Computing Skills in the Market Risk Management in the G-Sec Portfolio by the Banks in India

Das, Rituparna

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Volume II

Summary*

This project starts with the backdrop against which the practice of Market Risk Management in theTrading Book started in Indian Banks. The same is reflected in Chapter 1. A brief account of which component of Market Risk Management procedure is directed by which authority along with the terminologies in vogue with the Bankers' community in India is also laid out in this chapter.

The methods of computing yield to maturities (YTMs) and alternative return measures are explained in Chapter 2 with examples in MS Excel. The computational aspects of alternative return measures other than YTM like yield to call, realized compound yield, holding period return, spot rate and forward rate are delineated here.

Chapter 3 discusses threadbare the Duration and Modified Duration method methods of computing exposure of the securities to market risk, i.e. interest rate risk and the immunization strategies depending upon the price-yield relationship for the securities in the trading book and Modified Duration Gap in the banking book. This chapter also examines various facets of yield-price relationship under different combinations of coupon rate and maturity and ends with critique of Duration and need for am improved measurement, Value at Risk (VaR).

Chapter 4 treats different methods of VaR computation in details with their respective shortcomings and ends with focusing upon the need for validation of VaR. This chapter also throws light on how to calculate and validate VaR the case of small sample size.

Chapter 5 takes the up the issue of validation of VaR via Back Testing and Stress Testing. This chapter highlights failure of VaR measures and, hence, the need of Scenario Building. Here the Tail Losses beyond the VAR are estimated.

Chapter 6 is about computation of Market Risk Capital. With MS Excel examples, it discusses how capital is computed on the basis of duration method as per RBI and on the basis of VaR as per BIS. In duration based capital charge specific capital charge as well as horizontal and vertical risk allowances are described and, in VaR based capital charge it is explained how to mitigate risks of wrong models and breach of VaR estimate.

Lastly chapter 7 is about techniques of hedging interest rate risks like Interest Rate Swap (IRS) and Forward Rate Agreement (FRA) as balance sheet hedging techniques. The IRS process is described with numerical example as well as diagram and the FRA process is described in terms of forward rate determination from spot term structure and examples of the contract.

The Project ends with conclusion.

Chapter 1: Introduction

1.1. Background
Market Risk is defined by the Reserve Bank of India (RBI) as "the possibility of loss to a bank caused by changes in the market variables" and by the Bank of International Settlement (BIS) as the risk that the value of on- and off-balance-sheet positions will be adversely affected by movements in equity and interest rate markets, currency exchange rates and commodity prices. The need for Market Risk Management in the bond portfolio emerged out of bitter international experiences like those of Long Term Capital Management (LTCM), Savings and Loan Associations in America, Barings Bank, Orange County, crashes in the stock indices like Nikkei and Dow Jones and domestic experiences like losses in the trading books suffered by the Indian Banks post mid 2004. Several factors like defaults in G-Secs, wrong predictions about stock index, monetary policy actions of central banks and sizeable exposure to low-rated assets were responsible for above disasters. Therefore RBI gave the broad contours for management of credit, liquidity, interest rate, foreign exchange and operational risks in the circular DBOD.BP.SC. 98/21.04.103/99 dated 07-Oct-1999. As a step towards enhancing and fine-tuning the existing risk management practices in banks RBI further offered a draft guidance note on Market Risk Management vide circular BP./21.04.103/2001 dated 26-Mar-02. Thus Market Risk Management started gaining currency among the Indian Banks.

In the Indian context it is necessary to note that
i. A sizeable chunk of investment is bond portfolio at a public sector bank.
ii. Bond-investments by Public Sector Banks in India rose to 47% during 2003-04 when interest rates fell[i]. They primarily held GOI securities and few corporate bonds. RBI in order to discourage such high bond-investment confined SLR to 25%. On the other hand life insurers need invest a minimum of 25 per cent of their investible corpus in GOI securities where inclusive of sovereign guaranteed securities and public sector bonds, the minimum prescribed investments is 50 per cent. The non-life insurers need to invest the minimum prescribed 20 per cent in GOI securities and a minimum of 30 per cent in other guaranteed and state government securities[ii].
iii. G-sec market collapsed during the period from 2003-04 to 2004-05.
iv. PSBs made huge losses during the period from 2003-04 to 2006-07 when interest rates started rising.
v. Schedule 14, P/L account of the banks reveal that during the period from 02-03 to 03-04 trading profit varied in Rs. 000 between
   - 64,009,46 and 120,667,28 for Canbank
   - 31,070 and 47,500 for BOB[iii]
   but during the period from 03-04 to 06-07 trading profit varied in Rs. 000 between
   - 120,667,28 and 11,164,30 for Canbank
   - 475,00 and -104,254,43 for BOB[iv]
vi. Loan and deposits are like bonds in terms of notional valuation and sensitivity measures without tradability.

1.2. Importance of G-Sec Portfolios
In financial planning a great importance is attached to planning how to maintain G-Sec portfolio since incomes from bonds are less risky than their equity counterparts. Banks find more interest in G-Secs than loans since the former are default-free. Further banks are more interested in short term treasury bills because they are stores of liquidity[v]. On the other hand insurance companies are interested in long term securities because they are more concerned about reinvestment and less about liquidity. As per RBI the weighted average maturity of outstanding stock has gone up from 6.5 years in 1997-98 to 10.4 years in 2007-08 because insurance companies invested Rs 35,880 crore in the G-sec markets, which is over 173.06 per cent higher than the Rs 13,140 crore they invested in 2006-07 and in the process insurers have already emerged as the biggest domestic institutional player in the equity markets[vi]. Investment in bond portfolio can be planned in a manner such as to synchronize cash inflow there from with scheduled cash outflows like disbursement of mature TDs. A bank can invest the forex deposits in short term T-bills and meet sudden demand for forex[vii]. As per
But between 2001 and 04 when interest rates was falling banks did not invest in floating rate bonds, rather they were eager to invest in fixed rate coupon bonds. After May 2004 Banks' incomes began falling

banks incurred losses on fixed rate bonds because they did not get buyers for the same. As a matter of asset liability management (ALM) policy, the issuer should buy back the bonds when interest rate falls and refinance them at lower rates.

1.3. Procedure and Plan
Market Risk Management in the Indian banking industry has the following components in accordance with the guidelines from the appropriate authorities in parentheses
  i.   Valuation (FIMMDA)
  ii.   Sensitivity analysis – Modified duration (RBI)
  iii. Convexity analysis (CCIL)
  iv.   Value at risk (BIS)
  v.   Capital charge (RBI and BIS)
  vi.   Back testing and stress testing (RBI)
  vii. Capital charge for market risk (RBI)
  viii. Interest Rate Swap (RBI)
where FIMMDA – Federation of money market dealers association, CCIL – Clearing Corporation of India, BIS – Bank of International Settlement.
This report explains the computational aspects of the above components with working examples with reference to the guidelines.

1.4. Terms and Concepts
1. G-Sec or government securities in the Securities Contract (Regulation) Act mean securities issued by central and state governments for raising resources from the public per force of Public Debt Act 1944. They are also alternatively called 'bonds'. There are four types of G-Secs - dated securities or coupon bonds, T-bills or ZC bonds, floating rate bonds, option embedded bonds, partly paid stocks and capital index bonds[viii]. The common feature in all these securities is that their market value varies inversely with interest rates.
2. A 'bond' is defined as a debt investment in which an investor lends money to an entity (corporate or governmental) that borrows the funds for a defined period of time at a fixed or floating interest rate.
3. Bonds are used by corporates, municipalities, and governments to finance a variety of projects and activities.
4. 'Interest' on the coupon bonds is usually paid every six months (semi-annually) in India and many other countries e.g. Uganda. There are countries e.g. Poland, where fixed rate bonds pay annual coupons or floating rate bonds pay quarterly coupons[ix].
    A bond which pays once in its life time and at maturity is called a 'zero coupon (ZC) bond' or simply 'zero', e.g. T-bill. It is a good source of capital. When Punjab and Sind Bank was bailed out by GOI the latter bought ZC bonds from the former which the former disbursed on maturity by the time it became self sustaining.
5. Bonds are commonly referred to as 'fixed-income securities'.
6. A bond is called a 'long' if held on asset side and a 'short' if held on liability side.
7. The 'price of a bond' is the sum of the present values of the expected cash flows.
8. The interest rate used to compute the present value of an expected cash flow is called the 'yield'.
9. The required yield of a bond depends on comparable securities in the market, e.g. yield of a zero coupon bond may compare with the yield of a fixed deposit.
10. If a bond is held till maturity (HTM) after purchase or settlement the yield of that bond called is it's 'YTM'.
11. 'YTM' is sometimes called 'rate of return' or simple 'return' on a HTM bond.
12. The required yield for a semiannual bond is one half of the selected annual rate of interest with which to discount the cash flows.
13. Normally each cash flow is discounted by the same yield. If each cash flow is looked upon as a zero coupon bond with a maturity equal to the time interval between settlement and receipt of that cash flow a coupon bond can be viewed as a portfolio of zero coupon bonds.
    In such a case each cash flow of the coupon bond is discounted by the spot rate on the appropriate zero coupon rate.
14. A bond is called 'option free', when it is noncallable and nonputable.

15. A bond is called noncallable when the issuer, with an intention to save interest expenses, cannot ask the investor to submit the bond for redemption when the interest rate in the market plummets.
16. A bond is called nonputable when the investor, with an intention to earn more interest, cannot ask the issuer to redeem the bond when the interest rate in the market hikes.
17. Higher the possibility of default, the lower the credit rating of the bond and the higher the return is expected by the investor.
18. There are two kinds of values of a security: present value and market value.
19. Valuation means calculating the present value with the help of YTMs circulated by FIMMDA.
20. One 'basis point (bp)' equals 1% of 1%, i.e. 0.0001.
21. The number of days between two coupon payments is called 'day count convention'.
23. Valuation is concerned with non-traded bonds, for traded bonds market price is taken as the value. In India the total number of bonds outstanding is near 100 but around 15 bonds are traded on a particular trade[x]. Their prices are available in FIMMDA and CCIL websites.
    In India bonds below investment grade can not be traded.
24. If a bond is bought between two coupon periods there are two prices:
    a) 'Full or dirty price': The price calculated in usual present value method is called full or dirty price and it includes the accrued interest due to the seller.
    b) 'Flat or clean price': It is defined as the full price minus the accrued interest and it is not discounted, since it belongs to the past.
25. Investment portfolio consists of bonds, T-bills, debentures, shares, mutual funds, commercial papers and mortgaged backed securities.
26. As per RBI's "Master Circular – Prudential norms for classification, valuation and operation of investment portfolio by banks" dated 2 July 2007 investment portfolio is classified as
    i. Accounting Categories,
    ii. SLR and non SLR Categories and
    iii. Valuation Categories.
    i. **Accounting category**
        a) government securities
        b) other approved securities
        c) debentures and corporate bonds
    ii. **SLR and Non-SLR categories**
        a) SLR category
           GOI securities at 25% NDTL
        b) Non SLR category
           PSU, corporate bonds, debentures, shares, CPs etc.
    iii. **Valuation Categories**
        a) HTM – maximum 25% of NDTL, not MTM, classification at purchase or transfer from other categories maximum once a year
        b) AFS - MTM at minimum 90 days interval, classification at purchase or transfer from other categories. AFS category is like a vacation in the sense that the bank gets respite here from MTM or P/L for 2 more months.
        c) HFT – MTM at least every month[xi].
27. Present value: In the context of a bond present value of a cash flow realizable after t years is the amount multiplied by the discount factor $1/(1+y)^t$ or e^{-yt}; y being the market rate this means had I invested in an alternative scheme how much I should have realized at the market rate y over t years, alternatively how much I am loosing in compounded sum having been locked in this bond.
28. 'Term structure' is defined as the schedule of spot rates on ZC bonds of maturities ranging from 91 days to 30 years in India. It is calculated by NSE and CCIL every business day (5-day week) for zeros and freely distributed to the public. However term structures for derivatives e.g. interest rate swaps are calculated by private sector banks for their own businesses and hence not available to the public.
29. The symbol * means multiplication, e.g. x*y means xy; the symbol ^ means "to the power of", e.g. x^y means x^y.

For more see Appendix I.

Chapter 2: Valuation and Return measures

2.1. Determining Price

Price of a bond is calculated with the present value method (PV) applying yields given by FIMMDA, NSE or CCIL. There are two methods of computing PV –

(i) Arithmetic Method or discreet discounting, i.e. multiplying a cash flow by the discount factor $1/(1+y)^t$ where 'y' is yield and 't' is the period after which the cash flow will take place. MS Excel works with this method.

(ii) Geometric Method, i.e. continuous discounting, i.e. multiplying a cash flow by the discount factor e^{-yt} where 'y' is yield and 't' is the period after which the cash flow will take place.

The Geometric Method is more beneficial for an investor in the sense that this method offers a lower asset price and higher interest rate payments than what Arithmetic Method offers for the same values of YTM and maturity. At the same time Geometric Method is more prone to interest rate risk since the asset price fall more in this method following an upward yield shock than in the case of

Given the settlement date, maturity date, coupon rate, yield, redemption (par) value and frequency the price of a bond is computed in MS Excel using the formula

=price(settlement, maturity, rate, yield, redemption, frequency)

2.2. Example in MS Exel

The price of 9% semiannual coupon bond with 20 years to mature and a par value 100 for the required yield 12% is 77.43.

Settlement	10-Oct-07
Maturity	9-Oct-27
Rate	9%
Yield	12%
Redemption	100
frequency	2
Price	77.43062

Since the above calculation is done in MS Excel it is based on arithmetic compounding. Calculation in

geometric method the price is $[\sum_{t=1}^{3} 4.5 * e^{-6\% * t}] + (104.5 * e^{-6\% * 40}) = 75.242$

An upward yield shock by 100 bps reduces the price to 71.71 in the case of arithmetic discounting and 69.46 in the case of geometric discounting. In the latter case price is falling more by 0.065.

The price of a dated benchmark security in India is calculated on the basis of average of the daily polled YTMs. Since all the dated benchmark securities in India are option-free, the difference between YTMs of an option free security and a notionally option embedded but otherwise similar securities is called option adjusted spread.

2.3. What Does It Mean to Pay Rs. 77.43 for a Bond with Face Value Rs. 100/-?

Since the bond is offering a lower coupon than the market rate, the market is loosing 22.56.

So the market makes up the loss by paying 100 - 22.56 = 77.43. See below

Half year	CF	PV@9%	PV@12%	loss
1	4.5	4.30622	4.24528302	0.06093708
2	4.5	4.120785	4.00498398	0.1158008
3	4.5	3.943335	3.77828677	0.16504794
4	4.5	3.773526	3.56442148	0.20910456
5	4.5	3.61103	3.36266178	0.24836793
6	4.5	3.455531	3.17232243	0.28320839

7	4.5	3.306728	2.99275701	0.31397105
8	4.5	3.164333	2.82335567	0.3409774
9	4.5	3.02807	2.66354309	0.36452684
10	4.5	2.897675	2.5127765	0.38489807
11	4.5	2.772894	2.37054386	0.40235046
12	4.5	2.653487	2.23636214	0.41712526
13	4.5	2.539222	2.1097756	0.42944678
14	4.5	2.429878	1.99035434	0.43952354
15	4.5	2.325242	1.87769277	0.44754922
16	4.5	2.225112	1.77140828	0.45370368
17	4.5	2.129294	1.67113988	0.45815385
18	4.5	2.037602	1.57654706	0.4610546
19	4.5	1.949858	1.48730855	0.4625495
20	4.5	1.865893	1.40312127	0.4627716
21	4.5	1.785543	1.32369931	0.4618441
22	4.5	1.708654	1.24877294	0.45988105
23	4.5	1.635076	1.17808768	0.45698791
24	4.5	1.564666	1.11140347	0.45326216
25	4.5	1.497288	1.04849384	0.44879385
26	4.5	1.432811	0.98914513	0.44366605
27	4.5	1.371111	0.93315578	0.4379554
28	4.5	1.312068	0.88033564	0.43173247
29	4.5	1.255568	0.83050532	0.42506225
30	4.5	1.2015	0.78349559	0.41800448
31	4.5	1.149761	0.73914678	0.41061405
32	4.5	1.10025	0.69730829	0.40294132
33	4.5	1.05287	0.657838	0.39503243
34	4.5	1.007532	0.62060189	0.38692962
35	4.5	0.964145	0.58547348	0.37867151
36	4.5	0.922627	0.55233347	0.37029331
37	4.5	0.882896	0.52106932	0.36182713
38	4.5	0.844877	0.49157483	0.35330215
39	4.5	0.808495	0.46374984	0.34474488
40	104.5	17.96655	10.1597186	7.80683065
	∑PV	100	77.4305547	22.5694453

2.4. Rule of Thumb

Let us consider a bond with FV = RV = 100, coupon 10% and market rate 12%.

Price = ∑PV of cash flows = 96.6199

Why is the price exactly 96.6199?

In order to earn 12% one should invest exactly 96.6199 such that on redemption (110-96.6199) $\cong$ 12%

See Excel file or below.

t	CF	PV
1	10	8.928571
2	110	87.69133
	∑PV	96.6199
	return	0.121637

This is the rule of thumb to explain why and how a dealer would buy a security at a price more than face value in order to earn a rate of return more than the coupon rate.

2.5. FIMMDA-Bloomberg Method

In India valuation of G-Secs are performed as per Bloomberg method following FIMMDA Circulars[xii]. Value or price of a security implies the YTM here. There are six categories of G-Secs – most liquid benchmark securities, non-benchmark traded securities, semi-liquid securities, illiquid securities, floating rate securities and special category securities. The YTMs of the most liquid benchmark securities and some special category securities like Oil and UTI Bonds are determined everyday on the basis of polled prices. These prices called Bloomberg Generic Price (BGN) are characterized by consensus of 23 contributors ranging from ABN Amro Bank to UTI Bank. This is known as consensus pricing. But this method does not apply to the securities which are less or not traded. The YTM calculated with the method, which apply to all categories of securities in determining the theoretical fair prices is known as Bloomberg Fair Values or prices (BFV). In this method option adjusted spreads are also determined. For illiquid bonds spread are generated on polling basis on every Monday. Adding such spreads to BFV YTMs produce the YTM of illiquid securities called Bloomberg-FIMMDA (BFMD) prices. For floating rate bonds YTM is set equal to the coupon rate. The BFV Method is demonstrated in detail under the 'Bootstrapping' section.

2.6. Determining the YTM from Price and Other Information about the Coupon Bond

If the cash flows of a bond starting from settlement till maturity are known, the YTM could be calculated using the formulas like
=irr
Or
=rate
Or
=yield
See example below.

settlement	10-Oct-07	settlement	10-Oct-07	-89.1888
maturity	9-Oct-12	coupon date	10-Oct-08	9
rate	9%		10-Oct-09	9
yield	12%		10-Oct-10	9
redemption	100		10-Oct-11	9
frequency	1		10-Oct-12	109
Price	89.1887516	YTM		12%

What does it interpret by "the price of a bond is the sum of the present values of the expected cash flows in a well functioning market with sufficient liquidity unlike India"?

Interpretation

Present value of a future payment of 100 after 5 years is 65 at an yield of 9% interprets as if you invest 65 for 5 years at a compound rate of 9%, the mature value would be 100. In other words investing 65 at a compound rate of 9% for 5 years is equivalent to buying a security which would fetch 100 after 5 years when the market yield is 9%.

Let us apply the same reasoning to valuation of a coupon paying bond below. Suppose the market price of the bond is P, the yield y and maturity 2 years, coupon C and face or par value F.

1. The investor can invest P somewhere else for 2 years at a rate y such that the mature sum is $P*(1+y)^2$.
2. The investor can buy the bond and invest the cash flows at rate y till the period T, the future value of his earnings will be $\{C*(1+y)\} + \{(C+F)\}$

The investor would buy the bond if

$\{C*(1+y)\} + \{C+F\} \geq P*(1+y)^2$ and at least

$P*(1+y)^2 = \{C*(1+y)\} + \{C+F\}$

Now divide both sides of the above equality by (1+y)^2 such that
P = {C/(1+y)} + {(C+F)/(1+y)^2}, i.e. the price of the bond is equal to the present values of future cash flows.
What will happen if {C*(1+y)} + {C+F} > P*(1+y)^2?
Arbitrage will take place.
Arbitrage means earning something without investing anything of own.
Suppose you are an arbitrageur who knows that {C*(1+y)} + {C+F} > P*(1+y)^2.
You will borrow P for 2 years and buy bond[xiii].
At present your cash outflow nil.
At the end of 2 years you earn a positive arbitrage profit
= {C*(1+y)} + {C+F} − P*(1+y)^2 > 0.
Such profit would raise the demand for the bond such that price of the bond would rise towards wiping out the profit. See below.

Example

			settlement	10-Oct-07
			maturity	10-Oct-09
			rate	9%
			yield	8%
			redemption	100
			frequency	1
			Price	101.78
			P*(1+8%)^2	118.7162
T	CF	Reinvestment		
1	9	9.72		
2	109	109		
Total		118.7162		

What will happen if {C*(1+y)} + {C+F} < P(1+y)^2?
Apply the concept of short selling.
{C*(1+y)} + {C+F} < P(1+y)^2 ⇒ C + {(C+F)/(1+y)} < P(1+y)
Suppose C + {(C+F)/(1+y)} = P_1 > P(1+y)
Borrow the bond, sell at P and return P_1 in order to make arbitrage profit P − P_1.
Here P is not the theoretical price of an untraded bond, rather it is the market price of a traded bond. For an untraded bond the question of arbitrage does not arise.
2.7. Price vs. Par and Coupon vs. Yield
For a coupon bearing bond
1. market price > par value when coupon rate > yield and the converse
2. market price approaches par when maturity approaches zero.

Interpretation
Investment in a coupon bearing bond is expected to produce two components of cash flow –
    1. a finite number of coupons before maturity and
    2. redemption of par at maturity (T).
The finite number of coupons for T periods can be looked upon as the difference between two perpetual bonds a and b bearing same yield y and coupon rate c
Bond a: It is a perpetual long bond on asset side to be held today (c/y)
Bond b: It is another perpetual short bond on liability side to be held T period hence (c/y)/{(1+y)^T}.

The par (F) redemption at maturity part can be looked upon as a zero coupon bond held today $F/\{(1+y)^T\}$ on asset side to mature at T.

Hence a coupon bond is a package of two long bond and one short bond.

So $P = (cF/y) - (cF/y)/\{(1+y)^T\} + F/\{(1+y)^T\} = (cF/y)[1 - \{1/((1+y)^T)\}] + F/\{(1+y)^T\}$

Dividing both sides by F we get $(P/F) = (c/y)[1 - \{1/((1+y)^T)\}] + 1[1/\{(1+y)^T\}]$

This is called weighted average of $\{(c/y)$ and 1.

Thus the ratio of market value to par value (P/F) of a long bond is the weighted average of (c/F)/y and 1 since the sum of the weights $[1 - \{1/((1+y)^T)\}]$ and $\{1/((1+y)^T)\}$ is 1.

This is clear from the expression $(P/F) = \{(c/F)/y\}[1 - \{1/((1+y)^T)\}] + 1[1/\{(1+y)^T\}]$, that

1. (P/F) > 1 if (c/F)/y > 1 and the converse;
   thus there is an inverse relationship between yield and market value of a bond, other things remaining unchanged
2. (P/F) = 1 if T = 0 at maturity.

Example: Price > Par when Coupon > YTM and the Converse

Settlement	10-Oct-07	settlement	10-Oct-07
maturity	9-Oct-27	maturity	9-Oct-27
rate	9%	rate	9%
yield	12%	yield	6%
redemption	100	redemption	100
frequency	2	frequency	2
Price	77.43062	Price	134.6693

Another Interpretation

Let us surmise whether yield would rise exactly, more or less than the rise in coupon in order to keep the PV unchanged, cet par.

Consider our two period coupon bond for which $P(1+y)^2 = cF(1+y) + cF + F$

Holding P and F constant, we take changes in c and y below.

(cet par is small form of ceteris paribus meaning other things remaining unchanged)

The original expression

$P + 2Py + (y^2) = cF + cFy + cF + F$

Replacing y by Δy and c by Δc in above

$2P \Delta y + \{(\Delta y)^2\} = 2 \Delta c F + (\Delta c \Delta y F)$

Since $\Delta y, \Delta c < 1$, we can ignore $\Delta c \Delta y$ and $(\Delta y)^2$ for absolute changes smaller than any positive number however small may it be such that

$2P \Delta y = 2 \Delta c F$ i.e. $\Delta y/\Delta c = F/P$

This means when P > F, $\Delta y /\Delta c < 1$ and vice versa.

Example: $\Delta y/\Delta c < 1$, When P > F

settlement	10-Oct-07			CF
maturity	9-Oct-12		Price	-117.309
rate	9%		coupon	9.25
yield	5%		coupon	9.25
redemption	100		coupon	9.25
frequency	1		coupon	9.25
Price	**117.3088**		coupon+par	109.25
			yield	**5.23%**
	CF			*CF*

Price	-117.309		Price	-117.309
coupon	9.5		coupon	9.75
coupon	9.5		coupon	9.75
coupon	9.5		coupon	9.75
coupon	9.5		coupon	9.75
coupon+par	109.5		coupon+par	109.75
yield	**5.45%**		**yield**	**5.68%**

$\Delta y/\Delta c < 1$, when P > F

c	*Δc*	*y*	*Δy*	*Δy/Δc*
9%		5%		
9.25%	0.25%	5.23%	0.23%	0.92
9.50%	0.25%	5.45%	0.22%	0.88
9.75%	0.25%	5.68%	0.23%	0.92

2.8. How to interpret (P/F) = 1 if T = 0 at maturity

(P/F) = 1 if T = 0 at maturity interprets that
the coupon bond price converges to par happen what may before maturity (T).
As T approaches zero the number of coupons also approaches zero and at time of the last coupon what remains is redemption. Thus the time path of a bond converges to par value. If market price is above par before maturity it will fall to par on maturity and vice versa. This is called the pooled-to-par property of bond price. It applies to both coupon and ZC bonds.

Example of Pooled-to-Par from Premium at Fixed YTM

If market price is above par value before maturity the market price comes down to par value as maturity approaches zero.

settlement	10-Oct-07
maturity	9-Oct-17
rate	9%
yield	5%
redemption	100
frequency	1
Price	130.8797

Settlement	Price
10-Oct-07	130.87968
10-Oct-08	128.42369
10-Oct-09	125.84491
10-Oct-10	123.13718
10-Oct-11	120.29407
10-Oct-12	117.30881
10-Oct-13	114.17428
10-Oct-14	110.88302
10-Oct-15	107.4272
10-Oct-16	103.79826
8-Oct-17	100.00986

If market price is below par value before maturity the market price goes up to par value as maturity approaches zero.

Example of Pooled-to-Par from Discount at Fixed YTM

settlement	10-Oct-07
maturity	9-Oct-17
rate	9%

Settlement	Price
10-Oct-07	83.050479
10-Oct-08	84.016703

yield	12%
redemption	100
frequency	1
Price	83.05048

10-Oct-09	85.098874
10-Oct-10	86.310905
10-Oct-11	87.66838
10-Oct-12	89.188752
10-Oct-13	90.891568
10-Oct-14	92.798723
10-Oct-15	94.934736
10-Oct-16	97.325402
8-Oct-17	99.988679
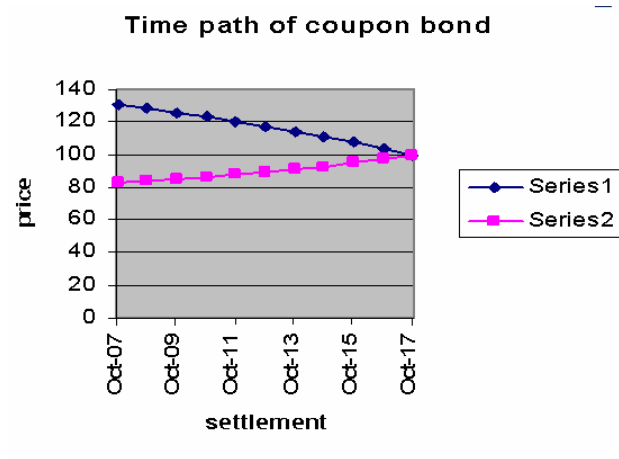
Series 1 indicates the bond price comes down to par on maturity if above par before.
Series 2 indicates the bond price goes

How Pooled-to-par Happens
Because of higher credit risk corporate bonds have lower price and G-secs have higher prices; in course of time the former witnesses rise in their prices while the latter witnesses the opposite because of pooled-to-par property. This happens as follows: as a strategy, the dealers buy corporate bonds when their prices are lower and then sell at higher prices; borrow the G-secs at higher prices, sell then and repurchase them at lower prices. Borrowing G-Sec and then selling is called short selling. Short selling is allowed in India.

Exception to the Pooled-to-Par
When Russia defaulted on Ruble denominated debts after mid 1998 the prices of corporate bonds issued from Russia and East European countries fell sharply while the prices of USA government bonds rose sharply. This was an exception to the pooled-to-par property of bond price.

2.9. Price-yield Relationship for An Option Free Coupon Bond
For a 20 year 100 par option free 9% semi annual coupon paying bond the relationship between price and yield are found to have a negative relationship.
Example of Price-yield Relationship for An Option Free Coupon Bond

settlement	10-Oct-07		y	P	ΔP	$\Delta\Delta P$
maturity	9-Oct-27		1%	244.6707		
rate	9%		2%	214.9083	29.76241	
yield	5%		3%	189.7382	25.17005	4.592358
redemption	100		4%	168.3822	21.35601	3.814045

frequency	2	5%	150.2012	18.18107	3.174939
Price	150.2012	6%	134.6693	15.53188	2.649186
		7%	121.3533	13.31601	2.215875
		8%	109.8953	11.45793	1.858075
		9%	99.99946	9.895878	1.562054
		10%	91.42024	8.579217	1.316662
		11%	83.95385	7.466389	1.112827
		12%	77.43062	6.523227	0.943163
		13%	71.70904	5.721586	0.801641
		14%	66.67079	5.038246	0.68334
		15%	62.21678	4.454016	0.58423
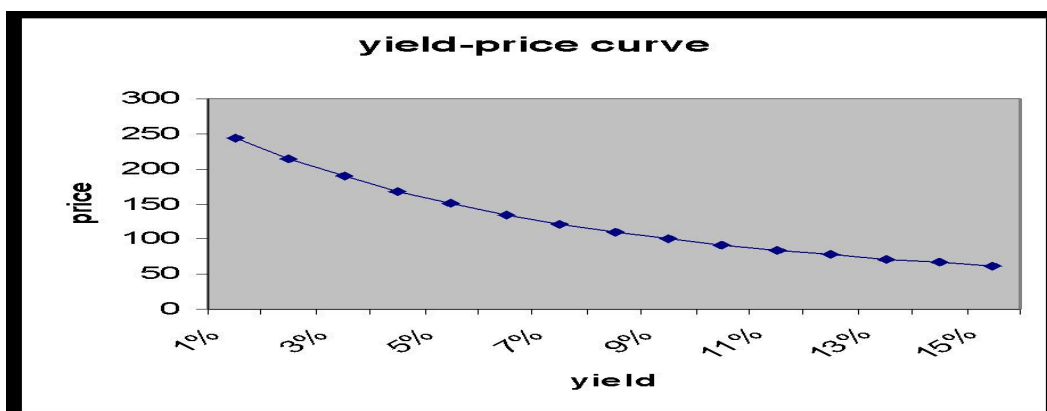
The yield-price curve is found to be linear for slender yield shocks and convex for robust yield shocks.

Interpretation
When bond price is sliding down continuously along the yield price curve, side by side the bond holders' hesitation to sell the bond is scaling up because they expect price to rise any time and hence the rate of fall in price slackens.

One to One Correspondence between Price and YTM
Based on the PV formula of bond pricing, for every YTM there is a corresponding unique price, i.e. there can't be more than one price for a particular YTM, *ceteris paribus*. This relationship is called one to one correspondence between price and YTM

2.10. Convexity
Convexity is the yield-price relationship for a security for large yield shocks, e.g. 700 bps. This means for 700 bps down yield shock price would rise more than expected and for 700 bps up yield shock price will fall less than expected. When there is upward yield shock three forces work – (i) NII (net interest income) goes up and reinvestment rates also go up, (ii) distant future cash flows are affected less than near future ones and (iii) MV (market value of the paper) falls. Here first two forces are favorable to the investor and partially neutralize the third force which is not favorable to the investor. Therefore in the wake of substantial upward yield movement depreciation in MV is less than expected. In the same line of logic it can be argued why appreciation in MV is more than expected in the wake of substantial downward yield movement. Graphically the price-yield relationship looks linear for small changes in yield and convex for large changes in yield.

Example of Convexity

t	CF@9%	PVCF@8%	PVCF@8.01%	PVCF@7.99%	PVCF@1%	PVCF@15%
1	9	8.3333333	8.3325618	8.33410501	8.910891089	7.826086957
2	9	7.7160494	7.714620683	7.717478479	8.822664445	6.805293006
3	9	7.1444902	7.142505956	7.146475117	8.735311331	5.917646092
4	9	6.6152687	6.612819143	6.617719342	8.6488231	5.14577921
5	109	74.183568	74.14923377	74.21792226	103.7097599	54.19226415
Price		**103.99271**	**103.9517414**	**104.0337002**	**138.8274499**	**79.88706941**
Price	**Yield%**	**ΔP/P**				
104.0337	7.99%	0.0003942				
103.9927	8%	0				
103.9517	8.01%	-0.000394				
Price	**Yield%**	**ΔP/P**				
138.8274	1%	0.3349729				
103.9927	8%					
79.88707	15%	-0.2318013				

The above table shows that when yield shock is marginal e.g. 0.01% upward or downward the absolute magnitude of price shock is almost equal, 0.0003942 - 0.000394 ≅ 0; this is called linearity in the price-yield relationship. But when yield shock is robust e.g. 7% the absolute magnitude of price shock is higher in the case of downward yield shock by 0.3349729 - 0.2318013 ≅ 10% than in the case of upward yield shock; this is called convexity in the price-yield relationship. The features of linearity and convexity in the
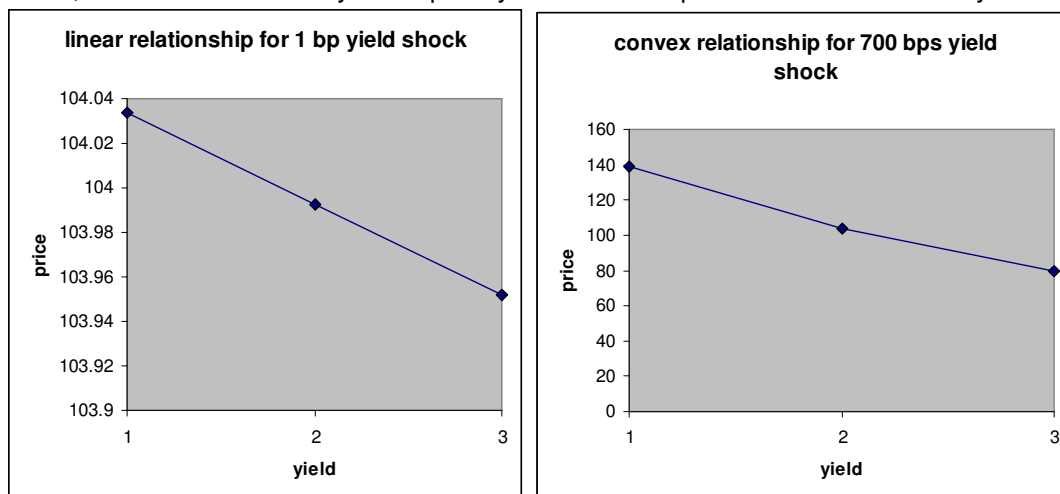
price-yield relationship are demonstrated by the above diagrams.

Convexity has serious implication for portfolio designing, which will be discussed later. CCIL (Clearing Corporation of India Limited) the central counterparty in all security transactions and the House for clearing and settlement for transactions in Money, GSecs and Foreign Exchange in India, calculates the convexity of every security and circulates amongst its members[xiv].

Why a Bond is More Convex than Another?
For two bonds with same par value, same coupon and same maturity convexity may differ depending on at what point on the price yield curve they are located. Suppose both of them have at present the same

price yield combination; also you have to take into consideration the profile, rating etc of the issuers; suppose they are issued by different entities. See diagrams

Location of Bond I on price-yield curve
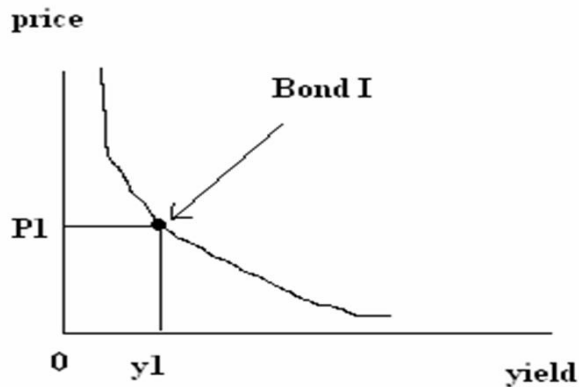
Location of Bond II on Price-yield Curve
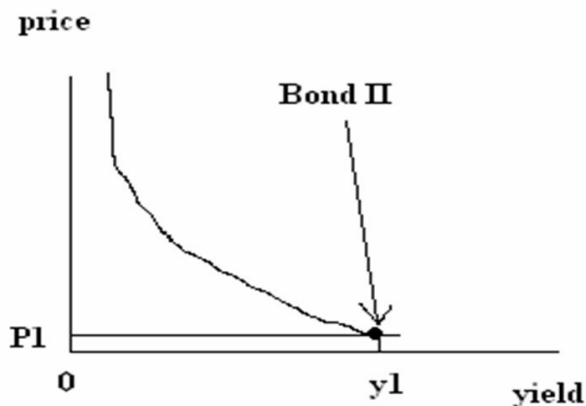
Interpretation
Though both the bonds have same price-yield combination bond I is located on relatively more elastic segment of the p-y curve compared to bond II. This means if yield increases further, price of bond II may fall drastically while price of bond II won't change, i.e. bond II holders are expecting a price rise any moment and so reluctant to sell it off, while bond I holders are expecting further price-fall and ready to dispose it. This implies bond II has better rating than bond I.
So the higher the rating or credibility of the issuer the less the convexity and the less the gain from risk-return game or strategies; after all less convexity means less price-volatility or risk, less risk means less return. More convex a portfolio higher the risk content.

2.11. Yield to Worst
Price-yield relationship for a callable bond is studied by comparing YTMs of a non-callable bond and a callable bond with the same coupon, maturity and par value; the call value can be same with or different than par value. In FIMMDA parlance yield to call is called the yield to the worst[xv].

Example of Yield to Worst

Suppose a bond with 8% coupon 30 year maturity sells for 115 and is callable in 10 years at a call price 110. Calculate YTM and ytc.

Non callable	
nper	30
pmt	8
pv	-115
fv	100
type	0
YTM	6.81%

Callable	
nper	10
pmt	8
pv	-115
fv	110
type	0
ytc	6.64%

2.12. Relaxing the Assumption of Coupon Bond: Valuation of ZC Bond

ZC bond is also called discount bond. Valuation of a ZC bond is akin to valuation of a FD without any intermediate cash flow. The fair value of a ZC is the discounted par value, since, ZC consists of a single cash flow on maturity.

Example of ZC Bond Valuation

The price of a 100 par ZC bond that matures after 10 years with expected return 8.6% is 44.

settlement	10-Oct-07
maturity	9-Oct-17
yield	0.086
redemption	100
Price	43.82296

What does it interpret by "the price of a 100 par ZC bond that matures after 10 years with expected return 8.6% is 43.83 $\cong$ 44"? "The price of a 100 par ZC bond that matures after 10 years with expected return 8.6% is 44" interprets "how much cash investment should compound into 100 after 10 years at 8.6% p.a". It can be shown easily that 43.83 would compound into 100 after 10 years at 8.6% p.a, *i.e.* $43.83*(1.086)^{10} = 100.02 \cong 100$

2.13. Rate of Change in Rise in Price of ZC Bond During its Life Time

The price of a ZC bond rises from the issue price to par as maturity approaches zero, *cet par*, because, with every one year elapsing in the bond's life, the par value will be discounted by one-year less discount factor and the valuation would be larger. Take the same ZC bond with 100 par, maturity 10 years, YTM 8.6% and issue value 44.

settlement	10-Oct-08
maturity	9-Oct-17
yield	0.086
redemption	100
Price	47.59174
settlement	10-Oct-10
maturity	9-Oct-17

settlement	10-Oct-09	
maturity	9-Oct-17	
yield	0.086	
redemption	100	
Price	51.68463	
Maturity	P	ΔP
10	43.82296	

yield	0.086		9	47.59174	3.768775
redemption	100		8	51.68463	4.09289
Price	56.12951		7	56.12951	4.444878

As maturity continues to diminish by extra one period every time the rise in price is more than in the previous rise.

t	Maturity	P	ΔP
0	10	43.82296	0
1	9	47.59174	3.768775
2	8	51.68463	4.09289
3	7	56.12951	4.444878
4	6	60.95664	4.827138
5	5	66.19892	5.242271
6	4	71.89202	5.693107
7	3	78.07474	6.182714
8	2	84.78916	6.714427
9	1	92.08103	7.291868
10	0	100	7.918969

The extent of rise in price when every time maturity decreases by one period every time
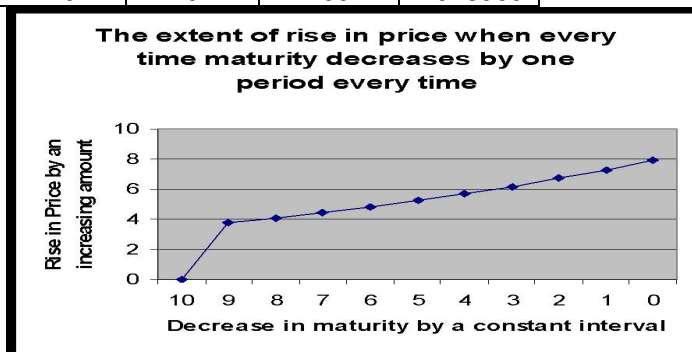
Interpretation

At period t price $P(t) = F/(1+y)^{T-t}$

At period t+1 price $P(t+1) = F/(1+y)^{T-t-1}$

Now $P(t+1)/P(t) = 1+y$

$\Rightarrow$ $P(t+1) - P(t) = y\, P(t)$

$\Rightarrow$ $P(t+2) - P(t+1) = y\, P(t+1)$

$\Rightarrow$ $P(t+2) - P(t+1) > P(t+1) - P(t)$

since $P(t+1) > P(t)$

Corollary

- A zero price is increasing by an increasing amount when maturity decreasing at a constant amount.
- Other things remaining unchanged, price of a 1 year bond today with one year hence to be mature is same as price of 10 year bond today with one year hence to be mature.
- In the case of the latter, since the coupons of last nine years are the dead past and only one coupon and redemption are left as if it is a 1 year bond.
- As bonds become old and gradually approach maturity coupons become less. Irrespective of whether coupons are high or low the price differences between various bonds reduce.
- Thus the price differences prevail in the early stages of bonds' lives compared to latter stages other things remaining the same.
- On maturity, other things remaining unchanged, prices of all bonds are same since by definition the difference between coupon and market rate does not exist.

In the India prices of all bonds maturing in 2007-08 are available in the 'financial markets watch' menu of www.rbi.org.in. Quoting and receiving orders by treasury officers take place on the NDS-OM platform. These are posted by CCIL and maintained by RBI.

2.14. Relaxing the Assumption - The First Coupon Payment is Due at the End of a Coupon Period
Often an investor purchases a bond between 2 coupon dated, i.e. next coupon is less than 6 months away. To value the bond we have to know
1. How many days are there until the next coupon payment?
2. How should we determine the present value of cash flows received over fractional periods?
3. What is the accrued interest due to the seller for the for the fraction of the coupon period he held the bond?

Steps
Compute
1. The number of days in the coupon period
2. The ratio w of the number of days between settlement and next coupon payment to the basis
3. The bond price P applying the formula $C\sum 1/(1+y)^{(w+j)} + F/(1+y)^{(w+T-1)}$, for j = 0, ..., T-1
   where
   C = coupon amount, F = par value, y = yield and T is maturity

Example of Accrued Interest
A corporate bond 100 par with a coupon rate of 10% is maturing on March 1, 2008 with a settlement date of July 17, 2002. What would be the price of this bond if the required yield is 6.5%?
1. The number of days in the coupon period is 180 since it is a corporate bond.
2. There are 44 days between the settlement and next coupon date, so w = 44/180 = 0.24444
3.

Date	t	CF	PV
1-Sep-02	0.24444	5	4.961063
1-Mar-03	1.24444	5	4.804903
1-Sep-03	2.24444	5	4.653659
1-Mar-04	3.24444	5	4.507176
1-Sep-04	4.24444	5	4.365304
1-Mar-05	5.24444	5	4.227897
1-Sep-05	6.24444	5	4.094816
1-Mar-06	7.24444	5	3.965923
1-Sep-06	8.24444	5	3.841088
1-Mar-07	9.24444	5	3.720182
1-Sep-07	10.24444	5	3.603082
1-Mar-08	11.24444	105	73.28302
			120.0281

The Accrued Interest Due to the Seller
The seller held the bond from March 1 to July 16 for 136 days.
So AI = 5*(136/180) = 3.777778

2.15. Realized Compound Yield (RCY)
RCY is that rate at which the compound growth value of an invested sum is equal to the sum of the compound value of coupon-annuity during the life of a bond and its redemption at maturity. Suppose you

buy a 100 par 2 year 10% bond at a par value 100. You know that you can reinvest the coupon @ 10% p.a.

At the end of two years

Coupon 1 reinvestment	11
Coupon 2	10
Redemption	100
Total CF	**121**

So $100*(1 + RCY)^2 = 121$ Or RCY = 10%

What If Coupons are Reinvested at a Rate Less than YTM?

RCY = YTM when coupons are reinvested at a rate equal to YTM.

Suppose coupons are reinvested @ 8%

At the end of two years

Coupon 1 reinvestment	10.8
Coupon 2	10
Redemption	100
Total CF	**120.8**

So $100*(1 + RCY)^2 = 1208$ Or RCY = 9.91% < YTM = 10%

Similarly RCY > YTM if coupon reinvestment rate > YTM

2.16. Total Return

Total or horizontal rate of return (TR) on a bond is defined as the RCY applicable to holding period, i.e. the investor sells the bond before maturity. For a semi annual bond, the RCY is the semi-annual return (ξ) that turns an investment P today by its owner into a future value V at horizon or holding period H years such that $P*(1+ \xi)^{2H} = V$

So $\xi = (V/P)^{(1/2H)} - 1$.

In order to compute ξ the following additional information is necessary:

1. the expected annual rate of coupon investment (ρ) during H years
2. expected annual YTM (θ) to be realized while reselling the bond at the end of H years.
3. The value of H in terms of years or semi annuals.

Therefore at the end of H years total cash flow (V) to the investor is compound value of coupon investment @ annually ρ + the bond price @ yield annually θ.

Example of Total Return on a Bond

For an investor H = 3 years, T = 20 years,

c/F = 8%, γ = 10%, ρ = 6%, θ = 7%, P = 82.84, frequency = 2. Compute TR.

Annual TR Calculation

coupon reinvestment cash flow			
t	c	ρ	CF
1	4	3%	4.6370963
2	4	3%	4.50203524
3	4	3%	4.370908
4	4	3%	4.2436
5	4	3%	4.12
6	4	3%	4
Total			25.8736395

settlement	20-Oct-10
maturity	20-Oct-27
rate	8%
yield	7%
redemption	100
frequency	2
price	**109.85**

Total CF after H years V		135.72
	Initial P	82.84
	H years	3

		$\xi = (V/P)^{\wedge}(1/2H) - 1$	0.08576561
		Annual TR = 2ξ	17.15%

TR analysis helps comparison of performances across bonds in different scenarios. This will be discussed under "Stress Testing in Different Scenarios".

2.17. Bond Portfolio Yield

The rate at which the sum of the PVs of all periodic cash flows from all bonds in the portfolio for all the periods equals the market value of the portfolio is called bond portfolio YTM at that period.

Example of Bond portfolio yield

Bond	c/F	T	F	P	YTM
A	7%	5 yrs	1000	920.9	9%
B	10.50%	7 yrs	2000	2000	10.50%
C	6%	3 yrs	3000	2805	8.50%
Total				5725.9	

t	Bond A	Bond B	Bond C	Portfolio
	CF+	CF+	CF+	CF
1	35	105	90	230
2	35	105	90	230
3	35	105	90	230
4	35	105	90	230
5	35	105	90	230
6	35	105	3090	3230
7	35	105		140
8	35	105		140
9	35	105		140
10	1035	105		1140
11		105		105
12		105		105
13		105		105
14		2105		2105

t	Portfolio
	CF
0	-5725.9
1	230
2	230
3	230
4	230
5	230
6	3230
7	140
8	140

9	140
10	1140
11	105
12	105
13	105
14	2105
YTM	**9.54%**

2.18. Relaxing the Assumption of Fixed Coupon

A security with floating coupon rate is called floating rate security or floater. Such a coupon rate is based on some reference rate like LIBOR. YTM of a floater cannot be calculated because future coupons are unknown. However, the potential return of a floater is computed. Rajwade (2007) observed that the present value based method adding the credit risk spread to the yield on a comparable benchmark security for the discounting purpose is not used much in India because it needs interpolation of the ZC spot rate at the time of computation and therefore mentioned an alternative method devised by Eugene Fama but however did not supplement that method with any computational example. But Fabbozi (2005) has done so with another alternative method. The method is given below.

Method

1. At every six month future coupons are decided on the basis of current spot rate (called reference rate) of the comparable fresh securities whose maturity equals the remaining maturity of a floater.
2. Alternative margins are considered, which the investor thinks feasible to earn above the reference rate.
3. For each of the above margin the bond PV is valued.
4. The margin for which the total PV equals current market value of the portfolio is called the discount margin. Example is given below or excel file.

Example of Return on a Floater

Compute the potential return of the bond with
T = 6 years
Current reference rate 10%
Coupon rate 10.8%
Alternative bps margins 80, 84, 88, 96, 100
F = 100
P = 99.398

t	ref rate	c/F	CF
1	10%	5.40%	5.4
2	10%	5.40%	5.4
3	10%	5.40%	5.4
4	10%	5.40%	5.4
5	10%	5.40%	5.4
6	10%	5.40%	5.4
7	10%	5.40%	5.4
8	10%	5.40%	5.4
9	10%	5.40%	5.4
10	10%	5.40%	5.4
11	10%	5.40%	5.4
12	10%	5.40%	105.4

Alternative bps margins

84	88	96	98
5.122368	5.121396	5.119454	5.118969
4.859009	4.857166	4.853483	4.852563
4.609191	4.606569	4.60133	4.600022
4.372217	4.368901	4.362277	4.360623
4.147427	4.143495	4.135644	4.133684
3.934193	3.929718	3.920785	3.918556
3.731923	3.726971	3.717089	3.714623
3.540052	3.534684	3.523975	3.521303
3.358046	3.352318	3.340894	3.338045
3.185398	3.179361	3.167324	3.164323
3.021625	3.015327	3.002772	2.999643
55.94541	55.8182	55.56472	55.50155
99.82686	99.65411	99.30975	99.2239

The selling price equals PV for the spread 96 bps.
Hence the potential return = 2 x 5.48% = 10.96%

2.19. Relationship between Maturity and Price
Other things remaining unchanged when maturity increases price also increases when coupon rate is more than YTM and the converse when coupon rate is less than YTM.

Example: Relationship between maturity and price when coupon rate is more than YTM
A bond with par 100, coupon rate 9%, YTM 5%, frequency 1

Maturity	*P*	*ΔP*
5	117.3088	
6	120.2941	2.985266
7	123.1372	2.843111
8	125.8449	2.707724

Example: Relationship between maturity and price when coupon rate is less than YTM
A bond with par 100, coupon rate 9%, YTM 12%, frequency 1

Maturity	*P*	*ΔP*
5	89.18875	
6	87.66838	1.520372
7	86.3109	1.357475
8	85.09887	1.212031

Interpretation
When T increases by one period the bond price increases to the extent of
$cF/(1+y)^{T+1} + (F/(1+y)^{T+1} - F/(1+y)^{T}) = \{(c-y)F\}/(1+y)^{T+1}$
This means $\Delta P = \{(c-y)F\}/(1+y)^{T+1} > 0$ when c > y and the reverse when c < y
Again ΔP is a decreasing function of T, hence every next change in P in absolute terms would be less than the previous when T rises each time by 1 period.

2.20. Relationship between maturity and YTM

For a particular bond when maturity rises

1. YTM rises, cet par, with initial c>y because of greater future cash flow at same capital expenditure
2. YTM falls, cet par, with initial c<y because of lesser amount in each future cash flow at same capital expenditure

Example: Relationship between maturity and YTM when coupon rate is more than YTM

settlement	10-Oct-07
maturity	9-Oct-15
rate	9%
yield	5%
redemption	100
frequency	1
Price	125.8449088

T	YTM
8	5.00%
9	5.31%
10	5.56%
11	5.76%

Example: Relationship between maturity and YTM when coupon rate is less than YTM

settlement	10-Oct-07
maturity	9-Oct-15
rate	9%
yield	12%
redemption	100
frequency	1
Price	85.09887364

T	YTM
8	12.00%
9	11.77%
10	11.59%
11	11.45%

2.21. Relaxing the Assumption of a Fixed YTM

A major shortcoming of YTM concept is that it is not constant for a bond as it seems to be. For different market prices of the same bond there are different YTMs. But it does not mean that YTM is influenced by price or coupon rate, rather issuers decide the coupon looking at the market rate. Further two bonds cannot be compared with help of a single YTM if a coupon bond is looked upon as a portfolio of ZC bonds where each different cash flow is compared with a mature ZC bond of a maturity equal to the interval between settlement and the time of the respective coupon payment.

Stripping vs. reconstitution

In US it is allowed to buy a whole bond at a certain price and sell each coupon as a ZC bond. This is called stripping, i.e. separating interests and principle. Stripping requires valuation using spot rates. In India all RBI and FIMMDA circulars prescribe valuation using YTM. Arbitrage equates prices computed using YTM and spot rates. If the spot rate based price is higher you can go to the treasury and exchange the whole bond for separate coupons. This is called reconstituting which is reverse to stripping. Stripping and constitution are not allowed in India.

2.22. Spot rate determination method in India

In India spot rate is determined in two ways – (i) Bootstrapping in Bloomberg Method performed by FIMMDA and (ii) Interpolation Method performed by NSE and CCIL

Bootstrapping Method

In this method, polls on YTMs of government securities are taken from at least 5 contributors like ABN AMRO Bank, ICICI Securities, UTI Bank etc. After filtering out the extreme values the polled data is averaged on the basis of such averages zero coupon spot rates are decided. In the following we have explained this method with the help of three securities for which polled average YTMs are available.

Our three securities are I year T-bill and dated securities of 2 and 3 year maturities paying annual coupons. From 1-year T-bill we can compute one year ZC price of face value 1. Next, in the present value

equation of the security of 2 years maturity we multiply the first coupon by the above one-year ZC price of face value 1 and solve the equitation for the 2 year ZC price of face value 1. The same method we apply to the PV equation of the security of 3-years maturity and obtain the 3 year ZC price of face value 1.

Bond	Price	Year 1	Year 2	Year 3
1	99	100	0	0
2	101	6	106	0
3	100	7	7	107
TTM	ZC prices	Spot rates (arithmetic compounding)	Spot rates (continuous compounding)	
1	0.99	0.010101	0.01005	
2	0.896792	0.055976	0.054465	
3	0.811144	0.072261	0.06977	

Now we have three ZC prices of maturities 1, 2 and 3 years with face value 1. Now taking the ith root of the reciprocal of each ZC price where 'i' is maturity produces the respective zero spot rate. This method is manual and hence laborious for large number of securities up to 30 years maturities as is the case of India. Therefore matrix method is used in solving the following 3 equations for 3 unknowns:

$100*Z_1 + 0*Z_2 + 0*Z_3 = 99$
$6*Z_1 + 106*Z_2 + 0*Z_3 = 101$
$7*Z_1 + 7*Z_2 + 107*Z_3 = 100$

The above system of equations is rewritten in matrix form as
CF*Z = P
Where CF is the cash flow matrix

99	100	0	0
101	6	106	0
100	7	7	107

Z is the ZC price matrix

0.99
0.896792
0.811144

and P is the price matrix

99
101
100

Solving for Z we get $Z = CF^{-1}*P$ which is the product of the inverse of cash flow matrix and price matrix. Assuming arithmetic or discreet compounding we get the spot rates

0.010101
0.055976
0.072261

and assuming continuous compounding we get the spot rates

0.01005
0.054465
0.06977

Following is the diagram of term structures:

Term Structure
Series 1: discreet compounding rates
Series 2: continuous compounding rates
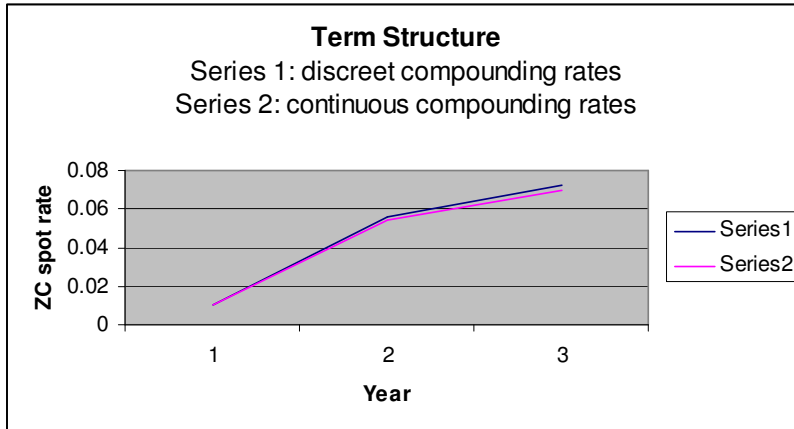
As usual continuous rates are less than arithmetic rates and the differences are pronounced in the long end of the yield curve.

Parametric Method

Parametric method estimates the term structure i.e. relationship between ZC rate and term to maturity with the help of different methods like interpolation, cubic interpolation, cubic spline model, Nelson-Siegel model and Svensson model. Interpolation method presumes a linear yield curve like below, which is often not the case:
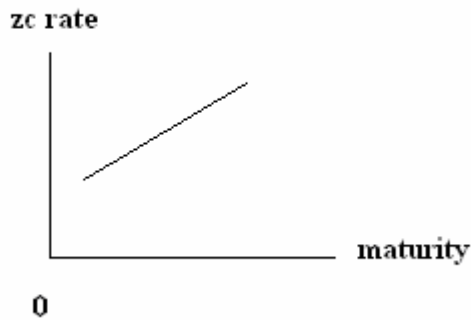
Cubic interpolation method takes into consideration presence of a hump like below:
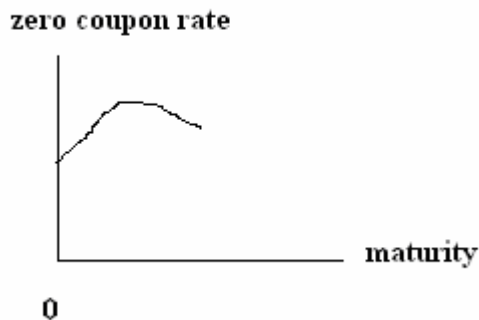
Cubic spline method is more realistic in the sense that it takes into account multiples humps and ups and downs in yield as below:
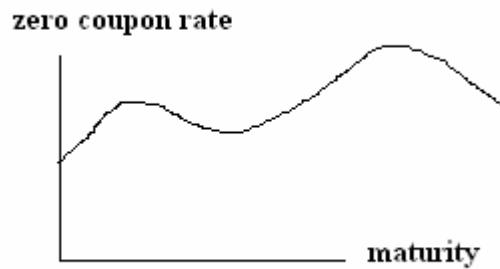
The cubic method and the cubic spline method need a lot of data points for determining the parameters, for example the cubic method prescribes the relationship between forward rate yield r and maturity m: $r = \beta_0 + \beta_1 m + \beta_2 m^2 + \tau m^3$ where β_0, β_1, β_2 and τ are four parameters in the cubic interpolation formula. This formula needs information on spot prices of four different maturities.

In India CCIL uses two such formula named "N-S" formula and "N-S Svensson" formula, where values of the parameters β_0, β_1, β_2 and τ are given on everyday basis.

Example: Spot rate Computation with NS Parameters

NS parameters	as on 11 Jan 08	m	y
β_0	8.04	1	6.61577
β_1	-1.7	2	6.61577
β_2	0.0001	3	6.833519
τ	2.74	4	7.006879
m		5	7.146031
		6	7.258664
		7	7.350611
		8	7.426319
		9	7.489194
		10	7.541858
		11	7.586338
		12	7.624213
		13	7.65672
		14	7.68483
		15	7.709314
		16	7.730787
		17	7.749739

		18	7.76657
		19	7.7816
		20	7.795095
		21	7.807271
		22	7.818308
		23	7.828354
		24	7.837536
		25	7.845959
		26	7.853711
		27	7.86087
		28	7.867501
		29	7.873659
		30	7.879393

30 year N-S yield curve
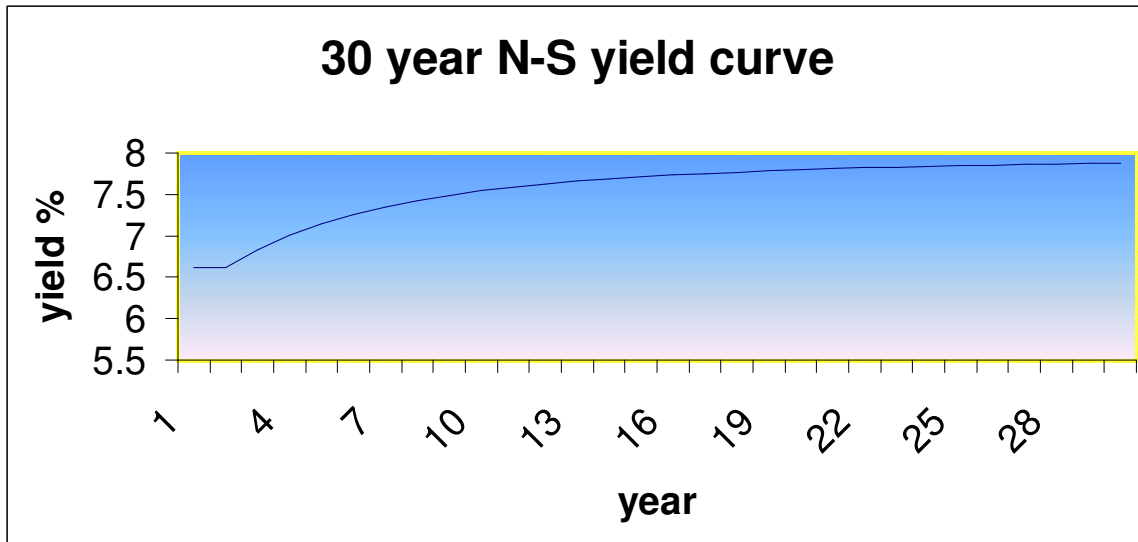
Chapter 3: Price Sensitivity: Duration, PVBP and Modified Duration

3.1. Background

The need for price sensitivity analysis reflected as early as on 10-Feb-1999 in "Asset-Liability Management System in Banks – Guidelines" circulated by RBI. A rise in YTM of a long bond leads to shrinkage in the asset side of the balance sheet. A fall in YTM of a short bond leads to swelling up of the liability side of the balance sheet. Either of these phenomena can adversely impact the economic value of equity, i.e. the difference between MVA (market value of assets) and MVL (market value of liabilities). In this context a strategy is required to ensure that a change in interest rates will not affect the value of a portfolio or leave it better off. Price sensitivity measurement is at the base of such strategy. Macaulay Duration, PVBP and Modified Duration are alternative measures of above kind of price sensitivity. Modified duration is prescribed by the RBI as a measure of interest rate risk[xvi]. This part would be taken care of under Market Risk Capital Charge.

3.2. Duration

Concept

RBI defined duration as a measure of the percent change in the economic value of a position that will occur given a small change in the level of interest rates in the 'Guidance Note on Market Risk Management' dated 26 March 2002. There are however two definitions of duration which are required to understand the implications of duration on assets and liabilities.

Definition I

Duration is defined as sensitivity of bond price to change in the discount factor when yield changes, i.e. $D = [-\Delta P/P] / [\Delta y/(1 + y)]$ i.e. price-change for 100 bps yield shock

Duration can be directly calculated in MS Excel by using the formula =duration (settlement, maturity, coupon, yield, frequency)

Duration is a direct measure of price shock.
- Rate of change in price = - D x yield shock
- $[\Delta P/P] = - D \times [\Delta y/(1 + y)]$, i.e. $\Delta P = P \times - D \times [\Delta y/(1 + y)]$
- If the value of Δy for a 10-day investment is such that any greater rate has a probability of 0.5%, $\Delta P = P \times - D \times [\Delta y/(1 + y)]$ is called value at risk (VAR) from the statistical perspective.

Here Duration is also known as Macaulay Duration.

Example of D

t	CF@9%	PVCF@8%	PVCF@8.01%	PVCF@7.99%
1	9	8.3333333	8.3325618	8.33410501
2	9	7.7160494	7.714620683	7.717478479
3	9	7.1444902	7.142505956	7.146475117
4	9	6.6152687	6.612819143	6.617719342
5	109	74.183568	74.14923377	74.21792226
Price		103.99271	103.9517414	104.0337002
Price	**Yield%**	**ΔP/P**	**Δy/(1+y)**	**D**
104.0337	7.99%	0.0003942	-9.26E-05	4.256970378
103.9927	8%	0	0	0
103.9517	8.01%	-0.000394	9.26E-05	4.254738642

duration-linear relationship for 1 bp yield shock
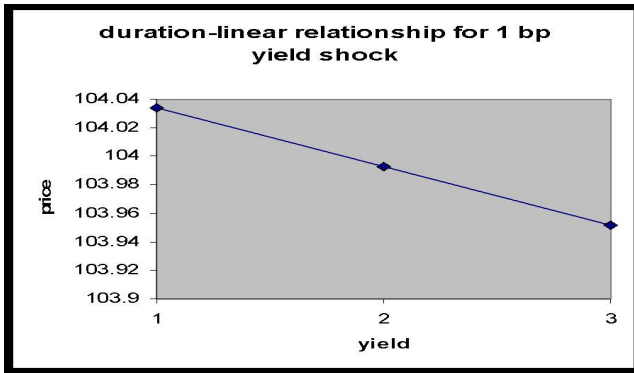
D = 0.3547, y = 8%, P = 103.9927, Δy = - 1 bps = - 0.01

Hence ΔP = - 0.3547 x (-0.01/1.08) x 103.9927 = 0.041 > 0

Once Duration value is known it is easy to predict price change.

This is called the linear relationship between price and yield; this is however true for small changes in yield like 1 bps.

3.3. Need for Modified Duration

In Duration calculation it would have been simpler were it every time not required to divide the factor $1 + \dfrac{y}{2}$ particularly in the case of semi annual bonds since most of the bonds in India are semi annual by nature. Calculation of ΔP/P would have been easier by simply multiplying the yield shock by some factor. Such a factor is known as Modified Duration. The terms 'Modified Duration' and 'Mduration' may be used alternatively.

3.4. Modified Duration - Concept

- Modified Duration is a linear transformation of D which is easier to calculate without error.
- Modified Duration is D divided by one plus yield and hence less than the latter. MD = D/(1+y)
- Now it is easier to calculate price change following an yield shock. Again if we know Duration we need not separately study Modified Duration. We can say MD is D indeed.

Example of MD

Semi annual bond price 98.2, yield 9%, MD 1.805, coupon 8%

For an yield shock Δy = - 0.01, new price = 99.98

One can also apply the formula =Mduration(date of settlement, date of maturity, coupon rate, yield, frequency);

Example Modified Duration of a semi annual bond

Settlement	1/1/2008
Maturity	1/1/2010
coupon rate	8%
yield	9%
frequency	2
redemption	100
Duration	1.886390721
Modified Duration	1.805158585
Price	98.20623715
ΔP/P for -1 bp y-shock	0.018051586

ΔP	1.772778321
New P	99.97901547

3.5. Portfolio Modified Duration and Duration Gap

Portfolio Modified Duration is the weighted average Modified Duration of the securities in the portfolio. The difference between weighted average Modified Durations of assets and liabilities is called Duration Gap. Duration Gap and Modified Duration Gap are same in result but different in expressions. Hence Modified Duration Gap is alternatively called Duration Gap. Modified Duration Gap is useful in assessing the impact of interest rate shock on Economic Value of Equity (EVE). If there is an upward yield shock and Modified Duration Gap is positive, market value of asset portfolio will fall more than market value of liability portfolio leading to fall in EVE and vice versa.

3.6. Definition II of Duration
Definition II

Duration is defined as the weighted average time to maturity where the present values of cash flows are respective weights or the time of recovery of investment or average payments timing. Since it is an average time, duration lies between settlement date and maturity for coupon paying bonds.

As per definition II, $D = (\Sigma t*PV)/\Sigma PV$ where $PV = CF/(1+y)t$, $t = 1,..,T$.

In this formula the par value F is present in all terms in both of numerator and denominator and hence cancels out. Therefore even without par value or redemption-input in the MS Excel Duration or Modified Duration calculation is possible. The MS Excel formulae of Duration and Modified Duration calculate duration and Modified Duration respectively of every one rupee of the market value of the instrument. Therefore the redemption value is not needed in calculating duration and modified duration.

Example: Calculation of Duration and modified duration of a 100 par annual, 6% coupon, 9% YTM and 4 years T in the MS Excel with the formulae '=duration(date of settlement, date of maturity, coupon rate, yield, frequency)' and '=Mduration(date of settlement, date of maturity, coupon rate, yield, frequency)' Average payment timing formula of D at initial y = 9%

t	CF	PV@9%	t x PV
1	6	5.50458716	5.504587
2	6	5.05007996	10.10016
3	6	4.63310088	13.8993
4	106	75.0930724	300.3723
	P	90.2808404	329.8763
	D	3.65389088	
	MD	3.35219347	

Alternative way of calculating D as average payment timing
$D = \Sigma t*w = 3.65$, $MD = D/1+y = 3.35$

t	CF	PVCF	w	t*w
1	6	5.504587	0.060972	0.060972
2	6	5.05008	0.055937	0.111875
3	6	4.633101	0.051319	0.153956
4	106	75.09307	0.831772	3.327088
		90.28084	**1**	**3.653891**
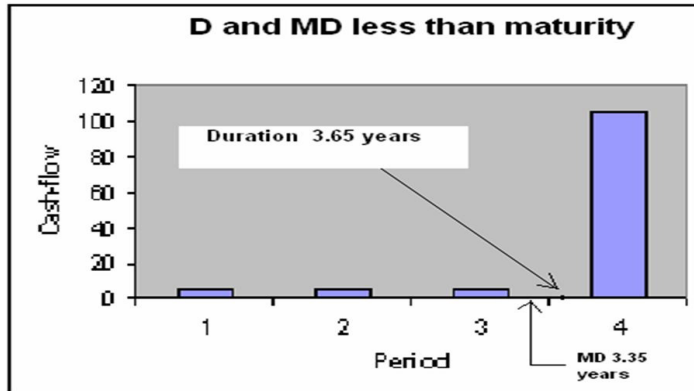
In the early stages of the bond-life capital gain or loss outweighs reinvestment loss or gain when yield falls or rises. In the later stages of the bond-life capital gain or loss falls short of reinvestment loss or gain when yield falls or rises. Only at the duration stage the two are equal.
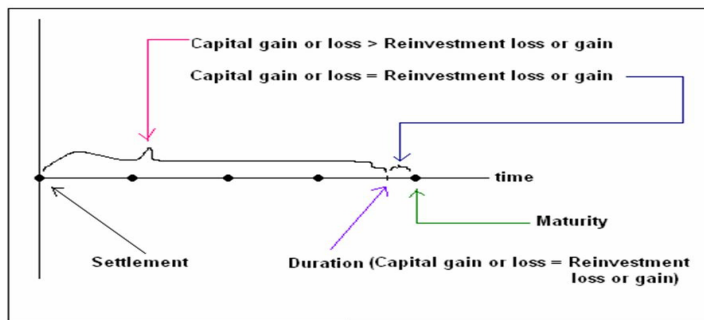
D as average payment timing 3.65 years means the investment would be recovered at 3.65 happen what may to YTM.

Consider the bond 9% 2009

Two different viewpoints of D

D is looked upon from two viewpoints:

1. yield elasticity of price as per definition I.
2. average payments timing as per definition II

Since both are equal the viewpoint does not matter for all practical purposes.

Suppose D = 1.91 years in terms of average payment timing for a bond.

Now there is y-shock 1% p.a.

So bond price will fall by 1.91 years x 1% per year = 1.91 % = D in terms of elasticity of bond price with respect to the DF = (1+y).

For a 2 year annual coupon paying there are only two cash flows 9 and 109.

D Analysis		t	CF	PVCF	w = PVCF/P	w*t
settlement	10-Oct-07	1	9	8.035714	0.084649	0.084649
maturity	10-Oct-09	2	109	86.89413	0.915351	1.830702
rate	9%			94.92985	1	1.915351
yield	12%		Total	Price		Duration
frequency	1					
D	1.91535102					
RV	100					

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P	94.9298469					

The first cash flow takes place after 1 year of investment such that there is a loss of 0.084649 x 1% before it is reinvested when y rises by 1%. Similarly there is loss on the second cash flow 0.915351 x 1% before it is reinvested when y rises by 1%. So total loss (0.084649 + 0.915351) x 1% = 1.91 % = D

How are both the viewpoints technically same?
If we differentiate the PV formula $P = (\Sigma C/(1+y)^T) + (F/(1+y)^T)$ with respect to y
we get $(dP/P)/\{-dy/(1+y)\} = (\Sigma t*PV)/(\Sigma PV)$ where $PV = CF/(1+y)^t$, t = 1, .., T.
So $(dP/P)/\{-dy/(1+y)\} = D$ since average payment timing is called $D = (\Sigma t*PV)/(\Sigma PV)$
Both the viewpoints are technically same because the price of the asset is recovered roughly at the end of the duration period, i.e. setting holding period equal to duration period produced three cash flows – accumulated reinvested coupon at the current YTM, resale value and accrued interest. The present value of sum of these cash flows at the current YTM is roughly equal to the current asset price. This is explained with the following example.

Example: Duration of a bond of 6 year maturity with 8% annual coupon

Duration calculation at par					
T	CF	Reinv@8%	Issue		03/01/2008
1	6	7.011229	Resale		05/09/2011
2	6	6.614367	maturity		03/01/2012
3	6	6.239969	rate		6%
3.673011949	103.88221	103.8822	yield		6%
Total		123.7478	redemption		100
		99.90533	frequency		1
			Resale		99.92220769
			duration		3.673011949
			price		100
Duration calculation at discount					
T	CF	Reinv	Issue		03/01/2008
1	6	7.186358	Resale		25/08/2011
2	6	6.716222	maturity		03/01/2012
3	6	6.276843	rate		6%
3.666695452	103.51918	103.5192	yield		7%
Total		123.6986	redemption		100
		96.52136	frequency		1
			Resale		99.55917895
			duration		3.666695452
			price		96.61278874
Duration calculation at premium					
T	CF	Reinv	Issue		03/01/2008
1	6	6.837906	Resale		25/08/2011
2	6	6.512291	maturity		03/01/2012
3	6	6.202182	rate		6%
3.679270543	104.2418	104.2418	yield		5%
Total		123.7942	redemption		100
		103.452	frequency		1
			Resale		100.2818049
			duration		3.679270543
			Price		103.5459505

Hence we can claim our investment will be recovered if HP = 3.67 years (app).						

A similar reasoning stands for MD.

Average payment timing formula of D at initial y = 9%

t	CF	PV@9%	t × PV	
1	6	5.50458716	5.504587	
2	6	5.05007996	10.10016	
3	6	4.63310088	13.8993	
4	106	75.0930724	300.3723	
	P	90.2808404	329.8763	
	D	3.65389088		
	MD	3.35219347		

Now y = 9.01% - an increase by 1 bp over initial 9%

t	CF		PV@9.01%	t × PV
1		6	5.50408219	5.504082
2		6	5.04915347	10.09831
3		6	4.63182595	13.89548
4		106	75.0655216	300.2621
	P		90.2505832	329.76
	D		3.65382629	
	MD		3.3518267	

New price 90.2505832 lower than before, D-change negligible

Sensitivity formula of D

ΔP/P	Δy/(1+y)	D	MD
-0.00034	9.17431E-05	3.65307869	3.351448

D calculation for an annual bond, 7% coupon, 5% YTM, 10 years T, D = Σt*w = 5.76, MD = D/1+y = 5.49

t	Earnings	PV	w	t x w
1	7	6.666667	0.043166	0.043166
2	7	6.349206	0.04111	0.08222
3	7	6.046863	0.039153	0.117458
4	7	5.758917	0.037288	0.149153
5	7	5.484683	0.035513	0.177563
6	7	5.223508	0.033821	0.202929
7	7	4.974769	0.032211	0.225477
8	7	4.737876	0.030677	0.245417
9	7	4.512262	0.029216	0.262946
10	107	65.68872	0.425325	4.253252
Total		115.4435		5.75958

It can be shown that duration rises with maturity other things remaining unchanged. For example if the coupon is 5% and YTM is 7.4% then duration would be as follows other things remaining unchanged.

Duration 5.76 years for maturity 10 years
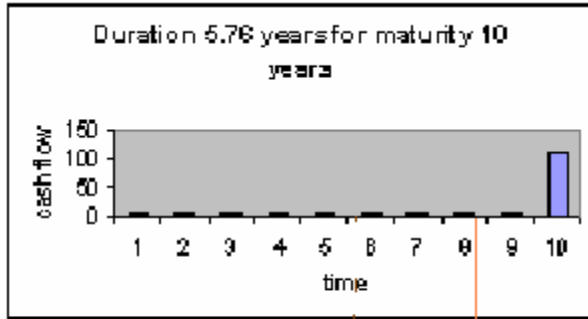
Duration 7.7 years

T rises to 15 from 10 years cet par

Duration 9.8 years for maturity 15 years
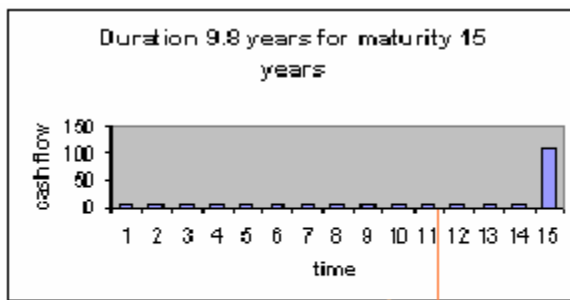
Duration 10.24 year

Same is the reasoning why MD is less for an amortizing loan than for an non-amortizing debt instrument like bond other things remaining the same. In the case of non-amortizing loan of 115.4435, 7% coupon, 5% YTM, 10 years T, MD = 5.49. For the same investment in a amortizing loan, MD = 4.86,

Example of Duration Calculation in the case of amortizing loan: Investment 115.4, YTM 5%, annual equal CF 14.95, T 10 years, without redemption, D = $\Sigma t*w$ = 5.09, MD = 4.86

t	pmt	PV	w	t*w
1	14.95046	14.23853	0.123338	0.123338
2	14.95046	13.56051	0.117464	0.234929
3	14.95046	12.91477	0.111871	0.335613
4	14.95046	12.29978	0.106544	0.426175
5	14.95046	11.71408	0.10147	0.507351
6	14.95046	11.15626	0.096638	0.57983
7	14.95046	10.62501	0.092036	0.644255
8	14.95046	10.11906	0.087654	0.70123
9	14.95046	9.637201	0.08348	0.751318
10	14.95046	9.178286	0.079505	0.795046
Total		**115.4435**		**5.099085**

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MD decreases with increase in yield because of the instant increase in reinvestment earnings and hence faster recovery of initial investment.

Example
For the same annual bond, 7% coupon, 5% yield, T 10 years, MD is 7.34 and reduces to 7.29 if y rises to 5.5%.

t	c	PV, y = 5%	w	t*w	
1	7	6.666666667	0.057748	0.057748	
2	7	6.349206349	0.054998	0.109997	
3	7	6.04686319	0.052379	0.157138	
4	7	5.758917324	0.049885	0.199541	
5	7	5.484683165	0.04751	0.237548	
6	7	5.223507776	0.045247	0.271484	
7	7	4.974769311	0.043093	0.301649	
8	7	4.737875534	0.041041	0.328325	
9	7	4.512262414	0.039086	0.351777	
10	107	65.68871813	0.569012	5.69012	
		115.4434699		7.705327	D
				7.338407	MD

t	c	PV, y = 5.5%	w	t*w	
1	7	6.63507109	0.059611	0.059611	
2	7	6.28916691	0.056503	0.113006	
3	7	5.961295649	0.053558	0.160673	
4	7	5.650517203	0.050765	0.203062	
5	7	5.355940477	0.048119	0.240594	
6	7	5.076720831	0.04561	0.273662	
7	7	4.81205766	0.043233	0.302628	
8	7	4.561192095	0.040979	0.32783	
9	7	4.323404829	0.038842	0.349581	
10	107	62.641072	0.56278	5.627803	
		111.3064387		7.658449	D
				7.293761	MD

3.7. When Coupon < YTM what will Happen to Duration and Modified Duration when Maturity rises?
MD decreases with increase in coupon rate because of the instant increase in cash flows and hence faster recovery of initial investment.

Example
For the same annual bond, 7% coupon, 5% yield, T 10 years, MD is 7.3 and reduces to 7.06 if coupon rises to 8%.

t	c = 7%	PV	w	t*w	
1	7	6.666667	0.057748	0.057748	
2	7	6.349206	0.054998	0.109997	
3	7	6.046863	0.052379	0.157138	
4	7	5.758917	0.049885	0.199541	
5	7	5.484683	0.04751	0.237548	
6	7	5.223508	0.045247	0.271484	
7	7	4.974769	0.043093	0.301649	
8	7	4.737876	0.041041	0.328325	
9	7	4.512262	0.039086	0.351777	
10	107	65.68872	0.569012	5.69012	
		115.4435		7.705327	D
				7.338407	MD

t	c = 8%	PV	w	t*w	
1	8	7.619048	0.06186	0.06186	
2	8	7.256236	0.058915	0.117829	
3	8	6.910701	0.056109	0.168328	
4	8	6.58162	0.053437	0.213749	
5	8	6.268209	0.050893	0.254463	
6	8	5.969723	0.048469	0.290815	
7	8	5.685451	0.046161	0.323128	
8	8	5.414715	0.043963	0.351704	
9	8	5.156871	0.04187	0.376826	
10	108	66.30263	0.538323	5.383227	
		123.1652		7.541931	D
				7.182792	MD

Interpretation

The formula of D can be derived as (1) and (2) below.

(1) $D = 1 + (i/y) + \{T(y - c/F) - (1+i)\}/[[\{(c/F)(1+i)^{\wedge}T\} - 1] + y]$

(2) Rise in c/F would lead to fall in denominator and rise in numerator, so D will fall.

Similarly from the following expression one sees that Duration falls when y rises.

Example

For an annual bond 6% coupon, 9% yield, T 4 years, MD reduces after 1 year elapses in the bond's life.

settlement	17-Oct-07
maturity	17-Oct-11
coupon	6%
yield	9%
frequency	1
D	3.653891
MD	3.352193
settlement	17-Oct-08
maturity	17-Oct-11
coupon	6%
yield	9%
frequency	1
D	2.82621
MD	2.592853

3.8. Price value of a Basis Point (PVBP)

Concept

PVBP is a special case of Duration.

PVBP is defined as the extent of the price shock caused by 1 bp yield shock

PVBP is measured as the absolute difference between the pre and post shock price.

Whether yield shock is 1 bps negative or positive, price-shock is uniform in the case of duration, therefore it is same in the case of PVBP also.

Example of PVBP: 5 year maturity bond of 9% coupon, yield shock 0.01%

t	CF@9%	PVCF@8%	PVCF@8.01%	PVCF@7.99%	
1	9	8.3333333	8.3325618	8.33410501	
2	9	7.7160494	7.714620683	7.717478479	
3	9	7.1444902	7.142505956	7.146475117	
4	9	6.6152687	6.612819143	6.617719342	
5	109	74.183569	74.14923377	74.21792226	
Price		**103.99271**	**103.9517414**	**104.0337002**	

Price	Yield%	ΔP/P	Δy/(1+y)	D	PVBP
104.0337	7.99%	0.0003942	-9.26E-05	4.256970378	0.041
103.9927	8%	0	0		
103.9517	8.01%	-0.000394	9.26E-05	4.254738642	0.041

3.9. Properties of Duration

1. Duration rises with maturity.
2. Capital gain for negative y-shock is more than capital loss for equivalent positive y-shock (example as follows).

t	CF@9%	PVCF@8%	PVCF@1%	PVCF@15%
1	9	8.333333333	8.9108911	7.826086957
2	9	7.716049383	8.8226644	6.805293006
3	9	7.144490169	8.7353113	5.917646092
4	9	6.615268675	8.6488231	5.14577921
5	109	74.18356848	103.70976	54.19226415
Price		**103.99271**	**138.82745**	**79.88706941**

Price	Yield%	abs(DP)	
138.8274	1%	34.83473988	capital gain
103.9927	8%		
79.88707	15%	24.10564063	capital loss

3. Duration rises with maturity, *cet par*.
4. With rise in maturity, D rises at a decreasing rate.

settlement	10-Oct-07		settlement	10-Oct-07
maturity	9-Oct-09		maturity	9-Oct-10
rate	9%		rate	9%
yield	8%		yield	8%
redemption	100		redemption	100
frequency	1		frequency	1

D	1.915349		D	2.75952086
settlement	10-Oct-07	**T**	**D**	**ΔD**
maturity	9-Oct-11	2	1.915349	
rate	9%	3	2.759521	0.84417195
yield	8%	4	3.536709	0.77718825
redemption	100			
frequency	1			
D	3.536709			

5. Duration varies inversely with coupon rate s
6. Duration varies inversely with YTM.

The first 5 properties of D are known as Malkiel's bond-pricing relationships.

The following properties are known as rules of Duration (D).
1. $D = T$ for a ZC bond, T means maturity
2. D varies inversely with c/F cet par.
3. D generally rises with T cet par for all traded bonds, but at a decreasing rate.
4. D varies inversely with YTM cet par.
5. $D = 1 + (1/y) = f(y)$ for perpetuity
6. $D = 1 + (1/y) - (T/(1+y)^T - 1) = \phi(y, T)$ for an annuity
7. $D = 1 + (1/y) - [(1+y) + \{T*((c/F) - y))\}]/ [\{(c/F)/(1+y)^T - 1\} + y] = \psi(c/F, y, T)$ for a coupon paying bond.
8. $D = 1 + (1/y) [1 - (1+y)^{(-T)}] = \xi(y, T)$ for a coupon paying bond when $F = P$

The relationship between D and its determinants c/F, y and T are explained with help of D-T graph of the following four bonds

Example of D-T relationship across different combinations of coupon and yield with same dates of maturity and settlement
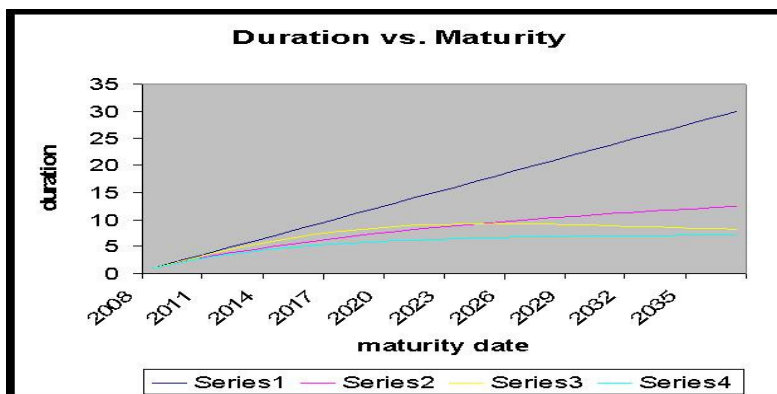Settlement of all bonds: 23-Oct-2007
Series 1: ZC bond
Series 2: 15% coupon, 6% YTM
Series 3: 3% coupon, 15% YTM
Series 4: 15% coupon, 15% YTM

Description
1. The dark blue straight line through origin $\Rightarrow D = T$ for ZC bond
2. The light blue D line below the yellow D line $\Rightarrow D\downarrow$ when $c/F\uparrow$ for c bond.
3. The slope of the D line for any coupon paying bond is positive slope but less than unity $\Rightarrow D\uparrow$ when $T\uparrow$

4. The pink D line is above the light blue D line $\Rightarrow$ D$\downarrow$ when y$\uparrow$ for c bond.
5. The light blue and yellow D lines are converging to the D of a perpetuity with 15% y as T is increasing $\Rightarrow$ D of a perpetuity depends only on y.

3.10. Computation and Use of Convexity
Computation
The linear relationship between price and yield is not however true for large changes in bps like 700 bps. Let us recall the earlier example of 9% 5 years security when YTM is 8%. CCIL measures convexity as the average price shock when there is equal yield shock up and down. This is done by applying the formula $(V_- + V_+ - 2V_0)/2V_0(\Delta y)^2$, V_- is the price after negative yield shock, V_+ is the price after positive yield shock, V_0 is the original price, Δy is the yield shock. International consultants like Frank Fabbozi follow this method. At YTM = 8%, V_0 = 109.896; at YTM = 7.8%, V+ = 112.05; at YTM = 8.2%, V- = 107.8.

settlement	19-Aug-08	19-Aug-08	19-Aug-08
maturity	19-Aug-28	19-Aug-28	19-Aug-28
coupon	9%	9%	9%
yield	8%	7.8%	8.2%
par	100	100	100
freq	2	2	2
Price	109.89639	112.0544581	107.80065

So the convexity measure is $(V_- + V_+ - 2V_0)/2V_0(\Delta y)^2$ = 70.89. Modified duration of the same security at 8% yield is 10.06. This means for any large yield shock Δy, the change in price
$\Delta P = -MD^*P^*\Delta y + CVX^*(\Delta y)^2$. The first term of the expression is positive or negative depending on negative or positive yields shock while the second term is always positive. Thus, for positive yield shock, price fall is partially neutralized by the convexity factor and for negative yield shock price rise is reinforced by the same. Therefore convexity is called a desirable property of investment portfolio. The portfolio should be as convex as possible

Use of Convexity in Portfolio Strategy
We have considered two hypothetical portfolios:
Portfolio (i) 9.25% 10 year security with convexity 55.45, weight 100%, yield 9.25%,
Portfolio (II) The above security combined with another security 8.5% 5 year with convexity 19.81 and yield 8.5% at a ratio 49.81:50.19 such that the combined convexity is 71.19 - more than portfolio (i).
Then we give yield shocks of different magnitudes ranging from -4% to +4% to both these portfolios and compute their returns after each yield shock. The portfolio with greater convexity is found to generate more return than the portfolio with less convexity. The differences in return for different yield shocks are given below:

Yield Shock	Difference
0	
-4%	0.020731933
-3.50%	0.014966895
-3%	0.010372365
-2.50%	0.006797072
-2%	0.004106606
-1.50%	0.002181631
-1%	0.000916288
-0.50%	0.000216761
-0.40%	0.00013727
-0.30%	7.64415E-05
-0.20%	3.36758E-05
-0.10%	8.38599E-06

-0.05%	2.11523E-06
0	0
0.05%	1.97096E-06
0.10%	7.95963E-06
0.20%	3.17204E-05
0.30%	7.07514E-05
0.40%	0.000124535
0.50%	0.000192565
1%	0.000729603
1.50%	0.001553927
2%	0.002615189
2.50%	0.003869141
3%	0.005276979
3.50%	0.006804741
4%	0.008422786

3.11. Convexity of Option Embedded Bonds

The nature of convexity depends on whether the bond is option free or has embedded options. So far what we have discussed belongs to option free bonds where neither can the investor put the bond nor can the issuer call the bond before maturity, happen what may to interest rate. Accordingly the shape of the yield-price curve varies. When yield declines, price of a callable bond rises less than a option free bond; the investor may not be able to reap capital gain if the bond is called back.

Example of Convexity of a Callable Bond: Suppose you want to calculate, before purchasing on behalf of your bank, the price in a 9% bond with par value 100. The bond will expire after 5 years and is expected to see a negative yield shock by 50 bps after two years of the settlement.
If the applicable yield is 8% for post-settlement years 1 and 2 and 7.5% for the remaining years of its life what will be the price of the bond if
   a.   If the bond is option free and hence expected to be held for next 5 years?
   b.   If the bond is callable and hence assumed to be called back at par immediately after receiving the yield shock?
Any sort of accrued interest is not assumed here.
The price of a 9% 100 par value bond with 5 years left to maturity at 8% YTM is 103.99271.

t	CF	PVF@8%	PVCF@8%
1	9	0.925926	8.33333333
2	9	0.857339	7.71604938
3	9	0.793832	7.14449017
4	9	0.73503	6.61526868
5	109	0.680583	74.1835685
Price			**103.99271**

As the expected yield falls to 7.5% after two years, the price rises to 105.958 if the bond is option free, i.e. held till maturity (HTM).

t	CF	PVF	PVCF
1	9	0.925926	8.33333333
2	9	0.857339	7.71604938

3	9	0.804961	7.24464513
4	9	0.748801	6.73920477
5	109	0.696559	75.9248909
Price			**105.958124**

### yield-price relationship for option free bond in the case of negative yield shock
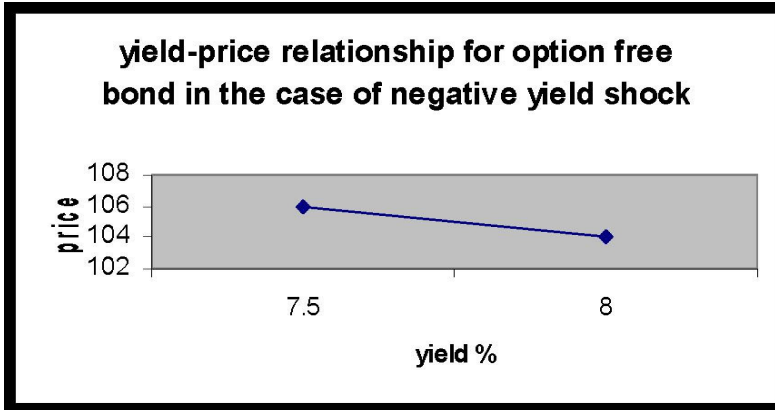
As the yield falls to 7.5% after two years, the price rises to 104.91 instead of 105.958 if the bond is called after third year.

t	CF	PVF	PVCF
1	9	0.925926	8.33333333
2	9	0.857339	7.71604938
3	9	0.804961	7.24464513
4	109	0.748801	81.6192577
Price			**104.913286**

yield-price relationship for a callable bond in the case of a negative yield shock
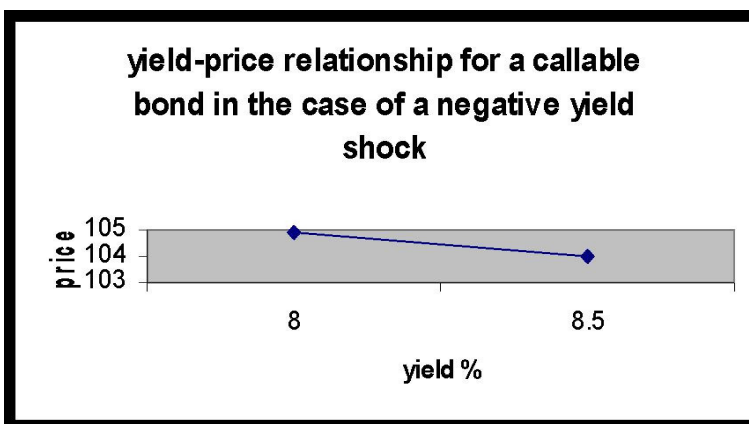
In the same manner tt is observed that the price-hike of a callable bond is less in an initially low yield position compared to an initially high yield position, cet par. This means as yields goes down and down the ratio of price-shock to yield shock becomes less and less.

Diagram II
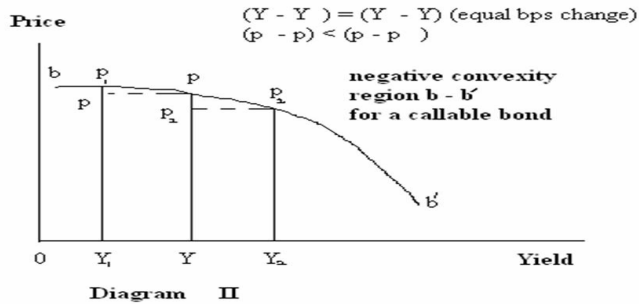
Intuition

Lower the initial yield the lower is expectation of further y-fall and hence lower is the willingness to buy the bond in anticipation of call-back by the issuer and hence the slower the process of price-hike. The anticipation of call-back does not exist with option free bond.

In the above diagram, the shape of the curve b′b is known as negative convexity since it is opposite to the convex yield-price curve of an option paying bond. The convexity of the yield-price curve of an option paying bond is also known as positive convexity.

Example: For a 100 par 9% bond 10 years to mature, the price rises from 106.7 to 108.857 when the initial yield 8% reduces to 7% after 4 years and the holder calls back at par after receiving the first yield shock. Any sort of accrued interest is not assumed here.

t	CF	PVCF 8%
1	9	8.333333333
2	9	7.716049383
3	9	7.144490169
4	9	6.615268675
5	9	6.125248773
6	9	5.671526642
7	9	5.251413557
8	9	4.862419961
9	9	4.502240704
10	109	50.4880902
Price		106.7100814

t	CF	PVCF
1	9	8.333333
2	9	7.716049
3	9	7.14449
4	9	6.615269
5	9	6.416876
6	109	72.6313
Price		108.8573

For the same bond the price rises from 114.04 to only 114.05 when the initial yield 7% reduces to 6% after 4 years of settlement, other things remaining unchanged.

t	CF	PVCF 7%
1	9	8.411214953
2	9	7.860948554
3	9	7.346680892
4	9	6.866056908
5	9	6.416875615
6	9	5.997080014
7	9	5.604747677
8	9	5.238081941
9	9	4.895403683
10	109	55.41007284

t	CF	PVCF
1	9	8.411215
2	9	7.860949
3	9	7.346681
4	9	6.866057
5	9	6.725324
6	109	76.8407
Price		114.0509

Price		114.0471631

Similarly it can be found when yield rises price for option free bond falls more than a putable bond since the issuer may not be able to save interest expenses in absence of any exit charge, rather, end up with more cash outflow if the investors exercises the put right.

3.12. How to Measure Convexity

Convexity (cx) is interpreted as the curvature of the price-yield curve as a fraction of bond price and accordingly measured below. Convexity is measured as

$$\frac{1}{P} * (d^2P/dy^2)$$

$$= \frac{1}{P * (1+y)^2} \sum_{t=1}^{T} [\frac{CF_t}{(1+y)^t} * t * (t+1)]$$

Since convexity is a measure of curvature it means rate of change in average cash flow timing the PV of cash flow needs to be multiplied by $(t+1)/(1+y)$.

Applying Taylor's series one can derive the price-shock formula for large y-shock as follows as shown by Bodie *et al* 2005

$$\frac{\Delta P}{P} = \{-MD * \Delta y\} + \{\frac{1}{2} * cx * (\Delta y)^2\}$$

Since convexity is the rate of change in average timing of cash flow we need two consecutive yield shocks in order to understand their impacts on price. Hence the convexity measure needs to be multiplied

by $\frac{(\Delta y)^2}{2}$.

Example

Consider the same bond;

When YTM rises from 8% to 10%, price changes to 811.46 from 1000; but

$\Delta P/P = -MD*\Delta y = -11.26 * 0.02 = -0.2252$ as per simple MD measure, while

$\Delta P/P = [-MD*\Delta y] + [(1/2)*cx* \{(\Delta y)^2\}]$

$= (-11.26*0.02) + [0.5*212.4*\{(0.02)^2\}]$

$=-0.1827$; this is far closer to actual price change.

Settle	10/23/2007
Mature	10/23/2037
Rate	8%
Y	8%
Redeem	1000
F	1
MD	11.25778334
P	1000
Settle	10/23/2007
Mature	10/23/2037
Rate	8%
Y	10%
Redeem	1000

F	1
MD	9.678982482
P	811.4617107
ΔP/P	-0.188538289

3.13. Effective Duration

Some investors prefer computing effective duration (ED) over D for option embedded bonds.

ED is percentage change in price for 100 bps change in market interest rate (r, not YTM), 50 bps change on each side (up and down) of the initial/current market rate of interest, since YTM is not relevant for premature bonds.

So ED = -(ΔP/P)/Δr,

where Δr = rise in r – fall in r

ΔP = P at higher r – P at lower r

Example

Suppose a bond selling today for P = 109.9.

If the y-curve shifts up by 0.5%, P = 103.2 and if y-curve shifts down by 0.5%, P = 106.66

Compute the effective duration.

Δr = rise in y – fall in y = 0.5% - (-0.5%) = 1% = 0.01

ΔP = P at higher y – P at lower y

= 103.20 – 106.66 = - 3.46

ED = - (ΔP/P)/Δr = - (-3.46/104.9)/0.01 = 3.29

Interpretation

How much the price varies over the rate interval of 100 bps around the current rate with 50 bps on each side.

In our example it is 3.29%.

3.14. PVBP and Convexity

PVBP is defined as price-change for 1 unit change in YTM, thus PVBP = ΔP/Δy.

Convexity cx = {Δ(dP/dy)/dy}(1/P)

{Δ(ΔP/Δy)/Δy}.(1/P) = {ΔPVBP/Δy}.(1/P)

Example

For a 2 year ZC with y = 10%, P = 82.6446, PVBP at 10% is 150.2425 and the PVBP at 10.1% is 150.2015.

So ΔPVBP/ Δy = (150.2425 -150.2015)/0.01% = 410

So cx = 410/82.6446 = 4.96

3.15. Modified Duration-Gap

Modified Duration Gap or MD-Gap is defined as the leverage adjusted gap between the modified duration of asset portfolio (MD_A) and the modified duration of liability portfolio (MD_L). The leverage or liability to asset ratio (κ) plays an important role in determining the magnitude of the impact of an average YTM shock (ΔY) on asset portfolio which transmits the shock to equity or net worth. The average YTM means the weighted average of YTMs of all assets and liabilities. Wider the D-Gap the stronger would be the interest rate sensitivity of equity and vice-versa. A positive D-Gap would lead to depletion of equity in a state of rising YTMs and vice versa.

The impact of D-Gap on equity is measured as ΔE = -(MD_A – κMD_L)*MVA* ΔY

In the simplest example, let us suppose MVA = 150, MVL = 125, MD_A = 4, MD_L = 2. Hence κ = (MVL/MVA) = (125/150) $\cong$ 0.8. Now MD-Gap = 4 – (0.8*2) = 2.4. If there is an upward average YTM shock worth 0.5%, the reduction in equity is ΔE = - 2.4*150*0.005 = - 1.8. If there is a downward average YTM shock worth 0.5%, the reduction in equity isΔE = - 2.4*150*(-0.005) = 1.8.

3.16. Shortcomings of Duration

The main problems with duration model are given below with their probable solutions.

Problem 1: Duration model does not throw light on chances of default.
Solution: Calculate Modified Duration using the weighted average formula. In the default cases add risk premium to the yield and then calculate Mduration and replace cash flows by expected cash flows. Instead of Y, discount with Y+P, where Y is yield and P is premium. The formula of discounted probable future cash flow pC/(1+Y+P), where C is future cash flow, p is the probability.

Example of Expected-default Bonds

Suppose the risk premium is forecast to be 2% such that the required rate of return should be 11% instead of 9%. Hence the MD price falls from 1.805 to 1.786.

settlement	1/1/2008
maturity	1/1/2010
coupon rate	8%
yield	9%
frequency	2
redemption	100
Modified Duration	1.805158585
settlement	1/1/2008
maturity	1/1/2010
coupon rate	8%
yield	11%
frequency	2
redemption	100
Modified Duration	1.785840649

Interpretation
Higher the risk, higher the expected return or yield and hence less MD.

Problem 2: Duration model does not throw light on floating rate bonds (FRB).
Solution: Use the concept of ZC bond.
A FRB could be looked upon as two ZC bonds maturing at the time of next coupon and hence having the average duration.

Example
Suppose you bought a bond of T years maturity now and expect 1st coupon payment as well as repricing of the bond after 6 month when the coupon rate will be reset in tune with 6 month treasury bond spot rate. So the value of the bond now has two parts – present value of the expected coupon receipt and present value of the expected resale proceeds. These two values should be a portfolio like two discount bonds of the same maturity. Since maturity equals duration for ZC bond, in the above case the portfolio duration is 6 months.

Problem 3: Duration model does not throw light on non-flat term structure.
Solution: Calculate Duration using the weighted average formula.
Discount every coupon payment with the spot rate of zero coupon bond of that particular maturity instead of a rate.

Example of Flat and Non-flat Term Structures
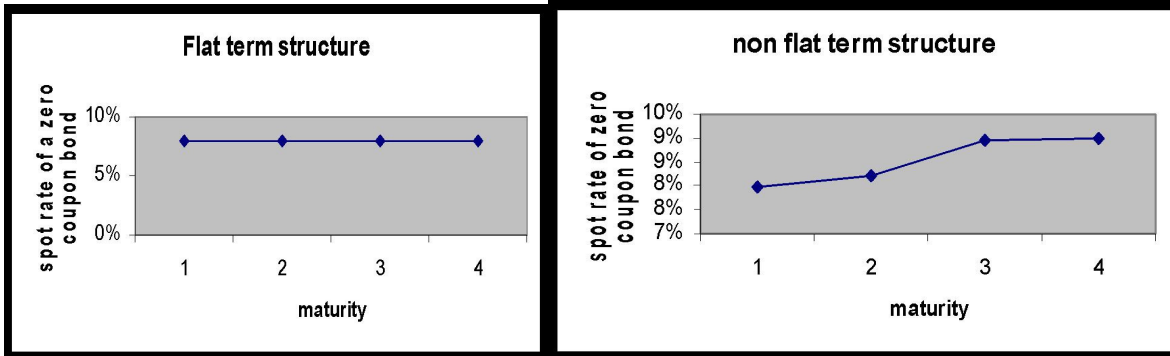
t	CF	y	PVF	PVCF
1	6	8%	1.08	5.5045872
2	6	8%	1.1664	5.05008
3	6	8%	1.259712	4.6331009
4	106	8%	1.360489	75.093072
	P			**90.28084**

t	CF@9%	y	PVF	PVCF
1	6	8%	1.0799	6.4794
2	6	8.20%	1.170724	7.024344
3	6	8.95%	1.293248	7.759486
4	106	9.01%	1.4121	149.68257
	P			**170.9458**

3.17. Critique of Modified Duration and Need for VaR

Though modified duration gives an estimate of capital loss in the phase of adverse interest rate movements, duration cannot be accepted as a measure of market risk because it does not
i. apply to other category instruments in portfolio like equity, commodity and forex and hence cannot help aggregation of risks across the asset categories in the portfolio.
ii. tell the probability of a particular yield shock
iii. help computation of MRAROC (market risk adjusted return on capital).
iv. capture the reality that yield shocks across the maturities can hardly be uniform
Further Modified Duration as a measure of price risk is not uniformly used by the Banks in other Countries[xvii]. Since Banks in India are in a nascent stage of development as per RBI, they are supposed to use Modified Duration as a measure of price risk in debt securities[xviii].
Therefore a better measure of market risk has been devised by J. P. Morgan called "value at risk" or VaR.

Chapter 4: Value at Risk

4.1. Definition
VaR is defined as the maximum prospective capital loss with certain probability or confidence like 99% or 95%. Thus 99% VaR is 100 crore means that there can be loss beyond 100 crore with a probability of 1%. Statistical concepts like confidence interval hypothesis testing required for understanding VaR is given in the Appendix II.

4.2. Methodologies of VaR Computation with Examples and Critiques
There are three methods of VaR portfolio – VaR-Covariance Method, Historical Simulation Method and Monte-Carlo simulation Method. Each method has two sub-methods – simple VaR and weighted VaR. In India the most popular method among the banks is simple Historical Simulation Method. But for understanding the nuances of VaR concept it is necessary to know VaR-Covariance Method. This knowledge helps designing portfolios characterised by negative correlation. CCIL, however, computes VaR following three methods for every security on every trading day.

4.2.1. Simple VaR-Covariance Method or Normal VaR Method
4.2.1.1. Normal Distribution
VaR corresponds to the lower limit of 99% confidence interval of the probability distribution of the return of an asset or portfolio. It is assumed that in a sufficiently liquid market where number of players are fairly large on both supply and demand sides and the market has other characteristics of perfect competition price is determined by sheer demand and supply forces the probability distribution return of an asset or portfolio follows normal distribution. Maximum number of return figures here lie around the average and any deviation from the average in either side, below or above is equiprobable. The average return is close to zero, i.e. no profit no loss position. Hence 3% positive and negative returns are equally likely and nobody can guarantee a higher probability of 3% profit than 3% loss. If such guarantee exists the market is not perfect and there is some manipulation. Such a probability distribution is called normal distribution. If the sample has average close to zero and standard deviation close 1 the probability distribution is called standard normal distribution. It is empirically found that ±2.33 are the lower and upper limits of symmetric 99% area under the standard normal distribution; similarly ±1.64 correspond to symmetric 95% area and so on.

4.2.1.2. Basel II
Basel II prescribes a sample size of 1000 daily return figures, data of 4 years each year having 250 trading days, and a probability of 99% for VaR computation. For every 1 simple (i.e. all past return figures are given equal weight 1 irrespective of whether they are near past or remote past) standard deviation (SD) and Re. 1 investment, the VaR of daily return with 99% confidence is 2.33. This means for Rs. 100 investment, there is probability of daily loss beyond Rs. 2.33 is 1%. If the sample of daily return has SD = 5%, the portfolio size is 100 crore then 99% daily VaR is 100*5%*2.33 = 11.65 crore. This method is known as Variance-Covariance VaR because VaR results in this method are influenced by the variance of the portfolio and covariance in the assets. If the assets are negatively correlated the portfolio VaR is less than the sum of the individual VaRs of the assets.

4.2.1.3. Example
We have taken a portfolio of 100 scrips of each of two banks – SBI and BOB. We collected samples of size 1000 return figures on the individual stocks as well as the portfolio from 28-Jun-04 to 13-Jun-08 in order to calculate VaR the next day as on 16-Jun-08. The simple SDs of SBI, BOB and portfolio are 0.022608, 0.02897 and 0.0226 and their averages are close to zero. Then we calculate 99% daily VaR of each of them as on 16-Jun-08 (the asset values were 132000, 24000 and 156000 for SBI, BOB and portfolio respectively) and found the 99% VaR figures –
2.33* 0.022608*132000 = 6942.382 for investment worth 132000 in SBI scrips.
2.33*0.2897*24000 = 1617.46 for investment worth 24000 in BOB scrips .
2.33*0.0226*156000 = 8201.841 for investment worth 156000 in the portfolio.

All of the above are 1-day VaR figures. If the holding period is more than 1 day like 10 days then the above figures are to be multiplied by √10 because the variance of 10 days returns is 10*variance of daily returns; variance of daily returns are uncorrelated each assumed to follow independently the standard normal distribution. Therefore the SD of 10 days returns is √(10*variance of daily returns) =(√10)*SD of daily returns

Weighted VaR-Covariance Method: Here the weighted standard deviation is calculated for the assets as well as the portfolio. The calculation of VaR is the same as above except replacement of simple SD by weighted SD. Calculation of the weighted SD follows J.P. Morgan's Risk Metric Method:

Weighted SD = $\sqrt{[(1-\lambda)\sum_{t=1}^{T}\lambda^{t-1}(r_t - \bar{r})^2]}$, where decay factor λ = 0.94 as suggested by J.P.

Morgan and T is sample size. In the avove sample we found weighted SDs are 0.000002153, 0.007098614, 0.000001766 respectively for SBI, BOB and portfolio. Weighted SDs are less than simple SDs since the weights of all return figures are less than 1 in the case of the former. In rest of the discussions we shall be discussing the simple versions of historical and Monte Carlo simulation.

Simple VaR-Covariance Method has certain disadvantages; these will be discussed under 'Stress Testing'.

4.2.2. Simple Historical Simulation Method
4.2.2.1. Process
In this method the entire sample is arranged in ascending order and a serial number from 1 to 1000 is allotted to each figure. The lowest figure is allotted number 1 and the highest figure is allotted number 1000. It is assumed the first 1% left area under standard normal probability distribution curve covers the return figures numbered 1-10, the second 1% left area covers the return figures numbered 11-20 and so on. So the figure numbered 11 is taken to be the lower limit of 99% confidence interval. This method is simple and does not require the knowledge of variance or covariance.

4.2.2.2. Example
In the sample of portfolio return, -0.060009786 is found to be daily return figure taken to be the lower limit of 99% confidence interval.

4.2.2.3. Critique
Simple historical method works better than simple VaR-Covariance Method in certain cases but it cannot work with small samples.

4.2.3. Monte Carlo Simulation (MCS) Method
This method involves building scenarios and generating returns under new scenarios and then compute VaR. This method has several versions depending upon the choice of the analysts. One such version consists of
1. ascertaining the probability distribution of the existing sample with the help of softwares like Bestfit® and noting down the parameter values
2. feed those parameter values in the same software or other softwares like @Risk® or Easyfit and generate as many samples required following the same probability distribution.
3. arranging each sample in ascending order and detect 99% VaR as per historical simulation method or calculate VaR as per VAR-COVAR method.
4. select the maximum, average, mode or median of the above VaR figures
The application of MCS method would be discussed under "Back Testing". MCS method is popular for computing VaR for derivatives.

4.3. Critique to VaR – Need for Back testing and Stress testing
Banks over the globe faced a number of short comings of VaR as follows:

i. Even the largest size of sample of asset return in an efficient market may not follow normal probability distribution. VaR model is too simplistic to assume normal or lognormal distribution of returns[xix].

ii. VaR is silent about the quantum of losses on the left side of the confidence interval.

iii. Under some circumstances VaR fails to detect market risk.

iv. If the sample is biased VaR estimation would be breached violating the level of confidence.

Therefore VaR should be validated or back tested and stress tested.

Chapter 5: Back testing and Stress Testing

5.1. Back testing
5.1.1. Concept and Procedure
Back testing of VaR means validating the result from one sample in other samples. In backtesting what is confirmed is whether the number of VaR breaches is more than 10 in other samples of size 1000. Basel II gave a schedule of penalty for different numbers of braches of VaR in every consecutive 250 data points. There is no penalty towards a number of violations of VaR up to 4 in 250 data points. This is done in two ways:
a. If sample size is sufficiently large, as in the developed countries like US, say a multiple of 1000, it is divided into a number of samples of size 1000; then VaR is calculated out of one sample and validated in other samples.
b. If sample size is small, the probability distribution of the sample as well as parameters of the distribution are extracted with the help of softwares like Bestfit or Easyfit. Then feeding those parameters in the same software several pseudo-random samples are generated. In one such sample VaR is calculated and verified in other samples.

5.1.2. Example
Let us refer to the earlier case of portfolio of stocks of SBI and BOB. Suppose our sample size is 1492, almost 1.5 times of 1000. The sample covers adjusted close prices of the two scrips from 12-Aug-2002 to 13-Jun-08. We divide it into two samples – sample I of size 1001 and sample II of size 491. The portfolio HS VaR on the basis of sample I is -0.060009786. This means validation requires maximum 12 (= 5+7) violations of above VaR in sample II – 1% data points of sample II is 4.9 combined with maximum 7 additional data points, total $11.9 \cong 12$ data points are allowed to breach VaR without any penalty. In this case the numbers of data points beyond ± 0.060009786 are found to be 7. Hence this model can be passed. The same process can also apply to computing HS VaR for individual scrips SBI and BOB.

Now let us assume that we have only one sample of size 1000 from 28-Jun-04 to 13-Jun-08 of the same portfolio. The portfolio HS VaR is -0.060009786. In order to validate it we need other samples. So we need other samples of size 1000 with same probability. Here it was assumed that probabilities of portfolio returns are following normal distribution with average = 0.00128854 and SD = 0.0226002. Feeding these parameters into the simulation software a number of samples can be generated for back testing. One such sample we have generated in MS Excel and found the number of breaches to be 5. This is called one round of simulation. Statistically 10000 rounds of simulation is prescribed.

5.2. Stress Testing
5.2.1. Concept and Procedure
Stress means impact of adverse moves in portfolio return on VaR. Stress testing means computing VaR in the hypothetical stress scenarios. Therefore stress testing presupposes scenario building. It was an individual desk level exercise prior to 1996but now is a Bank level exercise as per RBI guidelines on 26-Jun-07.
The need for stress testing arises out of the following weaknesses of VaR:
•   different methodologies give different VaR numbers in the same sample.
•   different samples give different VaR numbers with same methodologies
    VaR does not describe the vulnerability of the portfolio to
        i. breakdown of normal correlations like 23% crash in Dow Jones in October 1997
        ii. sudden decrease in liquidity – as happened in mid 2004 in India
        iii. concentration risk – recent sub prime crisis
        iv. macroeconomic risks – Orange County disaster December 1994 following rise in interest rate by Fed.
•   VaR does not describe the losses beyond the confidence limits.
Hence the need for stress testing and computation of extreme tail losses.

5.2.2. Benefits of Stress testing
- Gives future view about requirements of risk capital and liquidity
- Detects failure of VaR to determine risk capital for some specific compositions of pf.
- Detects vulnerability of pf to risk factors like sudden illiquidity.

5.2.3 Examples
Here we have built up three scenarios – 1st and 2nd scenarios demonstrate the failure of VaR to detect risk and the 3rd scenario demonstrates VaR computations amidst variations in portfolio return in diverse interest rate scenarios.

We have taken the same portfolio of 100 scrips of each of SBI and BOB 28-Jun-04 to 13-Jun-08.

I.   We compute VAR-COVAR 99% portfolio VaR and compare it with the portfolio VaR computed after giving 1.5 SD shock to each daily return figure. It is found that VaR remains unaffected. We assumed that all returns fell by 1.5 SD. Since SD is not influenced by change in origin VaR remains unchanged. This is a scenario of failure of VaR.

II.   Next we assume all return figures became half. It is found that VaR also became half. Since SD is influenced by change in scale of the variable in the same direction and to an equal extent VaR falls by 50% as all daily return fall by 50%. This is another scenario of failure of VaR.

III.   We take a portfolio of 2 semi annual bonds having 10 and 20 year maturities respectively. We compute portfolio returns for a holding period of 3 years. Holding period return (HPR) is the annualized rate of return on the initial investment on two bonds out of the 3 years' accumulated sum consisting of coupon reinvestment and resale of the bonds. The HPR varies directly with coupon reinvestment rate and inversely with the YTMs prevailing at the end of 3 years. We have imagined 100 combinations of coupon reinvestment rates and YTMs at the end of 3 years and calculated 100 HPS under those scenarios. Next we sorted the HPRs in ascending order and detect 99% VaR. This is a scenario where VaR is successful in detecting the impact of interest rate variations on portfolio return.

5.3. Extreme Tail Losses
Since VaR does not tell losses beyond the prescribed confidence limit we should compute those losses and find out their maximum values. Let us suppose the prescribed probability is 90%. There is need to compute VaR at each point on tail of the normal distribution curve between 95% and 100%. Dowd (2002) divided the left tail into 10 equal parts – 95%, 95.5%, 96%, …., 99.5% and computed daily VaR of one unit of currency investment with SD = 1 and found the average loss 1.986962 as follows:

95%	1.644853
95.50%	1.695397
96%	1.750686
96.50%	1.81191
97.00%	1.880793
98%	1.959963
98.00%	2.053748
98.50%	2.170089
99%	2.326347
99.50%	2.575831
Avg	1.986962

Thus for 10 equal parts of the tail the average is 1.986962, for 25 equal parts it is 2.0273 and so on. He found the following table of average losses for different numbers of divisions of the left tail:

n	Avg ETL
10	1.987
25	2.0273

50	2.0432
100	2.0521
250	2.058
500	2.0602
1000	2.0614
2500	20.621
5000	2.0624

The VaR indicative of maximum average loss is found to be 2.0624.
The above exercise can also be carried out for 95% and 99% VaRs.

Chapter 6: Market Risk Capital Charge

6.1. Duration-based Capital Charge

In India, as per RBI guidelines market risk capital charge for government securities is based on modified duration[xx]. As per, RBI The banks are suppose to make general market risk charge towards interest rate risk in the portfolio where long and short positions in different securities or instruments can be offset. In India RBI allows short position only in derivatives and Central Government Securities. RBI instructs the banks to provide the capital charge for interest rate risk in the trading book other than derivatives as per the guidelines given below for both specific risk and general risk after measuring the risk of holding or taking positions in debt securities and other interest rate related instruments in the trading book. Risk charges for individual securities based on yield shocks prescribed by the RBI for different maturity buckets as follows (Table 1) and vertical and horizontal disallowances (Table 2) in the trading book in order to immunize it against basis risk and yield curve risk.

Table (1)

Time Bands	Assumed Change in Yield
Zone 1	
1 month or less	1
1 to 3 months	1
3 to 6 months	1
6 to 12 months	1
Zone 2	
1.0 to 1.9 years	0.9
1.9 to 2.8 years	0.8
2.8 to 3.6 years	0.75
Zone 3	
3.6 to 4.3 years	0.75
4.3 to 5.7 years	0.7
5.7 to 7.3 years	0.65
7.3 to 9.3 years	0.6
9.3 to 10.6 years	0.6
10.6 to 12 years	0.6
12 to 20 years	0.6
over 20 years	0.6

Table (2)

Zones	Time band	Within the zones	Between adjacent zones	Between zones 1 and 3
Zone 1	1 month or less	40%		
	1 to 3 months			

Zone	Maturity bucket			
	3 to 6 months			
	6 to 12 months			
	1.0 to 1.9 years		40%	
	1.9 to 2.8 years			
Zone 2	2.8 to 3.6 years	30%		100%
	3.6 to 4.3 years			
	4.3 to 5.7 years			
	5.7 to 7.3 years			
	7.3 to 9.3 years		40%	
	9.3 to 10.6 years			
	10.6 to 12 years			
	12 to 20 years			
Zone 3	over 20 years	30%		

Vertical disallowance is made for handling basis risk between asset and liability yield curves within the same maturity bucket by keeping 5% as capital of whichsoever is smaller between asset and liabilities. Horizontal disallowance is made within a bucket or between two consecutive buckets by keeping 40% as capital of whichsoever is larger and between two distant buckets like 1st and 3rd buckets by keeping 100% as capital of whichsoever is larger since the difference between the new (post-shift) yield curve and the old (pre-shift) yield curve becomes larger towards the longer maturity.

Example
In what follows we have taken a hypothetical trading book where on the asset side there are four government securities and on the liability side there are fixed-pay obligations of swap contracts across three maturity buckets. There are however two government securities in the 1st maturity bucket whereas there is one instrument in every bucket of asset and liability. Total risk capital is coming to be around 0.6 crore as follows:

MVA(Cr.)	TTM (yrs)	Mduration	ΔYTM	ΔMVA=individual risk chanrge(Cr.)
165	0.5	0.4	1%	0.66
66	0.4	0.35	1%	0.2079
79	2.4	1.9	0.80%	1.2008
38	12.4	11.1	0.60%	2.5308

MVL(Cr.)	TTM (yrs)	Mduration	ΔYTM	ΔMVL=specific risk chanrge(Cr.)
181	0.75	0.7	1%	1.267
61	2.8	2.5	0.80%	1.22
36	15	14.2	0.60%	3.0672

	Vertical Disallowance (Cr.)	Horizontal Disallowance (Cr.)	zone(s)	Sum (Cr.)
	0.043395	0.5068	within	0.220078
	0.06004	0.024016	1&2	0.084056

	0.12654	0.050616	2&3	0.177156
		0.12654	1&3	0.12654
				Total = 0.60783

6.2. VaR-based Capital Charge

Though RBI does not require the Banks to provide VaR capital, Basel II does. As per Basel II Banks need to keep capital, in addition to specific risk capital, three times of whichsoever is larger between the current day's portfolio VaR and the average of last 60 days portfolio VaR[xxi]. The rationale behind the above multiplication by three lies in the fact that in real life the asset return series may not follow normal distribution. For example, in India, it is found that returns on stock prices of some Bank of Baroda follow Log-logistic distribution and in these cases VaR (8.25) is more than 3 fold of the VaR computed on normal distribution assumption (see the figure below). Secondly, in the case of normal distribution itself the 99% daily normal VaR for Re. 1 investment is 2.33 whose 100% counterpart is 7.93. Thus multiplying 99% normal VaR by 3 or more means acquiring 100% confidence that future loss won't exceed VaR.

Figure 1: 99% Daily VaR of discreet return on BOB of sample size 1000 from 25-Jun-04 to 13-Aug-08 follows Log-logistic distribution.

Logistic(0.00037515, 0.015519)
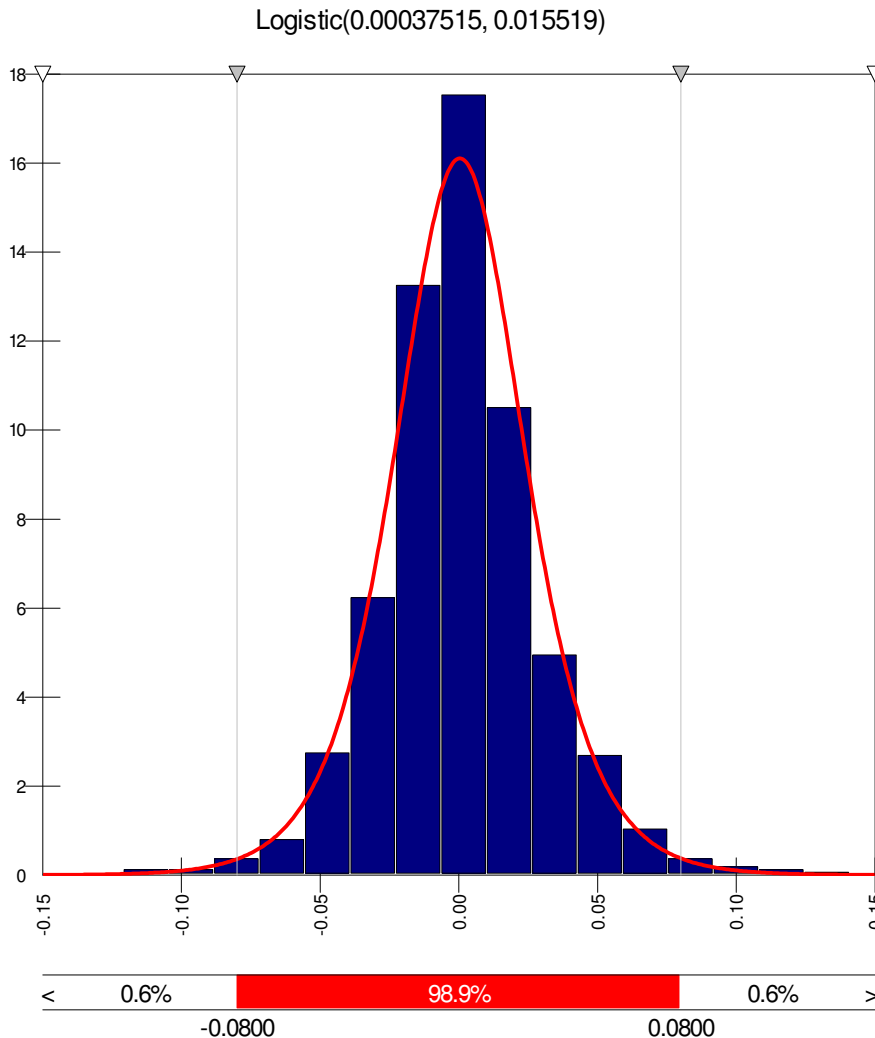
Chapter 7: Interest Rate Derivatives

7.1. Interest Rate Swap
Hedging market risk means entering interest rate derivative contracts – forward rate agreements (FRA) and interest rate swap (IRS) as per the 'Comprehensive Guidelines on Derivatives' of RBI dated 20-4-07. The inception of FRA and IRS in India dates back to 1999 as per RBI's "Forward Rate Agreements and Interest Rate Swaps – Guidelines" dated 07-07-1999[xxii].

7.1.1. Concepts
1. An interest rate swap is an agreement between two parties to exchange interest rate payments on a fixed (notional) amount of debt.
2. In a standard interest rate swap, one party to the swap agrees to pay a fixed interest rate in exchange for receiving a variable (floating) rate on the swap's notional amount.
3. The party agreeing to make fixed rate payment is called interest rate swap buyer; the party agreeing to make floating rate payment is called the interest rate swap seller.
4. An interest rate swap buyer seeks to convert the floating rate liabilities into fixed rate liabilities in order to better match the fixed earnings from its assets; an interest rate swap seller seeks to convert the fixed rate liabilities into floating rate liabilities in order to better match the variable earnings from its assets.
5. The three main types of swaps are
   a. coupon swaps - fixed rate to floating rate in the same currency,
   b. basis swaps - one floating rate index to another floating rate index in the same currency, like treasury bill rate for commercial paper
   c. cross-currency interest rate swaps - fixed rate in one currency to floating rate in another
6. Plain vanilla fixed-floating rate swaps are intermediates by a third party acting as an agent or broker bringing together two FIs who opposing interest rate risk exposure to enter into a swap agreement.
7. Floating rate security has shorter duration than fixed rate securities.
8. There are two kinds of hedging -
   a. Micro-hedging: Hedging a particular asset or liability through futures or forward contracts.
   b. Macro-hedging: Hedging all of assets and liabilities through futures or other derivatives like swap.

7.1.2. When to Buy or Sell an Interest rate swap?
Let us recall 3.15 and suppose a FI is facing a positive MD Gap. In order to hedge the above risk the FI or swap buyer wants to set $MD_A - \kappa MD_L = 0$ by reducing MD_A by transforming short term floating rate liabilities into long term fixed rate liabilities that better match the long term fixed rate asset portfolio. There are two ways of hedging:
   a) attract long term fixed rate deposits on balance sheet
   b) buy interest rate swap off balance sheet
In a reverse case, when MD Gap is negative, the swap seller wants to set $D_A - kD_L = 0$ by reducing D_L by transforming long term fix rate liabilities into short term floating rate liabilities that better match the short term floating rate asset portfolio. There are two ways of hedging:
   a) attract short term deposits on balance sheet
   b) sell interest rate swap off balance sheet

7.1.3. An Intuitive Example
1. Suppose Bank A finances fixed rate 100 cr. 10% long term loan on asset side with short term one year floating rate (indexed to say, MIBOR, for a global perspective one can think of LIBOR also) deposits.
2. Problem: fixed returns from asset side may not be enough to match the costs of the deposit when the market rate rises.
3. Suppose Bank A, instead of issuing long term fixed rate deposits decides to buy an interest rate swap.

4. Suppose Bank B finances short term 100 cr floating rate (indexed to say, MIBOR, for a global perspective one can think of LIBOR also) commercial loan on asset side with 10% 4 year deposits of the same maturity.
5. Problem: variable return on from asset side may not be enough to match fixed cost of the deposits when market rate falls.
6. Suppose Bank B, instead of issuing floating rate deposits, decides to sell an interest rate swap.
7. Suppose a third party brings Bank A and Bank B together who enter the following swap contract for the notional amount 100 cr.
8. Bank A may agree to pay 10% fixed rate on 100 cr. to Bank B and Bank B may agree to pay MIBOR + 2% to Bank A on 100 cr, when MIBOR is presently 8%.

7.1.4. A Small Example
A commercial bank invests in a MIBOR-linked instrument having raised resources through certificates of deposit. The bank would pay the proceeds from the investment in an overnight MIBOR swap contract in order to lock the margin between cash inflow from the swap and cash outflow to the holders of the above certificates of deposit.

7.1.5. Diagram of Fixed-Floating Rate Swap
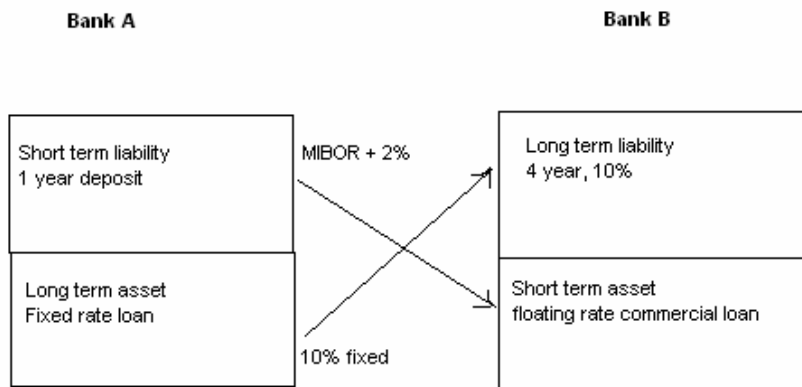
7.1.6. Table of the Cash flows in the swap process under the following circumstances:
a. the floating rate on the short term deposit is $MIBOR + 2\frac{1}{2}\%$ and
b. the fixed rate on long term deposits is 12% (this is only for example).

	Bank A	Bank B
cash outflow to liabilities	- (floating rate on 1 year deposit x 100 cr.)	- 10% x 100 cr.
cash inflow from swap	(MIBOR + 2%) x 100%	10% x 100 cr.
cash outflow from swap	- 10% x 100 cr.	-(MIBOR + 2%) x 100 cr.
net cash flow	-(8% + 1 year floating rate - MIBOR) x 100 cr.	-(MIBOR + 2%) x 100 cr.
rate available on		
floating rate debt		$MIBOR + 2\frac{1}{2}\%$
fixed rate debt	12%	

7.1.8. Determining the Notional Value of Swaps

The notional value of swap contracts (N_S) is that value at which change in the market value of swap equals the change in the value of equity.

i.e. $\Delta E = \Delta S$

i.e. $(D_A - kD_L) \times A \times \dfrac{\Delta R}{1+R} = (D_{fixed} - D_{float}) \times N_S \times \dfrac{\Delta R}{1+R}$

Suppose $D_A = 5$, $D_L = 3$ and $k = 0.9$, $A = 1$ billion, duration of a current 10 year fixed rate government security with 10 year and the same coupon as the fixed rate on the swap is 7 years and duration of a floating rate bond with annual repricing is 1 year, thus $D_{fixed} = 7$ years, $D_{float} = 1$ year, Now $N_S = 38,333,333$

If the size of 1 swap contract is 1 lakh total number of swap one has to buy is = 38,333,333 / 100000 = 383.33 $\cong$ 383

7.2. Forward Rate Agreement (FRA)

7.2.1. Concept

FRA also known as a Future Rate Agreement is an over-the-counter contract, mentioning the rates to be used along with the termination date and notional value, between parties that determines the rate of interest, to be paid or received on an obligation beginning at a future start date. On such an agreement, it is only the differential that is paid on the notional amount of the contract.

7.2.2. Determination of Forward Rates

An idea about a Forward Rate can be made determined on the basis of spot rate of ZC bonds. The reciprocal of a spot rate is the price of a ZC bond with face value Re. 1. Since ZC bond price keeps on increasing from the discount price to the par price, buying a ZC bond at present and selling it in future gives a positive rate of return. Such rate of return is called the forward rate for the period from buying the ZC bond to selling the ZC bond. In the example of Bootstrapping Method of ZC spot rate extraction we got the following prices of ZC bonds with face value Re. 1.

Maturity	ZC Price of Face Value Re. 1
0	1
1	0.99
2	0.896792
3	0.811144

Now dividing the every next period price by the previous period price we the 1 period forward rates

1 period Forward Rate	Maturity
	0
1.01%	1
10.39%	2
10.56%	3

Similarly every next 2nd period price by the previous price we the 1 period forward rates

2 Period Forward Rate	Maturity
	0
	1
11.51%	2
22.05%	3

7.2.3. Example of Forward Rate Agreement

Suppose Bank A makes projection on 01-12-07 that the rates on its floating rate liabilities would go up on 02-02-08 and would continue to be so till 02-05-08. So it enters a forward contract with Bank B for the minimum notional amount 25 crore where it is decided that Bank A would pay FRA

rate 10% and Bank B would pay 3 month MIBOR. If the 3 month MIBOR on 02-02-08 is more than 10% Bank B would pay the difference to Bank A at maturity of the contract on 02-05-08 and if the 3 month MIBOR on 02-02-08 is less than 10% Bank A would pay the difference to Bank B at maturity of the contract on 02-05-08. On 02-02-08 3 month MIBOR is 9.25%.

Interest rate payable by Bank A is = 25 Cr.*10%*(91/365) = 62,32,877 Cr.

Interest rate payable by Bank B is = 25 Cr.*9.25%*(91/365) = 57,65,411 Cr.

Since 3 month MIBOR on 02-02-08 is less than 10% Bank A will pay to Bank B at maturity of the contract on 02-05-08 the difference 62,32,877 – 57,65,411 = 4,67,466 Cr.

Chapter 8: A Live Example: Study of 8.24% 2018

8.1. Introduction

In this chapter we shall calculate price, duration, Mduration and convexity for a bond of approximately 10-year residual term to maturity traded in the Indian G-Sec market as an example of the concepts and methods of Market Risk Management in the G-Sec Portfolio of a bank and encourage the reader to try other methods described in this material like VaR calculation and stress testing. The 10-year risk free sovereign yield is considered to be one of the benchmark rates in G-Sec valuation. It is considered to contain sufficient information about inflation, monetary policy and liquidity condition in the G-Sec market[xxiii]. We take the trade day 24-Oct-08 with annualized YTM 7.67% and price 103.789.

8.2. Pricing

This price can be calculated by using the =price(date of settlement, date of settlement, rate, yield, redemption, frequency) formula in MS Excel. It can also be calculated manually in two alternative ways:

(i) assigning the numbers 1, 2, …, 19 to the successive semi annual coupon periods and discounting the semi annual cash flows with the semi-annual yield, or

(ii) taking the actual term to maturities of the cash flows in terms of years and discounting them with the annual yield.

Both (i) and (ii) are described below. (i) gives the results close those given by MS Excel and reported by CCIL.

Settlement	24-Oct-08
Maturity	22-Apr-18
rate	8.24%
yield	7.67%
Redeem	100
Frequency	2
MS Excel Price	103.789
CCIL Price	103.79

(i)

Date	Cash flow	TTM (Semi Annual)	PVCF
22-Apr-09	4.12	1	3.967833582
22-Oct-09	4.12	2	3.821287217
22-Apr-10	4.12	3	3.680153337
22-Oct-10	4.12	4	3.544232038
22-Apr-11	4.12	5	3.413330802
22-Oct-11	4.12	6	3.287264219
22-Apr-12	4.12	7	3.165853729
22-Oct-12	4.12	8	3.048927364
22-Apr-13	4.12	9	2.936319511
22-Oct-13	4.12	10	2.827870671
22-Apr-14	4.12	11	2.723427236
22-Oct-14	4.12	12	2.622841273
22-Apr-15	4.12	13	2.525970312
22-Oct-15	4.12	14	2.432677144
22-Apr-16	4.12	15	2.342829627
22-Oct-16	4.12	16	2.256300503
22-Apr-17	4.12	17	2.17296721

22-Oct-17	4.12	18	2.092711716
22-Apr-18	104.12	19	50.93338991
		Sum	**103.79619**

(ii)

Settlement	24-Oct-08		
Date	**Cashflow**	**TTM (Years)**	**PVCF**
22-Apr-09	4.12	0.49	3.972173
22-Oct-09	4.12	0.99	3.828078
22-Apr-10	4.12	1.49	3.689211
22-Oct-10	4.12	1.99	3.555381
22-Apr-11	4.12	2.49	3.426405
22-Oct-11	4.12	2.99	3.302109
22-Apr-12	4.12	3.49	3.182321
22-Oct-12	4.12	3.99	3.066879
22-Apr-13	4.12	4.49	2.955625
22-Oct-13	4.12	4.99	2.848406
22-Apr-14	4.12	5.49	2.745077
22-Oct-14	4.12	5.99	2.645497
22-Apr-15	4.12	6.49	2.549529
22-Oct-15	4.12	6.99	2.457042
22-Apr-16	4.12	7.49	2.36791
22-Oct-16	4.12	7.99	2.282011
22-Apr-17	4.12	8.49	2.199229
22-Oct-17	4.12	8.99	2.11945
22-Apr-18	104.12	9.49	51.61937
		Sum	**104.812**

(ii) Sensitivity Analysis

Duration and modified duration can be calculated in excel by using the formula =duration(date of settlement, date of settlement, rate, yield, redemption, frequency) formula in MS Excel. It can also be calculated manually. Here we are using that method which gives the result close to what is reported by CCIL. As on 24-Oct-08 the YTM was 7.67% with price 103.79, duration 6.82. Now 1 bp YTM-shock is given and prices and duration are recalculated.

			yld 7.67%	yld 7.68%	yld 7.66%
Date	**Cash flow**	**TTM(SA)**	**PVCF**	**PVCF**	**PVCF**
22-Apr-09	4.12	1	3.967833582	3.96764253	3.968024656
22-Oct-09	4.12	2	3.821287217	3.82091923	3.821655259
22-Apr-10	4.12	3	3.680153337	3.67962175	3.680685023
22-Oct-10	4.12	4	3.544232038	3.54354945	3.544914787
22-Apr-11	4.12	5	3.413330802	3.4125091	3.414152737
22-Oct-11	4.12	6	3.287264219	3.28631462	3.288214135
22-Apr-12	4.12	7	3.165853729	3.16478681	3.166921059
22-Oct-12	4.12	8	3.048927364	3.04775309	3.050102147
22-Apr-13	4.12	9	2.936319511	2.93504728	2.937592359
22-Oct-13	4.12	10	2.827870671	2.82650932	2.829232745
22-Apr-14	4.12	11	2.723427236	2.72198509	2.724870216

Date			YTM	YTM	YTM
22-Oct-14	4.12	12	2.622841273	2.62132617	2.62435733
22-Apr-15	4.12	13	2.525970312	2.5243896	2.527552085
22-Oct-15	4.12	14	2.432677144	2.43103775	2.434317717
22-Apr-16	4.12	15	2.342829627	2.34113805	2.344522505
22-Oct-16	4.12	16	2.256300503	2.25456284	2.258039589
22-Apr-17	4.12	17	2.17296721	2.17118918	2.174746787
22-Oct-17	4.12	18	2.092711716	2.09089867	2.094526425
22-Apr-18	104.12	19	50.93338991	50.8868127	50.98001198
			103.79619	103.727993	103.8644395
		DF	1.03835	1.0384	1.0383
		%Ch in P		-0.0006570	0.000657559
		%ch in DF		0.00004815	-0.00004815
		D		**13.64393**	**13.655532**
		MD		13.1400102	13.15118393

The list of YTMs, prices and durations are given below:

Price	YTM	Duration
103.86444	7.66%	6.82776592
103.79	7.67%	6.81923981
103.727993	7.68%	6.82196479

The Duration is found to be almost unchanged. Therefor the price yield curve is linear as follows:

Price-yield Relationship for 1 bp change in yield - linear
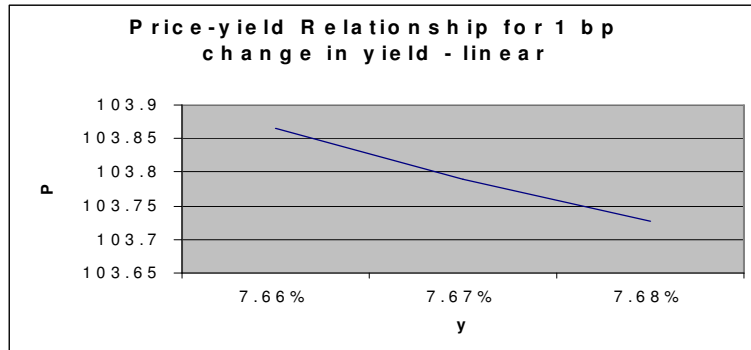
But it is necessary to examine whether duration remains unchanged even in stress scenarios like a robust YTM-shock to the extent of 7%.

Next we gave 7% YTM-shock and recalculate prices and durations. The result is as follows:

Date	Cashflow	TTM(SA)	YTM 14.67% PVCF	YTM 0.67% PVCF
22-Apr-09	4.12	1	3.8384497	4.1062441
22-Oct-09	4.12	2	3.5761399	4.0925341
22-Apr-10	4.12	3	3.3317556	4.0788699
22-Oct-10	4.12	4	3.1040719	4.0652513
22-Apr-11	4.12	5	2.8919476	4.0516782
22-Oct-11	4.12	6	2.6943192	4.0381504
22-Apr-12	4.12	7	2.5101963	4.0246677
22-Oct-12	4.12	8	2.3386559	4.0112301
22-Apr-13	4.12	9	2.1788381	3.9978373
22-Oct-13	4.12	10	2.0299419	3.9844893
22-Apr-14	4.12	11	1.8912209	3.9711858

22-Oct-14	4.12	12	1.7619797	3.9579268
22-Apr-15	4.12	13	1.6415705	3.944712
22-Oct-15	4.12	14	1.5293897	3.9315413
22-Apr-16	4.12	15	1.4248751	3.9184146
22-Oct-16	4.12	16	1.3275028	3.9053318
22-Apr-17	4.12	17	1.2367846	3.8922926
22-Oct-17	4.12	18	1.1522659	3.879297
22-Apr-18	104.12	19	27.129907	97.709663
			67.589813	169.56132
		DF	1.07335	1.00335
		%Ch in P	-0.348822	0.6335987
		%ch in DF	0.0337073	-0.0337073
		D	10.348547	18.797063

The list of YTMs, prices and durations are given below:

P	YTM	Duration
169.56	0.67%	9.3985316
103.79	7.67%	6.8192398
67.59	14.67%	5.1742734

The Duration is found to rise faster in the case of downward YTM-shock than in the case of upward YTM-shock. Therefor the price yield curve is convex as follows:

Price-yield relationship for 7% change in yield-convex
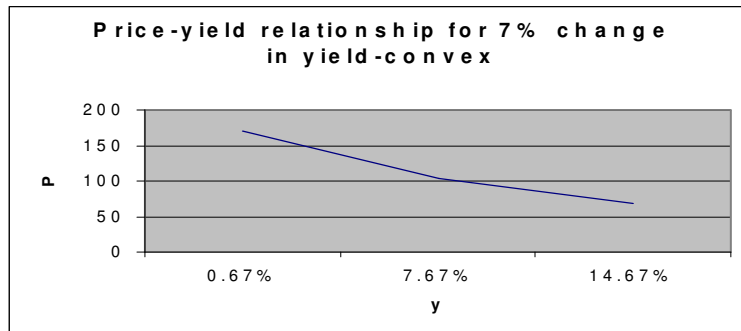
The CCIL calculates convexity following formula:$\{(P_+ - P0)+(P_- - P0)\}/\{2*P*(\Delta YTM)^2\}$ where $\Delta YTM = 1\%$.

As on 24-Oct-08 we calculate

YTM	P
6.67%	110.9127
7.67%	103.7914
8.67%	97.2505

Here P_+ = 97.2505, P_0 = 103.7914, P_- = 110.9127. Now convexity is {(97.2505 - 103.7914)+(110.9127 - 103.7914)}/(2*7.67%*1%*1%) = 27.95

Conclusion

Keeping in view the computational aspects of the procedure of Market Risk Management of G-Sec portfolio by the Banks in India described in Chapters 2-7, the following points of caution should be noted:

Market Risk Management Process in India is in an evolving process since the Banks in India are still in an early stage of development in the sense that they are lacking statistical database, equipped MIS and adequate supply of trained personnel. Many a good number of banks are suffering from breaches of VaR calculated following internal models. Further they are also finding difficulty in validation with small sizes of sample. As per BIS the prescribed sample size for VaR calculation and accordingly capital charge estimation is 1000 days. It is difficult to get return figures for such a long duration of return figures on any government security since there is rare possibility that a security is traded everyday. For example, in India, the maximum traded security during the 1st quarter of 2008-09 is the 10 year benchmark G-Sec; but nevertheless the return figures on a security of 10 year maturity are not available since the particular benchmark security has no more a maturity of 10 years after a single day elapsed.

Secondly, Duration method of sensitivity analysis fails in the case of robust interest rate shocks. The predicted yield shocks are provided by the RBI and, accordingly, the Banks are asked to make specific risk charge on G-Secs. Notional daily zero coupon yields up to 30 year maturities are given NSE and CCIL using the models like Nelson-Siegel and Nelson-Siegel-Svensson. The liquidity in Indian money and G-Sec markets is not sufficient to induce active trading in all instruments of all maturities such as to get an idea of yield movement in every maturity. Further, because of dearth of information about the market other models like Vasicek which are used in developed countries cannot be used in India. In this circumstance if yield shock prediction is not accurate, the capital charge may fall short of what is required.

Thirdly, in order to get the price of a non-traded security one can take it as a sum of the present values of future cash flows. The present values are calculated by multiplying the cash flow by the respective price of ZC bond of same maturity using discreet discounting. But use of continuous discounting gives a lower price with a greater volatility, further the spot rates extracted through discreet discounting becomes higher and higher above those extracted through continuous discounting as maturity increases. Therefore, using discreet compounding and discounting is not appropriate in valuation.

Fourthly the VaR method prescribed by the BIS is based on the assumption of Normal Distribution. Only in this method, the simple and probability weighted measures of average and standard deviation are same. But the asset returns in reality follow other distributions like beta and log-logistics where the simple and probability weighted measures of average and standard deviation are different and hence 99% VaR estimate is far above the estimate based on the assumption of Normal Distribution. Overlooking the aspect of probability distribution of portfolio VaR may lead to serious consequences.

Finally, it is not mandatory in India to compute VaR but Duration does not provide risk measurement across the categories of assets and, hence, aggregation of risk for the entire trading book. In terms of capital charge, however, the aggregation across the categories over the portfolio is possible.

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Appendix I

As far as G-Sec operations are concerned, following are some important activities/operations:

- **NDS-OM**
  A major part of trading is in G-Sec through negotiated dealing system order matching (NDS-OM) managed by Clearing Corporation of India (CCIL). The NDS has been operationalised since February 2002. The deals of the CCIL members with the counterparties (who are also CCIL members) are finalized on the screen and settled through CCIL. CCIL here works as central counterparty. This means for dealing between A and B, A is dealing with CCIL and CCIL in turn is dealing with B. In cash transactions, deals up to Rs. 20 crore is put through NDS. The dealers conclude deals. The dealer of the selling party enters deal details on NDS server. Dealer of the buying party approves the deal after scrutiny. The dealer prepares deal slip and sends to the back office. Once the buying dealer approves the deal, it is sent by the NDS server itself to Chief Dealers and settlers of both sides for acknowledgement and settlement respectively. The required margin here depends on value at risk of the security and deposited with CCIL. Here the deal ends. All G-Secs are physically back-checked by both the settlers for the deals not via NDS-OM, e.g. deals arranged through phone and fax. Since April 2004 G-Sec transaction settlement via CCIL has been taking place through DVP (delivery vs. payment) – III system so that each security is netted before delivery or receipt. Previously Previously such nettings were done only for funds via the DVP – II system.

- **Directions of Investment Committee**
  In the morning of every working day (Mon-Fri) the Investment Committee of the Bank forwards instructions in form of the minutes of meetings some directions regarding the elements like amount of purchase or sale of securities, range of prices for such purchase and relevant the accounting category (i.e. purchase for HFT/HTM category or SLR/excess-SLR), and the range of bids to be submitted in. If the dealer intends to do something which is outside the above directions she has to consult Chief Dealer who in turn has to consult the Investment Committee.

- **Arbitrage in the case of some particular security**
  For all G-Sec trading purposes 10 year G-Sec yield is taken as benchmark. For any longer term security in HFT category, e.g. 24 years maturity, some term premium like 50-60 bps is expected. If this spread at some point of time is more than above, the dealer expects the spread to narrow down and accordingly she releases that security for booking profit and uses the fund elsewhere.

- **Submission of bid**
  The front office maintains a calendar of G-Sec auctions. For example, every Wednesday 91D TB is auctioned. The auction screen opens at 9 A. M. morning. The dealer can give bid till 12.30 P. M. Before bid submission, the dealer collects information from other bank dealers through telephone and internet-chatting regarding what range of bid is being planned by them looking at the secondary market yields. Most of the time the dealer gives multiple bids if it is must for her to buy as per Investment Sub Committee directions. The dealer has the option not to participate in auction. In such a case the dealer compares the prospective primary market yield with those in the secondary market and rate of returns on money market.

- **CRR maintenance and G-Sec sale/purchase triggering by back office**
  The Back keeps data of daily NDTL and funds position and maintains CRR with RBI through RTGS. Any shortfall in CRR due to rise in NDTL leads to search for funds. If there is not enough surplus fund or any surplus fund, fund is raised from domestic or forex markets or via selling securities from AFS or HFT category. Again any excess CRR due to fall in NDTL leads to search for avenues for deployment in money market and/or investment G-Sec market. In making selling/buying decisions the Investment Committee keeps in mind YTM

along with funds position, current yield, prospective capital loss, modified duration, maturity bucket & liquidity of the securities.

- **G-Sec sale/purchase triggering by changes in NDTL**
  G-Secs are purchased for, among others, SLR requirements. Again SLR requirement is based on NDTL position. SLR is 24% of DTL as on the last Friday of the 2nd preceding fortnight. The size of SLR may fall short either for rise in NDTL or redemption of existing securities on maturity. In such a case it is mandatory to buy securities for meeting SLR limit. Such an incidence occurred on 10 September 2008 (http://www.thehindubusinessline.com/2008/09/11/stories/2008091152090600.htm accessed on 24 December 2008).

- **Monitoring Activity**
  In both the cases where G-Sec purchases/sales are triggered by changes in NDTL and fund positions, the limits of the trading limit per dealer, the sizes of deals and transactions through each broker are monitored by the mid-office.

- **Integration of domestic treasury and forex**
  Looking at interest rates at rupee market and forex market every morning, before starting front office operations, domestic and forex Chief Dealers decide which market to raise funds from and/or which market to deploy fund in. When there is surplus fund in the hand of domestic back office, forex market may be a place to deploy instead of domestic market depending on the rate of return. Similarly if there is paucity of funds for CRR maintenance, such funds can be raised from forex market instead of the domestic market depending on the cost of borrowing.

- **Valuing and Tracking market risk of G-Sec**
  Tracking market risk of G-Sec in HFT and AFS categories of different maturity buckets based on yield shocks prescribed by RBI is done applying modified duration method by the Mid-Office. The market risk reports are submitted to Risk Management Department. The Mid Office also conducts valuation of AFS and HFT categories. In HFT valuation it uses prices provided by FIMMDA. In AFS valuation it uses the yield curve, credit spreads and methods suggested by FIMMDA. The software packages like 'Credence' or 'Condor Plus' are used by the Mid Office is linked to FIMMDA and Reuter terminals and updated daily.

- **Limits**
  All domestic treasury operations – SLR and non-SLR - are subject to the limits given the Bank's Investment Policy Document based on RBI and SEBI guidelines and Banking Regulations Act 1949. These limits relate to G-Secs, other approved securities, shares, debentures and bonds, investment in associates and club deals, subsidiaries and joint ventures, mutual funds, certificate of deposits, commercial papers, PTCs and RIDF.

Time Value of Money

The Concepts

'Time value of money' is a premise that an investor prefers to receive a fixed amount of cash today, rather than in future, all else being equal. In other words, the present value of a certain cash amount 'a' is greater than the present value of the right to receive the same at a future period 't'. This is because the amount 'a' could be deposited in an interest-bearing bank account (or otherwise invested) from now for 't' periods and yield interest. This is why lenders demand interest payments for use of their capital. Among other motivations for demanding interest may be compensations for the risk of borrower default and the risk of inflation[1]. The concept of risk is inherent in the concept of time value of money and hence to be studied together with the latter.

Utility of the Concepts

The concepts of the time value of money and risk are extremely vital in financial decision making. If timing and risk of cash flows are not considered the firm make decisions that may allow it to miss its objective of maximizing the owners' interest. Again maximizing owners' interest depends how an investment decision is made. An investment decision means selecting one out of the alternative capital expenditure proposals on the basis of time value of money[2].

How to calculate time value

Calculation of time value happens through calculations of present value and future value.

Rationale behind Calculation of Present Value

Since we do not know the future we prefer receiving cash now rather than in future. The possible reasons behind such preferences are illness, death, inflation, currently available investment avenues wherein to put the case to earn further cash etc. Assuming even that there is no uncertainty in future we can put the money in bank and earn risk free interest. For example a rational individual who is offered Rs 100 now or in future after one year will obviously choose the first alternative and put the money in bank @ 'r' rate of interest per annum such that after one year his earning will be 100 (1 + r) > 100.

Rationale behind Calculation of Future Value

Calculation of future value facilitates comparison of alternative investment proposals in respect of what amount the same sum would grow to after a certain time interval when put in the available investment avenues. To make easy, assume that there are two alternative business proposals are available – 'A' and B and the investible cash be 'S'. Also assume that both the proposals have the same locking period 'N' years. Now the question is which proposal would yield a larger sum after 'N' years, i.e. which proposal would yield a higher rate of return. For a bank the problem takes this form. Suppose Mr. A and Mr. B are two businessmen and want loan from the bank. Also suppose the bank has some limited cash which it can lend to only one person. For the bank the problem is which one of the two could fetch more business to the bank in future. Banks are interested in more cash and hence in building up relationship with those entities where there is possibility of larger flow of cash. One easy way is to see between two businessmen whose business earns higher profit. Assuming that things will not change much in future, the bank would give loan to the businessman with higher rate of return.

Working Formulae for Calculating Present Value
For what cash amount invested today @ 'r' p.a. (i.e. the % interest rate per annum). shall I get 'F' cash amount after 't' number of years or in other words if I get 'F' after 't' years and the prevailing annual interest rate is 'r' what is the present value (PV) of 'F'? Let the PV amount be 'P'. Thus

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[1] http://en.wikipedia.org/wiki/Time_value_of_money accessed on 18 April, 2007
[2] Pandey (2005)

(a) for simple rate $F \equiv [P\{1 + (r*t)\}]$ or $P \equiv [F/\{1 + (r*t)\}]$;

(b) for compound rate $F \equiv [P\{(1 + r)^t\}]$ or $P \equiv [F/\{(1 + r)^t\}] = (F*PVF_{t,r})$ where $PVF_{t,r} = [1/\{(1 + r)^t]$ is called the PV factor for 't' years @ 'r' p.a. It is also called the PV of Re 1 year realizable after 't' years @ 'r' p.a. The product $F*PVF_{t,r}$ can be interpreted in the following manner:

If future value of Re 1 is calculated today to be $PVF_{t,r}$, future value F calculated today would be $F*PVF_{t,r}$ given 'r' and 't'.

In the banking industry the compound interest is in practice. So we shall concentrate on compound interest.

Following is the excel table of $PVF_{t,r}$ for different values of 't' and 'r':

t/r	1%	2%	3%	4%	5%	10%	15%	20%	25%	30%
1	0.99	0.98	0.971	0.962	0.952	0.909	0.87	0.833	0.8	0.769
2	0.98	0.961	0.943	0.925	0.907	0.826	0.756	0.694	0.64	0.592
3	0.971	0.942	0.915	0.889	0.864	0.751	0.658	0.579	0.51	0.455
4	0.961	0.924	0.888	0.855	0.823	0.683	0.572	0.482	0.41	0.35
5	0.951	0.906	0.863	0.822	0.784	0.621	0.497	0.402	0.33	0.269

Problems on PV

Problem 1: Extend the above table to t = 8 and r = 50%

Problem 2: Extend the above table to r = 40%

Problem 3: What is the PV of Rs 600 receivable at the end of 15 years @ 5% interest?

Working Formulae for Calculating Compound/Future Value per annum

What cash amount shall I get after 't' number of years having invested a cash amount 'P' today @ 'r' p.a or in other words if I invest 'P' for 't' years @ 'r' p.a what is the future value (FV) of 'P'? Let the FV amount be 'F'. Thus

(a) for simple rate of interest $F \equiv [P\{1 + (r*t)\}]$ or $P \equiv [F/\{1 + (r*t)\}]$;

(b) for compound rate of interest $F \equiv [P\{(1 + r)^t\}] \equiv (P*CVF_{t,r})$ where $CVF_{t,r} = \{(1 + r)^t$ is called the compound value (CV) factor for 't' years @ 'r' p.a. It is also called the CV of Re 1 year realizable after 't' years @ 'r' p.a. The product $P*CVF_{t,r}$ can be interpreted in the following manner:

If Re 1 is invested today the future value is $CVF_{t,r}$ given 'r' and 't'.

In the banking industry the compound interest is in practice. So we shall concentrate on compound interest. We shall use CV and FV synonymously.

Following is the excel table of $CVF_{t,r}$ for different values of 't' and 'r':

t/r	1%	2%	3%	4%	5%	10%	15%	20%	25%	30
1	101%	102%	103%	104%	105%	110%	115%	120%	125%	130
2	102%	104%	106%	108%	110%	121%	132%	144%	156%	169
3	103%	106%	109%	112%	116%	133%	152%	173%	195%	220
4	104%	108%	113%	117%	122%	146%	175%	207%	244%	286
5	105%	110%	116%	122%	128%	161%	201%	249%	305%	371

Compounding half-yearly, quarterly and monthly

For r p.a. $F = \{P*(1 + r)\}$ after 1 year.

For r half yearly $F = [P*\{1 + (r/2)\}]$ after 6 months and $F = [P*\{1 + (r/2)\}^2]$ after one year.

For r quarterly $F = [P*\{1 + (r/4)\}]$ after 1 quarter, $F = [P*\{1 + (r/4)\}^2]$ after 2 quarters, $F = [P*\{1 + (r/4)\}^3]$ after 3 quarters and $F = [P*\{1 + (r/2)\}^4]$ after 1 year.

For r monthly $F = [P*\{1 + (r/12)\}]$ after 1 month, $[P*\{1 + (r/12)\}^2]$ after 2 months, ………….......,
$F = [P*\{1 + (r/12)\}^{12}]$ after 12 months.

Assuming r = 8%

$F = \{P*(1.08)\}$ after 1 year in the case of annual compounding.

$F = [P*\{(1.04)^2\}] = \{P*(1.0816)\}$ after 1 year in the case of half yearly compounding.

$F = [P*\{(1.02)^4\}] = \{P*(1.082432)\}$ after 1 year in the case of quarterly compounding.

F = [P*{(1.006667)^{12}}] = {P*(1.083)} after 1 year in the case of quarterly compounding.

Problems on CV
Problem 1: Extend the above table to t = 6 and r = 40%
Problem 2: Extend the above table to r = 40%
Problem 3: What is the FV of Rs 600 receivable at the end of 15 years @ 5% p.a?
Problem 4: How long will it take for a sum of money to double @ 12% p.a.
Problem 5: How long will it take for a sum of money to double @ 1.01% p.a.

Trick: In the formula F = P {(1 + r)^t}
a.   if 'F' is unknown while 'P', 'r' and 't' are known it is a problem of CV.

b.   If 'P' is unknown while 'F', 'r' and 't' are known it is a problem of PV.

Estimation

In this topic we shall try to learn how sampling distribution is used to estimate population parameters.

Focus: We shall focus on estimation of the population mean in the cases where the sample-mean as well as the population-mean follow normal distributions.

References

1. Levin Richard I. and David S. Rubin (1997): *Statistics for Management*, Prentice Hall India, 7th Edition, Chapter 7.

or

Levin Richard I. and David S. Rubin (1997): *Statistics for Management*, Pearson Education, 7th Edition, Chapter 7; this book is available in library

2. http://www.isixsigma.com/library/content/c000709a.asp accessed on 15 August 2007.

Concepts

1. There are two kinds of estimates of a population parameter like population mean– point estimate and interval estimate.

2. The term 'point estimate' is defined as a single number computed to estimate an unknown population parameter.

3. The term 'interval estimate' is defined as a range of values computed to estimate an unknown population parameter.

4. The term 'estimator' is defined as a sample statistic computed to estimate an unknown population parameter.

5. The term 'estimate' is defined as a specific observed value of the sample statistic.

Degrees of Freedom (df)

Concept

1. In statistics, the term degrees of freedom (df) is a measure of the number of independent pieces of information on which the precision of a parameter-estimate is based.

2. It can also be thought of as the number of observations (values) which are freely available to vary after some parameters are already estimated.

3. The degrees of freedom for an estimate equals the number of observations (values) minus the number of parameters already estimated.

4. As we have to estimate more parameters, the degrees of freedom available decreases.

Example: In the sample variance formula $\sigma_s^2 = \{\sum(x - \overline{x})^2\}/(n-1)$ degrees of freedom is n-1.

Rationale of replacing 'n' by 'n – 1': Since sample variance formula contains sample mean, there is no need to take n sample values; given the value of sample mean the value of any sample unit is automatically determined by the values of n – 1 units. We can say that we have freedom to take only n -1 sample values because the nth sample value is hidden in the sample mean; so our degree of freedom is n-1.

Criteria of a good estimator

An estimator like sample mean is called a good estimator when it satisfies the following criteria:

1. Unbiasedness: An estimator is said to be unbiased when

a. the mean of the population distribution of sample means is equal to the population mean and

b. the number of sample means above the population mean is equal to the number of sample means below the population mean when large number of samples are drawn.

Example: For a normally distributed population, sample mean is empirically observed to be the unbiased estimator of population mean.

2. Efficiency: A statistic is called the efficient estimator of population parameter, when its sampling distribution has less SE compared to the SEs of the distributions of other statistics, provided that the sample size is uniform.

Example: For a normally distributed population sample mean is empirically found to be the efficient estimator of population median compared to sample median in large samples, since the latter is observed to have a smaller SE than the former has.

3. Consistency: A statistic is called a consistent estimator when it tends to equal the population parameter as the sample size gradually tends to equal population size.

Example: For a normally distributed population sample mean is empirically found to be the consistent estimator of population median.

4. Sufficiency: A statistic is called a sufficient estimator if, compared to other statistics, it is able to extract from the sample the maximum information about the population parameter.
Example: Sample mean is generally taken as the sufficient estimator of the central tendency of a normally distributed population as long as the sample is sufficiently large and representative.

Types of Estimates
There are two kinds of estimates of population parameter: point estimate and interval estimate.

Point Estimate
Concept
The value of the sample statistic is called the point estimator of the corresponding population parameter as long as the sample is sufficiently large and representative.
Example
1. Sample mean can be called a point estimate of population mean.
2. Sample proportion can be called a point estimate of population proportion.

More Examples of Point Estimate

S. No	Population in study	Population parameter to be estimated	Sample statistic to be used as an estimator	Observed estimate
1	Employees in an Indian private sector bank	Mean annual turnover	Mean monthly turnover	Annually 40%
2	Applicants for Credit Risk Manager in a foreign bank in India	Mean formal relevant prior experience (years)	Mean formal relevant prior experience (years) of every 4th applicant	2 years
3	Loan accounts of an Indian public sector bank	Proportion of NPA	Proportion of a sample of 50 branches	2.5%

Interval Estimate
Concept
1. An interval estimate describes a range of values within which a population parameter, say population mean, is likely to lie in a specific proportion of the samples (say, 95% of the samples).
2. In terms of z distribution an interval estimate means two values of z between which the value of z corresponding to population mean would lie in 95% cases
3. The interval estimate is written as $\bar{x} - 1.96\ SE \leq \mu \leq \bar{x} + 1.96\ SE$ or as $\bar{x} \pm 1.96\ SE$.
4. The interpretation is that if we select many random samples and estimate confidence intervals there is 95% probability of the population mean lying within $\pm 1.96\ SE$ of sample means.
5. The probability attached with an interval estimate is called the confidence level/area. This is denoted by β, while $1 - \beta$ is called the rejection or critical level/area and denoted by α. The confidence area β is bound by two half-rejection areas (i.e. $\alpha/2$ on each side).
The values of z demarcating the boundaries are $\pm z_{\alpha/2}$ and called critical values.
Illustration: See the following illustration
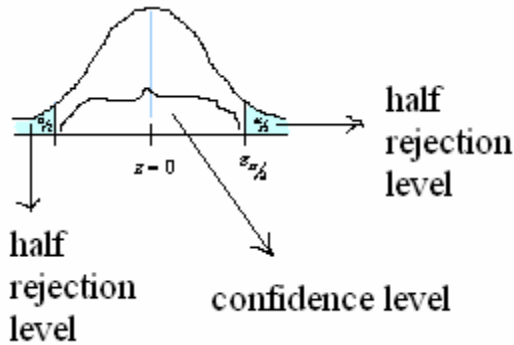
6. Commonly used confidence levels are 90%, 95% and 99% associated with the intervals respectively $\bar{x} \pm 1.64$ SE, $\bar{x} \pm 1.96$ SE and $\bar{x} \pm 2.58$ SE.

7. The smaller value of the interval is called lower limit and the larger value is called the upper limit.

8. The higher the confidence level the wider is the confidence interval and hence the less precise is the estimate.

Example

Suppose the central bank needs an estimate of the average holding period (AHP) of long term bonds floated by commercial banks. The research team selected a random sample of 200 dealers. A dealer is defined as an individual or corporate willing to buy or sell securities for own profit. The sample mean is 36 months. Using point estimate one can say that the sample-mean is the best estimator is population-mean. But the head of the team wants a reliable statement regarding the uncertainty that is likely to accompany the point estimate, i.e. a reliable statement about the range within which the unknown population parameter is supposed to lie. The previous study revealed the population SD is 10 months. If an estimate which is true in 95% cases is considered to be a reliable estimate, find the range.

Solution: SE of distribution of sample means = $\sigma/\sqrt{n} = 10/\sqrt{200} = 10/14.14 = 0.707 \cong 0.7$ months. The required confidence interval is $36 \pm 1.96 * 0.7$

Types of Problems

Following are types of the common problems.

1. Estimating confidence Interval of the mean when population SD is known and sample size is large.

Large sample means that the sample size is well above 30. Use either of the formulae $\bar{x} \pm 1.64$ SE, $\bar{x} \pm 1.96$ SE and $\bar{x} \pm 2.58$ SE depending on the prescribed confidence level 90%, 95% and 90% respectively, where SE = population SD/$\sqrt{n}$.

Example: Problem 2.

2. Estimating confidence Interval of the mean when population SD is unknown and sample size is large.

Use either of the formulae $\bar{x} \pm 1.64$ SE, $\bar{x} \pm 1.96$ SE and $\bar{x} \pm 2.58$ SE depending on the prescribed confidence level 90%, 95% and 90% respectively, where sample SD $\sigma_s = \sqrt{[\sum\{(x - \bar{x})^2\}/(n-1)]}$ would replace population SD σ in SE formula.

Example: Problem 3

3. Estimating confidence Interval of the mean when the population SD is unknown and the population follows normal distribution and sample size is small.

1. Small sample means that the sample size is below 30. Here one has to use t distribution instead of z distribution; the formula is $\bar{x} \pm t$ SE, the value of t depends on the prescribed probability as well as the degree of freedom of sample SD; SE = $\sigma_s/\sqrt{n}$; $\sigma_s = \sqrt{[\sum\{(x - \bar{x})^2\}/(n-1)]}$ is sample SD.

2. In contrary to z table, t table shows the chance that the population mean would lie outside the confidence interval against a particular degree of freedom.

3. t distribution has been developed as a remedy to the problems where z distribution cannot be used, i.e. when the population SD is unknown and the population follows normal or close to normal distribution and sample size is small.

Example: Problem 4

Note: Discussion of details of t distribution including properties and derivation is subject to time and interest constraints.

4. Estimating the sample size when the maximum permissible limit (say E) has been imposed on the absolute difference between sample mean and population mean.
Use the formula: $n = (z_{\alpha/2} * \sigma/E)^2$, where $1 - \alpha$ is the confidence level, α is called the critical or rejection level/region, σ is population variance and E is the maximum permissible absolute difference between sample mean and population mean.
Example: Problem 1

Problems
1. A central government undertaking bank in India, before launching internet banking service as a part of core banking would like to estimate the average Internet usage of customers in one week. How many customers must the research team randomly select to be 95% sure that the sample mean is within 1 minute of the population mean μ. A previous survey of customer usage has shown population SD σ = 6.95 minutes.
Solution: We are solving the problem for the sample size n.
A 95% degree confidence corresponds to $\alpha = 0.05$. Each of the shaded tails in the following figure has an area of $\alpha/2 = 0.025$. The region to the left of $z_{\alpha/2}$ and to the right of $z = 0$ is $0.5 - 0.025$, or 0.475. In the Table of the Standard Normal (z) Distribution, an area of 0.475 corresponds to a z value of 1.96. The critical value is therefore $z_{\alpha/2} = 1.96$.
The margin of error E = 1 and the standard deviation σ = 6.95. Using the formula for sample size, we can calculate n:
$n = (z_{\alpha/2} * \sigma/E)^2 = \{(1.96*6.95)/1\}^2 = (13.62)^2 = 185.55 \cong 186$
So we will need to sample at least 186 (rounded up) randomly selected customers. With this sample we expect to be 95% confident that the sample mean $\bar{x}$ will be within 1 minute of the true population of Internet usage.
Note: This particular example is taken from
http://www.isixsigma.com/library/content/c000709a.asp visited on 15 August 2007
Area between mean z and z value

z value	0	0.01	0.02	0.03	0.04	0.05
0	0	0.004	0.008	0.012	0.016	0.0199
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596
0.2	0.0793	0.0832	0.0871	0.091	0.0948	0.0987
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368
0.4	0.1554	0.1591	0.1628	0.1664	0.17	0.1736
0.5	0.1915	0.195	0.1985	0.2019	0.2054	0.2088
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422
0.7	0.258	0.2611	0.2642	0.2673	0.2704	0.2734

2. In a mood to enter banking industry Reliance Money looks for a 95% confident estimate of the range of the mean life of savings bank account (SBA) balances. Life of a SBA balance here means how long on average a customer does not withdraw the balance before there is a rise in market rate of interest. A rise in the market rate of interest spurs balance-withdrawal and using it for interest earning purpose. Previous studies prescribe a population SD of lives of SBA balances of 6 months. If the sample size is 100 with a sample mean of 21 months, determine the range.
Solution
Observation: it is a case of large sample from an infinite population.
Working out: Since the sample size is 100 we apply the formula $\bar{x} \pm 1.96$ SE;
where SE $= \sigma/\sqrt{n}$.
Answer: (19.82, 22.18) in months.

3. Mahindra Finance in order to tap vast rural banking opportunity by means of replacing private money lenders (mahajans) in West Bengal looks for a 90% confident interval estimate of average annual income of the families living in 700 panchayat areas. There is no previous study in this case and it is also costly and time consuming to draw samples from most of the villages because of poor/no communication infrastructure. So the research team decided to

proceed on the basis of a random sample with size 50 with a mean of Rs. 11800 and SD Rs. 950. Compute the interval.
Solution
Observations
1. It is a case of finite population and a sample size of more than 5% of the population size.
2. Again population SD is unknown.
Working out: We use the formula x ± 1.64 SE, where SE = $(\sigma_s/\sqrt{n})*\sqrt{\{(N-n)/(N-1)\}}$, where $\sigma_s = \sqrt{[\sum\{(x - \bar{x})^2\}/(n-1)]}$ is sample SD.
Answer: (11587.5, 12012.5)

4. The Banking Supervision Department (BSD) of the regulator of the banking industry is asked by the central government to submit in two days a 95% confident estimate cash balances banks maintain during festive seasons. The BSD research team facing paucity of time and without any knowledge of population SD took a sample of size 10 with a mean of Rs. 11400 cr. and SD Rs 700 cr. What is the interval estimate?
Solution
Observation: Here sample size is much below 30.
Working out: We apply the formula $\bar{x} \pm t$ SE, where SE = $\sigma_s/\sqrt{n}$, $\sigma_s = \sqrt{[\sum\{(x - \bar{x})^2\}/(n-1)]}$ is sample SD. Here df = n -1 = 9, the t value for 5% with df = 9 is 2.262.
The answer: (10899, 11901) in Rs. cr.

t table

df\p	0.4	0.25	0.1	0.05	0.025	0.01	0.005	0.0005
1	0.32492	1	3.077684	6.313752	12.7062	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.5407	5.84091	12.924
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.94318	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.306	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.36343	1.795885	2.20099	2.71808	3.10581	4.437
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.681	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.34503	1.76131	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.75305	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.015
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.25658	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.31946	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.68485	1.317836	1.710882	2.0639	2.49216	2.79694	3.7454
25	0.25606	0.68443	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75	3.646
inf	0.253347	0.67449	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905

Hypothesis Testing

In this topic we shall learn how to use sampling distribution to verify the estimates submitted by the research teams or agencies or claims, beliefs or hunches put forth by experts.

Focus: We shall focus on how to decide whether or not to reject an estimate of population mean where the population as well as the sample mean follow normal distributions.

Reference

1. Levin Richard I. and David S. Rubin (1997): *Statistics for Management*, Prentice Hall India, 7th Edition, Chapters 8 & 9.

<div align="center">or</div>

Levin Richard I. and David S. Rubin (1997): *Statistics for Management*, Pearson Education, 7th Edition, Chapters 8 & 9; this book is available in library

Note

1 In the event of any error/mistake/confusion in the handouts the above references will prevail over the handouts.

2. Study of the above reference is essential for learning and examination purposes; study of the handouts is optional.

Concepts

Hypothesis

Concept: In statistics a hunch or an estimate of the population whether or not explicit in the form of a claim or proposition is called a hypothesis.

Example: The claim of Indian Ministry of Finance (MOF) in page 61 of Economic Survey 2006-07 that public sector banks' income (sum total of interest and other income) as a proportion of assets have fallen to 8.4% in 2005-06 from 8.5% in 2004-05 can be called a hypothesis.

Hypothesis testing

Concept: In statistics the method of verifying a hypothesis with the help of sampling distribution for the purpose of rejecting or not rejecting is known as hypothesis testing.

Purpose: The purpose of hypothesis testing is to make a judgement about the difference between the estimated sample statistic and a claim or a hypothesized population parameter.

Example: The above MOF claim may be a matter of serious concern for a potential entrant to the Indian banking industry like Barclays Bank, who might like to verify it instead of relying on government estimates.

Null Hypothesis

Concept: The hypothesis the Barclays Bank wants to test is called null hypothesis and denoted by H_0.

Example: Suppose the Barclays Bank wants to test whether H_0: $\mu = 8.5\%$ is true or not, where μ is the mean of income-asset ratios of the banks.

Alternative Hypothesis

Concept

1. Any hypothesis which differs from the null hypothesis is called an alternative hypothesis and denoted by H_1.

2. The null hypothesis H_0 is tested against the alternative hypothesis H_1.

Example

The alternative hypothesis Barclays Bank likes to test is either of the following:

1. H_1: $\mu \neq 8.5\%$

2. H_1: $\mu < 8.5\%$

3. H_1: $\mu > 8.5\%$

Level of Significance

Concept: The probability of rejection of a null hypothesis is called the level of significance.

Illustration

Suppose we like to examine whether the null hypothesis is 95% true, we call it hypothesis testing at 5% level of significance. Following are the illustration of three types of hypothesis testing at 5% level of significance.

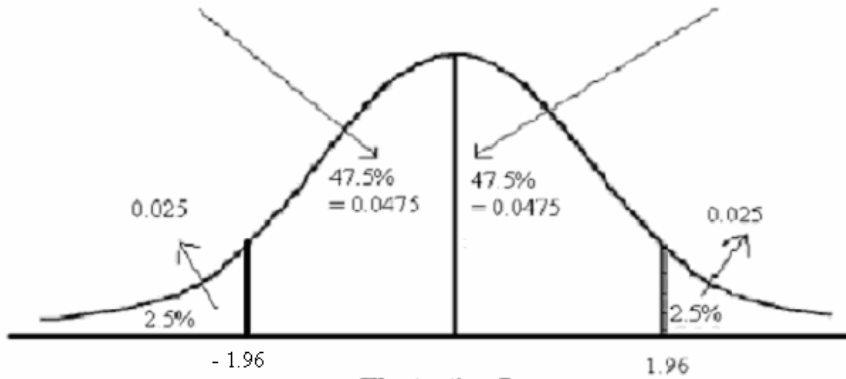**Two-tailed test at 5% level of significance**

Illustration I

Here H_0: $\mu = 8.5\%$ while H_1: $\mu \neq 8.5\%$; the values of $z = \pm 1.96$ are here called the critical values.
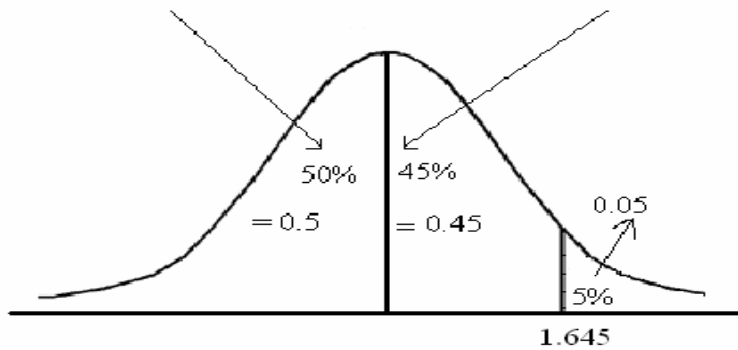
Right-tailed test at 5% level of significance

Illustration II

Here H_0: $\mu = 8.5\%$ while H_1: $\mu > 8.5\%$; the value of $z = 1.645$ is here called the critical value.
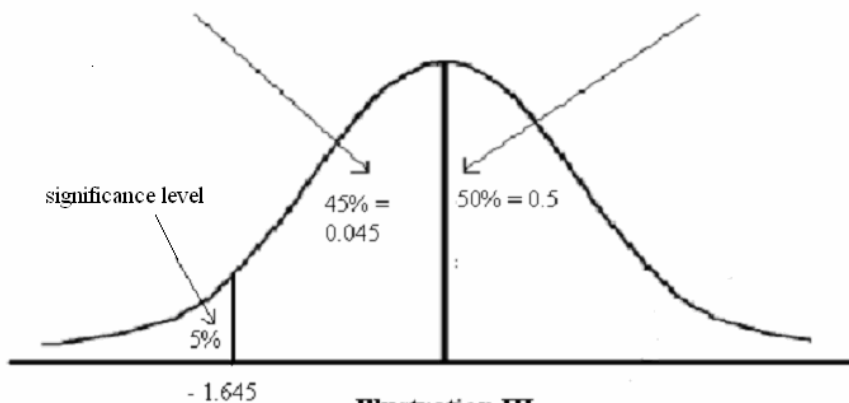
Left tailed test at 5% significance level

significance level

45% = 0.045

50% = 0.5

5%

- 1.645

Illustration III

Here H_0: μ = 8.5% while H_1: μ < 8.5%; the value of z = - 1.645 is here called the critical value.

How is z = 1.645 obtained from z table where z = 1.645 is not given?
It is through linear interpolation with the formula =forecast() shown in the following excel table

z	Prob Area
1.64	0.4495
1.65	0.4505
1.645	0.45

Instead, one can also use an approximate value 1.65.
Example: Suppose the Barclays Bank explores that, the sample mean 8.5% does not lie within 95% confidence level under the z curve; this means the sample mean 8.5% lies in the 5% critical or rejection level under the z curve; then 5% is called the level of significance/rejection.

Possible Errors in Hypothesis Testing: There are two types of errors which are often committed in course of hypothesis testing:
Type I Error
Concept
1. If the null hypothesis is rejected even when it is true, the level of significance is called 'Type I' error.
2. Rejection of the null hypothesis implies that the sample does not provide statistical evidence for non-rejection (i.e. acception).
3. Rejection of the null hypothesis should not be deemed as the proof of the fact that the null hypothesis is false.
4. The probability of type I error is the singnificance area α under the z curve.
5. The higher the significance level the higher the probability of type I error.

Example: Suppose the IBA rejected the Union Bank of India's claim that the Bank's total NPA to total advances ratio has improved (i.e. reduced) by 0.9% between 2005-06 and 2006-07. The null hypothesis is reduction in the NPA ratio for all the branches of the Bank H_0: μ = 0.9%; but in reality if the NPA level had come down to 2.94% from 3.84% during the period, rejection of the null hypothesis is an error of Type I.

Type II Error
Concept
1. If the null hypothesis is not rejected even when it is false, the level of confidence is called 'Type II' error.

2. Non-rejection of the null hypothesis implies that the sample does not provide statistical evidence for rejection.

3. Non-rejection of the null hypothesis should not be deemed as the proof of the fact that the null hypothesis is true.

4. The probability of type II error is the area under the z curve except the critical α; region and denoted by $\beta = 1 - \alpha$ such that $\alpha + \beta = 1$.

Example: Suppose only one of the branches of a commercial bank has NPA and may be the NPA level is formidable. But in the sample that branch is by chance not picked up. It is possible that the hypothesis $H_0: \mu = 0$ would not be rejected. This is an example of type II error.

Selection of z or t Distribution in Hypothesis testing of population mean
The rules for choosing a distribution is similar to those we came across in the topic on estimation.

Steps of hypothesis testing of the mean
Step 1. Describe the hypothesis.
Step 2. Decide whether two-tailed, right-tailed or left-tailed test to be conducted.
Step 3. Decide which distribution (z or t) is appropriate.
Step 4. Find out the critical values of z or t from the table if you need.
Step 5. Calculate SE of the sample mean.
Step 6. Convert the observed sample mean value to a standardized value if you like.
Step 7. Draw the probability distribution and mark the values of the sample statistic or the standardized sample statistics and the critical values.
Step 8. Decide whether or not to reject the hypothesis.

Common types of problems
Following are the common types of problems of hypothesis testing encountered in hypothesis testing.
1. Two-tailed tests of mean when population SD is known and sample size is large.
Solution
Reject Ho: $\mu = \mu_1$, against $H_1: \mu \neq \mu_1$ at α level of significance if the sample mean $\overline{x}$ lies outside the interval ($\mu \pm x_{\alpha/2}$*SE), i.e. ($\mu + x_{\alpha/2}$*SE, $\mu - z_{\alpha/2}$*SE), where μ is the population mean and $\pm x_{\alpha/2}$ are critical values; x is the original variable following normal distribution.

or

Reject Ho: $\mu = \mu_1$, against $H_1: \mu \neq \mu_1$ at α level of significance if the z value of the sample mean $\overline{x}$ lies outside the interval $\pm z_{\alpha/2}$, i.e. ($z_{\alpha/2}$, - $z_{\alpha/2}$) where μ is the population mean of the original variable following normal distribution and $\pm z_{\alpha/2}$ are critical values.

Example: Problem 1
Problem 1: In a study on daily purchase of US dollars by the banks' dealing rooms across the globe the population mean during the year 2006-07 is believed to be to the tune of $80000 billions with an SD of $4000 billions. The Federal Reserve Bank (Fed) selects a sample of 100 days and finds the mean value $79600 billions. If the Fed uses 5% significance level does the data validate the above belief?
Solution
1. Ho: $\mu = 80000$
2. $H_1: \mu \neq 80000$ since it is not mentioned whether the true population mean is suspected to be more or less than what is believed it is a two tailed test.
3. $\overline{x} = 79600$, $\sigma = 4000$, $n = 100$; so SE = $4000/\sqrt{100} = 400$
4. Testing Ho: $\mu = 80000$ against $H_1: \mu \neq 80000$ at 5% significance level means whether the sample mean $\overline{x} = 79600$ is lying within the interval 80000 ± 1.96*400 i.e. (80784, 79216). Clearly it is lying.
Conclusion: Ho: $\mu = 80000$ is not rejected.

2. Left-tailed tests of mean when population SD is known and sample size is large.
Solution
Reject Ho: $\mu = \mu_1$, against H_1: $\mu < \mu_1$ at α level of significance if the sample mean $\overline{x}$ is smaller than $\mu - x_\alpha * SE$, where μ is the population mean and $- x_\alpha$ is the critical value.

Or

Reject Ho: $\mu = \mu_1$, against H_1: $\mu > \mu_1$ at α level of significance if the sample mean $\overline{x}$ is greater than $\mu + x_\alpha * SE$, where μ is the population mean and x_α is the critical value.
Note: The same result can be obtained by converting x variable to the z variable.
**Example:** Problem 2
Problem 2: The minimum balance with which to allow to open a savings account (excluding salary accounts) without any cheque-book facility in any of the public sector banks in India is permitted to be Rs. 100 with SD = Rs 2. In a study 50 such accounts were examined in different banks at random in a particular state and the sample mean is found to be Rs. 99.75.
If the sample mean is too small at 10% significance level compared to what it is permitted to be it should be a matter of concern for the regulator. Is it really?
Solution
1. Ho: $\mu = 100$
2. H_1: $\mu < 100$ because the problem is about whether the sample mean is too **small** compared to what is permitted.
3. $\overline{x} = 99.75$, $\sigma = 2$, n = 50; so SE = $2/\sqrt{50}$ = 0.2829
4. The z value of sample mean $\overline{x}$ is (99.75 – 100)/0.2829 = -0.88
5. The required z area is 10% area to the left end of the z curve; the z value is – 1.28. See Illustration IV below:

Left tailed test of the mean at 10% significance level

Illustration IV
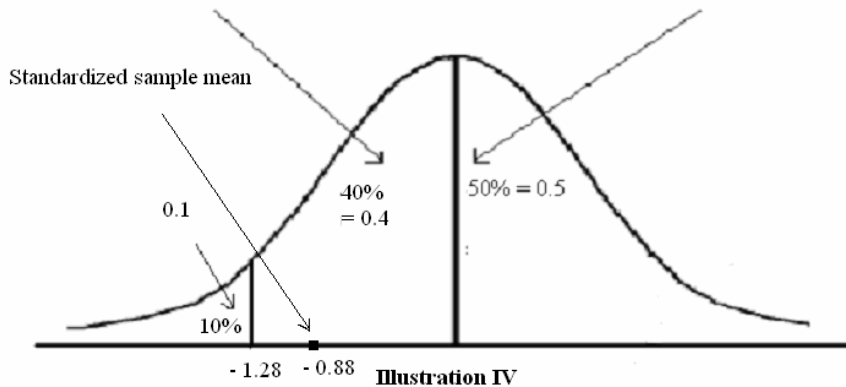
0.7	0.258	0.2611	0.2642	0.2673	0.2704	0.2734
0.8	0.2881	0.291	0.2939	0.2967	0.2995	0.3023
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289
1	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265
1.5	0.4332	0.4345	0.4357	0.437	0.4382	0.4394
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505

Conclusion: Since the standardized sample mean 0.88 lies within the 90% confidence level, the null hypothesis is not rejected and it should not be a matter of concern for the regulator.

3. Two-tailed tests of mean when population SD is unknown and sample size is small.

Solution: Reject Ho: $\mu = \mu_1$, against H_1: $\mu \neq \mu_1$ at α level of significance if the standardized t value of the sample mean $\overline{x}$ lies outside the interval $\pm t_{\alpha/2}$, i.e. $(t_{\alpha/2}, - t_{\alpha/2})$, where μ is the population mean and $\pm t_{\alpha/2}$ are critical values.

Example: Problem 3

Problem 3: A personnel specialist of a major foreign bank is recruiting a large number of dealing room employees in campus-placement programme of a top rated B school in India. During the recruitment process, the executive chairman of the bank asks what would be the average cost to company per annum. The specialist replied "90 lakh INR, I suppose". When management examines 20 new recruits the sample mean came to be 84 lakhs INR with SD 11 lakhs INR. The expert's history reflects 95% of her predictions coming true. Should management reject the expert's hypothesis?
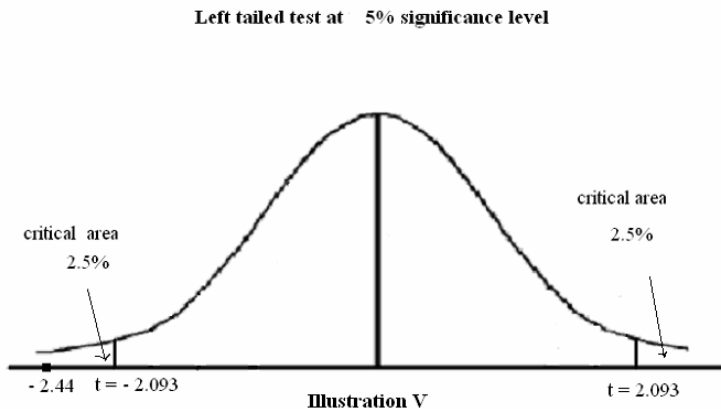
Solution

1. Ho: $\mu = 90$

2. H_1: $\mu \neq 90$ since it is not mentioned whether the true population mean is suspected to be more or less than what is believed it is a two tailed test.

3. $\overline{x} = 84$, $\sigma_s = 11$, n = 20; so SE = $11/\sqrt{20} = 2.46$. Testing Ho: $\mu = 90$ against H_1: $\mu \neq 90$ at 10% significance level means whether the standardized t value of sample mean $\overline{x} = 84$ is lying within the interval $\pm t_{5\%}$ for df = 19 i.e. (2.093, -2.093).

4. the standardized t value of sample mean $\overline{x} = 84$ is t = $(\overline{x} - \mu_{Ho})/SE = (84 - 90)/2.46 = -2.44$. Clearly it is lying within the rejection level. See illustration V below.

Left tailed test at 5% significance level

Illustration V

Conclusion: Ho: $\mu = 80000$ is rejected.

4. One-tailed tests of mean when population SD is unknown and sample size is small.

Solution: Reject Ho: $\mu = \mu_1$, against H_1: $\mu \neq \mu_1$ at α level of significance if the standardized t value of the sample mean $\overline{x}$ lies outside the interval $\pm t_\alpha$, i.e. $(t_\alpha, - t_\alpha)$, where μ is the population mean and $\pm t_\alpha$ are critical values, since in t table t values for areas in both tails combined are available.

Note: Before conducting one tailed test with t distribution one should check the availability of t values for the particular value of 2α.

See the t table below:

| 12	1.7823	2.1788	2.56	3.0545
13	1.7709	2.1604	2.5326	3.0123
14	1.7613	2.1448	2.5096	2.9768
15	1.7531	2.1315	2.4899	2.9467
16	1.7459	2.1199	2.4729	2.9208
17	1.7396	2.1098	2.4581	2.8982
18	1.7341	2.1009	2.445	2.8784
19	1.7291	2.093	2.4334	2.8609
20	1.7247	2.086	2.4231	2.8453

Example: Problem 4

Problem 4: Consider the problem 3. Suppose the management wants to test at 5% level of significance whether the specialist has overstated the figure, i.e. whether the actual mean is smaller than 90 lakh INR. What is the result of the test?

Solution:

1. As per the t table $t_{10\%} = 1.729$ for df = 19.
2. Again the sample mean is in the rejection level. See illustration VI below.

Left tailed test of the mean at 10% significance level

critical area

5%

critical area

5%

- 2.44   t = - 1.729   **Illustration VI**   t = 1.729
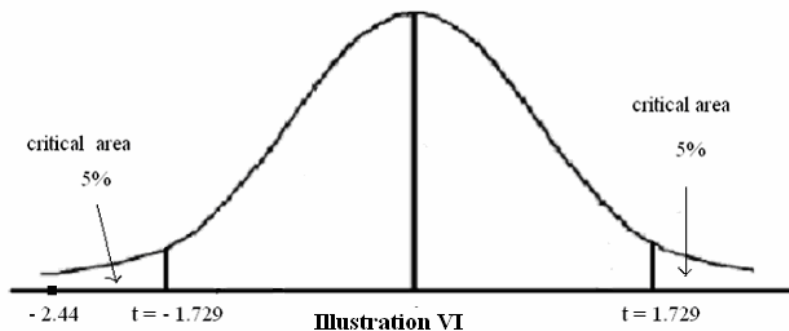
Conclusion: Again the null hypothesis is rejected.

Hypothesis testing for differences between means

Concept

1. Here we test $H_0: \mu_1 - \mu_2 = k$ against $H_0: \mu_1 - \mu_2 \neq k$ where k is a particular value, where $H_0: \mu_1$ is the mean of population 1 and μ_2 is the mean of population 2, both the variable having same unit and their sample means follow normal distribution independently.

2. Since sample mean of variable 1 follows a normal distribution with mean μ_1 and SE_1 and sample mean of variable 2 follows a normal distribution with mean μ_2 and SE_2, therefore the difference of the sample means follows a normal distribution with mean $\mu_1 - \mu_2$ and SE of the difference between two means = $\sqrt{(\sigma_1^2/n_1 + \sigma_2^2/n_2)}$.

3. If population variances σ_1 and σ_2 are unknown then they are estimated by respective sample SDs σ_{s1} and σ_{s2} as before.

4. The rest of the procedure is as before, e.g. we use original normal or z distribution for large sample; and t distribution for small sample and unknown population SD.

Example: Problem 5

Problem 5: The ICICI Bank wants to test at 5% significance the hypothesis that the mean business per banker is same between two metros Delhi and Mumbai. The populations are assumed to follow normal distribution though their SDs are unknown. The research team

took a sample from each of Delhi and Mumbai of sizes 200 and 175 respectively, sample means are 8.95 and 9.10 respectively in lakh INR and sample SDs are 0.4 and 0.6 respectively in lakh INR. Should the null hypothesis be rejected?

Solution

1. H_0: $\mu_1 - \mu_2 = 0$ against H_1: $\mu_1 - \mu_2 \neq 0$ since nothing in terms of 'small' or 'large' is given in comparison between H_0 difference and sample difference.

2. SE of difference = 0.053

3. So sample z value of $\mu_1 - \mu_2$ = - 2.83 which lies outside the confidence level $\pm$ 1.96

Conclusion: Reject H_0

Note: H_0: $\mu_1 - \mu_2$ could take any value, e.g. H_0: $\mu_1 - \mu_2$ = - 0.1; then sample z value of $\mu_1 - \mu_2$ = - 0.94 which lies within the confidence level $\pm$ 1.96; here H_0 would not be rejected.

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Warm Regards,

Rituparna Das

End Notes

[i] Interest rates mean rates in money market and debt market, i.e. benchmark rates, e.g. YTM of 10 year maturity Government of India's dated securities, YTM of 91 days maturity T-bill rate, MIBOR-Swap rates, call rates, monetary policy rates of RBI like CRR, repo rate, reverse repo rate and bank rate.

[ii] http://www.thehindubusinessline.com/2003/05/10/stories/2003051001691000.htm accessed on 5-Dec-07

[iii] http://www.canbankindia.com/shareholders/performance/a2004/a2004-ind.html http://www.bankofbaroda.com/fin/fin_fy2004_placcount.asp accessed on 5 Dec 07

[iv] http://www.canbankindia.com/shareholders/performance/a2006/a2006-ind.html http://www.bankofbaroda.com/fin/fin_fy2007_placcount.aspaccessed on 5 Dec 07

[v] In US a local football club was selling tickets at a very high price; being weekend that time everybody rushed to ATM and the bank collapsed since the Central Bank could be contacted on weekend.

[vi] http://www.business-standard.com/india/storypage.php?autono=332984 accessed on 01-Sep-08

[vii] At the individual level, many people invested in RBI bonds such that interest incomes there from match with LIC premium payment schedule. Insurance companies invest in long term bonds such as to match the mature policy disbursements with the interest receipts. Their motive is ALM not profit maximization.

[viii] http://www.pnbgilts.com/gsec.asp accessed on 18-Aug-08

[ix] http://www.gpw.pl/gpw.asp?cel=e_inwestorzy&k=4&i=/e_papiery/bonds/bonds&sky=1 accessed on 30-May-08 http://www.mbea.net/debt.php accessed on 18-Aug-08

[x] For example, as on 13-Aug-08, FIMMDA (Fixed Income Money Market and Derivatives Association of India) published prices and YTMs of 97 coupon bonds of the maturity dates ranging from 28-Aug-08 to 7-Jun-36. Out of 97 dated securities only 24 were traded on the same day. URLs: http://fimmda.org/, http://ccilindia.com/CCIL/DATA/Market%20Statistics/G-Sec%20Settlement/Daily%20volume%20wise/CCIL-106-13082008.pdf accessed on 18-Aug-08

[xi] http://rbi.org.in/scripts/BS_ViewMasCirculardetails.aspx?Id=3664&Mode=0 accessed on 18-Aug-08

[xii] www.fimmda.org/Information_Center/Circulars/Docs/**Valuation_method**.doc, http://www.fimmda.org/Information_Center/Circulars/Docs/ValuationCircular.pdf, http://fimmda.org/products_and_services/fair_value/misc/NoteonFIMMDAmethod.pdf accessed on 18-Aug-08

[xiii] Grandville (2001), Chapter 2

[xiv] www.**ccil**india.com/**CCIL**/DATA/Research/Historical%20Statistics/**CCIL**-131-20071012121316.xls accessed on 19-Aug-08

[xv] http://www.fimmda.org/Information_Center/Circulars/Docs/ValuationCircular.pdf dated 16-Mar-07 accessed on 18-Aug-08

[xvi] **rbi**docs.**rbi**.org.in/rdocs/PressRelease/PDFs/3204.pdf accessed on 01-Aug-08

[xvii] http://www.afdb.org/pls/portal/docs/PAGE/ADB_ADMIN_PG/DOCUMENTS/FINANCIALINFORMATION/MARKET%20RISK%20REVIEW%20ENGLISH.PDF accessed on 01-Aug-08

[xviii] http://rbi.org.in/scripts/BS_ViewMasCirculardetails.aspx?Id=3642&Mode=0 accessed on 01-Aug-08

[xix] http://www.rba.gov.au/PublicationsAndResearch/Bulletin/bu_dec96/bu_1296_1.pdf accessed on 01-Aug-08

[xx] http://rbi.org.in/scripts/BS_ViewMasCirculardetails.aspx?Id=3642&Mode=0#market accessed on 01-Aug-08

[xxi] Jorion (2007)

[xxii] http://rbidocs.rbi.org.in/rdocs/notification/PDFs/59361.pdf accessed on 01-Aug-08

[xxiii] http://www.rediff.com/money/2007/sep/28rbi.htm, http://www.thehindubusinessline.com/2009/01/03/stories/2009010351910100.htm, http://www.domain-b.com/finance/finance_diary/2004finance_diary/200408aug/20040809financediary.html, http://www.financialexpress.com/news/indias-10year-bonds-near-monthhigh/226923/ accessed on 25-Dec-08