Identifying the Shocks also Identifies the Constants: Implications for VAR analysis

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Identifying the Shocks also Identifies the Constants: Implications for VAR analysis*

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Abstract

Restrictions on the contemporaneous effects matrix used to identify fundamental shocks in a structural VAR, also determine the mapping from the structural constant terms to the reduced form constant terms. In some models one will have priors about these structural constant terms and these should therefore be included in a Bayesian estimation procedure. We illustrate the significance of this using a standard 3 variable VAR estimated in Baumeister and Hamilton (2018). We show that imposing priors over the structural constant terms can lead to a more intuitive estimated monetary policy rule and a larger role for monetary policy in describing the evolution of the data, particularly for inflation. (108 words)

Keywords: Vector Autoregressions, Historical Decompositions, Monetary Policy

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1 Introduction

The constant terms in structural vector autoregressions (VARs) have not received the same attention in the literature as the impulse response functions. Yet the structural constant terms are subject to the same identification transformations as the shock processes and can play a large role in determining the evolution of the variables in the VAR. Furthermore, economists will have priors over the contribution of these structural constant terms and these should be implemented in a Bayesian estimation procedure and so allowed to affect the VAR's results and interpretation. We illustrate this using Baumeister and Hamilton (2018)'s analysis of a standard three equation macroeconomic model. We show how implementing a prior that the structural constant of the monetary policy equation is significantly positive can significantly increase the estimated role of monetary policy, particularly in reducing inflation. However, otherwise the results remain close to the original, with the demand and supply equations the main drivers of the evolution of the macroeonomy.

This paper makes contributions to the literature in two ways; to the identification of VARs in general and to the specific literature on the effects of monetary policy shocks. The contribution to the literature on shock identification builds on the recent work of Bergholt et al. (2025). They showed that the reduced form constant term in a VAR is often estimated imprecisely, and that this in turn leads to an increased variance in the estimated contribution of the identified shocks. They showed how a single-unit-root prior can solve this problem by shrinking the uncertainty around the estimated deterministic component. Brignone and Piffer (2025b) show how this modelling of the deterministic terms can aid prediction. In this paper we build on this analysis by considering the role of the structural constants, which together make up the reduced form constant terms, following Mountford and Wadsworth (2025)[1]. In this paper we impose priors over the structural constants so that they are consistent with the economic intuition of the Taylor rule and Philips curve, and show how these priors affect the posterior parameter distributions and can alter the interpretation of the VAR.

The effects of monetary policy on the economy is a fundamental topic in macroeconomics with intrinsic policy implications. It consequently has a large literature spanning a variety of approaches. This paper contributes to the structural VAR strand of this literature, which tries to identify the effects of monetary policy using time series methods without imposing strong parametric assumptions.[2] A seminal paper in this field is Leeper, Sims and Zha (1996) which found, using US data, that shocks to monetary policy were responsible for only a small proportion of the variation in output and prices, and that most of the variation in the policy interest rate itself can be attributed to systematic reactions to other macroeconomic shocks. An intuitive result that is consistent with how one might expect a monetary authority to

[1]Mountford and Wadsworth (2025) analyze the labor market effects of immigration where the structural constant associated with the growth of immigration share in the labor market negatively effects the growth of native wages.

[2]We do not attempt to summarize here the large related literature deriving restrictions on VARs from DSGE models, see Baumeister and Hamilton (2018) and Canova (2007) for a discussion and Canova and Ferroni (2022) on the complexities of the mapping from DSGE to VARs.

behave. Of course, what constitutes a strong assumption is to some extent subjective, and this has been explored. Uhlig (2005) using apparently minimal identifying sign restrictions, also found only a small role for monetary policy in the variation of output and prices. However other studies,using alternative plausible sign restrictions, have found significant effects of monetary policy, such as Canova and De Nicolò (2002) for G7 economies, including the US, and Mumtaz and Surico (2009) for the UK economy, who found significant effects of foreign monetary policy shocks, which intuitively, should be exogenous to the UK economy.

Baumeister and Hamilton (2015) showed that the methodology used in the previous sign restriction literature imposed stronger restrictions on the parameters than had previously been believed. They proposed a new method for imposing informative priors on the structural parameters of a VAR, which incorporate beliefs about the size and sign of these structural parameters, as well as the uncertainty surrounding these beliefs. Baumeister and Hamilton (2018) applied and extended this methodology to the analysis of monetary policy in the United States. They found, in common with Leeper, Sims and Zha (1996) and Uhlig (2005), that monetary policy shocks had only a small influence on the evolution of income, inflation and interest rates, with demand and supply shocks responsible for most of the variation in the macroeconomy. In this paper we show that the identification in Baumeister and Hamilton (2018) implies a counter-intuitive monetary policy rule that sets interest rates insignificantly different from zero in the environment where output is at its potential and inflation is zero. We show that by imposing a prior such that the constant in the monetary policy rule is positive, the monetary policy rule becomes more intuitive and results in monetary policy having a greater estimated role in the evolution of the macroeconomy, particularly for inflation.

The paper is organized as follows. In section 2 we describe the relationship between the structural constants and the reduced form constants and the contribution of structural constants to a historical decompositions of the data. Section 3 then describes how we implement priors over the structural constants which are the focus of the paper. Section 4 applies this method to the model of Baumeister and Hamilton (2018) and section 5 extends the dataset to 2025(Q2) and compares its explanation of recent history to other recent structural VAR models.

2 Accounting for the constant terms

A structural VAR with n variables and p lags is described by equation (1),

$$Ay_t = C + B_1 y_{t-1} + B_2 y_{t-2} + \cdots + B_p y_{t-p} + u_t \tag{1}$$

where A, is an $(n \times n)$ matrix of contemporaneous effects, C, is a $(n \times 1)$ vector of constants, B_i, are the $(n \times n)$ matrix of parameters for lag i, and u_t, is the vector of the structural shocks on the VAR. The $(n \times n)$ variance covariance matrix for these shocks, $E[u_t u_t'] = D$, is assumed to be diagonal. The diagonal structure implies the shocks are 'fundamental' in the sense of having a zero covariance with each other.

The reduced form of the VAR in equation (1) is given by multiplying both sides by the

inverse of the A matrix, so that

$$\begin{aligned} y_t &= A^{-1}C + A^{-1}B_1 y_{t-1} + A^{-1}B_2 y_{t-2} + \cdots + A^{-1}B_p y_{t-p} + A^{-1}u_t \\ &= \kappa + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \cdots + \Phi_p y_{t-p} + \epsilon_t \end{aligned} \tag{2}$$

The reduced form coefficient matrices and error terms in equation (2) - κ, Φ_j and ϵ_t, - are their counterparts in the structural equation multiplied by A^{-1}.

Macroeconomic research has focussed on identifying the matrix A and, given A, on describing the effects of the structural shocks, ϵ_t on the time series using impulse response functions and historical decompositions. Following Hamilton (1994) impulse response functions can be calculated by rewriting equation (2) as a VAR of 1 lag,

$$\underset{(np\times 1)}{Y_t} = \underset{(np\times 1)}{K} + \underset{(np\times np)}{F}\underset{(np\times 1)}{Y_{t-1}} + \underset{(np\times 1)}{E_t} \tag{3}$$

where

$$Y_t = \begin{bmatrix} \underset{(n\times 1)}{y_t} \\ \vdots \\ \underset{(n\times 1)}{y_{t-p+1}} \end{bmatrix} K = \begin{bmatrix} \kappa_{n\times 1} \\ 0_{(n(p-1)\times 1)} \end{bmatrix} \quad F = \begin{bmatrix} \Phi_1 & \cdots & \Phi_p \\ I_{n(p-1)} & & 0_{n(p-1),n} \end{bmatrix} \quad E_t = \begin{bmatrix} \epsilon_t \\ 0_{n(p-1),1} \end{bmatrix}$$

The iteration of equation (3) backward from period $t+s$, implies that the observation Y_{t+s} in period $t+s$ can be decomposed into three contributions: the initial conditions, the constant terms and the innovations in the previous s periods. i.e.

$$Y_{t+s} = \underbrace{F^s Y_t}_{\text{Initial Conditions}} + \underbrace{F^{s-1}K + F^{s-2}K + \cdots + K}_{\text{Constant Terms}} + \underbrace{F^{s-1}E_{t+1} + F^{s-2}E_{t+2} + \cdots + E_{t+s}}_{\text{Innovations}} \tag{4}$$

where the variables of interest are the first n rows of this system. The contribution of the structural shocks at time period t, u_t, s periods after the shock, denoted H_s, is thus given by

$$H_s = F_n^{s-1} A^{-1} u_t \tag{5}$$

where F_n denoted the top $n \times n$ matrix of matrix F. The impulse response functions f are the H_s calculated for all periods s, for $s = 1, T^{\max}$, where $T^{\max}$ is an arbitrary time horizon sufficient to determine the impulse response function's shape.

Equation (4) shows that the F matrix, and so the A^{-1} matrix, determines the mapping of both the structural shocks and the structural constant terms into the observed times series. One can therefore apply the same analysis of equation (5) above, for the contribution of the structural constant term to Y_{t+s}, denoted J_s, so that

$$J_s = \sum_{l=0}^{s-1} F_n^i A^{-1} C \tag{6}$$

Furthermore the mean of the vector time series, $[y_t, \pi_t, r_t]$, denoted μ, is given by

$$\mu = [I_n - \Phi_1 - \Phi_2 - \ldots \Phi_p]^{-1} A^{-1} C \qquad \text{(Hamilton (Ch10))}$$

Thus the matrix A plays an important role in determining how the mean of each variable in the VAR is explained by the structural constant term.

In some models one may have priors about the contributions of the structural constant terms. Including these priors in the estimation process will therefore affect not only the contribution of the structural constants but also the contribution of the structural shocks. In the next section we describe how we implement the priors over the structural constants and in the following section we show the implications of these priors using the example of Baumeister and Hamilton (2018)'s analysis of a standard three equation macroeconomic model

3 Implementing priors over the structural constants

To implement priors over the structural constants we follow Baumeister and Hamilton (2015) and Bergholt et al. (2025) and apply the 'mixed estimation' method described in Theil (1971). We follow Baumeister and Hamilton's (2015) approach and notation closely in this paper. Their method is explained in detail in their paper and so our description here will be brief

Baumeister and Hamilton (2015) use Bayesian methods to estimate the structural VAR of equation (1). They employ natural conjugate priors for the autoregressive and variance parameters conditional on the parameters of the A matrix which are independent across equations. Specifically, conditional on A and D, the prior for the B coefficients of each equation i in equation (1) - denoted $b_i \equiv [B_1(i,:) \dots B_p(i,:)\ C(i)]$ with $k = np + 1$ - are independently distributed $\mathcal{N}(m_i, d_{ii}M_i)$ across equations. Each diagonal element, d_i, of matrix D, is independently inverse gamma distributed.

Baumeister and Hamilton (2015) show how, given the matrix A, the mean and variance of the posterior for the coefficients of the structural VAR can be estimated equation by equation, using an OLS regression where the dependent variable in each equation is $[a_i'y_1 \dots a_T'y_Y\ m_i'P_i]$ where P_i is the Cholesky factors of M_i^{-1}, and where the dependent variables are $[x_i \dots x_{T-1}\ P_i]$. Additional prior information, which can be represented by linear combinations on the parameters of one equation, can be implemented following Theil (1971) by further augmenting the data matrices. For example consider the case where there are h pieces of prior information for the parameters in equation i which can be represented by the equation, $r_i = h_i R_i b_i + \nu_i$, where ν_i is normally distributed error term $\mathcal{N}(0, V_i)$. This prior information can be included in the estimation by augmenting the data matrices so that the dependent and independent variables in the OLS regression, denoted, $\widetilde{Y}$ on $\widetilde{X}$, become,

$$\begin{array}{l} \underset{(1\times(T+k+h_i))}{\widetilde{Y}} = [\underset{(1\times n)(n\times 1)}{a_i'y_1} \quad \dots \quad \underset{(1\times n)(n\times 1)}{a_T'y_T} \quad \underset{(1\times k)(k\times k)}{m_i'P_i} \quad \underset{(1\times h_i)(h_i\times h_i)}{\mathbf{r_i'P_{Vi}}}\ ] \\ \underset{(k\times(T+k+h_i))}{\widetilde{X}} = [\underset{(k\times 1)}{x_i} \quad \dots \underset{(k\times 1)}{x_{T-1}} \quad \underset{(k\times k)}{P_i} \quad \underset{(k\times h_i)(h_i\times h_i)}{\mathbf{R_i'P_{Vi}}}\ ] \end{array} \qquad \text{(BH(2015) eq(47,48))}$$

where P_{Vi} is the Cholesky factors of V_i^{-1}. Thus a prior that the structural constant in equation i should be close to 0.1 is a single constraint,$(h_i = 1)$, with $r_i = 0.1$, R a $(k \times 1)$ vector with a value of 1 in the position of the constant term and 0 elsewhere and where the uncertainty about this prior is represented by the variance term, V_i.

However a prior over one structural constant will also effect the other structural constants

indirectly as we are also imposing the prior that the mean of the reduced form VAR is close to the sample means of the data series via a single-unit-root prior as in Bergholt et al. (2025). This single-unit-root prior is implemented by further augmenting the data matrices of both the independent variables and the dependent variables at all lags by μ_i/δ, where μ_j is the sample mean of variable j and where δ represents the precision of this prior, where we set $\delta = 0.01$. The implicit restrictions on the other structural constants is determined by the precision of these two priors i.e. if both the prior over the structural constants and the single-unit-root prior are very right then this also implies a tight prior on the remaining structural constants.

4 An example of identifying the monetary policy rule

We demonstrate the implications of including priors over the structural constants in macroeconomic models using Baumeister and Hamilton's (2018) analysis of a structural VAR in the output gap, y_t, inflation, π_t and the nominal interest rate, r_t. We first describe the model, emphasizing the economic interpretation of the constant terms. In section 4.2 we replicate Baumeister and Hamilton's results using the data described in section 4.1. We again emphasize the contribution of the structural constant terms and show how the posterior distribution for the constant of the monetary policy rule is centered close to zero. In section 4.2 we show that imposing a prior on this constant can produce a monetary policy rule with a positive structural constant, which is intuitive and also leads to a larger role for monetary policy in explaining the evolution of macroeconomic variables, without overturning Baumeister and Hamilton's (2018) main results.

Baumeister and Hamilton's (2018) estimate the following structural VAR, which has the same form as equation (1) above,[3]

$$\begin{bmatrix} 1 & -\alpha^s & 0 \\ 1 & -\beta^d & -\gamma^d \\ -\xi^y & -\xi^\pi & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ r_t \end{bmatrix} = \begin{bmatrix} \kappa^s \\ \kappa^d \\ \kappa^m \end{bmatrix} + \underset{(3 \times np)}{\mathbf{b}} \underset{(np \times 1)}{\mathbf{x}_{t-1}} + \begin{bmatrix} u_t^s \\ u_t^d \\ u_t^m \end{bmatrix} \tag{BH1}$$

where x_{t-1} is a matrix containing the lagged values of y_t, π_t and r_t.

The interpretation of the first equation is a supply relationship where contemporaneous supply responds non-negatively to inflation, as in the Phillips curve. Thus the parameter α^s in equation (BH1) is restricted to be non-negative in the prior. Interest rates are assumed to affect this relationship only with a lag, a common assumption in the macroeconomics literature, and so r_t is multiplied by a zero in this equation. The constant in this equation, κ^s, therefore has the interpretation as a determinant of supply unrelated to inflation or the interest rate i.e. an exogenous determinant of supply. Intuitively this constant should be negative, so that, *ceteris paribus*, if inflation and interest rates were zero then output would be below its potential, which corresponds to the intuition of the Phillips curve literature. Note, as with all the equations, the equilibrium output gap will depend on the other equations and

[3]Note we are following Baumeister and Hamilton's (2018) notation here where the structural constants are denoted by κ, not C as in equation (1).

the long run mean of the output gap will also depend on the structural constants, κ^d and κ^r as well as the autoregressive coefficients, b_i.

The second equation is interpreted as a demand equation where contemporaneous demand responds non-positively to the interest rate. Thus the parameter γ^d in equation (BH1) is restricted to be non-positive in the prior. Inflation could plausibly affect demand positively or negatively and so β^d's sign is not restricted in the prior. The constant in this equation, κ^d, has the interpretation as a determinant of demand which is unrelated to inflation or the interest rate i.e. an exogenous determinant of demand. A positive constant, κ^d, implies that the forces of aggregate demand cause output to run above potential at zero interest rates and inflation, which corresponds to the 'overheating' intuition of demand shocks.

The third equation is interpreted as the monetary policy rule of the Federal Reserve Board. It is assumed that the Federal Reserve will increase interest rates when inflation increases or output is high. Thus the parameters ξ^y and ξ^π in equation (BH1) are assumed to be non-negative in the prior.[4] The constant in this equation, κ^m therefore has the interpretation as a determinant of the interest rate unrelated to inflation or the output gap i.e. an exogenous determinant of the interest rate. A positive constant, κ^m, implies that if the economy is operating at potential with zero inflation then the interest rate would be positive. This corresponds to a positive rate of time preference, which is intutive.

4.1 Data

Baumeister and Hamilton use data on real GDP, real potential GDP, the PCE deflator and the Federal Funds Rate for 1986(Q1) -2008(Q3). In this section we use their original dataset. In section 5 we use the updated version of these series taken from FRED for the sample 1986(Q1)-2025(Q2). These are the series GDPC1, GDPPOT, DPCERD3Q086SBEA and DFF using quarterly averages. The output gap is the log of real gdp minus the log of real potential gdp, inflation is measured by the year on year difference of the log PCE deflator and the Federal Funds Rate is not transformed.

4.2 Replicating the results of Baumeister and Hamilton (2018)

In this section we display the results of a replication of Baumeister and Hamilton (2018) using a stacked bar graph to emphasize the contribution of the constant terms. Figure 1 plots the median historical decomposition of the three series into the contributions of the three structural shocks, where the contribution of the demand shock is colored green, the supply shock colored yellow and the monetary policy shock colored blue. The actual data series are plotted in red. This corresponds to Figure 5 in Baumeister and Hamlton (2018).

[4]Baumeister and Hamilton (2018) relate this equation to the Taylor rule of Taylor (1993) which is commonly used in the literature, see e.g. Lubik and Schorfheide (2004), and show how the contemporaneous coefficients of this equation are the product of the contemporaneous influences on monetary policy ϕ^y and ϕ^π and a smoothing parameter ρ. Their model estimates all of these parameters with ρ restricted to be between 0 and 1 so as not to change the signs of contemporaneous effects.

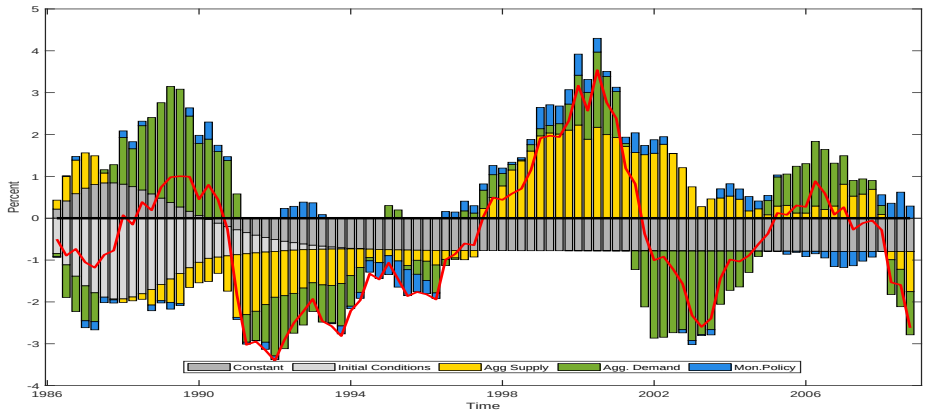
As noted by Baumeister and Hamilton (2018) monetary policy shocks, (blue), play very little role in the variation of output and inflation around their mean. The contribution of the initial conditions is colored light grey. This has a large contribution at the start of the sample. This is the inherited contribution of previous shocks, notably the higher than average inflation and interest rates in the years preceding the start of the sample. However, consistent with equation (4), this contribution dies out over time. The contribution of the reduced form constant terms are colored in darker grey. These show positive long run average levels of inflation and the interest rate and a negative long run output gap (potential GDP is higher than actual GDP), which is all intuitive.

Figure 2 plots the same data as in Figure 1, but this time decomposes the reduced form constant terms into their structural contributions. With the exception of the monetary policy constant, the results are still intuitive. The yellow bars represent the contribution of κ^s, the constant in the Phillips curve in equation (BH1). This has a negative contribution to the steady state output gap and a positive contribution to steady state inflation. Thus in the absense of the other equations, inflation would be positive but below the level needed for output to be at its potential. This is in line with textbook macroeconomic analysis based on e.g. Barro Gordon (1983). The green bars represent the contribution of κ^d, the constant in the demand equation, in equation (BH1). This has a positive effect on both steady state inflation and the output gap which implies the forces of aggregate demand cause output to run above potential at zero interest rates even if inflation was zero. This corresponds to the 'overheating' intuition of aggregate demand shocks. However the contribution of the constant in the monetary policy rule, κ^m, colored blue is not intuitive. Its contribution at the median, to both steady state inflation and the output gap is almost zero. The steady state interest rate itself is almost entirely determined by the constant in the demand equation, κ^d.

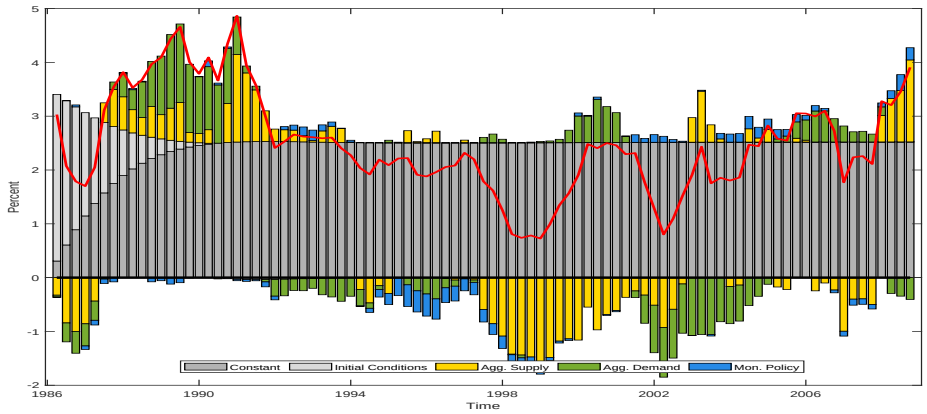
Figure 3a plots the posterior distribution of the structural constants for the replication of Baumeister and Hamilton (2018). This shows that the posterior distribution of κ^m has a small negative mode but has sizeable portions of the density above and below zero. A positive rate of time preference implies that κ^m should be positive. In any case, this paper argues that it is interesting to see the effects of implementing such a prior belief on the results. This is what we do below in section 4.3.

Figure 1: Historical decomposition of growth of the output gap, inflation and interest rate from a replication of Baumeister and Hamilton (2018)

(a) Output gap



(b) Inflation



(c) Interest Rate

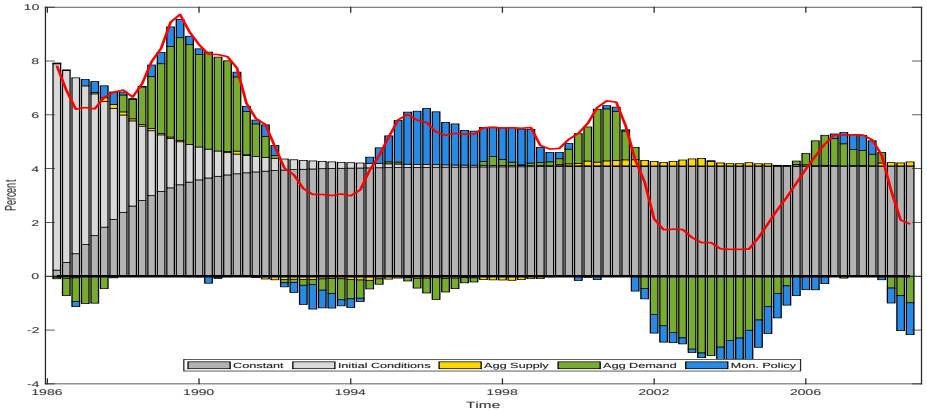
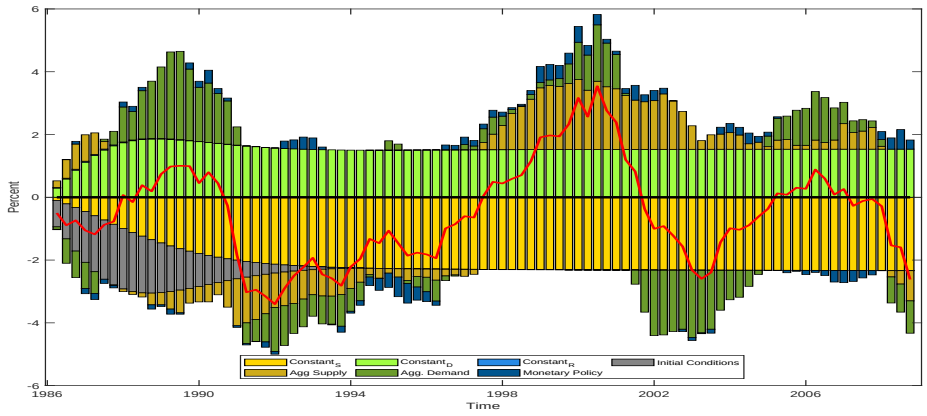


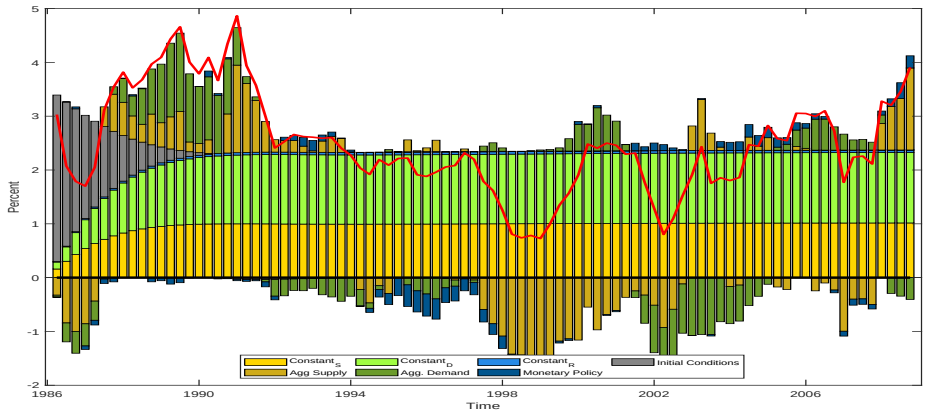
Figure 1 plots the median estimates of the historical decomposition of the output gap, inflation, and the interest rate from a replication of Baumeister and Hamilton (2018). The yellow bars show the contribution of the supply shocks, the green bars, the contribution of the demand shocks and the blue bars the contribution of the monetary policy shocks. In each panel the red line plots observed series. The contribution of the initial conditions is given by the light gtey bars and that for the reduced form constant in darker grey.

Figure 2: Historical decomposition of the output gap, inflation and interest rate including decomposing the constants from replication of Baumeister and Hamilton (2018)

(a) Output gap



(b) Inflation



(c) Interest Rate

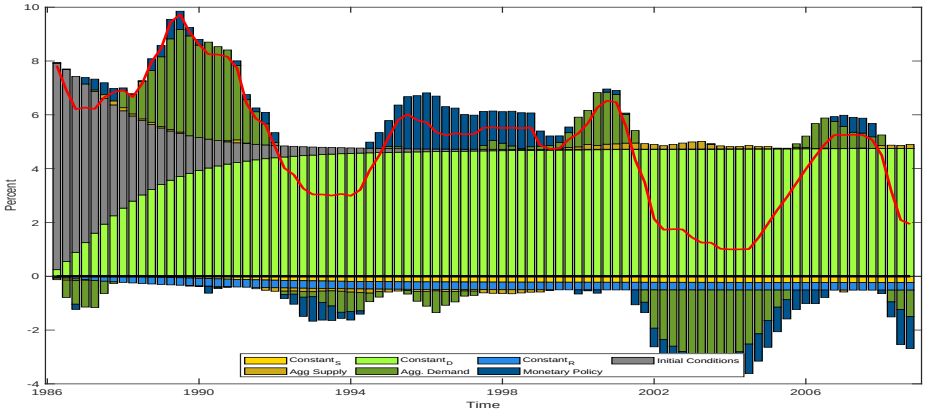
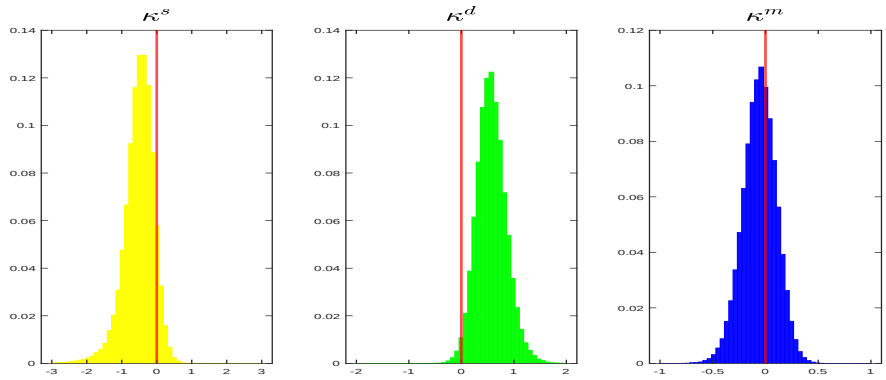


Figure 2 plots the median estimates of the historical decomposition of the output gap, inflation, and the interest rate from a replication of Baumeister and Hamilton (2018), where the constant terms are also decomposed into the parts associated with their structural equations. The yellow bars are the shocks and structural constant from the supply equation, the green bars the same from the demand equation and the blue bars the same from the monetary policy equation. The constant terms are given a lighter shade. In each panel the red line plots observed series.

Figure 3: Posterior distribution of the structural constants with and without additional priors from replication of Baumeister and Hamilton (2018)

(a) Without priors over structural constants



(b) With priors over structural constants

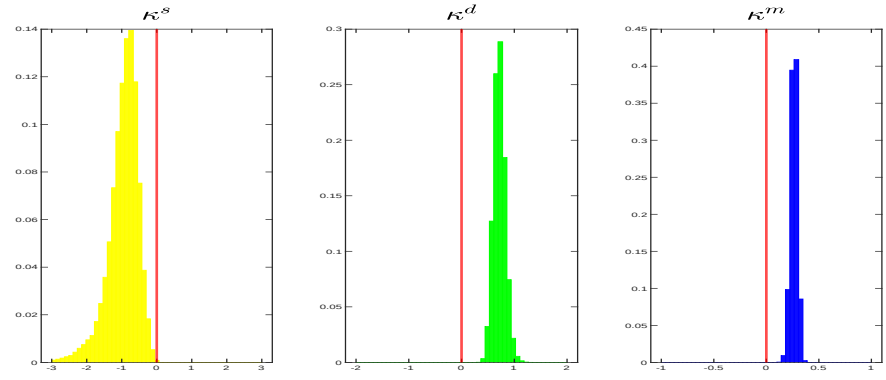


Figure 3 plots the posterior distribution of the structural constants for the case without additional priors over the structural constants, panel (a), and with an additional prior over the structural constants as described in section 4.3, panel (b).

4.3 Imposing priors on the structural constants in Baumeister and Hamilton (2018)

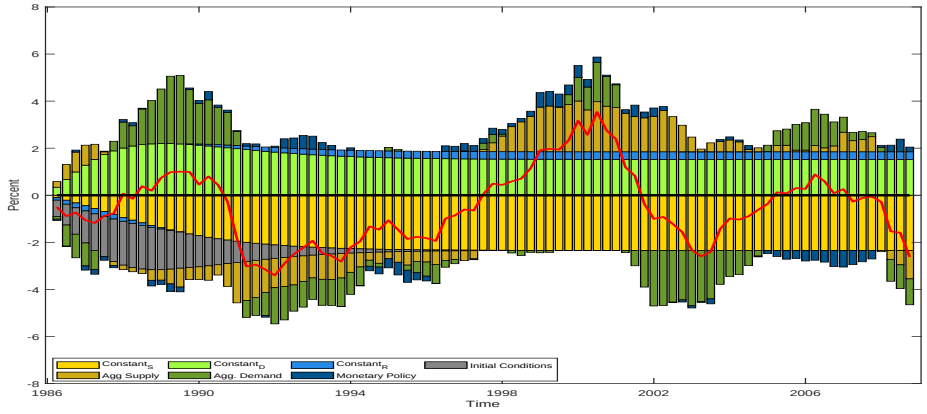
In this section we present the results from implementing a prior that the constant in the monetary policy rule equation, κ^m, be positive and that the means of the reduced form VAR equations are close to the sample means of the data series as recommended by Bergholt et al. (2025). Using the notation of section 3, the prior we impose sets $\kappa^m = 0.5$ with a variance, V_i of 0.1. The posterior for the structural constants is shown in Figure 3b. This shows that the posterior for κ^m is between the prior, which has a mode of 0.5, and the previous estimation, which had a mode close to zero. Furthermore, the variance of the posterior of all the structural constant terms has been reduced as in Bergholt et al. (2025). The posterior of κ^D has shifted to the right so that its mode is higher and because of the reduced variance it is now almost entirely greater than zero. Similarly the posterior of κ^S has shifted to the

left so that its mode is now lower with almost all of the distribution less than zero. Thus the effect of imposing the prior that $\kappa^m = 0.5$ has been to make the signs of all the structural constants much clearer and also to make their signs agree with economic intuition as outlined in section 4.

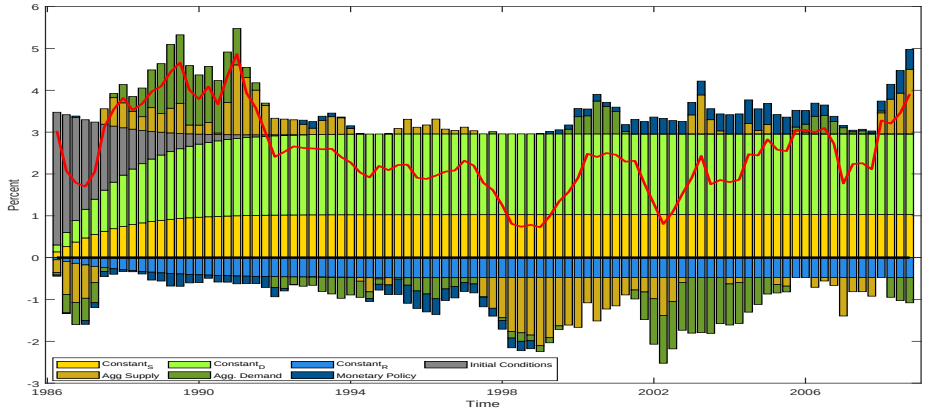
The effect of the prior on the historical decompositions is shown in Figure 4. The most apparent change is to the historical decomposition of the interest rate. Now monetary policy does significantly effect the long run interest rate. However it is still the case that the demand equation is the biggest influence on the interest rate, both in terms of variation and in its long run level, as in Figure 2. The historical decomposition for inflation is also significantly different from Figure 2 in that now the monetary policy constant has a significantly negative impact on long run inflation. This accords with intuition. The historical decomposition for inflation and the interest rate also show that interest rates were relatively tight in the late 1990's and relatively loose in the early 2000's and that this impacted inflation negatively in the late 1990s and positively in the early 2000's. Finally the historical decomposition for the output gap shows the same dominant roles of demand equation which is positive and the supply equation which is negative. However this is a very different narrative from Figure 2, as now the monetary policy equation is a significant force on the macroeconomy, yet despite this, still has little effect output. One thing to note here is the small positive influence of the monetary policy equation's constant on output. This is counterintuitive, though small, and may indicate that a three dimensional VAR is too small and is something to follow up in future work.

Figure 4: Historical decomposition of the output gap, inflation and interest rate after imposing prior on the structural constants

(a) Output gap



(b) Inflation



(c) Interest Rate

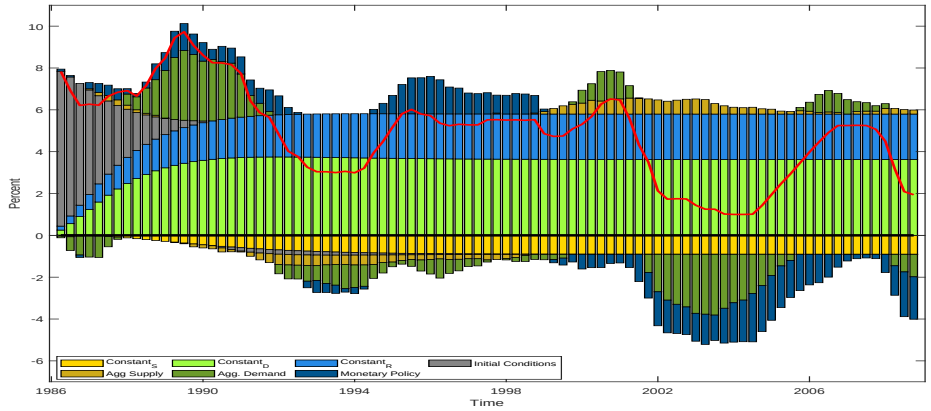


Figure 4 plots the median estimates of the historical decomposition of the output gap, inflation, and the interest rate with an additional prior over the structural constants as described in section 4.3. As with Figure 2 the yellow bars are the shocks and structural constant from the supply equation, the green bars the same from the demand equation and the blue bars the same from the monetary policy equation. The constant terms are given a lighter shade. In each panel the red line plots observed series.

4.4 How large are the effects? - a counterfactual analysis

The historical decompositions in Figure 4 shows how each data series can be decomposed into the constituent parts related to each equation in the structural model, equation (BH1) described in section 4 above, including the contributions of the structural constants. In this section we provide another quantification and depiction of the effects of these fundamental constituent parts on each observed data series. We do this for the estimated model of section 4.3 by setting the contribution of a structural equation to zero, both the constant and shocks, and then recalculating each time series as per equation (4) in section 2. The difference between this counterfactual time series and the observed time series highlights the contribution of the left-out structural equation to the observed time series. Note that this 'counterfactual' is done without reference to a deep structural model and so one should not use this analysis to make statements such as 'If the interest rate had been set x% lower then inflation would have been y% higher'. However these Figures do show the contribution of each structural equation to the each data series over the sample period in the estimated model.

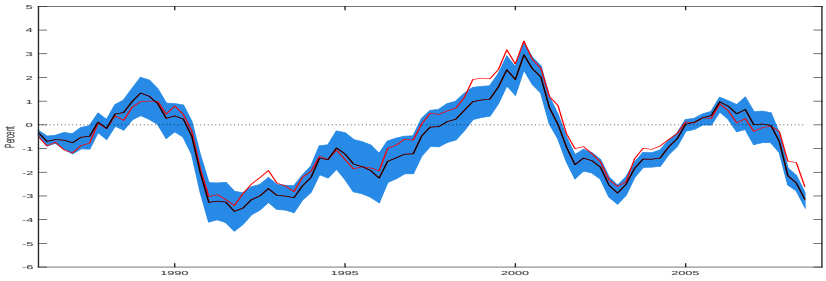
Figure 5 plots the counterfactual time series with the Taylor rule equation's constant and error terms set to zero. The median and the 2.5% and 97.5% quantile bands of the counterfactual are colored blue and the actual time series is colored red. For the interest rate the absence of the positive constant is obscured initially by the effects of the initial conditions as in Figure 4c but thereafter the interest rate in the counterfactual is significantly below that in the data. This causes inflation to significantly higher over most of the sample period with output being insignificantly different from the data for most of the sample but lower in the middle of the sample which is due to the small, but still counterintuitive, positive influence of the monetary policy rule constant on output discussed above.

The size of the effects looks small. A fall in the interest rate of over 1%, at the median, reduces inflation by less than 1%. However, one should bear in mind that the model is linear and is estimated using data where the monetary policy equation is hopefully near its optimum. Thus large non-linear explosive effects are not in the data, or the model. The interpretation of Figure 5 is therefore of one of the first order effects of perturbations to an optimal rule.

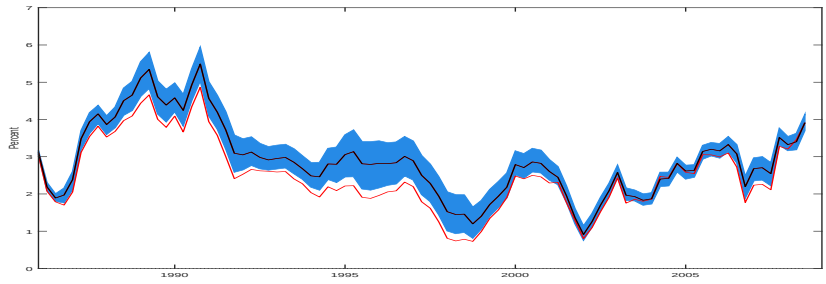
Figure 6 plots the equivalent counterfactual time series where the demand equation set to zero. The absence of demand has a large and significant negative effect across all the data series, indicating that demand equation has a persistently positive effect on output and inflation and consequently, via the monetary policy rule, interest rates. The absence of demand leads interest rates to become negative, particularly in the early 2000's, as there is no zero lower bound in our model. This counterfactual does seem consistent with the effects of the demand shocks following the financial crises in 2008 which we discuss in section 5 below.

Figure 5: Counterfactual with the monetary policy rule equation set to zero

(a) Output gap



(b) Inflation



(c) Interest Rate

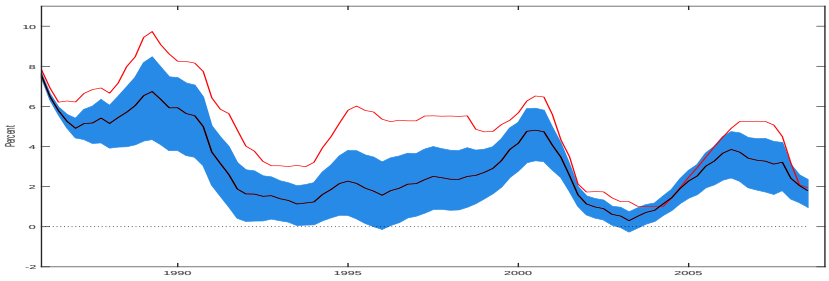
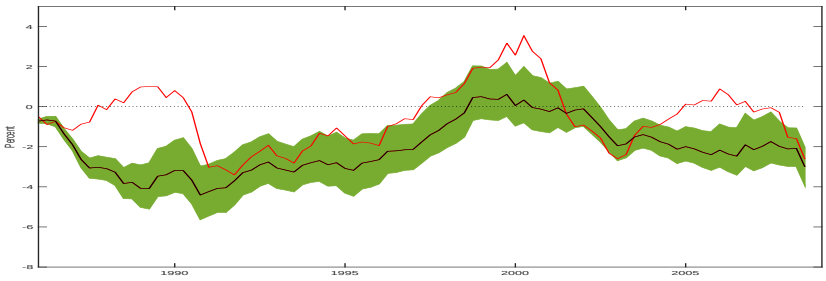


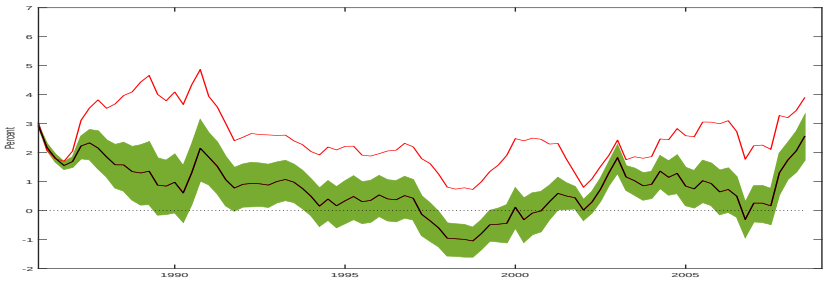
Figure 5 plots the counterfactual time series for the case where monetary policy rule equation is set to zero. The red line plots the observed data series and the shaded region shows the area between the 2.5% and 97.5% quantiles of the counterfactual posterior distribution with the median plotted in black. The shading is blue, consistent with the color used for the monetary policy equatioon in the other Figures. The distribution is mostly below the data for the interest rate over the sample period, illustrating that the monetary policy rule increases the interest rate above what it would otherwise be.

Figure 6: Counterfactual with the demand equation set to zero

(a) Output gap



(b) Inflation



(c) Interest Rate

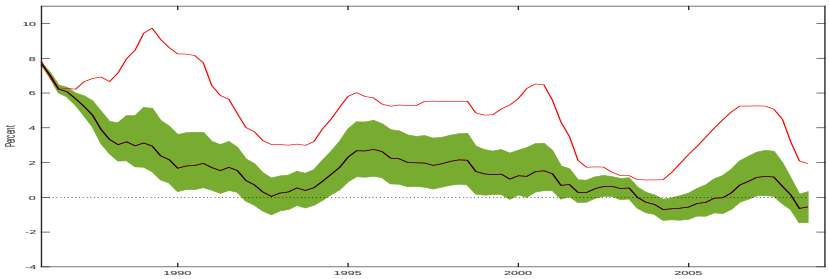


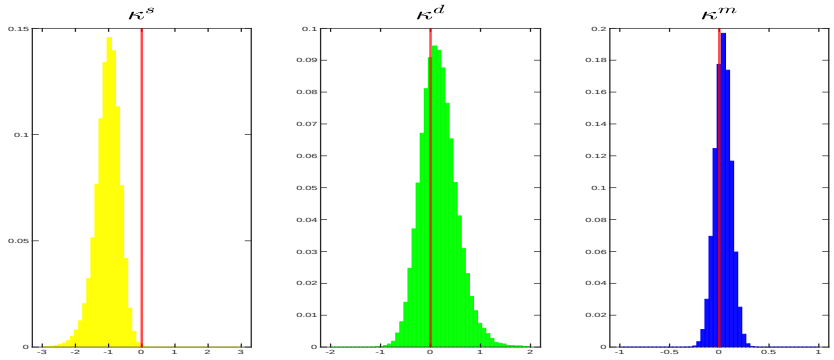
Figure 6 plots the counterfactual time series for the case where demand equation is set to zero. The red line plots the observed data series and the shaded region shows the area between the 2.5% and 97.5% quantiles of the counterfactual posterior distribution with the median plotted in black. The shading is green, consistent with the color used for the monetary policy equation in the other Figures. The distributions of all the counterfactuals are significantly below their respective data series, illustrating that the significantly positive influence of the demand equation in the macroeconomy.

5 Extending the sample to 2025

In this section we update the analysis of Baumeister and Hamilton (2018) by extending the sample to 2025(Q2) which allows for comparison with other recent structrual VAR analyses of inflation dynamics, including Berholt et (2025) and Gagliardone and Gertler (2024) for the US economy and Brignone and Piffer (2025a) for the UK economy. As in section 4.2 we

Figure 7: Posterior distribution of structural constants with and without additional priors from replication of Baumeister and Hamilton (2018)

(a) Without Priors over Structural Constants



(b) With Priors over Structural Constants

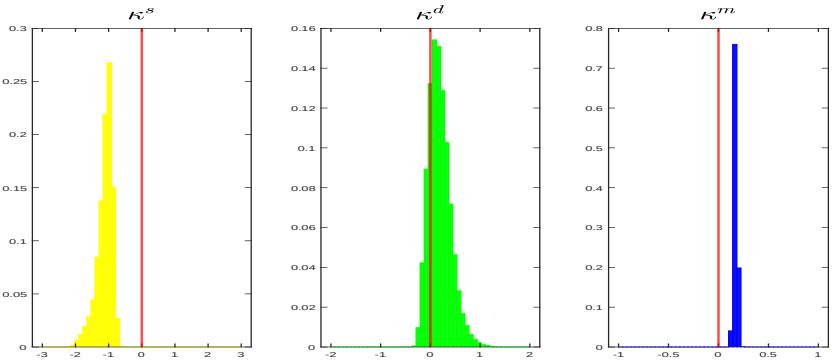


Figure 7 plots the posterior distribution of the structural constants for the case without additional priors over the structural constants, panel (a), and with an additional prior over the structural constants as described in section 4.3, panel (b).

will compare the historical decompositions for the cases where additional priors are, and are not, imposed on the structural constants.

Figure 7 shows the posterior distributions of the structural constants for the two cases considered. There are many similarities to Figure 3 for the original sample period. Figure 7a shows that without imposing the additional priors, the structural constant of the monetary policy equation has significant portions that are negative and positive. The mode this time being slightly positive. The structural constant for the supply equation is negative which is intuitive and that for the demand equation has a significant portions both positive and negative. When the prior that $\kappa^m = 0.5$ is imposed then Figure 7b shows that the structural constants now take the sign that one would expect, with κ^m, positive, κ^s negative and κ^d mostly positive

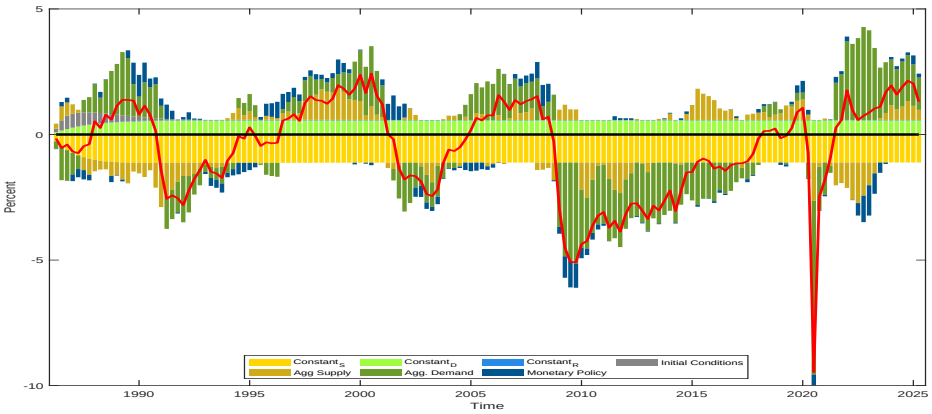
Figure 8 shows the historical decomposions for the extended sample without imposing priors over the structural constants. The picture tells a similar story to Figure 2 for the original sample, only in this case there is a small effect of the monetary policy equation on

the interest rate but again no noticeable effect of monetary policy on inflation or output. When priors are imposed over the structural constants, Figure 9 shows, similarly to Figure 4, that now the monetary policy equation plays a much bigger role in determining the interest rate and has a negative effect on inflation, although this is now very small indeed. The counterintuitive effect on output is again present. The smaller effects are perhaps not surprising in this extended sample, where interest rates were very close to the zero lower bound for a significant part of the sample and so unrelated, in the data, to changes in inflation. Thus in further work we should investigate using, say longer maturity interest rates, as well as a large scale VAR, to explore whether these model can explain the data without these counter-intuitive effects.

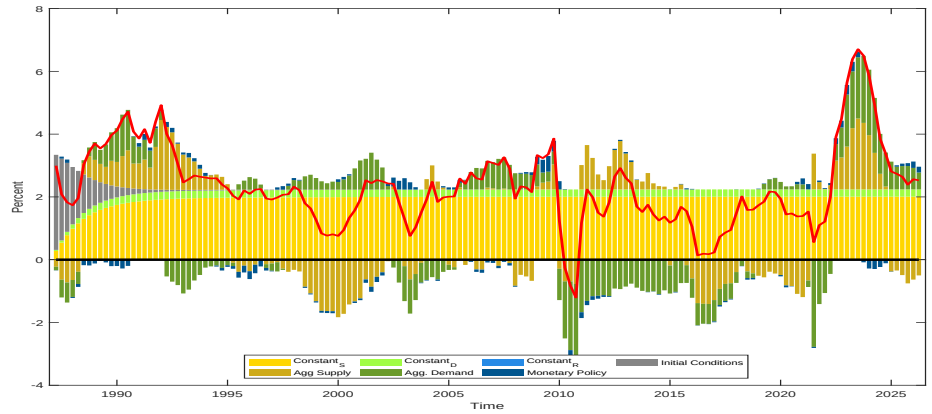
Nevertheless, the results are very comparable with the analysis of Giannone and Primiceri (2024) and Bergholt et al (2025) on the US economy, where demand and supply effects are modeled as shocks with a positive and negative correlation between output and inflation respectively. They identify these effects using the sign restriction identification strategy, of Canova and De Nicolò, (2002) and Uhlig (2005). They both find that demand and supply effects are complementary for the recent post crisis inflation and Giannone and Primiceri (2024) also find that they are offsetting for recent GDP growth, as we also find in Figures 8 and 9. Similar results have also been found for the UK economy by Brignone and Piffer (2025a) who, like Mumtaz and Surico (2009), also identify foreign demand effects. They find that world demand and world energy, i.e. supply, are complementary for historical decompsitions of UK inflation and offsetting for UK growth.

Figure 8: Historical decomposition of growth of Output Gap, Inflation and Interest Rate including decomposing the constants from replicating Baumeister and Hamilton (2018)

(a) Output gap



(b) Inflation



(c) Interest Rate

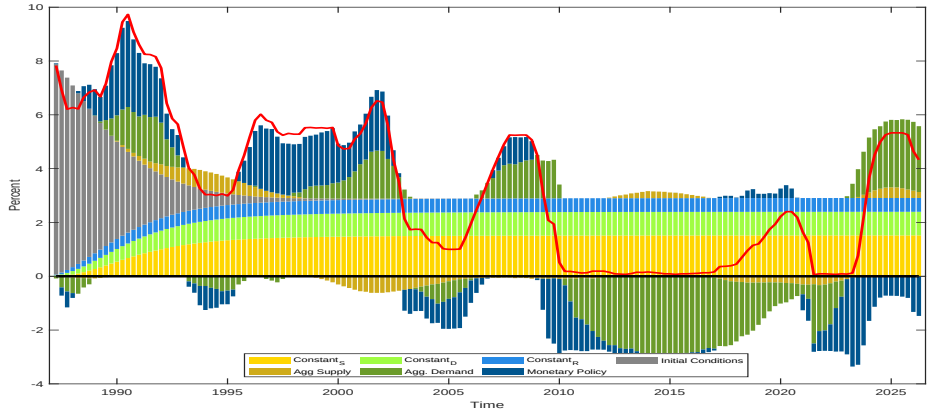
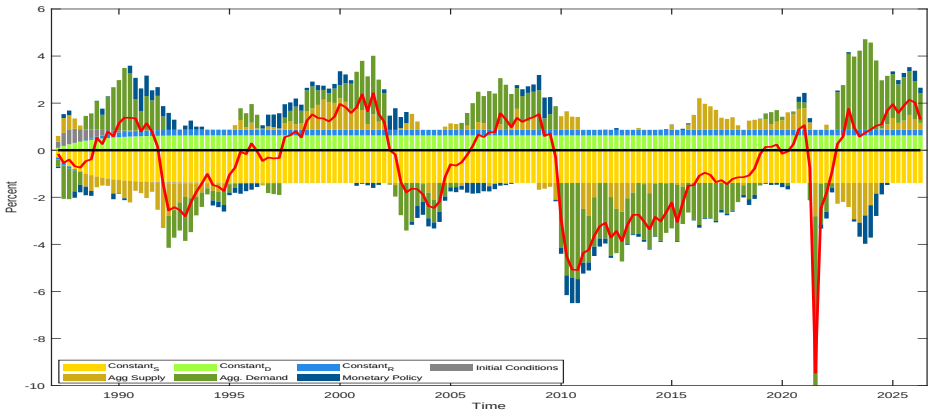


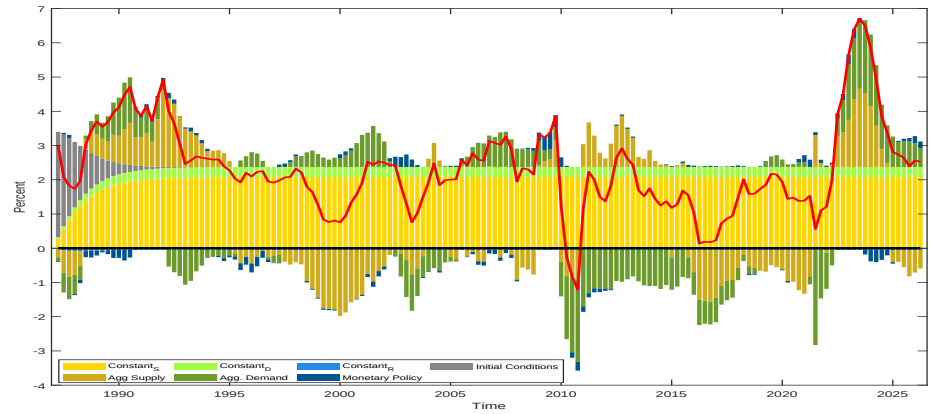
Figure 8 plots the median estimates of the historical decomposition of the output gap, inflation, and the interest rate from the update sample, where the constant terms are also decomposed into the parts associated with their structural equations. The yellow bars are the shocks and structural constant from the supply equation, the green bars the same from the demand equation and the blue bars the same from the monetary policy equation. The constant terms are given a lighter shade. In each panel the red line plots observed series.

Figure 9: Historical decomposition of growth of Output Gap, Inflation and Interest Rate after imposing prior on the structural constants

(a) Output gap



(b) Inflation



(c) Interest Rate

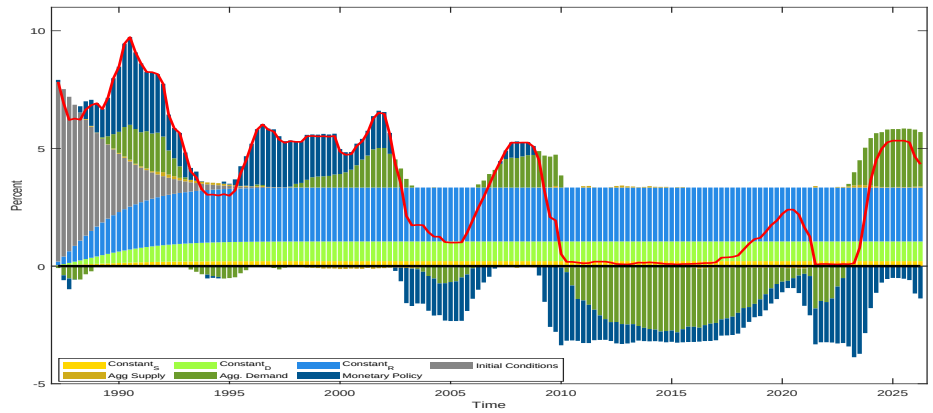


Figure 9 plots the median estimates of the historical decomposition of the output gap, inflation, and the interest rate from the updates sample with an additional prior over the structural constants as described in section 4.3. As with Figure 2 the constant terms are also decomposed into the parts associated with their structural equations. The yellow bars are the shocks and structural constant from the supply equation, the green bars the same from the demand equation and the blue bars the same from the monetary policy equation. The constant terms are given a lighter shade. In each panel the red line plots observed series.

6 Conclusion

In this paper we have demonstrated the importance of modeling the structural constant terms in a structural VAR. Structural constants can have clear economic interpretations and are subject to the same identification transformation as the fundamental shocks. Identification and estimation procedures should therefore take account of these effects. We have illustrated this using a standard 3 variable VAR that has been used to estimate the effects of monetary policy. We showed that imposing priors over the structural constant terms led to a more intuitive estimated monetary policy rule which gives a larger role for monetary policy in describing the evolution of the data, particularly for inflation.

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