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Shipping the Good Apples Out: Another Proof with A Graphical Representation

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Abstract

This paper provides a proof of Alchian and Allen's thesis—superior good is relatively more consumed in distant market—with some restrictions on the utility function. Comparing to the proof by Borcharding and Silberberg (1978), those assumptions may restrict the model in some sense but we will find the Alchian-Allen theorem is independent of the third law of Hicks (Hicks 1946, pp. 309-311). The theorem is given by natural characteristics of relative demand curves.

JEL Classifications: *D00; F19*

Keywords: *Fixed transportation cost; quality of goods; relative demand; Alchian-Allen theorem*

1. Introduction

Alchian and Allen suggested fixed transportation costs raise relative demand on superior good to inferior one in their textbook (Alchian and Allen 1967, pp. 63-64). Many scholars discuss the generality of their statement and the first study of Borcharding and Silberberg (1978) provided an algebraic proof of the Alchian-Allen theorem with two qualities and a Hicksian composite good. Currently, Bauman (2004) with multiple Hicksian goods and Saito (2006) with multiple qualities suggest variant models of Borcharding and Silberberg.

Empirically, the Alchian-Allen theorem is supported, for example, by Staten and Umbeck (1989) and Bertonazzi, Maloney, and McCormick (1993). We can also find an example in the history of the United States. From 1816 to 1857, the minimum valuation tariff and ad valorem tariff were imposed on British textiles. As a result, the average per yard FOB price of British textiles became much higher than the average per yard price of local textiles. According to Alchian and Allen, British manufacturers should ship finer products to American market than before. The history followed Alchian and Allen's thesis and manufacturers in the United States could specialize in the production of coarse textiles and gained their competitiveness during those periods, which resulted in some failures of British merchants to sell inferior quality textiles after the drastic tariff cut in 1857 (see Temin 1988, and Irwin and Temin 2001).

But there are some skeptics such as Gould and Segall (1969) that says which effect dominates the income or substitution and Alchian and Allen's thesis depends on the specification of the model. Umbeck (1980) argue the specification of Borcharding and

Silberberg and it insists the Alchian-Allen effect is more general concept. Regarding to Borcharding and Silberberg, Razzolini, Shughart II, and Tollison (2003) also discuss some opposite cases by which the bad apples are shipping out. Thus we cannot say there is a common theoretical consensus of the Alchian-Allen theorem despite empirical results and historical observations.

This paper explores another proof with some constraints on preferences of consumers and gives a graphical representation of the Alchian-Allen theorem with two qualities and a Hicksian composite good. I develop the argument as follows. In Section 2, the base model is constructed to prove the Alchian-Allen theorem and the graphical representation is also provided. Section 3 shows two specific examples; CES and Cobb-Douglas cases. Then Section 4 concludes the argument.

2. The Alchian-Allen Theorem

I consider a model of a good that has two qualities, fine and coarse represented by x_0 and x_1 respectively, and a Hicksian composite good y . I assume consumers of this economy have homothetic preferences and the social utility function is represented by

$$U = U[u(x_0, x_1), y]. \quad (1)$$

Further, I also assume the social utility function is differentiable, and the sub-utility function is concave, homothetic, and also differentiable.

Let p_0 and p_1 be the relative prices of x_0 and x_1 in terms of the Hicksian good such that $p_0 > p_1$ (this order reflects the quality of x_0 is finer than that of x_1). The budget constraint of this economy is

$$p_0x_0 + p_1x_1 + y = I, \quad (2)$$

where $I > 0$ is the social income measured by the Hicksian good. Hence the utility maximization problem of this economy is

$$\max_{p_0, p_1} U[u(x_0, x_1), I - p_0 x_0 - p_1 x_1]. \quad (3)$$

The first order conditions of this problem give

$$\frac{p_0}{p_1} = \frac{u_0(x_0, x_1)}{u_1(x_0, x_1)}, \quad (4)$$

where $u_i(\bullet)$ is the partial derivative of the sub utility function with respect to x_i . Let

$v(x_0, x_1)$ be a function of homogeneous of degree one and g be a monotonic

transformation that generate the sub-utility function such as $u(x_0, x_1) = g \circ h(x_0, x_1)$.

Then (4) is rewritten as

$$\frac{p_0}{p_1} = \frac{h_0\left(\frac{x_0}{x_1}, 1\right)}{h_1\left(1, \frac{x_1}{x_0}\right)} = \Phi\left(\frac{x_0}{x_1}\right), \quad (5)$$

where $h_i(\bullet)$ is the partial derivative of $h(\bullet)$ with respect to x_i . Assume (5) has its inverse

function, then, it solves

$$\frac{x_0}{x_1} = \Phi^{-1}\left(\frac{p_0}{p_1}\right), \quad (6)$$

where $\Phi^{-1}(\bullet)$ is the relative demand function of the fine good in terms of the coarse one.

Because both x_0 and x_1 are normal, $\Phi^{-1}(\bullet)$ shall be decreasing in its argument p_0/p_1 .

Consider a fixed transportation cost $t > 0$ that is imposed uniformly on x_0 and x_1 .

Then it follows from the assumption $p_0 > p_1$ that

$$\frac{\bar{p}_0}{\bar{p}_1} > \frac{\bar{p}_0 + t}{\bar{p}_1 + t}, \quad (7)$$

where \bar{p}_0 and \bar{p}_1 are respective prices before the transportation cost is imposed. Then it proves the Alchian-Allen theorem because the relative price with the transportation cost is lower than without the transportation cost. The relative demand curve can be depicted as the down sloping curve in Figure 1. This figure shows the Alchian-Allen theorem of as the transition of demands from A to B on the relative demand function by some fixed transportation costs.

Theorem 1 (Alchian-Allen Theorem): *Let there be two qualities, where both qualities are normal, and the finer quality priced higher than the coarse one. Suppose consumers have homothetic preferences over these two goods represented by a homothetic and differentiable sub-utility function, which is independent of the Hicksian composite good or the “all other goods.” If the social marginal rate of substitution function derived from their preferences has its inverse, then the Alchian-Allen theorem holds, that is, the fixed transportation cost raises the relative demand of the superior good.*

Comparing to the proof given by Borchering and Silberberg (1978), and their followers such as Bauman (2004) and my paper (Saito, 2006), the proof given by this study is more robust against demand elasticity and no matter whether researchers consider Marshallian demand or Hicksian demand functions. However, it requires some conditions on consumers' preferences. Therefore, it is not clearly stated which proof is more general and appropriate but it is true that the proof of this paper reinforces Alchian and Allen's thesis.

3. Some Specific Examples

In order to present some specific examples, two popular cases of sub-utility functions are examined, which are a constant elasticity case and a Cobb-Douglas case.

Constant Elasticity of Substitution

Suppose the sub-utility function is given by a CES form such that

$$u(x_0, x_1) = (x_0^\rho + x_1^\rho)^{\frac{1}{\rho}}, \quad (8)$$

where $0 < \rho < 1$ for its concavity and for cases x_0 or x_1 are zero. In this case, the relative demand function is given by

$$\frac{x_0}{x_1} = \frac{1}{(p_0/p_1)^{1/(1-\rho)}}. \quad (9)$$

Hence the graph of this relative demand function shows exactly the same shape as the function depicted by Figure 1. Consequently we find the existence of the Alchian-Allen effect in this CES-sub-utility model.

Cobb-Douglas

Next, suppose the sub-utility function is given by

$$u(x_0, x_1) = x_0^\alpha \cdot x_1^\beta, \quad (10)$$

where $\alpha, \beta > 0$. In this case, functions g and h are given by

$$g(h) = h^{\alpha+\beta} \quad \text{and} \quad h(x_0, x_1) = x_0^{\frac{\alpha}{\alpha+\beta}} \cdot x_1^{\frac{\beta}{\alpha+\beta}}. \quad (11)$$

Then the relative demand function is given by

$$\frac{x_0}{x_1} = \frac{\alpha/\beta}{p_0/p_1}. \quad (12)$$

Hence the graph of this relative demand function is again exactly the same shape as the function depicted by Figure 1 and the Alchian-Allen theorem holds.

4. Concluding Remarks

This paper showed the Alchian-Allen theorem for two qualities with some restrictions on the sub-utility function. The interpretation of this proof is that the theorem holds if the sub-utility function is independent of the “all other goods” and homothetic in addition to some other regular assumptions as such monotonicity and differentiability. In such cases, we can find corresponding graphical representations of the Alchian-Allen theorem in terms of relative demand functions.

Comparing to the proof by Borchering and Silberberg (1978), the proof of this paper requires rather strict assumptions such as the independence of the sub-utility function. However, under those assumptions, we can find the Alchian-Allen theorem regardless of elasticity of demand. This result comes from natural characteristics of relative demand curves; the relative demand of the fine good is decreasing in the relative price of itself. In this sense, the Alchian-Allen theorem is independent of the third law of Hicks (Hicks 1946, pp. 309-311).

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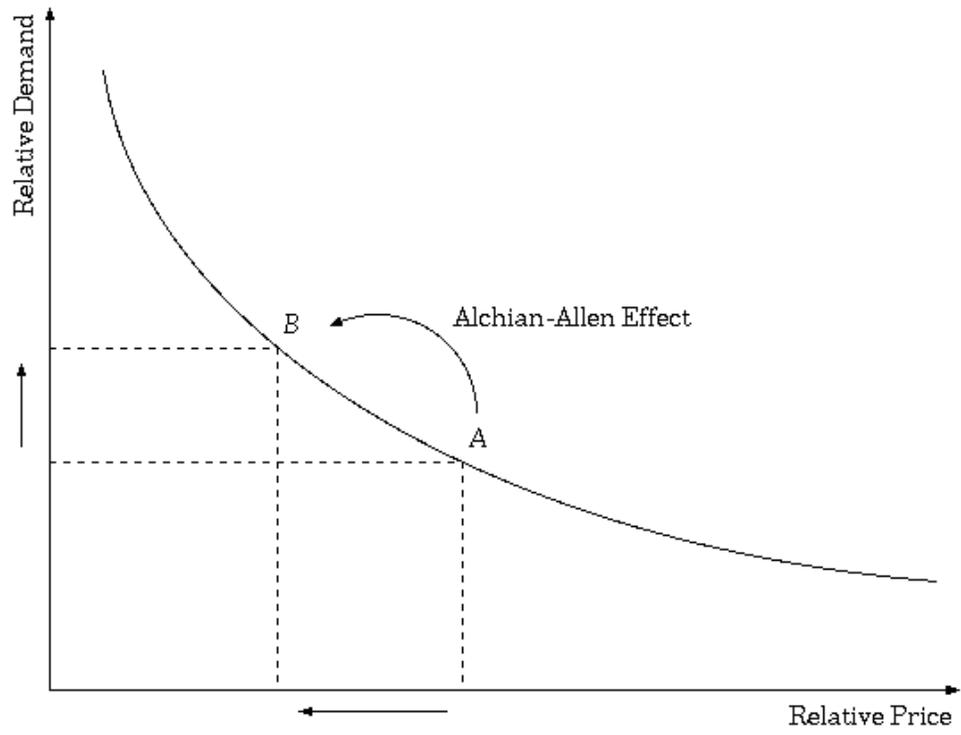


Figure 1: Relative Demand and Alchian-Allen Theorem