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## Internet network externalities

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**Abstract:** A driving force behind the emergence of the 'new' or information economy is the growth of the internet network capacity. A fundamental problem in mapping this dynamic is the lack of an acceptable theoretical framework through which to direct empirical investigations. Most of the models in the literature on network externalities have been developed in a static framework, with the externalities viewed as instantaneous or self-fulfilling. The model specified here builds on the received theory from several sources to extend these features and develops a dynamic model that is both capable of econometric estimation and which provides as an output a direct measure of the network effect. Accordingly, the main goal of this paper is to find the magnitude of the external effect on internet network growth. In addition, this paper illustrates the ability of the panel data to generate estimates of structural parameters capable of explaining internet host growth.

**Keywords:** information; network externalities; internet; growth.

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**Biographical notes:** Russel Cooper is a Professorial Research Fellow at Macquarie University, Australia. Research interests include intertemporal optimisation models in economics, the duality theory and its applications in economics, new growth economics, inter-industry modelling, applications of rational economic modelling in specific industries, applied consumer demand studies and spatial economics. Current research projects include the estimation of cost functions, the investment decisions of high-technology firms, the investigation of network externalities in the info-communications sector, forecasting with short time series and the economic modelling of technology transfer across the digital divide.

Professor Gary Madden's primary research area is the economic modelling of electronic networks. Within this gambit, his particular research fields encompass theoretically motivated short time series forecasting, the economics of disruptive technologies, digital divide issues, network externalities and the internet evolution and the welfare impact of economic growth. He is the author

of 78 peer-reviewed publications in these fields since 1993 and has attracted over \$1M to the University for his research since 1994. In particular, he is the Chief Investigator on four Australian Research Council (ARC) Discovery Project (Large) grants since 1998. He is a Consultant to the government and a Member of the Board of Directors of the International Telecommunications Society. He is currently the Associate Editor of the *International Journal of Management and Network Economics* and an Editorial Board Member of *The Open Communication Science Journal* and the *Journal of Media Economics*. He is a Member of the Scientific Council of *Communications and Strategies*.

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## 1 Introduction

The internet is a distribution system or conduit through which content is sent. Traditional telecommunications systems are specialised in that they (essentially) carry only two-way simultaneous voice messages along dedicated circuit-switched paths and they are not easily modified to do much else (Economides and White, 1994). What is different (and unique) about the internet network is that it is both two-way broadband and interactive. Just about any electronic signal can be sent, more or less, from anybody to anybody else (Faulhaber, 1999). Another distinguishing feature of internet traffic is that it is packet-switched, *i.e.*, no continuous path is devoted to the delivery of a message.

Recent internet network growth has created new and expanded existing markets for broadband (bandwidth) capacity to carry two-way interactive high-speed data transfers. Accordingly, the internet has the potential to increase productivity growth and generate wealth in a variety of distinct but mutually reinforcing ways (Litan and Rivlin, 2001). Given this potential, a recent OECD (2000) finding that indicates that the European Union (EU) is lagging behind the USA in terms of internet penetration is important. That study shows, *e.g.*, that in March 2000, there were 185 internet hosts per 1000 inhabitants in the USA compared to 41 per 1000 in the UK and 16 per 1000 in France. Further, it is suggested that internet access pricing structures may be a key factor in explaining penetration (Bourreau, 2001; Rappoport *et al.*, 2002). A fairly natural question then for economists to consider is whether the differential rates of internet system growth is due to internet access pricing structures or, more fundamentally, growth generated by direct network externalities after a critical system mass is achieved.

Direct network externalities occur when the utility of a consumer depends directly on the total number of compatible services (Gandal, 1995). Such direct network externalities have long been recognised in models explaining optimal telecommunications network size (Katz and Shapiro, 1986).<sup>1</sup> In this context subscribers' utility depends on the number of subscribers with compatible access (Economides, 1996). Rohlfs (1974) formulated the first model of the equilibrium number of telephone handsets in a population by focusing on individual constrained choice for telephone subscription incorporating parameters for consumer income and price. The equilibrium user set is the subscriber base resulting from the combined outcome of individual utility maximisation programmes. Multiple equilibriums may exist, with a small network making potential subscription relatively unattractive.

Economides and Himmelberg (1995) refine the notion of critical mass as the smallest size network that can be sustained in equilibrium. They argue that when the critical mass is substantial, market coverage will not be achieved – either the market does not exist or it is of insufficient coverage.<sup>2</sup> Accordingly, consumer willingness to adopt internet service is an increasing function of network size (Shy, 2001). The existence of network externalities in a dynamic setting increases the speed at which market demand grows in the presence of a downward trend for industry marginal cost. Given the possible existence of a network externality for internet connection (and e-commerce), estimates of the size of the network effect are critical for forecasting demand and in network planning. Accordingly, a model is developed here to describe the global internet market growth that provides a detailed analysis of the nature of the externality.

Bensaid and Lesne (1996) argue that most network externality models are developed in a static framework, with externalities viewed as either instantaneous or self-fulfilling. An Economides (1996) dynamic ‘macro’ approach is employed here to analyse the role network externalities have in explaining internet system growth in a continuous-time setting. The ‘macro’ approach simply assumes network externalities exist and attempts to model their consequences.<sup>3</sup> Further, here the notion of network externality is broadened to include those due to producer activity. Interaction between agents’ (consumers’ and firms’) decisions is considered by a representative agent model in which sustained growth is the result of positive externalities from investment in network input  $n$ . Agents are linked through income flows and endogenous growth in the internet network occurs through the inclusion of a network externality in the production argument in the ‘old economy’ firms’ production function and also in the consumer’s instantaneous utility function. The system is stochastic because the return to the representative consumer from Applications Sector (AS) investment is uncertain. The stochastic income specification leads to a stochastic inter-temporal optimisation problem. The resultant solution provides an optimised network growth equation for estimation. The model is estimated on cross-country panel data to yield a direct measure of the strength of the network effect.

The paper is organised as follows. Section 2 specifies a model to examine internet network growth that incorporates a network externality. In Section 3 data and variables used in estimation are presented and described. The empirical modelling strategy is explained in Section 4, and estimates of network externalities are reported. Concluding remarks and policy implications are provided in Section 5.

## **2 A dynamic model of internet network growth**

Consider a decentralised economy that consists of a representative household, and a representative old economy firm that behaves competitively. The firm controls both network and non-network input levels. A positive externality is associated with network investment through production activity. Internet network externalities can also arise through consumption. A representative consumer obtains utility from real total consumption and its current network size. The consumer has the option not to consume all her income. Non-consumption (or saving) defaults to network investment. The consumer can, moreover, elect to relinquish ownership of part of the network in exchange for ownership of an AS investment that provides a risky return.

### 2.1 Network production externalities

Let  $F(v, n, n^*)$  denote the production function of a representative firm where  $v$  is either an aggregate non-network input or a vector of non-network inputs, *e.g.*, labour and non-network capital. Let  $n^*$  represent a network externality generated through productive activity. This argument allows ‘endogenous growth’ to occur in the network growth equation, *viz.*, the production function exhibits decreasing returns in  $n$  (from the perspective of the firm) and increasing returns when  $n$  is equated to  $n^*$  post-optimisation. That is, during optimisation  $n^*$  is treated by the firm as exogenous, and post-optimisation  $n^*$  is equated to  $n$  when model equations are derived. Thus positive externalities arise from network capital and are a source of increasing returns in production. Let  $w$  represent the price of variable inputs. Illustration of the ‘optimising out’ process is provided for the case where  $v$  is a variable input. Consider the production function:

$$F(v, n, n^*) = v^\alpha n^{1-\alpha} (1 + n^*)^\beta \quad (1)$$

and the instantaneous variable profit function (conditional on network size,  $n$ ):

$$\Pi(w, n, n^*) = \max_v \langle F(v, n, n^*) - wv \rangle. \quad (2)$$

The solution for optimal  $v$  is:

$$\hat{v} = \alpha^{1/(1-\alpha)} w^{-1/(1-\alpha)} (1 + n^*)^{\beta/(1-\alpha)} n \quad (3)$$

where the linearity of  $\hat{v}$  in  $n$  follows from the linear homogeneity of the production function in  $(v, n)$ .

Conditional on the  $n$ , optimised output can then be constructed as a function of input prices:

$$\hat{F}(w, n, n^*) = \alpha^{\alpha/(1-\alpha)} w^{-\alpha/(1-\alpha)} (1 + n^*)^{\beta/(1-\alpha)} n. \quad (4)$$

Further, the linearity of optimised output in  $n$ , *i.e.*, from the point of view of the firm’s optimisation, without internalising the externality, is emphasised by writing:

$$\hat{F}(w, n, n^*) = R(w, n^*)n \quad (5)$$

where  $R(w, n^*)$  is the return per unit of network capital:

$$R(w, n^*) = \alpha^{\alpha/(1-\alpha)} w^{-\alpha/(1-\alpha)} (1 + n^*)^{\beta/(1-\alpha)}. \quad (6)$$

$\partial R(w, n^*)/\partial n^* > 0$  indicates the production network externality directly augments the return per unit of network capital or interest rate in this stylised model.

### 2.2 Network consumption externalities

Internet network externalities can also arise through consumption. Let  $U(c, n^*)$  denote the instantaneous utility function of a representative consumer where  $c$  is real total consumption and  $n^*$  is the current network size for an average firm (which is outside the control of the consumer). The point of departure here is the standard iso-elastic utility specification that emphasises the importance of the inter-temporal elasticity of substitution ( $IES = -\partial \ln c / \partial \ln U_c$ ). Temporarily setting aside the network effect, specify instantaneous utility iso-elastic:

$$U(c, \bullet) = c^\gamma. \quad (7)$$

The *IES* for (7) is:

$$IES = 1/(1-\gamma), \quad (8)$$

where  $-\infty < \gamma < 1$ . The *IES* indicates the willingness of a consumer to forego current consumption in favour of current saving and greater discounted future utility, viz., consumer flexibility. Here the network consumption externality is introduced through the *IES*, i.e., as income is received from  $n^*$  and as consumer flexibility might realistically be income dependent, it seems reasonable to suspect the *IES* is affected by  $n^*$ . The specification adopted here is:

$$IES = \theta_1 \left[ \frac{1}{1+n^*} \right] + \theta_2 \left[ \frac{n^*}{1+n^*} \right]. \quad (9)$$

In Equation (9) the *IES* ranges in value from  $\theta_1$  when there is no network rollout ( $n^* = 0$ ) and asymptotes to  $\theta_2$  as the network expands indefinitely ( $n^* \rightarrow \infty$ ). The *IES* is increasing in  $n^*$  if  $\theta_2 > \theta_1$ , viz., the consumer becomes more flexible. Accordingly, the utility function incorporating network externality effects is written as a function of network size  $G(n^*)$ :

$$U(c, n^*) = c^{G(n^*)} \quad (10)$$

where since  $IES = 1/[1-G(n^*)]$  or  $G(n^*) = 1-1/IES$ , and with the *IES* given by (9),  $G(n^*)$  is specified:

$$G(n^*) = 1 - \frac{1}{\theta_1 \left[ \frac{1}{1+n^*} \right] + \theta_2 \left[ \frac{n^*}{1+n^*} \right]}. \quad (11)$$

### 2.3 Income flows

The old economy firm produced output through Equation (5). This output is a source of income to the owners of the firm – the consumers. Income is also derived from a stochastic return to consumers from AS (new economy) investment obtained by selling  $x$  of the network  $n$ . In doing so the consumer foregoes a ‘sure’ rate of return  $R(w, n^*)xdt$  for receipt of risky return  $xdq/q$ . While uncertainty of income flow is expected for both new and old economy firms – it seems reasonable to assume that new economy firm returns are the more uncertain. To focus attention, uncertainty is isolated to the returns of the new economy firm. Here the risky asset is assumed to pay no dividend and provide only a capital gain or loss. The resulting flow of consumer income from production and investment sources is:

$$dy = R(w, n^*)ndt + [dq/q - R(w, n^*)dt]x \quad (12)$$

where the price of the risky asset,  $q$ , is modelled as following a geometric Brownian motion with drift  $\mu_q$  and volatility  $\sigma_q$ :

$$dq = \mu_q qdt + \sigma_q qdz_q \quad (13)$$

and  $dz_q$  is Brownian motion, with the properties  $E(dz_q) = 0$ ,  $E(dz_q)^2 = dt$ .<sup>4</sup>

#### 2.4 Expenditure

An alternative to consumption is the retention of earnings by the firm, which are employed to extend the network. Consequently, network expansion is stochastic and so the demand side of the income identity is:

$$dy = cdt + pdn, \quad (14)$$

where the network access price  $p$  converts the value of the network extension into units of the consumption good.

#### 2.5 Optimisation model

For the stochastic income specification Equation (12) through Equation (14), the representative consumer's inter-temporal optimisation problem is:

$$J(n_0, p_0, w_0) = \max_{\{c(t), x(t)\}} E_0 \int_0^{\infty} e^{-\delta t} U(c(t), n^*(t)) dt \quad (15)$$

subject to:

$$dn = \left[ \frac{R(w, n^*)n - c}{p} \right] dt + \left[ \frac{dq/q - R(w, n^*)dt}{p} \right] x \quad (16)$$

$$dq = \mu_q q dt + \sigma_q q dz_q \quad (17)$$

$$dp = \mu_p p dt + \sigma_p p dz_p \quad (18)$$

$$dw = \mu_w w dt + \sigma_w w dz_w \quad (19)$$

$$n^*(t) = n(t), t \in [0, \infty) \quad (20)$$

$$n(0) = n_0, p(0) = p_0, w(0) = w_0. \quad (21)$$

#### 2.6 Optimised network growth equation

Combining Equation (16) and Equation (17) the network growth equation can be characterised as a diffusion of the form:

$$dn = \left\{ \frac{R(w, n^*)n + [\mu_q - R(w, n^*)]x - c}{p} \right\} dt + \frac{\sigma_q x}{p} dz_q. \quad (22)$$

Due to the time-autonomous nature of Equation (15), the solutions for  $c$  and  $x$  may be obtained in feedback or synthesised form, expressing the controls as a function of the current values of the states of  $n$ ,  $p$  and  $w$ . To describe the solution, it is useful to define the latent variables:

$$r = R(w, n^*) \quad (23)$$

and

$$h = 1/[1 - G(n^*)], \quad (24)$$

interpretable as the interest rate and the *IES*, respectively. Also note that  $n = n^*$  in the optimised model.<sup>5</sup> Further, Cooper *et al.* (1995) show optimal  $c$  can be written:

$$\hat{c} = \left\{ h\delta + [1-h] \left[ r + \frac{1}{2} h(\mu_q - r)^2 / \sigma_q^2 \right] \right\} n \quad (25)$$

and optimal  $x$  as:

$$\hat{x} = h \left[ \frac{\mu_q - r}{\sigma_q^2} \right] n. \quad (26)$$

Utilising the synthesised solutions Equation (25) and Equation (26), and substituting into (22), provides the optimal network diffusion:

$$dn = h \left\{ \frac{r - \delta + \frac{1}{2} [h+1] (\mu_q - r)^2 / \sigma_q^2}{p} \right\} ndt + h \left\{ \frac{\mu_q - r}{\sigma_q p} \right\} ndz_q \quad (27)$$

where, in view of the specifications of technology and preferences, and setting  $n = n^*$ :

$$r = \alpha^{\alpha/(1-\alpha)} w^{-\alpha/(1-\alpha)} [1+n]^{\beta/(1-\alpha)} \quad (28)$$

and

$$h = \theta_1 \left[ \frac{1}{1+n} \right] + \theta_2 \left[ \frac{n}{1+n} \right]. \quad (29)$$

### 3 Data and variables

Equation (27), after substitution of Equation (28) and Equation (29), is estimated on a sample of 23 OECD countries.<sup>6</sup> Annual data from 1995 through 2000 are collected for Consumer Price Index (CPI), exchange rates, GDP (gross domestic product), internet access price, internet hosts and wages. CPI, GDP and internet host numbers (HOST) are obtained from the International Telecommunication Union (ITU) *World Telecommunication Development Report*.<sup>7</sup> Internet access price data (PRICE) are from the *OECD Communications Outlook* 1997, 1999 and 2001. PRICE is the price of internet access for 20 hours per month peak rate in US Dollars (USD) purchasing power parity. The price of internet access is comprised of the timed public switched telephone network charge and monthly internet service provider fee. Published PRICE data for 1996 are converted from USD to USD Purchasing Power Parity (PPP). PRICE data for 1997 are not available and are interpolated.<sup>8</sup> Unpublished price data for 1999 is obtained directly from the OECD. PRICE is deflated using an adjusted CPI index. The CPI (1995 = 1) is adjusted to maintain currency relativities by multiplying the annual CPI index by 1996 USD PPP. The CPI is converted into USD by dividing country adjusted CPI by the nominal exchange rate. New hosts ( $\Delta\text{HOST} = \text{HOST}_t - \text{HOST}_{t-1}$ ) are obtained by first-differencing HOST series. WAGE is the proportion of Compensation of Employees (OECD code: WSSS) in nominal GDP.<sup>9</sup>



Mean, standard deviation, minimum and maximum values for HOST,  $\Delta$ HOST, PRICE and WAGE are reported in Table 1. Host numbers (HOST) range in value from less than four thousand (Luxembourg) to in excess of 80 million (US). The mean addition to the HOST count ( $\Delta$ HOST), across both countries and time, is almost 800 000. Eleven countries recorded declines in host numbers, with the largest decline in France (2000).<sup>10</sup> PRICE, the listed price of dominant ISP and PSTN carriers, ranges in value from USD18.96 (US) to USD291.43 (Mexico). Average WAGE compensation is 48% of GDP and reflects considerable variation across the sample from 26% (Turkey) to 61% (Switzerland).

**Table 1** Summary statistics 1996–2000

<i>Variable</i>	<i>Mean</i>	<i>Standard deviation</i>	<i>Minimum</i>	<i>Maximum</i>
<i>Complete sample</i>				
HOSTS	2 043 942	9 130 292	3518	80 566 944
$\Delta$ HOSTS	737 982	3 397 557	–110 664	27 390 988
PRICE	53.05	33.73	18.96	291.43
WAGE	0.48	0.08	0.26	0.61
<i>Sample with Mexico and Turkey excluded</i>				
HOSTS	2 209 516	9 504 014	3518	80 566 944
$\Delta$ HOSTS	796 852	3 537 170	–110 664	27 390 988
PRICE	48.41	20.36	18.96	135.69
WAGE	0.50	0.06	0.32	0.61

Notes: HOST is host numbers.  $\Delta$ HOST =  $\text{HOST}_t - \text{HOST}_{t-1}$ . PRICE is the real price of internet access in USD purchasing power parity.

## 4 Model estimation

### 4.1 Functional form specification

The network growth equation was derived in Section 2 in continuous time as Equation (27) to Equation (29). Converting to discrete time, let  $dt = 1$ ,  $dn = n_t - n_{t-1} = \Delta n_t$  and  $dz_q = \varepsilon_q \sim N(0,1)$ . The estimating form becomes:

$$\begin{aligned}
 \frac{\Delta n_t}{n_{t-1}} = & \left\{ \theta_1 \left[ \frac{1}{1+n_{t-1}} \right] + \theta_2 \left[ \frac{n_{t-1}}{1+n_{t-1}} \right] \right\} \frac{\alpha^{\alpha/(1-\alpha)} w_t^{-\alpha/(1-\alpha)} [1+n_{t-1}]^{\beta/(1-\alpha)} - \delta}{p_t} \\
 & + \frac{1}{2} \left\{ \theta_1 \left[ \frac{1}{1+n_{t-1}} \right] + \theta_2 \left[ \frac{n_{t-1}}{1+n_{t-1}} \right] \right\} \left\{ (1+\theta_1) \left[ \frac{1}{1+n_{t-1}} \right] + (1+\theta_2) \left[ \frac{n_{t-1}}{1+n_{t-1}} \right] \right\} \quad (30) \\
 & \times \frac{(\mu_q - \alpha^{\alpha/(1-\alpha)} w_t^{-\alpha/(1-\alpha)} [1+n_{t-1}]^{\beta/(1-\alpha)})^2}{p_t \sigma_q^2} + \varepsilon_{n,t}
 \end{aligned}$$

with error term:

$$\varepsilon_{n,t} = \left\{ \theta_1 \left[ \frac{1}{1+n_{t-1}} \right] + \theta_2 \left[ \frac{n_{t-1}}{1+n_{t-1}} \right] \right\} \left[ \frac{\mu_q - \alpha^{\alpha/(1-\alpha)} w_t^{-\alpha/(1-\alpha)} [1+n_{t-1}]^{\beta/(1-\alpha)}}{p_t \sigma_q} \right] \varepsilon_{q,t} \quad (31)$$

It will prove useful to identify the components of Equation (30) that have direct economic interpretation. They are, the inter-temporal elasticity of substitution:

$$IES = h = \left\{ \theta_1 \left[ \frac{1}{1+n_{t-1}} \right] + \theta_2 \left[ \frac{n_{t-1}}{1+n_{t-1}} \right] \right\}, \quad (32)$$

the ‘interest rate’ (or rate of return to the network as a productive resource),  $r$ :

$$r = \alpha^{\alpha/(1-\alpha)} w_t^{-\alpha/(1-\alpha)} [1+n_{t-1}]^{\beta/(1-\alpha)} \quad (33)$$

and the Relative Risk Premium (RRP), defined as the normalised equity premium,  $(\mu_q - r)/\sigma$  relative to network access price,  $p$ :

$$RRP = \left[ \frac{\mu_q - r}{\sigma_q} \right] / p. \quad (34)$$

Potential heteroscedasticity is implied by Equation (31). The scale factor attached to the random error  $\varepsilon_{q,t}$  in (31) may be summarised, in view of Equation (32) and Equation (34), as  $IES \times RRP$ . This scale factor is itself a stochastic process and contains error variation that is partly predetermined (due to  $n_{t-1}$ ) and partly currently determined (due to  $w_t$  and  $p_t$ ). In addition,  $w_t$  and  $p_t$  contain partly systematic variation (since they have drifts  $\mu_w$  and  $\mu_p$ ) and partly random variation (in view of their specification as stochastic processes, *i.e.*, Equation (18) and Equation (19)). While a weighted correction procedure could be applied if all variation were predetermined or systematic, the idea of giving observations different weights because of random variation is problematic since it could induce inconsistency. An alternative approach is to note that the offending term in Equation (31), *viz.*,  $h(\mu_q - r)/(p\sigma_q)$  has a drift that, though complicated, may be derived from the underlying stochastic processes for  $n$ ,  $p$  and  $w$  by application of Ito’s Lemma. Borrowing methodology from finance theory, there exist synthetic probabilities which would force this complex drift to zero, so that the offending scale factor in Equation (31), while not a constant, could at least be modelled as a martingale under the synthetic probability measure. Here it is proposed to find maximum likelihood estimates for this case. This seems more acceptable than attempting to convert the scale factor to a constant when it contains random variation.

To employ the proposed correction procedure, a variable parameter specification for components of  $r$ ,  $h$ ,  $\mu_q$  and  $\sigma_q$  where they appear in Equation (30) is utilised. This may be interpreted as indirectly estimating probabilities associated with realisations of  $IES \times RRP$ . The variable parameter estimation is conducted jointly with estimation of the economic parameters of interest in Equation (30), using a maximum likelihood estimator that treats the variance of the error in Equation (30) as a constant. This procedure allows the maximum likelihood estimation technique to choose parameter values most compatible with a variance whose expected value is constant. In particular, the variable parameter specification allows for both country-specific and time-specific adjustment factors that augment Equation (30) to provide:

$$\begin{aligned}
\frac{\Delta n_t}{n_{t-1}} = & \left\{ \theta_{1,t} \left[ \frac{1}{1+n_{t-1}} \right] + \theta_2 \left[ \frac{n_{t-1}}{1+n_{t-1}} \right] \right\} \frac{A_c T_t w_t^{-\alpha/(1-\alpha)} [1+n_{t-1}]^{\beta/(1-\alpha)} - \delta}{p_t} \\
& + \frac{1}{2} \left\{ \theta_{1,t} \left[ \frac{1}{1+n_{t-1}} \right] + \theta_2 \left[ \frac{n_{t-1}}{1+n_{t-1}} \right] \right\} \left\{ (1+\theta_{1,t}) \left[ \frac{1}{1+n_{t-1}} \right] + (1+\theta_2) \left[ \frac{n_{t-1}}{1+n_{t-1}} \right] \right\} \\
& \times \frac{(\mu_t - A_c T_t w_t^{-\alpha/(1-\alpha)} [1+n_{t-1}]^{\beta/(1-\alpha)})^2}{p_t \sigma_t^2} + \varepsilon_t
\end{aligned} \quad (35)$$

where it is now assumed, as part of a method which employs a variable parameter specification to choose parameter estimates and indirectly generate probabilities for realisations of  $IES \times RRP$  most compatible with this assumption, that  $\varepsilon_t \sim IID N(0, \sigma_\varepsilon^2)$ .

Other adjustments to Equation (30), contained in Equation (35), include subsuming the constant parameter function  $\alpha^{\omega/(1-\alpha)}$  into the production function ‘intercept’ term  $A$ . The adjusted intercept is specified as the product of:

$$A_c = \alpha_a + \alpha_0 e^{\sum_{j=1}^{23} c_j d_j} \quad (36)$$

and

$$T_t = \tau_a + e^{\tau_b(t-1) + \tau_c(t-1)^2} / (1 + \tau_0 e^{\tau_b(t-1) + \tau_c(t-1)^2}) \quad (37)$$

where the  $c_j$  and  $d_j$  are country parameters and indicator variables ( $j = 1, \dots, 23$ ), respectively. After a grid search,  $\alpha_a$  and  $\alpha_0$  are pre-set at  $\alpha_a = 0.1$  and  $\alpha_0 = 0.01$ , and  $\tau_a$  and  $\tau_0$  are pre-set at  $\tau_a = 0.01$  and  $\tau_0 = 19$ . The remaining parameters,  $c_j$  in the case of the country scale factor  $A_c$ , and  $\tau_b$  and  $\tau_c$  in the case of the time scale factor  $T_t$ , are freely estimated in the non-linear maximum likelihood estimation routine.

Further,  $\theta_t$ ,  $\mu_t$  and  $\sigma_t$  are specified as time varying, and are denoted by  $\theta_{1,t}$ ,  $\mu_t$  and  $\sigma_t$ , respectively, as:

$$\theta_{1,t} = \theta_0 + \theta_c \left[ \frac{n_{t-1}^c - n_{t-1}^{US}}{n_{t-1}^{US}} \right], \quad (38)$$

$$\mu_t = \mu_a + e^{\mu_b(t-1) + \mu_c(t-1)^2} / (1 + \mu_0 e^{\mu_b(t-1) + \mu_c(t-1)^2}) \quad (39)$$

and

$$\sigma_t = \sigma_a + e^{\sigma_b(t-1) + \sigma_c(t-1)^2} / (1 + \sigma_0 e^{\sigma_b(t-1) + \sigma_c(t-1)^2}). \quad (40)$$

Following a grid search, parameter settings  $\mu_a = 0.01$ ,  $\mu_0 = 4$ ,  $\sigma_a = 0.05$  and  $\sigma_0 = 29$ , are imposed. Remaining parameters ( $\theta_0$ ,  $\theta_c$ ,  $\mu_b$ ,  $\mu_c$ ,  $\sigma_b$  and  $\sigma_c$ ) are freely estimated.

Because of the form of the non-linearity in Equation (35), free estimation of the time preference rate  $\delta$  is problematic. Accordingly, this parameter is set at  $\delta = 0.02$  after grid search. Additionally, experimentation with different forms of the network externality variable (internet host numbers versus an index of cumulative growth) and with different measures of the externality (world versus country network) is undertaken to improve estimation prospects given the non-linear specification. This experimentation led to an index approach, and to different preferred network externality measures for the consumption and production externalities. The resultant preferred specification is:

$$\frac{\Delta n_t}{n_{t-1}} = IES_t \frac{r_t - 0.02}{p_t} + \frac{1}{2} IES_t [1 + IES_t] RRP_t + \varepsilon_t \quad (41)$$

where  $IES_t$ ,  $RRP_t$ , and  $r_t$  are respectively:

$$IES_t = \left\{ \theta_0 + \theta_c \left[ \frac{n_{t-1}^c - n_{t-1}^{US}}{n_{t-1}^{US}} \right] \right\} \left[ \frac{1}{1 + 0.5n_{t-1}^W} \right] + \theta_2 \left[ \frac{0.5n_{t-1}^W}{1 + 0.5n_{t-1}^W} \right], \quad (42)$$

$$RRP_t = \frac{\left( 0.01 + \frac{e^{\mu_b(t-1) + \mu_c(t-1)^2}}{1 + 4e^{\mu_b(t-1) + \mu_c(t-1)^2}} - r_t \right)^2}{p_t \left( 0.05 + \frac{e^{\sigma_b(t-1) + \sigma_c(t-1)^2}}{1 + 29e^{\sigma_b(t-1) + \sigma_c(t-1)^2}} \right)^2} \quad (43)$$

and

$$r_t = \left[ 0.1 + 0.01e^{\sum_{j=1}^{23} c_j d_j} \right] \left[ 0.01 + \frac{e^{\tau_b(t-1) + \tau_c(t-1)^2}}{1 + 19e^{\tau_b(t-1) + \tau_c(t-1)^2}} \right] w_t^{-\alpha/(1-\alpha)} \times [1 + n_{t-1}^W d_{US}]^{\beta_w/(1-\alpha)} [1 + n_{t-1}^c (1 - d_{US})]^{\beta_c/(1-\alpha)}. \quad (44)$$

Network measures are constructed from internet host numbers by applying the rule:

$$n_{t-1}^c = \frac{HOST_{t-1}^c - HOST_0^c}{HOST_0^c},$$

where  $t = 1, \dots, 5$ ,  $c = 1, \dots, 23$ , with 0 denoting year 1995.

#### 4.2 Variable coefficient commentary

Before proceeding, an interpretation for the variable parameter specifications is provided. By construction,  $n_{t-1}^c = 0$  for  $t = 1$ . At  $t = 1$  the interest rate applicable to holding network stock is:

$$r_1 = \left[ 0.1 + 0.01e^{\sum_{j=1}^{23} c_j d_j} \right] 0.06w_1^{-\alpha/(1-\alpha)}$$

and variations in the interest rate cross country in the initial period reflects real wage conditions, differences in initial technology and network externality effects, all of which are captured by the  $c_j$ .

In this specification, the technology parameter  $T_t$  takes the value  $T_1 = 0.06$  for all countries at  $t = 1$ , 1996, and acts as a normalising constant. The specification:

$$T_t = 0.01 + \frac{e^{\tau_b(t-1) + \tau_c(t-1)^2}}{1 + 19e^{\tau_b(t-1) + \tau_c(t-1)^2}}$$

allows for non-monotonic behaviour of network stock efficiency in production, with common country behaviour determined by the freely estimated parameters  $\tau_b$  and  $\tau_c$ . When  $\tau_b(t-1) + \tau_c(t-1)^2$  takes a large negative value,  $T_t$  will tend to 0.01, the imposed lower bound on  $T_t$ . In this specification,  $T_t$  can rise above its value at  $T_1$ , but just. The upper bound of 0.0626 is imposed by the scaling constant value of  $\tau_0 = 19$ . Based on similar reasoning, the remaining constrained non-linear variable parameter functions are described below.

The country-specific effect:

$$A_c = 0.1 + 0.01e^{\sum_{j=1}^{23} c_j d_j}$$

has a lower bound of 0.1, but no upper bound. An estimated coefficient of  $-91.381$  for Greece implies the lower bound is binding. Other countries are not affected. An estimated value of  $c_j = 3$  suggests a corresponding parameter value of  $A_c = 0.3$ .

The expected return on the risky AS investment is modelled as:

$$\mu_t = 0.01 + \frac{e^{\mu_b(t-1) + \mu_c(t-1)^2}}{1 + 4e^{\mu_b(t-1) + \mu_c(t-1)^2}}.$$

This specification forces  $\mu_1 = 0.21$  but allows  $\mu_t$  to vary from 0.01 to 0.26, with values dependent on the freely estimated parameters  $\mu_b$  and  $\mu_c$ . In estimation neither bound is binding.

The volatility of risky AS investment is modelled as:

$$\sigma_t = 0.05 + \frac{e^{\sigma_b(t-1) + \sigma_c(t-1)^2}}{1 + 29e^{\sigma_b(t-1) + \sigma_c(t-1)^2}}.$$

This specification has a lower bound of 0.05 for  $\sigma_t$ . It also forces an initial value of  $\sigma_1 = 0.083$  and has an upper bound of 0.0845.  $\sigma_t$  is constrained to begin near its upper bound. In estimation,  $\sigma_t$  fell to the lower bound by the latter part of the sample.

These variable parameter specifications capture the fall in the expected return on the AS risky investment mid-sample, making some allowance for the Asian financial crisis and world financial conditions more generally. Additionally, from an econometric point of view, the accompanying but lesser fall in volatility leads to a reduced, though substantial, fall in the *RRP*, countervailing a substantial rise in the *IES* and providing some support for the approach that treats *IES*  $\times$  *RRP* as a martingale.

### 4.3 Maximum likelihood estimates

Non-linear maximum likelihood estimation of Equation (41) is performed using SHAZAM Version 8 (White, 1997). Parameter estimates and asymptotic *t*-statistics are presented in Table 2. The key results concern parameters associated with consumption and production network externalities. Consumption externalities are measured through the parameters  $\theta_0$ ,  $\theta_c$  and  $\theta_2$ . The non-US  $\theta_c$  is estimated as economically small in impact at  $-0.649$ .<sup>11</sup> For purpose of discussion, treat  $\theta_0 = \theta_1$ . The estimates of  $\theta_1$  and  $\theta_2$  suggest the *IES* is bounded from below by 3.321 at the beginning of the sample period and from above by 16.657 as world network grows indefinitely large. The difference between the lower and upper bounds indicates the importance of the network externality in consumption.

**Table 2** Estimation results

<i>Parameter</i>	<i>Estimate</i>	<i>t-Ratio</i>
$\theta_0$	3.321	3.608
$\theta_C$	-0.649	-0.699
$\theta_2$	16.657	2.675
$\alpha$	0.584	10.807
$\beta_w$	0.334	2.797
$\beta_C$	0.461	3.185
Australia	4.111	19.468
Austria	3.278	37.295
Belgium	1.987	5.427
Canada	3.890	26.161
Denmark	3.153	24.884
Finland	4.423	16.316
France	2.918	18.213
Germany	4.023	27.406
Greece	-91.381	-2.756
Iceland	4.217	25.194
Ireland	2.725	14.496
Italy	2.837	17.357
Japan	2.932	16.488
Luxembourg	3.278	26.042
The Netherlands	3.150	28.019
NZ	3.159	17.232
Norway	3.373	21.993
Portugal	2.683	15.568
Spain	2.463	11.018
Sweden	4.253	28.521
Switzerland	4.310	30.717
UK	3.965	30.520
USA	3.809	8.639
$\tau_B$	-6.169	-7.179
$\tau_C$	1.258	5.771
$\mu_B$	-2.215	-9.306
$\mu_C$	0.464	6.449
$\sigma_B$	2.324	0.084
$\sigma_C$	-2.294	-0.171
R <sup>2</sup> statistic	0.716	
L	18.489	

Notes: R<sup>2</sup> is the squared correlation coefficient between observed and predicted values. L is the log of the likelihood. t-Ratio is asymptotic.

Turning to the evidence concerning production externalities, the crucial parameters are  $\beta_w$  for externalities related to the size of the world network estimated at 0.334 and,  $\beta_c$  for externalities related to the size of specific country networks and estimated at 0.461.<sup>12</sup> The results imply effective increasing returns to scale due to the externality of 1.334 for the US (with the world network size providing the externality) and 1.461 for other countries (with the size of the country-specific stock providing the externality).

An ancillary production function parameter is  $\alpha$ . Estimated at 0.584, this indicates the variable factor input share of output income is 58%. Remaining parameter estimates control for country-specific effects in technology, the extent of externalities prior to 1996, for variation in the normalised risk premium and the returns to AS investment over time. Generally, these results indicate the importance of allowing for these variations in the pooled data set.

Table 3 reports variable parameter estimates and other functions that vary cross-country or time. Column (3) and Column (4) labelled  $A_C$  and  $T$ , respectively, provide estimates of the country-specific component and time-specific component that together define the scale factor for the interest rate, *viz.*,  $A_C T_t$  in the expression for  $r_t$ :

$$r_t = A_C T_t w_t^{-\alpha/(1-\alpha)} [1 + n_{t-1}^W d_{USA}]^{\beta_w/(1-\alpha)} [1 + n_{t-1}^c (1 - d_{USA})]^{\beta_c/(1-\alpha)}. \quad (45)$$

The interest rate, constructed according to Equation (45), is given in Column (7) of Table 3. Column (5) and Column (6) report the remaining variable parameter components of the normalised risk premium, *viz.*,  $\sigma$  and  $\mu$ . Comparison of Column (6) and Column (7) shows the risk premium is positive for most countries and time periods, with negative values reported for seven countries, and all in the final time period. Preliminary grid searches for economically sensible values of parameters controlling upper and lower limits on the allowable variation in estimates of  $T_t$ ,  $\mu_t$  and  $\sigma_t$ , and a lower limit for  $A_C$  are based on minimising the number of violations of non-positive risk premiums. Given these pre-set values, maximum likelihood estimation proceeded on the basis of generation of a minimal number of these economically problematic results. Column (8) reports the calculated *IES* values. In particular, the *IES* is rising through time to near its upper bound, implying that benefits from increased network size will be lower on further network expansion.

The presence of a country-specific effect leads to some minor variation across countries in the size of the *IES*. An interesting result is the rise in the *IES* through time. This rise is significant, as indicated by a likelihood ratio test on the difference in the underlying parameters controlling the variability in the *IES*, and is directly attributable to world network externalities in consumption. As a further aid to economic interpretation, Column (9) translates the estimated *IES* value to the implied value of  $\gamma$  in the utility function. Over the sample period, the power rises from approximately 0.75 in 1996 to 0.93 in 2000. Based on an estimated value of 16.657 for  $\theta_2$ , which is the estimated asymptotic limit for the *IES*, the  $\gamma$  in the utility function will asymptote to 0.94 as network size increases indefinitely. This suggests that the long-run optimal degree of consumption externality has almost been exhausted. That is, further network growth will not increase the consumption externality appreciably.

**Table 3** Variable parameter estimates

<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>
<i>Country</i>	<i>Year</i>	$A_c$	$T$	$\sigma$	$\mu$	$r$	$IES$	$\gamma$
Australia	1996	0.710	0.060	0.083	0.210	0.119	3.938	0.746
Australia	1997	0.710	0.016	0.083	0.112	0.057	7.280	0.863
Australia	1998	0.710	0.011	0.058	0.068	0.048	10.571	0.905
Australia	1999	0.710	0.011	0.050	0.073	0.060	12.112	0.917
Australia	2000	0.710	0.019	0.050	0.132	0.152	13.700	0.927
Austria	1996	0.365	0.060	0.083	0.210	0.054	3.965	0.748
Austria	1997	0.365	0.016	0.083	0.112	0.026	7.301	0.863
Austria	1998	0.365	0.011	0.058	0.068	0.021	10.579	0.905
Austria	1999	0.365	0.011	0.050	0.073	0.036	12.117	0.917
Austria	2000	0.365	0.019	0.050	0.132	0.101	13.703	0.927
Belgium	1996	0.173	0.060	0.083	0.210	0.026	3.967	0.748
Belgium	1997	0.173	0.016	0.083	0.112	0.017	7.302	0.863
Belgium	1998	0.173	0.011	0.058	0.068	0.019	10.579	0.905
Belgium	1999	0.173	0.011	0.050	0.073	0.040	12.117	0.917
Belgium	2000	0.173	0.019	0.050	0.132	0.121	13.702	0.927
Canada	1996	0.589	0.060	0.083	0.210	0.090	3.931	0.746
Canada	1997	0.589	0.016	0.083	0.112	0.042	7.276	0.863
Canada	1998	0.589	0.011	0.058	0.068	0.038	10.568	0.905
Canada	1999	0.589	0.011	0.050	0.073	0.055	12.110	0.917
Canada	2000	0.589	0.019	0.050	0.132	0.152	13.699	0.927
Denmark	1996	0.334	0.060	0.083	0.210	0.049	3.965	0.748
Denmark	1997	0.334	0.016	0.083	0.112	0.030	7.300	0.863
Denmark	1998	0.334	0.011	0.058	0.068	0.032	10.578	0.905
Denmark	1999	0.334	0.011	0.050	0.073	0.061	12.116	0.917
Denmark	2000	0.334	0.019	0.050	0.132	0.127	13.702	0.927
Finland	1996	0.933	0.060	0.083	0.210	0.148	3.948	0.747
Finland	1997	0.933	0.016	0.083	0.112	0.064	7.290	0.863
Finland	1998	0.933	0.011	0.058	0.068	0.068	10.574	0.905
Finland	1999	0.933	0.011	0.050	0.073	0.064	12.115	0.917
Finland	2000	0.933	0.019	0.050	0.132	0.120	13.702	0.927
France	1996	0.285	0.060	0.083	0.210	0.043	3.954	0.747
France	1997	0.285	0.016	0.083	0.112	0.019	7.294	0.863
France	1998	0.285	0.011	0.058	0.068	0.020	10.576	0.905
France	1999	0.285	0.011	0.050	0.073	0.030	12.114	0.917
France	2000	0.285	0.019	0.050	0.132	0.136	13.700	0.927
Germany	1996	0.659	0.060	0.083	0.210	0.090	3.920	0.745
Germany	1997	0.659	0.016	0.083	0.112	0.039	7.272	0.862
Germany	1998	0.659	0.011	0.058	0.068	0.045	10.564	0.905



**Table 3** Variable parameter estimates (continued)

<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>
<i>Country</i>	<i>Year</i>	<i>A<sub>C</sub></i>	<i>T</i>	$\sigma$	$\mu$	<i>r</i>	<i>IES</i>	$\gamma$
Germany	1999	0.659	0.011	0.050	0.073	0.059	12.107	0.917
Germany	2000	0.659	0.019	0.050	0.132	0.118	13.699	0.927
Greece	1996	0.100	0.060	0.083	0.210	0.030	3.970	0.748
Greece	1997	0.100	0.016	0.083	0.112	0.018	7.304	0.863
Greece	1998	0.100	0.011	0.058	0.068	0.020	10.581	0.905
Greece	1999	0.100	0.011	0.050	0.073	0.038	12.118	0.917
Greece	2000	0.100	0.019	0.050	0.132	0.105	13.703	0.927
Iceland	1996	0.778	0.060	0.083	0.210	0.120	3.970	0.748
Iceland	1997	0.778	0.016	0.083	0.112	0.049	7.304	0.863
Iceland	1998	0.778	0.011	0.058	0.068	0.050	10.581	0.905
Iceland	1999	0.778	0.011	0.050	0.073	0.069	12.118	0.917
Iceland	2000	0.778	0.019	0.050	0.132	0.145	13.703	0.927
Ireland	1996	0.253	0.060	0.083	0.210	0.047	3.969	0.748
Ireland	1997	0.253	0.016	0.083	0.112	0.030	7.304	0.863
Ireland	1998	0.253	0.011	0.058	0.068	0.032	10.580	0.905
Ireland	1999	0.253	0.011	0.050	0.073	0.046	12.118	0.917
Ireland	2000	0.253	0.019	0.050	0.132	0.099	13.703	0.927
Italy	1996	0.271	0.060	0.083	0.210	0.054	3.963	0.748
Italy	1997	0.271	0.016	0.083	0.112	0.031	7.298	0.863
Italy	1998	0.271	0.011	0.058	0.068	0.039	10.577	0.905
Italy	1999	0.271	0.011	0.050	0.073	0.062	12.115	0.917
Italy	2000	0.271	0.019	0.050	0.132	0.084	13.703	0.927
Japan	1996	0.288	0.060	0.083	0.210	0.040	3.942	0.746
Japan	1997	0.288	0.016	0.083	0.112	0.033	7.270	0.862
Japan	1998	0.288	0.011	0.058	0.068	0.035	10.563	0.905
Japan	1999	0.288	0.011	0.050	0.073	0.053	12.106	0.917
Japan	2000	0.288	0.019	0.050	0.132	0.152	13.696	0.927
Luxembourg	1996	0.365	0.060	0.083	0.210	0.053	3.970	0.748
Luxembourg	1997	0.365	0.016	0.083	0.112	0.031	7.305	0.863
Luxembourg	1998	0.365	0.011	0.058	0.068	0.028	10.581	0.905
Luxembourg	1999	0.365	0.011	0.050	0.073	0.050	12.118	0.917
Luxembourg	2000	0.365	0.019	0.050	0.132	0.114	13.703	0.927
The Netherlands	1996	0.333	0.060	0.083	0.210	0.048	3.952	0.747
The Netherlands	1997	0.333	0.016	0.083	0.112	0.022	7.292	0.863
The Netherlands	1998	0.333	0.011	0.058	0.068	0.022	10.575	0.905
The Netherlands	1999	0.333	0.011	0.050	0.073	0.038	12.114	0.917
The Netherlands	2000	0.333	0.019	0.050	0.132	0.107	13.701	0.927

**Table 3** Variable parameter estimates (continued)

<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>
<i>Country</i>	<i>Year</i>	<i>A<sub>C</sub></i>	<i>T</i>	$\sigma$	$\mu$	<i>r</i>	<i>IES</i>	$\gamma$
NZ	1996	0.336	0.060	0.083	0.210	0.064	3.965	0.748
NZ	1997	0.336	0.016	0.083	0.112	0.029	7.301	0.863
NZ	1998	0.336	0.011	0.058	0.068	0.041	10.578	0.905
NZ	1999	0.336	0.011	0.050	0.073	0.032	12.117	0.917
NZ	2000	0.336	0.019	0.050	0.132	0.119	13.703	0.927
Norway	1996	0.392	0.060	0.083	0.210	0.070	3.962	0.748
Norway	1997	0.392	0.016	0.083	0.112	0.035	7.298	0.863
Norway	1998	0.392	0.011	0.058	0.068	0.044	10.577	0.905
Norway	1999	0.392	0.011	0.050	0.073	0.049	12.116	0.917
Norway	2000	0.392	0.019	0.050	0.132	0.143	13.702	0.927
Portugal	1996	0.246	0.060	0.083	0.210	0.048	3.969	0.748
Portugal	1997	0.246	0.016	0.083	0.112	0.028	7.304	0.863
Portugal	1998	0.246	0.011	0.058	0.068	0.034	10.580	0.905
Portugal	1999	0.246	0.011	0.050	0.073	0.049	12.118	0.917
Portugal	2000	0.246	0.019	0.050	0.132	0.121	13.703	0.927
Spain	1996	0.217	0.060	0.083	0.210	0.032	3.965	0.748
Spain	1997	0.217	0.016	0.083	0.112	0.023	7.299	0.863
Spain	1998	0.217	0.011	0.058	0.068	0.027	10.578	0.905
Spain	1999	0.217	0.011	0.050	0.073	0.044	12.116	0.917
Spain	2000	0.217	0.019	0.050	0.132	0.123	13.702	0.927
Sweden	1996	0.803	0.060	0.083	0.210	0.101	3.955	0.747
Sweden	1997	0.803	0.016	0.083	0.112	0.051	7.294	0.863
Sweden	1998	0.803	0.011	0.058	0.068	0.051	10.576	0.905
Sweden	1999	0.803	0.011	0.050	0.073	0.056	12.115	0.917
Sweden	2000	0.803	0.019	0.050	0.132	0.141	13.702	0.927
Switzerland	1996	0.845	0.060	0.083	0.210	0.103	3.962	0.748
Switzerland	1997	0.845	0.016	0.083	0.112	0.049	7.299	0.863
Switzerland	1998	0.845	0.011	0.058	0.068	0.047	10.578	0.905
Switzerland	1999	0.845	0.011	0.050	0.073	0.064	12.117	0.917
Switzerland	2000	0.845	0.019	0.050	0.132	0.127	13.703	0.927
UK	1996	0.627	0.060	0.083	0.210	0.090	3.924	0.745
UK	1997	0.627	0.016	0.083	0.112	0.043	7.271	0.862
UK	1998	0.627	0.011	0.058	0.068	0.039	10.566	0.905
UK	1999	0.627	0.011	0.050	0.073	0.058	12.107	0.917
UK	2000	0.627	0.019	0.050	0.132	0.123	13.698	0.927
USA	1996	0.551	0.060	0.083	0.210	0.072	3.321	0.699
USA	1997	0.551	0.016	0.083	0.112	0.031	6.826	0.854
USA	1998	0.551	0.011	0.058	0.068	0.033	10.270	0.903
USA	1999	0.551	0.011	0.050	0.073	0.044	11.886	0.916
USA	2000	0.551	0.019	0.050	0.132	0.117	13.552	0.926

## 5 Conclusion

A driving force behind the emergence of the new or information economy is the growth of internet network capacity. However, a fundamental problem in mapping this dynamic is the lack of an acceptable theoretical framework through which to direct empirical investigation of internet network host evolution. Most of the models in the literature on network externalities have been developed in a static framework, with externalities viewed as instantaneous or self-fulfilling. They also only consider consumption externalities. The model specified here builds on received theory from several sources to include these features, and develops a model that is both capable of econometric estimation and which provides as an output a direct measure of the network effect. Accordingly, a goal of this paper is to find the magnitude of the externality effect on internet network growth. In addition, the paper illustrates how panel data can generate estimates of structural parameters capable of explaining internet host growth.

Estimates of an endogenous growth model in which sustained internet system growth are the result of consumption and production externalities are presented. Estimation on a sample of OECD Member States shows model results are compatible with internet host growth data. To summarise, both production and consumption externalities are strongly in evidence in model estimates. Production externalities although modelled reasonably simply, indicate substantial increasing returns to scale. On the consumption side, the possibility of the externality varying with network size is also examined. Over the estimation period, the consumption externality has strengthened and appears close to its maximum. This finding suggests that future internet growth will most likely be due to production-related externalities.

Several issues are raised as a result of this investigation. In particular, with consumption-side evidence suggesting its future effect will be relatively minor, closer attention needs to be paid to the production-side specification. This specification treats the production externality as a scale effect for a modified linearly homogeneous production function. A task remains to consider both effects in a more general setting, so as to allow examination of ultimate optimal network size. Finally, the model suggests that the traditional notion of critical mass needs to be modified, in the context of the internet, so as to allow for both local and global critical mass.

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**Notes**

- 1 Rohlfs (1974), Littlechild (1975) and Oren and Smith (1981) analyse network externalities in the context of a monopoly telecommunications network.
- 2 The field around the unstable critical mass point is 'critical' in the sense that smaller fluctuations can have a large effect upon the continued development of diffusion (Schoder, 2000). Industries with network externalities typically exhibit a positive critical mass, that is, small networks are not observed at any price (Economides and Himmelberg, 1995). The critical mass point can also be interpreted as the turning point between positive and negative returns to diffusion (Markus, 1990).
- 3 The 'micro' approach is more concerned with the actual configuration of the network so as to better understand the origin of any externalities (Economides, 1996).
- 4 In a more general formulation, if the equity investment is in new economy stocks, then the drift and volatility might be modelled as functions of the network size, leading potentially to another source of network externalities.
- 5 Since the externality is irrelevant to the private optimiser, the problem is formally equivalent to a stochastic inter-temporal optimisation of the type described by Cooper *et al.* (1995).
- 6 The 23 countries are comprised of: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, UK and the USA. Mexico and Turkey are not included as they are outliers. Czech Republic, Hungary, Poland and South Korea are excluded because of insufficient internet access price data.
- 7 Complete GDP data are not available for Ireland (2000) and New Zealand (1999, 2000) in the ITU database and are obtained directly from the Central Statistics Office (Ireland) and Statistics New Zealand.
- 8 A geometric procedure based on the rule  $PRICE_{1997} = PRICE_{1996} \times \left( \frac{PRICE_{1998}}{PRICE_{1996}} \right)^{\frac{1}{2}}$  is used to interpolate the PRICE series.
- 9 Compensation of Employees is obtained directly from the OECD.
- 10 The countries with declines in new hosts are: Belgium, Denmark, Finland, France, Italy, New Zealand, Portugal, Spain, Switzerland, Turkey and the UK.
- 11 Although the parameter shows up as insignificant according to the asymptotic  $t$ -statistic, a Likelihood Ratio (LR) test rejects the restriction that  $\theta_c = 0$  (LR=24.960, critical  $\chi_1^2(.01) = 6.63$ ). Therefore, the results with  $\theta_c$  freely estimated are reported. From an economic perspective, however, the country-specific effect is undoubtedly minor.
- 12 The world stock network externality is related to US hosts but not other country hosts, while the reverse is true for the country network size externality, which is relevant for countries other than the USA. At this point the significance of these effects is simply noted. A LR test of the joint null hypothesis  $\beta_w = 0, \beta_c = 0$  rejects the null (LR = 97.688, critical  $\chi_2^2(.01) = 9.21$ ).

## Appendix

Table 1 Source and constructed data

<i>Country</i>	<i>Year</i>	<i>Price</i>	<i>Wage</i>	<i>HOSTS</i>	$\Delta$ <i>HOSTS</i>	<i>HOSTS<sub>-1</sub></i>
Australia	1996	22.09	0.4801	514,760	205,198	309,562
Australia	1997	33.53	0.4859	665,403	150,643	514,760
Australia	1998	43.76	0.4916	792,351	126,948	665,403
Australia	1999	43.39	0.4810	1,090,468	298,117	792,351
Australia	2000	43.42	0.4787	1,615,939	525,471	1,090,468
Austria	1996	55.40	0.5281	88,811	35,467	53,344
Austria	1997	75.43	0.5215	108,473	19,662	88,811
Austria	1998	83.10	0.5230	172,569	64,096	108,473
Austria	1999	73.41	0.5229	262,632	90,063	172,569
Austria	2000	46.10	0.5178	483,208	220,576	262,632
Belgium	1996	46.81	0.5169	65,064	34,443	30,621
Belgium	1997	70.85	0.5119	106,808	41,744	65,064
Belgium	1998	87.60	0.5099	208,665	101,857	106,808
Belgium	1999	80.53	0.5110	339,357	130,692	208,665
Belgium	2000	56.90	0.5095	300,193	-39,164	339,357
Canada	1996	26.47	0.5142	603,325	230,434	372,891
Canada	1997	31.18	0.5185	839,141	235,816	603,325
Canada	1998	38.14	0.5262	1,119,172	280,031	839,141
Canada	1999	35.89	0.5115	1,669,664	550,492	1,119,172
Canada	2000	40.73	0.5062	2,364,014	694,350	1,669,664
Denmark	1996	35.20	0.5326	106,732	56,175	50,557
Denmark	1997	40.95	0.5334	169,368	62,636	106,732
Denmark	1998	38.94	0.5379	298,275	128,907	169,368
Denmark	1999	42.92	0.5399	338,239	39,964	298,275
Denmark	2000	25.87	0.5258	333,978	-4,261	338,239
Finland	1996	23.16	0.4996	314,141	98,437	215,704
Finland	1997	26.19	0.4864	486,811	172,670	314,141
Finland	1998	23.76	0.4841	459,568	-27,243	486,811
Finland	1999	29.38	0.4855	461,760	2,192	459,568
Finland	2000	30.43	0.4631	529,261	67,501	461,760
France	1996	31.72	0.5213	236,874	85,701	151,173
France	1997	47.72	0.5193	355,031	118,157	236,874
France	1998	59.30	0.5184	511,193	156,162	355,031
France	1999	54.75	0.5209	1,233,071	721,878	511,193
France	2000	34.44	0.5243	1,122,407	-110,664	1,233,071
Germany	1996	41.54	0.5559	691,864	217,489	474,375
Germany	1997	52.43	0.5417	1,132,174	440,310	691,864

**Table 1** Source and constructed data (continued)

<i>Country</i>	<i>Year</i>	<i>Price</i>	<i>Wage</i>	<i>HOSTS</i>	$\Delta$ <i>HOSTS</i>	<i>HOSTS</i> <sub>-1</sub>
Germany	1998	53.98	0.5321	1,449,915	317,741	1,132,174
Germany	1999	40.07	0.5328	1,635,067	185,152	1,449,915
Germany	2000	33.48	0.5343	2,040,437	405,370	1,635,067
Greece	1996	70.00	0.3207	16,738	8,997	7,741
Greece	1997	72.67	0.3299	28,131	11,393	16,738
Greece	1998	66.42	0.3420	49,904	21,773	28,131
Greece	1999	81.77	0.3411	75,088	25,184	49,904
Greece	2000	54.61	0.3427	110,608	35,520	75,088
Iceland	1996	24.52	0.5113	11,542	3,232	8,310
Iceland	1997	31.37	0.5015	18,520	6,978	11,542
Iceland	1998	35.72	0.5207	24,794	6,274	18,520
Iceland	1999	35.93	0.5288	29,872	5,078	24,794
Iceland	2000	28.62	0.5367	39,901	10,029	29,872
Ireland	1996	73.34	0.4474	26,895	13,460	13,435
Ireland	1997	82.27	0.4275	39,864	12,969	26,895
Ireland	1998	75.85	0.4039	55,859	15,995	39,864
Ireland	1999	64.92	0.4098	63,913	8,054	55,859
Ireland	2000	61.00	0.3938	110,545	46,632	63,913
Italy	1996	48.85	0.4252	147,873	72,497	75,376
Italy	1997	49.36	0.4273	254,296	106,423	147,873
Italy	1998	40.81	0.4091	386,632	132,336	254,296
Italy	1999	43.53	0.4119	301,528	-85,104	386,632
Italy	2000	38.06	0.4043	1,019,711	718,183	301,528
Japan	1996	22.90	0.5536	734,406	465,079	269,327
Japan	1997	31.68	0.5583	1,168,956	434,550	734,406
Japan	1998	35.17	0.5653	1,687,534	518,578	1,168,956
Japan	1999	24.25	0.5598	2,636,541	949,007	1,687,534
Japan	2000	22.87	0.5640	4,640,863	2,004,322	2,636,541
Luxembourg	1996	38.64	0.5317	3,518	1,638	1,880
Luxembourg	1997	53.91	0.5117	4,743	1,225	3,518
Luxembourg	1998	61.29	0.5067	7,737	2,994	4,743
Luxembourg	1999	84.24	0.5005	9,614	1,877	7,737
Luxembourg	2000	59.93	0.4920	11,814	2,200	9,614
The Netherlands	1996	45.16	0.5350	270,511	98,746	171,765
The Netherlands	1997	55.06	0.5315	391,228	120,717	270,511
The Netherlands	1998	54.45	0.5306	625,769	234,541	391,228
The Netherlands	1999	48.07	0.5169	959,083	333,314	625,769
The Netherlands	2000	52.90	0.5167	1,623,567	664,484	959,083
NZ	1996	55.41	0.4368	84,532	30,922	53,610
NZ	1997	58.55	0.4398	169,264	84,732	84,532
NZ	1998	56.61	0.4390	137,247	-32,017	169,264

**Table 1** Source and constructed data (continued)

<i>Country</i>	<i>Year</i>	<i>Price</i>	<i>Wage</i>	<i>HOSTS</i>	$\Delta$ <i>HOSTS</i>	<i>HOSTS</i> <sub>-1</sub>
NZ	1999	50.32	0.4441	271,003	133,756	137,247
NZ	2000	53.62	0.4430	345,107	74,104	271,003
Norway	1996	27.16	0.4596	150,130	65,836	84,294
Norway	1997	34.04	0.4678	292,382	142,252	150,130
Norway	1998	39.19	0.5017	318,993	26,611	292,382
Norway	1999	37.93	0.4964	438,961	119,968	318,993
Norway	2000	39.07	0.4444	452,677	13,716	438,961
Portugal	1996	135.69	0.4290	23,482	11,706	11,776
Portugal	1997	116.40	0.4313	42,447	18,965	23,482
Portugal	1998	79.35	0.4397	55,746	13,299	42,447
Portugal	1999	108.09	0.4279	77,761	22,015	55,746
Portugal	2000	74.05	0.4347	62,147	-15,614	77,761
Spain	1996	53.54	0.5225	113,227	61,771	51,456
Spain	1997	54.46	0.4980	196,403	83,176	113,227
Spain	1998	45.13	0.5003	306,559	110,156	196,403
Spain	1999	58.56	0.5026	469,587	163,028	306,559
Spain	2000	58.62	0.5061	455,487	-14,100	469,587
Sweden	1996	22.65	0.5895	237,832	92,988	144,844
Sweden	1997	31.59	0.5631	348,609	110,777	237,832
Sweden	1998	39.64	0.5648	379,455	30,846	348,609
Sweden	1999	33.76	0.5613	522,888	143,433	379,455
Sweden	2000	33.56	0.5612	595,698	72,810	522,888
Switzerland	1996	29.22	0.6034	132,925	52,791	80,134
Switzerland	1997	38.35	0.6063	189,175	56,250	132,925
Switzerland	1998	44.13	0.6074	245,409	56,234	189,175
Switzerland	1999	41.34	0.6044	269,812	24,403	245,409
Switzerland	2000	29.58	0.5942	262,510	-7,302	269,812
UK	1996	57.71	0.5351	719,333	279,565	439,768
UK	1997	59.96	0.5369	987,733	268,400	719,333
UK	1998	60.07	0.5437	1,449,315	461,582	987,733
UK	1999	52.87	0.5524	1,739,078	289,763	1,449,315
UK	2000	36.79	0.5577	1,677,946	-61,132	1,739,078
USA	1996	28.06	0.5756	10,112,888	4,057,929	6,054,959
USA	1997	32.17	0.5603	20,623,996	10,511,108	10,112,888
USA	1998	37.18	0.5689	30,489,464	9,865,468	20,623,996
USA	1999	32.18	0.5699	53,175,956	22,686,492	30,489,464
USA	2000	18.96	0.5659	80,566,944	27,390,988	53,175,956
World	1996			16,249,917		9,485,918
World	1997			30,127,576		16,249,917
World	1998			43,547,090		30,127,576
World	1999			72,010,326		43,547,090
World	2000			106,724,179		72,010,326