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Forward Guidance in the Nonlinear New Keynesian Model

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Abstract

The forward-guidance puzzle refers to the implausibly large effects of anticipated future interest rate changes on current output and inflation, as predicted by standard New Keynesian models. In this paper, we analyze both theoretically and numerically the nonlinear model that underlies the canonical linearized framework, explicitly tracking the full distribution of prices across firms. We show that large output expansions arise only under extreme and economically implausible circumstances: firms that are unable to reset prices are forced to sell below marginal cost while satisfying unbounded demand, thereby accumulating arbitrarily large losses. When we modify the model so that non-reoptimizing firms can at least set prices equal to marginal cost, output and inflation remain bounded and moderate. However, under this modification not all forward-guidance announcements are feasible in equilibrium. Our results identify a neglected microeconomic assumption as the root cause of the forward-guidance puzzle and clarify the limits of New Keynesian models in the analysis of large or persistent monetary shocks.

1 Introduction

The New Keynesian (NK) model has been highly successful in capturing the qualitative and quantitative responses of key macroeconomic variables to monetary policy shocks under normal business-cycle conditions (see, among others, Christiano, Eichenbaum, and Evans, 2005, or Smets and Wouters, 2007). Following the Global Financial Crisis, however, nominal interest rates in many advanced economies reached the zero lower bound, prompting central banks to rely on unconventional policy tools. One such tool was forward guidance: explicit announcements about the future path of policy rates, typically committing to keep rates at exceptionally low levels for an extended period.

Several authors have shown that standard New Keynesian models predict implausibly large effects of forward-guidance announcements on current output and inflation. In particular, Del Negro, Giannoni, and Patterson (2013) and Carlstrom, Fuerst, and Paustian (2015), building on the early work of Eggertsson and Woodford (2003), demonstrate that the impact of forward guidance grows exponentially with the length of the announced commitment. For standard parameter values, the model can generate double-digit output gaps following commitments that last only a few quarters.

These predictions stand in sharp contrast to the empirical evidence. During the period in which forward guidance was most actively used (roughly 2008–2015), its observed macroeconomic effects were modest (see e.g. Campbell, Fisher, Justiniano and Melosi, 2017). More broadly, estimates of the U.S. output gap since 1950 indicate that output has rarely deviated from potential by more than a few percentage points (see Figure 1). The extreme booms implied by the New Keynesian model therefore raise serious questions about its suitability for analyzing forward guidance and, more generally, large or persistent monetary interventions.

To understand how such outcomes arise, this paper moves beyond the standard linearized representation of the model and studies the underlying nonlinear economy. While the linearized three-equation system is analytically convenient and almost universally employed, it obscures the microeconomic mechanisms that generate extreme responses. We therefore analyze the behavior of firms in monopolistic competition subject to Calvo-style price rigidity, explicitly keeping track of the entire distribution of prices in the economy.

Our analysis reveals that the forward-guidance puzzle is driven by a rarely discussed but crucial assumption: firms that are unable to reoptimize prices are nevertheless required to satisfy all demand at their preset price, even when that price falls below marginal cost.

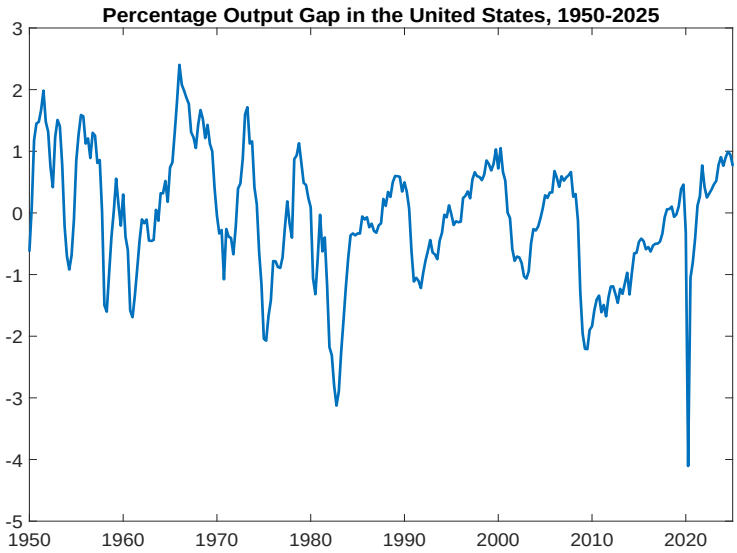

Figure 1: Percentage output gap in the United States, 1950-2025. Output gap is defined as the logarithm of the ratio between realized and potential GDP. Potential GDP is estimated by the Congressional Budget Office and retrieved from FRED database.

Firms set the price of their product at a positive markup to marginal cost when they can. With inflation, the prices that cannot be reoptimized fall below the optimal level, i.e. the markup shrinks. If inflation is not too large, the markup is compressed but remains positive, and the economy moves closer to the perfect-competition benchmark, generating a modest expansion. When inflation is large or persistent, however, some firms are driven to sell far below cost. Demand for their goods explodes, and the model sustains large output expansions only by implicitly forcing these firms to incur arbitrarily large losses.

We establish several theoretical results that clarify this mechanism. First, we show that in the nonlinear model inflation is bounded for any strictly positive degree of price rigidity, in stark contrast to the unbounded inflation predicted by the linearized system. Second, we prove that output can exceed the flexible-price benchmark only if the aggregate price level falls below marginal cost, implying negative aggregate profits. Third, we demonstrate that as the fraction of non-reoptimizing firms shrinks, positive output gaps can be sustained only if the prices of a vanishingly small set of firms collapse toward zero relative to marginal cost. These results explain both the magnitude of the forward-guidance effects and the counterintuitive finding,

previously noted by Werning (2012) and Cochrane (2017), that the puzzle intensifies as prices become more flexible.

The nonlinear framework also uncovers an asymmetry that is absent from the linearized model, and that, to our knowledge, has not been documented before. While expansionary forward guidance can generate extreme booms, sufficiently aggressive or prolonged contractionary policies may fail to admit an equilibrium altogether, as no price reduction is sufficient to induce a contraction in output of the required magnitude.

Finally, we consider a minimal modification of the model in which non-reoptimizing firms are allowed to adjust prices up to marginal cost. This modification leaves local dynamics unchanged in normal times but rules out equilibria that rely on firms selling below cost. In the modified model, output and inflation remain bounded; however, not all forward-guidance announcements are feasible.

Literature Review

Mckay, Nakamura and Steinsson (2016) focus on a slightly different aspect of the forward-guidance puzzle: the effect of an anticipated rate change of a given size and duration is increasing in the horizon of its implementation. They show that such effect is dampened in incomplete markets, in particular when agents are subject to borrowing constraints. Hagedorn, Luo, Manovskii and Mitman (2019) similarly find that the effect of forward guidance is mild in incomplete markets.

Another strand of the literature focuses on information imperfections: Campbell, Ferroni, Fisher and Melosi (2019) show that the effect of forward guidance is dampened if we take into account that central bank communication is imperfect and agents' expectations do not fully align with central bank's announcements; Angeletos and Lian (2018) find that absence of common knowledge about future policy and fundamentals make forward guidance less powerful. Eggertsson and Schule (2024) contend that the puzzle stems from an implausibly radical policy, in which the central bank would stick to its commitment about future rates regardless of the changes in the economic environment.

The remainder of the paper is organized as follows. Section 2 presents the basic New Keynesian model. Section 3 reviews the forward-guidance experiment in the linearized model. Section 4 analyzes key properties of the nonlinear economy. Section 5 studies the nonlinear response to forward guidance numerically. Section 6 introduces and evaluates a modified model. Section 7 concludes.

2 The Basic New Keynesian Model (BNK)

This section presents the baseline New Keynesian model that serves as the foundation for the analysis. The environment is standard, and the presentation follows closely Gali (2015). Since we focus on the forward guidance experiment, we choose to abstract from productivity or monetary policy shocks. There is perfect foresight after the initial monetary policy announcement.

2.1 Model Setup

2.1.1 Households

Households consume a continuum of goods of measure 1 labeled by i, that are imperfect substitutes. They derive utility from consumption and disutility from labor according to the separable utility function

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \tag{1}$$

where N_t is the work supplied and C_t is a consumption index given by

$$C_t = \left(\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \qquad \epsilon > 1 \tag{2}$$

where $C_t(i)$ is the consumption of good i.

The only saving instrument is a 1-period risk-free bond. One unit of the asset B_t can be purchased at $t-1$ at price Q_t and pays one unit of "money" at time t.[1] Households' budget constraint is

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t = B_{t-1} + W_t N_t \tag{3}$$

where $P_t(i)$ is the price of good i and W_t is the economy-wide wage.

2.1.2 Firms and Price Setting

Each good i is produced by a single firm, similarly labeled by i, with the production function[2]

$$Y_t(i) = N_t(i) \tag{4}$$

[1] In a cashless world, "money" is just a unit of account: one unit of money entitles the owner to $1/P$ units of a good with price P.

[2] In a slightly more general version of the basic model the production function could be decreasing-returns-to-scale: $Y_t(i) = zN_t(i)^{1-\alpha}$. We focus on the simplest case $\alpha = 0$. We also normalize the productivity factor z to 1, as the experiments we consider abstracts from productivity shocks.

where $N_t(i)$ is the amount of work employed by firm i at time t. Production function (4) implies that the marginal cost is equal to the wage. Notice that aggregate labor N_t is simply the sum of the labor for each firm.

$$N_t = \int_0^1 N_t(i)di \tag{5}$$

At any given time t, with probability θ firm i is constrained to keep the previous-period price for its product; with probability $1-\theta$ it is instead a reoptimizer. In the latter case, the firm chooses its price P_t^* to maximize the present value of all the profits it will make while the price remains effective, i.e. until the next time it can reoptimize. For example, with probability θ^k the price P_t^* is still effective at time $t+k$, in which case $t+k$-profits are given by

$$\Pi_{t+k} = P_t^* Y_{t+k;t} - W_{t+k} N_{t+k;t}$$

where $Y_{t+k;t}$ and $N_{t+k;t}$ are the demands for goods and labor at time $t+k$, respectively, for firms whose price has remained fixed at P_t^*. Firms are owned by households, hence profits are discounted at the households' stochastic discount factor $D_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma}$. The firm's problem is then

$$max_{P_t^*} \sum_{k=0}^{\infty} \theta^k \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} (P_t^* Y_{t+k;t} - W_{t+k} N_{t+k;t}) \tag{6}$$

2.1.3 Central Bank

The central bank sets the nominal interest rate $i_t \equiv -log(Q_t)$ at every t. In normal times it uses the Taylor rule:

$$i_t - \rho = \phi_\pi \pi_t + \phi_x x_t \tag{7}$$

where $\pi_t \equiv log\left(\frac{P_t}{P_{t-1}}\right)$ is the inflation rate; $x_t \equiv log\left(\frac{Y_t}{Y_{SS}}\right)$ is the *output gap*, i.e. the output log-deviation from its steady-state level Y_{SS}; $\rho \equiv -log(\beta)$ is the *natural interest rate*.

Exceptionally, at $t=1$, the central bank announces the (exogenous) rate i_t, $t=1,...,N$, it commits to set over these N periods, regardless of the value of the other economic variables (inflation, output, or other).[3]

[3]The situation that has most often been considered in the literature is one in which the central bank cannot apply the Taylor rule (7) at $t=1$ as it is constrained by the zero lower bound.

2.2 Model Equations

The derivation of all model equations is standard, see Gali (2015).

2.2.1 Households' first-order conditions

In addition to the Euler equation and intertemporal labor/consumption trade-off

$$C_t^{-\sigma} = \beta(1+i_t)\frac{P_t}{P_{t+1}}C_{t+1}^{-\sigma} \tag{8}$$

$$\frac{W_t}{P_t} = C_t^{\sigma}N_t^{\phi} \tag{9}$$

where P_t is a price index

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \tag{10}$$

we have the demand for each good i

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t \tag{11}$$

2.2.2 Firms' first-order conditions

Using the closed-economy resource constraint $C_t = Y_t$, the production function (4) and the demand function (11), it is straightforward to find that the optimal price P_t^*, i.e. the price that maximizes the profit function (6), is

$$P_t^* = \frac{\epsilon}{\epsilon-1}\sum_{k=0}^{\infty} w_k W_{t+k} \tag{12}$$

$$\text{with} \quad w_k = \frac{\theta^k\beta^k P_{t+k}^{\epsilon}C_{t+k}^{1-\sigma}}{\sum_{k=0}^{\infty}\theta^k\beta^k P_{t+k}^{\epsilon}C_{t+k}^{1-\sigma}} \tag{13}$$

Given the production function (4) and the demand function (11), we can also rewrite the labor supply (5), the wage (9) and thus the optimal price (12) as

$$N_t = C_t\mathcal{O}_t \tag{14}$$

$$W_t = P_t C_t^{\sigma+\phi}\mathcal{O}_t^{\phi} \tag{15}$$

$$P_t^* = \frac{\epsilon}{\epsilon-1}\frac{\theta^k\beta^k P_{t+k}^{1+\epsilon}C_{t+k}^{1+\phi}\mathcal{O}_t^{\phi}}{\sum_{k=0}^{\infty}\theta^k\beta^k P_{t+k}^{\epsilon}C_{t+k}^{1-\sigma}} \tag{16}$$

where

$$\mathcal{O}_t \equiv P_t^{\epsilon} \int_0^1 P_t(i)^{-\epsilon} di$$

is the **price-dispersion index**, a key variable in the model, that is equal to 1 if $P(i) = P \ \forall i$. The derivation of (12)-(16) is in Appendix A.

The two fundamental equations of BNK are the Euler equation (8) and the price-setting equation (16).

2.3 Equilibrium

We assume that the system is at the steady state at $t = 0$, with $P_0(i) = P_0 \ \forall i$, zero inflation and

$$C_0 = C_{SS} = \left(\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{\sigma+\phi}}$$

Given the sequence for the optimal price P_t^* for $t \geq 1$, we can reconstruct the whole distribution of prices in the economy at all times. The set of all prices at $t \geq 1$ is

$$\{P_0, P_1^*, P_2^*, ...P_t^*\}$$

with shares

$$\{\theta^t, (1-\theta)\theta^{t-1}, (1-\theta)\theta^{t-2}, ..., (1-\theta)\}$$

The price level P_t and the price-dispersion $\mathcal{O}_t$ are then obtained as

$$P_t = \left(\theta^t P_0^{1-\epsilon} + (1-\theta)\Sigma_{k=1}^t \theta^{t-k}(P_k^*)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \tag{17}$$

$$\mathcal{O}_t = P_t^{-\epsilon}\left(\theta^t P_0^{-\epsilon} + (1-\theta)\Sigma_{k=1}^t \theta^{t-k}(P_k^*)^{-\epsilon}\right) \tag{18}$$

An equilibrium is a sequence $\{P_t^*, P_t, \mathcal{O}_t, i_t, C_t\}_{t\geq 1}$ solving:

- The Euler equation (8) and the optimal price setting (16) for $t \geq 1$;
- The Taylor rule (7) for $t > N$;
- The exogenous i_t level announced by the central bank for $t = 1, ..N$;

with P_t and $\mathcal{O}_t$ given by (17)-(18) $\forall t$.

2.4 The flexible-price case

In the flexible-price case ($\theta = 0$), (12) becomes

$$P_t^* = \frac{\epsilon}{\epsilon - 1} W_t$$

and since all the firms reoptimize and choose this price, we have $P_t(i) = P_t^* \ \forall i$ hence $P_t = P_t^*$ and $\mathcal{O}_t = 1$. Inserting this in (32) we obtain that flexible-price consumption is equal to the steady-state value of consumption:

$$C_{FP} = C_{SS} = \left(\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{\sigma+\phi}} \tag{19}$$

Finally, in order for the Euler equation to be satisfied, we must have

$$\frac{P_{t+1}}{P_t} = \frac{1 + i_t}{\beta} \simeq 1 + i_t - \rho \tag{20}$$

The perfect-competition case is found for $\epsilon \to \infty$, in which goods are perfect substitutes. in this case consumption is $C_{PC} = 1$.

2.5 BNKL: the log-linearized version of BNK

The New Keynesian model is almost universally used in its log-linearized form, in our specification reducing to the equations for inflation π_t and the *output gap* $x_t \equiv log\left(\frac{C_t}{C_{SS}}\right)$:

$$\pi_t = \kappa x_t + \beta \pi_{t+1} \tag{21}$$

$$i_t - \rho = \sigma(x_{t+1} - x_t) + \pi_{t+1} \tag{22}$$

for the period in which the rate i_t is set exogenously, plus the Taylor rule (7) afterwards. (21) is the New Keynesian Phillips curve (NKPC), resulting from the linearization of the price-setting equation. (22) is the linearized Euler equation. The derivation of (21)-(22) is standard (see e.g. Gali (2015)). The equilibrium consists in a sequence $\{\pi_t, x_t, i_t\}_{t \geq 1}$ solving (21)-(22) and the Taylor rule for $t > N$.

3 The forward-guidance experiment in BNKL

This section reviews the forward-guidance experiment in the standard log-linearized New Keynesian model (BNKL). The objective is not to introduce new results, but to establish a benchmark and clarify the source of the forward-guidance puzzle in the canonical framework. The analysis closely follows Del Negro, Giannoni, and Patterson (2013) and Carlstrom, Fuerst, and Paustian (2015).

The baseline policy we focus on is what we call "E-Policy" (expansionary), in which the quarterly nominal rate is set 25 bps below the natural

rate for N periods: $\{i_t = \rho - 0.25\%, \}_{t=1,...N}$. This implies that the annualized interest rate is 1% below its steady-state value. At quarter $N+1$ the Taylor rule (7) is restored. We will also consider the specular "C-Policy" (contractionary), where i_t is set 25 bps above the natural rate for the whole duration of the experiment. Since we focus on the workings of the model, we abstract from the reasons behind the decision of the central bank to conduct this experiment.

Solving the linear model

It is well known that in BNKL, the economy returns to the steady state as soon as the Taylor rule is restored, provided that the Taylor rule parameters are such that $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$ (see Gali(2015)). Assuming that the latter condition holds, the system starts at the steady state at $t = 0$ and returns to the steady state at $t = N+1$. We can then use (21) and (22) and obtain x_t, π_t for $t = 1, ...N$ by backward induction from the boundary condition $x_{N+1} = \pi_{N+1} = 0$. A solution exists for every choice of the exogenous path $\{i_t\}_{t=1,...N}$.

The E-Policy equilibrium

It is easy to get an intuition about what the equilibrium is going to look like for the E-Policy $\{i_t = \rho - 0.025\%\}_{t=1,...,N}$. The fact that $\pi_{N+1} = x_{N+1} = 0$ implies that the real rate at $t = N$, $i_N - \pi_{N+1}$, is forced to be 25 bps below the natural rate ρ (or, equivalently, 1% below ρ in annual terms). In order to satisfy the Euler equation (22), we then need a positive output gap, $x_N = \frac{0.0025}{\sigma}$. But (21) tells us that, to have positive output gap at time N we need positive time-N inflation, $\pi_t = \kappa x_t$. Going backward to time $N-1$, the time-N results for inflation tell us that the time-$(N-1)$ real rate is *even lower* than the time-N real rate. To satisfy the Euler equation, we need x_{N-1} to be higher than x_N, which forces π_{N-1} to be higher than π_N. Proceeding backward, we obtain the explosive behavior for output gap and inflation that has been noted in previous literature.

To get a quantitative sense of the output gap and inflation path predicted by the model for this experiment, we choose standard values of the parameters, in line with those of Gali (2015). Taking one period to be a quarter, we set $\beta = 0.99$ – so that the annual natural rate is 4% – and we set the utility function parameters $\phi = 1$, $\sigma = 1$. For the production sector, we set $\epsilon = 6$ and $\theta = 2/3$. The latter value for the price-stickiness parameter θ implies

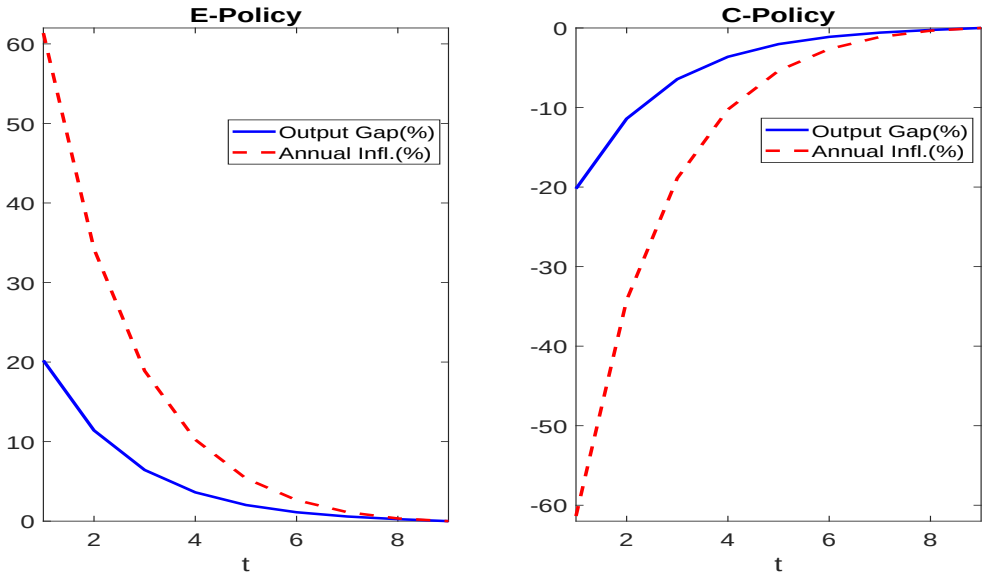

Figure 2: Response of the economy to the E-Policy (left panel) and C-Policy (right panel) experiment lasting 8 periods in the linear model.

that firms reset prices on average every three quarters (see e.g. Galí, Gertler, and López-Salido (2001) and Sbordone (2002)).

The left panel of Figure 2 shows the path of the economy for a forward guidance period of 8 quarters. While output gap and inflation are modest toward the end of the period (for example, in the last two quarters before the end of the forward guidance period, output gap is 59 bps and 29 bps and inflation is 1.13% and 34 bps, respectively), the values in the first period are enormous: 20% for output gap and 61% for inflation (in annual terms).

C-Policy Equilibrium

The equilibrium for the specular C-Policy is shown in the right panel of Figure 2. It is easy to see that, if $\{[x_t, \pi_t]\}_{t=1,\dots,N}$ is the solution for a policy $\{i_t\}_{t=1,\dots,N}$, the solution for the specular policy $\{i_t\}_{t=1,\dots,N}$ is just given by $\{[-x_t, -\pi_t]\}_{t=1,\dots,N}$. Hence, BNKL predicts an equally explosive deflationary and recessive behavior for the solution of the C-Policy experiment.

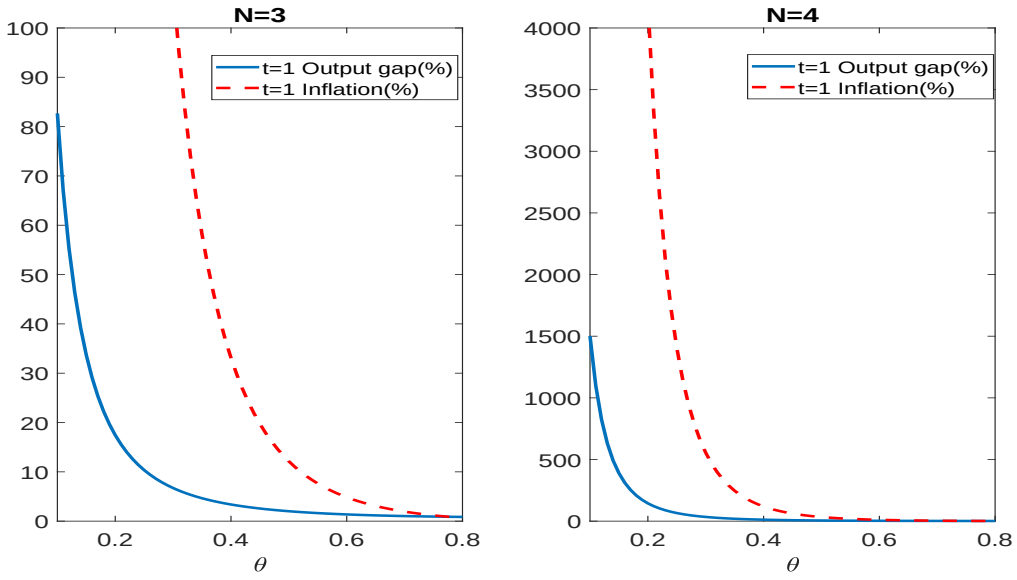

Figure 3: Initial-period output gap and inflation response to Policy E lasting 3 periods (left panel) and 4 periods (right panel), as a function of θ.

Sensitivity to Price Rigidity

Another unrealistic feature of the solution has been pointed out by Werning (2012) and Cochrane (2017): for a given length of the forward guidance period, the predicted inflation and output gap is larger the lower is θ, i.e. the less sticky are prices. This means that the closer we get to the flexible-price case, the farther the solution departs from the flexible-price solution. This is shown in Figure 3: The left (right) panel shows the initial-period output and inflation, as a function of θ, in response to a policy lasting 3 periods (4 periods). For $\theta = 0.1$, i.e. in case 90% of the firms are reoptimizers in any period, the initial output gap is 80% if the policy lasts 3 periods and 1500% if it lasts 4 periods! We will find an explanation of this aberrant behavior of the model in the next sections, by analyzing the nonlinear model.

4 Properties of BNK

This section studies the key properties of the nonlinear model (BNK) and contrasts them with those of its log-linearized counterpart. While previous work has shown that the forward-guidance puzzle persists quantitatively in the nonlinear model (e.g., Carlstrom, Fuerst, and Paustian, 2015), we show

that the nonlinear framework delivers qualitatively different insights once the full distribution of prices and the resulting price dispersion are taken into account.

Our analysis highlights three central features. First, inflation is bounded in the nonlinear model for any strictly positive degree of price rigidity, in sharp contrast to the linearized system. Second, output can exceed the flexible-price benchmark only if the aggregate price level falls below marginal cost, implying negative aggregate profits. Third, as price rigidity vanishes, negative output gaps are not feasible; positive output gaps, on the other hand, can be sustained only through increasingly extreme relative-price distortions, with a vanishingly small set of firms selling at prices arbitrarily close to zero. All detailed proofs and calculations are in Appendix A.

4.1 State Variables and Price Dispersion

Lemma 1: The price-dispersion index $\mathcal{O}_{t-1}$ is the only state variable in the nonlinear New Keynesian model.

Preset prices affect the time-t model equations through their contribution to the variables P_t and $\mathcal{O}_t$. As shown in Appendix A, we can rewrite P_t and $\mathcal{O}_t$ as

$$P_t = (\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon})^{\frac{1}{1-\epsilon}} \tag{23}$$

$$\mathcal{O}_t = P_t^{\epsilon}(\theta P_{t-1}^{-\epsilon}\mathcal{O}_{t-1} + (1-\theta)(P_t^*)^{-\epsilon}) \tag{24}$$

Since P_{t-1} can be normalized without loss of generality, the entire effect of preset prices $P_{t-1}(i)$ is summarized by the lagged dispersion variable $\mathcal{O}_{t-1}$. A direct implication is that the linearized model, which approximates the economy around a zero-inflation steady state where $\mathcal{O}_t = 1$ to first order, features no state variable.

Lemma 2: For all t, the price-dispersion variable satisfies $\mathcal{O}_t \geq 1$.

This result follows directly from Jensen's inequality.

4.2 Bounded Inflation in the Nonlinear Model

We begin by characterizing the behavior of inflation. In the nonlinear model, inflation is bounded above whenever there is any degree of price rigidity.

Proposition 1: For any $\theta > 0$, the gross inflation rate satisfies

$$\frac{P_{t+1}}{P_t} \leq \theta^{\frac{1}{1-\varepsilon}}$$

In contrast, inflation is unbounded in the linearized model.

One way to understand this result intuitively is to write the aggregate price level as the demand-weighted average of individual prices (as shown by (11), demand for good i is proportional to $\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}$)

$$P_t = \int P_t(i) \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di \tag{25}$$

As the prices chosen by reoptimizing firms rise, demand is increasingly reallocated toward firms with preset prices. This endogenous reweighting makes the aggregate price level progressively less sensitive to further increases in optimal prices, placing a tight upper bound on inflation. Linearization around a zero-inflation steady state eliminates this mechanism, allowing inflation to grow without bound.

4.3 Profits, Marginal Cost, and the Output Gap

We next relate output fluctuations to firms' profitability. Aggregate profits can be expressed in closed form as a function of consumption and price dispersion.

Lemma 3: Aggregate profits satisfy

$$\Pi_t = P_t C_t \big(1 - C_t^{\sigma+\varphi} \mathcal{O}_t^{1+\varphi}\big) \tag{26}$$

This expression makes clear that profits depend not only on the level of activity but also on the degree of price dispersion, which raises effective marginal costs.

Proposition 2: If the aggregate price level weakly exceeds the real wage, output cannot exceed the perfect-competition level. Specifically, if $P_t \geq W_t$ then $C_t \leq 1$, so that the output gap is bounded above by $x_t \leq \frac{1}{\sigma+\varphi} \log\left(\frac{\varepsilon}{\varepsilon-1}\right)$

The economic content of this result is transparent. To induce households to supply labor sufficient to produce output above the perfect-competition benchmark, real wages must rise above productivity. Firms can pay wages in excess of marginal revenue only by operating at a loss. Large output

expansions therefore require negative aggregate profits,[4] financed implicitly by households in their role as firm owners.

4.4 The Flexible-Price Limit

We now consider the behavior of the nonlinear model as price rigidity vanishes. This limit is of particular interest given the finding, emphasized by Werning (2012) and Cochrane (2017), that the forward-guidance puzzle becomes more severe as prices become more flexible.

Proposition 3: As $\theta \to 0$:

(i) Strictly negative output gaps are not feasible in equilibrium.

(ii) If there exists $\kappa > 0$ such that $P_t(i)/W_t \geq \kappa$ for all firms, then strictly positive output gaps are not feasible in equilibrium.

The first part establishes that contractionary monetary policies cease to be feasible in the limit in which prices become fully flexible. The proof exploits the fact that, for a given θ, there is a lower bound to the relative price $\frac{P^*}{P}$. This is due to the endogenous reweighting of the price level, as discussed in the context of Proposition 1. As $\theta \to 0$, $\frac{P^*}{P} \geq 1$. As we will see in the next section, with a contractionary monetary policy an equilibrium involves reoptimizing firms lowering their price relative to the general price level, but, as $\theta \to 0$, this becomes unfeasible.

The second part shows that positive output gaps can persist in the limit only if some firms charge prices that are arbitrarily small relative to marginal cost. In an equilibrium with strictly positive output gap, reoptimizing firms raise prices sharply, driving up wages, while a vanishing fraction of firms that cannot reoptimize absorb virtually all demand by selling at extremely low prices.

This result clarifies the paradoxical behavior of the model near the flexible-price limit. The divergence between the New Keynesian and flexible-price allocations relies on increasingly extreme and economically implausible relative-price distortions.

4.5 Implications

Taken together, the results in this section show that the nonlinear New Keynesian model places sharp theoretical restrictions on inflation, output, and

[4] This can be seen immediately from (26): since from Lemma 2 $\mathcal{O}_t \geq 1$, $C_t > 1$ implies necessarily negative aggregate profits.

equilibrium feasibility that are entirely absent from its linearized version. Extreme output responses to forward guidance are not generic implications of price rigidity, but are instead supported by equilibria in which some firms incur arbitrarily large losses. These properties provide the foundation for the numerical analysis in Section 5, which illustrates how such equilibria arise in practice under forward-guidance policies.

5 The Forward-Guidance Experiment in the Nonlinear Model

This section studies the forward-guidance experiment in the fully nonlinear New Keynesian model (BNK). The objective is twofold. First, we assess to what extent the extreme predictions of the linearized model persist once nonlinearities and price dispersion are taken into account. Second, we provide a detailed diagnosis of the microeconomic mechanisms that sustain large output responses, thereby clarifying the economic content of the equilibria generated by the model.

5.1 Solving the Nonlinear Model

The nonlinear equilibrium is characterized by the Euler equation and the optimal price-setting condition during the forward-guidance period, and by the Taylor rule thereafter. In contrast to the linearized model, the equilibrium cannot be computed by backward induction. Price dispersion generated during the forward-guidance period persists after the Taylor rule is restored, so that the economy returns to the steady state only asymptotically. Moreover, equilibrium allocations at time t depend on the inherited dispersion $\mathcal{O}_{t-1}$. We therefore solve the model using a recursive algorithm described in Appendix B. For robustness, we also consider a "semi-linear" version of the model in the economy is assumed to return immediately to the steady state after the end of the forward-guidance period, thereby shutting down the effects of post-guidance price dispersion. Whenever differences arise between the two approaches, they are explicitly discussed below.

5.2 A Short Expansionary Policy

We begin with a short expansionary forward-guidance policy in which the nominal interest rate is set one percentage point below the natural rate for four quarters. Figure 4 reports the resulting equilibrium paths.

For this experiment, the linear and nonlinear models yield very similar outcomes. Inflation and the output gap remain modest throughout the forward-guidance period, and results from the semi-linear model are virtually indistinguishable from those of the fully nonlinear economy. The bottom-left panel of Figure 4 shows that all prices in the economy—including the minimum price among firms—remain above the wage at all dates. As a result, markups are compressed but remain positive, and aggregate profits stay positive as well (bottom-right panel).

In this case, the expansion operates through the standard New Keynesian channel: price rigidity temporarily reduces markups, moving the economy closer to the perfectly competitive benchmark and generating a moderate increase in output. No firm is required to sell below marginal cost, and the equilibrium is economically well behaved.

5.3 A Longer Expansionary Policy

We now consider a longer expansionary policy in which the interest rate is kept one percentage point below the natural rate for eight quarters. The results are reported in Figure 5.

The behavior of inflation differs sharply between the linear and nonlinear models. In the nonlinear economy, inflation is bounded, as established in Proposition 1, and never exceeds the theoretical upper limit implied by price rigidity. In contrast, the linearized model predicts explosive inflation dynamics. Despite this stark difference, the behavior of the output gap is qualitatively similar across models and quantitatively even more extreme in the nonlinear case. In the first period of the forward-guidance announcement, the nonlinear model generates an output gap exceeding 40 percent.

Some differences are noticeable also between the solutions of the semi-linear and nonlinear model. In the non-linear model, inflation stays positive after $t = N$, while the output gap turns *negative* .[5]

The lower panels of Figure 5 reveal how such an expansion is sustained. Throughout most of the forward-guidance period, the aggregate price level lies below the wage, implying negative aggregate profits. While reoptimizing firms raise their prices aggressively, firms that cannot reoptimize see their

[5]The intuition for the coexistence of positive inflation and negative output gap when the Taylor rule is restored is the following: since the wage W_t is proportional to the price dispersion variable $\mathcal{O}$, price dispersion generated in the FGP drives real wages up. This drives the optimal price P^* above the price level, creating inflation. In response to inflation, the central bank, following the Taylor rule, increases the nominal interest rate *more than one-for-one* with inflation, inducing an increase of the real rate above the natural rate, hence a contraction of output.

relative prices fall. Demand for their goods increases sharply, proportional to the ϵ-power of their inverse relative price, and the model assumes that these firms satisfy all demand at their preset price. The resulting equilibrium is therefore supported by firms that sell below marginal cost and incur massive losses. In the first period alone, aggregate losses approach 120 percent of output.

These findings confirm Proposition 2: output expansions beyond the flexible-price benchmark require the aggregate price level to fall below marginal cost. The forward-guidance boom is thus not a consequence of intertemporal substitution alone, but of an extreme form of demand reallocation toward loss-making firms.

5.4 The Role of Price Rigidity

Figure 6 illustrates how the equilibrium response to an eight-period expansionary policy varies with the degree of price rigidity θ. As θ decreases, both inflation and the output gap in the first period increase sharply, even in the nonlinear model. Although inflation is bounded for any fixed $(\theta > 0)$, the bound itself diverges as $(\theta \to 0)$.

The bottom panels of Figure 6 provide a precise microeconomic interpretation. As prices become more flexible, the optimal price chosen by reoptimizing firms increases dramatically relative to the wage, while the minimum price in the economy collapses toward zero. In the limit, a vanishingly small fraction of firms accounts for an arbitrarily large share of demand by selling at prices that are arbitrarily low relative to marginal cost. This mechanism explains why the forward-guidance puzzle becomes more severe as the economy approaches the flexible-price limit, as formalized in Proposition 3.

5.5 Contractionary Forward Guidance

We next turn to contractionary forward-guidance policies, in which the nominal interest rate is set above the natural rate. Figure 7 compares the linear and nonlinear responses to a six-period contractionary policy. For moderate policy interventions, the two models produce similar dynamics. Prices rise relative to marginal cost, markups increase, and output contracts in a conventional manner.

However, unlike the linearized model, the nonlinear economy does not admit an equilibrium for arbitrarily aggressive or prolonged contractionary policies. Figure 8 illustrates this result. For each duration of the forward-guidance period, there exists an upper bound on the feasible increase in the nominal interest rate (shown in the left panel), and a lower bound on the

feasible contraction of outoput (middle panel). Beyond this bound, no equilibrium exists. As price rigidity decreases, the set of feasible contractionary policies shrinks, and in the limit of fully flexible prices (right panel) no contractionary policy is feasible at all, consistently with Proposition 3.

This non-existence result represents an asymmetry relative to the expansionary case that is entirely absent from the linearized model.

5.6 Summary

The nonlinear analysis shows that the forward-guidance puzzle survives linearization only in a qualified sense. While inflation dynamics are fundamentally altered once nonlinearities are taken seriously, extreme output responses persist. These responses, however, are supported by equilibria in which a subset of firms incurs arbitrarily large losses by selling below marginal cost. The puzzle is therefore not a generic implication of intertemporal substitution under price rigidity, but a consequence of equilibrium allocations that rely on economically implausible firm behavior.

6 Toward an Alternative Model

The analysis so far has shown that extreme responses to forward guidance in the New Keynesian model are supported by equilibria in which some firms sell below marginal cost while satisfying unbounded demand. In this section, we introduce a minimal modification of the model that blocks this mechanism. The purpose of the exercise is not to propose a new benchmark model, but to assess how sensitive the forward-guidance results are to the assumption that non-reoptimizing firms must satisfy all demand at their preset price. The modification is straightforward. Firms that are unable to reoptimize prices remain subject to Calvo rigidity, but they are allowed to adjust their price upward to marginal cost if their preset price would otherwise imply selling below cost. Formally, the price charged by a non-reoptimizing firm in period is given by

$$P_t(i) = max(P_{t-1}(i), W_t) \tag{27}$$

All other aspects of the model—preferences, technology, and monetary policy—are unchanged.

This modification preserves the local behavior of the economy around the steady state. When inflation is low and prices remain above marginal cost, the constraint (27) does not bind and the equilibrium coincides exactly with that of the baseline New Keynesian model. The modification therefore affects only those allocations in which the baseline model would

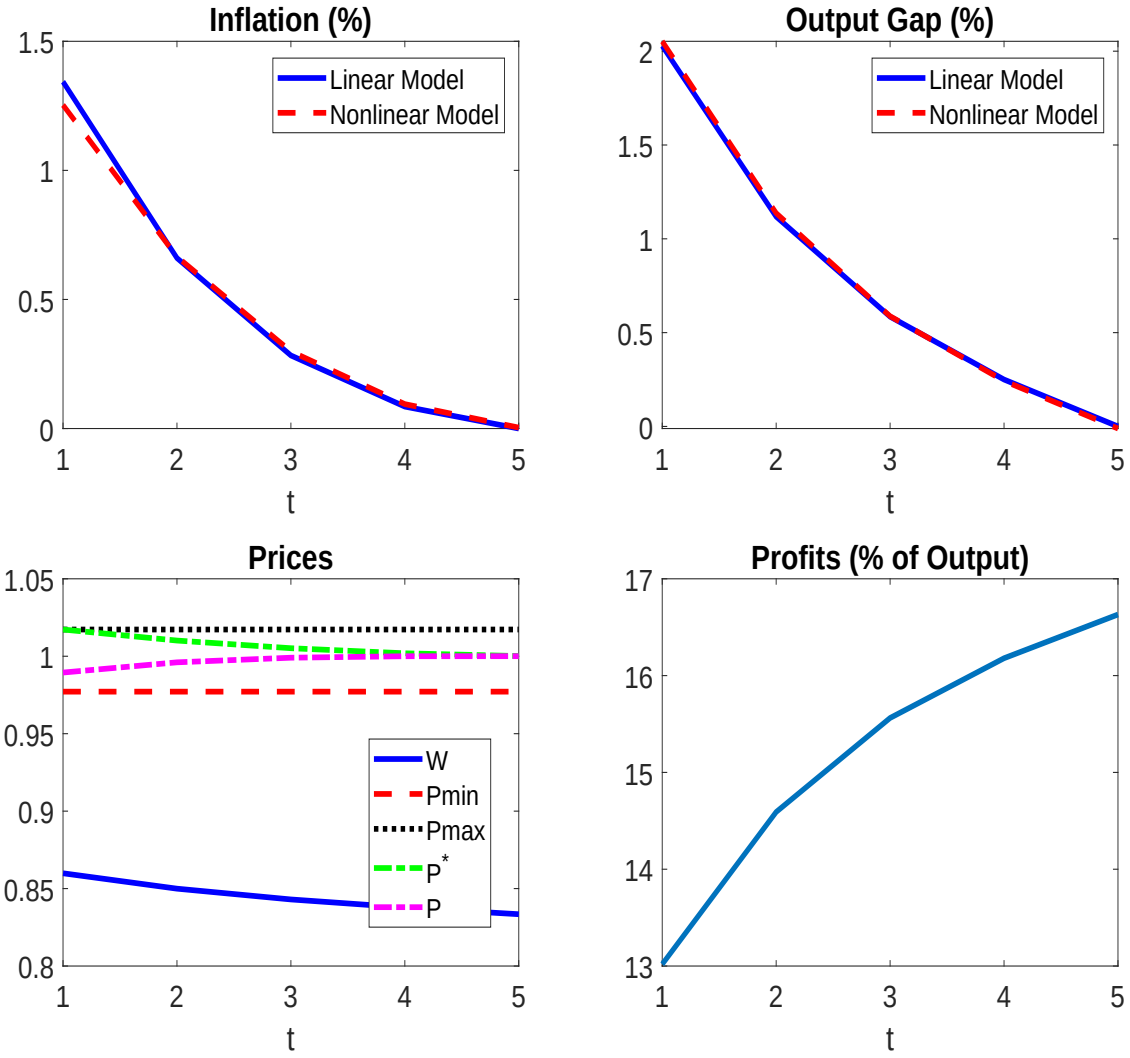

Figure 4: Response of the economy to a 4-period E-Policy experiment, as a function of time. At $t = 5$ the Taylor rule is restored.

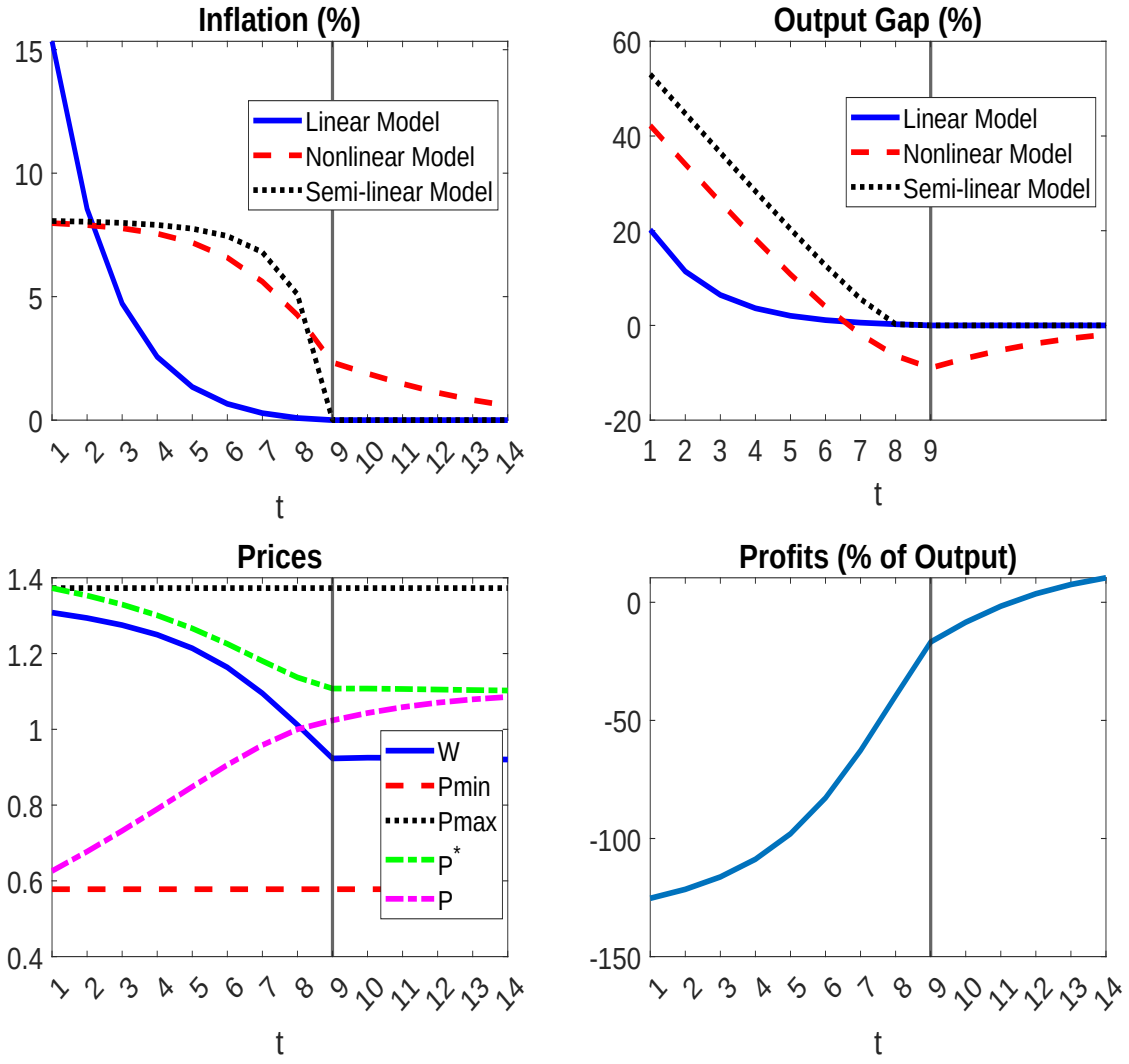

Figure 5: Response of the economy to an 8-period E-Policy experiment, as a function of time. The vertical grey line marks the return to the Taylor rule.

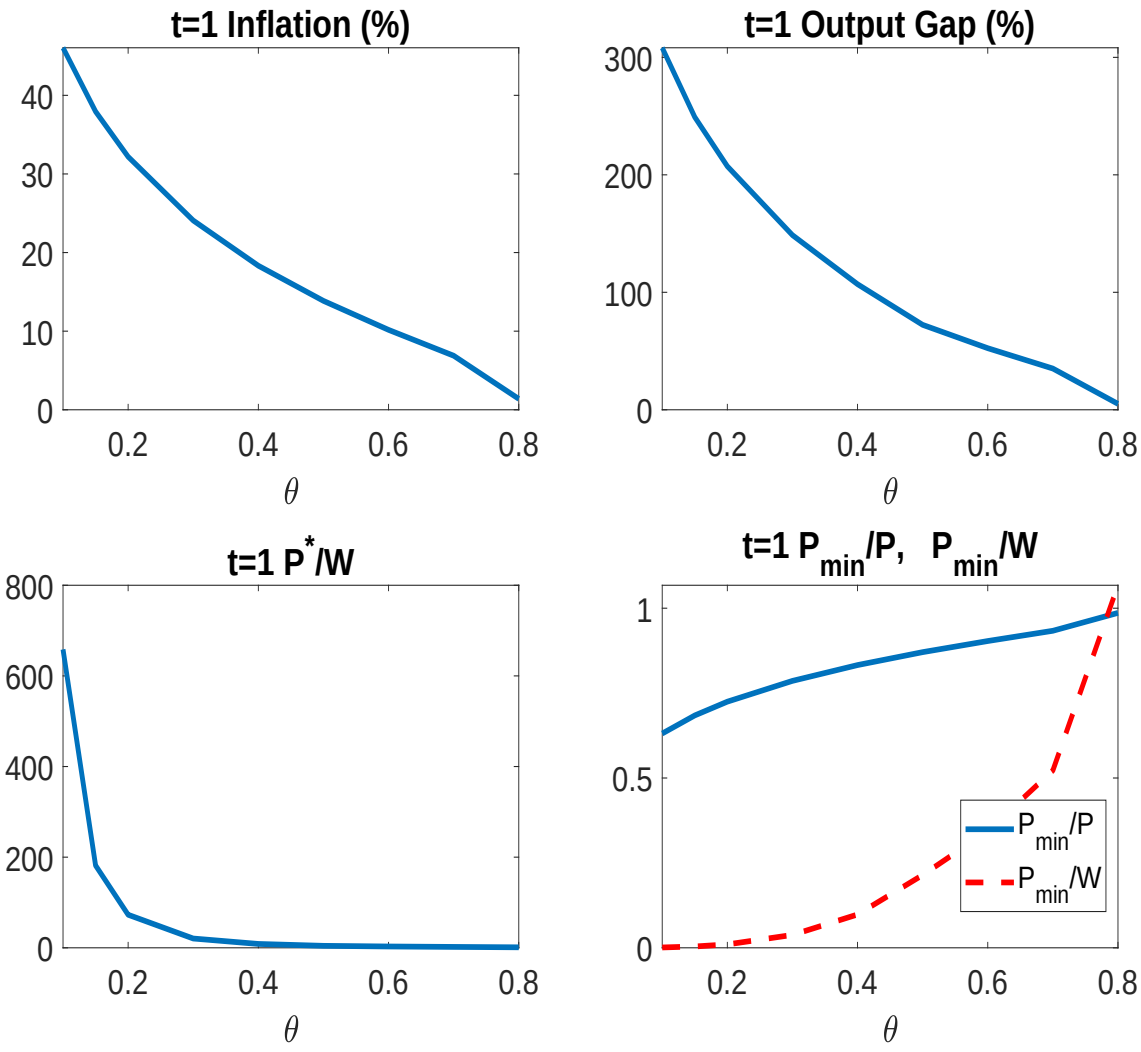

Figure 6: Response of the economy in the first period of an 8-period E-Policy as a function of θ. The top panels show the response of inflation and the output gap. The lower panels show the response of the optimal price as a fraction of the wage (bottom-left panel) and the minimum price as a fraction of the wage and of the general price level (bottom-right panel), all as a function of θ.

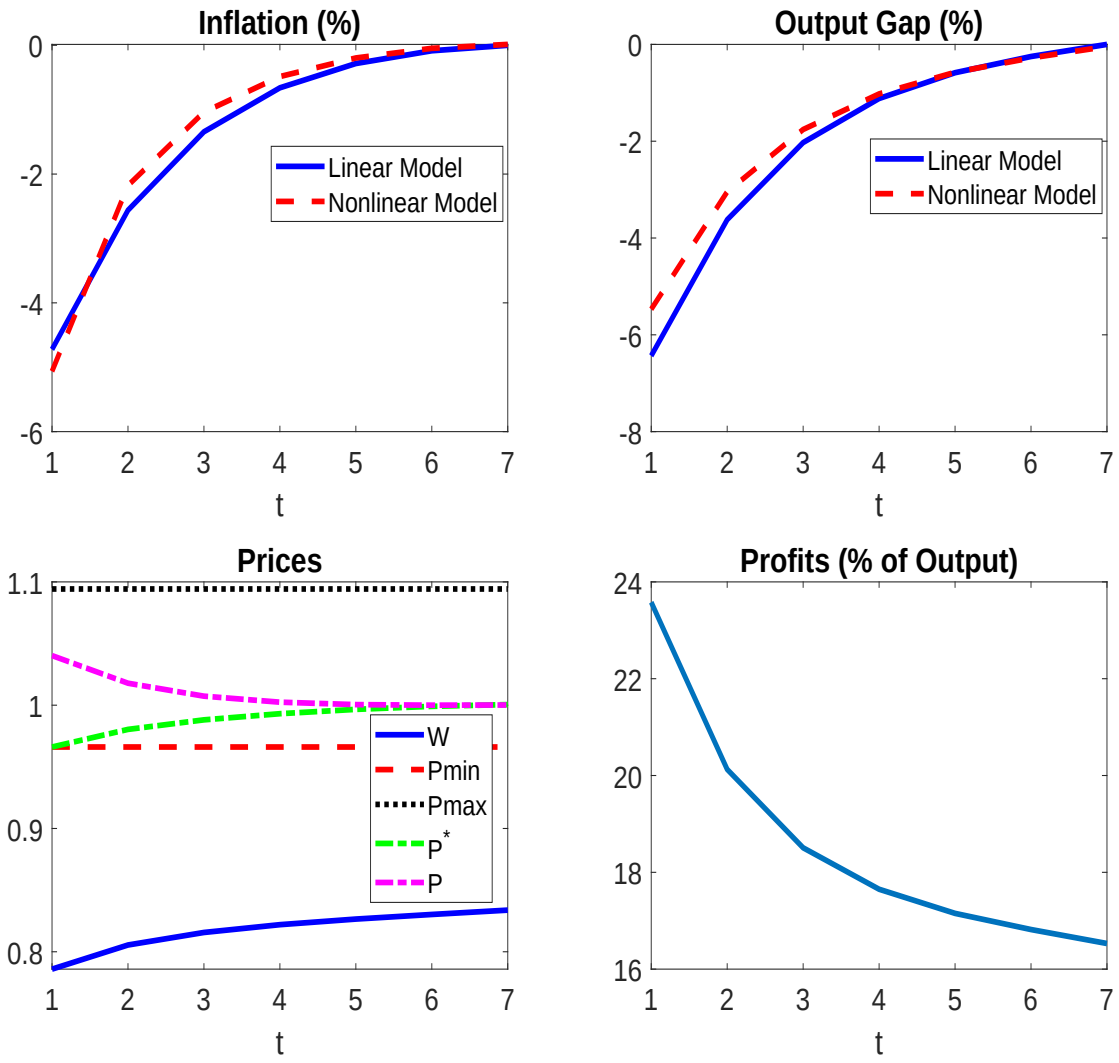

Figure 7: Response of the economy to a 6-period C-Policy experiment, as a function of time.

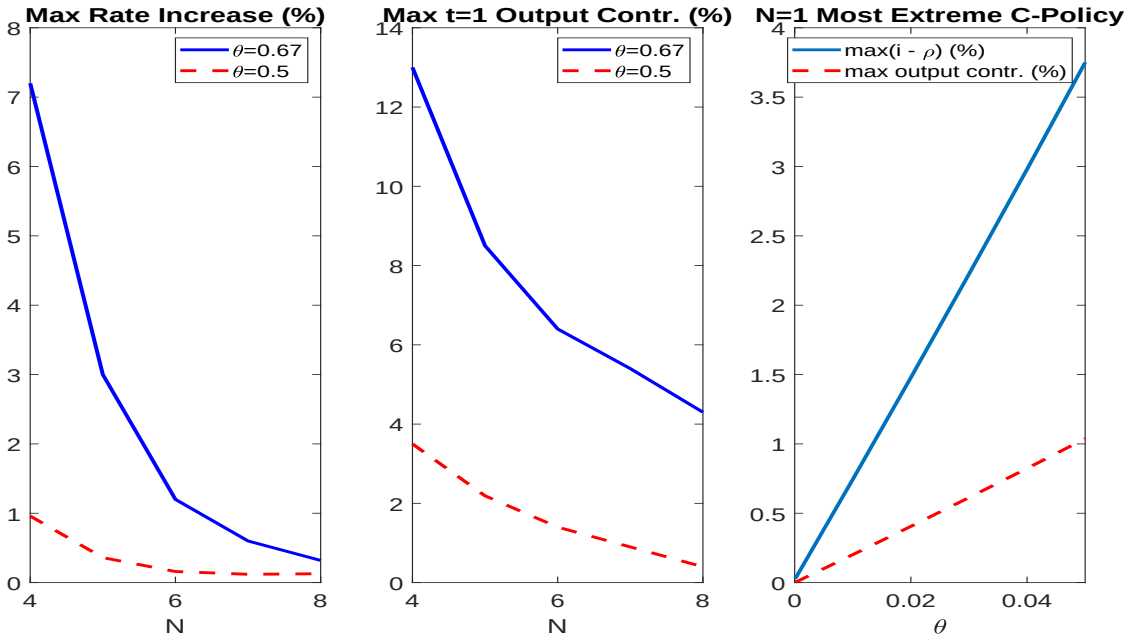

Figure 8: The left panel shows the highest feasible increase in the interest rate above the natural rate (in annnual terms), as a function of the duration of the policy, for two different values of θ. The middle panel shows the maximum $t = 1$ feasible contraction of the economy, as a function of the duration of the policy, for the same two values of θ. The right panel shows the maximum feasible rate increase ($v \equiv i - \rho$) and contraction of the economy for very small θ, with a policy lasting only 1 period.

require firms to operate at a loss. Constraint (27) has however immediate implications for equilibrium feasibility. In contrast to the baseline model, the modified economy does not admit equilibria associated with sufficiently aggressive forward-guidance policies.

As the simplest example, consider a 1-period expansionary policy. As a back-of-the-envelope estimation, let us neglect the effect of Period 1's price dispersion on the solution starting at $t = 2$, when the Taylor rule is restored. In other words, let us assume that the economy goes back to the zero-inflation steady state at $t = 2$. Actual calculations confirm that the latter is a very good approximation. We then have $C_2 = C_{SS}$ and $P_2 = P_1$. The Euler equation then implies

$$\left(\frac{C_{SS}}{C_1}\right)^{\sigma} = \beta(1 + i_1)$$

or, taking logs

$$\sigma x_1 \simeq \rho - i_1 \tag{28}$$

where x_1 is the output gap $x_1 \equiv log\left(\frac{C_1}{C_{SS}}\right)$. Since C_1 is upward-bounded, it is clear that i_1 is downward-bounded. If i_1 is below its downward bound, no equilibrium exists.

Figure 9 shows the maximum output gap that is achievable at $t = 1$ and the corresponding minimum value of the interest rate $v \equiv i_1 - \rho$ (there shown in annualized terms), as a function of θ. In this figure we see that, in the limit $\theta \to 0$, no positive output gap can emerge, and correspondingly no interest rate different from the natural rate can result in an equilibrium. This is consistent with Proposition 3: in the limit $\theta \to 0$, a positive output gap can be achieved only if some price of the economy falls to 0 relative to the wage; this situation is clearly ruled out in the modified model.

One final consideration is in order. We saw that in the modified model no output gap can emerge in the limit $\theta \to 0$, which is consistent with the behavior of the genuine flexible-price model. However, in the genuine ($\theta = 0$) flexible-price case, any forward guidance policy is feasible, since inflation, as discussed in Section 2.4, adjusts so as to keep the real rate equal to the natural rate $\rho = -log(\beta)$ and all the real variables at the steady-state level. But for this to happen, a rather peculiar price-setting behavior is required of firms: since the real rate is equal to the nominal rate minus the *next-period* inflation, for the real rate to be constant firms have to set their price reacting to the *previous-period* nominal rate. The latter could never occur if even an infinitesimal fraction of firms cannot reset their price: in this case firms resetting their price are able to affect their price relative to that of the non-optimizing firms and thus relative to the general price level. In this

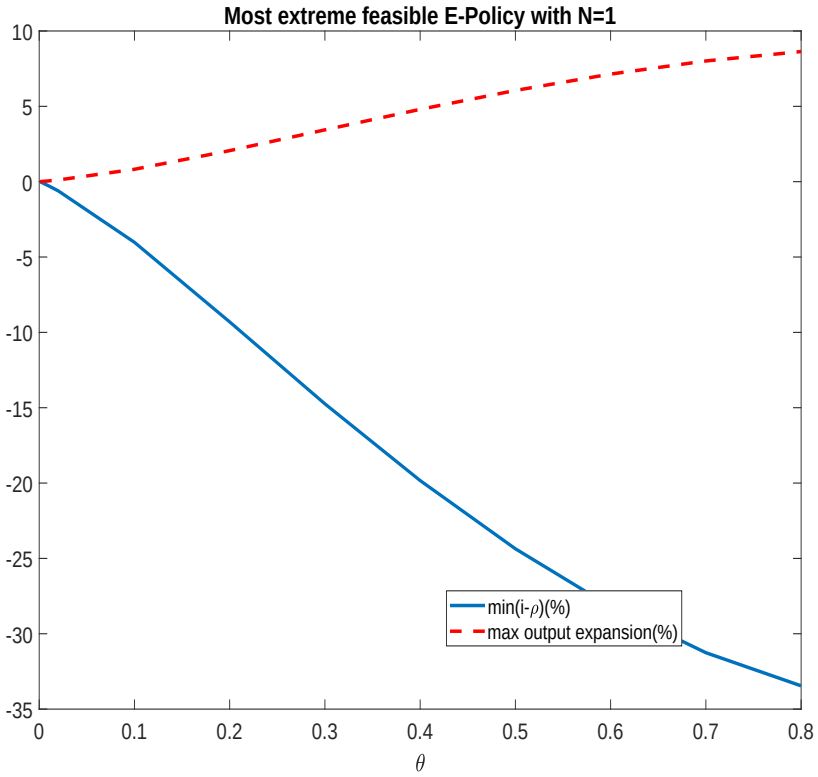

Figure 9: Minimum interest rate, in annual terms and in excess of the natural rate, that is feasible in equilibrium with a single-period E-Policy in the modified model, and maximum feasible output gap, as a function of θ.

case they make an optimal, *forward-looking* decision. We fiind therefore this discontinuous behavior of the model for $\theta = 0$: while in the limit $\theta \to 0$ only equilibria with the nominal rate equal to the natural rate (and zero inflation) are feasible, for $\theta = 0$ any nominal rate is feasible, as inflation adjust to ensure the equality of the *real* rate and the natural one.

7 Conclusion

This paper revisits the forward-guidance puzzle through a careful analysis of the nonlinear New Keynesian model that underlies its widely used linearized representation. By explicitly tracking the full distribution of prices and the resulting price dispersion, we show that the extreme responses of output and inflation to forward-guidance announcements do not arise from intertemporal

substitution per se, but from an assumption embedded in the Calvo pricing structure: non-reoptimizing firms are required to satisfy all demand at their preset price, even when that price falls far below marginal cost.

In the nonlinear model, large and persistent inflation compresses markups for constrained firms and can eventually drive their prices below marginal cost. When this occurs, the model generates expansions only by implicitly forcing these firms to absorb unbounded losses while meeting arbitrarily large demand. Moreover, as the degree of price rigidity vanishes, positive output gaps can be sustained only if the prices of a vanishingly small fraction of firms collapse toward zero relative to wages. These mechanisms explain both the magnitude of the forward-guidance effects and the counterintuitive result that the puzzle intensifies as prices become more flexible.

The broader lesson of the paper extends beyond forward guidance. In environments featuring large or persistent shocks that generate substantial inflation or deflation, the standard New Keynesian model can deliver quantitatively and conceptually misleading predictions unless its microeconomic assumptions are carefully scrutinized. Our results suggest that restoring economically plausible firm behavior is essential for using New Keynesian models to evaluate unconventional monetary policies. Future research should explore alternative forms of nominal rigidity and demand rationing that preserve tractability while avoiding the pathological outcomes identified here.

Appendix A – Proofs and derivations

Derivation of the price-setting equation and of (12)-(16)

Given the prduction function (4), $N_{t+k;t} = Y_{t+k;t}$ and given the demand equation (11) and the resource constraint $C_t = Y_t \ \forall t$,

$$N_{t+k;t} = Y_{t+k;t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} C_{t+k} \tag{29}$$

Inserting (29) into (6), we obtrain that the firm's problem at time t is

$$max_{P_t^*} P_t^{\epsilon} C_t^{\sigma} \Sigma_{k=0}^{\infty} \theta^k \beta^k \left((P_t^*)^{1-\epsilon} C_{t+k}^{1-\sigma} - (P_t^*)^{-\epsilon} C_{t+k}^{1-\sigma} W_{t+k}\right) \tag{30}$$

The first-order condition gives (12). Next, using $N(i) = Y(i) = C(i)$, where the first equality comes from the production function and the second equality comes from the resource constraint, and using (11), we have

$$N_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$$

hence aggregate labor (5) is

$$N = C_t P_t^{\epsilon} \int (P_t(i))^{-\epsilon} di = C_t \mathcal{O}_t \tag{31}$$

and finally wage (9) is

$$W_t = P_t C_t^{\sigma+\phi} \mathcal{O}_t^{\phi} \tag{32}$$

inserting (32) in (12) we have the price setting equation in the form (16).

Lemma 1 – Derivation of (23) and (24)

Define an indicator $\vartheta_t(i)$ that is equal to 1 if firm i *cannot* change its price at t and 1 otherwise. The time-t price for firm i can be written as

$$P_t(i)^{1-\epsilon} = \vartheta_t(i) P_{t-1}(i)^{1-\epsilon} + (1 - \vartheta_t(i))(P_t^*)^{1-\epsilon}$$

In addition we have

$$\theta = \int_0^1 \vartheta_t(i) di$$

Hence the price level can be written as

$$\begin{aligned} P_t &= \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}} \\ &= \left(\int_0^1 P_{t-1}(i)^{1-\epsilon} \vartheta_t(i) di + (1-\theta)(P_t^*)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \end{aligned} \tag{33}$$

Since the firms that can optimize are chosen randomly, the random variable $\vartheta(i)$ is independent of $P_{t-1}(i)$, so that

$$\int_0^1 P_{t-1}(i)^{1-\epsilon} \vartheta_t(i) di = E[P_{t-1}(i)^{1-\epsilon} \vartheta_t(i)] = E[P_{t-1}(i)^{1-\epsilon}] E[\vartheta(i)] = \theta P_{t-1}^{1-\epsilon} \tag{34}$$

Inserting (34) in (34) we obtain (23). Similarly, we have

$$\mathcal{O}_t = P_t^{\epsilon} \left(\int P_{t-1}(i)^{-\epsilon} \vartheta(i) di + \theta (P_t^*)^{-\epsilon} \right)$$

and

$$\int P_{t-1}(i)^{-\epsilon} \vartheta(i) di = (E[P_{t-1}(i)^{-\epsilon}] E[\vartheta(i)] = \theta \int P_{t-1}(i)^{-\epsilon} di$$

(24) follows.

Lemma 2: $\mathcal{O}_t \geq 1$

$$\begin{aligned} \mathcal{O}_t &= P_t^{\epsilon} \int_0^1 P_t(i)^{-\epsilon} di = P_t^{\epsilon} \int_0^1 \left(P_t(i)^{1-\epsilon}\right)^{\frac{\epsilon}{\epsilon-1}} di \\ &\geq P_t^{\epsilon} \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{\epsilon}{\epsilon-1}} = P_t^{\epsilon} P_t^{-\epsilon} = 1 \end{aligned} \tag{35}$$

where the inequality is due to Jensen inequality and the last equality uses (10).

Proposition 1:

From (33)

$$P_{t+1} = \left(\theta P_t^{1-\epsilon} + (1-\theta)(P_{t+1}^*)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \tag{36}$$

Given P_t, P_{t+1} is strictly increasing in P_{t+1}^* so its supremum is reached in the limit $P_{t+1}^* \to \infty$. We easily obtain $lim_{P_{t+1}^* \to \infty} \frac{P_{t+1}}{P_t} = \theta^{\frac{1}{1-\epsilon}}$.

Lemma 3

Profits of company i are given by

$$\Pi_t(i) = P_t(i) C_t(i) - W_t C_t(i) \tag{37}$$

where W_t is the economy-wide wage. Using (11)

$$\Pi_t(i) = P_t^{\epsilon} C_t (P_t(i)^{1-\epsilon} - W_t P_t(i)^{-\epsilon}) \tag{38}$$

intergrating across all firms and using (10)

$$\Pi_t = \int_0^1 \Pi_t(i) = P_t C_t (1 - \frac{W_t}{P_t} \mathcal{O}_t) \tag{39}$$

Finally, using (32) we obtain the result

$$\Pi_t = P_t C_t (1 - C_t^{\sigma+\phi} \mathcal{O}^{1+\phi})$$

Proposition 2

The result is immediate considering (32), that we repeat here for convenience.

$$W_t = P_t C_t^{\sigma+\phi} \mathcal{O}_t^{\phi}$$

Since, by Lemma 2, $\mathcal{O}_t \geq 1$, it is obvious that C_t can exceed 1 only if $W_t > P_t$. The result that the output gap is smaller than $\frac{1}{\sigma+\phi} log\left(\frac{\epsilon}{\epsilon-1}\right)$ unless $W_t > P_t$ is immediate given that $C_{SS} = \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\sigma+\phi}}$. □

Proposition 3

We express part a) of this proposition in a more general form:

Proposition 3 a): In the limit $\beta\theta \to 0$

$$C_t \geq \left(\frac{\epsilon - 1}{\epsilon} \frac{(1-\theta)^{\frac{1}{\epsilon-1}}}{\left(\theta^{\frac{1}{1-\epsilon}} \mathcal{O}_{t-1} + (1-\theta)^{\frac{1}{1-\epsilon}}\right)^{\phi}} \right)^{\frac{1}{\sigma+\phi}} > 0$$

.

Clearly this implies that in the limit $\theta \to 0$, with β fixed, $C_t \geq C_{SS}$, i.e. the output gap cannot be negative.

Proof of part a): First, consider the ratio $\frac{P_t}{P_t^*}$. It is easy to show that this is upward-bounded. Using (23)

$$\frac{P_t}{P_t^*} = \left(\theta \left(\frac{P_{t-1}}{P_t^*} \right)^{1-\epsilon} + (1-\theta) \right)^{\frac{1}{1-\epsilon}} < (1-\theta)^{\frac{1}{1-\epsilon}} \tag{40}$$

We then have $(P_t^*)^{-\epsilon} < P_t^{-\epsilon}(1-\theta)^{\frac{\epsilon}{1-\epsilon}}$. Using (24)

$$\begin{aligned} \mathcal{O}_t &= P_t^{\epsilon}(\theta P_{t-1}^{-\epsilon}\mathcal{O}_{t-1} + (1-\theta)(P_t^*)^{-\epsilon}) < \left(\theta \frac{P_t^{\epsilon}}{P_{t-1}^{\epsilon}} \mathcal{O}_{t-1} + (1-\theta)^{\frac{1}{1-\epsilon}} \right) \\ &< \left(\theta^{\frac{1}{1-\epsilon}} \mathcal{O}_{t-1} + (1-\theta)^{\frac{1}{1-\epsilon}} \right) \end{aligned} \tag{41}$$

where the last inequality uses the result of Proposition 1.

Moving now to the limit $\theta\beta \to 0$, we have

$$P_t^* \to \frac{\epsilon}{\epsilon - 1} P_t \mathcal{O}_t^{\phi} C_t^{\sigma+\phi} \tag{42}$$

So, in this limit we have

$$(1-\theta)^{\frac{1}{\epsilon-1}} < \frac{P_t^*}{P_t} = \frac{\epsilon}{\epsilon-1}\mathcal{O}_t^{\phi} C_t^{\sigma+\phi} < \frac{\epsilon}{\epsilon-1}\left(\theta^{\frac{1}{1-\epsilon}}\mathcal{O}_{t-1} + (1-\theta)^{\frac{1}{1-\epsilon}}\right)^{\phi} C^{\sigma+\phi} \tag{43}$$

where the first inequality uses (40), the second equality uses (42) and the third inequality uses (41). This finally establishes

$$C_t > \left(\frac{\epsilon - 1}{\epsilon} \frac{(1-\theta)^{\frac{1}{\epsilon-1}}}{\left(\theta^{\frac{1}{1-\epsilon}} \mathcal{O}_{t-1} + (1-\theta)^{\frac{1}{1-\epsilon}}\right)^{\phi}} \right)^{\frac{1}{\sigma+\phi}} \tag{44}$$

Proof of part b): For $\theta \to 0$ we have, as in part a) $P_t^* \to \frac{\epsilon}{\epsilon-1} W_t$.

The price level (10) and the dispersion index can be written as

$$\begin{aligned} P_t &= \left(\int_0^1 P_t(i)^{1-\epsilon} \vartheta_t(i) di + (1-\theta) \left(P_t^*\right)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \\ &= P_t^* \left(\int_0^1 \left(\frac{P_t(i)}{P_t^*} \right)^{1-\epsilon} \vartheta_t(i) di + (1-\theta) \right)^{\frac{1}{1-\epsilon}} \qquad (45) \\ \mathcal{O}_t &= P_t^{\epsilon} \left(\int_0^1 P_t(i)^{-\epsilon} \vartheta_t(i) di + (1-\theta) \left(P_t^*\right)^{-\epsilon} \right) \\ &= \left(\frac{P_t}{P_t^*} \right)^{\epsilon} \left(\int_0^1 \left(\frac{P_t(i)}{P_t^*} \right)^{-\epsilon} \vartheta_t(i) di + (1-\theta) \right) \qquad (46) \end{aligned}$$

where $\vartheta_t(i)$ is an indicator that takes value 1 if firm i *cannot* reoptimize at time t. If $P_t(i) > \kappa W_t \ \forall i$, then $P_t(i) > \tilde{\kappa} P_t^* \ \forall i$, with $\tilde{\kappa} = \frac{\epsilon-1}{\epsilon}\kappa$. hence

$$\int_0^1 \left(\frac{P_t(i)}{P_t^*} \right)^{1-\epsilon} \vartheta_t(i) di < \tilde{\kappa}^{1-\epsilon} \int_0^1 \vartheta_t(i) di = \tilde{\kappa}^{1-\epsilon} \theta$$

$$\int_0^1 \left(\frac{P_t(i)}{P_t^*} \right)^{-\epsilon} \vartheta_t(i) di < \tilde{\kappa}^{-\epsilon} \int_0^1 \vartheta_t(i) di = \tilde{\kappa}^{-\epsilon} \theta$$

Obviously both the above epressions tend to 0 as $\theta \to 0$. It is then immediate to see from (45), (46) that in this limit $P_t \to P_t^*$ and $\mathcal{O}_t \to 1$.

The latter also implies that $W_t = P_t C_t^{\sigma+\phi} \mathcal{O}_t^{\phi} \to P_t^* C_t^{\sigma+\phi}$. Hence

$$lim_{\theta \to 0} P_t^* = \frac{\epsilon}{\epsilon - 1} W_t = \frac{\epsilon}{\epsilon - 1} P_t^* C_t^{\sigma+\phi}$$

which shows that in the limit C_t tends to its steady-state value $\left(\frac{\epsilon}{\epsilon-1}\right)^{\frac{1}{\sigma+\phi}}$.

B – Solution Algorithm for the non-linear model

Given that the economy is at the steady state at $t = 0$, with $P_0(i) = P_0 \ \forall i$ and $C_0 = C_{SS}$, the solution of the non-linear BNK model for a FG policy lasting N periods is given by a sequence $\{P_t, P_t^*, C_t, \mathcal{O}_t\}_{t \geq 1}$ satisfying the Euler equation and the optimal price-setting equation:

$$\begin{aligned} C_t^{-\sigma} &= \beta \frac{1}{1+i_t} \frac{P_t}{P_{t+1}} C_{t+1}^{-\sigma} \qquad (47) \\ P_t^* &= \frac{\epsilon}{\epsilon - 1} \frac{\sum_{k=0}^{\infty} \theta^k \beta^k P_{t+k}^{1+\epsilon} C_{t+k}^{1+\phi} \mathcal{O}_{t+k}^{\phi}}{\sum_{k=0}^{\infty} \theta^k \beta^k P_{t+k}^{\epsilon} C_{t+k}^{1-\sigma}} \qquad (48) \end{aligned}$$

with i_t exogenous for $t = 1, ..N$, and additionally satisfying the Taylor rule for $t > N$; subject to the law of motion of the price level

$$P_{t+1} = \left(\theta P_t^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \tag{49}$$

and

$$\mathcal{O}_t = P_t^{\epsilon}(\theta P_{t-1}^{-\epsilon}\mathcal{O}_{t-1} + (1-\theta)(P_t^*)^{-\epsilon}) \tag{50}$$

with $\mathcal{O}_0 = 1$.

Finding a solution for the non-linear model is challenging because, since the equilbrium quantities for each period depend on the state variable $\mathcal{O}$, backward induction over time is not feasible. We use the recursive algorithm described below to find the solution starting at a generic time t_0 (i.e. the sequence $\{P_t, P_t^*, C_t, \mathcal{O}_t\}_{t \geq t_0}$) as a function of the state variable $\mathcal{O}_{t_0-1}$ and the number of periods n until the end of the FG policy. $n = 0$ refers to the case in which, at t_0, The FG policy is already over and the Taylor rule has been restored, but there is still some price dispersion in the economy, inherited grom the FG period. We call the sequence that constitutes the solution for this case the "Taylor sequence".

It is convenient to adopt the normalization in which the price level is 1 in the last period of the FG policy (or in the period before t_0 if $n = 0$). Our normalization is then $P_{t_0+n-1=1}$.

The algorithm comprises three different routines, for the three cases $n = 0$, $n = 1$ and $n > 1$, described in Section *B1, B2* and *B3*, respectively.

We also consider a simpler version of the algorithm, that we call "semi-linear", that differs from the main alforithm for the case $n = 0$: in this version, we ignore the effect of price dispersion once the Taylor rule is restored, and posit that as soon as the FG-policy period is over the system goes back to steady state immediately (as is the case for the solution of the linear model). This simpler version of the algorithm is detailed in Section *B4*.

B1: Solution for $n = 0$

After the Taylor rule is restored, the optimal price chosen by the unconstrained firms, P_t^*, must be a function of price dispersion in the previous period, $\mathcal{O}_{t-1}$, which is the only state variable. We postulate the following functional form

$$\frac{P_t^*}{P_{t-1}} = f(\mathcal{O}_{t-1}) = 1 + a(\mathcal{O}_{t-1} - 1) + b(\mathcal{O}_{t-1} - 1)^2 \tag{51}$$

where a and b are constant parameters. Obviously when $\mathcal{O}_{t-1} = 1$ (no previous-period price dispersion) $P_t^* = P_{t-1}$, which ensures that no price

dispersion is generated and inflation is zero. In this case the economy is at the steady state.

The algorithm to calibrate a and b is the following. Given a value of $\mathcal{O}_{t_0-1}$, and given our price normalization $P_{t_0-1} = 1$, choosing an arbitrary guess value for a and b we can compute a sequence $\{P_t, P_t^*, \mathcal{O}_t\}_{t \geq t_0}$ by

$$\begin{aligned} \frac{P_t^*}{P_{t-1}} &= f(\mathcal{O}_{t-1}) \\ P_t &= \left(\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \\ \mathcal{O}_t &= P_t^{\epsilon}(\theta P_{t-1}^{-\epsilon}\mathcal{O}_{t-1} + (1-\theta)(P_t^*)^{-\epsilon}) \end{aligned}$$

starting from $t = t_0$ and ending at T such that $\mathcal{O}_T$ is sufficiently close to 1. After T we can consider the economy to be at the steady state with constant prices, i.e. $P_{T+s} = P_{T+s}^* = P_T \ \forall s > 0$, and $\mathcal{O}_{T+s} = 1 \ \forall s > 0$.

Given the sequence for the price level P and the Taylor rule, we use the Euler equation to find a sequence $\{C_t\}_{t \geq t_0}$. We then compute a second sequence $\{\tilde{P}_t^*\}_{t \geq t_0}$ through (48), using the previously computed sequences $\{P_t, C_t, \mathcal{O}_t\}_{t \geq t_0}$ on the RHS. We finally select a and b as the values that minimize the distance between the sequences P_t^* and $\tilde{P}_t^*$.

B2: Solution for $n = 1$

In this case the initial time t_0 is only one period away from the restoration of the Taylor rule. Following our price normalization we have $P_{t0} = 1$. Given $\mathcal{O}_{t_0-1}$:

1. Guess a value of $\mathcal{O}_{t0}$ and obtain the corresponding "Taylor sequence", i.e. the sequence $\{P_t, P_t^*, C_t, \mathcal{O}_t\}_{t \geq t_0+1}$ given $\mathcal{O}_{t0}$, following the algorithm in section *B1*.

2. Obtain $\pi_{t_0+1} = log(P_{t_0+1})$, hence C_{t0} from the Euler equation.

3. Obtain $P_{t_0}^*$ from the firm's optimization problem (48), hence P_{t_0-1} using (49).

4. Using $\mathcal{O}_{t-1}$, P_{t-1} and $P_{t_0}^*$, obtain an "output" value of the price-dispersion variable, $\tilde{\mathcal{O}}_{t_0}$ (in general different from the guess value $\mathcal{O}_{t0}$) using (50).

5. Find the "fixed-point" value of the price dispersion variable, such that $\mathcal{O}_{t_0} = \tilde{\mathcal{O}}_{t_0}$

Notice that this algorithm generates a sequence $\{P_t, P_t^*, C_t, \mathcal{O}_t\}_{t \geq t_0}$, and in addition generates P_{t_0-1} and does so by calling the "$n = 0$"-routine described in section *B1* that generates the sequence $\{P_t, P_t^*, C_t, \mathcal{O}_t\}_{t \geq t_0+1}$.

B3 Solution for $n > 1$

Given $\mathcal{O}_{t_0-1}$:

- Guess a value of the state variable $\mathcal{O}_{t_0}$ and obtain the corresponding sequence $\{P_t, P_t^*, C_t, \mathcal{O}_t\}_{t \geq t_0+1}$ by calling the "$n-1$-routine" (if $n > 2$, this would be again the routine described in this section, otherwise it would be the routine described in section *B2*). It should be noticed that the "$n-1$-routine" also generates the price level P_{t_0-1}, consistent with the normalization $P_{t_0+n-1} = 1$.
- Obtain $\pi_{t_0+1} = log\left(\frac{P_{t_0+1}}{P_{t_0}}\right)$, hence C_{t_0} from the Euler equation.
- Obtain $P_{t_0}^*$ from the firm's optimization problem and P_{t_0-1} using (49).
- Using $\mathcal{O}_{t_0-1}$, P_{t_0-1} and $P_{t_0}^*$, obtain an "output" value of the price-dispersion variable, $\tilde{\mathcal{O}}_{t_0}$ using (50).
- Find the "fixed-point" value of the price dispersion variable, such that $\mathcal{O}_{t_0} = \tilde{\mathcal{O}}_{t_0}$

Again, this algorithm generates a sequence $\{P_t, P_t^*, C_t, \mathcal{O}_t\}_{t \geq t_0}$, and in addition generates P_{t_0-1}, and does so by calling the "$n-1$"-routine, which generates the sequence $\{P_t, P_t^*, C_t, \mathcal{O}_t\}_{t \geq t_0+1}$.

Given the routines described in section *B1, B2* and *B3*, we obtain the solution for the $N-$ period FG-policy problem by calling the $n = N$ routine with $\mathcal{O}_{t_0-1} = 1$.

Solution of the semi-linear model

For the semi-linear model, the algorithm is the same as above, except that the '$n = 0$' routine simply returns the steady-state solution, i.e. $P_t = P_t^* = 1 \ \forall t \geq t_0$, $C_t = C_{SS} \ \forall t \geq t_0$, regardless of the value of the state variable $\mathcal{O}_{t_0-1}$.

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