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Law of Motion for Economic Inequality: An Analogy to Newtonian Mechanics

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Abstract

This paper demonstrates that the dynamics of economic inequality follow a law of motion closely analogous to Newtonian mechanics. Within this framework, specific types of inherent household heterogeneity generate continuous "forces" that persistently exacerbate inequality, thereby necessitating government intervention as an essential counterforce. This study formally models how the force-based aggravation of economic inequality aligns with classical Newtonian laws—specifically the principles of inertia, acceleration, and action and reaction. Furthermore, the underlying real interest rate is identified as the primary point of application for these forces, where the interest-bearing period determines the transient deformation, or softness, of the economic system. Finally, the force driving inequality is conceptualized as a dynamic interaction among heterogeneous households. These findings provide a novel mathematical foundation for understanding and controlling the persistent widening of economic inequality.

JEL Classification: D63, H55, I30

Keywords: Economic inequality; Household heterogeneity; Law of motion; Newtonian mechanics; Sustainable heterogeneity

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1 INTRODUCTION

Economic inequality, as measured by income or wealth disparity, has long been a central focus of economic inquiry. It has recently attracted renewed attention following empirical evidence of rising inequality across developed countries since the 1980s (Piketty 2003, 2013; Piketty and Saez 2003; Atkinson et al. 2011; Parker 2014; Saez and Zucman 2016). Although diverse theories explain the origins of inequality (e.g., Kuznets 1955; Boix 2010; Piketty 2013; Milanovic 2016) and the mechanisms driving its recent surge (Katz and Murphy 1992; Autor et al. 1998, 2003; Card and DiNardo 2002; Leamer 1998; Goldberg and Pavcnik 2007; Helpman 2016; Piketty 2013), a definitive consensus remains elusive.

Furthermore, establishing universally accepted criteria for socially optimal allocation is complicated by the interpersonal incomparability of utility and Arrow's impossibility theorem (Arrow 1951). While Bergson (1938) and Samuelson (1947) attempted to resolve the incomparability problem, Arrow (1951) provided a fundamental critique of their frameworks. This critique can be circumvented by relaxing Arrow's original assumptions—for instance, by introducing single-peaked preferences (Black 1958)—thereby justifying the use of social welfare functions in economic analysis. Nonetheless, a consensus regarding the precise functional design of these social welfare functions is still lacking.

As an alternative approach to this design dilemma, Harashima (2010[1], 2012[2], 2014) introduced the concept of "sustainable heterogeneity (SH)." In this framework, heterogeneity is defined as sustainable if all optimality conditions for all heterogeneous households are satisfied indefinitely. In an economy characterized by household heterogeneity in the rate of time preference (RTP), the degree of risk aversion (DRA), persistent rent incomes (Harashima, 2020a[3], 2020b[4], 2023b), and investment success rates (Harashima, 2021a), SH can be attained through appropriate government intervention. Crucially, this state is achievable even under unilateral household behavior; conversely, absent proper intervention, economic inequality inevitably worsens. Although Harashima (2018a[5], 2021b, 2021c, 2022) demonstrated that achieving "pure" SH poses a significant challenge for governments, an "approximate" SH—wherein economic inequality does not

[1] Harashima (2010) is also available in Japanese as Harashima (2017a).

[2] Harashima (2012) is also available in Japanese as Harashima (2020c).

[3] Harashima (2020a) is also available in Japanese as Harashima (2021d).

[4] Harashima (2020b) is also available in Japanese as Harashima (2023a).

[5] Harashima (2018a) is also available in Japanese as Harashima (2019).

aggravate—can be realized with relative ease.

It is generally accepted that households behave by forming rational expectations to reach a steady state. However, Harashima (2018a) demonstrated an alternative mechanism for reaching a steady state, wherein households maintain their capital-wage ratio at a "maximum degree of comfortability" (MDC). Crucially, behavior under this MDC-based framework is equivalent to that under rational expectations—that is, it aligns with behavior guided by the rate of time preference (RTP) (Harashima 2018a, 2021b, 2021c, 2022). Because the MDC-based approach is far more accessible for households than the RTP-based alternative yet yields the identical steady state, Harashima (2022, 2023c, 2023d, 2023e, 2024, 2025a, 2025b) conducted a series of simulations to examine the steady states reached under various scenarios and conditions. While the law of motion for economic inequality can be derived equally from both the MDC- and RTP-based models, this paper derives it exclusively from the RTP-based model for analytical convenience, as the RTP-based framework is mathematically more tractable and intuitive.

In this paper, I demonstrate that the law of motion for economic inequality can be derived from the SH-based model by analogy to Newtonian mechanics. The SH framework illustrates that even if the RTP, DRA, persistent rent incomes, or investment success rates are heterogeneous across households, the government can achieve SH by transferring economic resources according to rules tailored to each specific heterogeneity. These redistribution rules can be interpreted as counteracting the "forces" that fundamentally drive economic inequality.

Given that this law of motion is force-based, it can be interpreted within the framework of Newtonian mechanics. Specifically, the dynamics of economic inequality can be understood through the concepts of velocity and acceleration; furthermore, the laws of inertia, acceleration, and action-reaction are shown to align closely with Newtonian principles. Harashima (2026) demonstrated that, similar to economic inequality, the law of motion for inflation can also be described by analogy with Newtonian mechanics.

While households exhibit many forms of heterogeneity, the majority of these factors are unlikely to be directly relevant to economic inequality. Only a few factors—specifically, heterogeneities in the RTP, DRA, persistent rent incomes, and investment success rates—persistently drive economic inequality. Despite their limited number, however, the consequences of these factors—namely, the structural and persistent rise in economic inequality—are of considerable significance. Therefore, government intervention to mitigate their effects remains indispensable.

2 SUSTAINABLE HETEROGENEITY

This section briefly outlines the conditions for SH, drawing on the framework developed by Harashima (2010, 2012, 2014). While the following analysis focuses exclusively on a two-economy setup, Harashima (2010, 2012, 2014) demonstrated that this model can be straightforwardly extended to a multi-economy setting (i.e., more than two economies), yielding results that are essentially analogous to those presented here.

2.1 Heterogeneity in RTP

2.1.1 Model

For simplicity, suppose a country consists of two economies—Economy 1 and Economy 2—which are identical except for their rates of time preference (RTP). Each economy is populated by identical households. Let θ_1 and θ_2 denote the RTP of households in Economies 1 and 2, respectively. The population growth rate is assumed to be zero in both economies. The two economies are perfectly integrated; goods, services, and capital move freely between them, whereas labor is immobile across borders.

The production function of Economy $i\ (=1,2)$ is given by

$$y_{i,t} = A_t^{\alpha} k_{i,t}^{1-\alpha}$$

where $y_{i,t}$ and $k_{i,t}$ represent per capita output and capital in Economy i in period t, respectively; A_t denotes the technology level in period t; and α ($0 < \alpha < 1$) is a constant. The current account balance in period t is τ_t for Economy 1 and $-\tau_t$ for Economy 2. The accumulated current account balance,

$$\int_0^t \tau_s ds$$

reflects capital flows between the two economies, wherein the economy running a current account surplus invests its excess funds in the other. Since the marginal products of capital, $\frac{\partial y_{1,t}}{\partial k_{1,t}}\ \left(=\frac{\partial y_{2,t}}{\partial k_{2,t}}\right)$, represent the real interest rate in period t (r_t)),

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds \ \text{ and } \ \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

represent the income receipts or payments on cross-border assets. Consequently, the balance on goods and services is given by

$$\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

for Economy 1, and

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds - \tau_t$$

for Economy 2. Because the current account balance mirrors capital flows, it can be expressed as a function of capital in both economies:

$$\tau_t = \lambda(k_{1,t}, k_{2,t}).$$

The government can intervene in economic activities by transferring financial resources between the two economies. The transfer from Economy 1 to Economy 2 in period *t* is denoted by g_t, which is assumed to depend on capital such that

$$g_t = \bar{g}_t k_{1,t}.$$

Here, $\bar{g}_t$ is exogenous to households and firms and is continuously adjusted by the government in each period to achieve SH. Under the condition that $k_{1,t} = k_{2,t}$ and $\dot{k}_{1,t} = \dot{k}_{2,t}$,

$$g_t = \bar{g}_t k_{1,t} = \bar{g}_t k_{2,t}.$$

Each household in Economy 1 maximizes its expected utility:

$$E \int_0^\infty u_1(c_{1,t}) \exp(-\theta_1 t) dt$$

subject to the budget constraint:

$$\frac{dk_{1,t}}{dt} = A_t^\alpha k_{1,t}^{1-\alpha} - c_{1,t} + (1-\alpha) A_t^\alpha k_{1,t}^{-\alpha} \left(\int_0^t \tau_s \, ds + z_0 \right) - \tau_t - \bar{g}_t k_{1,t},$$

while each household in Economy 2 maximizes:

$$E\int_0^{\infty} u_2(c_{2,t})\exp(-\theta_2 t)dt$$

subject to

$$\frac{dk_{2,t}}{dt} = A_t^{\alpha}k_{2,t}^{1-\alpha} - c_{2,t} - (1-\alpha)A_t^{\alpha}k_{2,t}^{-\alpha}\left(\int_0^t \tau_s\, ds + z_0\right) + \tau_t + \bar{g}_t k_{2,t},$$

where $c_{i,t}$ is the per capita consumption of Economy *i* in period *t*, u_i is the utility function, and *E* is the expectation operator.

2.1.2 SH

Harashima (2010, 2012, 2014) showed that SH is achieved if, and only if, the consumption growth rates in both economies converge to an identical constant asymptotically: $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant. If Economy 1 acts unilaterally—in the sense that it optimizes without regard to the optimality conditions of Economy 2—SH cannot be attained unless the government appropriately intervenes. When SH is successfully achieved, the long-run consumption growth rates in both economies are equal to:

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \varepsilon^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \frac{\theta_1+\theta_2}{2}\right] = \varepsilon^{-1}\left(r - \frac{\theta_1+\theta_2}{2}\right), \quad (1)$$

where *m*, *v*, and ϖ are positive constants, r denotes the real interest rate on the balanced growth path, and $\varepsilon = -\frac{c_{1,t}u_1''}{u_1'} = -\frac{c_{2,t}u_2''}{u_2'}$ represents the constant DRA.

2.1.3 Government intervention

Harashima (2012) demonstrated that if the government intervenes such that

$$\lim_{t\to\infty}\bar{g}_t = \frac{\theta_2-\theta_1}{2}, \quad (2)$$

then SH is achieved even under unilateral behavior by Economy 1, thereby satisfying Equation (1).

2.2 Heterogeneity in DRA

Economies 1 and 2 are assumed to be identical except for their DRAs. Let ε_1 and ε_2 denote the DRAs of households in Economies 1 and 2, respectively. Harashima (2010, 2012) showed that, to achieve SH, the government must intervene such that:

$$\lim_{t\to\infty} \bar{g}_t = \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2}(r - \theta), \tag{3}$$

where θ represents the common RTP of households in both economies.

2.3 Heterogeneity in persistent rent incomes

Harashima (2016a[6]) introduced a novel type of economic rent derived from ranking preferences, wherein monopoly profits (rents) enable certain athletes or artists to become superstars in the sports, art, and music industries (Harashima, 2016a, 2018c[7]) and allow top corporate executives to earn exceptionally high compensation (Harashima 2017b, 2018d). Furthermore, Harashima (2020b, 2023b) demonstrated that transactional errors in business dealings also generate persistent economic rents.

Suppose a household in Economy 1 obtains a persistent rent income of $z_t(>0)$ in period *t*, while the income of a household in Economy 2 is reduced by the same amount. Instead of consuming this rent, the household in Economy 1 lends the financial equivalent of z_t to a household in Economy 2. This rent income is assumed to be proportional to the capital stock such that:

$$z_t = \bar{z}k_{1,t},$$

where $\bar{z}\,(>0)$ is a constant. Harashima (2020a) demonstrated that, to attain SH, the government should intervene according to the following rule:

$$\lim_{t\to\infty} \bar{g}_t = \bar{z}. \tag{4}$$

2.4 Heterogeneity in the success rates of investment

Economies 1 and 2 are identical except for their capital investment success rates, defined as the ratio of successful capital investments to total capital investments. Let ρ_1 and ρ_2 denote the success rates of investment for households in Economies 1 and 2, respectively. Harashima (2021a) demonstrated that, to achieve SH, the government must intervene

[6] Harashima (2016) is also available in Japanese as Harashima (2018b).

[7] Harashima (2018c) is also available in Japanese as Harashima (2021e).

such that:

$$\lim_{t\to\infty}\bar{g}_t = \left(\frac{\rho_1-\rho_2}{\rho_1+\rho_2}\right)r. \tag{5}$$

2.5 Multiple heterogeneities

If Economies 1 and 2 are simultaneously heterogeneous in the RTP, DRA, persistent rent incomes, and investment success rates, while remaining identical in all other respects, the consumption growth rate in Economy 1 is given by:

$$\frac{\dot{c}_{1,t}}{c_{1,t}} = \varepsilon_1^{-1}\left\{\rho_1\left[r+\bar{z}+r\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}}-\frac{\partial\tau_t}{\partial k_{1,t}}-\bar{g}_t\right]-\theta_1\right\},$$

and that in Economy 2 is given by:

$$\frac{\dot{c}_{2,t}}{c_{2,t}} = \varepsilon_2^{-1}\left\{\rho_2\left[r-\bar{z}-r\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2,t}}+\frac{\partial\tau_t}{\partial k_{2,t}}+\bar{g}_t\right]-\theta_2\right\}.$$

Therefore, to achieve SH—that is, to ensure

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$$

under the condition $\int_0^t \tau_s ds = \tau_t = 0$—the government transfer must be implemented according to the following rule:

$$\lim_{t\to\infty}\bar{g}_t = \left(\frac{\varepsilon_2\rho_1-\varepsilon_1\rho_2}{\varepsilon_2\rho_1+\varepsilon_1\rho_2}\right)r+\left(\frac{\varepsilon_2\rho_1+\varepsilon_1\rho_2}{\varepsilon_2\rho_1+\varepsilon_1\rho_2}\right)\bar{z}+\frac{\varepsilon_1\theta_2-\varepsilon_2\theta_1}{\varepsilon_2\rho_1+\varepsilon_1\rho_2}. \tag{6}$$

For analytical convenience, let this target transfer rate be denoted by Λ:

$$\Lambda = \left(\frac{\varepsilon_2\rho_1-\varepsilon_1\rho_2}{\varepsilon_2\rho_1+\varepsilon_1\rho_2}\right)r+\left(\frac{\varepsilon_2\rho_1+\varepsilon_1\rho_2}{\varepsilon_2\rho_1+\varepsilon_1\rho_2}\right)\bar{z}+\frac{\varepsilon_1\theta_2-\varepsilon_2\theta_1}{\varepsilon_2\rho_1+\varepsilon_1\rho_2}. \tag{7}$$

2.6 Heterogeneity in productivities

Harashima (2009) argued that the innovative activities of average workers constitute an

essential element of productivity, leading to heterogeneous productivities across workers, firms, and economies. A production function incorporating these average workers' innovations has been shown to take a Cobb-Douglas form (Harashima 2009):

$$y_t = \bar{\sigma}\omega_A\omega_L A_t^{\alpha} k_t^{1-\alpha}.$$

where ω_A and ω_L are positive constant parameters capturing average workers' creative activities, and $\bar{\sigma}$ is a parameter representing a worker's locational accessibility limit to capital. Crucially, the parameters ω_A and ω_L are independent of the aggregate technology level A_t but depend on the creative efforts of average workers; thus, unlike A_t, they can be heterogeneous across workers, firms, and economies.

Following this specification, let $y_{1,t} = \bar{\sigma}\omega_1^{\alpha} A_t^{\alpha} k_t^{1-\alpha}$ be the production function of Economy 1, and $y_{2,t} = \bar{\sigma}\omega_2^{\alpha} A_t^{\alpha} k_t^{1-\alpha}$ be that of Economy 2, where ω_1 $(0 < \omega_1 \leq 1)$ and ω_2 $(0 < \omega_2 \leq 1)$ signify the respective productivities of Economies 1 and 2. Harashima (2010, 2012, 2014) demonstrated that under such heterogeneous productivities, government intervention is not required to achieve SH. In other words, SH is attained naturally, implying that:

$$\bar{g}_t = 0.$$

3 ANALOGY TO NEWTONIAN MECHANICS

3.1 Body (object), force, and economic inequality

3.1.1 Capital holdings

In the model outlined in Section 2, $k_{1,t}$ and $k_{2,t}$ do not represent household capital holdings; instead, they represent the capital utilized for production in Economies 1 and 2, respectively. Therefore, $k_{1,t} = k_{2,t}$ always holds in the model except in the case of heterogeneous productivities, where $k_{1,t} = k_{2,t}\frac{\omega_1}{\omega_2}$. Household capital holdings are distinct from $k_{1,t}$ and $k_{2,t}$ due to lending and borrowing activities. The capital holdings of Economies 1 and 2 (denoted as $\tilde{k}_{1,t}$ and $\tilde{k}_{2,t}$, respectively) are given by:

$$\tilde{k}_{1,t} = k_{1,t} + \int_0^t \tau_s ds$$

and

$$\tilde{k}_{2,t} = k_{2,t} - \int_0^t \tau_s ds.$$

It follows naturally that $\tilde{k}_{1,t} + \tilde{k}_{2,t} = k_{1,t} + k_{2,t}$.

3.1.2 Body (object)

Let κ_t be a variable indicating the position of the body (object) relative to economic inequality. In period *t*, this variable satisfies the following condition:

$$\frac{d\kappa_t}{dt} = k^*_{1,t} - k^*_{2,t}, \tag{8}$$

where

$$k^*_{1,t} = \frac{\tilde{k}_{1,t}}{\frac{\tilde{k}_{1,t} + \tilde{k}_{2,t}}{2}}$$

and

$$k^*_{2,t} = \frac{\tilde{k}_{2,t}}{\frac{\tilde{k}_{1,t} + \tilde{k}_{2,t}}{2}}.$$

3.1.3 Acceleration and force

Differentiating Equation (8) with respect to time yields:

$$\frac{d^2\kappa_t}{dt^2} = \frac{d\left(k^*_{1,t} - k^*_{2,t}\right)}{dt}. \tag{9}$$

By analogy with Newtonian mechanics, Equation (9) can be interpreted as the "acceleration" of κ_t. Furthermore, the force (F) exerted on the body (object) at position κ_t can be defined based on this acceleration as follows:

$$F = 2m_e \frac{d^2\kappa_t}{dt^2} = 2m_e \frac{d\left(k^*_{1,t} - k^*_{2,t}\right)}{dt}, \tag{10}$$

where m_e denotes the mass (scale) of each economy, both of which are identical as previously stated. Additionally, from Equation (8), the evolution of κ_t over time is described by:

$$\kappa_t = \kappa_0 + \int_0^t \left(k^*_{1,s} - k^*_{2,s}\right) ds. \tag{11}$$

3.1.4 Economic inequality

Evidently, the term $k^*_{1,t} - k^*_{2,t}$ in Equation (8) signifies the current level of economic inequality between the two economies. In contrast, κ_t itself does not serve as a standard measure of contemporaneous inequality. As demonstrated in Equation (11), κ_t represents the cumulative economic inequality inherited from previous periods rather than its present level. Consequently, as shown in Equation (9), an acceleration of κ_t is equivalent to a change in the level of economic inequality represented by $k^*_{1,t} - k^*_{2,t}$.

3.2 Force that increases inequality due to heterogeneities

3.2.1 Force due to heterogeneity in RTP

3.2.1.1 Two-economy case

Economies 1 and 2 are identical in all respects except for their RTP. Achieving SH requires the government to transfer g_t from Economy 1 to Economy 2 in period t according to Equation (2). Conversely, Equation (2) demonstrates that, in the absence of government transfers, $k^*_{1,t}$ increases by $\frac{\theta_2-\theta_1}{2}$, and $k^*_{2,t}$ decreases by $\frac{\theta_2-\theta_1}{2}$, in each period. Therefore, the level of economic inequality between the two economies, represented by $k^*_{1,t} - k^*_{2,t}$, increases according to the following equation:

$$\frac{d\left(k^*_{1,t} - k^*_{2,t}\right)}{dt} = 2\left(\frac{\theta_2 - \theta_1}{2}\right) = \theta_2 - \theta_1. \tag{12}$$

According to Equations (9) and (12), the acceleration of κ_t due to the heterogeneity in RTP is determined by:

$$\frac{d^2\kappa_t}{dt^2} = \theta_2 - \theta_1. \tag{13}$$

Based on Equations (10) and (13), the force originating from the heterogeneity in RTP (F_θ) is defined as:

$$F_\theta = 2m_e(\theta_2 - \theta_1).$$

Integrating Equation (13) with respect to t yields the velocity of κ_t driven by the heterogeneity in RTP:

$$\frac{d\kappa_t}{dt} = (\theta_2 - \theta_1)t, \tag{14}$$

under the initial condition $\left.\frac{d\kappa_t}{dt}\right|_{t=0} = 0$. Furthermore, integrating Equation (14) with respect to t, the trajectory of κ_t due to the heterogeneity in RTP is given by:

$$\kappa_t = \kappa_0 + \frac{1}{2}(\theta_2 - \theta_1)\, t^2$$

As indicated by Equations (8) and (14), we have:

$$k^*_{1,t} - k^*_{2,t} = (\theta_2 - \theta_1)t.$$

Differentiating this with respect to time yields:

$$\frac{dk^*_{1,t}}{dt} - \frac{dk^*_{2,t}}{dt} = \theta_2 - \theta_1. \tag{15}$$

Equation (15) can be rewritten as:

$$\frac{dk^*_{1,t}}{dt} - \frac{\theta_2 - \theta_1}{2} = \frac{dk^*_{2,t}}{dt} + \frac{\theta_2 - \theta_1}{2}. \tag{16}$$

Equation (16) illustrates that, as implied by Equation (2), if the government transfers

$$\left(\frac{\theta_2 - \theta_1}{2}\right)\left(\frac{\tilde{k}_{1,t} + \tilde{k}_{2,t}}{2}\right)$$

from Economy 1 to Economy 2, the level of inequality $k^*_{1,t} - k^*_{2,t}$ remains constant.

3.2.1.2 Multi-economy case

Suppose that a country consists of H economies (Economy 1, 2,…, H) that are identical except for their RTP, where θ_i denotes the RTP of Economy i. Suppose also that Economies 1, 2,…, $H-1$ achieve SH through government intervention, resulting in economic equality among them. In this sense, they can be treated as a combined economy with a total mass of $(H-1)\,m_e$ and an aggregate RTP given by:

$$\frac{1}{H-1}\sum_{q=1}^{H-1}\theta_q\,.$$

Let this combined entity be denoted as Economy $(1+2+\ldots+H-1)$, and its RTP be denoted as $\theta_{(1+\ldots+H-1)}$.

According to Equation (13), the following acceleration and its corresponding force (F_θ) exist between a $\frac{1}{H-1}$ fraction of Economy $(1+2+\ldots+H-1)$ and Economy H:

$$\frac{d^2\kappa_t}{dt^2}=\frac{1}{H-1}\sum_{q=1}^{H-1}\frac{dk^*_{q,t}}{dt}-\frac{dk^*{}_{H,t}}{dt}=\theta_H-\theta_{(1+\ldots+H-1)} \tag{17}$$

and

$$F_\theta=2m_e\left[\theta_H-\theta_{(1+\ldots+H-1)}\right].$$

According to Equation (17), we have:

$$\frac{1}{H-1}\sum_{q=1}^{H-1}\frac{dk^*_{q,t}}{dt}-\frac{\theta_H-\theta_{(1+\ldots+H-1)}}{2}=\frac{dk^*{}_{H,t}}{dt}+\frac{\theta_H-\theta_{(1+\ldots+H-1)}}{2}. \tag{18}$$

Equation (18) illustrates that if the government transfers

$$(H-1)\left[\frac{\theta_H-\theta_{(1+\ldots+H-1)}}{2}\right]\left(\frac{1}{H-1}\sum_{q=1}^{H-1}\tilde{k}_{q,t}+\tilde{k}_{H,t}\right)$$

$$=(H-1)\left[\frac{\theta_H-\theta_{(1+\ldots+H-1)}}{2}\right]\frac{1}{H}\sum_{q=1}^{H}\tilde{k}_{q,t}$$

from Economy (1 + 2 + … + H – 1) to Economy H in each period, SH is achieved for all H economies. Since the multi-economy cases for heterogeneities in other factors examined in the subsequent sections are analogous to the case for RTP discussed here, they are omitted in this paper.

3.2.2 Force due to heterogeneity in DRA

Economies 1 and 2 are identical in all respects except for their DRA. Achieving SH requires g_t to be implemented according to Equation (3). Therefore, as with the heterogeneity in RTP, the acceleration is given by:

$$\frac{d^2\kappa_t}{dt^2} = 2\left(\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2}\right)(r - \theta), \tag{19}$$

According to Equations (10) and (19), the force originating from the heterogeneity in DRA (F_ε) is defined as:

$$F_\varepsilon = 4m_e\left(\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2}\right)(r - \theta).$$

From Equation (19), the velocity is:

$$\frac{d\kappa_t}{dt} = 2\left(\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2}\right)(r - \theta)t,$$

under the initial condition $\left.\frac{d\kappa_t}{dt}\right|_{t=0} = 0$, and the trajectory is:

$$\kappa_t = \kappa_0 + \left(\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2}\right)(r - \theta)t^2.$$

In addition, we obtain:

$$\frac{dk^*_{1,t}}{dt} - \left(\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2}\right)(r - \theta) = \frac{dk^*_{2,t}}{dt} + \left(\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2}\right)(r - \theta).$$

Therefore, if the government transfers

$$\left(\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2}\right)(r - \theta)\left(\frac{\tilde{k}_{1,t} + \tilde{k}_{2,t}}{2}\right)$$

from Economy 1 to Economy 2, the level of inequality $k^*_{1,t} - k^*_{2,t}$ remains constant, as indicated in Equation (3).

3.2.3 Force due to heterogeneity in persistent economic rents

Economies 1 and 2 are identical in all respects except obtaining persistent rent incomes. Achieving SH requires g_t to be implemented according to Equation (4). Therefore, as with the heterogeneity in RTP, the acceleration is given by:

$$\frac{d^2\kappa_t}{dt^2} = 2\bar{z}. \tag{20}$$

According to Equations (10) and (20), the force originating from the heterogeneity in persistent economic rents ($F_{\bar{z}}$) is defined as:

$$F_{\bar{z}} = 4m_e\bar{z}.$$

From Equation (20), the velocity is:

$$\frac{d\kappa_t}{dt} = 2\bar{z}t,$$

under the initial condition $\left.\frac{d\kappa_t}{dt}\right|_{t=0} = 0$, and the trajectory is:

$$\kappa_t = \kappa_0 + \bar{z}t^2.$$

In addition, we obtain:

$$\frac{dk^*_{1,t}}{dt} - \bar{z} = \frac{dk^*_{2,t}}{dt} + \bar{z}.$$

Therefore, if the government transfers

$$\bar{z}\left(\frac{\tilde{k}_{1,t} + \tilde{k}_{2,t}}{2}\right)$$

from Economy 1 to Economy 2, the level of inequality $k^*_{1,t} - k^*_{2,t}$ remains constant, as indicated in Equation (4).

3.2.4 Force due to heterogeneity in investment success rates

Economies 1 and 2 are identical in all respects except for their investment success rates. Achieving SH requires g_t to be implemented according to Equation (5). Therefore, as with the heterogeneity in RTP, the acceleration is given by:

$$\frac{d^2\kappa_t}{dt^2} = 2\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)r. \tag{21}$$

According to Equations (10) and (21), the force originating from the heterogeneity in investment success rates (F_ρ) is defined as:

$$F_\rho = 4m_e\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)r.$$

From Equation (21), the velocity is:

$$\frac{d\kappa_t}{dt} = 2\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)rt,$$

under the initial condition $\left.\frac{d\kappa_t}{dt}\right|_{t=0} = 0$, and the trajectory is:

$$\kappa_t = \kappa_0 + \left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)rt^2$$

In addition, we obtain:

$$\frac{dk^*_{1,t}}{dt} - \left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)r = \frac{dk^*_{2,t}}{dt} + \left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)r.$$

Therefore, if the government transfers

$$\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)r\left(\frac{\tilde{k}_{1,t} + \tilde{k}_{2,t}}{2}\right)$$

from Economy 1 to Economy 2, the level of inequality $k^*_{1,t} - k^*_{2,t}$ remains constant, as indicated in Equation (5).

3.2.5 Force due to multiple heterogeneities

As outlined in Section 2.5, when Economies 1 and 2 are simultaneously heterogeneous in RTP, DRA, persistent rent incomes, and investment success rates, while remaining identical in all other respects, the government transfer required to achieve SH must comply with Equations (6) and (7). Therefore, as with the previously discussed individual factors, the acceleration is given by:

$$\frac{d^2\kappa_t}{dt^2} = 2\Lambda. \tag{22}$$

According to Equations (10) and (22), the force originating from multiple heterogeneities (F_κ) is defined as:

$$F_\kappa = 4m_e\Lambda. \tag{23}$$

From Equation (22), the velocity is:

$$\frac{d\kappa_t}{dt} = 2\Lambda t,$$

under the initial condition $\left.\frac{d\kappa_t}{dt}\right|_{t=0} = 0$, and the trajectory is:

$$\kappa_t = \kappa_0 + \Lambda t^2. \tag{24}$$

In addition, we obtain:

$$\frac{dk^*_{1,t}}{dt} - \Lambda = \frac{dk^*_{2,t}}{dt} + \Lambda.$$

Therefore, if the government transfers Λ from Economy 1 to Economy 2, the level of inequality $k^*_{1,t} - k^*_{2,t}$ remains constant, as indicated in Equations (6) and (7).

3.3 Instantaneous force that generates inequality due to

heterogeneity in productivity

Economies 1 and 2 are identical in all respects except for their productivity. As outlined in Harashima (2010, 2012, 2014) and Section 2.6, achieving SH does not require government intervention when productivities are heterogeneous. Therefore, $g_t = 0$ for all t. This implies that:

$$\frac{d\left(k^*_{1,t} - k^*_{2,t} \right)}{dt} = 0,$$

and

$$\frac{d^2\kappa_t}{dt^2} = 0,$$

which yields:

$$F_\omega = 0,$$

where F_ω represents the force originating from the heterogeneity in productivity.

Importantly, however, at the exact instant $t = 0$, an instantaneous force F_ω is exerted such that:

$$\frac{d^2\kappa_t}{dt^2} = \frac{\omega_1 - \omega_2}{\omega_1 + \omega_2}.$$

and

$$F_\kappa = 2m_e\left(\frac{\omega_1 - \omega_2}{\omega_1 + \omega_2}\right).$$

Consequently, economic inequality is generated by this productivity differential. Because F_ω is exerted only instantaneously and momentarily at $t = 0$, it remains $F_\omega = 0$ for all $t > 0$. F_ω functions solely as an instantaneous force. Due to this instantaneous exertion of F_ω, the resulting economic inequality remains constant thereafter, satisfying:

$$\frac{d\kappa_t}{dt} = \frac{\omega_1 - \omega_2}{\omega_1 + \omega_2}.$$

In contrast to the aforementioned heterogeneities, productivity differentials do not generate a continuous force that persistently widens economic inequality.

3.4 Interaction

It is possible to reinterpret the force F_κ as a result of an interaction between Economies 1 and 2. Let $\kappa_{1,t}$ and $\kappa_{2,t}$ be defined as:

$$\frac{d\kappa_{1,t}}{dt} = k^*_{1,t} \tag{25}$$

and

$$\frac{d\kappa_{2,t}}{dt} = k^*_{2,t}. \tag{26}$$

According to Equations (8), (25), and (26), we have:

$$\frac{d\kappa_t}{dt} = \frac{d\kappa_{1,t}}{dt} - \frac{d\kappa_{2,t}}{dt}. \tag{27}$$

Additionally, extending Equation (7), let

$$\Lambda_{i,j} = \left(\frac{\varepsilon_j\rho_i - \varepsilon_i\rho_j}{\varepsilon_j\rho_i + \varepsilon_i\rho_j}\right)r + \left(\frac{\varepsilon_j\rho_i + \varepsilon_i\rho_j}{\varepsilon_j\rho_i + \varepsilon_i\rho_j}\right)\bar{z} + \frac{\varepsilon_i\theta_j - \varepsilon_j\theta_i}{\varepsilon_j\rho_i + \varepsilon_i\rho_j}, \tag{28}$$

where $i, j \in \{1, 2\}$ and $i \neq j$. It is evident that

$$\Lambda_{i,j} = -\Lambda_{j,i}. \tag{29}$$

As outlined in Equations (22), (27), and (28), the second derivative yields:

$$\frac{d^2\kappa_t}{dt^2} = \frac{d\kappa_{1,t}^2}{dt^2} - \frac{d\kappa_{2,t}^2}{dt^2} = 2\Lambda_{i,j}. \tag{30}$$

According to Equations (29) and (30), it follows that:

$$\frac{d\kappa_{i,t}^2}{dt^2} = \Lambda_{i,j}. \tag{31}$$

Therefore, according to Equations (10) and (31), the force exerted on Economy i due to the interaction between Economies 1 and 2 (denoted as $F_{\kappa,i}$) can be defined as follows:

$$F_{\kappa,i} = 2m_e \Lambda_{i,j}. \tag{32}$$

According to Equations (23), (28), (29), and (32), we obtain:

$$\begin{aligned} F_{\kappa,1} - F_{\kappa,2} &= 2m_e \Lambda_{1,2} - 2m_e \Lambda_{2,1} = 4m_e \Lambda_{1,2} \\ &= F_{\kappa}. \end{aligned}$$

3.5 Law of motion for economic inequality

The law of motion for economic inequality, grounded in the force (F_{κ}) due to the heterogeneities illustrated in Section 3.2, is analogous to Newtonian mechanics. The following principles define this analogy:

- **The First Law (Inertia)**: The body representing the economic inequality between two economies, whose position is measured by κ_t, remains in a state of uniform motion unless acted upon by an external force.

- **The Second Law (Acceleration)**: At any given moment, the force acting on the body is equivalent to the product of the economy's mass (scale), m_e, and its acceleration, $\frac{d^2\kappa_t}{dt^2}$.

- **The Third Law (Action and Reaction)**: If two segments of the body representing economic inequality exert forces on each other, these forces are equal in magnitude and opposite in direction.

The First Law holds because, if $\frac{d^2\kappa_t}{dt^2} = 0$, then $\frac{d\kappa_t}{dt}$ remains constant. Regarding the Third Law, suppose that the body representing economic inequality is divided into two segments (A and B). When $\frac{d\kappa_t}{dt}$ is constant, we have:

$$m_{e,A} \frac{d^2\kappa_{A,t}}{dt^2} + m_{e,B} \frac{d^2\kappa_{B,t}}{dt^2} = 0,$$

where $\kappa_{A,t}$ and $\kappa_{B,t}$ represent the positions κ_t of segments *A* and *B*, and $m_{e,A}$ and $m_{e,B}$ are their respective masses (scales). This implies that the acceleration of κ_t caused by a force is independent of any relative internal fluctuations between $\kappa_{A,t}$ and $\kappa_{B,t}$.

4 COUNTERFORCE

4.1 Objectives

As outlined in Section 3, in the absence of a continuous force F_κ, κ_t moves by inertia in accordance with the First Law of motion, similar to the movement attributed to $F_{\boldsymbol{\omega}}$**.** In this case, the value of $k^*_{1,t} - k^*_{2,t}$ remains constant. Conversely, if F_κ is persistently exerted, it continuously intensifies economic inequality if left unaddressed, which may eventually compel government intervention. Because the heterogeneities discussed in Sections 2 and 3 are inevitable and inherent to the economies, government interventions are essential to prevent economic inequality from escalating indefinitely.

However, a fundamental question remains: what is the ultimate objective of such intervention? There are three potential objectives:

$$(1)\ \left.\frac{d^2\kappa_t}{dt^2}\right|_{t=t_2} = \left.\frac{d\left(k^*_{1,t} - k^*_{2,t}\right)}{dt}\right|_{t=t_2} = 0,$$

$$(2)\ \left.\frac{d\kappa_t}{dt}\right|_{t=t_2} = k^*_{1,t_2} - k^*_{2,t_2} = 0,$$

and

$$(3)\ \kappa_{t_2} = \kappa_{t_1} + \int_{t_1}^{t_2} \left(k^*_{1,s} - k^*_{2,s}\right) ds = 0,$$

where $t = t_1$ represents the initial period when the government commences its counterforce measures, and $t = t_2$ represents the period when the government achieves its objective.

Objective (1) aims to arrest any further increase in economic inequality while leaving the current elevated level of inequality as it is. Consequently, the level of economic inequality given by

$$k^*_{1,t} - k^*_{2,t} = \left.\frac{d\kappa_t}{dt}\right|_{t=t_1} = k^*_{1,t_1} - k^*_{2,t_1}$$

persists. Objective (2) seeks to reduce and entirely eliminate contemporaneous economic inequality. Objective (3) consists of two parts: the elimination of contemporaneous economic inequality and compensation to Economy 2 for past economic inequality.

It is not possible to determine *a priori* which objective is socially superior. In a democratic society, the objective is chosen by its citizens through elections. Harashima (2018a, 2021b, 2021c, 2022) demonstrates that the standard equilibrium is not a pure, but rather an approximate SH. Even after implementing income redistribution measures, economic inequality remains significant, with no clear indication of a rapid escalation. This empirical and theoretical observation suggests that, in many democratic countries, Objective (1) serves as the primary goal, with Objective (2) playing a secondary or auxiliary role.

4.2 Counterforce to achieve Objective (1)

The counterforce ($-F_c$) exerted by the government to mitigate the impact of the continuous force F_κ (as well as F_θ, F_ε, $F_{\bar{z}}$, and F_ρ) is defined as follows:

$$F_c = -2m_e\Gamma_c.$$

One effective method for achieving Objective (1) is to transfer g_t in every period such that, for F_κ,

$$\Gamma_c = \frac{d^2\kappa_t}{dt^2} = 2\Lambda,$$

according to Equation (23). With this value of Γ_c, the objective $\frac{d^2\kappa_t}{dt^2} = 0$ is immediately achieved. However, if there is a sufficient time window until $t = t_2$, the initial amount of g_t can be reduced below the fully required amount. The transfer can then be gradually increased to the necessary level shown above by $t = t_2$.

4.3 Counterforce to achieve Objective (2)

To achieve Objective (2), a greater amount of fiscal transfer is required. However, it appears to be an unrealistic goal to instantaneously eliminate the entire level of inequality, given by

$$\left.\frac{d\kappa_t}{dt}\right|_{t=t_1} = {k^*}_{1,t_1} - {k^*}_{2,t_1}$$

in period $t = t_1$. A more reasonable approach is to reduce ${k^*}_{1,t_1} - {k^*}_{2,t_1}$ linearly. For F_κ, this implies that between t_1 and t_2, Γ_c satisfies:

$$\Gamma_c = \frac{d^2\kappa_t}{dt^2} + \frac{{k^*}_{1,t_1} - {k^*}_{2,t_1}}{t_2 - t_1} = 2\Lambda + \frac{{k^*}_{1,t_1} - {k^*}_{2,t_1}}{t_2 - t_1},$$

and after t_2,

$$\Gamma_c = \frac{d^2\kappa_t}{dt^2} = 2\Lambda,$$

according to Equation (23). Through this manipulation, the velocity between t_1 and t_2 becomes:

$$\frac{d\kappa_t}{dt} = {k^*}_{1,t_1} - {k^*}_{2,t_1} - \left(\frac{{k^*}_{1,t_1} - {k^*}_{2,t_1}}{t_2 - t_1}\right)(t - t_1),$$

and after t_2,

$$\frac{d\kappa_t}{dt} = 0.$$

Consequently, in every period between t_1 and t_2, the gap ${k^*}_{1,t} - {k^*}_{2,t}$ is reduced by $\frac{{k^*}_{1,t_1} - {k^*}_{2,t_1}}{t_2 - t_1}$. This ultimately achieves Objective (2) (i.e., ${k^*}_{1,t_2} = {k^*}_{2,t_2}$) at t_2, which is maintained thereafter.

5 ACTION OF FORCE

5.1 Point of application

The force F_κ (as well as F_θ, F_ε, $F_{\bar{z}}$, and F_ρ) primarily affects κ_t. As indicated by Equation (11), κ_t is a function of capital. Therefore, F_κ is closely related to capital, and consequently to the real interest rate (r_t). It is crucial to note that, despite facing a common

r_t, households react to the real interest rate differently regarding their capital holdings due to their respective heterogeneities, as outlined in Sections 2 and 3. This divergence implies that the point of application of these forces is r_t. When households encounter a common r_t, a force is generated if their capital accumulation behaviors diverge. Beyond this aspect, r_t plays a pivotal role in economic activities, serving as the primary catalyst for capital accumulation, where the distribution of capital acts as a key indicator of economic inequality.

In the case of F_θ, Economies 1 and 2 respond differently to the common r_t. Consequently, they adjust their capital holdings in distinct ways, operating to achieve $r_t = \theta_1$ and $r_t = \theta_2$, respectively. In the case of F_ε, they adjust their holdings so that the growth rate of consumption—which is proportional to $r_t - \theta$—is consistent with ε_1 and ε_2, respectively. In the cases of $F_{\bar{z}}$ and F_ρ, they do so because the actual, realized returns on capital are expected to deviate from r_t. Furthermore, these returns are expected to differ between the two economies due to the presence of $\bar{z}$ and the disparity between ρ_1 and ρ_2, respectively.

As stated in Section 3.3, heterogeneity in productivity does not lead to a sustained increase in economic inequality. This is because all households react to r_t in an identical manner when accumulating capital. When households first encounter r_t, their capital holdings diverge because of the different output levels resulting from heterogeneous productivities, allowing r_t to remain uniform across both economies. Crucially, however, their behavioral responses to r_t do not differ. Let θ represent the common RTP of identical households that differ only in terms of their productivities. They all behave in a way that leads to $r_t = \theta$. Consequently, at the steady state, their capital holdings are fixed, resulting in a constant level of inequality, $k^*_{1,t} - k^*_{2,t}$.

This implies that heterogeneity in productivity does not persistently affect households' actions with respect to r_t (i.e., the point of application). This provides the underlying reason why F_ω does not persistently increase economic inequality. When the instantaneous force F_ω initially encounters r_t, it is exerted on r_t momentarily, resulting in different initial capital holdings. However, F_ω exerts no further influence on households' subsequent actions with respect to r_t.

5.2 Softness of the body (object)

The real interest rate (r_t), which serves as the point of application, necessitates a positive (i.e., non-zero) maturity date or interest-bearing period (δ). Otherwise, $r_t = 0$, meaning it becomes non-existent, as is the case with cash. In such a scenario, the force F_κ would be unable to function due to the absence of its point of application. Therefore, by analogy with the law of motion for inflation presented in Harashima (2026), the following

relationship holds:

$$\int_{t-\delta}^{t}\int_{s}^{s+\delta}\kappa_v\,dvds=\kappa_t+\frac{\frac{d^2\kappa_t}{dt^2}}{\chi}, \tag{33}$$

and

$$\lim_{t\to\infty}\frac{\chi}{\delta}\left(\int_{t-\delta}^{t}\int_{s}^{s+\delta}\kappa_v dvds-\kappa_t\right)=Z, \tag{34}$$

where χ and Z are constants unique to each force (F_κ, F_θ, F_ε, $F_{\bar{z}}$, and F_ρ), and

$$\kappa_t=\kappa_0+\int_0^t\left(k^*{}_{1s}-k^*{}_{2,s}\right)ds.$$

In Equations (33) and (34), δ represents the degree of softness of the body (object) whose position is measured by κ_t. As δ increases, the body undergoes a greater degree of deformation when a force is initially exerted. That is, immediately after a force is applied, κ_t does not precisely follow the trajectory described by Equation (24). However, as time progresses, this initial deformation diminishes and ultimately disappears; that is, κ_t eventually tracks Equation (24) exactly.

This initial deformation occurs because r_t remains temporarily sticky or constant during the interval δ even after a force is exerted. The value of δ for κ_t is expected to be equivalent to that for inflation (π_t) whose law of motion is demonstrated in Harashima (2026). This equivalence arises from the fact that δ commonly and fundamentally corresponds to the maturity date or the interest-bearing period inherent in financial contracts.

6 CONCLUDING REMARKS

Economic inequality, as measured by income or wealth disparities, remains a long-standing and central issue in economics. While a wide variety of explanations regarding the origins of inequality have been proposed, universally agreed-upon criteria for socially optimal allocation remain elusive due to the interpersonal incomparability of utility, Arrow's impossibility theorem, and other theoretical barriers. Harashima (2010, 2012, 2014) introduced an alternative approach based on the concept of sustainable harmony

(SH), demonstrating that achieving SH requires appropriate government intervention. Conversely, in the absence of such intervention, economic inequality inevitably intensifies.

In this paper, I have demonstrated that the law of motion for economic inequality can be derived from the SH-based model by analogy to Newtonian mechanics. The policy rules for government transfers necessary to achieve SH can be interpreted as mirroring the "forces" that drive economic inequality. Because this law of motion is fundamentally force-based, its dynamics can be effectively framed within Newtonian mechanics—specifically through the concepts of velocity and acceleration. Furthermore, the principles governing economic inequality have been shown to align with the classical laws of inertia, acceleration, and action-reaction.

While households exhibit numerous types of heterogeneity, the majority of these factors are unlikely to be directly relevant to economic inequality. Those that persistently drive economic inequality are few in number, primarily including heterogeneities in RTP, DRA, persistent rent incomes, and the success rate of investment. Despite their limited number, the consequences of their continuous exertion—namely, the persistent widening of economic inequality—are of profound socioeconomic significance. Therefore, the role of government intervention in countering and containing these forces is indispensable.

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