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Multibrand Price Dispersion

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Abstract

We study a market in which firms each might supply a number of variants, or “brands”, of fundamentally the same product. Consumers differ in the sets of brands they consider, and firms compete using (multi-dimensional) mixed pricing strategies. We show when firms apply uniform pricing across their brands, and when they use segmented pricing so that one “discount” brand is priced below another “premium” brand. We study the case of symmetric brands in particular, and discuss the impact of a firm introducing a new brand, of imposing a requirement to set uniform prices across brands, and of mergers between firms.

Keywords: Price dispersion, price discrimination, multiproduct firms, mixed strategies, oligopoly, multibranding, multi-channel selling.

1 Introduction

Consumers looking to purchase an item often consider only a restricted subset of options from among all those available in the relevant market. For example, as in Varian’s (1980) classic paper, while local diners may be aware of all restaurants in the area, tourists might just visit a single random restaurant. In models of these situations Bertrand equilibrium involves mixed strategies and price dispersion, as different firms choose different prices. Firms trade off the incentive to set high prices to their captive consumers and low prices to attract the pool of price-sensitive consumers. Armstrong and Vickers (2022) provide an analysis of price competition in this kind of situation, and an account of the rich literature about how patterns of consumer consideration determine patterns of price competition.

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These models assume that each firm supplies a single product. That is, in the context of Varian’s model each restaurant is separately owned. In some situations, though, a firm might supply several similar products, or might supply a single product through different sales channels. Thus, a number of restaurants in the area might be jointly owned, and this will affect incentives to choose the prices in these restaurants. For instance, the owner might use one restaurant to set a high price to exploit captive tourists while another might be used to compete for local diners.

In this paper we study a framework in which firms might each supply several similar products. Consumers wish to buy a single item, and differ in the set of products they consider for purchase. There are a number of familiar reasons why consumers have different consideration sets. Some consumers might be better informed about market options than others, for instance because they are “local” (as in Varian’s model) or because a prior stage of advertising or consumer search means that some consumers are aware of a different set of brands than other consumers. Different consumers might have different physical outlets located near to them, and consumers might also differ in their willingness to use an online sales channel. There might be horizontal brand preferences, such that only a subset of products meet a consumer’s perceived needs or tastes.

In many markets distinct brands are jointly owned, a practice sometimes known as “multibranding”. For example, the Volkswagen group currently supplies a number of distinct car brands (such as Skoda and SEAT as well as the Volkswagen brand itself), and cars from different brands have underlying mechanical similarities. Several brands of premium ice cream are jointly owned, as are different brands of washing powder, car insurance, cigarettes, trainers, pet food, dating platforms, and beer.¹ Sometimes, a firm might even retail what is the *same* product via different brands—e.g., a garment which differs only by its brand label—and at different prices.² Similarly, a pharmaceutical company might supply both branded and more generic versions of essentially the same drug at the same time, at

¹For instance, Heineken currently owns more than three hundred brands of beer. McDonald and Wren (2018) study a comparison website for car insurance, and observe that of the 83 brands advertised on the site just 21 were single-brand firms, while the remaining 62 brands were supplied by 16 multibrand firms. See Table 1 in Grubb and Westphal (2025) for further evidence of multibranding.

²For instance, the UK clothing manufacturer Boohoo sells its products under several brands as well as under its own label. A number of media stories suggest that sometimes the same garment is retailed with different brand labels at very different prices—see for instance the report <https://www.bbc.co.uk/news/business-56653060> which suggests that the same coat was selling for £55 under one brand and £89 under another.

different prices.³ In these various markets, some consumers are loyal (“captive”) to one brand or another while others might be more willing to consider buying any of several brands. In the pharmaceutical context, for instance, some patients would only consider taking the branded drug, while others would be equally happy with a generic. In the case of soft drinks, some consumers will only consider the diet version, while others do not care.

Another situation where our model applies is when a firm supplies the same product but through different sales channels—e.g., through bricks-and-mortar stores in different locations, or through a bricks-and-mortar channel and an online channel—with potentially different prices at each outlet. Different outlets might be more convenient for different groups of consumers, and might compete with different sets of outlets from rival sellers too. Shoppers on the high street, say, might enter individual stores somewhat randomly, while online shoppers might be more inclined to check prices of multiple suppliers before purchase. One further situation covered by our analysis is when a firm supplies a product with distinct attributes, where some consumers care about these attributes while others do not. For instance, some travellers care about whether they have an aisle or a window seat on a plane, while others simply want the cheapest seat.

In this paper we present a model in which a firm might supply a number of similar products, or, as we refer to them henceforth, *brands*. In order to focus as sharply as possible on pricing incentives arising from multibranding, we assume that these are variants of fundamentally the same product. Consumers differ in the set of brands they consider, and wish to buy the cheapest brand from among the brands they consider. Just as with models with single-brand firms, except in extreme cases, equilibrium between multiple firms involves firms choosing their price(s) according to mixed strategies, so there is price dispersion across firms. However, in contrast to the single-brand case, a firm might set different prices across its brands, in which case there is also *intra-firm* price dispersion.

We begin our analysis in section 2 with a single firm that supplies two brands. This is of interest in its own right, and we also use the analysis as an important ingredient when we study competition between firms. We find that the incentive to set the same price or distinct prices for the two brands depends on how the elasticity of demand for a low-priced brand compares with the elasticity of a more-expensive brand. The firm prefers to offer a uniform price when—if it offers a low price for one brand and a higher price for

³In such cases the generic is usually termed an “authorized generic” — see Gulsen Oner and Polli (2018) for details.

the other—its low-price demand is less elastic than high-price demand. This situation can occur when demands for the two brands are asymmetric, rather than being a knife-edge occurrence when elasticities are precisely equal. By contrast, the firm prefers to set distinct prices when demand for its cheaper brand is *more* elastic, and this can happen even when the two brands are symmetric.

In section 3 we study the case where there are two competing firms, one of which supplies one brand and the other supplies two. This situation can be fully solved and, except in cases of exact ties in the relevant elasticities, the equilibrium is unique. Rich patterns of pricing emerge even in this simple case. In qualitative terms, three pricing patterns for the multibrand firm can exist in equilibrium: uniform pricing (where prices for the two brands are exactly equal), “segmented” pricing (in which the price for one brand is below any price used for the other brand), and “disjoint” pricing (when one brand has intermediate prices while the price for the other is either low or high). These various pricing patterns resemble those seen in real markets. Uniform pricing can be interpreted as a firm offering price promotions in a coordinated way across its product line, segmented pricing corresponds to a situation in which one (“fighting” or “discount”) brand is systematically cheaper than another brand supplied by the firm, while disjoint pricing corresponds to having an “everyday value” brand alongside a more expensive brand that has periodic sales.⁴

In section 4 we study an alternative model in which brands are symmetric, in the sense they are equally likely to be considered by a consumer. This pattern of brand consideration is similar to that of “noisy search” in Burdett and Judd (1983, section 3.3), and it includes as a special case the Varian (1980) pattern of consideration mentioned above. This section generalizes Burdett and Judd’s analysis to allow firms to supply multiple brands. In contrast with section 3, there might be more than two firms in the market, there might be several multibrand firms, and a firm might supply more than two brands. We focus on two situations, one where relevant demand ratios are increasing, when multibrand firms use segmented pricing, and the other where these ratios decrease, when firms use uniform pricing. Whether these demand ratios increase or decrease corresponds to whether a firm’s cheaper brands have more or less elastic demand than its more expensive brands.

In section 5, we use this analysis to discuss applications of the multibrand framework.

⁴In marketing parlance, these two pricing strategies are sometimes referred to respectively as *everyday low pricing* (or EDLP) and *high-low pricing*.

We discuss how a firm has an incentive to introduce a new brand, both because this could facilitate price discrimination in the form of segmented pricing and as a means by which to deter entry. We discuss the impact of restricting a firm’s ability to set different prices across its brands, and show how a firm might wish to commit to use uniform pricing to relax competition. We also show that the impact on consumer surplus of an industry-wide requirement to use uniform pricing depends on the pattern of ownership, and such a policy is more likely to benefit consumers when the market is more symmetric. We also discuss the impact of mergers in these markets, and show that when firms use segmented pricing with symmetric brands, any profitable merger will reduce consumer surplus. Section 6 concludes.

Related literature: Our monopoly analysis in section 2 is in the tradition of Salop’s (1977) model of intra-firm price dispersion, in which a monopolist that supplies a single product might offer that product through different sales channels at different prices in order to be able to engage in price discrimination. In his model, consumers need to search to find a low price from the firm, and if consumers with relatively high search costs are also willing to pay more for the product, it is profitable for the firm to offer different prices in different channels.

In our oligopoly models we use the consideration set framework in Armstrong and Vickers (2022), modified so that firms might supply more than one brand. That paper aimed to provide a unifying framework for the prior literature that focussed on specific patterns of consumer consideration, and we proposed a family of consideration patterns—“symmetric interactions”—which included previous models such as Burdett and Judd (1983), Narasimhan (1988) and Varian (1980) as special cases. Outside this family, we showed how the pattern of price competition might take novel forms, notably a “segmented” form of competition where only two firms competed within a given price range. This pricing pattern also plays a major role in the current paper.

Other papers have examined pricing by multiproduct firms when consumers differ in their consideration sets, where firms use multi-dimensional mixed strategies to price *complementary* goods, such as a washing machine and a drier. See McAfee (1995), Shelegia (2012), Sinitsyn (2012), Zhou (2014) and Kaplan *et al.* (2019) for models in this spirit. For example, in the version of Shelegia’s model with perfect complements, all consumers value

the product combination equally but differ in terms of which retailer they can buy from: some (captive) consumers must buy both elements from a given retailer while others can mix and match across retailers. Equilibrium often exhibits negative correlation between the two prices at a retailer—the sum of prices is kept high to exploit the captives, while one (random) price is set low to attract non-captives for that product. Thus, even if products are symmetric, they may be sold at distinct prices. Sinitsyn (2012), in a somewhat different framework in which the non-captives gain extra utility if they buy both elements with the same brand name from the same retailer, often finds price promotions to be positively correlated in equilibrium. He also describes empirical evidence that complementary products with the same brand name (such as shampoo and conditioner, or cake mix and cake frosting) often have price promotions being offered at the same time from a given manufacturer.

Sinitsyn (2016) studies a model closer to ours, in that firms supply multiple substitute products. His model has two firms, each of which supplies two products, and consumers wish to buy just one of the four products. Some consumers are captive to a product, while the remainder are willing to buy any of the four products, with horizontal differentiation between products modelled in logit fashion, with different substitutabilities for products within a firm and across firms. His analysis, which is done numerically, shows that when intra-firm brand substitution is strong (relative to inter-firm brand substitution), firms in equilibrium offer price promotions for only one product at a time.⁵

Perhaps the paper most closely related to ours is Ireland (2007), who studies a model with symmetric brands. He assumes a pattern of consideration such that a fraction of consumers consider one random brand and the remaining consumers consider a random *pair* of brands. He studies ownership structures whereby all brands are separately owned, where just one pair of brands is jointly owned, and where all sellers own a pair of brands. He shows that this pattern of consideration leads to multibrand firms using uniform pricing. We extend this analysis to cover all ownership structures, and show how his pattern of consideration fits into a broader class of demand systems.

⁵Sinitsyn (2016, Table 1) shows how a supplier with several brands of biscuits rarely offers a price promotion for more than one brand in a given week. Sinitsyn (2020) studies a model of a merger between single-brand firms, within a similar framework where some consumers are captive to a brand while the remainder have logit preferences over all three brands. This demand system implies that after the merger the two-brand firm sets the price of one brand at a high level and competes against the rival using its other brand.

Our model makes two important simplifying assumptions. First, consideration sets are exogenous and a consumer does not search for further brands if she is unsatisfied with her existing options. If consumer search is modelled explicitly, the issue arises whether a consumer knows *which* brands are jointly owned and can thereby direct her search towards (or away from) multibrand sellers. Janssen and Moraga-González (2007) study a model in which a consumer knows which brands are jointly owned. The search technology is such that a two-brand seller uses segmented pricing, with one of its prices set at the consumer reservation price, and this discourages consumers from investigating either of the merged entity’s brands. The merged firm then has an incentive to remove one brand from the market. Alternatively, Ireland (2007, section IV) extends his basic model to allow consumers to choose in advance whether or not to see two brands or one. As discussed, in his setting a two-brand firm uses uniform pricing. Consumers understand that pairs of brands are jointly owned, but not which brands are jointly owned. The fact that pairs of prices are perfectly correlated then affects a consumer’s incentive to observe two prices. Finally, Grubb and Westphal (2024) consider search incentives in a three brand framework, both when consumers know that two brands are jointly owned (but not which two) and when consumers mistakenly believe that all brands set prices independently.

The second simplifying assumption is that we abstract away from quality differences, and firms supply essentially the same product under different branding or through different sales channels. By contrast, a rich literature studies how firms compete by offering a product line with distinct qualities, where lower-quality products are often designed to appeal to more price-sensitive consumers. For instance, Johnson and Myatt (2003) discuss a model in which a firm might react to new competition either by introducing a new low-quality “fighting brand” or by retreating to higher-quality products by pruning its low-quality products. Despite abstracting from these important issues, our parsimonious framework nevertheless yields rich patterns of pricing.

2 Monopoly with two brands

In this section we study optimal pricing by a two-brand monopolist. While this has some independent interest, we will use this analysis mostly as an ingredient for subsequent oligopoly analysis. Since that analysis involves mixed pricing strategies, where firms are indifferent over a set of optimal prices, in this section we pay special attention to situations

in which the monopoly firm has multiple optimal prices.

In more detail, suppose that a monopolist supplies two substitute brands, 1 and 2. Some consumers are aware of (or care about) only brand $i = 1, 2$, and have continuous decreasing demand $x_i(p_i)$, while other consumers are willing or able to buy either brand and will buy the cheaper of the two, with continuous decreasing demand $x_{12}(\min\{p_1, p_2\})$. Unit cost is the same for each product, which implies that the firm does not mind which brand the “non-choosy” consumers choose to buy with uniform pricing, and for simplicity we set this cost to zero. We wish to understand which price pairs can be optimal for the firm.

Define

$$L_i(p) \equiv x_i(p) + x_{12}(p) , \quad H_i(p) \equiv x_i(p)$$

to be respectively the demand for brand i when that brand has the lower price and when it has the higher price. Therefore, when $p_i \leq p_j$ the firm’s total profit takes the additively separable form $p_i L_i(p_i) + p_j H_j(p_j)$. Since $L_1(p) + H_2(p) \equiv L_2(p) + H_1(p)$, whenever the two prices are equal it makes no difference which brand is notionally taken to have the lower price in this profit expression, and profit is continuous in (p_1, p_2) . It is useful to introduce notation for the following ratios:

$$z_1(p) = \frac{H_2(p)}{L_1(p)} ; \quad z_2(p) = \frac{H_1(p)}{L_2(p)} ; \quad l(p) = \frac{L_2(p)}{L_1(p)} \text{ and } h(p) = \frac{H_2(p)}{H_1(p)} . \quad (1)$$

The firm’s incentive to choose distinct prices for its two brands depends on whether or not these functions are increasing or decreasing in p , i.e., (in the smooth case) on the relative elasticities of the four functions L_1 , L_2 , H_1 and H_2 . A key insight is the following result. (See the appendix for this and subsequent proofs.)

Lemma 1 *If the firm optimally chooses a price pair (p_1, p_2) such that $p_1 > p_2$, then (i) $z_2(p_1) \geq z_2(p_2)$ and (ii) either $l(p_2) \geq l(p_1)$ or $h(p_2) \geq h(p_1)$ (or both).*

This result has the following immediate corollary:

Corollary 1 *If (i) z_2 is strictly decreasing or (ii) both l and h are strictly increasing, then only price pairs such that $p_1 \leq p_2$ can be optimal.*

In particular, if z_2 decreases then brand 1 cannot be the more expensive brand. Clearly, a parallel result holds for z_1 decreasing. As another corollary, we see that if both z_1 and

z_2 are strictly decreasing then only uniform prices, i.e., price pairs such that $p_1 = p_2$, can be optimal, as reported here:

Corollary 2 *If z_1 and z_2 are both strictly decreasing then optimal price pairs are uniform.*

For instance, if the two brands are symmetric (so that $x_1 \equiv x_2$), then $z_1 \equiv z_2$ is decreasing if the non-choosy demand x_{12} is less elastic than the choosy demand $x_1 = x_2$, in which case only uniform prices can be optimal. More generally, Corollary 2 applies in wide variety of situations, and the incentive to set a uniform price is not a knife-edge result that applies only when the two brands are symmetric. It applies when both L_1 is less elastic than H_2 and L_2 is less elastic than H_1 .⁶

Corollary 1 provided conditions ensuring that all optimal price pairs lie in the region $p_1 \leq p_2$. Within this region, the firm's profit is $p_1 L_1(p_1) + p_2 H_2(p_2)$. If demand L_1 is more elastic than H_2 , i.e., if z_1 is strictly increasing, then if p_1 maximizes $pL_1(p)$ and p_2 maximizes $pH_2(p)$ then $p_1 \leq p_2$ and the constraint is automatically satisfied. The set of optimal price pairs (p_1, p_2) is then the "rectangle" consisting of Cartesian product of the set of prices p_1 that maximize $pL_1(p)$ and the set of prices p_2 that maximize $pH_2(p)$.

This discussion is summarised in the following result:

Lemma 2 *If z_1 is strictly increasing then for optimal price pairs within the region $p_1 \leq p_2$ every optimal brand 1 price is weakly below every optimal brand 2 price. In particular, there can be at most one uniform price pair that is optimal.*

It is possible that the set of optimal price pairs includes pairs with $p_1 < p_2$ and pairs with $p_1 > p_2$. For instance, if the two brands are symmetric, then $z_1 \equiv z_2$ is increasing if the non-choosy demand x_{12} is more elastic than the choosy demand $x_1 = x_2$. Then the set of optimal price pairs consists of a low price that maximizes $pL(p)$ and a high price that maximizes $pH(p)$, and it makes no difference to profits which brand has the lower price. In such cases, it is optimal to set different prices for symmetric brands.⁷

When the conditions for Corollary 1 hold, so that only price pairs with $p_1 \leq p_2$ can be optimal, then Lemma 2 implies that when z_1 is increasing any optimal price p_1 used by the

⁶For example, suppose $x_1 = 1 - p$, $x_2 = (1 - p)^2$, and $x_{12} \equiv 1$ for $p \leq 1$ (and $x_{12} = 0$ for $p > 1$). Then both z_1 and z_2 are strictly decreasing, and only a uniform price can be optimal.

⁷For instance, if $x_1 = x_2 = 1$ for all $p \leq 1$ (and $x_1 = x_2 = 0$ for higher prices) and $x_{12} = \frac{1-p}{p}$, then the set of optimal price pairs consists of those (p_1, p_2) such that $\max\{p_1, p_2\} = 1$.

firm is weakly below any optimal price p_2 . We refer to this pricing pattern as *segmented pricing*. (See Figure 2 below for illustrations of uniform pricing and segmented pricing.) In this situation, unlike the situation with uniform pricing, one can assign a profit to each brand, and brand 1 generates profit equal to the maximized value of $pL_1(p)$ and brand 2 generates profit equal to the maximized value of $pH_2(p)$.

Corollary 1 provided two separate conditions ensuring $p_1 \leq p_2$ is optimal. However, when z_1 increases the first of these conditions implies the second.⁸ So only part (ii) of the Corollary is relevant when z_1 is increasing, and we have the following result:

Corollary 3 *If z_1 , l and h all strictly increase, then segmented pricing is optimal, i.e., any price p_1 used by the firm for brand 1 is weakly below any price p_2 it uses for brand 2.*

Suppose that l is a strictly increasing function, so that brand 1's demand when it is the cheaper brand is more elastic than brand 2 when that brand is cheaper. From the previous footnote, in this case if z_1 decreases then so does z_2 , in which case uniform pricing is optimal. On the other hand, if z_1 increases then if h also increases there is segmented pricing, with prices offered for brand 1 being below all prices offered for brand 2. Among situations where the functions (1) are monotonic, therefore, the remaining case is when z_1 increases and h decreases (which then implies that z_2 increases).

This final case again allows the possibility that the set of optimal price pairs includes pairs with $p_1 < p_2$ and pairs with $p_1 > p_2$. Since z_1 is increasing, the most profitable price pairs within the constrained upper region $p_1 \leq p_2$ consists of the rectangle equal to the Cartesian product of the set of prices p_1 that maximize $pL_1(p)$ and the set of prices p_2 that maximize $pH_2(p)$. Let p_L^{\min} and p_L^{\max} denote respectively the smallest and the highest lower price used (i.e., the price p_1 for brand 1) in this upper region, and similarly let p_H^{\min} and p_H^{\max} denote the smallest and highest higher price used (i.e., p_2) in this region. Since z_2 is also increasing, the set of optimal price pairs within the constrained lower region $p_1 \geq p_2$ consists of the rectangle equal to the Cartesian product of the set of prices p_1 that maximize $pH_1(p)$ and the set of prices p_2 that maximize $pL_2(p)$. Let P_L^{\min} and P_L^{\max} denote respectively the smallest and highest lower price used (i.e., p_2) in the lower region, and let P_H^{\min} and P_H^{\max} denote the smallest and the highest higher price (i.e., p_1) used in the lower

⁸To see this, suppose that z_1 is increasing. Note that l can be written as $l = (1 + z_1)/(1 + z_2)$. (This uses the equality $L_1 + H_2 = L_2 + H_1$.) Therefore, if z_2 is decreasing then l is increasing. Likewise, we can write $1/h = (1 + 1/z_1)/(1 + 1/z_2)$, and so if z_2 decreases then h also increases.

region. Clearly, the most profitable price pairs across both regions consists either of the first rectangle or the second, depending on which of them yields the higher profit, or of both rectangles if the two profits coincide.

Any optimal lower price in the upper region maximizes $pL_1(p)$ and any optimal lower price in the lower region maximizes $pL_2(p)$. Since l is increasing, it follows that any such price in the lower region weakly exceeds that in the upper region, so that

$$p_L^{\min} \leq p_L^{\max} \leq P_L^{\min} \leq P_L^{\max} .$$

Likewise, since h is decreasing we have

$$P_H^{\min} \leq P_H^{\max} \leq p_H^{\min} \leq p_H^{\max} .$$

Since z_2 is increasing, any price that maximizes $pH_1(p)$ is weakly greater than any price that maximizes $pL_2(p)$, and so $P_L^{\max} \leq P_H^{\min}$, in which case

$$p_L^{\min} \leq p_L^{\max} \leq P_L^{\min} \leq P_L^{\max} \leq P_H^{\min} \leq P_H^{\max} \leq p_H^{\min} \leq p_H^{\max} . \quad (2)$$

In particular, over these two rectangles every higher price is above every lower price, which is a kind of generalized “segmented pricing”. See Figure 3 below for one depiction of the arrangement of these two rectangles. Finally, observe that if a uniform price pair (p, p) is optimal, it lies in both rectangles such that $p_L^{\max} = P_H^{\min} = p$, and (2) then implies that the set of optimal price pairs lies on the horizontal line $p_2 \equiv p$.

The monopoly results in this section will now be used to solve oligopoly models.

3 Duopoly with three brands

3.1 Preliminaries

Consider next the situation with three brands, $i = 1, 2$ and 3, where brands 1 and 2 are jointly owned by an integrated firm m while firm s supplies the single brand 3. Using similar notation as used in Armstrong and Vickers (2022), for each subset of brands $B \subset \{1, 2, 3\}$ a specified number of consumers α_B have the consideration set B , as depicted on Figure 1.

Let σ_i denote the *reach* of brand $i = 1, 2, 3$, which is the number of consumers who consider buying this brand. (So $\sigma_1 = \alpha_1 + \alpha_{12} + \alpha_{13} + \alpha_{123}$, say.) The number of consumers who are *captive* to brand i , i.e., who consider only that brand for their purchase, is α_i , and

let $\rho_i = \alpha_i/\sigma_i$ denote the *captive-to-reach ratio* of brand i . Since brands 1 and 2 are jointly owned by m , it is also convenient to define

$$\alpha_m = \alpha_1 + \alpha_2 + \alpha_{12} ; \sigma_m = \sigma_1 + \sigma_2 - (\alpha_{12} + \alpha_{123}) ; \rho_m = \frac{\alpha_m}{\sigma_m}$$

respectively for the fraction of consumers who are captive to firm m as whole, the fraction reached by firm m as a whole, and for firm m 's captive-to-reach ratio. We suppose that each of m 's brands contributes something to its overall reach, i.e., that $\sigma_i < \sigma_m$ for $i = 1, 2$.

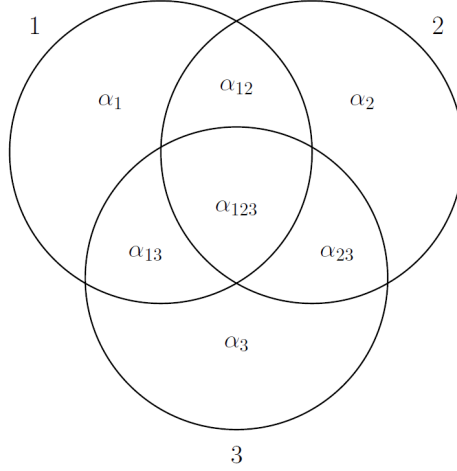


Figure 1: The structure of consumer choice sets

As in Armstrong and Vickers (2022), suppose that a consumer is willing to pay up to 1 for any product, and buys the cheapest product she considers provided that that lowest price is no greater than 1.⁹ We write

$$\rho_i^L = \frac{\alpha_i + \alpha_{12}}{\sigma_i} \text{ and } \rho_i^H = \frac{\alpha_i}{\sigma_m - \sigma_j}$$

to be respectively the the captive-to-reach ratio of brand $i = 1, 2$ given that it is the cheaper of firm m 's two brands and given that it is the more expensive of its two brands. Without loss of generality, label the two brands so that $\rho_1^L \leq \rho_2^L$.

Firm m 's overall captive-to-reach ratio ρ_m is an average of that of its two brands in the sense that

$$\frac{\sigma_1}{\sigma_m} \rho_1^L + \frac{\sigma_m - \sigma_1}{\sigma_m} \rho_2^H = \frac{\sigma_2}{\sigma_m} \rho_2^L + \frac{\sigma_m - \sigma_2}{\sigma_m} \rho_1^H = \rho_m . \quad (3)$$

⁹As in our earlier paper, the equilibrium analysis that follows is not affected if each consumer has a downward-sloping demand function $x(p)$ for each brand, provided revenue $px(p)$ is an increasing function up to the monopoly price. However, welfare would then no longer depend just on the total number of consumers served but also on the price they pay, and our welfare analysis (e.g., in our discussion of mergers and brand proliferation) requires adjustment with downward-sloping demand.

In particular, $\rho_i^L \leq \rho_j^H$ if and only if $\rho_i^L \leq \rho_m$. Moreover, we have

$$\rho_m \geq \frac{\alpha_1 + \alpha_2}{\sigma_1 + \sigma_2} = \frac{\sigma_1}{\sigma_1 + \sigma_2} \rho_1 + \frac{\sigma_2}{\sigma_1 + \sigma_2} \rho_2 ,$$

where the inequality follows from $\alpha_m \geq \alpha_1 + \alpha_2$ and $\sigma_m \leq \sigma_1 + \sigma_2$. It follows that at least one individual brand has a weakly lower captive-to-reach ratio than that of the two brands combined.

Since industry profit is continuous in the three brand prices (p_1, p_2, p_3) , Theorem 5* in Dasgupta and Maskin (1986) can be used to show that a Bertrand equilibrium exists in this market.¹⁰ Except in extreme configurations (e.g., if the two firms do not overlap at all in their reaches), an equilibrium will exist only in mixed pricing strategies. Firm s will never choose a price below ρ_3 , as if it did it would have profit below α_3 , which is no greater than its profit when it chooses $p = 1$. Therefore, firm m will never choose a price below ρ_3 for either of its brands, and so the minimum price in the market for any brand cannot be below ρ_3 . Likewise, firm m will never choose a price below ρ_i for brand $i = 1, 2$. (If it did, it would make strictly more profit from this brand by choosing $p_i = 1$, and this would also boost the demand and profit for its other brand.) However, as will be seen, firm m might choose a price below ρ_m for one of its brands while choosing a high price for the other brand. Finally, standard arguments (see for instance Lemma 1 in Armstrong and Vickers (2022)) show that in equilibrium neither firm can choose a particular price $p < 1$ with positive probability and neither firm can have a gap in the set of prices they offer.¹¹

Suppose that in some equilibrium firm s chooses its single price according to the CDF $G(p)$. The support of G is an interval $[P_0, 1]$, where P_0 is also the minimum price used by m , and G is continuous and strictly increasing on $[P_0, 1)$. By contrast, firm m supplies two brands and so its price support lies within the square $[P_0, 1]^2$. It follows a strategy of *uniform pricing* if it always chooses the same price for the two brands, in which case

¹⁰This result can be invoked by making the following definitions. Firm m 's strategy set is $A_1 = [0, 1]^2$ and firm s 's strategy set is $A_2 = \{(p, p) \mid 0 \leq p \leq 1\}$, both of which are compact, convex subsets of \mathbb{R}^2 . (Firm s 's profit is a function only of a single price, p , as it only supplies a single brand.) If the price pair used by m is denoted (p_1, p_2) and the price "pair" used by s is denoted $(\tilde{p}_1, \tilde{p}_2)$, then the profit of either firm is only discontinuous at strategies in $A_1 \times A_2$ such that $p_1 = \tilde{p}_1$ or $p_2 = \tilde{p}_2$, which satisfies the requirements of Theorem 5*. Note that we have duplicated firm s 's single price p as (p, p) so that strategy sets lie in the same two-dimensional space for each firm and so that discontinuities occurs when the first component of the two strategies are equal and when the second component of the two strategies are equal, in which case we can satisfy assumption (A1) in their paper.

¹¹For firm m , this means that the set of prices it might offer for one brand or the other can have no gaps, although, as we will see, it might be that one of its brands uses low and high prices but not intermediate prices.

its price support lies on the diagonal $p_1 \equiv p_2$. (This is shown on the left-hand diagram on Figure 2, where firm m chooses its price pair on the bold diagonal line.) Alternatively, it might make one brand always cheaper than the other, so its price support lies to one side of this diagonal. One might then refer to the cheaper brand as a “discount brand”, although in our model there are no quality differences across brands. A particular way to make one brand cheaper is *segmented pricing*, where the price of one brand never exceeds the minimum price chosen for the other brand. When firm m engages in segmented pricing, firm s competes against one brand when it chooses a low price and competes against the other brand when it chooses a high price. (This is depicted on the right-hand diagram on Figure 2, where firm m chooses its pair of prices within the shaded rectangle.)

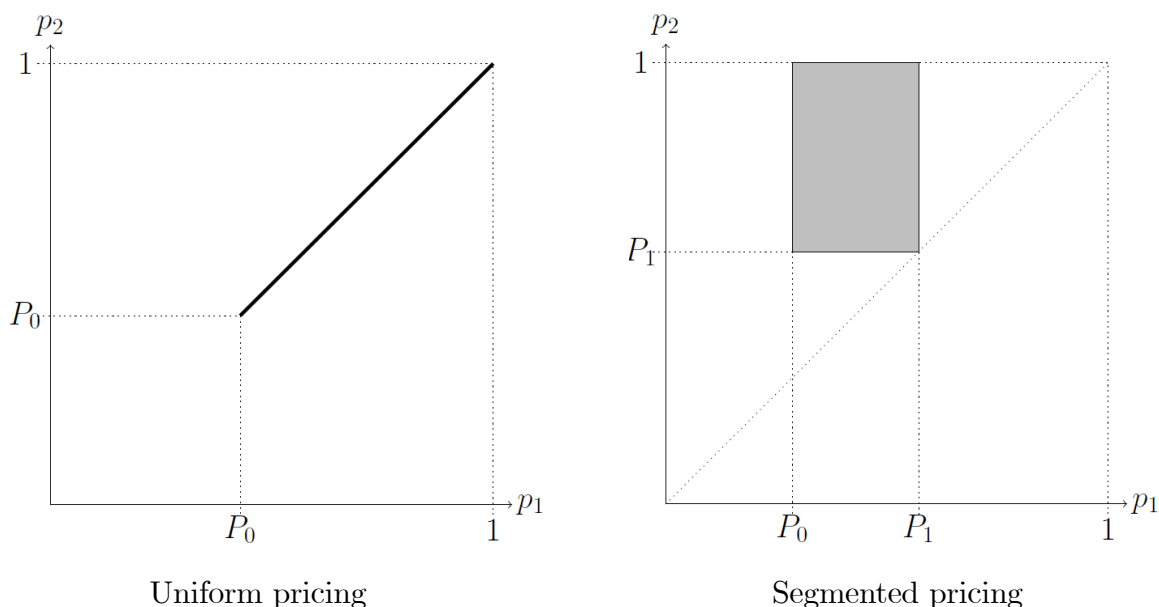


Figure 2: Pricing patterns for firm m

A third pricing pattern, which we term *disjoint pricing*, is illustrated on Figure 3. Here firm m chooses an intermediate price for one brand (brand 2 on the figure) and either a high price (above any price used for brand 2) or a low price (below any price used for brand 2) for its other brand. Clearly, with this strategy either brand might end up being the cheaper brand. With disjoint pricing, firm m chooses its prices to be negatively correlated: a low price for brand 2 is associated with a high price for brand 1 and *vice versa*. In the following analysis we show that in equilibrium firm m will choose one of the three patterns of pricing shown on these figures.

Given firm s 's price strategy $G(p)$, from Figure 1 one can see that demand for firm m 's

lower-priced brand i is

$$L_i(p) = \sigma_i[1 - (1 - \rho_i^L)G(p)] \quad (4)$$

while demand for its higher-price brand j is

$$H_j(p) = (\sigma_m - \sigma_i)[1 - (1 - \rho_j^H)G(p)] .$$

It follows that $z_i(p) \equiv H_j(p)/L_i(p)$ is decreasing if and only if $\rho_i^L \geq \rho_j^H$, i.e., if $\rho_i^L \geq \rho_m$ (and is strictly decreasing if the inequality is strict). Likewise, our standing assumption $\rho_1^L \leq \rho_2^L$ implies that $l(p) = L_2(p)/L_1(p)$ is increasing (and strictly increasing if the inequality is strict), while $h(p) = H_2(p)/H_1(p)$ strictly increases if $\rho_1^H < \rho_2^H$. In particular, the functions (1) are necessarily monotonic in this duopoly market.

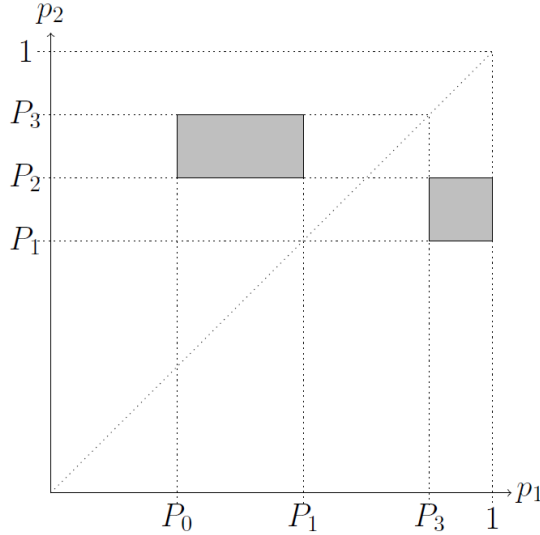


Figure 3: Disjoint pricing

3.2 Uniform pricing

Our earlier Corollary 2 immediately implies the following condition for firm m to use uniform pricing for its two brands.

Proposition 1 *It is the unique equilibrium for firm m to charge a uniform price for its two brands if*

$$\rho_1^L > \rho_m . \quad (5)$$

When firm m uses uniform pricing, the details of the equilibrium construction follows standard duopoly analysis, where the equilibrium has minimum price P_0 equal to the

higher of the two firms' captive-to-reach ratios and a firm's profit equals its reach times P_0 . That is, equilibrium profits for firm m and firm s are respectively $\sigma_m P_0$ and $\sigma_s P_0$, where $P_0 = \max\{\rho_m, \rho_s\}$ is the higher captive-to-reach ratio of the two firms.¹²

If (5) holds then $\rho_j \leq \rho_j^H < \rho_m$, and so both ρ_1 and ρ_2 are below ρ_m and the captive-to-reach ratio of each individual brand is lower than that of the brands combined. As discussed earlier, in general at least one ρ_i must be below ρ_m , although not necessarily both of them. Thus if one ρ_i exceeds ρ_m then uniform pricing by firm m cannot occur in equilibrium.

To illustrate, suppose that the three brands are symmetric. That is, following Burdett and Judd (1983), for each $1 \leq n \leq 3$ there are β_1 consumers who are captive to each of the three brands, β_2 consumers who consider each of the three pairs of brands, and β_3 consumers who consider all three brands. (This case of symmetric brands is discussed in detail in section 4.) Then from Figure 1 one sees that

$$\rho_1^L = \rho_2^L = \frac{\beta_1 + \beta_2}{\beta_1 + 2\beta_2 + \beta_3}, \quad \rho_1^H = \rho_2^H = \frac{\beta_1}{\beta_1 + \beta_2}.$$

Therefore, Proposition 1 applies if and only if

$$\beta_2^2 \geq \beta_1 \beta_3, \quad (6)$$

i.e., if there are enough consumers who consider two brands relative to the number of other consumers.¹³ The incentive of the two-brand firm to engage in uniform pricing is not systematically associated with consumers being more or less attentive or informed. Increasing the number of consumers who consider two brands at the expense of the number who are captive will tend to push the firm to use uniform pricing, but increasing the number of consumers who consider all three brands at the expense of those who consider two will tend to reduce this incentive.

3.3 Non-uniform pricing

Consider next situations with non-uniform pricing by firm m . Given the previous discussion, a straightforward situation where uniform pricing by firm m is not an equilibrium is

¹²See Narasimhan (1988) for this analysis.

¹³This is consistent with Ireland (2007), who finds that uniform pricing is used when brands are symmetric and consumers consider at most two brands (so that $\beta_3 = 0$). This is also consistent with the merger analysis in Grubb and Westphal (2024) which uses this symmetric three-brand framework, and as here they find that when (6) holds the post-merger firm will use uniform pricing.

when brands are symmetric and when $\beta_2 = 0$, in which case (6) fails. (This is the pattern of consideration in Varian (1980), where a consumer either considers all three brands or just one.) The equilibrium here is easily seen to have firm m choosing the price $p \equiv 1$ for one brand. For if m chooses prices $p_i \leq p_j < 1$ for its two brands, then regardless of its rival's strategy it can strictly increase its profit by increasing p_j to 1. Since brand j is undercut by the firm's other brand, it will sell only to customers who are captive to brand j , and in that case it does best to set the maximum price $p_j = 1$. Thus the firm will set one price equal to 1 and compete against its rival with the other price. In equilibrium the firm offers a discount on only one brand at a time.¹⁴ The equilibrium where brand 2, say, is priced deterministically at $p_2 \equiv 1$ is an extreme instance of a segmented pricing pattern illustrated on Figure 2.¹⁵

More generally, Corollary 3 implies the following result:

Proposition 2 *Suppose that $\rho_1^L < \rho_2^H$, $\rho_1^L < \rho_2^L$ and $\rho_1^H < \rho_2^H$. Then in equilibrium firm m 's prices are segmented in the sense that there are threshold prices $P_0 < P_1 \leq 1$ such that brand 1 uses prices in the interval $[P_0, P_1]$ and brand 2 uses prices in the interval $[P_1, 1]$.*

Under the conditions of Proposition 2, Corollary 3 implies there is segmented pricing in the sense that any price offered for brand 1 is below any price offered for brand 2. However, in the current duopoly context, one can say further that for some $P_1 \leq 1$ firm m chooses price in a range $[P_0, P_1]$ for brand 1 and price in a range $[P_1, 1]$ for brand 2, so that the maximum p_1 is equal to the minimum p_2 . (The right-hand panel in Figure 2 depicts this situation.) For if the maximum p_1 were strictly below the minimum p_2 then firm m would have a gap in the range of prices it offered, and this cannot occur in equilibrium. In the appendix we derive expressions for the threshold prices, P_0 and P_1 , as well as the profits of the two firms. Note that, while profits and consumer surplus are uniquely determined when Proposition 2 applies, there remains some indeterminacy about the details of firm m 's pricing strategy. With segmented pricing as in Figure 2, all that matters are the *marginal* distributions for p_1 and p_2 , and any correlation between these prices has no impact. In

¹⁴Since brands 1 and 2 are symmetric, which brand has $p = 1$ is not determined, and it is also an equilibrium for brand 1 sometimes to set $p_1 = 1$ and for brand 2 sometimes to set $p_2 = 1$. Sinitzyn (2020) considers a variant of this Varian pattern of consideration, where the non-captive consumers have logit preferences over the three brands, and also finds that the two-brand firm always sets one price to be high.

¹⁵In fact, for the symmetric three-brand framework, of which the Varian pattern is a special case, our analysis in section 4.2 shows that this extreme segmented pricing (where the more expensive brand has price $p \equiv 1$) occurs more generally when β_2 is positive but small enough that expression (6) does not hold.

particular, it is an equilibrium for m to choose these prices independently, or for them to be perfectly positively or negatively correlated.

A condition which ensures that Proposition 2 holds is

$$\rho_1^L < \rho_2^H \text{ and } \rho_2^L > \rho_1^H . \quad (7)$$

This is the situation where z_1 increases and z_2 decreases.¹⁶ Patterns of consideration where this stronger condition holds include when brand 1's reach lies inside that of brand 3. In this case, brand 1 has no captive customers even if its price is below that of brand 2, so provided that m has some captive customers it follows that $\rho_1^L = \rho_1^H = 0$ and $\rho_2^L, \rho_2^H > 0$. In particular, if brand 1's reach almost coincides with brand 3's reach, so that these brands are close substitutes, then m will wish to use segmented pricing rather than uniform pricing. Another configuration where (7) holds is when m 's brands have disjoint reach, and where brand 1 has a lower captive-to-reach ratio than brand 2. In this case $\rho_1^L = \rho_1^H < \rho_2^L = \rho_2^H$. A third configuration where (7) holds is when brand 2 has disjoint reach from brand 3, in which case $\rho_2^L = \rho_2^H = 1$ and $\rho_1^L, \rho_1^H < 1$. (In this situation firm m will set $p_2 \equiv 1$.)

We have so far discussed when firm m 's strategy is uniform or segmented pricing. The remaining situation (barring knife-edge cases) reverses the ordering of ρ_1^H and ρ_2^H in Proposition 2, so that $\rho_1^L < \rho_2^H$, $\rho_1^L < \rho_2^L$ and $\rho_1^H > \rho_2^H$. This configuration simplifies to $\rho_1^L < \rho_2^L < \rho_2^H < \rho_1^H$. (The reason is that $\rho_1^L < \rho_2^H$ is equivalent to $\rho_2^H > \rho_m$, and since ρ_2^L and ρ_1^H average to ρ_m and $\rho_1^H > \rho_m$ we must have $\rho_2^L < \rho_m < \rho_2^H < \rho_1^H$.) Here, broadly speaking, brand 2 has an intermediate number of captives, while brand 1 has extreme numbers of captives depending on whether that brand is its cheaper or more expensive brand. It is then intuitive that the equilibrium involves brand 2 using intermediate prices and brand 1 using extreme prices, so there is disjoint pricing as shown on Figure 3. This is verified in the next result.

Proposition 3 *If $\rho_1^L < \rho_2^L < \rho_2^H < \rho_1^H$, equilibrium takes the disjoint form in which there exist price thresholds $P_0 < P_1 \leq P_2 \leq P_3 \leq 1$ such that firm m chooses price pairs both in $(p_1, p_2) \in [P_0, P_1] \times [P_2, P_3]$ and in $(p_1, p_2) \in [P_3, 1] \times [P_1, P_2]$.*

It is straightforward to construct a pattern of consideration that leads to a disjoint pricing equilibrium. A key requirement is that $\rho_i^L < \rho_i^H$ for $i = 1, 2$, and by examining

¹⁶As explained in section 2, if z_1 is increasing, then z_2 decreasing implies that both l and h are increasing.

Figure 1 one can see that this is the case when α_{12} is small. Another requirement is that ρ_1^H is large, which can be ensured by choosing α_{13} also to be small. With this in mind, an example with disjoint pricing is

$$\alpha_{12} = \alpha_{13} = \alpha_3 = 0, \quad \alpha_1 = 1, \quad \alpha_{23} = \alpha_{123} = 2, \quad \alpha_2 = 4,$$

when we have $\rho_1^L = \frac{1}{3}$, $\rho_2^L = \frac{1}{2}$, $\rho_2^H = \frac{2}{3}$ and $\rho_1^H = 1$. Since brand 1 serves only its captive customers if it is m 's more expensive brand, it will set $p_1 \equiv 1$ whenever it is the more expensive brand, and so in equilibrium we have $P_3 = 1$. More detailed analysis shows that the equilibrium has other threshold prices $P_0 \approx 0.44$, $P_1 \approx 0.84$, and $P_2 \approx 0.92$. The firm chooses to make brand 1 more expensive with only a small probability, approximately equal to 0.05.

Except for knife-edge cases where captive-to-reach ratios for different brands exactly coincide, these three propositions find essentially unique price equilibria for all possible patterns of consideration in Figure 1, and these correspond to the three pricing patterns shown on Figures 2 and 3.

4 Symmetric brands

4.1 Preliminaries

Suppose now that are symmetric, in the sense they are equally likely to be considered by a consumer. Following Burdett and Judd (1983), and extending the symmetric three-brand analysis in section 3, suppose there are N brands in total, and for $1 \leq k \leq N$ suppose that β_k consumers consider each of the $\binom{N}{k}$ sets consisting of k distinct brands, where

$$\binom{N}{k} = \frac{N!}{k!(N-k)!}$$

is the Binomial coefficient.

The analysis can be conducted in terms of the parameters $\{q_i\}$, where q_i denotes the demand for a brand when that brand has the i th lowest price from the N prices offered. Thus, q_i is weakly decreasing in i , and q_1 is the reach of a brand and q_N is the number of consumers captive to a brand. One can express q_i in terms of the underlying consideration parameters $\{\beta_i\}$ as

$$q_i = \sum_{k=1}^{N+1-i} \binom{N-i}{k-1} \beta_k. \quad (8)$$

Furthermore, for $i > 1$ incremental demand $q_{i-1} - q_i$ is

$$q_{i-1} - q_i = \sum_{k=1}^{N+1-i} \binom{N-i}{k-1} \beta_{k+1} ,$$

which takes the same form as q_i in (8) except with β_{k+1} instead of β_k , and which also weakly decreases with i . For instance, with four brands the demand system is

$$q_1 = \beta_1 + 3\beta_2 + 3\beta_3 + \beta_4 ; q_2 = \beta_1 + 2\beta_2 + \beta_3 ; q_3 = \beta_1 + \beta_2 ; q_4 = \beta_1 . \quad (9)$$

The N brands are divided among the firms in some way. Specifically, there are $2 \leq K \leq N$ firms, where firm i has n_i brands and $n_1 + \dots + n_K = N$. For simplicity, label firms so that $n_1 \leq \dots \leq n_K$. It is useful to note that the captive-to-reach ratio of a firm with more brands is greater than one with fewer brands. A firm with n brands has total reach (that is, its demand when its brands are all cheaper than those of its rivals) given by

$$\sigma_n = q_1 + \dots + q_n , \quad (10)$$

while the number of its captive customers (its demand when its brands are the most expensive in the market) is $q_{N+1-n} + \dots + q_N$, and so its captive-to-reach ratio, denoted ρ_n , is

$$\rho_n = \frac{q_{N+1-n} + \dots + q_N}{q_1 + \dots + q_n} . \quad (11)$$

Then we have

$$\rho_{n+1} = \frac{q_1 + \dots + q_n}{q_1 + \dots + q_n + q_{n+1}} \cdot \rho_n + \frac{q_{n+1}}{q_1 + \dots + q_n + q_{n+1}} \cdot \frac{q_{N-n}}{q_{n+1}}$$

which is an average of ρ_n and q_{N-n}/q_{n+1} . Each term in the numerator of ρ_n in (11) is less than q_{N-n} , while each term in its denominator is greater than q_{n+1} , and so $\rho_n \leq q_{N-n}/q_{n+1}$ and hence $\rho_{n+1} \geq \rho_n$ as claimed.

In the following analysis, it is often useful to work with the quantity *ratios*, which for $1 \leq i < N$ are defined to be

$$r_i \equiv \frac{q_{i+1}}{q_i} . \quad (12)$$

Thus $r_i \leq 1$ measures the proportional demand reduction for a brand when it is undercut by one more brand. The analysis is particularly simple when these ratios are monotonically ordered. As we will see in the situations we study, when

$$r_1 > \dots > r_{N-1} \quad (13)$$

then it is an equilibrium for multibrand firms to use uniform pricing. One can show that a sufficient, but not necessary, condition for (13) to hold is for the ratio β_{i+1}/β_i to decrease with i . Ireland's (2007) pattern of consideration has consumers considering at most two brands, so that $\beta_k = 0$ for $k > 2$. Then (8) simplifies to

$$q_i = \beta_1 + (N - i)\beta_2 \quad (14)$$

which satisfies (13).

The reverse applies when

$$r_1 \leq \dots \leq r_{N-1}, \quad (15)$$

in which case it is an equilibrium for multibrand firms to engage in segmented pricing. A sufficient condition for (15) is for the ratio β_{i+1}/β_i to increase with i . Varian's (1980) pattern of consideration has consumers either being captive or considering all N brands, so that $\beta_i = 0$ for $1 < i < N$, in which case

$$\beta_1 + \beta_N = q_1 > q_2 = \dots = q_N = \beta_1$$

which satisfies (15). In the case of independent reach, where a consumer considers any particular brand with independent probability σ say, we have

$$q_i = \sigma(1 - \sigma)^{i-1} \quad \text{and} \quad \beta_i = \sigma^i(1 - \sigma)^{N-i}, \quad (16)$$

Here, $r_i = 1 - \sigma$, and so independent reach is a knife-edge situation where demand ratios are constant.

One further preliminary observation concerns how a firm's demand for its various brands is generated. Suppose a firm offering n brands chooses prices $p_1 \leq \dots \leq p_n$, so that p_1 is the price of its cheapest brand and p_n is the price of its most expensive brand. (These prices are equal if the firm uses uniform pricing.) Note that with brand symmetry it makes no difference to a firm or its rivals *which* of its brands is the cheapest or the most expensive, and it could be that sometimes a particular brand is the cheapest and other times the most expensive. In this analysis with symmetric brands, therefore, when we refer to a two-brand firm (say) using segmented pricing, we mean that its lower price is never higher than any higher price it offers, not necessarily that the price of one of its brands is never higher than any price for the other brand.

Given the pricing strategies of its rivals, let $X^j(p)$ be the expected demand for the firm's brand with the j th lowest price when that price is p . The firm's profit is additively separable in its prices p_1, \dots, p_n , with total profit given by

$$\pi = p_1 X^1(p_1) + \dots + p_n X^n(p_n) . \quad (17)$$

The firm's optimal response to the rival strategies is to maximize π subject to the monotonicity constraint $p_1 \leq \dots \leq p_n$. If the demands X^j become less elastic as j is larger, then the firm can ignore the monotonicity constraints, and the optimal response is segmented pricing, while if X^j is more elastic for larger j then the monotonicity constraints will be binding and uniform pricing is used.

This firm's rivals together supply $N - n$ brands. Given their pricing strategies, for each $0 \leq m \leq N - n$ suppose $\phi_m(p)$ is the probability that exactly m rival brands undercut the price p . Then we have

$$X^1(p) = \phi_0(p)q_1 + \dots + \phi_{N-n}(p)q_{N-n+1} \quad (18)$$

and

$$X^2(p) = \phi_0(p)q_2 + \dots + \phi_{N-n}(p)q_{N-n+2} = \phi_0(p)q_1 r_1 + \dots + \phi_{N-n}(p)q_{N-n+1} r_{N-n+1} .$$

Therefore, the demand ratio X^2/X^1 takes the form

$$\frac{X^2(p)}{X^1(p)} = w_0(p)r_1 + \dots + w_{N-n}(p)r_{N-n+1} , \quad (19)$$

where w_k is given by

$$w_k(p) = \frac{\phi_k(p)q_{k+1}}{\phi_0(p)q_1 + \dots + \phi_{N-n}(p)q_{N-n+1}}$$

which together sum up to 1. Thus, X^2/X^1 is a weighted average of the ratios $\{r_i\}$, where the weights depend on the price p . (Similar expressions hold for the other demand ratios X^3/X^2 , and so on.) Loosely speaking, increasing p means more brands undercut the price, and so this tends to increase those w_k with large k at the expense of w_k with small k . If the ratios r_i decrease, this will tend to decrease X^2/X^1 , so that demand X^2 is more elastic than X^1 , while the reverse is the case when the ratios increase. This is the reason why the distinction between decreasing and increasing demand ratios is so central to the analysis that follows.

4.2 Segmented pricing

Suppose that demand ratios are increasing, so that (15) holds. In this case, regardless of the pattern of brand ownership, there is an equilibrium in which all multibrand firms engage in segmented pricing. Moreover, the pricing pattern takes a simple form in which all firms offer a cheap brand, while larger firms with more brands also offer more expensive brands alongside their cheap brands.

In more detail:

Proposition 4 *Let $M = n_{K-1}$ denote the number of brands supplied by the second-largest firm. Then when (15) holds an equilibrium exists of the following form:*

- (i) *There are threshold prices $P_0 < P_1 \leq \dots \leq P_M = 1$;*
- (ii) *A firm with $m \leq M$ brands chooses its prices in segmented fashion in the m lower intervals $[P_0, P_1], [P_1, P_2], \dots, [P_{m-1}, P_m]$, so that its cheapest brand has price $p_1 \in [P_0, P_1]$, its second cheapest brand (if it has at least two brands) has price $p_2 \in [P_1, P_2]$, and so on;*
- (iii) *If $n_K > M$, so that the largest firm has more brands than the second-largest firm, then firm K sets the prices for $n_K - M$ of its brands equal to 1 and chooses the prices for its remaining M brands in segmented fashion in the intervals $[P_0, P_1], \dots, [P_{M-1}, P_M = 1]$.*

When demand ratios are increasing, this result shows that otherwise symmetric brands are endogenously sorted into different price categories: all firms offer a “budget” brand in the same low price range, firms with more than one brand offer a second brand in a higher, adjacent price interval, and so on until the two or more firms with the most brands offer a “premium” brand in the highest price range. Brands in lower price ranges make greater expected profit than those in higher price ranges.

A special case of Proposition 4 is when there is just one multibrand firm, so that $M = 1$. In this case, when (15) holds there is an equilibrium in which the multibrand firm sets all but one of its prices equal to 1, and competes symmetrically with one brand against its single-brand rivals using the same price interval $[P_0, 1]$.

To illustrate Proposition 4, suppose there are three firms, where firm 1 has one brand, firm 2 has two brands and firm 3 has three brands, so there are six brands in all. Suppose the demand system is $q = (16, 8, 4, 2, 1, 1)$ which satisfies (15). Then all firms offer a budget brand with price in the range $[P_0, P_1]$, firms 2 and 3 offer an intermediate brand in the range $[P_1, 1]$, while firm 3 offers its remaining premium brand with price $p = 1$. The proof

of Proposition 4 shows that $P_0 = 1/8$ and $P_1 = 1/2$, while the profit from a budget brand is $\pi_1 = 2$, the profit from an intermediate brand is $\pi_2 = 1$, and firm 3 also makes profit $\pi_3 = 1$ from its premium brand.

4.3 Uniform pricing

The previous section studied the situation with increasing demand ratios in which firms used segmented pricing. Brands were endogenously sorted into a number of essentially separate sub-markets: a market for budget brands, a market for intermediate brands, and so on. Equilibrium was derived for all patterns of consideration with increasing ratios and for all patterns of brand ownership. In situations where firms use uniform pricing, by contrast, there is no way to separate brand segments in this way. A conjecture is that a parallel result to Proposition 4 applies, namely that when demand ratios are decreasing equilibrium exists in which all firms use uniform pricing and where all firms use low prices but only firms with more brands use higher prices. However, we have not been able to establish this result in general, and instead we present a number of natural special cases that illustrate this outcome.

The first case allows for an arbitrary pattern of ownership but assumes a restrictive pattern of consideration, which is that consumers consider no more than two brands as in Ireland's (2007) model. As discussed in section 4.1, demand q_i is then given by (14) which satisfies (13).

Suppose the $K - 1$ rivals to firm i choose their prices according to some given strategies, and that the resulting expected demand for firm i 's lowest-price brand when that brand has price p is denoted $X^1(p)$, a decreasing function. The demand for its second-cheapest brand, $X^2(p)$, is just the same, except that in any outcome for its rivals' prices it is undercut by one more brand (its own cheapest brand), and so $X^2(p) = X^1(p) - \beta_2$, and similarly for its more expensive brands (if any). This structure of demand with constant differences implies that it is optimal for the firm to use uniform pricing, regardless of the rival pricing strategies. For if not, then suppose an optimal set of prices is $p_1 \leq \dots \leq p_{n_i}$, with strict inequality between at least two of them, say $p_s < p_{s+1}$. Then Lemma 1 implies that we must have $X^{s+1}(p_s)/X^s(p_s) \leq X^{s+1}(p_{s+1})/X^s(p_{s+1})$, which requires $X^s(p_s) \leq X^s(p_{s+1})$, which is a contradiction. We conclude that with this pattern of brand consideration, any equilibrium involves all firms using uniform pricing.

In fact, we can say more about the pattern of pricing in equilibrium, which is that all firms use the same minimum price P_0 , which is therefore equal to the highest captive-to-reach ratio, ρ_K .

Proposition 5 *Suppose consumers consider no more than two brands. Then in equilibrium each firm uses uniform pricing with the same minimum price $P_0 = \rho_K$, and firm $i = 1, \dots, K$ obtains profit $P_0\sigma_i$.*

More detailed analysis—similar to that in Armstrong and Vickers (2022, Proposition 1)—shows that equilibrium takes the form whereby firm i chooses its uniform price in the interval $[P_0, P_i]$, where $P_0 < P_1 \leq \dots \leq P_{K-1} = P_K = 1$, so that firms with fewer brands only use low prices, while firms with the most brands use prices all the way to 1.

To illustrate, suppose again there are three firms, where firm 1 has one brand, firm 2 has two brands and firm 3 has three brands. Suppose that all consumers consider two brands (i.e., where $\beta_2 = 1$ and otherwise $\beta_i = 0$), in which case the demand system is $q = (5, 4, 3, 2, 1, 0)$. The minimum price is $P_0 = 1/4$, which is the captive-to-reach ratio of firm 3. All three firms offer prices in the range $[P_0, P_1]$, where the highest price used by firm 1 is $P_1 = 7/12$, and then only firms 2 and 3 offer prices in the upper range $[P_1, 1]$.

Going beyond this “Ireland” pattern of consideration, the following result provides useful insight into the equilibrium profits of firms when demand ratios decrease.

Lemma 3 *Suppose (13) holds, and let P_0 be the smallest price used by any brand in equilibrium. Then (i) if firm i uses P_0 for its cheapest brand it uses the uniform price (P_0, \dots, P_0) and its profit is $\sigma_{n_i}P_0$, and (ii) any firm with at least as many brands as firm i also uses the uniform price (P_0, \dots, P_0) .*

This result shows that if a firm uses the minimum P_0 for its cheapest brand it uses that price uniformly for all its brands, and it further shows that any other firm with at least as many brands also prices uniformly at P_0 . Since standard arguments show that at least two firms must have P_0 in the support of their cheapest brands, the following corollary is immediate.

Corollary 4 *Suppose (13) holds and the pattern of ownership takes the form*

$$n_1 = \dots = n_{K-1} \leq n_K .$$

Then all firms price uniformly at P_0 , and obtain profit equal to P_0 times their reach, where the minimum price P_0 is equal to the captive-to-reach ratio of firm K .

This result applies when there is one weakly larger firm competing against a number of equal sized rivals. As special cases this includes markets with just one multibrand firm, with two asymmetric firms, and when all firms have the same number of brands. The next result shows that in these three cases, equilibrium exists in which multibrand firms use uniform pricing throughout, not just for the lowest price.

Proposition 6 *Suppose that (13) holds. Then an equilibrium exists in which firms use uniform pricing if (i) there is just one multibrand firm, (ii) there are just two firms, or (iii) all firms are symmetric.*

While this result shows there is an equilibrium with uniform pricing, Corollary 4 shows that, in payoff terms at least, this equilibrium is unique.

4.4 Non-monotonic demand ratios

So far we have discussed situations where demand ratios were either increasing, when firms used segmented pricing, or decreasing, when they used uniform pricing. Clearly there are other, less regular, situations in which these ratios are non-monotonic. A plausible outcome with such patterns of consideration is that a multibrand firm will offer a *mixture* of uniform and segmented pricing patterns.

To illustrate this possibility in a simple setting, suppose that there is one multibrand firm (which supplies n brands) that competes against one single-brand rival. (Therefore, $N = n + 1$.) The earlier analysis shows that with decreasing ratios the multibrand firm sets a uniform price for all brands, while with increasing ratios the multibrand firm competes with just one brand. More generally, we now show that for some $1 \leq k \leq n$ the multibrand firm sets a uniform price for k brands, which compete against the rival, while the remaining brands (if any) set price $p \equiv 1$.

For each $1 \leq i \leq n$ it is useful to define

$$c_i = \frac{\sum_{j=1}^i q_j r_j}{\sum_{j=1}^i q_j} \quad (20)$$

for the respective ‘‘captive-to-reach’’ ratios in this context. If firm m uses uniform pricing for k of its brands, when this uniform price is below the rival’s price its demand is $q_1 + \dots + q_k$,

while if its price is undercut by the rival its demand is $q_2 + \dots + q_{k+1} = q_1 r_1 + \dots + q_k r_k$, and so c_k is the relevant captive-to-reach ratio. Since c_{i+1} in (20) is a weighted average of c_i and r_{i+1} , if (15) holds then $c_1 \leq \dots \leq c_n$ and if (13) holds then $c_1 \geq \dots \geq c_n$.

Equilibrium in this duopoly market is described in the following result:

Proposition 7 *Suppose that c_k is the smallest of the captive-to-reach ratios c_i in (20). Then an equilibrium exists in which the multibrand firm uses a uniform price for k of its brands and sets price $p \equiv 1$ for its remaining brands (if any). The minimum price offered is $P_0 = c_k$ and the multibrand firm makes its captive profit.*

To illustrate this result, suppose that there are four brands in all, where the consideration parameters are $\beta_1 = 1$, $\beta_2 = 0$, $\beta_3 = 2$ and $\beta_4 = 0$ which induce the demand system $q = (7, 3, 1, 1)$. Then (20) yields $c_1 = 3/7$, $c_2 = 2/5$ and $c_3 = 5/11$ of which the smallest is c_2 . The three-brand firm will therefore price two of its brands uniformly, which compete against the single-brand rival using prices in the range $[2/5, 1]$, and it will choose $p = 1$ for its remaining brand.¹⁷

5 Further topics

5.1 Brand proliferation

We have so far taken as given each firm's set of brands. Our analysis can however be used to consider a firm's incentive to introduce a new brand. Introducing a new brand can be modelled in our consideration framework as the addition of a new "circle" on the Venn diagram describing the pattern of consideration, as on Figure 4. (Doing so implies that the addition of a brand does not affect whether a consumer continues to consider any of the existing brands.) One straightforward motive to add a new brand is simply to reach a wider set of potential consumers. More subtle motives, which apply even if the firm's total reach is not increased by the new brand, include an increased ability to engage in price discrimination and the ability to deter entry by a rival not yet present in the market.

¹⁷One can also study equilibrium pricing in this context of non-monotonic ratios with other ownership structures. For example, in a symmetric market with two firms, each of which supplies two brands, one can show that an equilibrium exists in which a firm prices uniformly with some probability and otherwise uses segmented pricing. With the above demand system $q = (7, 3, 1, 1)$, say, a firm prices uniformly on the interval $[1/3, 1]$ with probability $1/3$, and otherwise uses segmented pricing (where the higher price is equal to $p = 1$).

First, though, note that brand proliferation need *not* benefit a firm, even if it is costless to introduce the new brand and it increases the total reach of the firm. To see this, suppose initially that there are two asymmetric single-brand firms. In equilibrium, the firm with the larger reach will obtain its captive profit (and so cannot lose by costlessly adding a brand) while the smaller firm obtains profit that exceeds its number of captive customers. Suppose that this smaller firm introduces a new brand that reaches exactly the same set of consumers as the rival's brand. In this case, equilibrium prices for the new brand and the larger existing brand are driven down to cost (zero), while the (now) multibrand firm sets price equal to 1 for its existing brand. As a result the profit of the multibrand firm falls to its captive profit, and the new brand harms its owner's (as well as the rival firm's) profit. Since total welfare is unchanged with this new brand (as no new consumers are reached), consumer surplus rises after the introduction of this new brand. In economic terms, when the firm adds a second brand that is a close substitute with the rival brand, the rival competes more aggressively, and this harms the profits from the firm's existing brand. This form of brand proliferation *intensifies* competition between the firms.

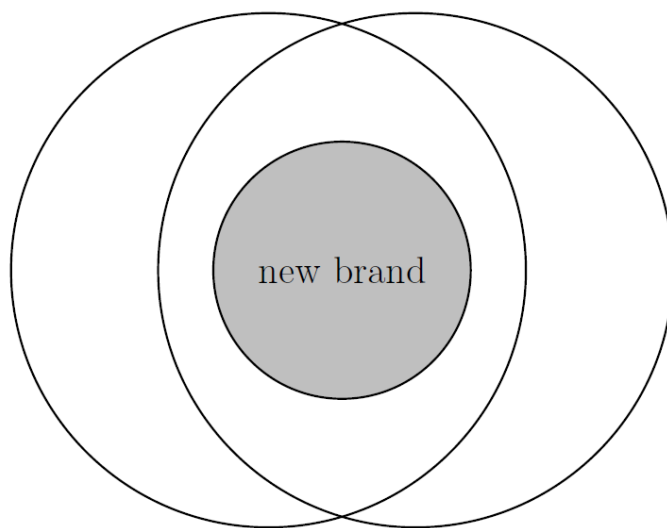


Figure 4: New brand considered by consumers who consider both existing brands

By contrast, with other patterns of consideration, adding a new brand can increase the profits of all firms. To see this, suppose initially there are two symmetric firms, which therefore make exactly their captive profit and use minimum price equal to their captive-to-reach ratio, say ρ . Suppose one of these firms introduces a new brand, which is considered only by consumers who already consider both the original brands as shown on Figure 4. The

single-brand firm will not price below ρ , its unchanged captive-to-reach ratio. Therefore the multibrand firm can make at least its old captive profit (by choosing $p = 1$ for its old brand) plus ρ times the reach of the new brand (if it chooses $p = \rho$ for its new brand), and so adding this brand is strictly profitable for the two-brand firm. It is also strictly profitable for the rival single-brand firm: the two-brand firm will now choose $p = 1$ with positive probability for its initial brand, and this implies that the single-brand firm now makes more than its previous captive profit. Since industry profits strictly increase, and total welfare is unchanged, consumers are strictly worse off with the introduction of the new brand. This form of brand proliferation therefore *relaxes* competition between firms.

Finally, asymmetric situations can arise where the firm introducing a new brand benefits while its rival is harmed. In such cases, brand proliferation can act as a way to deter entry by a potential rival that is not yet in the market. Judd (1985) provides an early analysis of when brand proliferation can deter entry. A theme in that paper was how the credibility of the incumbent's policy was a constraint on whether the policy was effective. If the incumbent had an incentive to withdraw the new brand in the event of entry, then the entrant may foresee the incumbent's post-entry incentive and choose to enter. However, in our context, if adding a new brand would harm the rival but benefit the two-brand firm, then brand proliferation could act as a credible entry deterrent.

To see this, suppose the pattern of consideration for three potential brands is again qualitatively as depicted on Figure 4. Suppose an incumbent firm has an initial brand with reach 3. If the rival firm enters, which incurs a sunk cost, then that firm has reach 2 and the overlap between these two brands has size 1. If these two brands compete alone, the entrant makes profit $4/3$ and the incumbent makes its captive profit 2. Suppose that the incumbent firm can, before the entrant decides to enter, introduce a new brand, the reach of which is exactly the overlap between the two larger brands. Then (as shown in more detail in the Appendix) with all three brands in the market there would be segmented pricing, where the incumbent two-brand firm sets $p = 1$ for sure for its original brand, and competes against the rival with its new brand. The resulting minimum price is $P_0 = 1/2$ and the entrant now makes only its captive profit 1 while the incumbent makes combined profit $5/2$. Thus the incumbent firm has no incentive to withdraw its new brand if entry occurs, and if the entrant's sunk cost lies between 1 and $4/3$ then entry occurs if and only if the potential new brand is absent. If entry is deterred, the incumbent is a monopolist

and makes profit 3, but if entry takes place the incumbent still prefers to offer the new brand alongside its existing brand. In such cases, the incumbent can credibly engage in brand proliferation to deter entry. Because of segmented pricing, the new brand intensifies competition with the rival brand without unduly reducing the profit that can be extracted from the incumbent’s original brand.

5.2 Constraints on price discrimination

An important question about a firm that supplies several similar products is whether it should be permitted to set distinct prices for them. How does a constraint on the ability to set non-uniform prices affect each firm’s profit or consumer surplus?

In many situations, the ability of a firm to set distinct prices for similar products will—except in knife-edge cases—induce the firm to do so. However, in the markets studied in this paper, we have seen that uniform pricing is not a knife-edge phenomenon, and occurs in the many situations in which lower-priced brands are less elastic than higher-priced brands. In such cases, a constraint to set uniform prices will have no impact. In the following discussion, therefore, we focus on situations in which firms would, if unconstrained, use forms of non-uniform pricing such as segmented pricing.

Unilateral constraints on uniform pricing: In some markets, when a firm that would otherwise engage in price discrimination commits in advance to set uniform pricing, this could boost the firm’s equilibrium profit if it weakens the intensity of its rivals’ competition. In the current context of multibranding, a firm might be able to commit to uniform pricing if it “merged” its brands into one new unified brand, for which it is most natural to set a uniform price.

To see this, consider a market in which there are four symmetric brands, and two firms each supply two of them. If (15) holds, then the firms will engage in segmented pricing. If one of these firms commits in advance to use uniform prices, this might boost that firm’s equilibrium profit. To be concrete, suppose the awareness parameters are $\beta_1 = \beta_2 = 1$, $\beta_3 = 2$ and $\beta_4 = 10$, which induce demands $q = (20, 5, 2, 1)$. Without a unilateral constraint on uniform pricing by one firm, the two firms compete symmetrically using segmented pricing, with price thresholds $P_0 = 1/8$ and $P_1 = 1/2$ and each firm makes profit $7/2$. If one firm commits to uniform pricing, the rival will continue to use segmented pricing, but

with *higher* price thresholds given by $P_0 = 7/45$ and $P_1 = 5/9$, and the uniform pricing firm makes profit $35/9$, which is higher than $7/2$, while the unconstrained rival makes still greater profit of $37/9$.¹⁸ Thus, when one firm commits to use uniform pricing, this relaxes competition in the market (in part by making the market less symmetric). Of course, if *both* firms committed to uniform pricing, equilibrium profits would be driven down to the level of captive profit, 3, which is worse for both firms than the unconstrained market.¹⁹

This effect seems harder to achieve when the market is less symmetric, however. For instance, consider the framework in section 3 with three brands, two of which are supplied by firm m . If the profit of firm m rises after imposing the constraint, then m as a whole must have fewer captives than its single-brand rival s , that is, $\rho_m < \rho_3$, in which case its profit with uniform pricing is $\sigma_m \rho_3$. (If m has more captives than s then it will make exactly its captive profit under the policy, which is a lower-bound on the profit it obtains in any equilibrium without the constraint.) But, as discussed in section 3, the minimum price P_0 in any equilibrium without the constraint cannot be below ρ_3 , and so m 's profit without the constraint must be at least $\sigma_m \rho_3$.

Industry-wide constraint to use uniform prices: A second scenario is that all firms in the market are required by policy to use uniform prices. One insight is that, if when constrained to use uniform prices firms are *symmetric*, then permitting them to engage in price discrimination can only boost each firm's profit and reduce consumer surplus.²⁰ The reason is that when symmetric firms compete with uniform prices they obtain exactly their captive profit, and in any equilibrium without a constraint on uniform prices each firm must obtain *at least* their captive profit.

It is straightforward to see that a requirement that all firms use uniform pricing can sometimes reduce consumer surplus in less symmetric markets. For instance, suppose there is a single multibrand firm competing against a number of single-brand rivals. As shown in section 4.2, if demand ratios are increasing then the multibrand firm will set $p = 1$

¹⁸These calculations use the analysis of segmented pricing in section 3.3, where the firm which uses uniform pricing plays the role of the single-brand firm s .

¹⁹The same qualitative features of this numerical example apply whenever the ratios are increasing and satisfy the additional condition $r_2(r_3 - r_1) < r_2 - r_1$.

²⁰Note that it is possible for firms to be symmetric when they use uniform pricing but not be symmetric otherwise. For instance, on Figure 4 if one firm supplies the ‘inner’ brand as well as one of the ‘outer’ brands, the two firms are symmetric with uniform pricing (when the inner brand plays no role for the two-brand firm) but not if the two-brand firm uses segmented pricing.

for all but one of its brands and compete with its remaining brand in symmetric fashion against its rivals. To be concrete, suppose the multibrand firm has n brands and there are m single-brand firms, so that $N = m + n$. The multibrand firm will make its captive profit with or without the uniform pricing constraint, so the impact on industry profit and consumer surplus depends on how the minimum price P_0 is affected by the constraint. Without the constraint it is as if there are $m + 1$ single-brand firms competing, so that $P_0 = q_{m+1}/q_1$. With the uniform price constraint, the minimum price P_0 is equal to the multibrand firm's captive-to-reach ratio, ρ_n in (11). Given increasing demand ratios (15) it follows that ρ_n , which is an average of $q_{m+1}/q_1, q_{m+2}/q_2, \dots, q_N/q_n$, is greater than q_{m+1}/q_1 , and so the minimum price is higher after the constraint. Thus industry profit is higher, and consumer surplus lower, when the multibrand firm is constrained to use uniform pricing.²¹

5.3 Mergers between firms

Within our framework, it is natural to model a merger between two firms as resulting in a new firm supplying the combined brands of the two separate firms, and with the same pattern of consumer consideration for those brands.

First observe that, when there is segmented pricing, a merger between two firms might have no impact on prices or profits, even if the brands supplied by these firms overlap in their reach. For example, suppose there are initially three single-brand firms with a pattern of consideration of an asymmetric ‘‘Varian’’ type, where each brand has some captive customers and there is pool of consumers who consider all three brands. Specifically, suppose brand 1 has ‘‘few’’ captives, brand 2 has an ‘‘intermediate’’ number of captives, while brand 3 has ‘‘many’’ captives. Then previous analysis shows that brand 3 will set $p = 1$, while the two smaller brands will compete. (See Baye *et al.*, 1992.) If brands 1 and 3 merge to become a two-brand firm, this firm will engage in segmented pricing and will again set $p = 1$ for its brand 3, and brands 1 and 2 will compete exactly as before.²²

For the remainder of this section we use the framework of symmetric brands from section 4. We discuss the situations with decreasing demand ratios (with uniform pricing) and increasing demand ratios (with segmented pricing) in turn.

²¹In a consideration set framework, but with a very different form of price discrimination, Armstrong and Vickers (2019) also showed how price discrimination was more likely to harm consumers when the market was relatively symmetric.

²²If brands 1 and 2 merge, however, this does affect competition. Such a merger is profitable if the combined number of captive consumers of brands 1 and 2 is still fewer than that of brand 3.

Suppose the initial market consists of symmetric firms engaging in uniform pricing. (In particular, this includes the case where all firms initially supply a single brand.) Then a merger between two or more of them must be profitable, and harm consumers. For in a symmetric market with uniform pricing all firms make exactly their captive profit, while after a merger the number of captives of the merged entity rises, and this provides a lower bound on its post-merger profit and so a merger must be profitable. Since a non-merging firm's profit cannot fall below its captive profit, which is unaffected by the merger, total industry profit must rise as well, and consumer surplus falls.

Moving beyond symmetric firms, suppose the pattern of consideration is of the “Ireland” form, whereby a consumer considers at most two brands. Proposition 5 describes the outcome in this market, which applies both before and after a merger between two firms. If two relatively small firms merge, in the sense that even their combined number of brands is no greater than that of the largest firm K , then the minimum price P_0 is unaffected by the merger. Such a merger must be unprofitable, as the reach of the merged firm is always below the sum of the reaches of the two pre-merger firms. Therefore, a merger is profitable only if the merged firm is the largest firm in the post-merger market, which will raise the minimum price P_0 and the profits of the non-merging firms and hence of industry profit overall. Therefore, with this pattern of consideration, any profitable merger will boost industry profit and reduce consumer surplus.

Under the conditions of Corollary 4, a merger between the larger firm K and a smaller firm is always profitable. (Such a merger leads to a market structure which still has one larger firm and so the Corollary still applies.) Both before and after the merger, the joint profit of the two merging firms is the captive profit of the n_K -brand firm, plus the reach of the smaller firm times the relevant minimum price P_0 . But the minimum price P_0 increases after the merger, and so joint profits rise. The effect on the non-merging firms is also profitable for the same reason, and so such a merger will harm consumer surplus.

Turning next to situations with segmented pricing, Proposition 2 describes equilibrium with any pattern of brand ownership in a market when ratios are increasing, and so this applies to the initial market and the market post merger. In this context, one can show that the impact of a merger on the non-merging firms is always profitable:

Claim 1 *Suppose (15) holds and that firms use the segmented pricing strategies described in Proposition 4. Then a merger between two firms will increase the profits of every non-*

merging firm. Therefore, any profitable merger will increase industry profit and reduce consumer surplus.

To illustrate, suppose initially there are N single-brand firms, which each therefore make captive profit q_N . If two of them merge, then from Proposition 2 the two-brand firm will set price $p = 1$ for one brand and compete symmetrically with the remaining single-brand firms with its other brand. As such, the two-brand firm will now make higher profit $q_{N-1} + q_N$ and this is a profitable merger, while the non-merging brands each make higher profit q_{N-1} . In addition, the pricing pattern in the market changes qualitatively after the merger, where instead of all brands choosing prices in a similar manner, after the merger one of the brands ceases to compete at all.²³

In other situations, mergers are unprofitable for the merging firms. For example, suppose there are four brands with increasing ratios, and the initial pattern of brand ownership is that there is one two-brand firm and two single-brand firms. Proposition 4 shows that the two-brand firm will set $p = 1$ for one brand and compete symmetrically with its two rivals with its other brand, with resulting industry profit given by $3q_3 + q_4$. If the two single-brand firms merge, the market becomes a symmetric duopoly, and one can check that industry profit becomes $2q_4(1 + q_2/q_3)$. It is ambiguous which of the two market structures is more profitable, and the merger reduces industry profit if and only if $3r_2 > 2r_3 + r_2r_3$, which is true if r_2 is close enough to r_3 . In such cases, since industry profit falls and the non-merging firm's profit rises, the merger is unprofitable.²⁴

6 Conclusion

Firms supply multiple brands of similar products for a variety of reasons. In this paper we have examined multibrand pricing in simple settings using a consideration set framework,

²³This accords with Sinitsyn (2020), who studies a model with three symmetric brands with a demand structure which is similar to Varian's, and finds that after a merger one of the merged brands sets a high price while the other competes symmetrically against the un-merged brand. In their account of five US mergers, Ashenfelter and Hosken (2010, p. 439) observe that "in the four mergers for which we find substantial evidence of a price increase, manufacturers did not increase all of their prices uniformly. Instead, the merged firm chose to increase the price of one of its products (or a set of products) while holding the other prices more or less fixed. For example, the combined Pennzoil/Quaker State substantially increased the price of its Quaker State brand of motor oil while leaving the Pennzoil brand's price essentially unchanged."

²⁴Putting this another way, starting from a symmetric duopoly where each firm has two brands, one firm has an incentive to dis-integrate into two single-brand firms. This change will harm the other firm and harm consumers in aggregate too.

in which different brands are perfect substitutes for consumers able to consider them. Equilibrium pricing in oligopoly with multibrand firms was shown to depend on patterns of consumer consideration of brands. The key to our analysis was how the ratio of demand for a firm's more expensive brand to its cheaper brand varied with its price. When a cheaper brand has less elastic demand than a more expensive brand then uniform pricing—i.e., forgoing the ability to engage in price discrimination—is the optimal strategy. Otherwise, rich pricing patterns that resemble real-world pricing practices can emerge even in our simple settings—in particular “segmented” pricing with one brand always priced higher than the other, and “disjoint” pricing with one brand priced high or low and the other in between.

Further results were obtained when brands were symmetric and equally likely to be considered by consumers. When demand ratios were increasing—an assumption which corresponds to cheaper brands having more elastic demand—equilibrium with a simple form of segmented pricing exists for any pattern of brand ownership: brands are endogenously sorted into price categories, where all firms (even single-brand firms) supply a “budget” brand, while firms with more brands also supply more expensive brands. The very largest firm does not attempt to compete at all with some of its brands, and focusses instead on extracting profit from its captive customers. We found situations in which a firm might benefit from a commitment to use uniform pricing, as this softens competition from its rivals. When demand ratios were decreasing, in the natural situations we study we find firms use uniform pricing, and larger firms compete less hard than those firms with fewer brands.

The paper could usefully be extended in a variety of directions. For instance, it would be valuable to make endogenous the pattern of consumer consideration, either by allowing consumers to search or firms to advertise. Allowing consumers to search raises the issue of whether or not they understand the pattern of brand ownership, and how that affects their propensity to search. Another route would be to allow firms to invest in advertising, while maintaining the assumption that consumers cannot affect their consideration sets. In any such extension, the analysis in the present paper will be useful ingredient.

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Appendix

Proof of Lemma 1: (i) In the region of prices such that $p_1 \geq p_2$ the firm’s profit is $H_1(p_1) + L_2(p_2)$. Suppose that (p_1, p_2) with $p_1 > p_2$ maximizes profit. Then by revealed preference

$$p_2 L_2(p_2) \geq p_1 L_2(p_1) \text{ and } p_1 H_1(p_1) \geq p_2 H_1(p_2) , \quad (21)$$

and after dividing we obtain $z_2(p_1) \geq z_2(p_2)$ as claimed.

(ii) The firm cannot gain by reversing its prices, so that brand 1 has price p_2 and brand 2 has price p_1 , and so

$$p_1 H_1(p_1) + p_2 L_2(p_2) \geq p_1 H_2(p_1) + p_2 L_1(p_2) .$$

Since $L_1(p) + H_2(p) \equiv L_2(p) + H_1(p)$, this condition can be re-written both as

$$p_2L_2(p_2) - p_1L_2(p_1) \geq p_2L_1(p_2) - p_1L_1(p_1) \quad (22)$$

and as

$$p_1H_1(p_1) - p_2H_1(p_2) \geq p_1H_2(p_1) - p_2H_2(p_2) . \quad (23)$$

The left-hand sides of (22) and (23) are both non-negative from (21) above.

Suppose contrary to the statement of the lemma that $l(p_2) < l(p_1)$ and $h(p_2) < h(p_1)$. The right-hand side of (22) can be written

$$p_2L_1(p_2) - p_1L_1(p_1) = \frac{p_2L_2(p_2)}{l(p_2)} - \frac{p_1L_2(p_1)}{l(p_1)} > \frac{p_2L_2(p_2) - p_1L_2(p_1)}{l(p_2)} ,$$

where the inequality follows by $l(p_2) < l(p_1)$. Therefore, since the left-hand side of (22) is positive it follows that $l(p_2) > 1$. But $l(p_2) > 1$ is equivalent to $x_2(p_2) > x_1(p_2)$, which is equivalent to $h(p_2) > 1$. Similarly, the right-hand side of (23) is

$$p_1H_2(p_1) - p_2H_2(p_2) = p_1h(p_1)H_1(p_1) - p_2h(p_2)H_1(p_2) > h(p_1)[p_1H_1(p_1) - p_2H_1(p_2)] ,$$

which implies $h(p_1) < 1$, which therefore contradicts $h(p_2) < h(p_1)$. This completes the proof.

Details for the segmented equilibrium in section 3.3: We here solve for threshold prices, P_0 and P_1 , and resulting profits, when Proposition 2 applies, so there is a segmented equilibrium in the three-brand market. First, if $\sigma_1 \geq \sigma_3$ it is an equilibrium for firm m always to set $p_2 = 1$ so that $P_1 = 1$ and $P_0 = \rho_1^L$. In effect there is duopoly equilibrium between brands 1 and 3 while brand 2 separately gets its captive profit α_2 . Firm m obtains profit α_m (as it would with uniform pricing), and firm s gets profit $\sigma_3\rho_1^L$.

Second, if $\sigma_1 < \sigma_3$ then $P_1 \leq 1$ and for brand 3 to get the same expected profit at prices P_0 and P_1 requires $\frac{P_1}{P_0} = \frac{\sigma_3}{\alpha_3 + \alpha_{23}}$. Using this relation between P_0 and P_1 implies that for brand 1 to get the same expected profit at prices P_0 and P_1 we require $G(P_1) = \frac{\sigma_1}{\sigma_3}$. Firm m 's payoff, which is obtained in particular at the uniform price $p_1 = p_2 = P_1$, can be expressed as

$$\begin{aligned} \pi_m &= \left[1 - (1 - \rho_m) \frac{\sigma_1}{\sigma_3} \right] \sigma_m P_1 \\ &= \left[1 + \frac{(\rho_m - \rho_1^L) \sigma_1}{\alpha_3 + \alpha_{23}} \right] \sigma_m P_0 > \sigma_m P_0 . \end{aligned} \quad (24)$$

There are two subcases. If $\alpha_2 \geq \rho_3(\sigma_3 - \sigma_1) > 0$ then firm m sets $p_2 = 1$ with positive probability and brand 2 generates its captive profit α_2 . Since brand 2 gets the same profit at P_1 and at 1, it follows that P_1 is given by

$$\begin{aligned} P_1 &= \frac{\rho_2^H}{1 - (1 - \rho_2^H)\frac{\sigma_1}{\sigma_3}} \\ &> \frac{\rho_1^L}{1 - (1 - \rho_1^L)\frac{\sigma_1}{\sigma_3}}, \end{aligned}$$

where this inequality is equivalent to $P_0 > \rho_1^L$ and so brand 1 makes more than its captive profit. Firm m 's payoff is

$$\pi_m = \alpha_m + \frac{\left(1 - \frac{\sigma_1}{\sigma_3}\right)(\rho_2^H - \rho_m)\sigma_m}{1 - (1 - \rho_2^H)\frac{\sigma_1}{\sigma_3}} > \alpha_m$$

and

$$\pi_s = (\alpha_3 + \alpha_{23})P_1 = \alpha_3 + \frac{(1 - \rho_2^H)[\alpha_2 - \alpha_3\left(1 - \frac{\sigma_1}{\sigma_3}\right)]}{1 - (1 - \rho_2^H)\frac{\sigma_1}{\sigma_3}} > \alpha_3,$$

where the inequality follows from the condition $\alpha_2 \geq \rho_3(\sigma_3 - \sigma_1)$, and so at least one firm gets strictly more than with uniform pricing.

Finally, if $\alpha_2 < \rho_3(\sigma_3 - \sigma_1)$, which implies $\sigma_3 > \sigma_m$ given $\rho_1^L < \rho_m$, then $P_0 = \rho_3$ in equilibrium and firm s sets $p = 1$ with positive probability and $\pi_s = \alpha_3$. From (24), $\pi_m > \sigma_m\rho_3$, which is firm m 's payoff with uniform pricing. Consumers would therefore do better with uniform pricing.

Proof of Proposition 3: We first show that the only uniform price pair which might be a best response for firm m is $(1, 1)$. Suppose to the contrary that (P, P) is a best response with $P < 1$. Since z_1, l and h are all strictly increasing, the discussion at the end of section 2 shows that any other price pairs which are best responses all must lie on the horizontal line $p_2 \equiv P$. This means that the firm chooses price $p_2 = P < 1$ for sure, which cannot occur in equilibrium.

Thus the only possible uniform price pair in m 's price support is $(1, 1)$. As above, if this is the case all optimal price pairs lie on the line $p_2 = 1$. If P_0 is the minimum price for p_1 in m 's support on this line, then, since m can have no gaps in the prices it offers, m 's support consists of all price pairs $(p_1, 1)$ with $P_0 \leq p_1 \leq 1$. This satisfies the statement of the proposition with $P_1 = P_2 = P_3 = 1$.

The remaining possibility is that there is *no* uniform price pair which is a best response for m . This implies that m 's price support includes price pairs where $p_1 < p_2$ and where $p_1 > p_2$. To see this is so, note that Lemma 2 shows that for best-response price pairs in the region $p_1 < p_2$ the maximum p_1 is strictly below the minimum p_2 . (If the maximum p_1 was *equal* to the minimum p_2 then there would be uniform price pair that was a best response.) Likewise, for best-response price pairs in the region $p_1 > p_2$ the maximum p_2 is strictly below the minimum p_1 . If m 's support was contained in only *one* of these regions, then it would have a gap in the set of prices if offered, which cannot occur in equilibrium. Therefore, its support includes prices both where brand 1 is cheaper and where it is more expensive.

Thus, following the discussion of the two rectangles at the end of section 2, in this duopoly context both the rectangles are optimal for the firm, and the extremities of the rectangles satisfy (2). But since firm m can have no gaps in the prices it offers in the range P_0 up to 1, it follows that

$$P_0 = p_L^{\min} < p_L^{\max} = P_L^{\min} \leq P_L^{\max} = P_H^{\min} \leq P_H^{\max} = p_H^{\min} \leq p_H^{\max} = 1 .$$

(Since firm m can have no "atoms" in its prices except at $p = 1$, at least the first of the above inequalities is strict.) Writing $P_1 = p_L^{\max} = P_L^{\min}$, $P_2 = P_L^{\max} = P_H^{\min}$ and $P_3 = P_H^{\max} = p_H^{\min}$ shows that the two rectangles are arranged as depicted on Figure 3. This completes the proof.

Proof of Proposition 4: The aim is to show for each firm $i = 1, \dots, K$ that firm i 's best response to its rivals' strategies as described in the Proposition is to do the same. We calculate firm i 's demand, X^j , for its brand with its j th lowest price, p_j , for each $1 \leq j \leq n_i$. As in (17), the firm's profit takes the form $p_1 X^1(p_1) + \dots + p_{n_i} X^{n_i}(p_{n_i})$. In general, one would find its optimal pricing policy by maximizing this profit subject to the constraint $p_1 \leq \dots \leq p_{n_i}$. However, to prove the result we ignore this monotonicity constraint on the price order and simply maximize $p_j X^j(p_j)$ for each j , and we will show that the firm wishes to choose its j th lowest price in the appropriate interval $[P_{j-1}, P_j]$, in which case its prices automatically satisfy the monotonicity constraint.

In more detail, let b_m be the number of firms that have at least m brands. Then with the stated pricing pattern there are $b_1 (= K)$ brands that are priced in the lowest interval $[P_0, P_1]$, there are b_2 brands that have prices in the next interval $[P_1, P_2]$, and so on in a

decreasing sequence until there are $b_M \geq 2$ brands in the upper interval $[P_{M-1}, 1]$. (Note that $b_1 + \dots + b_{n_K} = N$, the total number of brands.) It is useful to define $B_m = b_1 + \dots + b_m$. Since $b_m \geq 2$ for $m \leq M$ it follows that $B_m - B_{m-1} \geq 2$ for $m \leq M$. A brand in the lowest interval makes expected profit $P_0 q_1 = P_1 q_{b_1} = P_1 q_{B_1}$, where the equality follows since at price P_0 a brand will sell to its entire reach q_1 , while at price P_1 the brand is sure to be undercut by exactly $b_1 - 1$ other brands and so has demand q_{b_1} , and a firm needs to be indifferent between these two prices for this brand. Likewise, a brand in the next interval $[P_1, P_2]$ makes profit $P_1 q_{[B_1+1]} = P_2 q_{B_2}$, and so on up to a brand in the highest interval $[P_{M-1}, 1]$ that makes profit $P_{M-1} q_{[1+B_{M-1}]} = q_{B_M}$. Let π_m denote the equilibrium profit of a brand in the m th lowest price segment.

Thus the threshold prices are calculated recursively as

$$P_{M-1} q_{[1+B_{M-1}]} = q_{B_M} = \pi_M$$

and so on down to

$$P_0 q_1 = P_1 q_{B_1} = \pi_1 ,$$

where the expression for the m th segment (where $m > 1$) is

$$P_{m-1} q_{[1+B_{m-1}]} = P_m q_{B_m} = \pi_m . \quad (25)$$

Thus for $m < M$ we have

$$\frac{\pi_{m+1}}{\pi_m} = \frac{q_{[1+B_m]}}{q_{B_m}} = r_{B_m} , \quad (26)$$

so that more expensive brands generate lower expected profit. Given (15) and that B_m increases with m , this profit ratio (26) increases with m .

To check that this is an equilibrium we need to show that firm i wishes to price each of its n_i brands in the corresponding price interval, given that the other firms are following the strategies described. First, we derive the demand in the candidate equilibrium for a brand in the m th segment, $[P_{m-1}, P_m]$. In this equilibrium, there are b_m brands competing within this segment and each brand in this segment has demand function $X_m(p)$, say, when its price is p , where this satisfies the indifference condition $pX_m(p) \equiv \pi_m$, where π_m as above is the equilibrium profit from a brand in this segment. If we write $F_m(p)$ for each brand's CDF for price in this segment, then $X_m(p)$ is given by

$$X_m = y_0 + \dots + y_{b_m-1} , \quad (27)$$

where

$$y_k = \binom{b_m - 1}{k} (1 - F_m)^{b_m - 1 - k} F_m^k q_{[B_{m-1} + 1 + k]} . \quad (28)$$

(To see this, note that $q_{[B_{m-1} + 1 + k]}$ is the brand's demand when it is undercut by all the brands in lower segments, which total B_{m-1} , and also by $0 \leq k \leq b_m - 1$ of its rivals in that price segment, where the probability that exactly k of its rivals undercut it is $\binom{b_m - 1}{k} (1 - F_m)^{b_m - 1 - k} F_m^k$.) Note that when $p = P_{m-1}$, the lowest price in the segment, then $F_m = 0$ and so $X_m = q_{[1 + B_{m-1}]}$, while at $p = P_m$ we have $X_m = q_{B_m}$, as consistent with the demands in (25).

Next consider firm i 's optimal choice of p_j , its j th lowest price (where $j \leq n_i$). If it chooses this price to lie in the designated interval $[P_{j-1}, P_j]$ it will make profit π_j from this brand. If $j > 1$, the firm might deviate and instead choose p_j to lie in a lower interval $m < j$ given by $[P_{m-1}, P_m]$. If it does so, the demand for the firm's brand with the j th lowest price, denoted X say, is modified from (27) to be

$$X = \sum_{k=0}^{b_m - 1} \binom{b_m - 1}{k} (1 - F_m)^{b_m - 1 - k} F_m^k q_{[B_{m-1} + 1 + k + (j-m)]} . \quad (29)$$

If we write

$$\tilde{r}_k = r_{[B_{m-1} + 1 + k]} \times \dots \times r_{[B_{m-1} + (j-m) + k]}$$

for the demand reduction for a brand from having an extra $j - m$ brands undercutting it, then demand in this segment for the firm's j th lowest-price brand is

$$X = y_0 \tilde{r}_0 + \dots + y_{b_m - 1} \tilde{r}_{b_m - 1} .$$

The ratio of the two demands X and X_m is

$$\frac{X}{X_m} = \frac{y_0 \tilde{r}_0 + \dots + y_{b_m - 1} \tilde{r}_{b_m - 1}}{y_0 + \dots + y_{b_m - 1}} \leq \tilde{r}_{b_m - 1} , \quad (30)$$

where the inequality arises since (15) implies that the parameters \tilde{r}_k increase with k .

Thus the profit obtained from the j th lowest-price brand if the firm chooses its j price somewhere in the interval $[P_{m-1}, P_m]$ satisfies

$$\begin{aligned} p_j X(p_j) &\leq p_j X_m(p_j) \tilde{r}_{b_m - 1} \\ &= \pi_m \tilde{r}_{b_m - 1} \\ &= \pi_m [r_{[B_m]} \times \dots \times r_{[B_m + (j-m-1)]]} \\ &\leq \pi_m [r_{[B_m]} \times \dots \times r_{[B_{j-1}]}] = \pi_j . \end{aligned} \quad (31)$$

Here, the second inequality arises as there are the same number of r terms ($j - m$ of them) in both expressions, the subscript of a typical term in the first expression is $B_m + k$ while the corresponding subscript in the second expression is B_{m+k} which is higher (since $B_{i+1} - B_i \geq 2$), and so the inequality follows as r_i increases with i . The final equality arises from (possibly) repeated use of (26). We deduce that the firm is not better off if it chooses its j th lowest price to be in a lower price interval $[P_{m-1}, P_m]$.

This argument applies, but requires some elaboration, in the case where firm i is the largest firm and when $n_K > M$ so that the firm has some of its prices equal to 1. Here, if $j > M$ and firm K chooses its price p_j to lie in the m th interval $[P_{m-1}, P_m]$ the resulting profit from this brand as before is no greater than $\pi_m \tilde{r}_{b_{m-1}}$. Moreover, expression (26) continues to hold even when $m \geq M$. This firm's $n_K - M$ highest prices satisfy $p_{M+1} = \dots = p_{n_K} = 1$ and together generate captive profit $q_{[B_M+1]} + \dots + q_{[B_M+n_K-M]}$. We can therefore notionally think of this firm's $(M + 1)$ th highest-price brand as having profit $\pi_{M+1} = q_{[B_M+1]}$, its next highest-price brand (if any) as having profit $\pi_{M+2} = q_{[B_M+2]}$, and so on. For $M < m \leq n_K$ we have $b_m = 1$ and so $B_M + 1 = B_{M+1}$, $B_M + 2 = B_{M+2}$, and so on. Therefore, expression (26) holds when $m \geq M$. Finally, since $B_{i+1} - B_i \geq 1$ for all $i < n_K$, the chain of relations in (31) continues to hold even if $j > M$. Therefore, firm K has no incentive to lower one of its $n_K - M$ highest prices from 1 down to a lower price.

Next consider upward price deviations for firm i . We wish to show that firm i is no better off putting its j th lowest price into a higher price segment $[P_{m-1}, P_m]$, where $m > j$. Suppose first that $m \leq n_i$ so that the firm has at least m brands. Then demand for the firm's j th lowest-price brand in that segment, denoted X , is again given by (29), although now $(j - m)$ is negative. If we write

$$\hat{r}_k = r_{[B_{m-1}+1+k-(m-j)]} \times \dots \times r_{[B_{m-1}+k]}$$

then we can write this demand as

$$X = \frac{y_0}{\hat{r}_0} + \dots + \frac{y_{b_{m-1}}}{\hat{r}_{b_{m-1}}} .$$

Since $1/\tilde{r}_k$ decreases with k we see that $X/X_m \leq 1/\tilde{r}_0$, and similarly to (31) we have

$$\begin{aligned} p_j X(p_j) &\leq \frac{\pi_m}{\hat{r}_0} \\ &= \frac{\pi_m}{r_{[B_{m-1}+1-(m-j)]} \times \dots \times r_{[B_{m-1}]}} \\ &\leq \frac{\pi_m}{r_{[B_j]} \times \dots \times r_{[B_{m-1}]} } = \pi_j . \end{aligned}$$

Therefore, this price deviation is not profitable.

Next, suppose $m > n_i$ so that firm i chooses p_j above the price of its most expensive brand in the candidate equilibrium. We have already shown that a firm with m or more brands would not wish to raise its j th price to the m th segment. The firm with $n_i < m$ brands has even less incentive to do so, for two reasons. First, in the m th interval the firm has b_m rivals whereas the firm with m or more brands has $b_m - 1$. Second, if k rivals in the m th segment undercut the firm with n_i brands, the total number of brands that undercut it is $B_{m-1} - n_i + k + j - 1$, which exceeds the $B_{m-1} - (m - 1) + k + j - 1$ that undercut the firm with m or more brands in that case.

In sum, the firm has no incentive to choose p_j in any higher interval than the designated interval $[P_{j-1}, P_j]$. Note that this analysis allows for the possibility that firm i chooses $p_j = 1$, as that option exists within the highest interval $[P_{M-1}, 1]$. We conclude that firm i has no profitable deviation from the stated strategy.

Proof of Proposition 5: The following argument is related to that used in Armstrong and Vickers (2002, p. 165). Suppose to the contrary that some firms have a higher minimum price in equilibrium than others. Specifically, suppose there is a strict subset A of the K firms that use the minimum price P_0 and suppose firm i is the firm not in A which uses the next lowest minimum price, say $P > P_0$.

With this pattern of consideration, the total reach of firm i is

$$\sigma_{n_i} = n_i(q_1 - \frac{1}{2}(n_i - 1)\beta_2) , \quad (32)$$

and the number of consumers who consider at least one brand from firm i and at least one brand from firm j is $n_i n_j \beta_2$. When firm k uses the CDF F_k for its uniform price, firm j 's total demand takes the form

$$Q_j(p) = \sigma_{n_j} - \beta_2 n_j \sum_{k \neq j} n_k F_k(p) .$$

Then for any firm $j \in A$ we have $P_0 \sigma_{n_j} \geq P Q_j(P)$ so that

$$\frac{P_0}{P} \sigma_{n_j} \geq Q_j(P) = \sigma_{n_j} - \beta_2 n_j \sum_{k \in A/j} n_k F_k(P) .$$

(Note that $F_k(P) = 0$ if $k \notin A$.) If we divide this expression by n_j and add up over all

firms in A we obtain

$$\theta \equiv \beta_2 \sum_{j \in A} n_j F_j(P) \geq \frac{\sum_{j \in A} \frac{\sigma_{n_j}}{n_j}}{k_A - 1} \left(1 - \frac{P_0}{P}\right)$$

where $k_A \geq 2$ is the number of firms in A .

For firm i we have instead $P_0 \sigma_i \leq P Q_i(P) = P(\sigma_{n_i} - n_i \theta)$, and so putting these two inequalities together we have

$$(k_A - 1) \frac{\sigma_{n_i}}{n_i} \geq \sum_{j \in A} \frac{\sigma_{n_j}}{n_j}. \quad (33)$$

However, inequality (33) cannot hold because (32) implies $\sigma_{n_i}/n_i \leq q_1$ and so (33) implies

$$\frac{1}{2} \beta_2 \sum_{j \in A} (n_j - 1) \geq q_1$$

which cannot be true as $q_1 = \beta_1 + (N - 1)\beta_2$ and $N > \sum_{j \in A} n_j$. Therefore, in equilibrium all firms use the same P_0 , which is equal to ρ_K .

Proof of Lemma 3: (i) Let $X^j(p)$ denote demand for firm i 's j th cheapest brand. A best response for firm i cannot have prices (p_1, p_2) with $p_2 > p_1 = P_0$ for the firm's two cheapest brands because by Lemma 1 that would entail

$$\frac{X^2(p_2)}{X^1(p_2)} \geq \frac{X^2(P_0)}{X^1(P_0)} = \frac{q_2}{q_1} = r_1. \quad (34)$$

But from (19) $X^2(p_2)/X^1(p_2)$ is an average of the r_j with not all weight on r_1 . With strictly decreasing r_j the average is therefore less than r_1 and so (34) cannot hold. So any best response with $p_1 = P_0$ must also have $p_2 = P_0$. By a similar argument it follows that $p_3 = P_0$ whenever $p_2 = P_0$ and hence that it is a best response for firm i to price all its brands at P_0 . Its payoff is therefore $\sigma_{n_i} P_0$.

(ii) For some other firm which has $n \geq n_i$ brands, let P_1 be the lowest price used for its cheapest brand, and suppose for a contradiction that $P_1 > P_0$. Suppose that optimal prices in equilibrium that go with this lowest price P_1 are (P_1, P_2, \dots, P_n) . The firm's profit from its n_i cheapest brands at prices (P_1, \dots, P_{n_i}) must be at least $\sigma_{n_i} P_0$ because it could achieve that profit on those brands by pricing them uniformly at P_0 (and such a deviation does not affect the profit from its more expensive brands, if any). Firm i would make strictly greater profit at prices (P_1, \dots, P_{n_i}) than the other firm – and therefore strictly

more than $\sigma_i P_0$ – because the other firm never undercuts firm i at those prices whereas firm i sometimes undercuts the other firm, and they face the same rival prices otherwise. But that is inconsistent with firm i making profit $\sigma_i P_0$, as established in part (i) of the proof.

Proof of Proposition 6: For a firm with n brands, and given the pricing strategies of its rivals, recall from section 4.1 that $X^j(p)$ denotes the demand of the firm's j th cheapest brand when that brand's price is p . If for all $j = 1, \dots, n - 1$

$$\frac{X^{j+1}(p)}{X^j(p)} \text{ is strictly decreasing in } p \quad (35)$$

then we claim that the firm will price uniformly. For if the firm's prices $p_1 \leq \dots \leq p_n$ where $p_j < p_{j+1}$ we would have

$$p_j X^j(p_j) \geq p_{j+1} X^{j+1}(p_{j+1}) ; p_{j+1} X^{j+1}(p_{j+1}) \geq p_j X^{j+1}(p_j)$$

and hence

$$\frac{X^{j+1}(p_{j+1})}{X^j(p_{j+1})} \geq \frac{X^{j+1}(p_j)}{X^j(p_j)} ,$$

which contradicts (35). (This is essentially the argument in Lemma 1.)

As a general matter $\frac{X_{j+1}(p)}{X_j(p)}$ can be expressed as a weighted average of r_i ratios, as explained in section 4.1. If the r_i are decreasing and higher p increases weight on r_i with higher i then $\frac{X_{j+1}(p)}{X_j(p)}$ will decrease with p , and we can therefore deduce the firm will use uniform pricing. We show how this argument applies to the three cases in question.

(i) Suppose a firm with n brands competes against m single-brand rivals, each of which uses the CDF $F(p)$ for its price. Then the multibrand firm's demand X^1 is (18) where

$$\phi_k(p) = \binom{m}{k} (F(p))^k (1 - F(p))^{m-k}$$

is the probability that exactly k of the m rivals undercut its price p . Then X^2/X^1 is given by (19) where

$$w_k(p) = \frac{\phi_k(p)q_{k+1}}{\phi_0(p)q_1 + \dots + \phi_m(p)q_{m+1}} .$$

Note that the weight ratio w_{k+1}/w_k is proportional to $F/(1 - F)$ which strictly increases with F , and hence with p . If p and $p' > p$ are two prices with associated weights $\{w_k\}$ and $\{w'_k\}$, this implies that w'_k is larger than w_k for all large enough k (and otherwise w'_k is

smaller than w_k).²⁵ Suppose then that the integer k^* is such that $w'_k \geq w_k$ for all $k \geq k^*$ and $w'_k < w_k$ for all $k < k^*$. Then

$$\begin{aligned} \frac{X^2(p')}{X^1(p')} - \frac{X^2(p)}{X^1(p)} &= (w'_0 - w_0)r_1 + \dots + (w'_m - w_m)r_m \\ &= \{(w'_0 - w_0)r_1 + \dots + (w'_{k^*-1} - w_{k^*-1})r_{k^*-1}\} \\ &\quad + \{(w'_{k^*} - w_{k^*})r_{k^*} + \dots + (w'_m - w_m)r_m\} \\ &< \{(w'_0 - w_0)r_{k^*} + \dots + (w'_{k^*-1} - w_{k^*-1})r_{k^*}\} \\ &\quad + \{(w'_{k^*} - w_{k^*})r_{k^*} + \dots + (w'_m - w_m)r_{k^*}\} = 0 . \end{aligned}$$

Here, the inequality follows from (13). Therefore, X^2/X^1 decreases with p , and a similar argument shows that (35) holds for all j . We deduce that the multibrand firm will offer uniform prices when it faces single-brand rivals.

(ii) Next, suppose the n -brand firm faces a single rival with m brands, where the latter firm uses uniform pricing with CDF $F(p)$. Then

$$X^1(p) = (1 - F(p))q_1 + F(p)q_{m+1}$$

and

$$\frac{X^2}{X^1} = \left[\frac{(1 - F)q_1}{(1 - F)q_1 + Fq_{m+1}} \right] r_1 + \left[\frac{Fq_{m+1}}{(1 - F)q_1 + Fq_{m+1}} \right] r_{m+1}$$

which clearly decreases with F and p given (13). A similar argument shows that (35) holds for all j , and so the n -brand firm uses uniform pricing if its m -brand rival does, and *vice versa*.

(iii) Finally, suppose that the market consists of $K + 1$ firms which each have m brands. Consider one of these firms, and suppose that each of its rivals uses uniform pricing with the uniform price chosen with CDF $F(p)$. Let

$$\phi_k(p) \equiv \binom{K}{k} (1 - F)^{K-k} F^k$$

be the resulting probability that $0 \leq k \leq K$ of those rivals choose prices that undercut p , so that

$$X^j = \phi_0 q_j + \phi_1 q_{j+m} + \phi_2 q_{j+2m} + \dots + \phi_K q_{j+Km} .$$

²⁵To see this, note that w_{k+1}/w_k strictly increasing implies that

$$w'_{k+1} - w_{k+1} > \frac{w_{k+1}}{w_k} (w'_k - w_k) ,$$

and so if $w'_k > w_k$ then the same is true for all higher k .

For $0 \leq k \leq K$ define

$$w_k = \frac{\phi_k q_{j+km}}{X^j}$$

so that

$$\frac{X^{j+1}(p)}{X^j(p)} = \sum_{k=0}^K w_k(p) \frac{q_{j+1+km}}{q_{j+km}} = \sum_{k=0}^K w_k(p) r_{j+km} .$$

Then as with part (i) the ratio w_{k+1}/w_k increases with p , and since the sequence $r_j, r_j + m, r_{j+2m}$ is decreasing given (13), condition (35) and the firm will use uniform pricing if its rival do so. This completes the proof.

Proof of Proposition 7: Suppose the single-brand firm uses the CDF F for its price. If firm m chooses prices $p_1 \leq \dots \leq p_n$ its total profit is

$$\pi_m = \sum_{i=1}^n p_i q_i [1 - (1 - r_i)F(p_i)] . \quad (36)$$

If firm m prices $1 \leq k \leq n$ brands uniformly and the remaining brands (if any) have price equal to 1, then to keep m indifferent over its uniform price, the single brand firm's CDF F is given by

$$p[1 - (1 - c_k)F(p)] = c_k , \quad (37)$$

so the minimum price is $P_0 = c_k$. Expression (37) implies $p - c_k = (1 - c_k)pF(p)$, and so m 's profit with arbitrary prices in (36) can be expressed as

$$\pi_m = \sum_{i=1}^n q_i [c_k - (c_k - r_i)p_i F(p_i)] .$$

If we write $\pi_m^* = \sum_{i=1}^n q_i r_i$ for firm m 's captive profit, we then have

$$\begin{aligned} \pi_m - \pi_m^* &= \sum_{i=1}^n q_i (c_k - r_i) [1 - p_i F(p_i)] \\ &= \sum_{j=1}^n \left(\sum_{i=1}^j q_i (c_k - r_i) [p_{j+1} F(p_{j+1}) - p_j F(p_j)] \right) \\ &= \sum_{j=1}^n \left(\sum_{i=1}^j q_i (c_k - c_j) [p_{j+1} F(p_{j+1}) - p_j F(p_j)] \right) . \end{aligned} \quad (38)$$

(In the above we used (20) and the notation $p_{n+1}F(p_{n+1}) = 1$.)

By assumption, $c_k \leq c_i$ for all i , and in addition $p_{j+1}F(p_{j+1}) \geq p_j F(p_j)$. It follows from (38) that profit π_m is maximized (and then equal to captive profit π_m^*) when $p_{j+1}F(p_{j+1}) =$

$p_j F(p_j)$ for all $j \neq k$. This outcome is achieved, in particular, by setting $p_j = 1$ for $j > k$ (if $k < n$) and by setting $p_j = p_k$ for $j < k$ (if $k > 1$). That is, by choosing a uniform price for k brands and setting $p = 1$ for all other brands (if any).

Proof of Claim 1: Recall that n_K is the number of brands supplied by the largest firm in the pre-merger market. There are therefore n_K price segments in equilibrium, where some of these might be individual prices $p = 1$ when $n_K > n_{K-1}$. Recall from the proof of Proposition 4 that equilibrium profits in the various segments are related by (26), where this formula applies even for segments when the firm sets $p = 1$. Since the profit in the highest price segment is $\pi_{n_K} = q_N$, it follows by iterated use of (26) that the profit of the cheapest brand is given by

$$\pi_1 = \frac{q_N}{r_{B_1} r_{B_2} \dots r_{B_{n_K-1}}}, \quad (39)$$

and similarly for π_2 and so on.

Now consider a merger between firms i and j , where $i < j$. This reduces the number of firms to $K - 1$, and possibly (if the merged firm is the largest firm in the post-merger market) increases the number of brands supplied by the largest firm. The impact on the b_m is to (i) reduce b_m by 1 for $m \leq n_i$, (ii) to keep b_m unchanged for $n_i < m \leq n_j$, (iii) to increase b_m by 1 for $n_j < m \leq n_i + n_j$, and (iv) to keep b_m unchanged for all higher m . Here the number of negative increments equals the number of positive increments. It follows that the impact of the merger on the B_m is initially to reduce B_m for $m < n_i + n_j$, while B_m is unchanged for $m \geq n_i + n_j$. The profit in the new lowest price segment still takes the form (39), but where each B_i is weakly smaller, and hence r_{B_i} is weakly smaller, and where there are possibly more terms r_{B_i} in the denominator. It follows that π_1 is higher after the merger, and a similar argument shows that all other price segments also have weakly higher profit after the merger.

This implies that the external impact of the merger on non-merging firms is always positive (as all firms use the lower price segments), so any profitable merger will boost industry profit.