Representation-Constrained Canonical Correlation Analysis: A Hybridization of Canonical Correlation and Principal Component Analyses

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I. Introduction: We begin this paper with reference to a dataset that, when subjected to the classical canonical correlation analysis, gives us the leading (first or largest) canonical correlation which is misleading. It is misleading in the sense that, in this example, the canonical correlation (which is the coefficient of correlation between the two canonical variates, each being a linear weighted combination of the variables in the associated dataset) is, indeed, not a measure of the true association of the variables in the two datasets, but, instead, the datasets have been hijacked by a lone couple of variables across the two datasets.

In Table-1.1 the dataset X is presented which is a pooled set of two datasets, X_1 and X_2, such that $X=[X_1|X_2]$. The first dataset has m_1 (=4) variables and the second dataset has m_2 (=5) variables, each in n (=30) observations. These seemingly normal datasets, when subjected to the classical canonical correlation analysis, yield canonical correlation between the composite variables, z_1 and z_2 (the canonical variates), $r(z_1, z_2) = 1.0$: $z_1 = \sum_{j=1}^{4} w_j x_{1j}; x_{ij} \in X_1$; $z_2 = \sum_{j=1}^{5} w_j x_{2j}; x_{2j} \in X_2$. The weight vectors are: w_1=(1, 0, 0, 0, 0) and w_2=(0, 0, 0, 0, 1). This anomalous situation has arisen due to the fact that x_{25} is perfectly linearly dependent on x_{11} and the canonical correlation, $r(z_1, z_2)$, is in fact $r(x_{11}, x_{25})$. Other variables have no contribution to z_1 or z_2. It follows, therefore, that z_1 and z_2 do not represent other variables in X_1 and X_2. Nor is the canonical correlation, $r(z_1, z_2)$, a correlation between the two sets, X_1 and X_2, in any relevant or significant sense. Thus, the leading canonical correlation may deceive us if we are only a little less careful to look into the correlation matrix encompassing all variables.

Table-1.1: Simulated Dataset-1 for Canonical correlation

Sl No.	X_1 or Dataset-1				X_2 or Dataset-2				
	X_{11}	X_{12}	X_{13}	X_{14}	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}
1	0.7	2.6	0.1	1.7	0.2	0.8	1.6	0.5	1.6
2	1.5	1.7	1.2	1.5	1.6	2.4	2.3	1.4	3.2
3	2.3	0.3	2.7	1.2	2.5	2.9	0.6	1.3	4.8
4	0.6	2.0	0.9	2.8	2.8	2.5	1.1	1.8	1.4
5	0.1	0.9	1.6	1.8	2.2	2.7	2.1	0.2	0.4
6	1.9	1.1	1.7	2.6	1.5	2.2	2.2	2.0	4.0
7	1.0	2.7	2.4	2.7	1.0	0.2	2.0	0.4	2.2
8	1.8	2.9	1.4	0.9	1.7	1.0	1.8	1.2	3.8
9	2.8	0.1	1.8	0.4	2.3	0.6	1.7	0.6	5.8
10	1.4	0.6	2.8	1.4	2.6	1.8	0.8	1.7	3.0
11	1.2	2.5	2.9	0.8	2.1	0.7	1.4	2.3	2.6
12	1.1	1.3	0.2	2.5	0.7	1.5	1.0	2.2	2.4
13	3.0	1.9	1.1	1.6	0.1	0.1	2.7	3.0	6.2
14	2.0	0.8	0.6	1.3	1.9	0.5	0.4	0.8	4.2
15	1.6	2.2	2.6	1.9	1.4	1.3	1.3	2.5	3.4
16	2.9	0.7	1.9	2.9	2.4	1.2	2.5	2.1	6.0
17	1.3	1.4	2.0	0.2	1.8	2.8	0.3	2.6	2.8
18	0.8	0.2	2.3	2.0	2.9	1.4	3.0	0.7	1.8
19	1.7	0.5	1.3	0.1	2.0	0.9	2.9	1.5	3.6
20	2.1	2.4	0.7	0.5	0.9	2.3	0.7	0.3	4.4
21	2.5	1.0	3.0	2.2	1.2	2.6	2.6	1.0	5.2
22	2.2	2.8	2.5	0.7	3.0	3.0	0.2	1.9	4.6
23	0.5	0.4	0.8	1.0	0.8	0.4	0.1	1.1	1.2
24	2.7	2.1	1.5	2.3	1.1	1.1	0.9	2.7	5.6
25	2.4	1.8	0.5	0.3	2.7	1.6	2.8	0.1	5.0
26	0.2	1.6	0.3	1.1	0.6	0.3	2.4	2.8	0.6
27	0.9	2.3	0.4	0.6	1.3	1.7	1.5	2.4	2.0
28	2.6	3.0	2.2	3.0	0.5	1.9	1.9	1.6	5.4
29	0.4	1.2	1.0	2.4	0.4	2.0	0.5	2.9	1.0
30	0.3	1.5	2.1	2.1	0.3	2.1	1.2	0.9	0.8

Such examples may be multiplied *ad infinitum*. If one is cautious, the anomalous cases can be detected. However, such cases, if not detected, make scientific analysis and interpretation of empirical results rather hazardous. One may easily be misled to a conclusion that such two datasets are highly correlated while the truth may be quite far from it.

II. Objectives of the Present Work: We intend here to propose an alternative measure of association between two sets of variables that will not permit the greed of a select few variables in the datasets to prevail upon the fellow variables so much as to deprive the latter of contributing their say and share to the representative variables (ς_1 and ς_2), which they make by their participation in the linear combination. We may not call $\varsigma_1 = \sum_{j=1}^{m_1} \omega_{1j} x_{1j}$ and $\varsigma_2 = \sum_{j=1}^{m_2} \omega_{2j} x_{2j}$ the canonical variables (defined before as $z_1 = \sum_{j=1}^{4} w_j x_{1j};\ z_2 = \sum_{j=1}^{5} w_j x_{2j}$ obtained from the classical canonical correlation analysis).

In the classical canonical correlation analysis the objective is to maximize $r^2(z_1, z_2): z_1 = \sum_{j=1}^{m_1} w_{1j} x_{1j}; z_2 = \sum_{j=1}^{m_2} w_{2j} x_{2j}$ irrespective of $r(z_1, x_{1j}): x_{1j} \in X_1$ and $r(z_2, x_{2j}): x_{2j} \in X_2$, and, therefore, $r^2(z_1, z_2)$ is subject to an unconstrained maximization. However, in the method that we are proposing here, the objective will be to maximize $r^2(\varsigma_1, \varsigma_2)$: $\varsigma_1 = \sum_{j=1}^{m_1} \omega_{1j} x_{1j}$ and $\varsigma_2 = \sum_{j=1}^{m_2} \omega_{2j} x_{2j}$ with certain constraints in terms of $r(\varsigma_1, x_{1j}): x_{1j} \in X_1$ and $r(\varsigma_2, x_{2j}): x_{2j} \in X_2$. These constraints would ensure the representativeness of ς_1 to X_1 and that of ς_2 to X_2. Hence, the proposed method may be called the *Representation-Constrained Canonical Correlation Analysis*.

III. The Nature and Implications of the Proposed Constraints: There are a number of ways in which the canonical variates can be constrained insofar as their association and concordance with their fellow variables in their respective native datasets are concerned. In other words, their representativeness to their native datasets can be defined variously. We discuss here some of the alternatives in terms of correlation as a measure of representativeness.

(i) Mean absolute correlation principle: A (constrained) canonical variate $\varsigma_a = \sum_{j=1}^{m_a} \omega_{aj} x_{aj}; x_{aj} \in X_a$ is a better representative of X_a if the mean absolute correlation, $\sum_{j=1}^{m_a} | r(\varsigma_a, x_{aj}) |$, is larger. This approach is equalitarian in effect.

(ii) Mean squared correlation principle: A (constrained) canonical variate $\varsigma_a = \sum_{j=1}^{m_a} \omega_{aj} x_{aj}; x_{aj} \in X_a$ is a better representative of X_a if the mean squared correlation, $\sum_{j=1}^{m_a} r^2(\varsigma_a, x_{aj})$, is larger. This approach is elitist in effect, favouring dominant members.

(iii) Minimal absolute correlation principle: A (constrained) canonical variate $\varsigma_a = \sum_{j=1}^{m_a} \omega_{aj} x_{aj}; x_{aj} \in X_a$ is a better representative of X_a if the minimal absolute correlation, $\min_j[| r(\varsigma_a, x_{aj}) |]$, is larger. A larger $\min_j[| r(\varsigma_a, x_{aj}) |]$ implies that the minimal squared correlation, $\min_j[r^2(\varsigma_a, x_{aj})]$, is larger. This approach is in favour of the weak.

These three approaches lead to three alternative objective functions:

(i). Maximize $r^2(\varsigma_1,\varsigma_2) + \lambda[\sum_{j=1}^{m_1} | r(\varsigma_1, x_{1j}) | / m_1 + \sum_{j=1}^{m_2} | r(\varsigma_2, x_{2j}) | / m_2] : \varsigma_1 = \sum_{j=1}^{m_1} \omega_{1j} x_{1j};\ \varsigma_2 = \sum_{j=1}^{m_2} \omega_{2j} x_{2j}$.

(ii). Maximize $r^2(\varsigma_1,\varsigma_2) + \lambda[\sum_{j=1}^{m_1} r^2(\varsigma_1, x_{1j}) / m_1 + \sum_{j=1}^{m_2} r^2(\varsigma_2, x_{2j}) / m_2] : \varsigma_1 = \sum_{j=1}^{m_1} \omega_{1j} x_{1j};\ \varsigma_2 = \sum_{j=1}^{m_2} \omega_{2j} x_{2j}$.

(iii). Maximize $r^2(\varsigma_1,\varsigma_2) + \lambda\left[\min_j[| r(\varsigma_1, x_{1j}) |] + \min_j[| r(\varsigma_2, x_{2j}) |]\right] : \varsigma_1 = \sum_{j=1}^{m_1} \omega_{1j} x_{1j};\ \varsigma_2 = \sum_{j=1}^{m_2} \omega_{2j} x_{2j}$.

In these objective functions, the value of λ may be chosen subjectively. If $\lambda = 0,$ the objective function would degenerate to the classical canonical correlation analysis, but λ has no upper bound. Also note that if the first term is $|r(\varsigma_1, \varsigma_2)|$ rather than $r^2(\varsigma_1, \varsigma_2)$ and $\lambda \neq 0,$ its implied weight vis-à-vis the second term increases since $|r(\varsigma_1, \varsigma_2)| > r^2(\varsigma_1, \varsigma_2)$ for $|r| < 1.$

IV. The Method of Optimization: The classical canonical correlation analysis (Hotelling, 1936) sets up the objective function to maximize $r^2(\varsigma_1, \varsigma_2) : \varsigma_1 = \sum_{j=1}^{m_1} \omega_{1j} x_{1j};\ \varsigma_2 = \sum_{j=1}^{m_2} \omega_{2j} x_{2j}$ and using the calculus methods of maximization resolves the problem to finding out the largest eigenvalue and the associated eigenvector of the matrix, $[X_1'X_1]^{-1} X_1'X_2 [X_2'X_2]^{-1} X_2'X_1.$ The largest eighen-value turns out to be the leading $r^2(z_1, z_2) : z_1 = \sum_{j=1}^{m_1} w_{1j} x_{1j};\ z_2 = \sum_{j=1}^{m_2} w_{2j} x_{2j}$, and the standardized eigenvector is used to obtain w_1 and w_2. However, a general calculus-based method cannot be applied to maximize the (arbitrary) objective function set up for the constrained canonical correlation analysis. At any rate, the first and the third objective functions are not amenable to maximization by the calculus-based methods.

We choose, therefore, to use a relatively new and more versatile method of (global) optimization, namely, the Particle Swarm Optimization (PSO) proposed by Eberhart and Kennedy (1995). A lucid description of its foundations is available in Fleischer (2005). The PSO is a biologically inspired population-based stochastic search method modeled on the ornithological observations, simulating the behavior of members of the flocks of birds in searching food and communicating among themselves. It is in conformity with the principles of decentralized decision making (Hayek, 1948; 1952) leading to self-organization and macroscopic order. The effectiveness of PSO has been very encouraging in solving extremely difficult and varied types of nonlinear optimization problems (Mishra, 2006). We have used a particular variant of the PSO called the Repulsive Particle Swarm Optimization (Urfalioglu, 2004).

V. Findings and Discussion: We have subjected the data in Table-1.1 to the representation-constrained canonical correlation analysis with the three alternative objective functions elaborated in section-III. The first term, measuring the degree of association between the two datasets, X_1 and X_2, is in the squared form, that is $r^2(\varsigma_1, \varsigma_2)$, although we have reported its positive square root (=$|r(\varsigma_1, \varsigma_2)|$) in Table-1.2. The three objective functions have been optimized for the different values of $\lambda,$ varying from zero to 50 with an increment of 0.5. For the first objective function, the values of $|r(\varsigma_1, \varsigma_2)|,$ mean absolute $r(\varsigma_1, x_1)$ and mean absolute $r(\varsigma_2, x_2)$ at different values of λ have been plotted in Fig.-1.1. Similarly, for the second objective function, the values of $|r(\varsigma_1, \varsigma_2)|,$ mean squared $r(\varsigma_1, x_1)$ and mean squared $r(\varsigma_2, x_2)$ at different values of λ have been plotted in Fig.-1.2. Fig.-1.3 presents $|r(\varsigma_1, \varsigma_2)|$, minimum absolute $r(\varsigma_1, x_1)$ and minimum absolute $r(\varsigma_2, x_2)$ relating to the 3rd objective maximized at different values of $\lambda.$

From Fig.-1.1 and Fig.-1.2 it is clear that for increasing values of $\lambda,$ the value of $|r(\varsigma_1, \varsigma_2)|$ decreases monotonically, while the values of mean absolute (or squared) $r(\varsigma_1, x_1)$ and mean absolute (or squared) $r(\varsigma_2, x_2)$ increase monotonically. All of them exhibit asymptotic tendencies. However, for the third objective function the monotonicity of all the correlation functions is lost (shown in Fig.-1.3). Of course, the trends in minimum absolute $r(\varsigma_1, x_1)$ and minimum absolute $r(\varsigma_2, x_2)$ are clearly observable. These observations may be useful to the choice of $\lambda.$ For the case that we are presently dealing with, the value of λ need not exceed 10 to assure a fairly satisfactory representation of the two datasets by the corresponding canonical variates.

In particular, optimization of the second objective function has shown that the values of mean squared $r(\varsigma_1,x_1)$ and mean squared $r(\varsigma_2,x_2)$ exhibit asymptotic tendencies . For λ=50, the mean squared $r(\varsigma_1,x_1)$ is 0.3176 while the mean squared $r(\varsigma_2,x_2)$ is 0.2870.

Now, let us digress for a while to compute the first principal components of X_1 and X_2 (from the data given in Table-1.1). We find that for X_1 the sum of squared correlation (component loadings) of the component score (ξ_1) with its constituent variables is 0.317757. In other words, the first eigenvalue of the inter-correlation matrix R_1 obtained from X_1 is 1.271029, which divided by 4 (order of R_1) gives 0.317757. This is, in a way, a measure of representation of X_1 by its first principal component. Similarly, for X_2 the sum of squared correlation of the component score (ξ_2) with its constituent variables is 0.287521.

We resume our discussion for comparing these results (obtained from the Principal Component Analysis) with the results of our proposed representation-constrained canonical correlation analysis. We observe that the asymptotic tendencies of mean squared $r(\varsigma_1,x_1)$ and mean squared $r(\varsigma_2,x_2)$ clearly point to the explanatory powers of the first principal components of X_1 and X_2 respectively.

However, if we compute the coefficient of correlation between the two component scores ($r(\xi_1,\xi_2)$=0.390767) and compare it with the constrained canonical correlation ($r(\varsigma_1,\varsigma_2)$ =0.4480 for λ=50) we find that the latter is larger. Then, is the constrained canonical correlation analysis a hybrid of the classical canonical correlation and principal component analyses which has better properties of representation of data than its parents?

We conduct another experiment with the dataset presented in Table-2.1. We find that ξ_1 for X_1 has the representation power 0.333261 (eigenvalue=1.333042) while ξ_2 for X_2 has the representation power 0.382825 (eigenvalue=1.914123). The $r(\xi_1,\xi_2)$ is 0.466513. On the other hand, results of the constrained canonical correlation (for λ=49) are: mean squared $r(\varsigma_1,x_1)$ = 0.33317; mean squared $r(\varsigma_2,x_2)$ =0.38270 and the representation-constrained canonical correlation, $r(\varsigma_1,\varsigma_2)$ = 0.48761. These findings are corroborative to our earlier results with regard to the dataset in Table-1.1.

We conduct yet another experiment with the dataset presented in Table-3.1. We find that ξ_1 for X_1 has the representation power 0.661265 (eigenvalue=2.645058) while ξ_2 for X_2 has the representation power 0.752979 (eigenvalue=3.764895). The $r(\xi_1,\xi_2)$ is 0.922764. Against these, results of the constrained canonical correlation (for λ=49) are: mean squared $r(\varsigma_1,x_1)$ = 0.661261; mean squared $r(\varsigma_2,x_2)$ =0.752966 and the constrained canonical correlation, $r(\varsigma_1,\varsigma_2)$ = 0.923647. These results are once again corroborative to our earlier findings.

VI. A Computer Program for RCCCA: We append here the computer program (in FORTRAN) that we have developed and used for solving the problems in this paper. Its main program (RCCCA) is assisted by 13 subroutines. The user needs setting the parameters in the main program as well as in the subroutines CORD and DORANK. Parameter setting in RPS may seldom be required. This program can be used for obtaining Ordinal Canonical Correlation (Mishra, 2009) also. Different schemes of rank-ordering may be used (Wikipedia, 2008).

VII. Concluding Remarks: Our proposed Representation-Constrained Canonical correlation (RCCCA) Analysis has the classical canonical correlation analysis (CCCA) at its one end (λ=0) and the Classical Principal Component Analysis (CPCA) at the other (as λ tends to be very large). In between it gives us a compromise solution. By a proper choice of λ, one can avoid hijacking of the representation issue of two datasets by a lone couple of highly correlated variables across those datasets. This advantage of the RCCCA over the CCCA deserves a serious attention by the researchers using statistical tools for data analysis.

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Table-1.2. Relationship between Constrained Canonical Correlation and Representation Correlation between Canonical Variates and their Constituent Variables for Different Values of λ										
Sl No.	λ	Canonical	Mean Absolute		Canonical	Mean Squared		Canonical	Minimum Absolute	
		$\lvert r(\varsigma_1,\varsigma_2)\rvert$	$r(\varsigma_1,x_1)$	$r(\varsigma_2,x_2)$	$\lvert r(\varsigma_1,\varsigma_2)\rvert$	$r(\varsigma_1,x_1)$	$r(\varsigma_2,x_2)$	$\lvert r(\varsigma_1,\varsigma_2)\rvert$	$r(\varsigma_1,x_1)$	$r(\varsigma_2,x_2)$
1	0.0	1.0000	0.3342	0.2814	1.0000	0.2668	0.2121	1.0000	0.0234	0.0246
2	0.5	0.9831	0.3942	0.3254	0.9990	0.2717	0.2152	0.9755	0.1615	0.1361
3	1.0	0.9440	0.4434	0.3785	0.9961	0.2763	0.2183	0.8223	0.3921	0.2671
4	1.5	0.8942	0.4772	0.4188	0.9916	0.2805	0.2214	0.5072	0.5302	0.4618
5	2.0	0.8432	0.4992	0.4479	0.9855	0.2843	0.2244	0.4662	0.5319	0.4853
6	2.5	0.7975	0.5128	0.4679	0.9780	0.2878	0.2275	0.4556	0.5337	0.4889
7	3.0	0.7597	0.5210	0.4813	0.9691	0.2909	0.2306	0.4473	0.5338	0.4917
8	3.5	0.7298	0.5259	0.4902	0.9590	0.2938	0.2338	0.4296	0.5337	0.4968
9	4.0	0.7060	0.5290	0.4962	0.9477	0.2964	0.2369	0.4349	0.5334	0.4954
10	4.5	0.6870	0.5310	0.5005	0.9352	0.2987	0.2401	0.4230	0.5335	0.4978
11	5.0	0.6715	0.5323	0.5036	0.9217	0.3008	0.2433	0.4342	0.5337	0.4955
12	5.5	0.6590	0.5333	0.5058	0.9073	0.3027	0.2464	0.4359	0.5338	0.4950
13	6.0	0.6483	0.5339	0.5076	0.8921	0.3044	0.2495	0.4404	0.5338	0.4940
14	6.5	0.6394	0.5345	0.5089	0.8762	0.3059	0.2525	0.3743	0.5389	0.4963
15	7.0	0.6318	0.5348	0.5100	0.8599	0.3072	0.2554	0.4170	0.5337	0.4994
16	7.5	0.6251	0.5351	0.5108	0.8434	0.3083	0.2581	0.4175	0.5338	0.4992
17	8.0	0.6193	0.5354	0.5115	0.8270	0.3094	0.2607	0.4278	0.5338	0.4970
18	8.5	0.6142	0.5356	0.5121	0.8106	0.3102	0.2630	0.4167	0.5335	0.4990
19	9.0	0.6098	0.5357	0.5126	0.7945	0.3110	0.2652	0.4293	0.5337	0.4967
20	9.5	0.6056	0.5358	0.5130	0.7789	0.3117	0.2672	0.4206	0.5339	0.4986
21	10.0	0.6019	0.5360	0.5133	0.7641	0.3122	0.2690	0.3746	0.5389	0.4962
22	10.5	0.5988	0.5360	0.5136	0.7495	0.3127	0.2706	0.2904	0.5023	0.4748
23	11.0	0.5958	0.5361	0.5139	0.7359	0.3132	0.2721	0.4167	0.5338	0.4990
24	11.5	0.5931	0.5362	0.5141	0.7227	0.3136	0.2734	0.4201	0.4789	0.4281
25	12.0	0.5906	0.5362	0.5143	0.7103	0.3139	0.2746	0.4206	0.5338	0.4987
26	12.5	0.5884	0.5363	0.5144	0.6985	0.3142	0.2756	0.5150	0.4781	0.3664
27	13.0	0.5861	0.5363	0.5146	0.6872	0.3145	0.2766	0.4167	0.5337	0.4993
28	13.5	0.5842	0.5364	0.5147	0.6764	0.3148	0.2774	0.3745	0.5389	0.4964
29	14.0	0.5826	0.5364	0.5148	0.6665	0.3150	0.2782	0.3742	0.5390	0.4963
30	14.5	0.5807	0.5364	0.5150	0.6570	0.3152	0.2789	0.4022	0.4648	0.4532
31	15.0	0.5791	0.5365	0.5151	0.6478	0.3154	0.2795	0.4170	0.5338	0.4991
32	15.5	0.5778	0.5365	0.5151	0.6390	0.3155	0.2801	0.4179	0.5003	0.4860
33	16.0	0.5765	0.5365	0.5152	0.6310	0.3157	0.2806	0.2791	0.5387	0.4990
34	16.5	0.5751	0.5365	0.5153	0.6231	0.3158	0.2810	0.3992	0.4764	0.4347
35	17.0	0.5739	0.5365	0.5154	0.6158	0.3159	0.2815	0.3742	0.5388	0.4964
36	17.5	0.5728	0.5366	0.5154	0.6088	0.3160	0.2819	0.0285	0.4457	0.4501
37	18.0	0.5715	0.5366	0.5155	0.6021	0.3161	0.2822	0.2794	0.5389	0.4992
38	18.5	0.5706	0.5366	0.5155	0.5960	0.3162	0.2825	0.3811	0.4744	0.4599
39	19.0	0.5697	0.5366	0.5156	0.5898	0.3163	0.2828	0.3741	0.5389	0.4963
40	19.5	0.5688	0.5366	0.5156	0.5840	0.3164	0.2831	0.3743	0.5389	0.4962
41	20.0	0.5680	0.5366	0.5157	0.5783	0.3165	0.2834	0.3345	0.4838	0.3983
42	20.5	0.5671	0.5366	0.5157	0.5732	0.3166	0.2836	0.2795	0.5389	0.4994
43	21.0	0.5663	0.5366	0.5157	0.5682	0.3166	0.2838	0.4194	0.4718	0.4439
44	21.5	0.5655	0.5366	0.5158	0.5632	0.3167	0.2840	0.3746	0.5389	0.4963
45	22.0	0.5650	0.5367	0.5158	0.5587	0.3167	0.2842	0.5496	0.5103	0.3823
46	22.5	0.5643	0.5367	0.5158	0.5542	0.3168	0.2843	0.2539	0.5138	0.4743
47	23.0	0.5635	0.5367	0.5158	0.5499	0.3168	0.2845	0.2795	0.5390	0.4993
48	23.5	0.5630	0.5367	0.5159	0.5459	0.3169	0.2846	0.2865	0.4643	0.4394
49	24.0	0.5623	0.5367	0.5159	0.5419	0.3169	0.2848	0.3688	0.5389	0.4944
50	24.5	0.5618	0.5367	0.5159	0.5383	0.3170	0.2849	0.2490	0.5347	0.4720
51	25.0	0.5612	0.5367	0.5159	0.5347	0.3170	0.2850	0.2792	0.5387	0.4994

52	25.5	0.5607	0.5367	0.5159	0.5312	0.3170	0.2851	0.4305	0.4684	0.3653
53	26.0	0.5603	0.5367	0.5160	0.5280	0.3171	0.2852	0.2793	0.5387	0.4993
54	26.5	0.5597	0.5367	0.5160	0.5249	0.3171	0.2853	0.4418	0.5176	0.4731
55	27.0	0.5592	0.5367	0.5160	0.5219	0.3171	0.2854	0.3741	0.5388	0.4963
56	27.5	0.5589	0.5367	0.5160	0.5186	0.3171	0.2855	0.5795	0.4661	0.4031
57	28.0	0.5584	0.5367	0.5160	0.5160	0.3172	0.2856	0.2335	0.5213	0.4604
58	28.5	0.5581	0.5367	0.5160	0.5131	0.3172	0.2857	0.2335	0.5213	0.4604
59	29.0	0.5575	0.5367	0.5161	0.5103	0.3172	0.2858	0.2790	0.5388	0.4993
60	29.5	0.5572	0.5367	0.5161	0.5080	0.3172	0.2858	0.1922	0.5023	0.4015
61	30.0	0.5568	0.5367	0.5161	0.5054	0.3173	0.2859	0.4223	0.5119	0.4564
62	30.5	0.5564	0.5367	0.5161	0.5030	0.3173	0.2859	0.3929	0.5016	0.4801
63	31.0	0.5561	0.5367	0.5161	0.5008	0.3173	0.2860	0.2795	0.5390	0.4993
64	31.5	0.5558	0.5367	0.5161	0.4987	0.3173	0.2861	0.3260	0.5081	0.4567
65	32.0	0.5555	0.5367	0.5161	0.4964	0.3173	0.2861	0.2140	0.5156	0.4897
66	32.5	0.5549	0.5367	0.5161	0.4942	0.3173	0.2862	0.2793	0.5389	0.4992
67	33.0	0.5547	0.5367	0.5161	0.4921	0.3174	0.2862	0.4277	0.4566	0.4137
68	33.5	0.5545	0.5367	0.5161	0.4902	0.3174	0.2863	0.2794	0.5389	0.4993
69	34.0	0.5542	0.5367	0.5161	0.4883	0.3174	0.2863	0.4708	0.5056	0.3723
70	34.5	0.5539	0.5367	0.5162	0.4865	0.3174	0.2863	0.2787	0.5388	0.4988
71	35.0	0.5539	0.5367	0.5162	0.4846	0.3174	0.2864	0.3639	0.5312	0.4787
72	35.5	0.5534	0.5367	0.5162	0.4830	0.3174	0.2864	0.2793	0.5389	0.4992
73	36.0	0.5532	0.5367	0.5162	0.4814	0.3174	0.2864	0.4560	0.5133	0.4533
74	36.5	0.5528	0.5367	0.5162	0.4796	0.3174	0.2865	0.3375	0.5282	0.4788
75	37.0	0.5524	0.5368	0.5162	0.4780	0.3174	0.2865	0.2504	0.5345	0.4600
76	37.5	0.5524	0.5368	0.5162	0.4765	0.3175	0.2865	0.2784	0.5380	0.4988
77	38.0	0.5521	0.5368	0.5162	0.4749	0.3175	0.2866	0.0886	0.5222	0.4078
78	38.5	0.5520	0.5368	0.5162	0.4733	0.3175	0.2866	0.2791	0.5372	0.4631
79	39.0	0.4469	0.5394	0.5163	0.4721	0.3175	0.2866	0.2795	0.5389	0.4992
80	39.5	0.4468	0.5394	0.5163	0.4707	0.3175	0.2866	0.0385	0.5148	0.4071
81	40.0	0.4467	0.5394	0.5163	0.4693	0.3175	0.2867	0.2028	0.5160	0.4721
82	40.5	0.4463	0.5394	0.5163	0.4681	0.3175	0.2867	0.0080	0.5182	0.4812
83	41.0	0.4463	0.5394	0.5163	0.4666	0.3175	0.2867	0.3389	0.4771	0.4282
84	41.5	0.4461	0.5394	0.5163	0.4653	0.3175	0.2867	0.2795	0.5389	0.4994
85	42.0	0.4460	0.5394	0.5163	0.4644	0.3175	0.2868	0.3389	0.4771	0.4282
86	42.5	0.4458	0.5394	0.5163	0.4631	0.3175	0.2868	0.0338	0.5248	0.4897
87	43.0	0.4456	0.5394	0.5163	0.4617	0.3175	0.2868	0.2793	0.5389	0.4993
88	43.5	0.4454	0.5394	0.5163	0.4606	0.3175	0.2868	0.1597	0.4139	0.3977
89	44.0	0.4453	0.5394	0.5163	0.4593	0.3176	0.2868	0.0338	0.5248	0.4897
90	44.5	0.4452	0.5394	0.5163	0.4586	0.3176	0.2869	0.2794	0.5389	0.4994
91	45.0	0.4451	0.5394	0.5163	0.4576	0.3176	0.2869	0.1880	0.5229	0.4274
92	45.5	0.4450	0.5394	0.5163	0.4564	0.3176	0.2869	0.2733	0.5300	0.4848
93	46.0	0.4448	0.5394	0.5163	0.4555	0.3176	0.2869	0.2786	0.5389	0.4991
94	46.5	0.4447	0.5394	0.5163	0.4547	0.3176	0.2869	0.2822	0.5354	0.4665
95	47.0	0.4445	0.5394	0.5163	0.4535	0.3176	0.2869	0.2898	0.5252	0.4905
96	47.5	0.4444	0.5394	0.5163	0.4527	0.3176	0.2869	0.2796	0.5389	0.4993
97	48.0	0.4444	0.5394	0.5163	0.4510	0.3176	0.2870	0.3372	0.4676	0.4344
98	48.5	0.4442	0.5394	0.5163	0.4509	0.3176	0.2870	0.2768	0.5389	0.4985
99	49.0	0.4440	0.5394	0.5163	0.4500	0.3176	0.2870	0.2792	0.5388	0.4993
100	49.5	0.4439	0.5394	0.5163	0.4491	0.3176	0.2870	0.2790	0.5389	0.4993
101	50.0	0.4438	0.5394	0.5163	0.4480	0.3176	0.2870	0.2784	0.5390	0.4989

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Table-2.1: Simulated Dataset-2 for Canonical correlation

Sl	X_1 or Dataset-1				X_2 or Dataset-2					Sl	X_1 or Dataset-1				X_2 or Dataset-2				
No.	X_{11}	X_{12}	X_{13}	X_{14}	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	No.	X_{11}	X_{12}	X_{13}	X_{14}	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}
1	2.7	1.9	2.4	1.2	2.6	2.3	1.5	0.1	6.6	16	1.4	1.4	0.2	0.4	1.1	2.2	2.2	2.6	2.4
2	1.7	0.1	0.8	1.8	0.4	0.2	2.3	0.2	0.1	17	1.5	0.4	2.2	1.9	1.9	0.6	2.1	1.9	5.5
3	0.2	2.8	2.6	0.9	1.3	2.0	2.0	0.3	7.3	18	0.6	1.3	2.5	2.8	2.8	2.5	2.7	2.2	4.9
4	0.4	0.3	1.1	0.2	1.5	1.3	1.1	1.8	3.3	19	2.5	1.1	0.1	1.1	2.5	1.0	1.0	2.9	3.8
5	0.9	1.8	1.6	1.4	0.8	1.2	2.4	2.4	5.7	20	1.0	2.3	1.8	1.5	2.9	1.8	1.6	2.0	5.8
6	0.5	0.9	2.7	0.7	1.4	1.6	1.2	3.0	6.2	21	0.8	1.7	1.0	1.6	1.6	2.4	0.6	1.4	4.5
7	2.0	1.0	2.9	1.7	0.3	0.1	0.4	1.1	5.4	22	0.3	1.2	2.1	0.3	2.0	1.9	0.7	0.9	4.5
8	0.1	1.6	0.5	2.7	0.7	2.1	1.3	1.7	3.1	23	1.3	0.7	1.3	2.4	2.2	0.7	0.8	1.0	3.4
9	1.2	0.6	2.8	1.0	0.1	0.9	0.1	0.8	3.7	24	2.6	1.5	2.3	0.6	1.7	2.9	2.9	2.5	7.3
10	2.9	2.1	0.4	0.8	0.5	0.3	1.7	0.4	4.7	25	3.0	2.6	1.2	3.0	2.7	2.6	2.8	1.5	7.2
11	0.7	0.5	0.6	1.3	2.1	0.5	0.3	0.7	0.7	26	1.1	2.2	0.7	2.5	2.4	0.8	2.6	1.2	3.8
12	2.8	2.5	1.5	2.9	2.3	2.8	3.0	1.6	6.5	27	1.8	2.0	1.9	2.2	1.8	1.7	1.8	0.6	6.1
13	2.2	0.2	1.7	2.3	3.0	1.1	0.5	2.7	3.9	28	1.9	2.7	3.0	2.0	1.0	1.4	1.4	0.5	9.5
14	2.1	0.8	0.9	2.6	0.9	2.7	2.5	2.1	3.6	29	1.6	2.4	0.3	0.5	0.2	0.4	0.2	2.3	6.5
15	2.3	3.0	1.4	0.1	0.6	3.0	0.9	2.8	8.4	30	2.4	2.9	2.0	2.1	1.2	1.5	1.9	1.3	7.6

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Table-2.2. Relationship between Constrained Canonical Correlation and Representation Correlation between Canonical Variates and their Constituent Variables for Different Values of λ (Dataset in Table-2.1)

Sl	λ	Canonical	Mean Squared		Sl	λ	Canonical	Mean Squared	
No.		$\lvert r(\varsigma_1,\varsigma_2)\rvert$	$r(\varsigma_1,x_1)$	$r(\varsigma_2,x_2)$	No.		$\lvert r(\varsigma_1,\varsigma_2)\rvert$	$r(\varsigma_1,x_1)$	$r(\varsigma_2,x_2)$
1	0.0	0.95772	0.28514	0.23519	26	25.0	0.50983	0.33290	0.38230
2	1.0	0.94904	0.29212	0.26011	27	26.0	0.50804	0.33293	0.38234
3	2.0	0.91881	0.29987	0.28983	28	27.0	0.50638	0.33296	0.38238
4	3.0	0.86701	0.30782	0.31901	29	28.0	0.50475	0.33298	0.38241
5	4.0	0.80425	0.31506	0.34197	30	29.0	0.50340	0.33300	0.38244
6	5.0	0.74448	0.32066	0.35709	31	30.0	0.50203	0.33302	0.38247
7	6.0	0.69541	0.32452	0.36618	32	31.0	0.50086	0.33304	0.38249
8	7.0	0.65777	0.32703	0.37155	33	32.0	0.49966	0.33305	0.38252
9	8.0	0.62930	0.32867	0.37482	34	33.0	0.49858	0.33306	0.38254
10	9.0	0.60764	0.32976	0.37690	35	34.0	0.49755	0.33308	0.38256
11	10.0	0.59071	0.33052	0.37828	36	35.0	0.49659	0.33309	0.38257
12	11.0	0.57730	0.33106	0.37924	37	36.0	0.49571	0.33310	0.38259
13	12.0	0.56634	0.33146	0.37993	38	37.0	0.49492	0.33311	0.38260
14	13.0	0.55733	0.33176	0.38044	39	38.0	0.49409	0.33311	0.38261
15	14.0	0.54983	0.33199	0.38083	40	39.0	0.49333	0.33312	0.38262
16	15.0	0.54338	0.33217	0.38113	41	40.0	0.49265	0.33313	0.38263
17	16.0	0.53786	0.33232	0.38137	42	41.0	0.49193	0.33314	0.38264
18	17.0	0.53310	0.33244	0.38156	43	42.0	0.49132	0.33314	0.38265
19	18.0	0.52896	0.33253	0.38171	44	43.0	0.49074	0.33315	0.38266
20	19.0	0.52523	0.33262	0.38185	45	44.0	0.49018	0.33315	0.38267
21	20.0	0.52196	0.33268	0.38195	46	45.0	0.48958	0.33316	0.38268
22	21.0	0.51904	0.33274	0.38205	47	46.0	0.48912	0.33316	0.38268
23	22.0	0.51638	0.33279	0.38212	48	47.0	0.48852	0.33317	0.38269
24	23.0	0.51395	0.33283	0.38219	49	48.0	0.48812	0.33317	0.38270
25	24.0	0.51184	0.33287	0.38225	50	49.0	0.48761	0.33317	0.38270

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Table-3.1: Simulated Dataset-3 for Canonical correlation

Sl No.	X_1 or Dataset-1				X_2 or Dataset-2					Sl No.	X_1 or Dataset-1				X_2 or Dataset-2				
	X_{11}	X_{12}	X_{13}	X_{14}	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}		X_{11}	X_{12}	X_{13}	X_{14}	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}
1	0.7	1.1	1.6	1.3	1.1	0.1	2.7	1.5	1.8	16	2.1	2.7	2.7	1.8	1.8	0.9	6.4	1.6	2.3
2	1.3	1.2	0.8	1.0	0.3	1.2	1.1	0.4	0.3	17	2.2	2.9	2.6	2.3	3.0	3.0	6.9	1.9	2.8
3	1.7	2.5	1.8	1.1	1.9	2.3	5.5	2.6	2.1	18	2.6	1.9	2.8	2.2	2.5	2.8	5.0	2.4	3.0
4	2.9	2.4	1.9	2.8	2.7	2.2	4.9	2.3	1.9	19	1.6	1.3	2.4	3.0	1.7	2.1	4.3	2.0	2.9
5	0.4	1.0	0.2	0.9	0.5	2.5	1.7	1.0	0.1	20	1.9	0.9	2.9	1.9	1.5	2.0	3.6	2.1	1.4
6	0.6	0.4	2.2	0.4	1.0	0.8	2.4	1.7	0.4	21	1.5	0.2	0.4	0.7	1.3	1.6	0.8	0.2	0.9
7	0.8	0.1	0.7	0.8	0.1	0.2	0.4	0.1	0.5	22	1.8	0.5	1.1	0.5	1.2	1.4	2.1	0.8	1.0
8	2.3	2.8	3.0	2.6	2.6	2.9	6.7	2.5	2.7	23	1.4	0.7	0.5	1.6	0.4	1.9	2.6	2.2	1.5
9	1.2	2.0	0.9	1.7	2.4	0.7	3.2	1.8	2.0	24	1.0	1.6	0.3	0.1	0.7	1.1	1.2	0.3	0.7
10	2.4	2.1	2.5	2.5	2.1	1.5	3.9	2.7	1.7	25	0.5	1.8	1.4	2.7	0.2	1.8	3.3	1.3	1.3
11	0.2	0.6	0.1	1.5	1.4	0.4	0.6	0.5	0.2	26	3.0	2.6	2.3	2.4	2.0	1.7	5.1	3.0	2.2
12	2.0	1.5	0.6	0.3	0.8	0.6	1.5	0.6	1.6	27	2.5	1.4	1.3	2.1	2.3	2.6	3.8	2.9	2.4
13	0.9	0.3	1.7	2.0	1.6	0.5	2.5	0.9	0.8	28	0.3	2.2	1.2	0.2	0.9	1.3	2.3	1.1	0.6
14	2.7	3.0	2.1	2.9	2.8	2.7	7.0	2.8	2.6	29	2.8	2.3	2.0	1.4	2.9	2.4	5.8	1.4	2.5
15	1.1	0.8	1.0	1.2	0.6	0.3	2.0	1.2	1.2	30	0.1	1.7	1.5	0.6	2.2	1.0	4.2	0.7	1.1

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Table-3.2. Relationship between Constrained Canonical Correlation and Representation Correlation between Canonical Variates and their Constituent Variables for Different Values of λ (Dataset in Table-3.1)

Sl No.	λ	Canonical $\lvert r(\varsigma_1,\varsigma_2)\rvert$	Mean Squared $r(\varsigma_1,x_1)$	Mean Squared $r(\varsigma_2,x_2)$	Sl No.	λ	Canonical $\lvert r(\varsigma_1,\varsigma_2)\rvert$	Mean Squared $r(\varsigma_1,x_1)$	Mean Squared $r(\varsigma_2,x_2)$
1	0.0	0.94813	0.63711	0.69854	26	25.0	0.92443	0.66125	0.75293
2	1.0	0.93901	0.65895	0.74485	27	26.0	0.92437	0.66125	0.75294
3	2.0	0.93449	0.66036	0.74955	28	27.0	0.92431	0.66125	0.75294
4	3.0	0.93199	0.66076	0.75106	29	28.0	0.92426	0.66125	0.75294
5	4.0	0.93039	0.66094	0.75175	30	29.0	0.92421	0.66125	0.75294
6	5.0	0.92927	0.66104	0.75212	31	30.0	0.92417	0.66126	0.75295
7	6.0	0.92844	0.66110	0.75235	32	31.0	0.92412	0.66126	0.75295
8	7.0	0.92780	0.66114	0.75250	33	32.0	0.92409	0.66126	0.75295
9	8.0	0.92729	0.66116	0.75260	34	33.0	0.92405	0.66126	0.75295
10	9.0	0.92687	0.66118	0.75267	35	34.0	0.92401	0.66126	0.75295
11	10.0	0.92653	0.66119	0.75272	36	35.0	0.92398	0.66126	0.75295
12	11.0	0.92624	0.66121	0.75276	37	36.0	0.92394	0.66126	0.75296
13	12.0	0.92598	0.66121	0.75279	38	37.0	0.92392	0.66126	0.75296
14	13.0	0.92577	0.66122	0.75282	39	38.0	0.92389	0.66126	0.75296
15	14.0	0.92558	0.66123	0.75284	40	39.0	0.92386	0.66126	0.75296
16	15.0	0.92541	0.66123	0.75286	41	40.0	0.92383	0.66126	0.75296
17	16.0	0.92526	0.66123	0.75287	42	41.0	0.92381	0.66126	0.75296
18	17.0	0.92513	0.66124	0.75288	43	42.0	0.92378	0.66126	0.75296
19	18.0	0.92501	0.66124	0.75289	44	43.0	0.92376	0.66126	0.75296
20	19.0	0.92490	0.66124	0.75290	45	44.0	0.92374	0.66126	0.75296
21	20.0	0.92481	0.66124	0.75291	46	45.0	0.92372	0.66126	0.75296
22	21.0	0.92472	0.66125	0.75291	47	46.0	0.92370	0.66126	0.75296
23	22.0	0.92464	0.66125	0.75292	48	47.0	0.92368	0.66126	0.75297
24	23.0	0.92455	0.66125	0.75292	49	48.0	0.92366	0.66126	0.75297
25	24.0	0.92450	0.66125	0.75293	50	49.0	0.92365	0.66126	0.75297

.

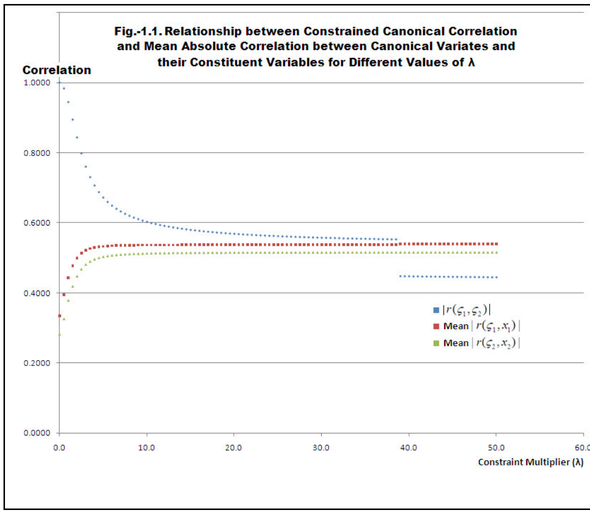

Fig.-1.1. Relationship between Constrained Canonical Correlation and Mean Absolute Correlation between Canonical Variates and their Constituent Variables for Different Values of λ

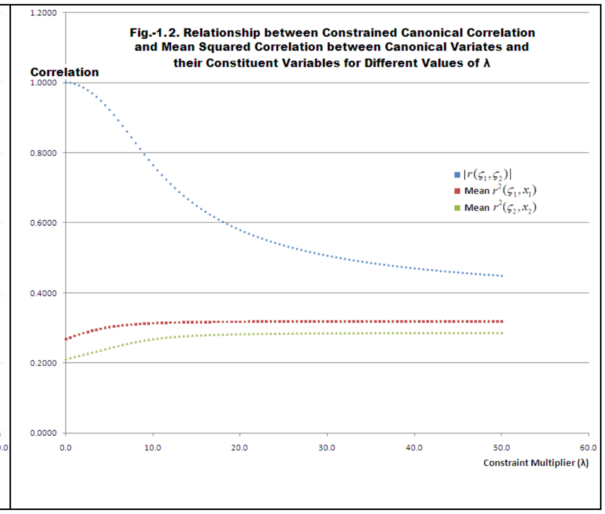

Fig.-1.2. Relationship between Constrained Canonical Correlation and Mean Squared Correlation between Canonical Variates and their Constituent Variables for Different Values of λ

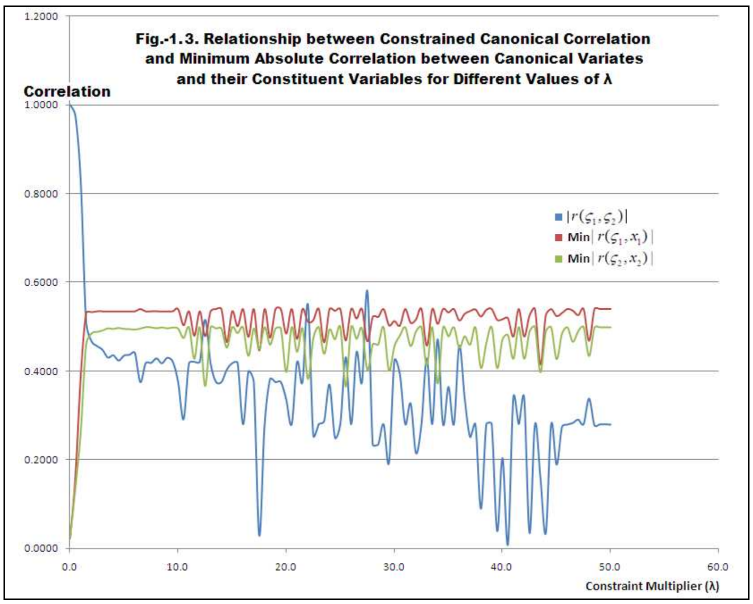

Fig.-1.3. Relationship between Constrained Canonical Correlation and Minimum Absolute Correlation between Canonical Variates and their Constituent Variables for Different Values of λ

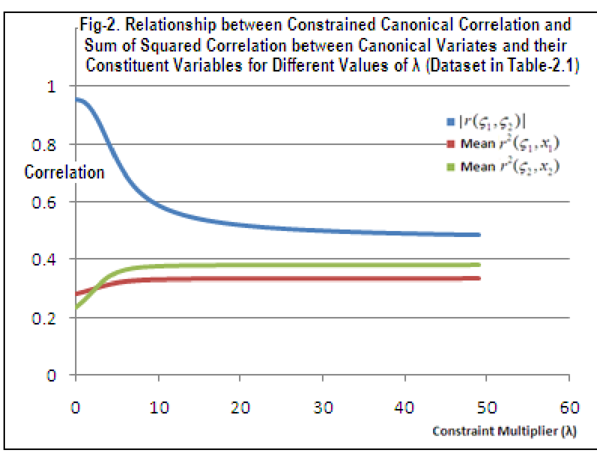

Fig-2. Relationship between Constrained Canonical Correlation and Sum of Squared Correlation between Canonical Variates and their Constituent Variables for Different Values of λ (Dataset in Table-2.1)

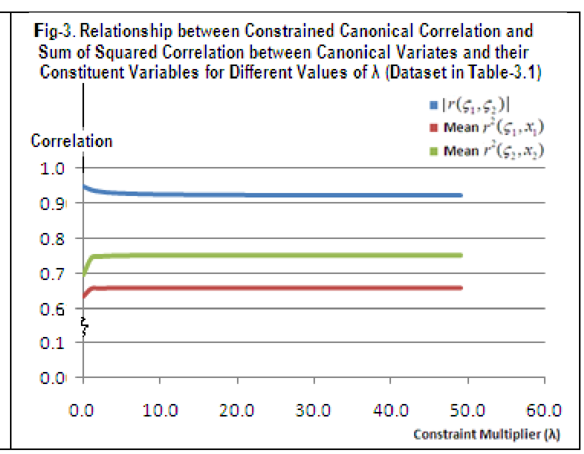

Fig-3. Relationship between Constrained Canonical Correlation and Sum of Squared Correlation between Canonical Variates and their Constituent Variables for Different Values of λ (Dataset in Table-3.1)

```fortran
C      !----------------- MAIN PROGRAM : RCCCA ----------------------
C          PROVIDES TO USE REPULSIVE PARTICLE SWARM METHOD TO OBTAIN
C      THE REPRESENTATION-CONSTRAINED CANONICAL CORRELATION & VARIATES
C      PRODUCT MOMENT AS WELL AS ABSOLUTE CORRELATION (BRADLEY, 1985) MAY
C      BE USED.
C                    PROGRAM BY SK MISHRA, DEPT. OF ECONOMICS,
C                  NORTH-EASTERN HILL UNIVERSITY, SHILLONG (INDIA)
C      -----------------------------------------------------------------
C                     ADJUST THE PARAMETERS SUITABLY
C              IN THIS MAIN PROGRAM AND IN THE SOBROUTINE CORD
C      WHEN THE PROGRAM ASKS FOR ANY OTHER PARAMETERS, FEED THEM SUITABLY
C      -----------------------------------------------------------------
       PROGRAM RCCCA
       PARAMETER(NOB=30,MVAR=9)!CHANGE THE PARAMETERS HERE AS NEEDED.
C      -----------------------------------------------------------------
C        NOB=NO. OF CASES AND MVAR=NO. OF VARIABLES IN ALL M= (M1+M2)
C        NOB AND MVAR TO BE ADJUSTED IN SUBROUTINE CORD(M,X,F) ALSO.
C       SET NRL TO DESIRED VALUE IN SUBROUTINE DORANK FOR RANKING SCHEME
C      -----------------------------------------------------------------
       IMPLICIT DOUBLE PRECISION (A-H, O-Z)
       COMMON /KFF/KF,NFCALL,FTIT ! FUNCTION CODE, NO. OF CALLS & TITLE
       CHARACTER *30  METHOD(1)
       CHARACTER *70 FTIT
       CHARACTER *40 INFILE,OUTFILE
       COMMON /CANON/MONE,MTWO
       COMMON /CORDAT/CDAT(NOB,MVAR),QIND1(NOB),QIND2(NOB),R(1),NORM,NCOR
       COMMON /XBASE/XBAS
       COMMON /RNDM/IU,IV ! RANDOM NUMBER GENERATION (IU = 4-DIGIT SEED)
       COMMON /GETRANK/MRNK
       COMMON /MRCCA/OWNR(2,MVAR),FROH,SOWNR1,SOWNR2,MRCC
       INTEGER IU,IV
       DIMENSION XX(3,50),KKF(3),MM(3),FMINN(3),XBAS(1000,50)
       DIMENSION ZDAT(NOB,MVAR+1),FRANK1(NOB),FRANK2(NOB),RMAT(2,2)
       DIMENSION X(50)! X IS THE DECISION VARIABLE X IN F(X) TO MINIMIZE
C        M = DIMENSION OF THE PROBLEM, KF(=1) = TEST FUNCTION CODE AND
C               FMIN IS THE MIN VALUE OF F(X) OBTAINED FROM RPS
       WRITE(*,*)'====================      WARNING     =============== '
       WRITE(*,*)'ADJUST PARAMETERS IN SUBROUTINES RPS IF NEEDED '
C      ------------------- OPTIMIZATION BY RPS METHOD -------------------
       NORM=2!WORKS WITH THE EUCLIDEAN NORM (IDENTICAL RESULTS IF NORM=1)
       NOPT=1 ! ONLY ONE FUNCTION IS OPTIMIZED
       WRITE(*,*)'=================================================== '
       METHOD(1)=' : REPULSIVE PARTICLE SWARM OPTIMIZATION'
C      INITIALIZE. THIS XBAS WILL BE USED TO INITIALIZE THE POPULATION.
       WRITE(*,*)' '
       WRITE(*,*)'---------- FEED RANDOM NUMBER SEED, AND NCOR ---------'
       WRITE(*,*)' '
       WRITE(*,*)'FEED SEED [ANY 4-DIGIT NUMBER] AND NCOR[0,1]'
       WRITE(*,*)'NCOR(0)=PRODUCT MOMENT; NCOR(1)=ABSOLUTE CORRELATION'
       WRITE(*,*)' '
     1 READ(*,*) IU,NCOR
       IF(NCOR.LT.0.OR.NCOR.GT.1) THEN
       WRITE(*,*)'SORRY. NCOR TAKES ON[0,1] ONLY. FEED SEED & NCOR AGAIN'
       GOTO 1
       ENDIF
       WRITE(*,*)'WANT RANK SCORE OPTIMIZATION? YES(1); NO(OTHER THAN 1)'
       READ(*,*) MRNK
       WRITE(*,*)'INPUT FILE TO READ DATA:YOUR DATA MUST BE IN THIS FILE'
       WRITE(*,*)'CASES (NOB) IN ROWS ; VARIABLES (MVAR) IN COLUMNS'
       READ(*,*) INFILE
       WRITE(*,*)'SPECIFY THE OUTPUT FILE TO STORE THE RESULTS'
       READ(*,*) OUTFILE
       OPEN(9, FILE=OUTFILE)
       OPEN(7,FILE=INFILE)
       DO I=1,NOB
       READ(7,*),CDA,(CDAT(I,J),J=1,MVAR)
       ENDDO
```

```
      CLOSE(7)
      DO I=1,NOB
      DO J=1,MVAR
      ZDAT(I,J+1)=CDAT(I,J)
      ENDDO
      ENDDO
      WRITE(*,*)'DATA HAS BEEN READ. WOULD YOU UNITIZE VARIABLES? [YES=1
     & ELSE NO UNITIZATION]'
      WRITE(*,*)'UNITIZE MEANS TRANSFORMATION FROM X(I,J) TO UNITIZED X'
      WRITE(*,*)'[X(I,J)-MIN(X(.,J))]/[MAX(X(.,J))-MIN(X(.,J))]'
      READ(*,*) NUN
      IF(NUN.EQ.1) THEN
      DO J=1,MVAR
      CMIN=CDAT(1,J)
      CMAX=CDAT(1,J)
      DO I=2,NOB
      IF(CMIN.GT.CDAT(I,J)) CMIN=CDAT(I,J)
      IF(CMAX.LT.CDAT(I,J)) CMAX=CDAT(I,J)
      ENDDO
      DO I=1,NOB
      CDAT(I,J)=(CDAT(I,J)-CMIN)/(CMAX-CMIN)
      ENDDO
      ENDDO
      ENDIF
C     -----------------------------------------------------------------
C     THIS XBAS WILL BE USED AS INITIAL X
      DO I=1,1000
      DO J=1,50
      CALL RANDOM(RAND)
      XBAS(I,J)=RAND ! RANDOM NUMBER BETWEEN (0, 1)
      ENDDO
      ENDDO
C     -----------------------------------------------------------------
      WRITE(*,*)' ****************************************************'
C     -----------------------------------------------------------------
      K=1
      WRITE(*,*)'PARTICLE SWARM PROGRAM TO OBTAIN CANONICAL CORRELATION'
      CALL RPS(M,X,FMINRPS,Q1) !CALLS RPS AND RETURNS OPTIMAL X AND FMIN
      WRITE(*,*)'RPS BRINGS THE FOLLOWING VALUES TO THE MAIN PROGRAM'
      WRITE(*,*)(X(JOPT),JOPT=1,M),' OPTIMUM FUNCTION=',FMINRPS
      IF(KF.EQ.1) THEN
      WRITE(9,*)'REPULSIVE PARTICLE SWARM OPTIMIZATION RESULTS'
      WRITE(9,*)'THE LARGEST CANONICAL R BETWEEN THE SETS OF VARIABLES'
      WRITE(9,*)' ABS(R)=',DABS(R(1)),'; SQUARE(R)=',R(1)**2
      IF(NCOR.EQ.0) THEN
      WRITE(*,*)'NOTE: THESE ARE KARL PEARSON TYPE CORRELATION (NCOR=0)'
      WRITE(9,*)'NOTE: THESE ARE KARL PEARSON TYPE CORRELATION (NCOR=0)'
      ELSE
      WRITE(*,*)'NOTE: THESE ARE BRADLEY TYPE CORRELATION (NCOR=1)'
      WRITE(9,*)'NOTE: THESE ARE BRADLEY TYPE CORRELATION (NCOR=1)'
      ENDIF
      WRITE(*,*)'_______________________________________________________'
      WRITE(9,*)'_______________________________________________________'

      DO II=1,NOB
      FRANK1(II)=QIND1(II)
      FRANK2(II)=QIND2(II)
      ENDDO
      ENDIF
      FMIN=FMINRPS
C     -----------------------------------------------------------------
      DO J=1,M
      XX(K,J)=X(J)
      ENDDO
      KKF(K)=KF
      MM(K)=M
      FMINN(K)=FMIN
```

```
      WRITE(*,*)' '
      WRITE(*,*)' '
      WRITE(*,*)'---------------------- FINAL RESULTS=================='
      WRITE(*,*)'FUNCT CODE=',KKF(K),'  FMIN=',FMINN(K),' : DIM=',MM(K)
      WRITE(*,*)'OPTIMAL DECISION VARIABLES : ',METHOD(K)
      WRITE(*,*)'FOR THE FIRST SET OF VARIABLES WEIGHTS ARE AS FOLLOWS'
      WRITE(9,*)'FOR THE FIRST SET OF VARIABLES WEIGHTS ARE AS FOLLOWS'
      WRITE(9,*)(XX(K,J),J=1,MONE)
      WRITE(*,*)(XX(K,J),J=1,MONE)
      WRITE(*,*)'FOR THE SECOND SET OF VARIABLES WEIGHTS ARE AS FOLLOWS'
      WRITE(9,*)'FOR THE SECOND SET OF VARIABLES WEIGHTS ARE AS FOLLOWS'
      WRITE(9,*)(XX(K,J),J=MONE+1,M)
      WRITE(*,*)(XX(K,J),J=MONE+1,M)
      WRITE(9,*)'CANONICAL R=',FROH,' OWN CORRELATIONS=',SOWNR1,SOWNR2
      WRITE(*,*)'CANONICAL R=',FROH,' OWN CORRELATIONS=',SOWNR1,SOWNR2
      WRITE(*,*)'//////////////////////////////////////////////////////'
      WRITE(*,*)'OPTIMIZATION PROGRAM ENDED'
      WRITE(*,*)'******************************************************'
      WRITE(*,*)'MEASURE OF EQUALITY/INEQUALITY'
      WRITE(*,*)'RPS: BEFORE AND AFTER OPTIMIZATION = ',Q0,Q1
      WRITE(*,*)' '
      WRITE(*,*)'RESULTS STORED IN FILE= ',OUTFILE
      WRITE(*,*)'OPEN BY MSWORD OR EDIT OR ANY OTHER EDITOR'
      WRITE(*,*)' '
      WRITE(*,*)'NOTE:VECTORS OF CORRELATIONS & INDEX(BOTH TOGETHER) ARE
     & IDETERMINATE FOR SIGN &   MAY BE MULTIPLED BY (-1) IF NEEDED'
      WRITE(*,*)'THAT IS IF R(J) IS TRANSFORMED TO -R(J) FOR ALL J THEN
     &THE INDEX(I) TOO IS         TRANSFORMED TO -INDEX(I) FOR ALL I'
      WRITE(9,*)' '
      WRITE(9,*)'NOTE: VECTORS OF CORRELATIONS AND INDEX (BOTH TOGETHER)
     & ARE IDETERMINATE FOR SIGN AND MAY BE MULTIPLED BY (-1) IF NEEDED'
      WRITE(9,*)'THAT IS IF R(J) IS TRANSFORMED TO -R(J) FOR ALL J THEN
     &THE INDEX(I) TOO IS TRANSFORMED TO -INDEX(I) FOR ALL I'
      CALL DORANK(FRANK1,NOB)
      CALL DORANK(FRANK2,NOB)
      DO I=1,NOB
      ZDAT(I,1)=FRANK1(I)
      ZDAT(I,2)=FRANK2(I)
      ENDDO
      IF(NCOR.EQ.0) THEN
      CALL CORREL(ZDAT,NOB,2,RMAT)
      ELSE
      CALL DOCORA(ZDAT,NOB,2,RMAT)
      ENDIF
      WRITE(9,*)'==================================================== '
      WRITE(*,*)'==================================================== '
      WRITE(9,*)'1ST 2 ARE CANONICAL SCORES AND LAST 2 ARE THEIR RANK'
      WRITE(*,*)'1ST 2 ARE CANONICAL SCORES AND LAST 2 ARE THEIR RANK'
      WRITE(9,*)'==================================================== '
      WRITE(*,*)'==================================================== '
      DO I=1,NOB
      IF(MRNK.EQ.1) THEN
      QIND1(I)=0.D0
      QIND2(I)=0.D0
      DO J=1,MONE
      QIND1(I)=QIND1(I)+ZDAT(I,J+1)*XX(NOPT,J)
      ENDDO
      DO J=MONE+1,MVAR
      QIND2(I)=QIND2(I)+ZDAT(I,J+1)*XX(NOPT,J)
      ENDDO
      ENDIF
      WRITE(9,2)I,QIND1(I),QIND2(I),(ZDAT(I,J),J=1,2)
      WRITE(*,2)I,QIND1(I),QIND2(I),(ZDAT(I,J),J=1,2)
      ENDDO
    2 FORMAT(I5,2F15.6,2F10.3)
      WRITE(9,*)'SQUARE OF CANONICAL CORRELATION =',RMAT(1,2)**2
      WRITE(*,*)'SQUARE OF CANONICAL CORRELATION =',RMAT(1,2)**2
```

```
      WRITE(9,*)'ABSOLUTE OF CANONICAL CORRELATION =',DABS(RMAT(1,2))
      WRITE(*,*)'ABSOLUTE OF CANONICAL CORRELATION =',DABS(RMAT(1,2))
      IF(NCOR.EQ.0) THEN
      WRITE(*,*)'NOTE: THESE ARE KARL PEARSON TYPE CORRELATION (NCOR=0)'
      WRITE(9,*)'NOTE: THESE ARE KARL PEARSON TYPE CORRELATION (NCOR=0)'
      ELSE
      WRITE(*,*)'NOTE: THESE ARE BRADLEY TYPE CORRELATION (NCOR=1)'
      WRITE(9,*)'NOTE: THESE ARE BRADLEY TYPE CORRELATION (NCOR=1)'
      ENDIF
      IF(MRCC.NE.0) THEN
      WRITE(9,*)'THE REPRESENTATION CORRELATIONS OF THE TWO SETS ARE:'
      WRITE(9,*) (OWNR(1,J),J=1,MONE)
      WRITE(9,*) (OWNR(2,J),J=1,MTWO)
      WRITE(*,*)'THE REPRESENTATION CORRELATIONS OF THE TWO SETS ARE:'
      WRITE(*,*) (OWNR(1,J),J=1,MONE)
      WRITE(*,*) (OWNR(2,J),J=1,MTWO)
      WRITE(9,*)'CANONICAL R=',FROH,' OWN CORRELATIONS=',SOWNR1,SOWNR2
      WRITE(*,*)'CANONICAL R=',FROH,' OWN CORRELATIONS=',SOWNR1,SOWNR2
      ENDIF
      CLOSE(9)
      WRITE(*,*) 'THE JOB IS OVER'
      END
C     -----------------------------------------------------------------
      SUBROUTINE RPS(M,ABEST,FBEST,G1)
C     PROGRAM TO FIND GLOBAL MINIMUM BY REPULSIVE PARTICLE SWARM METHOD
C     WRITTEN BY SK MISHRA, DEPT. OF ECONOMICS, NEHU, SHILLONG (INDIA)
C     -----------------------------------------------------------------
      PARAMETER (N=100,NN=20,MX=100,NSTEP=11,ITRN=10000,NSIGMA=1,ITOP=1)
      PARAMETER (NPRN=50) ! DISPLAYS RESULTS AT EVERY 500 TH ITERATION
C     PARAMETER(N=50,NN=25,MX=100,NSTEP=9,ITRN=10000,NSIGMA=1,ITOP=3)
C     PARAMETER (N=100,NN=15,MX=100,NSTEP=9,ITRN=10000,NSIGMA=1,ITOP=3)
C     IN CERTAIN CASES THE ONE OR THE OTHER SPECIFICATION WORKS BETTER
C     DIFFERENT SPECIFICATIONS OF PARAMETERS MAY SUIT DIFFERENT TYPES
C     OF FUNCTIONS OR DIMENSIONS - ONE HAS TO DO SOME TRIAL AND ERROR
C     -----------------------------------------------------------------
C     N = POPULATION SIZE. IN MOST OF THE CASES N=30 IS OK. ITS VALUE
C     MAY BE INCREASED TO 50 OR 100 TOO. THE PARAMETER NN IS THE SIZE OF
C     RANDOMLY CHOSEN NEIGHBOURS. 15 TO 25 (BUT SUFFICIENTLY LESS THAN
C     N) IS A GOOD CHOICE. MX IS THE MAXIMAL SIZE OF DECISION VARIABLES.
C     IN F(X1, X2,...,XM) M SHOULD BE LESS THAN OR EQUAL TO MX. ITRN IS
C     THE NO. OF ITERATIONS. IT MAY DEPEND ON THE PROBLEM. 200(AT LEAST)
C     TO 500 ITERATIONS MAY BE GOOD ENOUGH. BUT FOR FUNCTIONS LIKE
C     ROSENBROCKOR GRIEWANK OF LARGE SIZE (SAY M=30) IT IS NEEDED THAT
C     ITRN IS LARGE, SAY 5000 OR EVEN 10000.
C     SIGMA INTRODUCES PERTURBATION & HELPS THE SEARCH JUMP OUT OF LOCAL
C     OPTIMA. FOR EXAMPLE : RASTRIGIN FUNCTION OF DMENSION 30 OR LARGER
C     NSTEP DOES LOCAL SEARCH BY TUNNELLING AND WORKS WELL BETWEEN 5 AND
C     15, WHICH IS MUCH ON THE HIGHER SIDE.
C     ITOP <=1 (RING); ITOP=2 (RING AND RANDOM); ITOP=>3 (RANDOM)
C     NSIGMA=0 (NO CHAOTIC PERTURBATION);NSIGMA=1 (CHAOTIC PERTURBATION)
C     NOTE THAT NSIGMA=1 NEED NOT ALWAYS WORK BETTER (OR WORSE)
C     SUBROUTINE FUNC( ) DEFINES OR CALLS THE FUNCTION TO BE OPTIMIZED.
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /RNDM/IU,IV
      COMMON /KFF/KF,NFCALL,FTIT
      INTEGER IU,IV
      CHARACTER *70 FTIT
      DIMENSION X(N,MX),V(N,MX),A(MX),VI(MX),TIT(50),ABEST(*)
      DIMENSION XX(N,MX),F(N),V1(MX),V2(MX),V3(MX),V4(MX),BST(MX)
C     A1 A2 AND A3 ARE CONSTANTS AND W IS THE INERTIA WEIGHT.
C     OCCASIONALLY, TINKERING WITH THESE VALUES, ESPECIALLY A3, MAY BE
C     NEEDED.
      DATA A1,A2,A3,W,SIGMA /.5D00,.5D00,.0005D00,.5D00,1.D-03/
      EPSILON=1.D-12 ! ACCURACY NEEDED FOR TERMINATON
C     --------------------CHOOSING THE TEST FUNCTION -------------------'
      CALL FSELECT(KF,M,FTIT)
C     -----------------------------------------------------------------
```

```fortran
      FFMIN=1.D30
      LCOUNT=0
      NFCALL=0
      WRITE(*,*)'4-DIGITS SEED FOR RANDOM NUMBER GENERATION'
      READ(*,*) IU
      DATA FMIN /1.0E30/
C     GENERATE N-SIZE POPULATION OF M-TUPLE PARAMETERS X(I,J) RANDOMLY
      DO I=1,N
        DO J=1,M
        CALL RANDOM(RAND)
         X(I,J)=RAND
C     WE GENERATE RANDOM(-5,5). HERE MULTIPLIER IS 10. TINKERING IN SOME
C     CASES MAY BE NEEDED
         ENDDO
        F(I)=1.0D30
      ENDDO
C     INITIALISE VELOCITIES V(I) FOR EACH INDIVIDUAL IN THE POPULATION
      DO I=1,N
      DO J=1,M
      CALL RANDOM(RAND)
       V(I,J)=(RAND-0.5D+00)
C       V(I,J)=RAND
      ENDDO
      ENDDO
      DO 100 ITER=1,ITRN
C     WRITE(*,*)'ITERATION=',ITER
C     LET EACH INDIVIDUAL SEARCH FOR THE BEST IN ITS NEIGHBOURHOOD
        DO I=1,N
           DO J=1,M
           A(J)=X(I,J)
           VI(J)=V(I,J)
           ENDDO
           CALL LSRCH(A,M,VI,NSTEP,FI)
           IF(FI.LT.F(I)) THEN
            F(I)=FI
            DO IN=1,M
            BST(IN)=A(IN)
            ENDDO
C     F(I) CONTAINS THE LOCAL BEST VALUE OF FUNCTION FOR ITH INDIVIDUAL
C     XX(I,J) IS THE M-TUPLE VALUE OF X ASSOCIATED WITH LOCAL BEST F(I)
             DO J=1,M
             XX(I,J)=A(J)
             ENDDO
             ENDIF
        ENDDO
C      NOW LET EVERY INDIVIDUAL RANDOMLY COSULT NN(<<N) COLLEAGUES AND
C      FIND THE BEST AMONG THEM
      DO I=1,N
C     ------------------------------------------------------------------
      IF(ITOP.GE.3) THEN
C     RANDOM TOPOLOGY ******************************************
C     CHOOSE NN COLLEAGUES RANDOMLY AND FIND THE BEST AMONG THEM
          BEST=1.0D30
           DO II=1,NN
                 CALL RANDOM(RAND)
               NF=INT(RAND*N)+1
                IF(BEST.GT.F(NF)) THEN
                 BEST=F(NF)
                NFBEST=NF
                 ENDIF
            ENDDO
      ENDIF
C-----------------------------------------------------------------------
      IF(ITOP.EQ.2) THEN
C     RING + RANDOM TOPOLOGY ******************************************
C     REQUIRES THAT THE SUBROUTINE NEIGHBOR IS TURNED ALIVE
       BEST=1.0D30
```

```fortran
          CALL NEIGHBOR(I,N,I1,I3)
          DO II=1,NN
                IF(II.EQ.1) NF=I1
                 IF(II.EQ.2) NF=I
                  IF(II.EQ.3) NF=I3
                      IF(II.GT.3) THEN
                     CALL RANDOM(RAND)
                      NF=INT(RAND*N)+1
                     ENDIF
                  IF(BEST.GT.F(NF)) THEN
                  BEST=F(NF)
                  NFBEST=NF
                 ENDIF
             ENDDO
       ENDIF
C---------------------------------------------------------------------
      IF(ITOP.LE.1) THEN
C     RING TOPOLOGY *************************************************
C     REQUIRES THAT THE SUBROUTINE NEIGHBOR IS TURNED ALIVE
        BEST=1.0D30
           CALL NEIGHBOR(I,N,I1,I3)
              DO II=1,3
              IF (II.NE.I) THEN
             IF(II.EQ.1) NF=I1
              IF(II.EQ.3) NF=I3
                  IF(BEST.GT.F(NF)) THEN
                   BEST=F(NF)
                   NFBEST=NF
                  ENDIF
                  ENDIF
            ENDDO
       ENDIF
C---------------------------------------------------------------------
C     IN THE LIGHT OF HIS OWN AND HIS BEST COLLEAGUES EXPERIENCE, THE
C     INDIVIDUAL I WILL MODIFY HIS MOVE AS PER THE FOLLOWING CRITERION
C     FIRST, ADJUSTMENT BASED ON ONES OWN EXPERIENCE
C     AND OWN BEST EXPERIENCE IN THE PAST (XX(I))
           DO J=1,M
           CALL RANDOM(RAND)
           V1(J)=A1*RAND*(XX(I,J)-X(I,J))

C     THEN BASED ON THE OTHER COLLEAGUES BEST EXPERIENCE WITH WEIGHT W
C     HERE W IS CALLED AN INERTIA WEIGHT 0.01< W < 0.7
C     A2 IS THE CONSTANT NEAR BUT LESS THAN UNITY
           CALL RANDOM(RAND)
           V2(J)=V(I,J)
           IF(F(NFBEST).LT.F(I)) THEN
           V2(J)=A2*W*RAND*(XX(NFBEST,J)-X(I,J))
           ENDIF
C     THEN SOME RANDOMNESS AND A CONSTANT A3 CLOSE TO BUT LESS THAN UNITY
           CALL RANDOM(RAND)
           RND1=RAND
           CALL RANDOM(RAND)
            V3(J)=A3*RAND*W*RND1
C            V3(J)=A3*RAND*W
C     THEN ON PAST VELOCITY WITH INERTIA WEIGHT W
           V4(J)=W*V(I,J)
C     FINALLY A SUM OF THEM
           V(I,J)= V1(J)+V2(J)+V3(J)+V4(J)
           ENDDO
      ENDDO
C     CHANGE X
      DO I=1,N
      DO J=1,M
      RANDS=0.D00
C     -----------------------------------------------------------------
      IF(NSIGMA.EQ.1) THEN
```

```
         CALL RANDOM(RAND) ! FOR CHAOTIC PERTURBATION
         IF(DABS(RAND-.5D00).LT.SIGMA) RANDS=RAND-0.5D00
C        SIGMA CONDITIONED RANDS INTRODUCES CHAOTIC ELEMENT IN TO LOCATION
C        IN SOME CASES THIS PERTURBATION HAS WORKED VERY EFFECTIVELY WITH
C        PARAMETER (N=100,NN=15,MX=100,NSTEP=9,ITRN=100000,NSIGMA=1,ITOP=2)
        ENDIF
C       -----------------------------------------------------------------
        X(I,J)=X(I,J)+V(I,J)*(1.D00+RANDS)
        ENDDO
        ENDDO
         DO I=1,N
           IF(F(I).LT.FMIN) THEN
           FMIN=F(I)
           II=I
           DO J=1,M
           BST(J)=XX(II,J)
           ENDDO
           ENDIF
           ENDDO

        IF(LCOUNT.EQ.NPRN) THEN
        LCOUNT=0
        WRITE(*,*)'OPTIMAL SOLUTION UPTO THIS (FUNCTION CALLS=',NFCALL,')'
        WRITE(*,*)'X = ',(BST(J),J=1,M),' MIN F = ',FMIN
C        WRITE(*,*)'NO. OF FUNCTION CALLS = ',NFCALL
        DO J=1,M
        ABEST(J)=BST(J)
        ENDDO
        IF(DABS(FFMIN-FMIN).LT.EPSILON) GOTO 999
        FFMIN=FMIN
        ENDIF
        LCOUNT=LCOUNT+1
  100 CONTINUE
  999 WRITE(*,*)'------------------------------------------------------'
        DO I=1,N
        IF(F(I).LT.FBEST) THEN
        FBEST=F(I)
        DO J=1,M
        ABEST(J)=XX(I,J)
        ENDDO
        ENDIF
        ENDDO
        CALL FUNC(ABEST,M,FBEST)
        CALL GINI(F,N,G1)
        WRITE(*,*)'FINAL X = ',(BST(J),J=1,M),' FINAL MIN F = ',FMIN
        WRITE(*,*)'COMPUTATION OVER:FOR ',FTIT
        WRITE(*,*)'NO. OF VARIABLES=',M,'  END.'
        RETURN
        END
C       -----------------------------------------------------------------
        SUBROUTINE LSRCH(A,M,VI,NSTEP,FI)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        CHARACTER *70 FTIT
        COMMON /KFF/KF,NFCALL,FTIT
        COMMON /RNDM/IU,IV
        INTEGER IU,IV
        DIMENSION A(*),B(100),VI(*)
        AMN=1.0D30
        DO J=1,NSTEP
           DO JJ=1,M
           B(JJ)=A(JJ)+(J-(NSTEP/2)-1)*VI(JJ)
           ENDDO
        CALL FUNC(B,M,FI)
          IF(FI.LT.AMN) THEN
          AMN=FI
          DO JJ=1,M
          A(JJ)=B(JJ)
```

```
          ENDDO
          ENDIF
        ENDDO
        FI=AMN
        RETURN
        END
C       ----------------------------------------------------------------
C       THIS SUBROUTINE IS NEEDED IF THE NEIGHBOURHOOD HAS RING TOPOLOGY
C       EITHER PURE OR HYBRIDIZED
         SUBROUTINE NEIGHBOR(I,N,J,K)
         IF(I-1.GE.1 .AND. I.LT.N) THEN
         J=I-1
         K=I+1
         ELSE
         IF(I-1.LT.1) THEN
         J=N-I+1
         K=I+1
         ENDIF
         IF(I.EQ.N) THEN
         J=I-1
         K=1
         ENDIF
         ENDIF
         RETURN
         END
C       ----------------------------------------------------------------
C       RANDOM NUMBER GENERATOR (UNIFORM BETWEEN 0 AND 1 - BOTH EXCLUSIVE)
        SUBROUTINE RANDOM(RAND1)
         DOUBLE PRECISION  RAND1
         COMMON /RNDM/IU,IV
        INTEGER IU,IV
         IV=IU*65539
         IF(IV.LT.0) THEN
         IV=IV+2147483647+1
         ENDIF
         RAND=IV
         IU=IV
         RAND=RAND*0.4656613E-09
         RAND1= DBLE(RAND)
         RETURN
         END
C       ----------------------------------------------------------------
        SUBROUTINE GINI(F,N,G)
        PARAMETER (K=1) !K=1 GINI COEFFICENT; K=2 COEFFICIENT OF VARIATION
C       THIS PROGRAM COMPUTES MEASURE OF INEQUALITY
C       IF K =1 GET THE GINI COEFFICIENT. IF K=2 GET COEFF OF VARIATIONE
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DIMENSION F(*)
        S=0.D0
        DO I=1,N
        S=S+F(I)
        ENDDO
        S=S/N
        H=0.D00
        DO I=1,N-1
        DO J=I+1,N
        H=H+(DABS(F(I)-F(J)))**K
        ENDDO
        ENDDO
        H=(H/(N**2))**(1.D0/K)! FOR K=1 H IS MEAN DEVIATION;
C                                FOR K=2 H IS STANDARD DEVIATION
        WRITE(*,*)'MEASURES OF DISPERSION AND CENTRAL TENDENCY = ',G,S
        G=DEXP(-H)! G IS THE MEASURE OF EQUALITY (NOT GINI OR CV)
C       G=H/DABS(S) !IF S NOT ZERO, K=1 THEN G=GINI, K=2 G=COEFF VARIATION
        RETURN
        END
C       ----------------------------------------------------------------
```

```
      SUBROUTINE FSELECT(KF,M,FTIT)
      COMMON /CANON/MONE,MTWO
C      THE PROGRAM REQUIRES INPUTS FROM THE USER ON THE FOLLOWING ------
C     (1) FUNCTION CODE (KF), (2) NO. OF VARIABLES IN THE FUNCTION (M);
      CHARACTER *70 TIT(100),FTIT
      NFN=1
      KF=1
      WRITE(*,*)'---------------------------------------------------'
      DATA TIT(1)/'COMPUTE CANONICAL CORRELATION FROM 2 DATA SETS'/
C     -----------------------------------------------------------------
      DO I=1,NFN
      WRITE(*,*)TIT(I)
      ENDDO
      WRITE(*,*)'---------------------------------------------------'
      WRITE(*,*)'SPECIFY NO. OF VARIABLES IN SET-1[=M1] AND SET-2[=M2]'
      READ(*,*) MONE, MTWO
      M=MONE+MTWO
      FTIT=TIT(KF) ! STORE THE NAME OF THE CHOSEN FUNCTION IN FTIT
      RETURN
      END
C     -----------------------------------------------------------------
      SUBROUTINE FUNC(X,M,F)
C     TEST FUNCTIONS FOR GLOBAL OPTIMIZATION PROGRAM
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /RNDM/IU,IV
      COMMON /KFF/KF,NFCALL,FTIT
      INTEGER IU,IV
      DIMENSION X(*)
      CHARACTER *70 FTIT
      NFCALL=NFCALL+1 ! INCREMENT TO NUMBER OF FUNCTION CALLS
C     KF IS THE CODE OF THE TEST FUNCTION
      IF(KF.EQ.1) THEN
      CALL CORD(M,X,F)
      RETURN
      ENDIF
C     =================================================================
      WRITE(*,*)'FUNCTION NOT DEFINED. PROGRAM ABORTED'
      STOP
      END
C     ------------------------------------------------------------------
      SUBROUTINE CORD(M,X,F)
      PARAMETER (NOB=30,MVAR=9)! CHANGE THE PARAMETERS HERE AS NEEDED.
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER (MU=2,NF=1,ALAMBDA=5.0D0)!ALAMBDA BETWEEN[0, INDEFINITE)
C     MU=1 ABSOLUTE CANONICAL R ; MU=2 SQUARED CANONICAL R;
C     NF=1 SUM OF OWN FELLOW ABSOLUTE CORRELATION
C     NF=2 SUM OF OWN FELLOW SQUARED CORRELATION
C     NF=3 MINIMUM OF OWN FELLOW ABSOLUTE (OR SQUARED) CORRELATION
C     ------------------------------------------------------------------
C     NOB=NO. OF OBSERVATIONS (CASES) & MVAR= NO. OF VARIABLES
      COMMON /CANON/MONE,MTWO
      COMMON /RNDM/IU,IV
      COMMON /CORDAT/CDAT(NOB,MVAR),QIND1(NOB),QIND2(NOB),R(1),NORM,NCOR
      COMMON /GETRANK/MRNK
      COMMON /MRCCA/OWNR(2,MVAR),FROH,SOWNR1,SOWNR2,MRCC
      INTEGER IU,IV
      DIMENSION X(*),Z(NOB,2)
      MRCC=1 ! FOR CLASSICAL=0; FOR MOST REPRESENTATIVE=1
      DO I=1,M
      IF(X(I).LT.-1.0D0.OR.X(I).GT.1.0D0) THEN
      CALL RANDOM(RAND)
      X(I)=(RAND-0.5D0)*2
      ENDIF
      ENDDO
C     NORMALIZATION OF WEIGHTS
      XNORM=0.D0
      DO J=1,M
```

```
      XNORM=XNORM+X(J)**2
      ENDDO
      XNORM=DSQRT(XNORM)
      DO J=1,M
      X(J)=X(J)/XNORM
      ENDDO
C     CONSTRUCT INDEX
      DO I=1,NOB
      QIND1(I)=0.D0
      QIND2(I)=0.D0
      DO J=1,MONE
      QIND1(I)=QIND1(I)+CDAT(I,J)*X(J)
      ENDDO
      DO J=MONE+1,M
      QIND2(I)=QIND2(I)+CDAT(I,J)*X(J)
      ENDDO
      ENDDO

C     -----------------------------------------------------------------
C      !FIND THE RANK OF QIND
      IF(MRNK.EQ.1) THEN
      CALL DORANK(QIND1,NOB)
      CALL DORANK(QIND2,NOB)
      ENDIF
C     -----------------------------------------------------------------
C     FIND CORRELATION OF QIND1 WITH ITS OWN MEMBER VARIABLES
      SOWNR1=0.D0
      SOWNR2=0.D0
      IF(MRCC.NE.0) THEN
        DO J=1,MONE
        DO I=1,NOB
        Z(I,1)=QIND1(I)
        Z(I,2)=CDAT(I,J)
        ENDDO
        CALL CORLN(Z,NOB,RHO)
        OWNR(1,J)=RHO
        ENDDO
C     FIND CORRELATION OF QIND2 WITH ITS OWN MEMBER VARIABLES
        DO J=MONE+1,M
        DO I=1,NOB
        Z(I,1)=QIND2(I)
        Z(I,2)=CDAT(I,J)
        ENDDO
        CALL CORLN(Z,NOB,RHO)
        OWNR(2,J-MONE)=RHO
        ENDDO
C     FORMULATION OF CONSTRAINTS
          DO J=1,MONE
          IF(NF.EQ.1) SOWNR1=SOWNR1+DABS(OWNR(1,J))
          IF(NF.EQ.2) SOWNR1=SOWNR1+OWNR(1,J)**2
          ENDDO
          SOWNR1=SOWNR1/MONE
          IF(NF.EQ.3) THEN
          SOWNR1=1.D0
          DO J=1,MONE
          IF(SOWNR1.GT.DABS(OWNR(1,J))) SOWNR1=DABS(OWNR(1,J))
          ENDDO
          ENDIF

          DO J=1,MTWO
          IF(NF.EQ.1) SOWNR2=SOWNR2+DABS(OWNR(2,J))
          IF(NF.EQ.2) SOWNR2=SOWNR2+OWNR(2,J)**2
          ENDDO
          SOWNR2=SOWNR2/MTWO
          IF(NF.EQ.3) THEN
          SOWNR2=1.D0
          DO J=1,MTWO
```

```
          IF(SOWNR2.GT.DABS(OWNR(2,J))) SOWNR2=DABS(OWNR(2,J))
          ENDDO
          ENDIF
      ENDIF
C     ----------------------------------------------------------------
C     COMPUTE CORRELATIONS
        DO I=1,NOB
        Z(I,1)=QIND1(I)
        Z(I,2)=QIND2(I)
        ENDDO
        IF(NCOR.EQ.0) THEN
        CALL CORLN(Z,NOB,RHO)
        ELSE
        CALL CORA(Z,NOB,RHO)
        ENDIF
        R(1)=RHO
        FROH=DABS(R(1))
      F=FROH**MU+ALAMBDA*(SOWNR1+SOWNR2)
C     -----------------------------------------------------------------
      F=-F
      RETURN
      END
      SUBROUTINE CORLN(Z,NOB,RHO)
C     NOB = NO. OF CASES
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION Z(NOB,2),AV(2),SD(2)
      DO J=1,2
      AV(J)=0.D0
      SD(J)=0.D0
      DO I=1,NOB
      AV(J)=AV(J)+Z(I,J)
      SD(J)=SD(J)+Z(I,J)**2
      ENDDO
      ENDDO
      DO J=1,2
      AV(J)=AV(J)/NOB
      SD(J)=DSQRT(SD(J)/NOB-AV(J)**2)
      ENDDO
C      WRITE(*,*)'AV AND SD ', AV(1),AV(2),SD(1),SD(2)
      RHO=0.D0
      DO I=1,NOB
      RHO=RHO+(Z(I,1)-AV(1))*(Z(I,2)-AV(2))
      ENDDO
      RHO=(RHO/NOB)/(SD(1)*SD(2))
      RETURN
      END
C     ----------------------------------------------------------------
      SUBROUTINE CORA(Z,N,R)
C     COMPUTING BRADLEY'S ABSOLUTE CORRELATION MATRIX
C     BRADLEY, C. (1985) "THE ABSOLUTE CORRELATION", THE MATHEMATICAL
C     GAZETTE, 69(447): 12-17.
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION Z(N,2),X(N),Y(N)
C     -----------------------------------------------------------------
C     PUT Z INTO X AND Y
      DO I=1,N
      X(I)=Z(I,1)
      Y(I)=Z(I,2)
      ENDDO
C     ARRANGE X ANY IN AN ASCENDING ORDER
      DO I=1,N-1
      DO II=I+1,N
      IF(X(I).GT.X(II)) THEN
      TEMP=X(I)
      X(I)=X(II)
      X(II)=TEMP
      ENDIF
```

```
      IF(Y(I).GT.Y(II)) THEN
      TEMP=Y(I)
      Y(I)=Y(II)
      Y(II)=TEMP
      ENDIF
      ENDDO
      ENDDO
C     FIND MEDIAN
      IF(INT(N/2).EQ.N/2.D0) THEN
      XMED=(X(N/2)+X(N/2+1))/2.D0
      YMED=(Y(N/2)+Y(N/2+1))/2.D0
      ENDIF
      IF(INT(N/2).NE.N/2.D0) THEN
      XMED=X(N/2+1)
      YMED=Y(N/2+1)
      ENDIF
C     SUBTRACT RESPECTIVE MEDIANS FROM X AND Y AND FIND ABS DEVIATIONS
      VX=0.D0
      VY=0.D0
      DO I=1,N
      X(I)=X(I)-XMED
      Y(I)=Y(I)-YMED
      VX=VX+DABS(X(I))
      VY=VY+DABS(Y(I))
      ENDDO
C     SCALE THE VARIABLES X AND Y SUCH THAT VX=VY
      IF(VX.EQ.0.D0.OR.VY.EQ.0.D0) THEN
      R=0.D0
      RETURN
      ENDIF
      DO I=1,N
      X(I)=X(I)*VY/VX
      ENDDO
C     COMPUTE CORRELATION COEFFICIENT
      VZ=0.D0
      R=0.D0
      DO I=1,N
      VZ=VZ+DABS(X(I))+DABS(Y(I))
      R=R+DABS(X(I)+Y(I))-DABS(X(I)-Y(I))
      ENDDO
      R=R/VZ
      RETURN
      END
C     ----------------------------------------------------------------
      SUBROUTINE DORANK(X,N)! N IS THE NUMBER OF OBSERVATIONS
      PARAMETER (NRL=0) ! THIS VALUE IS TO BE SET BY THE USER
C                         !THE VALUE OF NRL DECIDES THE SCHEME OF RANKINGS
C     !THIS PROGRAM RETURNS RANK-ORDER OF A GIVEN VECTOR
      PARAMETER (MXD=1000)! MXD IS MAX DIMENSION FOR TEMPORARY VARIABLES
      ! THAT ARE LOCAL AND DO NOT GO TO THE INVOKING PROGRAM
      ! X IS THE VARIABLE TO BE SUBSTITUTED BY ITS RANK VALUES
C     NRULE=0 FOR ORDINAL RANKING (1-2-3-4 RULE);
C     NRULE=1 FOR DENSE RANKING (1-2-2-3 RULE);
C     NRULE=2 FOR STANDARD COMPETITION RANKING (1-2-2-4 RULE);
C     NRULE=3 FOR MODIFIED COMPETITION RANKING (1-3-3-4 RULE);
C     NRULE=4 FOR FRACTIONAL RANKING (1-2.5-2.5-4 RULE);
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION X(N),NF(MXD),NCF(MXD),RANK(MXD),ID(MXD),XX(MXD)
C     GENERATE ID(I),I=1,N
      DO I=1,N
      ID(I)=I
      NF(I)=0
      ENDDO
C     ARRANGE DATA (X) AND THE IDS IN ASCENDING ORDER
      DO I=1,N-1
      DO II=I,N
      IF(X(II).LT.X(I)) THEN
```

```fortran
      TEMP=X(I)
      X(I)=X(II)
      X(II)=TEMP
      ITEMP=ID(I)
      ID(I)=ID(II)
      ID(II)=ITEMP
      ENDIF
      ENDDO
      ENDDO
C     MAKE DISCRETE UNGROUPED FREQUENCY TABLE
      K=0
      J=1
    1 K=K+1
      XX(K)=X(J)
      NF(K)=0
      DO I=J,N
      IF(XX(K).EQ.X(I)) THEN
      NF(K)=NF(K)+1
      ELSE
      J=I
      IF(J.LE.N) THEN
      GOTO 1
      ELSE
      GOTO 2
      ENDIF
      ENDIF
      ENDDO
    2 KK=K
      DO K=1,KK
      IF(K.EQ.1) THEN
      NCF(K)=NF(K)
      ELSE
      NCF(K)=NCF(K-1)+NF(K)
      ENDIF
      ENDDO
      DO I=1,N
      RANK(I)=1.D0
      ENDDO

      IF(NRL.GT.4) THEN
      WRITE(*,*)'RANKING RULE CODE GREATER THAN 4 NOT PERMITTED',NRL
      STOP
      ENDIF

      IF(NRL.LT.0) THEN
      WRITE(*,*)'RANKING RULE CODE LESS THAN 0 NOT PERMITTED',NRL
      STOP
      ENDIF

      IF(NRL.EQ.0) THEN
      DO I=1,N
      RANK(I)=I
      ENDDO
      ENDIF
C     -----------------------------------------------------------------
      IF(NRL.GT.0) THEN
       DO K=1,KK
       IF(K.EQ.1) THEN
       K1=1
       ELSE
       K1=NCF(K-1)+1
       ENDIF
       K2=NCF(K)
       DO I=K1,K2
       SUM=0.D0
       DO II=K1,K2
       SUM=SUM+II
```

```fortran
       ENDDO
       KX=(K2-K1+1)
       IF(NRL.EQ.1)RANK(I)=K ! DENSE RANKING (1-2-2-3 RULE)
       IF(NRL.EQ.2)RANK(I)=K1!STANDARD COMPETITION RANKING(1-2-2-4 RULE)
       IF(NRL.EQ.3)RANK(I)=K2!MODIFIED COMPETITION RANKING(1-3-3-4 RULE)
       IF(NRL.EQ.4)RANK(I)=SUM/KX !FRACTIONAL RANKING (1-2.5-2.5-4 RULE)
       ENDDO
       ENDDO
      ENDIF
C     -----------------------------------------------------------------
      DO I=1,N
      X(ID(I))=RANK(I) ! BRINGS THE DATA TO ORIGINAL SEQUENCE
      ENDDO
      RETURN
      END
C     ---------------------------------------------------------------
      SUBROUTINE CORREL(X,N,M,RMAT)
      PARAMETER (NMX=30)!DO NOT CHANGE UNLESS NO. OF VARIABLES EXCEED 30
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION X(N,M),RMAT(2,2),AV(NMX),SD(NMX)
      DO J=1,2
      AV(J)=0.D0
      SD(J)=0.D0
      DO I=1,N
      AV(J)=AV(J)+X(I,J)
      SD(J)=SD(J)+X(I,J)**2
      ENDDO
      AV(J)=AV(J)/N
      SD(J)=DSQRT(SD(J)/N-AV(J)**2)
      ENDDO
      DO J=1,2
      DO JJ=1,2
      RMAT(J,JJ)=0.D0
      DO I=1,N
      RMAT(J,JJ)=RMAT(J,JJ)+X(I,J)*X(I,JJ)
      ENDDO
      ENDDO
      ENDDO
      DO J=1,2
      DO JJ=1,2
      RMAT(J,JJ)=RMAT(J,JJ)/N-AV(J)*AV(JJ)
      RMAT(J,JJ)=RMAT(J,JJ)/(SD(J)*SD(JJ))
      ENDDO
      ENDDO
      RETURN
      END
C     -----------------------------------------------------------------
      SUBROUTINE DOCORA(ZDAT,N,M,RMAT)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION ZDAT(N,M),RMAT(2,2),Z(N,2)
      DO I=1,N
      Z(I,1)=ZDAT(I,1)
      Z(I,2)=ZDAT(I,2)
      ENDDO
      CALL CORA(Z,N,R)
      RMAT(1,2)=R
      RMAT(2,1)=R
      DO J=1,2
      RMAT(J,J)=1.D0
      ENDDO
      RETURN
      END
```