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Abstract

We investigate a differentiated mixed duopoly in which private and public firms can choose to strategically set prices or quantities by facing a union bargaining process. For the case of a unionized mixed duopoly, only public firm is able to choose a type of contract based on the degree of substitutability in the equilibrium. Focusing on the case of substitute goods, we show that Bertrand (respectively, Cournot) competition entails higher social welfare than Cournot (respectively, Bertrand) competition if the degree of substitutability is relatively small (respectively, large). Thus, there are multiple Nash equilibria in the contract stage of the game. As a result, Singh and Vives' ranking of social welfare is reversed in a range of substitution values for which it is a dominant strategy for public firm to choose either quantity or price contracts.

JEL: C7, D43, H44, J51, L13.
Keywords: Wage Bargaining, Union, Cournot-Bertrand Competition, Mixed Duopoly.

1 Introduction

Recently, the economic implications of the mixed oligopoly market have been an issue with respect to the change in competition for market structure efficiency. This means that public firms still play an important role in most economic realms. This mixed oligopoly with private firms is common in many countries; industries such as oil, heavy manufacturing, telecommunications, and tourism are good examples of mixed oligopolies. There is a lot of existing literature that describes studies of mixed oligopolies[1]; however, that literature has largely ignored the strategic variables that private and public firms can choose in order to set prices or quantities. The few studies that have considered strategic variables are Singh and Vives (1984), Zanchettin (2006), López and Naylor (2004) and López (2007), who discuss *pure* Cournot and Bertrand competition when all firms compete among private firms[2]. These works, which deal with the choice of strategic variables for prices or quantities, suggest important implications in the determination of market outcome. However, none of these papers has considered the case in which both private and public firms choose to set prices or quantities in a mixed duopoly. Thus, the present paper will be modeled around the noncooperative game, in which the choice of strategic variables is set in a *unionized* mixed duopoly.

In a pure duopoly, Singh and Vives (1984) first show that Bertrand competition is more efficient than Cournot competition when goods are differentiated. Their study determines that Cournot equilibrium profits are greater than Bertrand equilibrium profits when goods are substitutes, and vice versa when goods are complements. They establish that when private firms play the downstream duopoly game without unions' wage bargaining, it creates a dominant strategy for private firms in a pure duopoly to choose quantity contracts if goods are substitutes. In game

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[1]See De Fraja and Delbono (1990) and Nett (1993) for general reviews of the mixed oligopoly models. For recent literature on mixed oligopoly, see Barcena-Ruiz (2007), Matsumura (1998), Matsumura and Kanda (2005), Matsumura and Matsushima (2004, 2006), Lu and Poddar (2006), etc.

[2]In this paper, we use "*pure Cournot and Bertrand competition* in duopoly" when all firms are private firms in order to distinguish the concept of mixed duopoly.

theory terms, a variety of strategic settings has emerged based on the choice of strategic variables, such as occurs in Cournot versus Bertrand equilibrium. Based on the framework of Singh and Vives (1984), Cheng (1985) establishs more general results in geometric analysis. Häckner (2000) extends the standard model of vertical product differentiation. In addition, by adding different assumptions into Singh and Vives' (1984) framework, Dastidar (1997) explores the fact that in a homogeneous product market, Bertrand equilibrium prices may not be lower than Cournot equilibrium prices if they fall under the equal sharing rule with asymmetric costs. Many works that address oligopoly models include those of Qiu (1997), Lambertini (1997), Okuguchi (1987), Amir and Jin (2001), among others. On the other hand, by enlarging the parameter space considered by Singh and Vives (1984) to allow for a wider range of cost and demand asymmetry, Zanchettin (2006) finds that Singh and Vives' (1984) result that firms always make larger profits under quantity competition than under price competition fails to hold[3].

Along these lines, we address the issue of whether or not the standard results of the ranking of Cournot and Bertrand equilibrium outcomes under differentiated duopoly hold up in the case of a unionized mixed duopoly. More specifically, we illustrate the way in which the choice of strategic variables to set prices or quantities affects social welfare in a mixed duopoly. In addition, a comparison is made between the social welfare of a unionized mixed duopoly and the choice of strategic variables.

There have been some attempts to introduce union utility into a model of the choice of strategic variables to set prices or quantities, namely López (2007) and López and Naylor (2004). López and Naylor (2004) compare Cournot and Bertrand equilibria in a downstream differentiated duopoly, in which wages are paid by each downstream in the outcome of a strategic bargain with its upstream labor union. They show that Singh and Vives' (1984) result holds unless unions are powerful and place considerable weight on the wage argument in their utility function. López (2007) analyzes the more general case of profit-maximizing upstreams that sell input to a duopoly in exchange for a negotiated input price[4]. The papers that are closest to the present model of unionized mixed duopoly are authored by De Fraja (1993a), Haskel and Sanchis (1995) and Haskel and Szymanski (1993). Furthermore, Ishida and Matsushima (2008a) analyzed the optimal framework, focusing on wage regulation that is imposed on both the public firm and the union.

Yet, recent developments in literature have not investigated the issue of how private and public firms play the noncooperative game of choosing strategic variables in the context of wage bargaining in a mixed duopoly. Consequently, our paper differs from previous works on unionized mixed oligopoly, which focused on privatization without public and private firms' choice of strategic variables. This paper investigates a mixed duopoly in which the private and public firms can choose to strategically set prices or quantities by facing a union bargaining process.

[3] Wang (2008) shows that while profit ranking between price and quantity competition can be partially reversed the traditional result by Singh and Vives (1984) that firms always choose a quantity in a two-stage game continues to hold in the enlarged parameter space.

[4] On the other hand, Manasakis and Vlassis (2006) extends Singh and Vives' (1984) framework that there is no ex-ante commitment over the type of contract which each firm will offer consumers. In the context of a unionized symmetric duopoly, they argue that the mode of competition which in equilibrium emerges is the one that entails the most beneficial outcome for both the firm and its labour union, given the choice of the rival firm/union pare. In addition, motivated by the institutional diversity of unionization structures and the growth of foreign direct investment (FDI), the current bargaining process between firms and unions has been developed independently. As identified by Naylor (1998, 1999), Zhao (1995), Skaksen and Sørensen (2001), Haucap and Wey (2004), and Leahy and Montagna (2000), the amount of domestic production is decided through union bargaining when a firm undertakes FDI. In another related paper, the relationship between the amount of production and the union has been explored among domestic private firms (Naylor, 2002). There are many studies considering unionized international oligopoly, see for instance, Straume (2003), Ishida and Matsushima (2008b), Mukherjee and Suetrong (2007) and references therein.

In accordance with Singh and Vives (1984) and López (2007), we analyze a noncooperative three-stage game in which two firms produce differentiated goods. Although previous analyses such as López and Naylor (2004) and López (2007) on the union's utility focused on collective bargaining in pure ologopoly, they did not analyze union utility in the mixed duopoly.

The timing of the game of the present paper is as follows. In the first stage, the private and public firms simultaneously commit to choosing a strategic variable for either price or quantity, that is a type of contract, to set in the unionized mixed duopoly. In the second stage, each union independently bargains over its wages, keeping in mind each strategic variable of the private and public firms. In the third stage, each firm chooses its quantity or price simultaneously, in order to maximize its objective knowledge of the strategic variable of the public and private firms and of the wage levels in previous stages. Given this three-stage game model, we show that only public firm is able to choose a type of contract based on the degree of substitutability in the equilibrium, regardless of the choice of the private firms' strategic variable. Moreover, such strategic choice commitment by public firm can either worsen or improve social welfare when all firms choose different types of contracts. While there is a dominant strategy for the public firm to choose Bertrand or Cournot contracts contingent on the degree of substitution, there is not a dominant strategy for private firms. This result contrasts with Singh and Vives' conclusion (1984), in which a dominant strategy for the private firms in a pure duopoly is to choose the quantity contract if goods are substitutes. This occurs because we relinquish Singh and Vives' assumption and treat the type of contract as it exists in a unionized mixed duopoly. Thus, a private firm's profit is determined by a public firm's choice of contract. As a result, the endogenous type of contract is determined by public firms, regardless of their choice of private firms' strategy. There is thus a range of substitution values for which it is a dominant strategy for public firm to choose either a quantity or price contract; hence, we show that when goods are substitutes, Bertrand (respectively, Cournot) competition entails higher social welfare than Cournot (respectively, Bertrand) competition if the degree of substitutability is relatively small (respectively, large). Furthermore, there are multiple Nash equilibria in the contract stage of the game.

To the best of the author's knowledge, only one work has attempted to compare Bertrand and Cournot outcomes in the mixed oligopoly: Ghosh and Mitra (2008). More specifically, Ghosh and Mitra (2008) derive their results comparing Counot and Bertrand competition in the mixed oligopoly where the endogenous type of contract is not determined by public firm, and there exists no union trade in the mixed oligopoly. Hence, Ghosh and Mitra (2008) show that the ranking of social welfare is exogenously determined. The theoretical results of the present study, however, treat the problem at the differentiated mixed duopoly in which private and public firms can choose to strategically set prices or quantities by facing a union bargaining process. Therefore, our paper differs from the existing literature in at least two important ways. First, the existing studies in mixed oligopoly considered exogenous type of contract rather than endogenous type of contract. Second, previous works focused on reversal result in Cournot-Bertrand profit differential in pure oligopoly market, while our paper investigates not only the case in which both private and public firms choose to set prices or quantities depending on the degree of substitutability but also how social welfare are affected by the type of contrat structure.

The organization of the paper is as follows. In Section 2, we describe the model. Section 3 presents fixed-timing games regarding the type of contract and determines firms' endogenous choice of strategic variable. Concluding remarks appear in Section 4.

2 The Basic Model

The basic structure of our research is a differentiated duopoly model, which is a simplified version of Singh and Vives' model (1984). Consider two single-product firms producing differentiated products that are supplied by a public firm 0 and a private firm 1. We assume that the representative consumer's utility is a quadratic function given by

$$U = x_i + x_j - \frac{1}{2}\left(x_i^2 + 2cx_ix_j + x_i^2\right),$$

where x_i denotes the output of firm $i = 0, 1$. The parameter $c \in (0, 1)$ is a measure of the degree of substitutability among goods. Thus, the inverse demand is characterized by

$$p_i = 1 - cx_j - x_i; \quad i \neq j, i = j = 0, 1, \tag{1}$$

where p_i is firm i's market price and x_i denotes the output of firm $i = 0, 1$. Hence we can obtain the direct demands as

$$x_i = \frac{(1-c) + cp_j - p_i}{1 - c^2} \tag{2}$$

provided that quantities are positive.

To analyze the union's wage bargaining, we also assume that the public and private firms are unionized, and wages $w_i, i = 0, 1$ are determined as a consequence of bargaining between firms and unions. Let $\overline{w}$ denote the reservation wage. Thus, we assume that the union sets the wage while public and private firms decide the level of employment unilaterally. Taking $\overline{w}$ as given, the union's optimal wage setting strategy w_i regarding firm i is defined as

$$u_i = (w_i - \overline{w})^{\theta} x_i \quad \text{where} \quad i = 0, 1. \tag{3}$$

As Haucap and Wey (2004), Leahy and Montagna (2000) and Lommerud *et al.* (2003) suggested, we assume that the union possesses full bargaining power $\theta = 1$ and $\overline{w} = 0$ to show our results in the simple way[5].

The firms are homogeneous with respect to productivity. Each firm adopts a constant returns-to-scale technology where one unit of labor is turned into one unit of the final good. The price of labor (i.e., wage) that firm i has to pay is denoted by $w_i, i = 0, 1$.

To specify the public firm 0's objective function SW, and each firm's profit π_i, as

$$SW = U - \sum_{i=0}^{1} px_i + \sum_{i=0}^{1} (\pi_i + u_i),$$
$$\pi_i = (p_i - w_i)x_i, \quad i = 0, 1,$$

where $U - \sum_{i=0}^{1} px_i$ is consumer surplus, and each firm π_i is the profit of both the private and public firm, and u_i is the union's utility for both the private and public firm. The objective function of the public firm is the sum of consumer surplus, profit of all firms and the union's utility for all the firms.

This study consider that each firm can make two types of binding contracts with consumers as described by Singh and Vives (1984) and López (2007). Thus, a three-stage game is conducted. The timing of the game is as follows: In the first stage, the private and public firms

[5] As Naylor (1998, 1999), Haucap and Wey (2004), Leahy and Montagna (2000) and Lommerud *et al.* (2003) suggested, this is because wages claims are decided by the elasticity of labor demand rather than firm's profit. Furthermore, see also Oswald and Turnbull (1995).

simultaneously commit to choose the strategy variable, either price or quantity i.e., a type of contract, to set in the unionized mixed duopoly. In the second stage, if each firm's union is allowed to bargain collectively, each union bargains over its wage w_i simultaneously. In the third stage, each firm decides its optimal production knowing each union's wage level and the type of contract. As in Singh and Vives (1984) and López (2007), we adopt the same assumption that there are prohibitively high costs associated with changing the type of contract in the first stage.

3 Bargaining in a Mixed Duopoly

3.1 Results: Fixed Contract Motives with Solutions

Before using the type of contract to apply the model and identify the point of equilibrium, four different cases of contract games are explained. In Bertrand competition, firms set prices; in Cournot competition, firms set quantities. In mixed cases, firm 0 sets the price and firm 1 sets the quantity, and vice versa. Thus, it is solved by backward induction, i.e., the solution concept used is the subgame perfect Nash equilibrium (SPNE).

3.1.1. [**Competition Game in Cournot**]: Assume that each firm i faces the inverse demand functions given by $p_i = 1 - cx_j - x_i$. In the third stage, the public firm's objective is to maximize welfare which is defined as the sum of consumer surplus, each firm's profit, and each union's utility:

$$SW = U - p_0x_0 - p_1x_1 + \pi_1 + u_1 + \pi_0 + u_0 = U.$$

Given w_1 for the private firm, the public firm's maximization problem is as follows:

$$\max_{x_0} \quad SW = U \quad \text{s.t.} \quad (p_0 - w_0)x_0 \geq 0.$$

The constraint implies that there is some lowerbound restriction on the public firm's profit, i.e., the public firm faces a budget constraint[6].

Denoting the multiplier of the budget constraint λ^{cc}, the Lagrangian equation can be written as

$$L = x_1 + x_0 - \frac{(x_1^2 + x_0^2 + 2cx_0x_1)}{2} + \lambda^{cc}(x_0 - x_0^2 - cx_1x_0 - w_0x_0).$$

Taking w_1 as given, the first-order conditions are given by

$$\frac{\partial L}{\partial x_0} = 1 - cx_1 - x_0 + \lambda(1 - 2x_0 - cx_1 - w_0) = 0 \tag{4}$$

$$\frac{\partial L}{\partial \lambda^{cc}} = 1 - cx_1 - x_0 - w_0 = 0. \tag{5}$$

On the other hand, the first-order condition for the private firm is given by

$$\frac{\partial \pi_1}{\partial x_1} = 0 \Leftrightarrow x_1 = \frac{1}{2}(1 - cx_0 - w_1). \tag{6}$$

[6] If both unions aim at maximizing wage level simultaneously and the public firm's union does not face the budget constraint with a simple Stone-Geary utility function $u_i = (w_i - \overline{w})^\theta x_i$, the public firm's union can unlimitedly raise its wage because the optimal output level of the public firm is independent of the wage.

Solving the first-order conditions (5) and (6), we obtain,

$$x_0 = \frac{2 - c - 2w_0 + cw_1}{2 - c^2}, \tag{7}$$

$$x_1 = \frac{1 - c - w_1 + cw_0}{2 - c^2}, \tag{8}$$

$$\lambda^{cc} = \frac{(2 - c^2)w_0}{2 - c - 2w_0 + cw_1}. \tag{9}$$

To solve for Lagrangian equation, the budget constraint is momentarily binding. We check ex-post that the omitted this constraint is binding.

In the second stage of this case, each wage is set to maximize its firm's union utility: $u_i = x_i w_i$. To do this, the two independent maximization problems should be considered simultaneously. Using (7) and (8), the problem for union i is defined as

$$\max_{w_0} \ u_0 = w_0 x_0 = \frac{w_0(2 - c - 2w_0 + cw_1)}{2 - c^2},$$

$$\max_{w_1} \ u_1 = w_1 x_1 = \frac{w_1(1 - c - w_1 + cw_0)}{2 - c^2}.$$

This implies the following first-order condition

$$w_0 = \frac{2 - c + cw_1}{4}, \quad w_1 = \frac{1 - c + cw_0}{2}. \tag{10}$$

Straightforward computation yields each an equilibrium wage, denoted as w_i^{cc} is obtained by solving (10), and substituting w_i^{cc} into (7) and (8) yields the equilibrium output x_i^{cc}. Thus, we have the following result which is the same results as in Ishida and Matsushima (2008a)[7].

Lemma 1 (Ishida and Matsushima, 2008a): *Suppose that each firm's union is allowed to engage in decentralized bargaining. Then, the equilibrium wages and output levels are given by*

$$w_0^{cc} = \frac{4 - c - c^2}{8 - c^2}, \quad w_1^{cc} = \frac{4 - 2c - c^2}{8 - c^2};$$

$$x_0^{cc} = \frac{2(4 - c - c^2)}{(2 - c^2)(8 - c^2)}, \quad x_1^{cc} = \frac{4 - 2c - c^2}{(2 - c^2)(8 - c^2)}.$$

Finally, noting that $SW^{cc} = U^{cc}$ and π_1^{cc}, we can compute the social welfare and private firm's profit, SW^{cc} and π_1^{cc} as follows;

$$SW^{cc} = \frac{304 - 144c - 256c^2 + 92c^3 + 67c^4 - 12c^5 - 6c^6}{2(8 - c^2)^2(2 - c^2)^2} \tag{11}$$

$$\pi_1^{cc} = \frac{(4 - 2c - c^2)^2}{(8 - c^2)^2(2 - c^2)^2}. \tag{12}$$

3.1.2. [Competition Game in Bertrand]; Consider that firm i faces the following direct demand function

$$x_i = \frac{(1 - c) + cp_j - p_i}{1 - c^2}.$$

[7] In Ishida and Matsushima (2008a), the calculation of $\lambda^{cc} = \frac{(2-c^2)w_0}{2c - 2w_0 + cw_1}$ is not correct. However, we can check that substituting lemma 1 into (9) yields $\lambda^{cc} = \frac{2(1+c)(4-3c)+c^2(c+c^2)}{8-2c} > 0$, which the budget constraint is binding.

In the third stage, by maximization social welfare (respectively, profit) each firm sets its price as a best response to any price chosen by its private firm (respectively, the public firm). The public firm's objective is given as in the previous case as follows:

$$
\begin{aligned}
\max_{p_0} U = & \frac{(1-c)+cp_1-p_0}{1-c^2} + \frac{(1-c)+cp_0-p_1}{1-c^2} \\
& - \frac{1}{2}\left\{\frac{[(1-c)+cp_1-p_0]^2+[(1-c)+cp_0-p_1]^2+2c[(1-c)+cp_0-p_1][(1-c)+cp_1-p_0]}{(1-c^2)^2}\right\} \\
\text{s.t.} \quad & \frac{(p_0-w_0)[(1-c)+cp_1-p_0]}{1-c^2} \geq 0.
\end{aligned}
$$

Denoting the multiplier of the budget constraint λ^{bb} and repeating the same process as in Competition Game in Cournot yields the first-order conditions of the Lagrangian equation with respect to λ^{bb} and p_0:

$$
\frac{\partial L}{\partial p_0} = 0 \Leftrightarrow \lambda^{bb} = \frac{(3-2c)c^2+(2-4c^2)p_0+(2c^3-2c)p_1-2c^2(1-c)}{2(1-c+cp_1-p_0)(1-c^2)}, \tag{13}
$$

$$
\frac{\partial L}{\partial \lambda^{bb}} = p_0 - w_0 = 0. \tag{14}
$$

On the other hand, the first-order condition for the private firm is given by

$$
\frac{\partial \pi_1}{\partial p_1} = 0 \Leftrightarrow p_1 = \frac{1-c+cp_0+w_1}{2}. \tag{15}
$$

By using x_i and solving the these two equations (14) and (15) problems yields

$$
x_0 = \frac{(1-c)(2+c)+cw_1-(2-c^2)w_0}{2(1-c^2)}, \tag{16}
$$

$$
x_1 = \frac{1-c-w_1+cw_0}{2(1-c^2)}. \tag{17}
$$

In the second stage of this case, each wage is set to maximize the its own firm's union: $u_i = x_i w_i$. In the analysis that follows, we again focus on the union's maximization problem. Using (16) and (17), the problems for union i are defined as

$$
\max_{w_0} \ u_0 = w_0 x_0 = \frac{w_0[(1-c)(2+c)+cw_1-(2-c^2)w_0]}{2(1-c^2)}, \tag{18}
$$

$$
\max_{w_i} \ u_1 = w_1 x_1 = \frac{w_1(1-c-w_1+cw_0)}{2(1-c^2)}. \tag{19}
$$

The best reply functions for the public firm 0 and the private firm 1 are $w_0 = \frac{(1-c)(2+c)+cw_1}{2(2-c^2)}$ and $w_1 = \frac{1-c+cw_0}{2}$, respectively. Thus, straightforward computation yields each an equilibrium wage, denoted as w_i^{bb} is obtained by maximizing (18) and (19), and substituting w_i^{bb} into (16) and (17) yields the equilibrium output x_i^{bb} and price p_i^{bb}. Thus, we have the following result.

Lemma 2: *Suppose that each firm's union is allowed to engage in decentralized bargaining. Then, the equilibrium wage, output and price levels are given by*

$$w_0^{bb} = \frac{4-c-3c^2}{8-5c^2}, \quad w_1^{bb} = \frac{4-2c-3c^2+c^3}{8-5c^2};$$
$$x_0^{bb} = \frac{8-2c-10c^2+c^3+3c^4}{2(1-c^2)(8-5c^2)}, \quad x_1^{bb} = \frac{4-2c-3c^2+c^3}{2(1-c^2)(8-5c^2)};$$
$$p_0^{bb} = \frac{4-c-3c^2}{8-5c^2}, \quad p_1^{bb} = \frac{12-6c-9c^2+3c^3}{2(8-5c^2)}.$$

Substituting lemma 2 into (13) then we have

$$\lambda^{bb} = \frac{(1-c^2)(8-2c-10c^2+c^3+3c^4)}{16+4c^2+38c^3+6c^6-28c-4c^4-18c^5} > 0$$

which shows that the budget constraint is binding. Finally, noting that $SW^{bb} = U^{bb}$ and π_1^{bb}, we can compute the social welfare and private firm's profit as SW^{bb} and π_1^{bb} respectively;

$$SW^{bb} = \frac{304-144c-844c^2+316c^3+795c^4-224c^5-330c^6+51c^7+48c^8}{8[(1-c^2)(8-5c^2)]^2}, \tag{20}$$
$$\pi_1^{bb} = \frac{(4-2c-3c^2+c^3)^2}{4(1-c^2)(8-5c^2)^2}. \tag{21}$$

3.1.3. [Firm 0 sets price, firm 1 sets quantity]: Let firm 0 optimally choose its price as a best response to any quantity chosen private firm 1, and let private firm 1 optimally choose its quantity as a best response to any price chosen public firm 0. The demand function that each firm i faces are given by

$$x_0 = 1 - cx_1 - p_0 \quad \text{and} \quad p_1 = 1 - c + cp_0 - (1-c^2)x_1. \tag{22}$$

In stage three, by maximization social welfare (respectively, profit) each firm sets its price as a best response to any price chosen by its private firm (respectively, the public firm). The public firm's objective is given as in the previous case as follows:

$$\max_{p_0} U = 1 - cx_1 - p_0 + x_1 - \frac{1}{2}[(1-cx_1-p_0)^2 + x_1^2 + 2cx_1(1-cx_1-p_0)]$$
$$\text{s.t.} \quad (p_0 - w_0)(1 - cx_1 - p_0) \geq 0.$$

Denoting the multiplier of the budget constraint λ^{bc} and repeating the same process as in previous cases yields the first-order conditions of the Lagrangian equation with respect to p_0 and λ^{bc}:

$$\frac{\partial L}{\partial p_0} = -p_0 + \lambda^{bc}(1 - cx_1 - 2p_0 + w_0) = 0, \tag{23}$$
$$\frac{\partial L}{\partial \lambda^{bc}} = p_0 - w_0 = 0. \tag{24}$$

In the third stage of this case, we obtain the pair (x_1, p_0) written as

$$x_1 = \frac{1 - c + cp_0 - w_1}{2(1-c^2)}, \tag{25}$$

$$p_0 = w_0. \tag{26}$$

Substituting the pair (x_1, p_0) into the pair (x_0, p_1) yields

$$x_0 = \frac{(1-c)(2+c) + cw_1 - (2-c^2)w_0}{2(1-c^2)}, \tag{27}$$

$$p_1 = \frac{1 - c + cw_0 + w_1}{2}. \tag{28}$$

Then, optimal wages are set to maximize union's firm including the public union's utility: $u = x_i u_i$. The best reply functions for the private firm 1 and the public firm 0 are as follows.

$$w_1 = \frac{1 - c + cw_0}{2} \quad \text{and} \quad w_0 = \frac{(1-c)(2+c) + cw_1}{2(2-c^2)}.$$

Straightforward computation yields the same results as in lemma 2 since best reply functions are the same, i.e., (16) equals to (27) and (17) equals to (25). Thus, $w_i^{bc} = w_i^{bb}, x_i^{bc} = x_i^{bb}$ and $p_i^{bc} = p_i^{bb}$. Therefore, we obtain the same social welfare and the private firm's profit, i.e., $SW^{bc} = SW^{bb}$ and $\pi_1^{bc} = \pi_1^{bb}$. Substituting equilibrium values into (23) yields

$$\lambda^{bc} = \frac{p_0^{bc}}{1 - cx_1 - 2p_0^{bc} + w_0^{bc}} = \frac{2(1-c^2)(4-c-3c^2)}{8 - 2c - 10c^2 + c^3 + 3c^4} > 0. \tag{29}$$

3.1.4. [Firm 1 sets price, firm 0 sets quantity]: Let firm 1 optimally choose its price as a best response to any quantity chosen private firm 0, and let private firm 0 optimally choose its quantity as a best response to any price chosen public firm 1. The demand function that each firm i faces is given by

$$x_1 = 1 - cx_0 - p_1 \quad \text{and} \quad p_0 = 1 - c + cp_1 - (1-c^2)x_0. \tag{30}$$

Thus, the public firm's objective is given as in the previous case as follows:

$$\max_{x_0} U = 1 - cx_0 - p_1 + x_0 - \frac{[(1 - cx_0 - p_1)^2 + x_0^2 + 2cx_0(1 - cx_0 - p_1)]}{2}$$
$$\text{s.t.} \quad [1 - c + cp_1 - x_0(1-c^2) - w_0]x_0 \geq 0$$

Denoting the multiplier of the budget constraint λ^{cb}, the Lagrangian equation can be written as

$$L = 1 - cx_0 - p_1 + x_0 - \frac{[(1 - cx_0 - p_1)^2 + x_0^2 + 2cx_0(1 - cx_0 - p_1)]}{2} + \lambda^{cb} x_0[1 - c + cp_1 - (1-c^2)x_0 - w_0].$$

Taking w_i as given, the first-order conditions are given by

$$\frac{\partial L}{\partial x_0} = -x_0 - c^2 x_0 + 2cx_0 - 2cp_0 + p_1 + \lambda^{cb}[1 - c + cp_1 + 2c^2 x_0 - w_0] = 0, \tag{31}$$

$$\frac{\partial L}{\partial \lambda^{cb}} = 1 - c + cp_1 - 2(1-c^2)x_0 - w_0 = 0. \tag{32}$$

In the second stage of this case, from profit and social welfare optimization for each firm, we obtain the pair (p_1, x_0) as

$$p_1 = \frac{1 + w_1 - cx_0}{2} \quad \text{and} \quad x_0 = \frac{1 - c + cp_1 - w_0}{1 - c^2}. \tag{33}$$

Substituting (p_1, x_0) into (30) yields

$$p_1 = \frac{1 - c + w_1 - c^2 w_1 + c w_0}{2 - c^2} \quad \text{and} \quad x_0 = \frac{2 - c - 2w_0 + c w_1}{2 - c^2}. \tag{34}$$

In addition, substituting p_1 into x_1 yields

$$x_1 = \frac{1 - c + c w_0 - w_1}{2 - c^2}. \tag{35}$$

Then, optimal wages are set to maximize union's firm including the public union's utility: $u = x_i u_i$. This implies the following first-order condition

$$w_0^{cb} = \frac{4 - c - c^2}{8 - c^2}, \quad w_0^{cb} = \frac{4 - 2c - c^2}{8 - c^2}.$$

Thus,

$$p_0^{cb} = \frac{8 - 2c - 6c^2 + c^3 + c^4}{(2 - c^2)(8 - c^2)}, \quad p_1^{cb} = \frac{12 - 6c - 7c^2 + 2c^3 + c^4}{(2 - c^2)(8 - c^2)}.$$

Similar to the case of 3.1.3. [Firm 0 sets price, firm 1 sets quantity], straightforward computation yields the same results as in lemma 1 since best reply functions in lemma 1 equal to (34) and (35). Thus, $w_i^{cc} = w_i^{cb}$ and $x_i^{cc} = x_i^{cb}$. Therefore, we obtain the same social welfare and the private firm's profit, i.e., $SW^{cc} = SW^{cb}$ and $\pi_1^{cc} = \pi_1^{cb}$. Substituting equilibrium values into (31) then we have

$$\lambda^{cb} = \frac{x_0(1 - c)^2 + 2cp_0 - p_1}{1 - c + cp_1 + 2c^2 x_0 - w_0} = \frac{-4 + 6c + 13c^2 - 26c^3 - c^4 + 2c^5}{-8 - 6c^2 + 2c + 2c^3 + 2c^4} > 0.$$

3.2 Comparative Statics for Social Welfare in a Mixed Duopoly

Having derived the equilibriums for four fixed types of contracts and social welfare levels in the previous subsection, the type of contract can be determined endogenously by taking each social welfare level and each private firm's profit as givens.

In order to employ the three-stage game, let "C" and "B" represent, respectively, Cournot and Bertrand competition with regard to each firm's choice. In this subsection, the SPNE will be found in the first stage for any given pair of competitions. Thus, the payoff matrix for the contract game can be represented by the following Table 1.

Table 1: Contract Game

Firm 0

		C	B
Firm 1	C	π_1^{cc}, SW^{cc}	π_1^{bc}, SW^{bc}
	B	π_1^{cb}, SW^{cb}	π_1^{bb}, SW^{bb}

Since comparing each SW^{lm}, $lm = cc, cb, bb, bc$ in Table 1 becomes complicated, so we need to use numerical examples to illustrate the impact of the degree of substitutability.

Table 2: Numerical Examples in Contract Game

value of c	$SW^{bb} = SW^{bc}$	$SW^{cc} = SW^{cb}$
$\vdots$	$\vdots$	$\vdots$
0.2821	0.530495786	0.530493668
0.2822	0.530476688	0.530476015
0.2823	0.53045759	0.530458364
0.2824	0.530438492	0.530440716
0.2825	0.530419394	0.530423071
$\vdots$	$\vdots$	$\vdots$

Using this computation, numerical analysis shows that social welfare is greater with Bertrand competition when $c \leq 0.2823$. Hence the cutoff level $\hat{c} \fallingdotseq 0.2823$ such that $SW^{bb} = SW^{bc} > SW^{cc} = SW^{cb}$ when $c \leq \hat{c} \fallingdotseq 0.2823$. We can therefore show that $SW^{bb} = SW^{bc} > SW^{cc} = SW^{cb}$ if the goods are substitutes for $c \leq \hat{c} \fallingdotseq 0.2823$. Thus, for the public firm 0, choosing Bertrand (respectively, Cournot) is strictly dominated by choosing Cournot (respectively, Bertrand). If $c > \hat{c}$ (respectively, $c \leq \hat{c}$), so the public firm 0 will not choose a Bertrand (respectively, Cournot) type of contract. Clearly, there are multiple sustained SPNEs in this stage: (C, C) and (C, B) if $c > \hat{c}$ since $\pi_1^{bb} = \pi_1^{bc}$ and $\pi_1^{cc} = \pi_1^{cb}$. Otherwise it would be (B, B) and (B, C). Multiple SPNEs of the three-stage game in a mixed duopoly are found and stated in the following proposition.

Proposition 1: *Suppose that goods are substitutes and all firms' unions are allowed to use decentralized bargaining. In that case, the public firm chooses a Bertrand type of contract and the private firm chooses either a Bertrand or a Cournot type of contract if* $c \leq \hat{c} \fallingdotseq 0.2823$ *in the first stage. Otherwise, the public firm chooses a Cournot type of contract and the private firm chooses either a Bertrand or Cournot type of contract if* $c > \hat{c}$ *in the first stage.*

Restricting attention to the subgame perfect Nash equilibrium of the three-stage game, one significant result can be derived from Proposition 1: only public firm chooses a type of contract depending on the degree of substitutability in the equilibrium. The sustaining of multiple SPNEs from $\pi_1^{bb} = \pi_1^{bc}$ and $\pi_1^{cc} = \pi_1^{cb}$ plays an important role in the derivation of the result. Moreover, such a choice of strategic commitment by public firm can worsen or improve social welfare when all firms choose different types of contracts, since there is a dominant strategy for the public firm to choose a Bertrand or Cournot type of contract that is contingent on the value of c, and there is not a dominant strategy for the private firm. Hence, when goods are substitutes, Bertrand (respectively, Cournot) competition entails higher social welfare than Cournot (respectively, Bertrand) competition if the degree of substitutability is relatively small (respectively, large), i.e., $c \leq \hat{c}$ (respectively, $c > \hat{c}$). This result contrasts with Singh and Vives' conclusion (1984) in the sense that, in the setting of Singh and Vives (1984), in which the dominant strategy for the private firm in a pure duopoly is to choose the quantity contract if goods are substitutes regardless of the degree of substitutablility. This will hold true in our paper if $c > \hat{c} \fallingdotseq 0.2823$ because we relinquish Singh and Vives' assumption and treat the type of contract as if it exists in a unionized mixed duopoly. Thus, private firm's profit is determined by public firms' choosing the type of contract.

In this setting, the endogenous type of contract is determined by the public firm regardless of the private firm's choice of strategy. As a result, there is a range of c values for which it holds that it is a dominant strategy for the public firm to choose either quantity or price contract; hence there are multiple SPNEs in the contract stage of the game.

The proposition 1 is in contrast to those reported by Ghosh and Mitra (2008), who analyzed a case of comparing Counot and Bertrand competition in the mixed oligopoly without both trade union and endogenous type of contract. Our main results show that reversal social welfare in Cournot-Bertrand when there is sufficiently large the degree of substitutability in the contract stage of the game. Consequently, we found that there are multiple Nash equilibria in the contract stage of the game when there is unionized mixed duopoly. Even though our result is partially similar to the report of Ghosh and Mitra (2008) when there is sufficiently small the degree of substitutability, we found that our main resluts differ from Ghosh and Mitra (2008) based on the exgoneous type of contract.

4 Concluding Remarks

We investigated a differentiated mixed duopoly in which private and public firms can choose to strategically set prices or quantities by facing a union bargaining process. A choice of strategic variables was proposed endogenously in the first stage. For the case of a unionized mixed duopoly, only public firm is able to choose a type of contract based on the degree of substitutability in the equilibrium. This occurs because there is a range of substitution values for which it is a dominant strategy for public firm to choose either quantity or price contracts. Thus, there are multiple Nash equilibria in the contract stage of the game. This result contrasts with the findings of Singh and Vives (1984), in whose research it is a dominant strategy for private firms in a pure duopoly to choose a quantity contract if goods are substitutes. This holds true in our paper if the degree of substitution is relatively large. This occurs because we relinquish Singh and Vives' assumption and deal with the type of contract as it exists in a unionized mixed duopoly. Thus, our paper also suggests that private firm's profit is determined by public firm's choice of contract.

In conclusion, we will discuss the limitations of our paper. We have used the simplifying assumption that private and public firms are symmetric due to identical production costs and a decentralized unionization structure. By making these assumptions, we neglected to take into account any cost difference that may arise due to the mixed bargaining that occurs between private and public firms. It is worth noting that the centralized (or, conversely, decentralized) wage setting process assumes that the centralized (or, conversely, decentralized) union sets a single (or, conversely, different) wage for all firms when public firms are less efficient than private firms. Thus, this paper does not investigate whether or not the degree of centralization of union bargaining matters for private firms when choosing different bargaining regimes and using strategic variables. Moreover, we did not extend the model to consider a situation where the public firm competes with both domestic and foreign private firms. Also, in this paper we do not analyze the policy analysis to privatization; in addition, a broader range of policies – such as such as lump-sum, taxation, and subsidies – is worth considering. Extending our model to these direction remains for future research.

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