Munich Personal RePEc Archive

Geopolitics, National Sovereignty and Trade Policy in Historical Perspective

Brezis, Elise

Bar Ilan University

28 April 2026

Online at https://mpra.ub.uni-muenchen.de/128967/
MPRA Paper No. 128967, posted 05 Jun 2026 14:46 UTC

April 2026

Geopolitics, National Sovereignty and Trade Policy in Historical Perspective

Abstract

The historical record of the past 350 years reveals recurrent cycles in the international distribution of power: periods of hegemonic leadership, in which a dominant state shapes the international order, alternate with periods of balance-of-power rivalry, when no single state is preeminent and major powers compete on more equal terms. Trade policy also follows a cyclical pattern, with phases of liberalization giving way to periods of protectionism.

This paper examines the relationship between these two cycles, and analyzes how the international power structure changes the optimal trade regime.. It argues that hegemonic periods, such as 1870–1910 and 1945–2015, are more conducive to free trade, because the hegemon provides security guarantees, access to markets, and institutional linkages that reduce the risks of liberalization and increase the benefits of openness. By contrast, during balance-of-power periods, such as 1910–1945 and the period since 2015, intensified rivalry among great powers strengthens nationalism, heightens concerns over sovereignty, and encourages protectionist policies.

The paper proposes an explanation for this correlation grounded in theories of national identity and sovereignty. More broadly, it shows that shifts in the structure of the international system affect not only interstate relations but also domestic policy choices. The empirical analysis supports this argument: hegemonic eras are associated with waves of trade liberalization, whereas balance-of-power eras are linked to higher tariffs and a stronger turn toward protectionism.

Keywords: Balance of Power, Geopolitics, Hegemony, National Sovereignty, Trade Policy, Free Trade, Protectionism, Social Identity.

JEL: B17, B27, E12, F30.

* Head, ACEP Center, Bar-Ilan University www.elisebrezis.com elise.brezis@biu.ac.il

I. Introduction

This paper analyzes how the international power structure affects the optimal trade regime. I show that in the presence of a hegemonic power, nations tend to opt for free trade. By contrast, in a balance-of-power context, in which no single actor possesses hegemonic status, countries are more likely to choose protectionism.

Over the centuries, the international system has displayed a pendulum movement. The historical record of the past 350 years reveals cycles in which a nation-state is dominant and exercises leadership, alternating with periods of balance of power in which no single state dominates and several nations possess comparable levels of power.

This paper develops a simple paradigm to explain the correlation between the structure of the international system and trade policy. Its main innovation is to incorporate social identity into the utility function. At the level of nations, the most salient form of identity is national sovereignty. Tajfel's social identity approach emphasizes that individuals derive part of their utility from membership in a group and evaluate their group relative to others, generating in-group attachment and out-group comparison. Even minimal group distinctions can produce systematic favoritism toward one's own group, suggesting that relative standing and not only absolute welfare, matters for behavior. (see Tajfel and Turner, 1979).

The us/them behavior emphasized by Tajfel-type models is particularly dominant at the level of nations: individuals care about the relative power of their nation vis-à-vis others. Incorporating national identity, that is, national sovereignty, into utility is the first key element of the model.

The second element is the structure of the international system. There are periods in which one country is richer and more powerful than the others, and periods without a hegemon. A first relationship explored in the model concerns how the importance of national sovereignty depends on the international system. I show that when there is a hegemon, the national sovereignty term remains present but is quantitatively small, so that standard economic components such as consumption become relatively more important.

A key mechanism is that, under hegemony, the hegemon supplies a common defense good that compresses relative-power differences across countries; as a result, sovereignty matters, but only to a limited extent. Under balance of power, by contrast, relative-power gaps are no longer compressed, and sovereignty becomes a first-order determinant of individual utility.

A second mechanism is that protectionism can raise sovereignty when it weakens the rival more than oneself: individuals may accept self-harm if it harms the out-group

even more, because relative power yields utility through identity. When there is no hegemon, national sovereignty becomes essential, since relative-power differences are no longer mitigated by a global public good.

The paper is structured as follows. Section II presents a brief historical perspective on international systems and trade policy. Section III reviews the relevant literature, Section IV introduces the model, and Section V concludes.

II. Hegemony, Balance of Power and Trade Policy

The historical record of the past 350 years shows cycles in which there are periods where a nation-state is dominant and has leadership, while there are periods of 'balance of power' in which there is no dominant state, and many nations are similar in their power.

The data are presented in Figures 1-4. From 1850 to 1910, there is *Pax Britannica*; the UK has leapfrogged the Dutch, and is the hegemonic power (see Figure 1). After 1945, we are in period of *Pax Americana*, and the US has leapfrogged the UK. From 1945 until 2015, the US is the hegemonic power of the world (see Figure 2).[1]

But, during the period 1910-40, the data display that GDP per capita among the main powers are very similar, (although Germany has higher GDP than France from 1900 and on). This is a pure 'balance of power' system which will be de facto until 1945, the end of World War II (see Figure 3). In consequence, the international system is very different at time of the Treaty of Versailles, compared to the end of World War II.

About trade policy, the Navigation Act of 1651 is widely held to have represented the end of effective Dutch commercial hegemony in Europe and to have marked the beginning of the British challenge.

By the end of the second decade of the 19th century, Britain had set about dismantling its own protectionist apparatus and had initiated an era of free trade which lasted as long as the *Pax Britannica* endured.[2]

The rise in German power in the 1870s went hand in hand with protectionism driven by nationalistic fervor.[3] The rise of American power in the pre-World War II

---

[1] Brezis, Krugman and Tsiddon (1993) present a model explaining endogenously the changes in hegemony, and about the data on the Balance of payments, see Brezis (1995).

[2] Gilpin (1975) writes that: "Britain's interest lay in universal free trade and the removal of all barriers to the exchange of goods."(p.84).

[3] Lawson (1983) summarizes the views of Gilpin and Krasner on this issues as follows: "Gilpin and Stephen Krasner suggest that after the 1870s Britain was no longer in a position to prevent Germany, France, and Italy from adopting protective tariffs and destroying an international order in which free trade had been the rule." (pp.317-339).

years was associated with a succession of protectionist measures, whereas the establishment of American hegemony after the war led to the pursuit of free-trade.[4]

III. Related Literature

3.1. National Identity and National Sovereignty

One of the main shifts in the history of economic theory lie in the introduction of Behavioral Economics in mainstream theory, which opened the floodgates to many other innovative lines of thinking. In consequence, it not only became 'kosher' to speak of culture and psychology in the economic realm, it became 'in'.[5]

Sociology also slowly penetrated the field of economic theory. In 2000, Akerloff and Kranton introduced 'identity' as an element affecting economic choices. In the field of social psychology, Tajfel developed the 'theory of social identity'. He contended that individuals have an inherent tendency to categorize themselves into one or more "in-groups", building a part of their identity based on membership in that group and enforcing boundaries with other groups.

The social identity theory (SIT) posits the fact that a person's self-concept and self-esteem derive not only from personal identity and accomplishments, but from the status and accomplishment of the groups to which s/he belongs (Tajnel and Turner, 1979). In their experiments, they have shown that humans have a need for 'us/them' distinctions. Thus, social identity theory suggests that people identify with groups in such a way as to *maximize positive distinctiveness*.

They have shown that social identity leads to discrimination. When being divided into two groups, and asked to split budgets between the two groups, individuals had the choice between giving $100 to each member of both groups, or receiving $50 for each person in their group, but only $10 to the other group. Most individuals chose the second option. Making the other group worse off is more important than making your group better.[6]

Recall that group selection was chosen almost randomly, and in some of the experiments, it was chosen completely randomly by tossing a coin. Yet, the results were significant. If these are the results for a random selection of groups, what happens

---

[4] See, for example Calleo and Rowland (1973). Cline (1980) argues that tendencies toward what he calls "neomercantilism" were visible at the end of the seventies, which corresponds to the end of American hegemony in the world system; also see Malmgren (1970).

[5] In reference to Temin (1997): "Is it Kosher to talk about Culture?"

[6] Moreover, it was shown that those who had been allowed to engage in intergroup discrimination had higher self-esteem that those who had not been given the opportunity to discriminate (Lemyre and Smith 1985, Oakes and Turner 1980).

when the group has some sort of common trait or culture, such as being white? Or being French?

Metzl (2019) discussed discrimination between whites and non-whites. He shows that individuals belonging to a given group can make decisions that are not "rationally" productive for them, just to feel part of the group and to separate themselves from the other group. An example he gives is that young white individuals belonging to the working class who were sick and in need of Medicare, chose "to be broke, but not to let the other group get it for free". [7] They are willing to make choices that harm themselves in order to maintain their class identity (see also Reicher, 2015).

Another clear grouping is the nation itself. One speaks today about the identity of being American: While "Being an American is an element of the self-concept of most Americans", it leads to discrimination and to in-group favoritism.

Robert Reich, former Secretary of Labor from 1993 to 1997 in the cabinet of President Bill Clinton and professor at Harvard, asked his students to choose between two scenarios. Choice A: the US grows by 3% per year, and China grows by 4% per year. Choice B: the US grows by 2% per year, and China grows by 1% per year. Approximately 70% of the students chose B.

In 2019, while I was a visiting professor at Nanjing University, I asked 80 Chinese students in my class to choose between Choice A and Choice B, swapping the growth rates for China and the US. All the students chose the option of having less economic growth if it meant that the opponent (the US) would also have less growth. Some students even added, "We want to become the next superpower, even at the cost of lower growth."

In consequence, the us/then distinction leads to some behavior, which would seem irrational for individuals who only maximize their consumption. Because our identity is tied to the group, the group's success feels like a personal win, while its failure can feel like a personal threat. National identity is an element of the utility of individuals.

What are the 'markers' of national identity of the nation? Following Metzl (2019), the main elements defining nation's identity are the symbols of nationalism and power. Thus, national sovereignty and power influence the sense of well-being of the individual.

[7] Medicare might improve life for all groups, but as it would improve the well-being of the non-white group more, working class 'white' Americans actively oppose it.

3.2. The International System

A. The Classical Theories

The relationship between protectionist commercial policy, power, nationalism, and the balance of power has long been explored by political scientists. This literature offers different perspectives on hegemony and its implications for international relations, emphasizing the role of power, interests, and institutions in shaping the international system, as well as the challenges associated with the presence or absence of a hegemonic leader.

Morgenthau (1949) has established the "balance of power" as a central concept, highlighting how the system relies on flexible alliances to maintain peace, but did not have a specific theory on the whole international system.

One of the most influential paradigms linking the structure of the international political system to commercial policy is the theory of hegemonic stability. Kindleberger (1973), following Frohlich, Oppenheimer, and Young (1971), argues that international stability and security are collective goods. The central claim is that a single dominant state, or hegemon, is needed to maintain order in the international system by providing public goods such as security, market access, and a stable economic framework.

Gilpin developed this perspective in a realist direction. In *U.S. Power and the Multinational Corporation* (1975) and later in *War and Change in World Politics* (1981), he argued that the rise and decline of hegemonic powers follow a recurrent pattern in international relations. He emphasized that a liberal international economy is not self-sustaining, but depends on the power, interests, and leadership of dominant states. In this view, hegemonic decline is likely to generate instability and conflict. Gilpin (1975) writes that "a liberal international economy cannot come into existence and be maintained unless it has behind it the most powerful state(s) in the system.… A liberal economic system is not self-sustaining, but is maintained only through the actions -- initiatives, bargaining, and sanctions of the dominant power(s)." [8]

Krasner (1976) further examined the relationship between state power and openness in international trade. He argued that hegemonic states tend to support free trade, whereas weaker states are more likely to prefer protectionism. His contribution was fundamental in showing how the international distribution of power shapes economic relations among states.

Stein (1984), for his part, focused on the costs of hegemony and the dilemmas faced by dominant powers in supplying international public goods. He highlighted the trade-offs involved in maintaining leadership and the tensions between collective benefits and national burdens.

---

[8] Gilpin (1975), p.85. See also Coleman (1969).

In contrast, Keohane (1984) challenged the view that hegemony is a necessary condition for international cooperation. He argued that even after hegemonic decline, states may still cooperate through institutions and international regimes. This important critique opened the way to a broader literature showing that cooperation may survive beyond hegemonic leadership. Snidal (1985) pushed this argument further by showing that hegemonic stability theory contains distinct variants, coercive and benevolent, and that its explanatory power is more limited than often assumed. In particular, he argued that hegemony is neither necessary nor always sufficient for cooperation.

A related literature, typified by Gallagher and Robinson (1953), considers the "imperialism of free trade."[9] This process, which purports to explain the development of the world trading system in the presence of a hegemonic power, argues that "the hegemonic power is expected to extend its control, and the open international trading order, over local economies by informal or indirect means whenever possible, since this constitutes the cheapest way to create and maintain its predominant position in the world."

A related contribution is Lake (1993), who reexamined the concepts of leadership and hegemony in the international economy. He argued that the theory of hegemonic leadership had often been stated too broadly, and that neither leadership nor hegemony alone could fully explain patterns of international openness. This line of work is useful because it helps distinguish more precisely between the existence of a dominant state and the specific mechanisms through which it shapes trade policy.

B. Theories of Unipolarity and Multipolarity

A second strand of the literature shifts the focus from hegemony in general to the consequences of different polarity structures, in particular unipolarity, bipolarity, and multipolarity. The central question is whether the number of major powers in the system changes the incentives of states to cooperate, to compete, or to adopt defensive and commercial policies shaped by geopolitical concerns.

An initial reference for the polarity literature is Waltz (1979), who argued that the structure of the international system depends on the distribution of capabilities across major powers, and that different structures generate different strategic constraints and forms of behavior. In this view, bipolar systems are inherently more stable than multipolar ones because they reduce the risk of miscalculation.

Some theorists put the emphasis on the positive effects of unipolarity. Ikenberry explores how the U.S. used its unipolar power to build a "liberal, rules-based order,"

[9] Also see MacDonagh (1962), Moore (1964), Mathew (1968), and Platt (1968, 1973).

arguing that international institutions can help sustain unipolarity by making it more acceptable to other states. Wohlforth (1999) argued that a unipolar world may be more stable than balance-of-power theorists had expected. In his view, when one state is clearly dominant, rival powers have fewer incentives to challenge it directly, and large-scale balancing becomes less likely.

This claim was later discussed more broadly in Ikenberry, Mastanduno, and Wohlforth (2011), which examined the consequences of unipolarity. Moreover, Krauthammer (1990), shortly after the Cold War, identifies the emergence of a unipolar world with the United States as the sole, unchallenged superpower.

Other scholars were more skeptical about the durability of unipolarity. Layne (1993) argued that unipolarity is likely to be temporary, because new great powers eventually rise and balancing pressures reappear. Mearsheimer (2001) similarly emphasized that durable peace is difficult in a world of great powers and that the distribution of power inevitably creates incentives for rivalry and competition. Mearsheimer argues that the anarchic nature of the international system forces states to maximize their power, often leading to intense competition in multipolar systems.

A further major contribution is Monteiro (2014), who asked whether unipolarity is durable, whether it is peaceful, and what forms of conflict it generates. Monteiro argued that unipolarity does not eliminate competition; rather, it changes its form. This insight is especially useful for the present paper, because it suggests that even when one state is dominant, geopolitical tensions remain relevant and can shape the incentives of other states.

Gowa (1989) links trade openness to the polarity of the international system. She argued that bipolar systems are more conducive to free trade than multipolar ones because alliances are more stable and the security externalities of trade are easier to internalize. Gowa and Mansfield (1993) extended this argument by showing that trade flows are strongly influenced by security relations and alliance patterns. Their contribution is central because it connects commercial openness directly to geopolitical structure rather than treating trade as a purely economic choice.

This literature has therefore examined not only whether one state is preponderant, but also whether the compression or widening of relative-power gaps changes the incentives of all other states. In this respect, the literature on polarity is highly relevant to the present paper, because it suggests that the strategic environment faced by states depends on whether relative-power differences are large or small. Yet most of this literature remains at the systemic level and does not explain through which utility mechanism these geopolitical configurations affect trade-policy choices.

In conclusion, the literature relating trade policy to the international system is vast. But broad ideas such as the "imperialism of free trade" or the provision of a "collective good" remain too general to identify the precise mechanism linking commercial policy to the structure of the international system. There is therefore a need to isolate the essential elements that connect trade policy to geopolitical order. In this vein, this paper develops a model in which a hegemon provides a public good and in which trade policy is shaped by social identity, national sovereignty, and relative power. In this way, the paper provides a micro-foundation for the historical correlation between shifts in polarity and shifts in trade policy.

IV. The Model

4.1 Introduction

The main idea of this paper is that international security is a collective good. Since free trade is optimal only in a climate of international security, a regime of free trade is most likely to arise when one power is sufficiently dominant to provide and enforce that security. In the absence of a hegemon, weaker states can no longer rely on a common defense umbrella, and protectionism becomes more attractive.

The main insight of the model is to incorporate social identity into the utility function. At the level of nations, the main identity is national sovereignty. The us/them behavior discussed in the previous section, following Tajfel, is a central feature of social identity and is particularly strong at the level of nations.

Individuals care about the relative power of their nation vis-à-vis others, as the us/them approach suggests. Incorporating national identity, that is, national sovereignty, into the utility function of individuals is therefore the first key element of the model.

The second element is the structure of the international system. There are periods in which one country is richer and more powerful than the others, and there are periods without a hegemon. This paper presents a model that explains why, when there is a hegemon, the utility function is less affected by national sovereignty and national identity, whereas when there is no hegemon and states are engaged in a struggle for power, these considerations have a much stronger impact on utility.

A key mechanism is that, under hegemony, the hegemon supplies a common defense good that compresses relative-power differences across countries; therefore, sovereignty matters, but its quantitative importance is limited. Under balance of power, by contrast, relative-power gaps are no longer compressed, and sovereignty becomes a first-order determinant of individual utility. A second mechanism is that

protectionism can raise sovereignty when it weakens the rival more than oneself: individuals may accept self-harm if it harms the outgroup even more, because relative power yields utility through identity. Let us present the framework.

4.2. The countries and the international system

There is one potential hegemon *H* and two countries i ∈ {1,2}. Countries 1 and 2 choose trade policy and then they choose how much to spend on defense.

The international system state can be either with a hegemon (H) and the hegemon exists, because its wealth W is high, or balance of power, *B*, because W is low. In consequence:

$$s \in [H, B] \qquad (1)$$

The model is a Stackelberg framework with three stages, which will be solved by backward induction.

Stage 1 The hegemon's choice.

If s=H, the hegemon chooses defense provision $D^H \geq 0$ (and thus provides a security umbrella to the world). If s=B, there is no hegemon and $D^H=0$. D^H is an endogenous variable.

Stage 2 Trade-policy of the countries

Countries 1 and 2 simultaneously choose $a_i \in$ {F,P}, when *F* denotes free trade, and *P* denotes protectionism.

Stage 3

Countries 1 and 2 simultaneously choose $D_i \geq 0$, their private investments in defense. It has a main effect on the utility through the element of national identity, and of course the probability of war, and of losses due to the war.

4.3. Income, trade policy, and defense costs

The output of each country can be spent either on consumption, c, net exports (NX) or government spending on military defense, *K(D).*

$$c_i = Y_i - K(D_i) - NX_i \qquad (2)$$

Trade policy affects output, and depending on the trade policy of each country, and the existence of a hegemon, we denote these reduced-form effects as $E_i(\cdot)$, we discuss below. In consequence:

$$Y_i = y_i + E_i(a_i, a_j; s) \qquad (3)$$

where $E_i(a_i, a_j; s)$ is the reduced form effects of trade policy when country i chooses

a_i (either F or P), and country 2 chooses a_j; the international system s is either H (hegemon) or B (balance of power).

About defense costs, we assume that they are convex, so for simplicity purpose, the costs of defense are:

$$K(D_i) = D_i + \frac{\kappa}{2} D_i^2 \quad \kappa > 0 \tag{4}$$

Hence we get that consumption is:

$$c_i = y_i + E_i(a_i, a_j; s) - K(D_i) - NX_i \tag{5}$$

The countries can choose a free trade policy (F), or a protectionist policy (P). Based on trade theory, we assume that free trade leads to gains, and a common affects the gains from trade good (see section III). So, in a schematic way, we have:

$$E_i(F, F; B) = T_0 \qquad T_0 > 0 \tag{6}$$
$$E_i(F, F; H) = T_1 \qquad T_1 > T_0 > 0 \tag{7}$$
$$E_i(P, P; s) = 0 \qquad \text{for both s=}H\text{ and s=}B. \tag{8}$$
$$E_i(F, P; B) = T_0 - \chi p(D_1, D_2; D^H = 0) \qquad \chi > 0 \tag{9}$$

When there is no hegemon and one side is open while the other is protectionist, openness may create vulnerability. Indeed equation (9) captures the idea that unilateral openness is vulnerable when there is no hegemon.

4.4. Relative power and national sovereignty

Based on the literature (see section III above), national sovereignty is modeled as a component that depends on relative power. The main mechanism in this paper is that relative power aggregates domestic defense and the hegemon's umbrella contribution.

So the relative power for country i is:

$$\Delta_i = D_i + \omega D^H \quad 0 \le \omega \le 1 \tag{10}$$

where Δ_i is power of country i, D_i is the defense level of country i, and D^H, the defense level of the hegemon, which is an umbrella contribution to the world. It is important to understand that when there is no hegemon, then D^H=0, as will be shown below.

National identity in this model takes the form of national sovereignty, S_i . Based on the theories of international system, we define national sovereignty as a function of the relative power:

$$S_i = \bar{S} + v \ln(\Delta_i / \Delta_j) \qquad v > 0 \quad i \neq j \tag{11}$$

Note that when there is a hegemon, the common term ωD^H is added to both sides, compressing the ratio Δ_i / Δ_j toward 1. Indeed, a key mechanism for understanding the differences between hegemony and balance of power is that, under hegemony, the hegemon supplies a common defense good that compresses relative-power differences across countries; therefore, sovereignty matters but is quantitatively small. Under balance of power, relative-power gaps are not compressed, and sovereignty becomes a first-order driver of individual utility. Let us present the utility function of individual in each country.

4.5. The utility functions

This paper put emphasis on the fact that national sovereignty is an essential element of the will of individuals. In consequence, the utility of individuals in countries 1 and 2 is a function of consumption plus a national sovereignty term, minus expected war losses. Indeed, war probability is an element which affects the behavior of countries and therefore should be included in models of trade policy. The utility function of country i is:

$$U_i = c_i + \alpha S_i - p(D_1, D_2; D^H)\Omega, \quad \alpha > 0. \tag{12}$$

where c_i is consumption, S_i is national identity, $p(.)$ is the probability of war, and Ω is the loss when there is war. The probability of war can take various forms. In this paper, war risk depends on private defense and on the hegemon's umbrell:.

$$p(D_1, D_2; D^H) = \bar{p} - \theta[D^H + \mu(D^H)(D_1 + D_2)], \tag{13}$$

with $\theta > 0$, $\mu'(\cdot) < 0$, and $0 < \mu \leq 1$. We assume that the hegemon's umbrella reduces war risk directly and crowds out private defense ($\mu' < 0$).

Equation (13) is a reduced-form of 'security production function'. It is consistent with a collective defense supplied by a dominant ally which both improves security directly and reduces the marginal role of smaller allies' own defense effort. It is based on Olson and Zeckhauser (1966) model in which the hegemon's defense has a 'public goods' aspect, which creates incentives for smaller members to rely on the dominant ally's effort. In the same work, they emphasize that "the burden of an alliance will be borne

in a disproportional way," with larger members paying more, which is the logic captured by $\mu'(D^H) < 0$ (the free-riding effect) in equation (13).

In consequence, expected war loss for i is: $p(D_1, D_2; D^H)\Omega \qquad \Omega > 0$.

For the hegemon, when it exists, (defined by a high income) the utility function differs, because he provides a common good, defense: D^H, which costs him $k \cdot D^H$, but gives him the proud of being a hegemon (see the literature above). The hegemon income is:

$$Y^H = \bar{y} + W \quad \text{and} \quad W > kD^H \tag{14}$$

The hegemon's utility is:

$$U^H = \bar{y} + W + \beta\Pi(D^H) - kD^H - p\Omega_H. \tag{15}$$

The term $\beta\Pi(D^H)$ captures the strategic and status benefit from providing the security umbrella. Let us check the size of the defense level provided by the hegemon.

Lemma 1

The hegemon chooses D^H to maximize (15). The interior solution, $D^{H*} > 0$, which satisfies equation (16) is the amount of defense provided by the hegemon.

Proof.

Assume U^H is concave in D^H. The hegemon solves the maximization problem, with respect to D^H which yields

$$\partial U^H / \partial D^H = \beta\Pi'(D^H) - k - \partial p / \partial D^H (D_1, D_2; D^H)\Omega_H = 0 \tag{16}$$

Since U^H is concave, the derivative is decreasing in D^H. Therefore, if it is positive at $D^H = 0$, there exists a unique interior level D^{H*} at which it becomes zero. If the derivative of U^H at $D^H = 0$ is non-positive, then the optimum is the corner solution $D^{H*} = 0$. Then, it is not a hegemon.

4.6 The equilibrium

We solve the model by backward induction. Stage 3 determines private defense, Stage 2 determines trade policy, and Stage 1 determines the hegemon's defense umbrella.

Since D is the main element affecting power and national sovereignty, we first focuses on the relationship between states, 1 and 2, and national sovereignty, S_i.

Proposition 1

a. Due to the common good provided by the hegemon, the equilibrium private-defense level under hegemony is lower than the equilibrium private defense level under balance of power. In other words, $D^*_{i,h} < D^*_{i,b}$.

b. the optimal choices of private defense, $D^*_{i,h}$ *and* $D^*_{i,b}$ *do not depend on trade policy.*

Note that $D^*_{i,h}$ is country i's optimal private defense under hegemony, and $D^*_{i,b}$ is country i's optimal private defense under balance of power.

Proof.

Country i finds the defense level which brings its utility (equation 12) to maximum.

Substituting (5), (11), and (13) into (12), country i solves max U: [10] The first-order condition is:

$$\partial U / \partial D_i = -1 - \kappa D_i + \alpha v / (D_i + \omega D^H) - \partial p / \partial D_i (D_1, D_2; D^H)\Omega = 0$$

So from (13), we get: $\partial p / \partial D_i (D_1, D_2; D^H) = -\theta\mu(D^H)$

Hence the first-order condition becomes:

$$1 + \kappa D_i = \alpha v / (D_i + \omega D^H) + \theta\mu(D^H)\Omega \qquad (17)$$

Let us start by proving part b.

Equation (17) depends on the international regime through D^H, but it does not depend on the trade-policy profile (a_i, a_j). Therefore, once the regime is fixed, the optimal private-defense level is the same, regardless of whether countries choose free trade or protectionism.

This fact implies that, within a given regime, any comparison between trade-policy payoffs in Stage 2 is not affected by a change in defense choice. Defense is determined only by whether there is a hegemon or not.

The proof of part a is presented in appendix 1.

Thus Proposition 1 underlines that private defense is lower under hegemony:

$D^*_{i,h} < D^*_{i,b}$.

We should emphasize that this inequality holds in equilibrium, despite the fact that in a symmetric equilibrium, sovereignty itself is $S^*_i = \bar{S}$ in both regimes.[11]

---

[10] which is:

$$U_i = y_i + E_i(a_i, a_j; s) - D_i - (\kappa/2)D_i^2 + \alpha[\bar{S} + v\ln((D_i + \omega D^H)/(D_j + \omega D^H)) - p(D_1, D_2; D^H)\Omega$$

[11] In other words, under hegemony what is lower is the marginal importance of private defense for sovereignty. Indeed:

This inequality in defense level is a central element of the model. Since the hegemon supplies a common defense good, countries spend less on private defense and can therefore achieve higher consumption under hegemony than under balance of power.

We now turn to Stage 2 and analyze trade-policy choice.

Proposition 2

Free trade is a Nash equilibrium under hegemony.

Protectionism is a Nash equilibrium under balance of power, under the assumption that: $\chi \cdot p(B) \geq T_0$.

The proof is presented in Appendix 2.

This proposition shows that under hegemony a necessary condition for free trade to be a Nash equilibrium is $T_1 \geq 0$, which given assumption (7), is automatically satisfied.

A necessary condition for protectionism to be a Nash equilibrium under balance of power is:

$$\chi p(.) \geq T_0 \qquad (18)$$

The proof is presented in Appendix 2.

Equation (18) is equivalent to $E_i(F, P; B) \leq 0 = E_i(P, P; B)$ (see equations (8) and (9)). Hence, under balance of power, unilateral free trade against a protectionist rival yields a lower trade payoff than remaining protectionist, which is the necessary condition for protectionism to be a Nash equilibrium.

Note that standard trade theory does not generally imply that unilateral openness is necessarily welfare-reducing. In the present model, however, unilateral openness is suboptimal, because it is under balance of power, which creates a vulnerability cost that outweighs the direct gain from trade. Thus, the result does not follow from conventional trade theory alone, but from the interaction between trade policy, security, and national sovereignty.

In conclusion, proposition 2 states that under hegemony, unilateral deviation from protectionism to free trade leads to a gain, although, it does not change defense, sovereignty, or war risk in the present model; since the main difference is the direct positive trade gain T_1.

Under balance of power, unilateral deviation from free trade to protectionism leaves defense unchanged; the relevant comparison is between the direct trade gain T_0 and the vulnerability cost χp. Protectionism is a Nash equilibrium because the costs are higher than the gain from unilateral openness.

$\frac{\partial S_i}{\partial D_i} = \frac{v}{D_i + \omega D^H}$,

In consequence, the incentive to build private defense for sovereignty reasons is weaker.

We now turn to the stage 1 decision of the hegemon:

Proposition 3

When there is no hegemon, with W sufficiently high, then $D^{H*} = 0.$. *When there is a hegemon, the hegemon chooses* D^{H} *to maximize (16). A condition for the existence of a hegemon is that wealth is sufficiently high and the marginal benefit of hegemonic defense exceeds its marginal cost.*

Proof.

This proposition follows immediately from Lemma 1 together with the resource feasibility condition. If W is too low, positive hegemonic defense cannot be financed and the only feasible choice is $D^{H*} = 0.$.

If W is sufficiently high and $\beta\Pi'(0) - k - \partial p / \partial D^{H}(D_1, D_2; D^{H})\Omega_H > 0$ then the derivative of the hegemon's utility at zero is positive. Under concavity, there exists a unique positive level D^{H*} at which the derivative becomes zero. Hence a hegemon arises only when wealth is high enough and the marginal status-security benefit of hegemonic defense exceeds its marginal cost.

In conclusion, the model shows that trade policy and defense spending differ systematically between periods of hegemony and periods of balance of power. Under hegemony, the common defense good compresses relative-power differences, lowers private defense, and makes free trade more likely. Under balance of power, sovereignty and vulnerability become more salient, and protectionism becomes more attractive. In consequence, consumption is higher under hegemony than balance of power.

We now turn to a numerical illustration.

4.7. Numerical illustration

Consider a symmetric calibration with identical countries, so that $D_1 = D_2 = D$ in equilibrium. Let $y = 1000$, $W = 300$, $\kappa = 0.02$, $\theta = 0.01$, $\Omega = 50$, $\omega = 1$, $\mu(0) = 1$, $\mu(150) = 0.6$, $\alpha v = 150$, T_1 = 150. The hegemon chooses D^{H}optimally. Under this calibration, the hegemon's maximization problem yields $D^{H*} = 150$. Given this equilibrium umbrella, the Stage-3 first-order condition for the two symmetric countries yields $D^{H,F} \approx 11.45$. Under balance of power, $D^{H*} = 0$, and the same condition yields $D^{B,P} = 75$.

The results are:

Summary of results

Regime	D^H	D_i	c_i	S_i
Hegemony + free trade	150	11.45	1137.24	$\overline{S}$
Balance of power + protectionism	0	75	868.75	$\overline{S}$

Note: With symmetric countries, the relative sovereignty term is zero in equilibrium. A numerical difference in sovereignty across regimes requires either asymmetry or a regime-specific baseline sovereignty term.

Hence, under hegemony countries consume more, and free-ride on the hegemon's defense umbrella, whereas under balance of power they consume less and invest much more in their own defense.

4.8. Conclusion

This model has shown that the optimal trade regime depends on the structure of the international system. When a hegemon provides security and compresses relative-power differences, sovereignty becomes less salient and free trade is optimal. When no hegemon exists, relative power and national sovereignty become central, and protectionism may be preferred despite its economic cost. The model thus provides a unified explanation for the historical correlation between hegemonic periods and free trade, on the one hand, and balance-of-power periods and protectionism, on the other.

V. Conclusion

The history of nations and civilizations may perhaps be summarized in one sentence: "Empires long divided must unite; empires long united must divide." This is the opening line of the celebrated 14th-century Chinese novel *Romance of the Three Kingdoms*.[12] Already in the 14th century, Chinese historians understood that world history moves like a pendulum, alternating between periods in which one great power dominates and periods marked by conflict and competition among states, that is, periods of balance of power.

But the history of nations has also displayed cycles in trade policy, and these cycles are closely correlated with cycles in international leadership. Hegemony is associated

[12] It is a 14th-century historical novel attributed to Luo Guanzhong. It is set in the turbulent years towards the end of the Han dynasty and the Three Kingdoms period in Chinese history, starting in 169 AD and ending with the reunification of the land in 280 by Western Jin.

with periods of free trade, whereas protectionism tends to arise in periods of balance of power.

This paper has shown that, once relative power and national sovereignty are incorporated into the analysis alongside consumption, this correlation becomes clear. Factors such as war, power, and national sovereignty shape political and economic decision-making in fundamental ways. The paper shows that during hegemonic periods, free trade tends to be the optimal policy, whereas under balance of power, countries prepare for possible conflict and are more likely to choose protectionism.

Since the rise of the nation-state, the locus of hegemonic power has shifted periodically, from Spain to the Netherlands, then to England, and finally to the United States. In each instance, the attainment of hegemonic status has been associated with the adoption of a free-trade policy. By contrast, interregna, in which no single state exercised hegemony over the international system, were accompanied by periods in which nationalism and mercantilism became central determinants of political and economic behavior. Why does free trade seem to require unified leadership, whereas an anarchic international system gives rise to mercantilism and protectionism?

This paper has offered an answer to that question by clarifying the link between the international distribution of power and commercial policy. The model shows that when a hegemon provides a common defense good, relative-power differences are compressed, and the sovereignty component of utility becomes less important; in that setting, the gains from consumption dominate, and free trade becomes optimal.

By contrast, when no hegemon exists, national sovereignty and relative power become central, so that protectionism may be chosen even when it reduces aggregate consumption. In this sense, commercial policy cannot be understood independently of geopolitics. Finally, and at the risk of adding to the already long list of paraphrases of Clausewitz's celebrated dictum, we may conclude that "commercial policy is the pursuit of political aims by other means."

What, then, does this paper teach us? There are two main conclusions.

The first is that, if we ask why 1945 was so different from 1918, and why the post-1945 peace settlements differed so sharply from the 1919 Treaty of Versailles, the answer lies in the structure of the international system. In 1945, there was a hegemonic state, the United States, whereas in 1919 the world remained in a balance-of-power system. As this paper has shown, that difference fundamentally altered countries' attitudes toward free trade. Under a hegemonic order, there is no "Clemenceau" demanding harsh reparations in the name of national prestige and identity. Instead, there is room for coordination, for institutions such as the UN and the GATT, and for policies such as the

Marshall Plan. There is less room for tariffs, punitive reparations, and the hoarding of foreign reserves.

The second main conclusion is that the framework developed in this paper helps us understand the broader transformation of U.S. policy. This policy is the political expression of a world that has moved back toward balance of power, especially in the rivalry between the United States and China. The era of U.S. hegemony, which lasted roughly from 1945 to 2015, has ended. Since then, we have returned to a period of geopolitical rivalry involving the United States, China, and Russia. As a result, growing numbers of individuals across countries oppose free trade. We are witnessing the return of nationalism, populism, protectionism, and trade wars. And we may also be entering an era of real wars.

References

Akerlof, G.A and R. Kranton. 2010. *Identity Economics*. Princeton University Press

Akerlof, G.A and R. Kranton. 2000. "Economics and Identity". *The Quarterly Journal of Economics.* 115 (3):715-753.

Amir, R., J. Y. Jin, and M. Troege. 2022. "On the limits of free trade in a Cournot world: When are restrictions on trade beneficial?. *Canadian Journal of Economics* 55:4, 2036-2057

Adam A. Ambroziak. Review of the Literature on the Theory of Industrial Policy 3-38. No

Baldwin, RE. 1989. "The Political Economy of Trade Policy". *Journal of Economic Perspectives* 3:4, 119-135.

Bar-Siman Tov, Y. 2004. *From Conflict Resolution to Reconciliation* Oxford University Press.

Bhagwati, J. 1989. Is free trade passé after all?. *Review of World Economics* 125:1, 17-44.

Binkley, 1929. "Ten Years of Peace Conference History" *The Journal of Modern History* 4: 607-29.

Christopher J. Boudreaux. 2015. The Evolutionary Effects of Democracy: In the Long Run, We are All Trading?. The International Trade Journal 29:5, 376-396.

Brezis, E.S. 2023. "Why was Keynes opposed to reparations and Carthaginian peace?" in G. Corsetti (ed.) *Keynes and Economic Consequences of the Peace*. Cambridge University Press.

Brezis, E.S. 2003. "Mercantilism", *Oxford Encyclopedia of Economic History*.

Brezis, E.S. and P. Temin. 1999. *Elites, Minorities and Economic Growth*, eds., North Holland, Elsevier.

Brezis, E.S. 1995. "Foreign Capital Flows in the Century of Britain's Industrial Revolution: New Estimates, Controlled Conjectures", *Economic History Review*, February, pp.46-67.

Brezis, E. S., and Krugman, P. 1997. "Technology and the Life Cycle of Cities". *Journal of Economic Growth*, 2(4), 369–383.

Brezis, E. S., Krugman P. and D. Tsiddon. 1993. "Leapfrogging in International Competition", *American Economic Review*, December, pp.1211-19.

Brito, D., and Intriligator, M. 1976. "Strategic Weapons and the Allocation of International Rights," in John V. Gillespie and Dino A. Zinnes, *Mathematical Systems in International Relations Research* (New York: Praeger).

Calvo-Pardo, H. 2009. "Are the antiglobalists right? Gains-from-trade without a Walrasian auctioneer". *Economic Theory* 38:3, 561-592.

Calleo, D. P. and Rowland, B. M. 1973. *America and the World Political Economy* Bloomington: Indiana UP.

Ciampani, A. and R. Tolomeo. 2015. "The need for an interpretive framework in the history of European Élites." In *National Identities and Transnational European Élites*, edited by A. Ciampani and R.Tolomeo, 5-13. Soveria Mannelli: Rubbettino Editore.

Cline, R.S. 1980. *World Power Trends and US Foreign Policy for the 1980s*. Routledge.

Coleman, J. S. 1986. "Social Theory, Social Research and a Theory of Action," *American Journal of Sociology* 91(6), 1309-1335.

Cowling, K. and R. Sugden. 1998. "Strategic Trade Policy Reconsidered: National Rivalry vs Free Trade vs International Cooperation". *Kyklos* 51:3, 339-357.

Gallagher, J. and R. Robinson. 1953. "The Imperialism of Free Trade," *Economic History Review*, 6.

Gilpin, R. 1975. *US Power and the Multinational Corporation*.

Gilpin, R. 1977. "Economic Interdependence and National Security in Historical Perspective," in Klaus Knorr and Frank N. Trager, eds., *Economic Issues and National Security* (Lawrence: Regents Press of Kansas)

Gilpin, R. 1981. *War and Change in World Politics* (New York: CUP)

Gowa, Joanne (1989), "Bipolarity, Multipolarity, and Free Trade," *American Political Science Review*, 83(4), 1245–1256.

Gowa, Joanne, and Edward D. Mansfield (1993), "Power Politics and International Trade," *American Political Science Review*, 87(2), 408–420.

Heckscher, E. F. 1965. *Mercantilism* (2nd edition, ed. E. F. Söderlund).

Hirschman, A. 0. 1945. *National Power and the Structure of Foreign Trade* (Berkeley: Univesity of CP).

Ikenberry, G. John (2001), *After Victory: Institutions, Strategic Restraint, and the Rebuilding of Order After Major Wars*. Princeton: Princeton University Press.

Ikenberry, G. John, Michael Mastanduno, and William C. Wohlforth, eds. (2011), *International Relations Theory and the Consequences of Unipolarity*. Cambridge: Cambridge University Press.

Keohane, R. and Nye. 1977. *Power and Interdependence: World Politics in Transition* (Boston: Little Brown).

Keohane, R. 1980. "The Theory of Hegemonic Stability and Changes in International Economic Regimes, 1967-1977," in Ole R. Holsti, Randolph M. Siverson, and Alexander L. George, eds., *Change in the International System* (Boulder: Westview).

Keohane, Robert O. 1984. After Hegemony: Cooperation and Discord in the World Political Economy. Princeton, NJ: Princeton University Press.

Keynes, J. M. 1919. *The Economic Consequences of the Peace.*

Kinder, H., and Hilgemann, W. 1964. *The Anchor Atlas of World History* (New York: Anchor).

Kindleberger, C. 1973. *The World in Depression*, 1929-1939 (Berkeley: UCP).

Kindleberger, C. 1975. "The Rise of Free Trade in Western Europe, 1820-1875," *Journal of Economic History*, XXXV, 20-55.

Krauthammer, Charles (1990/1991), "The Unipolar Moment," *Foreign Affairs*, 70(1), 23–33

Krasner, Stephen D. 1976. "State Power and the Structure of International Trade," *World Politics* 28.

Krugman, P. 1987. Is free trade passe? *Journal of Economic Perspectives*, 1(2): 131-144.

Lake, D.(1983), "International Economic Structures and American Foreign Economic Policy, 1887-1934," *World Politic*s, 35: 517-543.

Lampton, D. 2006. "The Faces of Chinese Power" Foreign Affairs 86(1)

Lawson, Fred H. 1983, "Hegemony and the Structure of international trade reassessed: a view from Arabia," *International Organization*, vol. 37, p.317-339.

Layne, Christopher (1993), "The Unipolar Illusion: Why New Great Powers Will Rise," *International Security*, 17(4), 5–51.

Lemyre, L. and P. Smith. 1985. "Intergroup discrimination and self-esteem in the minimal group paradigm" *Journal of Personality and Social Psychology* 49: 660-670.

MacDonagh, Oliver 1962, "The Anti-Imperialism of Free Trade," *Economic History Review*, 2d series, 14.

MacEwan, A. 1978. "The Development of the Crisis in the World Economy," in B. Steinberg *et al*, eds., *US Capitalism in Crisis* New York: Union for Radical Political Economists.

Malmgren, H. 1970. "Coming Trade Wars? Neo-Mercantilism and Foreign Policy," *Foreign Policy*, 115-143.

Mantoux, E. 1946. *The Carthaginian Peace: or the Economic Consequences of Mr. Keynes*. New York: Oxford University Press

Mathew, W. M. 1968. "The Imperialism of Free Trade: Peru, 1820-70," *Economic History Review*, 2d series, 21.

Metzl J. 2019. *Dying of Whiteness: How the Politics of Racial Resentment is Killing America's Heartland*. Hachette

Mearsheimer, John J. (2001), *The Tragedy of Great Power Politics*. New York: Norton.

Moore, R. J. 1964-5. "Imperialism and 'Free Trade' Policy in India, 1853-4," *Economic History Review*, 2d series, 17.

Monteiro, Nuno P. (2014), *Theory of Unipolar Politics*. Cambridge: Cambridge University Press.

Hans Morgenthau. 1948. *Politics Among Nations*

Ngono Fouda. R. 2012. "Protectionism & Free Trade: A Country's Glory or Doom?. *International Journal of Trade, Economics and Finance* 351-355.

Oakes, PJ and JC Turner. 1989. "Social Categorisation and Intergroup behavior" *European Journal of Social Psychology* 10: 295-301.

Olson, Mancur and Richard Zeckhauser, 1966. An Economic Theory of Alliances. Santa Monica, CA: RAND Corporation,

Organsky, A. F. K., 1968. *World Politics* New York: Knopf.

Platt, D. C. M. 1968. "The Imperialism of Free Trade: Some Reservations," *Economic History Review*, 2d series, 21.

Platt, D. C. M. 1973. "Further Objections to an 'Imperialism of Free Trade,' 1830-1860," *Economic History Review*, 2d series, 26.

Pompeo Della Posta. 2021. "An analysis of the current backlash of economic globalization in a model with heterogeneous agents". *Metroeconomica* 72:1, 101-120.

Rowland, B.M. 1976. *Balance of Power or Hegemony: The Interwar Monetary System* - New York: New York University Press.

Sai-wing Ho. P. 2015. "External Economies in Trade and Development: Contrasting Arguments for Industry Promotion Under Alternative Analytical Frameworks". *OEconomia* :5-3, 363-396.

Salvatore, D. 1992. "Recent trends in U.S. protectionism". *Open Economies Review* 3:3, 307-321.

Skidelsky. 1976. "Balance of Power or Hegemony" in *The Interwar Monetary System* - edited by B. M. Rowland. New York: New York University Press.

Spencer, B.J. and James A. Brander. *Strategic Trade Policy* 13173-13180.

Smith, A. 1937. *The Wealth of Nations* (New York: Modern Press).

Snidal, Duncan (1985), "The Limits of Hegemonic Stability Theory," *International Organization*, 39(4), 579–614.

Stein, Arthur A. 1984. "The Hegemon's Dilemma: Great Britain, the United States, and the International Economic Order." *International Organization* 38, no. 2: 355–386.

Tajfel, H. and J. Turner. 1979. "An Integrative theory of inter-group conflict" in WG Austin and S Worchel (eds), *The Social Psychology of Intergroup Relations*. Monterey, Brooks.

Tajfel, H. (1974), "Social Identity and Intergroup Behaviour," *Social Science Information*, 13(2), 65–93.

Taussig, F. (1892). *The Tariff History of the United States* New York: Capricorn Books, 1964 edition.

Temin, P. 1997. "Is it Kosher to Talk about Culture?" *The Journal of Economic History*, 57 (2): 267-287

Veblen, T. 1899. *The Theory of the Leisure Class*. New York: Macmillan.

Viner, J. 1930. "English Theories of Foregn Trade Before Adam Smith," *Journal of Political Economy*, XXXVIII, 239- 310 and 404-57.

Viner, J. 1955. *Studies in the Theory of International Trade* (London: Allen and Unwin).

Viotti, P. and Kauppi. M. 1987. *International Relations Theory* (New York: MacMillan).

Wallerstein, I. 1980. *The Modern World System, vol.2: Mercantilism and the Consolidation of the European World Economy, 1670-1750* New York: Academic Press. Weede, E. 1994. "Constraints, states, and wars." *European Journal of Political Research* 26:2,171-192.

Waltz, Kenneth N. (1979), *Theory of International Politics*. Reading, MA: Addison-Wesley.

Whitman, M. v. N. 1975. "Leadership without Hegemony," *Foreign Policy*, no. 20.

Wilson, C. 1949. "Treasure and Trade Balances: the Mercantilist Problem" *Economic History Review*, 2.

Wohlforth, William C. (1999), "The Stability of a Unipolar World," *International Security*, 24(1), 5–41.

Zirgulis, A. and T. Hashimoto. 2023. "Trade Theory Vs Reality: The Banana Trade War". *The International Trade Journal* 37:5, 502-518.

Figure 1: Hegemony of Britain: 1850-1910

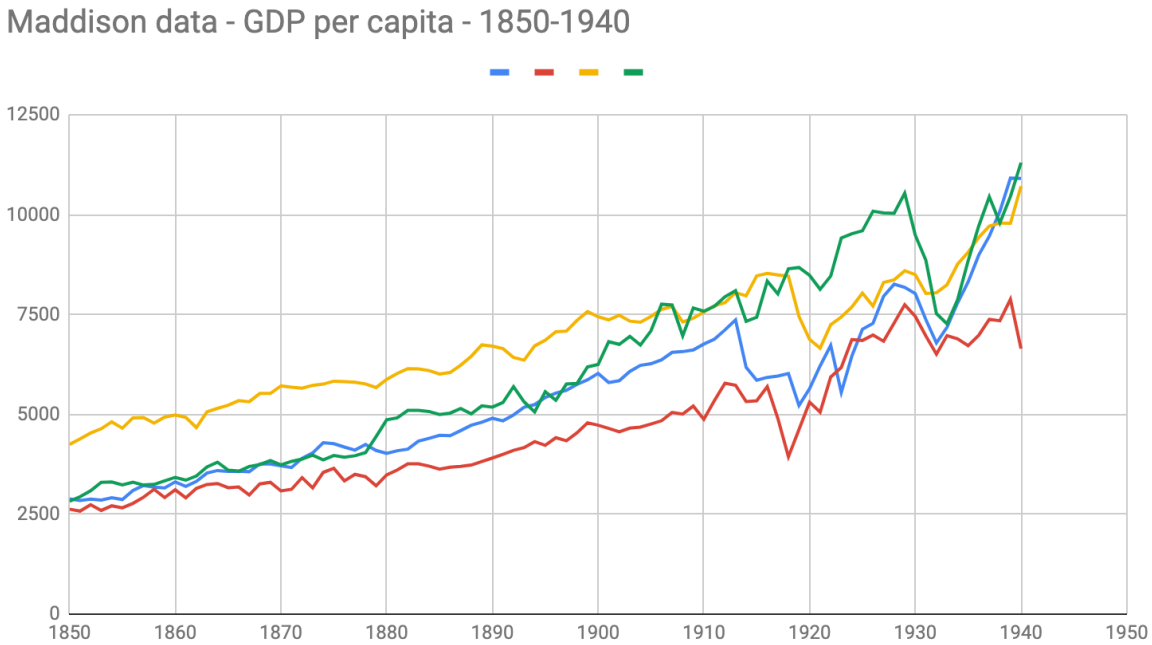


Figure 2: Hegemony of the US: 1945-2015.

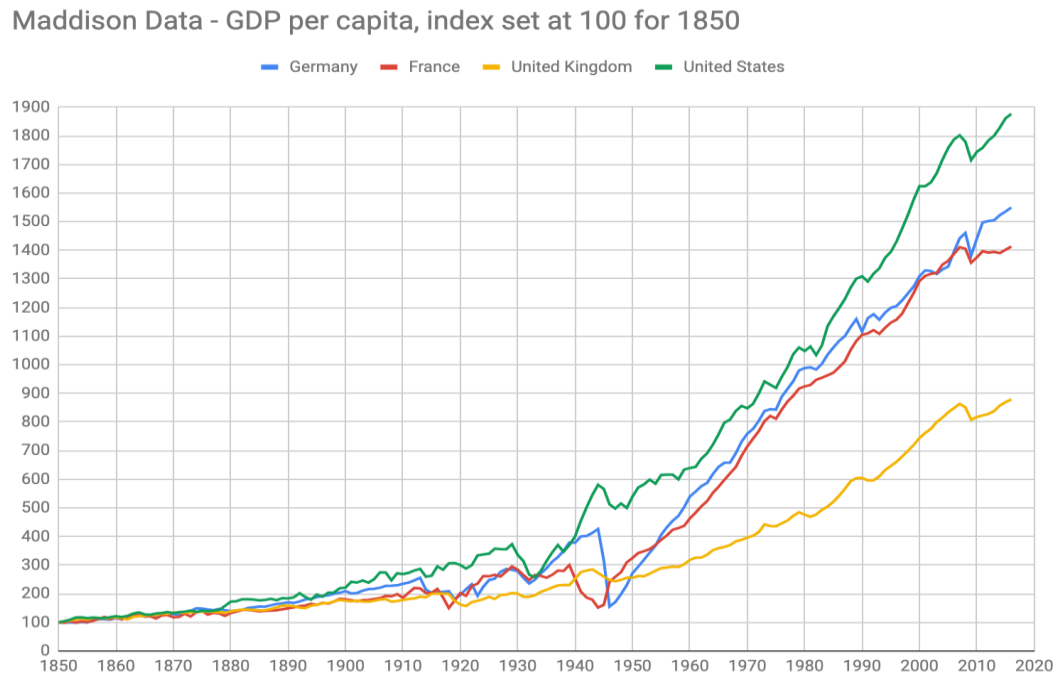


Figure 3. Balance of Power: 1910-1945.

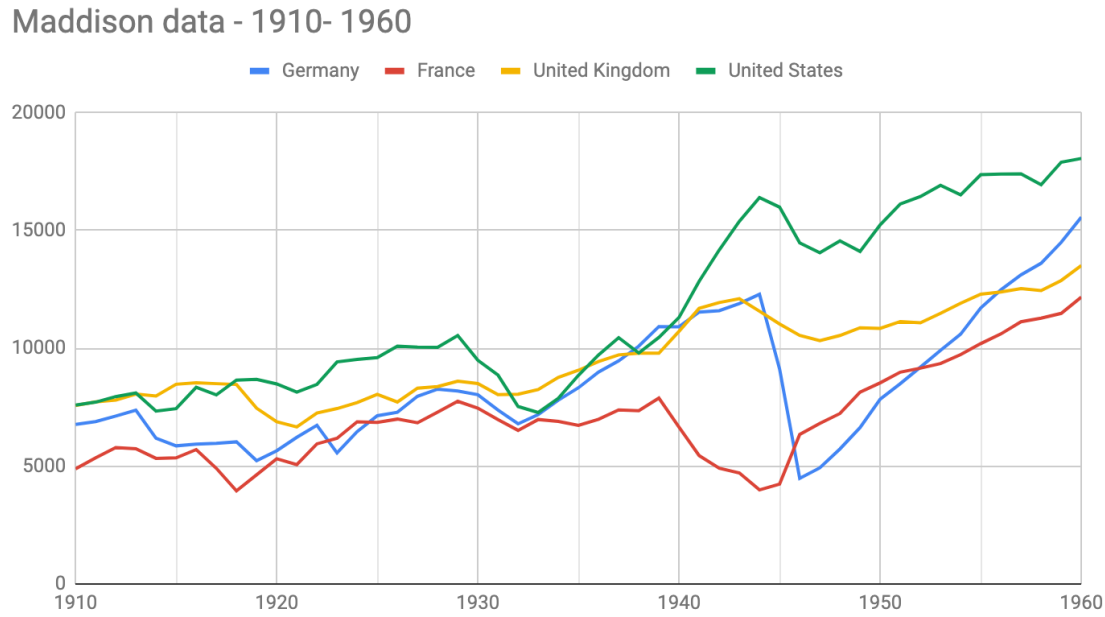
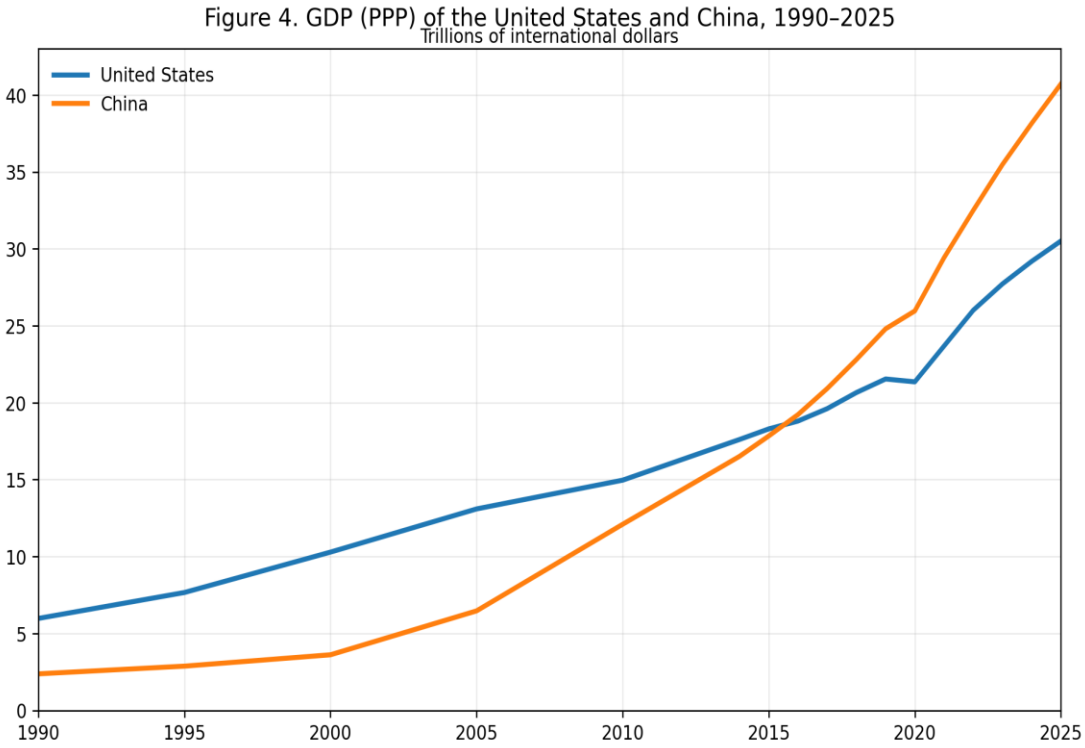


Figure 4 – The GDP of the US and China



Appendix 1.

Let us assume that both countries have the same size so that, $Y_1 = Y_2 = Y$. In conseqjence, $D_1 = D_2 = D$ and *NX-0*. Note that (17) characterizes the equilibrium:

$$1 + \kappa D_i = \alpha v/(D_i + \omega D^H) + \theta\mu(D^H)\Omega \qquad (17)$$

The left-hand side is strictly increasing in D, while the right-hand side is strictly decreasing in D.

Now compare the right-hand side under hegemony ($D^H > 0$) and under balance of power ($D^H = 0$).

We have that for any given D, the right-hand side of (17) is smaller under hegemony than under balance of power. [13] Since the left-hand side is increasing in D, the unique solution must satisfy $D^*_{i,h} < D^*_{i,b}$.

This is so, because (i) $\alpha v/(D + \omega D^H) < \alpha v/D$. And (ii) because $\mu'(D^H) < 0$, so that: $\mu(D^H) < \mu(0)$.

Appendix 2.

Let us start with the free trade case. In order to be a Nash equilibrium, a necessary condition is that when country j chooses F, country i does not wish to deviate from F to P. Thus, $U_i(F,F;H) \geq U_i(P,F;H)$

Indeed from equations (12), (7), (8) and part *b* of Proposition 1, we obtain that the difference between these utilities is T_1 .[14]

About the protectionism case, in order to be a Nash equilibrium, a necessary condition is that when country j chooses P, country i does not wish to deviate from P to F. Thus, $U_i(P,P;B) \geq U_i(F,P;B)$.

Indeed from equations (12), (6), (8) and part b of propostition 1, we obtain that the difference between these utilities is $\chi p(.) - T_0$.[15]

---

[13] This is so, because (i) $\alpha v/(D + \omega D^H) < \alpha v/D$. And (ii) because $\mu'(D^H) < 0$, so that: $\mu(D^{*H}) < \mu(0)$.

[14] $[y_i + T_1 - K(D_i(H)) + \alpha S_i(H) - p(H)\Omega] - [y_i - K(D_i(H)) + \alpha S_i(H) - p(H)\Omega] = T_1$.

[15] $[y_i - K(D_i(B)) + \alpha S_i(B) - p(B)\Omega] - [y_i + T_0 - \chi p(B) - K(D_i(B)) + \alpha S_i(B) - p(B)\Omega] = \chi p(B) - T_0$

.