dynamic Equations

The dynamics between serious crimes (C_H) and minor offenses (C_L) are described in discrete time, with bidirectional interactions of symbiosis and the effects of public policies. Criminal activity arises from the populations of adults (A) and youths (Y), while targeted policing (u_H, u_L), judicial efficiency (J), and prison stock (P) operate as deterrence/incapacitation mechanisms. For $t = 0, 1, 2, \dots$, the laws of motion are given by:

$$\begin{aligned} C_H(t+1) &= (1 - \delta_H) C_H(t) + \alpha_H A(t) + \tilde{\alpha}_H Y(t) + \sigma_{HL} C_L(t) \\ &\quad - [\beta_H u_H(t) + \gamma_H J(t) + \theta_H P(t)] C_H(t), \end{aligned} \quad (1)$$

$$\begin{aligned} C_L(t+1) &= (1 - \delta_L) C_L(t) + \alpha_L A(t) + \tilde{\alpha}_L Y(t) + \sigma_{LH} C_H(t) \\ &\quad - [\beta_L u_L(t) + \gamma_L J(t) + \theta_L P(t)] C_L(t). \end{aligned} \quad (2)$$

The populations of adults and youths evolve through entries driven by criminal opportunities and exits resulting from conviction, incarceration or custodial measures, and natural desistance:

$$\begin{aligned} A(t+1) &= (1 - \phi_A) A(t) + \eta_A C_H(t) + \tilde{\eta}_A C_L(t) \\ &\quad - \pi_A \tau(t) [\omega_H C_H(t) + \omega_L C_L(t)], \end{aligned} \quad (3)$$

$$\begin{aligned} Y(t+1) &= (1 - \phi_Y) Y(t) + \eta_Y C_H(t) + \tilde{\eta}_Y C_L(t) \\ &\quad - \pi_Y \tau(t) [\tilde{\omega}_H C_H(t) + \tilde{\omega}_L C_L(t)]. \end{aligned} \quad (4)$$

Judicial capacity and efficiency evolve through the accumulation of institutional investment and depreciation, while the prison stock increases with new admissions resulting from convictions and decreases through natural releases and rehabilitation:

$$J(t+1) = (1 - \delta_J) J(t) + v(t), \quad (5)$$

$$P(t+1) = (1 - \rho) P(t) + i_H \tau(t) \omega_H C_H(t) + i_L \tau(t) \omega_L C_L(t) - r(t) P(t). \quad (6)$$

The terms $\sigma_{HL} \geq 0$ and $\sigma_{LH} \geq 0$ capture the notion of symbiosis: minor offenses feed into the incidence of serious crimes (1), while shocks in serious crimes legitimize and propagate minor offenses (2). Targeted policing ($\beta_H u_H, \beta_L u_L$), judicial efficiency (γJ), and the prison stock (θP) reduce effective incidence through deterrence and incapacitation. The parameters $\alpha, \tilde{\alpha}$ measure the criminal productivity of each group; $\eta, \tilde{\eta}$ capture entries through opportunity or co-optation; $\pi, \tau(\cdot) \times \omega$ represent the conversion of crime into conviction/admission; and ϕ, δ, ρ denote exit and depreciation rates. The directly controllable policies are $u_H(t), u_L(t), \tau(t), r(t),$ and $v(t)$, all restricted to their usual domains (non-negativity and, when applicable, the interval $[0, 1]$).

Remark (steady state). Under constant controls, an interior steady state x^* solves the equilibrium versions of (1)–(6). This characterization allows for: (i) comparative statics for example, $\partial C_H^* / \partial u_L < 0$ when $\sigma_{HL} > 0$; and (ii) local linearization $\Delta x_{t+1} \approx A \Delta x_t + B \Delta u_t$ for stability analysis and policy simulations.

4.4 Planners Problem and First-Order Conditions

The system described in equations (1)–(6) generates a classical trade-off in criminal policy: selective repression versus social costs. The social planner chooses a trajectory of controls (u_H, u_L, τ, r, v) in order to minimize the discounted aggregate social cost:

$$\begin{aligned} \min_{\{u_H, u_L, \tau, r, v\}} \quad & \sum_{t=0}^{\infty} \beta^t \left[\phi_H C_H(t) + \phi_L C_L(t) + \kappa_P P(t) + \kappa_J v(t) \right. \\ & \left. + \frac{1}{2} \kappa_H u_H(t)^2 + \frac{1}{2} \kappa_L u_L(t)^2 + \kappa_\tau \tau(t)^2 + \kappa_r r(t)^2 \right], \end{aligned} \quad (7)$$

subject to the laws of motion. The discrete-time Hamiltonian is constructed with multipliers $\lambda_{CH,t+1}, \lambda_{CL,t+1}, \dots$, and the first-order conditions take the form:

$$\kappa_H u_H(t) = \beta_H \lambda_{CH,t+1} C_H(t), \quad \kappa_L u_L(t) = \beta_L \lambda_{CL,t+1} C_L(t),$$

with analogous rules for τ, r, v . These equations indicate that optimal policing increases with the product of the crime level and the shadow value of the state, reflecting that resources should be allocated according to both severity and marginal effectiveness.

4.5 Steady-State Equilibrium (Essential Characterization)

Assume constant controls (u_H, u_L, τ, r, v) and the existence of a finite steady state x^* . The stationary equations solve (example for C_H and C_L):

$$\delta_H C_H^* = \alpha_H A^* + \tilde{\alpha}_H Y^* + \sigma_{HL} C_L^* - [\beta_H u_H + \gamma_H J^* + \theta_H P^*] C_H^*, \quad (8)$$

$$\delta_L C_L^* = \alpha_L A^* + \tilde{\alpha}_L Y^* + \sigma_{LH} C_H^* - [\beta_L u_L + \gamma_L J^* + \theta_L P^*] C_L^*. \quad (9)$$

Combining (8)–(9) with the stationarity of A^*, Y^*, J^*, P^* (see equations in the Appendix) yields a linear/quadratic system that can be solved numerically.

From the stationary system it follows (intuitively):

- If $\sigma_{HL} > 0$, an exogenous increase in C_L raises C_H^* (ambience effect). Therefore, policies that reduce C_L (e.g., increasing u_L) indirectly reduce C_H as well.
- If $\sigma_{LH} > 0$, shocks to serious crime legitimize minor offenses testable prediction: peaks in violent crime precede increases in petty crime (minor offenses).
- Increases in J (through v) and in P (through τ and ω) negatively affect both C_H and C_L if $\gamma, \theta > 0$.
- Parameters governing youth transition $(\eta_Y, \tilde{\eta}_Y, \phi_Y, \pi_Y)$ determine intergenerational persistence: reducing Y (through effective rehabilitation r) mitigates the symbiosis.

These predictions motivate the empirical specifications (panel GMM, panel VAR, quasi-exogenous shocks).

4.6 Linearization and Local Policy Experiments

Let the aggregate state vector be $x_t \in \mathbb{R}^5$ and let $u_t \in \mathbb{R}^5$ represent the policy instruments. Around a steady state (x^*, u^*) , define the deviations:

$$\hat{x}_t = x_t - x^*, \quad \hat{u}_t = u_t - u^*.$$

The reduced-form law of motion is given by:

$$x_{t+1} = F(x_t, u_t),$$

with $F : \mathbb{R}^5 \times \mathbb{R}^5 \rightarrow \mathbb{R}^5$ continuously differentiable. A first-order approximation yields:

$$\hat{x}_{t+1} = A\hat{x}_t + B\hat{u}_t + \varepsilon_t, \quad (10)$$

where

$$A = \left. \frac{\partial F}{\partial x} \right|_{(x^*, u^*)}, \quad B = \left. \frac{\partial F}{\partial u} \right|_{(x^*, u^*)},$$

and $\varepsilon_t = o(\|\hat{x}_t, \hat{u}_t\|)$ collects neglected nonlinearities.

Stability. Local asymptotic stability requires purely that the eigenvalues of A satisfy:

$$\rho(A) < 1,$$

with $\rho(\cdot)$ denoting the spectral radius. This condition ensures that crime dynamics return to equilibrium if no persistent shock is applied.

Structure of propagation. Partitioning the state as

$$x_t = (CH_t, CL_t, A_t, Y_t, P_t)',$$

the matrix A can be written as:

$$A = \begin{pmatrix} a_{HH} & a_{HL} & \cdots \\ a_{LH} & a_{LL} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix},$$

where a_{HL} and a_{LH} encode the *criminal symbiosis*: reinforcement between high-severity and low-severity offenses.

Policy transmission. The control block B reveals the direct elasticities of the system with respect to enforcement (u_H, u_L), punishment (τ), rehabilitation (r), and public integrity (v). Thus, (10) provides a transparent mapping:

$$\Delta \hat{x}_{t+1} \approx A \Delta \hat{x}_t + B \Delta \hat{u}_t,$$

supporting counterfactual exercises under alternative policing portfolios.

Numerical protocol. Given horizon T and initial \hat{x}_0 , the updating rule becomes:

$$\hat{x}_{t+1} = A \hat{x}_t + B \hat{u}_t, \quad t = 0, \dots, T - 1.$$

Simulation relies on baseline controls and localized perturbations (e.g., temporary increases in u_L).

Illustrative parameters (for demonstration only; calibration must align with Brazilian sources DataSUS, SENASP, IPEA):

$$\delta_H = 0.19, \delta_L = 0.24, \phi_A = 0.09, \phi_Y = 0.13, \rho = 0.11,$$

$$\sigma_{HL} = 0.04, \sigma_{LH} = 0.05, \beta_H = 0.018, \beta_L = 0.027, \gamma_H = \gamma_L = 0.012,$$

$$\alpha_H = 0.0012, \tilde{\alpha}_H = 0.0007, \alpha_L = 0.0022, \tilde{\alpha}_L = 0.0013.$$

Computational representation.

$$\mathbf{X}_{t+1} = \underbrace{A}_{\text{persistence \& contagion}} \mathbf{X}_t + \underbrace{B}_{\text{policy channels}} \mathbf{U}_t,$$

which highlights two classes of mechanisms:

- (i) *Contagion and learning*: the off-diagonal elements of A determine how minor violence sustains major violence and vice-versa.
- (ii) *Policy leverage*: the high-sensitivity columns of B identify short-run levers for reducing persistent violence.

Policy-relevant insights. Experiments under (10) typically reveal:

- (i) Small offenses are a diffusion vector for severe crime, consistent with a contagion-type interpretation.
- (ii) Focused enforcement on low-severity crime can yield meaningful declines in homicide even without stronger policing of severe offenses.
- (iii) Rehabilitation generates nonlinear welfare gains through the $Y \rightarrow A$ channel, especially when coordinated with deterrence.

5 Conclusion

The evidence suggests that *criminal symbiosis* is a plausible yet contingent phenomenon. Its manifestation depends on institutional conditions, economic environments, and mental health factors. This approach broadens the understanding of the crimedisorder nexus and contributes to the design of more comprehensive and effective public policies.

The concept of *criminal symbiosis* offers a unified framework for analyzing mutually reinforcing dynamics between minor and serious crimes. The dynamic model yields testable predictions and informs the design of integrated policies that combine focused policing, judicial efficiency, and rehabilitation strategies.

Technical Appendix: Optimal Control and Local Dynamics

A. Necessary Optimality Conditions

At time t , the planner selects $u_t = (u_H, u_L, \tau, r, v)$ to maximize discounted welfare

$$W = \sum_{t=0}^{\infty} \delta^t [\Phi(x_t) - \Psi(u_t)],$$

where

$$\Phi(x_t) = \varphi_H C_H(t) + \varphi_L C_L(t) + \varphi_P P(t), \quad \Psi(u_t) = \frac{1}{2} \eta_H u_H(t)^2 + \frac{1}{2} \eta_L u_L(t)^2 + \theta_\tau \tau(t)^2 + \theta_r r(t)^2.$$

The current-value Hamiltonian is:

$$\mathcal{H}_t = \Phi(x_t) - \Psi(u_t) + \lambda_t^\top F(x_t, u_t),$$

with $F(\cdot)$ representing the state transitions.

The stationarity conditions become:

$$\frac{\partial \mathcal{H}_t}{\partial u_H} = \eta_H u_H(t) - \lambda_{C_H, t+1} \beta_H C_H(t) = 0,$$

$$\frac{\partial \mathcal{H}_t}{\partial u_L} = \eta_L u_L(t) - \lambda_{C_L, t+1} \beta_L C_L(t) = 0,$$

$$\begin{aligned}\frac{\partial \mathcal{H}_t}{\partial \tau} &= 2\theta_\tau \tau(t) - \lambda_{A,t+1} \vartheta_H C_H(t) - \lambda_{A,t+1} \vartheta_L C_L(t) = 0, \\ \frac{\partial \mathcal{H}_t}{\partial r} &= 2\theta_r r(t) - \lambda_{P,t+1} P(t) = 0, \quad \frac{\partial \mathcal{H}_t}{\partial v} = \chi_J - \lambda_{J,t+1} = 0.\end{aligned}$$

Costate dynamics follow the recursive form:

$$\lambda_{x,t} = \frac{\partial \mathcal{H}_t}{\partial x_t} + \delta \lambda_{x,t+1}, \quad x \in \{C_H, C_L, A, Y, J, P\}.$$

B. Local Linear Approximation

Let the steady state satisfy $\bar{x} = F(\bar{x}, \bar{u})$. Define deviations $\hat{x}_t = x_t - \bar{x}$ and $\hat{u}_t = u_t - \bar{u}$. Using the Jacobians:

$$A = \left. \frac{\partial F}{\partial x} \right|_{(\bar{x}, \bar{u})}, \quad B = \left. \frac{\partial F}{\partial u} \right|_{(\bar{x}, \bar{u})},$$

the system evolves locally as

$$\hat{x}_{t+1} = A\hat{x}_t + B\hat{u}_t. \quad (11)$$

Stability holds if and only if:

$$\rho(A) < 1.$$

C. Computational Strategy

Given an initial vector \hat{x}_0 and a control path $\{\hat{u}_t\}_{t=0}^{T-1}$, iterate

$$\hat{x}_{t+1} = A\hat{x}_t + B\hat{u}_t.$$

Suggested experiments:

- (i) assess impulse responses to short-lived shocks in u_L or τ ;
- (ii) evaluate persistent changes in rehabilitation efforts (r);
- (iii) compare equilibria under different deterrence regimes.

This framework isolates the structural channels through which selective enforcement and social policies propagate crime dynamics.

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