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Cross-Ownership, Loan Commitment, Managerial Delegation and Conflicting Interests

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Abstract: We investigate the intertwined of cross-ownership and managerial delegation on corporate profits, consumer surplus and social welfare under product market imperfections and loan commitments. We find that in the absence of managerial delegation, cross-ownership has a negative effect on interest loan rate, firm output, consumer surplus and social welfare, while it has a positive effect on corporate profit; in the presence of managerial delegation, the effect of cross-ownership is non-monotone dependent on the degree of cross-ownership. Furthermore, cross-ownership may strengthen conflicting interests between the consumers and owners, while managerial delegation can mitigate it when the degree of cross-ownership is low.

***Keywords*:** Cross-ownership; Sales delegation; Loan commitment; Conflicting interests

***JEL Classifications*:** G32, G34, J53, L21

1. Introduction

Loan commitment is a contract whereby commercial banks or other financial institutions lend a certain amount of funds to firms. Among the linkages between financial markets and product markets, loan commitment served as an important financial instrument supporting firm investment and market competition (Brander and Lewis, 1986; Maksimovic, 1990), ensuring the borrower access to credit via signing loan commitment contract at a relatively low-interest rate in the future. Existing literature discussed the issue of bank loan commitment from the perspective of strategic choice (Berkovitch and Greenbaum, 1991; Coscollá and Granero, 2003) and influencing factors (Graham et al., 2008; Mitani, 2014; Hollander and Verriest, 2016; Qian et al. 2021; Zhang et al. 2024; Wu et al. 2024).

In contemporary economy, businesses operation are observed under symmetric or asymmetric cross-ownership structure among enterprises and alternative managerial delegation contracts. These aspects have gained significant attention from scholars and the government. Cross-ownership refers to the situation where a company holds a minority stake in its rivals without controlling them. This reduces the firms' motivation to compete aggressively, which in turn leads to higher product prices and lower industry output (Reynolds and Snapp, 1986). However, it can also lead to improved social welfare, despite hindering market competition (Farrell and Shapiro, 1990). Note that some studies under cross-ownership focus on the emergence of implicit collusion between managerial firms and examine the possibility of cross-ownership resulting in collusion (Spagnolo, 2005; Gilo et al., 2006), and others investigate the welfare effect of cross-ownership under different ownership structure (Chen et al., 2024; Hu et al., 2021; Lestage, 2021; Li and Shuai, 2022; Alipranti et al., 2022; Sun and Wang, 2023). Buccella et al., (2025), in particular, showed that in a network Cournot duopoly with passive unilateral cross-ownership, either the industry profitability may not be increased or social welfare will be increased, depending on the degree of product compatibility.[1]

[1] In the absence of cross-ownership, Toshimitsu (2016) considered the efficiency of price and quantity

Another stream of literature on strategic managerial delegation has grown significantly since the seminal contributions of Vickers (1985), Fershtman (1985), Fershtman and Judd (1987), and Sklives (1987) (VFJS), the outcome is that the sale delegation in Cournot market will lead to the "Prisoner's Dilemma" situation. There is still a lack of theoretical and empirical studies exploring the intertwined of cross-ownership and managerial delegation on corporate profits, consumer surplus and social welfare under product market imperfections and bank loan commitment. Wang et al. (2024) constructed a vertical product differentiation duopoly model incorporating cross-ownership and managerial delegation, showing that there exists a U-shaped relationship between cross-ownership and endogenous managerial delegation coefficients in the absence of loan commitment. That is, cross-owned firms producing different quality products couple with hiring own manager may push up the product price and raise firms' profits, while reducing consumer surplus and social welfare.

In real economy since firms are running back up by a set of stakeholder's contracts, the conflicts of interest among stakeholders are hardly to avoid. Mitigating stakeholder conflicts turns out to be a key concern of stakeholders-oriented corporate governance. Related studies scrutinized the role of labor unions in the market (Le Pape and Wang, 2018; 2020; Lopez and Naylor, 2004). In an extended model of Lopez and Naylor (2004), Wu et al. (2024) examined when the owners bargain with the union and delegate production to managers, showing that an increasing degree of cross-ownership exacerbates the conflicting interests between the owners and consumers. There are scant theoretical and empirical studies paying attention to intertwined environment of cross-ownership, managerial delegation and loan commitment, it motivates us to investigate the effect of cross-ownership with loan commitment and managerial delegation on corporate profit and consumer welfare, and delineate the conflicting interest between stakeholders.

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competition in a network products market paring with network externalities and product compatibility (network compatibility effects). He showed that if network compatibility effects between firms are sufficiently asymmetric, the Cournot equilibrium is more efficient than the Bertrand equilibrium in terms of larger consumer, producer and total surpluses.

In this paper, we take cross-ownership and managerial delegation into account on loan commitment decisions in Cournot duopoly model.[2] Our findings reveal that when cross-ownership exists among firms, an increasing degree of cross-ownership leads to a decrease in the loan interest rate, firm output, consumer surplus, and social welfare. This result aligns with the traditional wisdom, which refers to the welfare-reduction effect of cross-ownership. This effect primarily arises from the fact that cross-ownership strengthens collusion between firms. However. the concurrent impact of cross-ownership and managerial delegation on equilibrium outcome depends on the degree of cross-ownership. At relatively low degrees of cross-ownership, the traditional wisdom remains hold. While when the degree of cross-ownership is relatively high, our results indicate that a rise of cross-ownership may have a welfare-improving effect. Hence, cross-ownership is not always harmful for the society's interests because the market interest rate will mitigate the harmful effect of cross-ownership.

The rest of this paper is organized as follows. Section 2 provides basic model setup. Section 3 explores the interwind of cross-ownership, sales delegation and loan commitment in Cournot product market. Section 4 compares the subgame perfect Nash equilibrium (SPNE) of two scenarios (cross-ownership vs. cross-ownership with managerial delegation) and shows the conflicting interests between the owners and consumers. Section 5 concludes the paper.

2. Basic Model

We consider Cournot competition between two firms producing a homogeneous product with quantity q_i ($i=1,2$). The inverse demand function is given by

[2] See Lian et al. (2023) showed that, compared with no network externalities, firms always obtain higher profits and social welfare in the presence of positive network externalities, irrespective of the managerial delegation contracts. Furthermore, they showed that whether the owner chooses market share delegation or sales delegation contracts relies on the type and strength of network externalities. Wu et al. (2024) integrated the product market, financial market, and fiscal policy addressing the important issues of cost pass-through and tax incidence. They pointed out that the linkage with the financial market plays an important role analyzing tax pass-through and tax incidence, and the result amplifies the relative difference of pass-through and tax incidences between ad valorem tax and the specific tax.

$p = a - Q$, where p is market price and $Q = q_1 + q_2$ is the total output. For simplicity, we assume that the marginal production costs of both firms are constant and equal to 1.

Production is financed by loans from the competitive banking sector. We assume the market interest rate is r . If firm i borrows at the interest rate of r , the profits of firms are as follows:

$$\pi_1 = (a - 1 - q_1 - q_2 - r)q_1 \tag{1a}$$

$$\pi_2 = (a - 1 - q_1 - q_2 - r)q_2 \tag{1b}$$

Consisting with Qian et al. (2021) and Lian et al. (2023), we assume that firm can obtain capital financing at interest rate r_i through loan commitment. Assuming $r_i \in [0, r]$ and $r < a - 1 < 7r/3$ to ensure a positive output and interest rate level. Upfront fee $F_i > 0$ is determined by the break-even constraint of representative competing banks, which means the price of loan commitments is reasonable. Thus, the profit of firm i under the loan commitment r_i is

$$\pi_1 = (a - 1 - q_1 - q_2 - r_1)q_1 - F_1 \tag{2a}$$

$$\pi_2 = (a - 1 - q_1 - q_2 - r_2)q_2 - F_2 \tag{2b}$$

Consumer surplus and social welfare are respectively:

$$CS = \frac{(q_1 + q_2)^2}{2} \tag{3}$$

$$SW = \sum_{i=1}^{2} \pi_i + \sum_{i=1}^{2} \left(F_i + r_i q_i\right) + CS \tag{4}$$

Based on a Cournot duopoly model incorporating loan commitment, we study the following two cases. We first consider the case of cross-ownership and then consider the cross-ownership with sales delegation contract. In the presence of cross-ownership among firms, we examine how it affects the bank loan interest rate, firm output and profits, consumer surplus and social welfare. For simplicity, we assume that the two

firms hold the same proportion of each other which is a and $a < \frac{1}{2}$. Thus, either firm can share in its rival's profits but not involving their production decisions. In this case, the profit of firm i is

$$\Pi_1^{CO} = (1-a)\pi_1 + a\pi_2 \tag{5a}$$

$$\Pi_2^{CO} = (1-a)\pi_2 + a\pi_1 \tag{5b}$$

where the superscript "*CO*" denotes the case of cross-ownership.

The timing order of this game is as follows. In the first stage, firm owners arrange financing simultaneously by acquiring loan commitments from competitive banks. In the second stage, the firms engage in quantities competition and choose output simultaneously. The SPNE is solved by backward induction.

In second stage, the firms choose output q_i to maximize profit Π_i^{CO}. From the first-order condition, we have

$$\frac{\partial \Pi_1^{CO}}{\partial q_1} = a - q_2 + 2q_1(a-1) + (1+r_1)(a-1) - aa = 0 \tag{6a}$$

$$\frac{\partial \Pi_2^{CO}}{\partial q_2} = a - q_1 + 2q_2(a-1) + (1+r_2)(a-1) - aa = 0 \tag{6b}$$

From (6a) and (6b), we obtain the following equilibrium output level

$$q_1^{CO} = \frac{(1-a)(a-1+r_2+2r_1(a-1)+2a(1-a))}{3+4a(a-2)} \tag{7a}$$

$$q_2^{CO} = \frac{(1-a)(a-1+r_1+2r_2(a-1)+2a(1-a))}{3+4a(a-2)} \tag{7b}$$

It indicates that the loan interest rate determined by loan bank will harm the initiative of producing more ($\partial q_i^{CO} / \partial r_i < 0, i = 1,2$); however, a rise in interest rate of one firm will incentivize the other's output ($\partial q_i^{CO} / \partial r_j > 0, i \neq j, i, j = 1,2$). This is because an increase in the loan interest rate for one firm can reduce the motivation to increase output by capturing more of the firm's profits. It will leave more market share available, encouraging the other firm to produce more under strategic substitution.

Furthermore, we find that a higher degree of cross-ownership will strengthen the effect of loan interest rate on firm's output. Specifically, a higher degree of cross-ownership can enable the owner to obtain more benefits from the other firm and recoup the losses due to a rise in loan interest rate, further decreasing firm's output ($\partial^2 q_i^{CO} / \partial r_i \partial a < 0, i = 1,2$). Cross-ownership via a positive impact on the effect of loan interest rate affecting the other firm's output ($\partial^2 q_i^{CO} / \partial r_j \partial a > 0, i \neq j, i, j = 1,2$). A rise in loan interest rate, for one firm, loses some market share. The other firm will produce more and fills the market gap. In addition, since a higher degree of cross-ownership results in a division of firm's profit, the other firm is forced to produce more.

In first stage, firm owners choose r_i to maximize Π_i^{CO}. Substituting $F_i = (r - r_i)q_i$ into Π_i^{CO}, we have

$$\frac{\partial \Pi_1^{CO}}{\partial r_1} = \frac{(a-1)(a-1-6r+4r_1+r_2-r_2a+r(7-2a)a+2r_1a(a-3))}{(3-2a)^2} = 0 \qquad (8a)$$

$$\frac{\partial \Pi_2^{CO}}{\partial r_2} = \frac{(a-1)(a-1-6r+r_1+4r_2-r_1a+r(7-2a)a+2r_2a(a-3))}{(3-2a)^2} = 0 \qquad (8b)$$

From (8a) and (8b), we obtain the optimal rate of loan commitment

$$r^{CO} = r_1^{CO} = r_2^{CO} = \frac{\left(2a^2-7a+6\right)r-a+1}{(2a-5)(a-1)} \qquad (9)$$

In (9), two firms will choose the same interest rate. Cross-ownership has a negative effect on the equilibrium interest rate ($\partial r^{CO} / \partial a < 0$), while a higher market interest rate will weaken this negative effect ($\partial^2 r^{CO} / \partial a \partial r > 0$). The above results is attributed to a higher degree of cross-ownership, which strengthens the monopolistic power of these two firms. It leads to a stronger bargaining power on interest rates, resulting in lower interest rates for the two firms. In addition, a higher market interest rate will also incentivize loan bank to raise interest rate to increase bank's profit.

Substituting (9) into (7a) and (7b), we obtain the SPNE outcomes. The output,

profit, consumer surplus and social welfare are as follows

$$q^{CO} = q_1^{CO} = q_2^{CO} = \frac{(a-2)(a-r-1)}{2a-5}$$

$$\Pi^{CO} = \Pi_1^{CO} = \Pi_2^{CO} = \frac{(1-a+r)^2(2-a)}{(5-2a)^2}$$

$$CS^{CO} = \frac{2(1-a+r)^2(a-2)^2}{(5-2a)^2} \tag{10}$$

$$SW^{CO} = \frac{2(a-1-r)(2-a)(3a-3+2r-(a-1+r)a)}{(5-2a)^2}$$

From (10), we have Proposition 1.

Proposition 1: *A higher degree of cross-ownership will reduce the firm's output, consumer surplus and social welfare, while it is beneficial to the firm's profit. Market interest rate alleviates the effect of cross-ownership, and the impact of cross-ownership on social welfare is dependent on the market interest rate.*

Proof: See in Appendix.

From Proposition 1, we can easily understand that when there is a higher degree of cross-ownership (like collusion), it strengthens the monopolistic power. As a result, various negative consequences arise, such as a decrease in the firm's output, an increase in market price, a decrease in consumer surplus, and an increase in the firm's profit. Ultimately, it leads to a decline on social welfare. These findings align with the widely accepted belief that when there is passive cross-ownership without managerial delegation, firms become less motivated to compete aggressively. Consequently, it leads to a higher product prices, lower industry output, and a negative impact on social welfare (See, Reynolds and Snapp, 1986).

Furthermore, an increase in market interest rate will squeeze firm's profit and incentivize them to produce more. At the meanwhile, under a given market price, more firm's output will decrease firm's profit and improve consumer surplus. Hence, market interest rate has a positive impact on firm's output and consumer surplus, while it has a negative impact on firm's profit. For the whole society, the impact of market interest rate is ambiguous. In particular, if market interest rate is relatively higher, a rise in market interest rate will cause an enhancement effect on consumers

and a squeezing effect on firm's profit, which can weaken the negative effect of cross-ownership. The losses on firm's profit and consumer surplus cannot be offset by the benefits obtained by the bank, resulting in a lower social welfare. However, if market interest rate is lower relatively, a higher level of market interest rate also has the enhancement effect and squeezing effect. But, these effects are limited and weaker than the positive effect on the bank, improving social welfare.

3. Cross-Ownership, Managerial Delegation and Loan Commitment

In this section, we further examine whether the equilibrium outcomes hold when the managerial delegation is introduced. In particular, we assume that firm i (i=1,2) hires a manager with a sales delegation contract which the weight λ_i is non-negative.

The manager is motivated to choose q_i to maximize

$$M_1 = \pi_1 + \lambda_1 q_1 \tag{11a}$$

$$M_2 = \pi_2 + \lambda_2 q_2 \tag{11b}$$

The timing order of this game is as follows. In first stage, firm owners arrange financing simultaneously by acquiring loan commitments from competitive banks. In second stage, each firm's owner adopts a sales delegation contract to compensate the manager. In third stage, the firms choose output simultaneously. The SPNE is solved by backward induction.

In third stage, the manager i (i=1,2) choose output q_i to maximize his payoff function M_i^{CS}. From the first-order conditions, we have

$$\frac{\partial M_1^{CS}}{\partial q_1} = a - 1 - 2q_1 - q_2 - r_1 + \lambda_1 = 0 \tag{12a}$$

$$\frac{\partial M_2^{CS}}{\partial q_2} = a - 1 - q_1 - 2q_2 - r_2 + \lambda_2 = 0 \tag{12b}$$

where the superscript "*CS*" denotes the case of cross-ownership with managerial

delegation.

From (12a) and (12b), we obtain the following equilibrium output level of each firm

$$q_1^{CS} = \frac{a-1-2r_1+r_2+2\lambda_1-\lambda_2}{3} \tag{13a}$$

$$q_2^{CS} = \frac{a-1+r_1-2r_2-\lambda_1+2\lambda_2}{3} \tag{13b}$$

We find that the direct effect of loan interest rate on own firm's output is negative , $\partial q_i^{CS} / \partial r_i < 0, i = 1,2$ and on the competitor's output is positive $\partial q_i^{CS} / \partial r_j > 0, i \neq j, i, j = 1,2$, which are the same with the above case. However, in the presence of managerial delegation, we find that the degree of managerial delegation has a positive association with firm's output, $\partial q_i^{CS} / \partial \lambda_i > 0, i = 1,2$, while it has negative impact on the competitor's output, $\partial q_i^{CS} / \partial \lambda_j < 0, i \neq j, i = 1,2$. It is because a higher degree of managerial delegation chosen by the owner will incentivize the manager to produce more, which will seize more market share from the competitor and reduce the competitor's output. Notably, the effect of managerial delegation is influenced via the loan interest rate, which is the other impact channel and the indirect effect of loan interest rate.

In second stage, the owner i maximize Π_i^{CS} by choosing λ_i

$$\frac{\partial \Pi_1^{CS}}{\partial \lambda_1} = \frac{a-1-2r_1+r_2+3a-3aa+3r_2a-4\lambda_1+6a\lambda_1-\lambda_2}{9} = 0 \tag{14a}$$

$$\frac{\partial \Pi_2^{CS}}{\partial \lambda_2} = \frac{a-1+r_1-2r_2+3a-3aa+3r_1a-4\lambda_2+6a\lambda_2-\lambda_1}{9} = 0 \tag{14b}$$

From (14a) and (14b), we have

$$\lambda_1^{CS} = \frac{1}{4}(2(a-1-r_2)+\frac{3(2-2a+r_1+r_2)}{5-6a}+\frac{3(r_1-r_2)}{2a-1}) \tag{15a}$$

$$\lambda_2^{CS} = \frac{1}{4}(2(a-1-r_1)+\frac{3(2-2a+r_1+r_2)}{5-6a}+\frac{3(r_2-r_1)}{2a-1}) \tag{15b}$$

According to (15a) and (15b), we find how loan interest rate affects the degree of managerial delegation. Specifically, a higher loan interest rate will make the bank obtain more profit from the firm, and the owner can only decrease the degree of managerial delegation for loss-reducing ($\partial\lambda_i^{CS}/\partial r_i < 0, i = 1,2$). The gap in market share will be left to the competitor under strategic substitution, and the competitor will raise the managerial delegation coefficient to improve firm's profit ($\partial\lambda_i^{CS}/\partial r_j > 0, i \neq j, i, j = 1,2$).

Furthermore, a higher degree of cross-ownership will strengthen the alliance between the two owners. The firms will improve firm's profit by choosing a lower output level, and further decrease the degree of managerial delegation ($\partial^2\lambda_i^{CS}/\partial r_i \partial a < 0, i = 1,2$). More interestingly, since the alliance between them is unstable and each firm has a motivation to increase output in order to obtain more benefits; a higher degree of cross-ownership will incentivize the competitor to raise the degree of managerial delegation ($\partial^2\lambda_i^{CS}/\partial r_j \partial a > 0, i \neq j, i, j = 1,2$). By summarizing the above statements, we find that cross-ownership can strengthen the effect of loan interest rate on managerial delegation.

In first stage, firm owners choose r_i to maximize Π_i^{CS}

$$\frac{\partial \Pi_1^{CS}}{\partial r_1} = \frac{1}{(5-6a)^2}(18(r_1 - r)a^3 + 3(2a - 2 + 25r - 24r_1 - 3r_2)a^2 - 2(a - 1 + 43r - 38r_1 - 6r_2)a - 2(a - 1 - 15r + 12r_1 + 2r_2)) = 0 \quad (16a)$$

$$\frac{\partial \Pi_2^{CS}}{\partial r_2} = \frac{1}{(5-6a)^2}(18(r_2 - r)a^3 + 3(2a - 2 + 25r - 3r_1 - 24r_2)a^2 - 2(a - 1 + 43r - 6r_1 - 38r_2)a - 2(a - 1 - 15r + 2r_1 + 12r_2)) = 0 \quad (16b)$$

Solving (16a) and (16b), we obtain the bank's loan interest rate:

$$r^{CS} = r_1^{CS} = r_2^{CS} = r + \frac{1 + r - a}{4 - 6a} - \frac{(a - 1 - r)(6a - 5)}{28 + 2a(6a - 23)} \quad (17)$$

From (17), we obtain the same qualitative results as mentioned in the absence of managerial delegation. That is, cross-ownership is negative related to the equilibrium

interest rate ($\partial r^{CS}/\partial a < 0$), and a higher market interest rate will weaken this negative effect ($\partial^2 r^{CS}/\partial a \partial r > 0$).

Substituting (17) into (13) and (15), we have the SPNE outcomes. The degree of managerial delegation, output, profit, consumer surplus and social welfare are obtained as follows:

$$\lambda^{CS} = \lambda_1^{CS} = \lambda_2^{CS} = \frac{(a-1-r)(3a-1)(6+a(3a-10))}{(3a-2)(14+a(6a-23))}$$

$$q^{CS} = q_1^{CS} = q_2^{CS} = \frac{(a-1-r)(6+a(3a-10))}{14+a(6a-23)}$$

$$\prod^{CS} = \prod_1^{CS} = \prod_2^{CS} = \frac{(1+r-a)^2(2-3a)(6+a(3a-10))}{(14+a(6a-23))^2} \tag{18}$$

$$CS^{CS} = \frac{2(1+r-a)^2(6+a(3a-10))^2}{(14+a(6a-23))^2}$$

$$SW^{CS} = \frac{2(a-1-r)\begin{pmatrix}8a-8+6r+(13-13a-10r)a\\+3(a-1+r)a^2\end{pmatrix}(6-a(10-3a))}{(14+a(6a-23))^2}$$

From (18), we have Proposition 2.

Proposition 2: *A higher degree of cross-ownership coupling sales delegation will decrease the degree of managerial delegation, while the effect of cross-ownership on output, profit, consumer surplus and social welfare are non-monotone, dependent on the degree of cross-ownership. Interest loan rate weaken the negative effect of cross-ownership.*

Proof: See in Appendix.

From Proposition 2, we find that when the degree of cross-ownership is low relatively, a higher degree of cross-ownership will intensify the collusion between two firms and decrease the coefficient of managerial delegation to increase market concentration. The firms will reduce the output and improve firm's profit. Furthermore, consumer surplus and social welfare will be decreased. These results are consistent with the traditional wisdom obtained in existing literature. However, when

the degree of cross-ownership is high relatively, the opposite results will be obtained. Specifically, a higher degree of cross-ownership will decrease the coefficient of managerial delegation. Moreover, since the manager's payoff function only includes firm itself profit and so, exacerbating a separation of management and ownership in a firm, especially under a higher degree of cross-ownership. It leads a rise in manager's benefit at the expense of the owner's loss. The manager increases the output will intensify market competition, thereby decreasing market price. On the bright side, a rise in total output will improve consumer surplus, and this positive effect on consumer surplus exceeds the owners' loss, improving social welfare.

In addition, we further take interest loan rate into consideration and find more interesting results: interest loan rate will weaken the effect of cross-ownership regardless of the degree of cross-ownership, which is similar with the case without managerial delegation.

In order to illustrate the above results, we use **Figure 1** and **2** to depict the effect of cross-ownership on equilibrium output, profit, consumer surplus and social welfare.

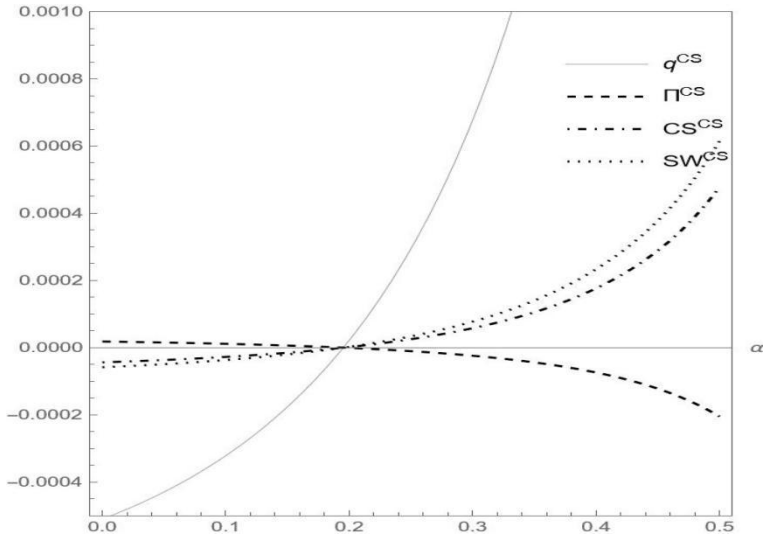

Figure 1. The effect of cross-ownership on equilibrium output, profit, consumer surplus and social welfare when $a-1=2r, r=0.05$.

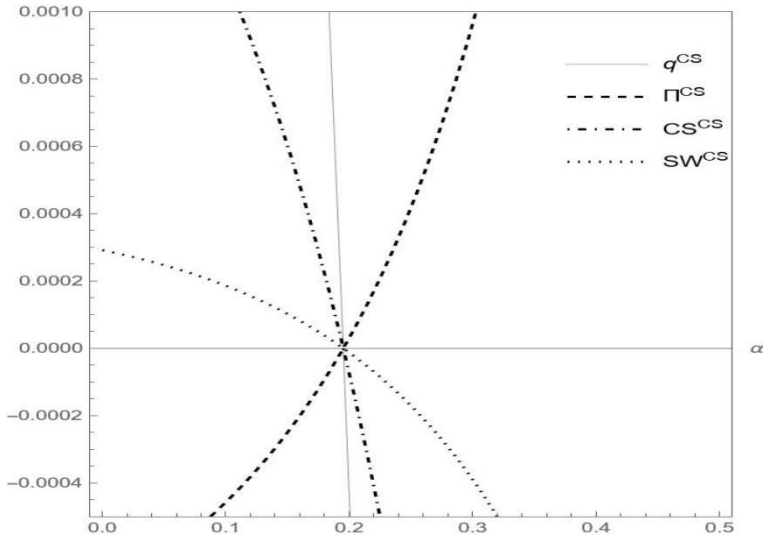

Figure 2. The impact of interest loan rate on the effect of cross-ownership when $a-1=2r, r=0.05$.

Summarizing Proposition 1 and 2 suggests that cross-ownership generally leads to a reduction in social welfare, as indicated in existing literature. However, when managerial delegation and the degree of cross-ownership is relatively high, cross-ownership may create the welfare-enhancement effect. The results occur due to the intensification of separation of management and ownership. Regardless of the welfare-reduction or welfare-enhancement effects, a change in market interest rate may weaken the negative effects of cross-ownership. Hence, cross-ownership is not always harmful. Under a different scenario, the coordination of market-regulatory and financial authority may need to adjust market interest rate to balance the effect of cross-ownership under product market imperfection.

4. Equilibrium Outcomes and Conflicting Interests

4.1 Comparison of Equilibrium Outcomes

We now compare the equilibrium outcomes of above two scenarios, and have the following results:

$$\begin{gathered} r^{CS} > r^{CO}, \text{ if } 0 < a < a^{*};\ r^{CS} < r^{CO}, \text{ if } a^{*} < a < \frac{1}{2} \\ q^{CS} > q^{CO} \\ \Pi^{CS} < \Pi^{CO} \end{gathered} \tag{19}$$

$$CS^{CS} > CS^{CO}$$

$$SW^{CS} > SW^{CO}$$

Summarizing the above results, we have Proposition 3.

Proposition 3: *In Cournot duopoly with sales delegation and loan commitment contract, the firm's output, consumer surplus and social welfare are greater under managerial delegation, while the equilibrium profit is lower by contrary. Moreover, the relative magnitude of the bank loan interest rate between two cases is dependent on the degree of cross-ownership.*

Of note, consume surplus and social welfare under managerial delegation is greater, because the puzzling "Prisoner's Dilemma" remains. The owner in delegated firms wants to decrease the coefficient of managerial delegation in the presence of cross-ownership, while the manager has an incentive to produce aggressively. Hence, firm's output, consumer surplus and social welfare are greater in the presence of managerial delegation; however, firm's profit is lower. When the degree of cross-ownership is low relatively, interest loan rate with managerial delegation is higher. Because the separation of management and ownership is not serous, the manager will produce more to increase firm's profit and obtain more benefit, resulting in the bank charging a higher interest loan rate. On the contrary, with a relatively higher degree of cross-ownership, the separation of management and ownership does not affect the manager's choice of producing aggressively. Hence, non-delegated firms will obtain more benefit relative to delegated firms, which results in a higher interest loan rate in the absence of managerial delegation.

4.2 Conflicting Interests

Through the above comparative static analysis, we have discovered that there exists conflicting interests between firms and consumers. Hence, we aim to delve deeper into this issue by examining the conflicting interests between consumers and firms from a corporate governance perspective. Our main focus is to understand how cross-ownership impacts the conflicting interests between the consumers and firms.

As defined by Le Pape and Wang (2018, 2020), the relationship between

stakeholder 1 and stakeholder 2 is in conflict caused by the variation of the degree of cross-ownership, α, if an increase in α favors S1 (S2), but at the same time damages S2 (S1). Specifically, when there is a conflicting interest between stakeholders, $\eta_{(S1/S2),\alpha} = \frac{\partial(S1/S2)}{\partial\alpha}\frac{\alpha}{(S1/S2)} \neq 0$, measures the degree of the conflicting interest between stakeholder 1 and 2.

As shown in Proposition 1 and 2, we find that the effects of cross-ownership on consumer surplus and firm's profit are different, which means that there exists a conflicting interest between the consumers and firm.

We have the following Proposition 4.

Proposition 4: *The conflicting interest between the consumers and firm is as follows:*

i) Under the case without managerial delegation, $\eta_{\left(CS^{CO}/\Pi^{CO}\right),a} = -\frac{a}{2-a} < 0$;

ii) Under the case with managerial delegation, $\eta_{\left(CS^{CS}/\Pi^{CS}\right),a} = \frac{-a\left(2-3a\left(4-3a\right)\right)}{\left(2-3a\right)\left(6-a\left(10-3a\right)\right)} < (>) 0$ *when* $a < (>) \frac{1}{3}\left(2-\sqrt{2}\right)$.

It implies that an increase in the degree of cross-ownership will first decrease the ratio CS/Π , when the degree of cross-ownership is low. But when the degree of cross-ownership is getting higher, an increase in the degree of cross-ownership will increase the ratio CS/Π . The conflicting interest is non-monotone as shown in **Figure 3**. Moreover, in the absence of managerial delegation, the conflicting interest between the consumers and firm is increasing in the degree of cross-ownership as depicted in **Figure 3**. In the case without managerial delegation, an increase in the degree of cross-ownership will further increase firm's profit and then decrease consumer surplus, aggravating conflicting interest between the consumers and firm. The results imply that managerial delegation may not eliminate the conflicting interest between the consumers and firm, but can mitigate it when the degree of cross-ownership is low. More importantly, when the degree of managerial delegation and cross-ownership are getting higher, an increase the degree of cross-ownership will further magnify the conflicting interests relative to the case without managerial

delegation.

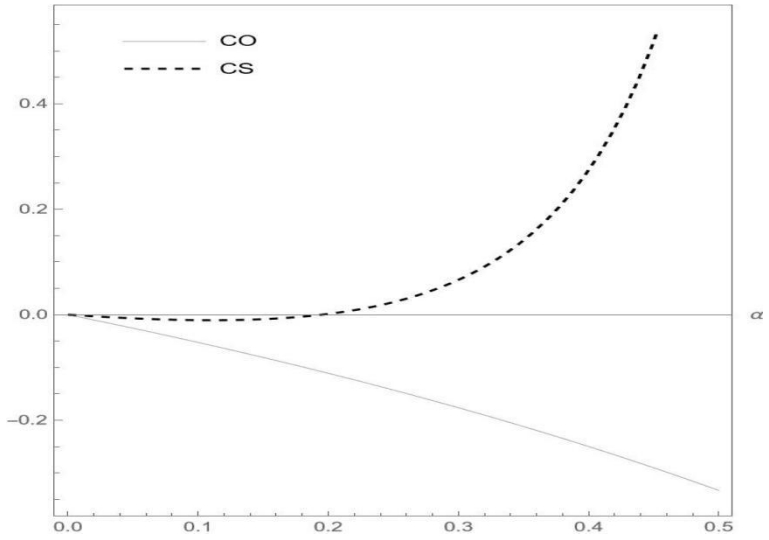

Figure 3. The conflicting interest between consumer and firm under two cases.

5. Conclusions

In this paper, we develop a Cournot duopoly setting to examine the impact of cross-ownership with managerial delegation on the loan interest rate, firm output and profits, consumer surplus and social welfare. We firstly find that in the absence of managerial delegation, cross-ownership has a negative effect on interest loan rate, firm output, consumer surplus and social welfare, while a positive effect on firm's profit. Secondly, in the presence of managerial delegation, the effect of cross-ownership is non-monotone (positive or negative), dependent on the degree of cross-ownership. Under both scenarios, market interest rate alleviate the negative effect of cross-ownership.

As commonly known, managerial delegation can increase firm's output, consumer surplus and social welfare, while reducing firm's profit due to the separation of management and ownership. Considering that cross-ownership between two firms can be regarded as a collusion, a change in the degree of cross-ownership will affect firm's output and profit, causing a conflicting interest between the consumers and firm. We find that there exists a conflicting interest between the consumers and firm.

In the absence of managerial delegation, cross-ownership will strengthen the conflicting interest. However, in the presence of managerial delegation, managerial delegation can mitigate the conflicting interest even though not eliminating it, when the degree of cross-ownership is low. The conflicting interest will be further magnified with the degree of cross-ownership. The results of this study have important implications for antitrust and financial policies. The antitrust authority should constantly monitor the variation of cross-ownership among firms and adopt active policies to mitigate the negative effect of cross-ownership.

Appendix

Proof of Proposition 1.

From (10), taking the derivative with respect to a and r yield:

$$\frac{\partial q^{CO}}{\partial a}=\frac{1+r-a}{(5-2a)^2}<0,\frac{\partial^2 q^{CO}}{\partial a\partial r}=\frac{1}{(5-2a)^2}>0$$

$$\frac{\partial \Pi^{CO}}{\partial a}=\frac{(1+r-a)^2(2a-3)}{(2a-5)^3}>0,\frac{\partial^2 \Pi^{CO}}{\partial a\partial r}=\frac{2(1-a+r)(3-2a)}{(5-2a)^3}<0$$

$$\frac{\partial CS^{CO}}{\partial a}=\frac{4(1+r-a)^2(2-a)}{(2a-5)^3}<0,\frac{\partial^2 CS^{CO}}{\partial a\partial r}=\frac{8(a-1-r)(2-a)}{(5-2a)^3}>0$$

$$\frac{\partial SW^{CO}}{\partial a}=\frac{2(a-1-r)(a-1+2r(2-a))}{(2a-5)^3}<0,$$

$$\frac{\partial^2 SW^{CO}}{\partial a\partial r}=\frac{8r(2-a)-(a-1)(6-4a)}{(5-2a)^3}\begin{cases}<0, & r>r^*\\ >0, & 0<r<r^*\end{cases}$$

where $r^*=\frac{(a-1)(3-2a)}{4(2-a)}$.

Proof of Proposition 2.

Taking the derivative with respect to a yields

$$\frac{\partial \lambda^{CS}}{\partial a}=\frac{(1+r-a)(256+3a(a(3a(15a-64)+356)-292))}{(2-3a)^2(14+a(6a-23))^2}<0$$

$$\frac{\partial^2 \lambda^{CS}}{\partial a\partial r}=\frac{256-3a\left(292-a\left(356-3a\left(64-15a\right)\right)\right)}{\left(2-3a\right)^2\left(14+a\left(6a-23\right)\right)^2}>0$$

$$\frac{\partial q^{CS}}{\partial a}=\frac{(1+r-a)(3a(3a-4)+2)}{(14+a(6a-23))^2}\begin{cases}>0, & \left(2-\sqrt{2}\right)/3<a<1/2\\ <0, & 0<a<\left(2-\sqrt{2}\right)/3\end{cases}$$

$$\frac{\partial^2 q^{CS}}{\partial a\partial r}=\frac{2-3a\left(4-3a\right)}{\left(14+a\left(6a-23\right)\right)^2}\begin{cases}>0, & 0<a<\left(2-\sqrt{2}\right)/3\\ <0, & \left(2-\sqrt{2}\right)/3<a<1/2\end{cases}$$

$$\frac{\partial \Pi^{CS}}{\partial a}=\frac{(1+r-a)^2(a-2)(6a-5)(3a(3a-4)+2)}{(14+a(6a-23))^3}\begin{cases}>0, & 0<a<\left(2-\sqrt{2}\right)/3\\ <0, & \left(2-\sqrt{2}\right)/3<a<1/2\end{cases}$$

$$\frac{\partial^2 \Pi^{CS}}{\partial a\partial r}=\frac{2(1-a+r)(2-a)(5-6a)(2+3a(3a-4))}{\left(14+a\left(6a-23\right)\right)^3}\begin{cases}>0, & \left(2-\sqrt{2}\right)/3<a<1/2\\ <0, & 0<a<\left(2-\sqrt{2}\right)/3\end{cases}$$

$$\frac{\partial CS^{CS}}{\partial a}=\frac{4(1+r-a)^2\left(a(10-3a)-6\right)\left(3a(3a-4)+2\right)}{\left(14+a(6a-23)\right)^3}\begin{cases}>0, & \left(2-\sqrt{2}\right)/3<a<1/2\\ <0, & 0<a<\left(2-\sqrt{2}\right)/3\end{cases}$$

$$\frac{\partial^2 CS^{CS}}{\partial a\partial r}=\frac{8(1-a+r)\left(a(10-3a)-6\right)\left(2+3a(3a-4)\right)}{\left(14+a(6a-23)\right)^3}\begin{cases}>0, & 0<a<\left(2-\sqrt{2}\right)/3\\ <0, & \left(2-\sqrt{2}\right)/3<a<1/2\end{cases}$$

$$\frac{\partial SW^{CS}}{\partial a}=\frac{2(a-1-r)\left(2-3a(4-3a)\right)\left(2-3a-a(2-3a)-2r\left(6-a(10-3a)\right)\right)}{\left(14+a(6a-23)\right)^3}\begin{cases}>0, & \left(2-\sqrt{2}\right)/3<a<1/2\\ <0, & 0<a<\left(2-\sqrt{2}\right)/3\end{cases}$$

$$\frac{\partial^2 SW^{CS}}{\partial a\partial r}=-\frac{2\left(2+3a(3a-4)\right)\left(10a-10-24r+(17-17a+40r)a+(6a-6-12r)a^2\right)}{\left(14+a(6a-23)\right)^3}\begin{cases}>0, & 0<a<\left(2-\sqrt{2}\right)/3\\ <0, & \left(2-\sqrt{2}\right)/3<a<1/2\end{cases}$$

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