Privatization, Government's Preference and Unionization Structure: A Mixed Oligopoly Approach

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Abstract

By introducing the government's preference for tax revenues into the theoretical framework of unionized mixed oligopolies, this study investigates the efficiency of privatization. The results show that (i) regardless of the government's preference for tax revenues, its incentive to privatize a public firm depends on the number of the private firms and (ii) social welfare can decrease with an increase in the number of firms depending on the level of government's preference for tax revenue. Moreover, if the number of private firms and the government's preference for tax revenue are sufficiently small, then social welfare under a unionized privatized oligopoly is greater than under a unionized mixed oligopoly while the government has an incentive not to privatize the public firm, and vice versa if only the number of firms is sufficiently large.

JEL: C7, D43, H44, J51, L13.

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1 Introduction

Recently, the economic implications of mixed oligopoly markets have been an issue with respect to the change in competition for both market structure efficiency and privatization. This means that public firms still play an important role in most economic realms. There are several studies of mixed oligopolies[1]. In such models, a public firm traditionally maximizes social welfare, while the private firms compete with the public firm maximizing their own profits.

From the perspectives on public choices, when governmental intervention, such as a production subsidy, is incorporated into the mixed oligopoly, White (1996), Poyago-Theotoky (2001), and Myles (2002) showed that all firms' profits and social welfare are identical before and after the privatization of the public firm in a mixed oligopoly, irrespective of whether the public firm moves simultaneously with the private firms or the public firm acts as a Stackelberg leader or all firms behave as profit-maximizers. On the other hand, Fjell and Heywood (2004) demonstrated that when the public leader is privatized and becomes the private leader, the optimal subsidy, output and social welfare are reduced. Moreover, by introducing taxes (ad valorem or specific) in a mixed oligopoly, Mujumdar and Pal (1998) showed that privatization can increase both social welfare and tax revenues, where an increase in tax does not change the total output but increases the output of the public firm and the tax revenue.

In all the abovementioned studies that consider both subsidies and taxation in a mixed oligopoly market, the public firm as well as the government maximize social welfare, which is defined as the sum of the tax revenue or subsidy, consumers' and producers' surplus. However, in the real world there exist some conflicts of interest between the public firm and the government. Most existing studies cannot appropriately evaluate these situations[2]. To evaluate privatization

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[1]Many of the issues concerning mixed oligopolies have been analyzed: partial privatization, capacity choice, and endogenous timing and so on. For the recent literature on the mixed oligopoly models, see Barcena-Ruiz (2007), Matsumura (1998), Matsumura and Matsushima (2004) and references therein. See also De Fraja and Delbono (1990), Bös (1991) and Nett (1993) for the general reviews of the mixed oligopoly model.

[2]Exceptions are, to the best of the author's knowledge, Matusumura (1998), Saha and Sensarma (2008) and Kato (2008) who explicitly incorporate different objective function between the public firm and the government in the mixed duopoly. We will mention later several differences. See footnote 5 for more on this point.

programs, we have to deviate from the framework of traditional models that involves a monolithic entity that seeks to maximize social welfare. It has been argued in the literature that there is another way to limit the discretionary power of governments when a Leviathan government exists (see Brennan and Buchanan, 1980). For example, Oates (1985) and Zax (1989) found empirical support for Leviathan, while Forbes and Zampelli (1989) rejected the Leviathan[3]. Therefore, this literature contains a number of puzzles for which fiscal centralization and the size of the public sector (Oates, 1989). These two contrasting views clearly reflect different perceptions of policy-making. Firstly, government is a benevolent maximizer of social welfare. Secondly, it intrinsically is a tax-revenue maximizer.

The main purpose of this paper is to provide a framework within which the above two contrasting views regarding welfare can be modeled and compared. We assume that the public firm gives full weight to the social welfare, while the government attaches weight to both its social welfare and preference for tax revenues. This assumption is appropriate because in reality, the government and a public firm do not function as a coherent entity[4]. To the best of the author's knowledge, Matusumura (1998), Saha and Sensarma (2008) and Kato (2008) attempted to analyze the differing objective functions of the government and public firm in a mixed duopoly setting[5]. More specifically, Kato (2008) showed that *without* the presence of unions, the government's privatization of the public firm depends on *its preference for tax revenues*. This is because the government is assumed to give more weight to tax revenue than to social welfare, whereas the public firm is only concerned with maximizing social welfare. To study the effects that arise when the objective functions of the government and a public firm are different, we extend Kato's (2008) model, which focuses on the efficiency of privatization by allowing firms to collectively bargain through their unions, rather than the framework used in Matusumura (1998) and Saha and Sensarma (2008).

The theoretical results of the present study, however, treat the problem of a mixed oligopoly in which the government can choose to privatize the public firm by facing a union-bargaining process. Kato's (2008) findings indicated that the government has no incentive to privatize the public firm if it sufficiently prefers tax revenues. In contrast, our paper shows that regardless of its preference for tax revenues, the government's incentive to privatize the public firm depends upon the number of the private firms, when all firms, including the public firm, has an incentive to opt for decentralized bargaining. Moreover, in terms of comparing a mixed duopoly with a privatized duopoly, Kato (2008) focused only on the government's payoff while our paper investigates how properties of social welfare are affected by the government's preference for tax revenues because there may be conflicts between the public firm and government with regard to the efficiency of privatization. First, we find that social welfare can *decrease* with an increase in the number of firms depending on the government's preference for tax revenues. Second, some numerical calculations show that if both the number of private firms and the government's preference for tax revenues are sufficiently small, the social welfare under a unionized privatized

[3]In theoretical studies of the Leviathan government, Edwards and Keen (1996) and Rauscher (2000) used formalized tax-competition models to address the issue and showed that the results of tax competition are ambiguous. For more detailed treatment of the Leviathan government, recent theoretical as well as empirical studies include Keen and Kotsogiannis (2002) and Brülharty and Jamettiz (2007, 2006).

[4]According to Wilson (1989) and Tirole (1994), "government agencies generally pursue multiple goals. Moreover, many of these goals are hard to measure, and incentives based on measurable goals must be limited to not completely jeopardize the nonmeasurable dimensions of social welfare. Indeed, several missions can be pursued by different officials of the same agencies. Composite missions that reflect the several goals optimization may not fit the officials self interest."

[5]Saha and Sensarma (2008) showed that if the government is producers' profit oriented, it will accommodate the private firm's aggression and cut back the public firm's output through partial privatization. Considering partial privatization, Matusumura (1998) assumed that the government puts more a larger weight on consumer surplus than on producers' surplus.

oligopoly is greater than that under a unionized mixed oligopoly where the government has an incentive not to privatize the public firm. On the other hand, if only the number of private firms is sufficiently large, the government always has an incentive to privatize the public firm regardless of the government's preference for tax revenues, while the social welfare under a unionized mixed oligopoly is greater than that under a unionized privatized oligopoly. These main results in our paper are in contrast to the findings of De Fraja and Delbono (1989) in the mixed oligopoly that the privatization can enhance social welfare when the number of existing private firms is relatively large.

The organization of the paper is as follows. In Section 2, we describe the model. Section 3 presents the results of unionized mixed and privatized oligopoly market. Section 4 presents the comparisons of social welfare and government's payoff with the privatization. Section 5 closes the paper.

2 The Model

Consider a mixed oligopoly situation for a homogeneous good that is supplied by a public firm and private firms. Firm i $(i = 1, ..., n)$ is a profit-maximizing private firm and firm 0 is a public firm that maximizes social welfare. Assume that the inverse demand is characterized by

$$p = 1 - x_0 - \sum_{i=1}^{n} x_i,$$

where x_0 is the output level of the public firm and x_i is the output level of the private firm i.

On the demand side of the market, the representative consumer's utility is a quadratic function given by

$$U = x_0 + \sum_{i=1}^{n} x_i - \frac{1}{2}\big(x_0 + \sum_{i=1}^{n} x_i\big)^2.$$

The firms are homogeneous with respect to productivity. Each firm adopts a constant returns-to-scale technology where one unit of labor is turned into one unit of the final good. The price of labor (i.e., wage) that firm j has to pay is denoted by $w_j, j = 0, ..., n$.

To analyze the union's wage bargaining, we also assume that the public and private firms are unionized and that wages $w_j, j = 0, 1, ..., n$ are determined as a consequence of bargaining between firms and unions. Let $\overline{w}$ denote the reservation wage. Taking $\overline{w}$ as a given, the union's optimal wage-setting strategy regarding firm j, w_j, is defined as

$$\max_{w_j} u_j = (w_j - \overline{w})^{\theta} x_j; j = 0, 1, ..., n, \tag{1}$$

where θ is the bargaining power for wages. As Haucap and Wey (2004), Leahy and Montagna (2000) and Lommerud *et al.* (2003) suggested, we assume that the union possesses full bargaining power ($\theta = 1$) and $\overline{w} = 0$ to show our results in a simple way[6]. Thus, we assume that the union sets the wage, while public and private firms unilaterally decide the level of employment.

Each firm's profit is as following function

$$\pi_j = (p - w_j)x_j - tx_j, \quad j = 0, 1, ..., n$$

[6]The papers that are closest to our representation of the unions' utilities are Naylor (1998, 1999), Haucap and Wey (2004), Leahy and Montagna (2000), and Lommerud *et al.* (2003). As they suggest, the monopoly union sets the wages but the firm unilaterally decides the level of employment. This is because the wage claims are decided by the elasticity of labor demand rather than the firm's profit. See also Oswald and Turnbull (1995). De Fraja (1993) also adopted this kind of unions' utilities.

where t is the specific tax rate. On the other hand, the public firm's objective, W is to maximize welfare, which is defined as the sum of the consumer surplus, the profits of individual firms, and the utilities of unions less the tax revenue. Thus, the public firm aims to maximize social welfare, which is defined as

$$\begin{aligned} W &= U - \sum_{j=0}^{n} px_j + \sum_{i=1}^{n}(\pi_i + u_i) + \pi_0 + u_0 \\ &= U - T, \end{aligned} \tag{2}$$

where $U - \sum_{j=0}^{n} px_j$ is the consumer surplus, $T = t(x_0 + \sum_{i=1}^{n} x_i)$ is tax revenue, and π_j is the profit of firm j (where j indexes the private firms and the public firm), u_j is the utility of union j (where j indexes the private firms and the public firm).

In the manner of Kato (2008), we also assume that the government's payoff is given by

$$G = W + (1 + a)T \qquad \text{where} \quad T = t(x_0 + \sum_{i=1}^{n} x_i), \tag{3}$$

where a is the parameter that represents the weight of the government's preference for tax revenues. Here $a \geq 0$, i.e., the government values tax revenues T more than social welfare W.

Finally, a three-stage game is conducted. The timing of the game is as follows. In the first period, the government sets the specific tax. In the second period, if each firm's union is allowed to bargain collectively, union j chooses its wage, w_j. In the third period, each firm simultaneously chooses its quantity x_j to maximize its respective object knowing each union's choice of the wage level.

3 Results

Before comparing the government's payoff and social welfare, we first consider all firms and the government's maximization problems. In this paper, since we focus on symmetric Nash equilibrium, we assume that all private firms choose the same type of bargaining. Thus, the game is solved by backward induction, i.e., the solution concept used is the subgame perfect Nash equilibrium.

3.1 The Unionized Mixed Oligopoly

In this case, the public firm's objective is to maximize welfare which is defined as the sum of the consumer surplus, individual firms' profits, and unions' utilities less the tax revenues. Thus, given w_j and t for each firm j ($j = 0, ..., n$), the public firm's maximization problem is as follows:

$$\max_{x_0} \quad W = U - T \quad \text{s.t.} \quad (p - w_0 - t)x_0 \geq 0.$$

As in Ishida and Matsushima (2008), the constraint implies there is some lower-bound restriction on the public firm's profit, i.e., the public firm faces a budget constraint[7].

Denoting the multiplier of the budget constraint λ, the Lagrangian equation can be written as

$$L(x_0, \lambda) = \sum_{i=1}^{n} x_i + x_0 - tx_0 - \sum_{i=1}^{n} tx_i - \frac{(\sum_{i=1}^{n} x_i + x_0)^2}{2} + \lambda(x_0 - x_0^2 - \sum_{i=1}^{n} x_i x_0 - w_0 x_0 - tx_0). \tag{4}$$

[7] In this model, if the public firm's union does not face the budget constraint with a simple Stone-Geary utility function $u_i = (w_i - \overline{w})^{\theta} x_i$, the public firm's union can indefinitely raise its wage because the optimal output level of the public firm is independent of the wage.

Taking as w_0 and t, by solving the first-order conditions (4), we obtain

$$\frac{\partial L}{\partial x_0} = 1 - nx_i - x_0 - t + \lambda(1 - 2x_0 - nx_i - w_0 - t) = 0, \tag{5}$$

$$\frac{\partial L}{\partial \lambda} = 1 - nx_i - x_0 - w_0 - t = 0. \tag{6}$$

On the other hand, the optimal output for the private firm is given by

$$\frac{\partial \pi_i}{\partial x_i} = 0 \Leftrightarrow x_i = \frac{1}{n+1}(1 - x_0 - w_i - t). \tag{7}$$

Given these results, we now obtain the output level for each firm. By solving the first-order conditions (6) and (7), we obtain,

$$x_0 = (n+1)(1 - w_0 - t) - n(1 - w_i - t), \tag{8}$$

$$x_i = w_0 - w_i, \tag{9}$$

$$\lambda = \frac{nx_i + x_0 + t - 1}{1 - 2x_0 - nx_i - w_0 - t}. \tag{10}$$

To solve the first-order conditions of the Lagrangian equation, the budget constraint is momentarily binding. We check ex-post whether this omitted constraint is binding.

Next, a case where each union's wage is determined as a result of collective bargaining between the firm and the union is considered. To do this, the two independent maximization problems should be considered simultaneously. Using (8) and (9), the problems for union j are defined as

$$\max_{w_0} \; u_0 = w_0 x_0 = [(n+1)(1 - w_0 - t) - n(1 - w_i - t)]w_0,$$

$$\max_{w_i} \; u_i = w_i x_i = (w_0 - w_i)w_i.$$

Straightforward computation yields each firm's reaction function as follows:

$$w_0 = \frac{1 + nw_i - t}{2(n+1)}, \quad w_i = \frac{w_0}{2}. \tag{11}$$

Then, an equilibrium wage, denoted as $w_j^*, j = 0, ..., n$ is obtained by solving (11); the substitution of each (11) into (8) and (9) yields the equilibrium output, x_j^*. The equilibrium wage and output, w_j^* and x_j^*, respectively, can be obtained as:

$$w_0^* = \frac{2 - 2t}{3n + 4}, \quad w_i^* = \frac{1 - t}{3n + 4}; \tag{12}$$

$$x_0^* = \frac{(2n+2)(1-t)}{3n+4}, \quad x_i^* = \frac{1 - t}{3n + 4}. \tag{13}$$

We now move to the first stage of the game. From (12) and (13), the government's payoff, G, in the mixed oligopoly can be rewritten as follows:

$$\max_t \; G = \frac{(1-t)(3n+2)[2(3n+4)(1+at) - (1-t)(3n+2)]}{2(3n+4)^2}.$$

Straightforward computation yields the optimal tax rate as follows:

$$t^* = \frac{a(3n+4) - 2}{(3n+2) + 2a(3n+4)}. \tag{14}$$

If the weight of the government's preference for tax revenues is sufficiently large in the case of $a > \frac{2}{3n+4}$, the optimal tax rate becomes positive. Conversely, when it is small in the case of $a < \frac{2}{3n+4}$, the optimal tax rate becomes negative; in the case of $a = \frac{2}{3n+4}$, the optimal tax rate is zero. We find that the greater the weight of the government's preference for tax revenues, the higher is the tax rate that the government imposes[8]. Thus, by using (14), we have the following result.

Lemma 1: *Suppose that each firm's union is allowed to bargain collectively. Then, the equilibrium wages and output levels are given by*

$$w_0^* = \frac{2(1+a)}{(3n+2)+2a(3n+4)}, \quad w_i^* = \frac{1+a}{(3n+2)+2a(3n+4)};$$
$$x_0^* = \frac{(2n+2)(1+a)}{(3n+2)+2a(3n+4)}, \quad x_i^* = \frac{1+a}{(3n+2)+2a(3n+4)}.$$

By substituting Lemma 1 into (10), we obtain

$$\lambda = \frac{1}{n+1} > 0,$$

which shows that the budget constraint is binding. Using lemma 1, and noting that $G^* = W^* + (1+a)T^*$ and $W^* = U^* - T^*$, we can compute the government's payoff, G^*, and social welfare, W^* as follows;

$$G^* = \frac{(1+a)^2(3n+2)}{2[3n+2+2a(3n+4)]}, \tag{15}$$
$$W^* = \frac{(1+a)^2(3n+2)(6+3n)}{2[3n+2+2a(3n+4)]^2}. \tag{16}$$

As shown, all equilibrium outcomes depend on both a and n. Thus, we now investigate how the properties of social welfare vary with the number of firms and the government's preference for tax revenues in the unionized mixed oligopoly. Differentiating (16) with respect to n, we obtain

$$\frac{\partial W^*}{\partial n} = 6(1+a)^2 \left\{ \frac{a^2(32+24n) - a(8+24n+18n^2) - (2+3n)^2}{[3n+2+2a(3n+4)]^4} \right\} \tag{17}$$

for which the sign changes according to the numerator of (17). Let the numerator of (17) denote $f(a)$. We can determine the sign of $f(a)$ by applying $f(a)$ to a discriminant, since the numerator of (17) in the quadratic function is parabolic. Thus, ignoring the negative solution for a^* by the assumption $a > 0$, we have the solution a^* with $n > 1$ is as follows[9].

$$a^* = \frac{(8+24n+18n^2) + \sqrt{(8+24n+18n^2)^2 + 4(32+24n)(2+3n)^2}}{2(32+24n)}.$$

[8]Differentiating (14) with respect to n, we obtain $\frac{\partial t^*}{\partial n} = \frac{(1+a)(3n+4)^2}{[(3n+2)+2a(3n+4)]^2} > 0$.

[9]For tedious calculations, the formal solutions of these results are available from author upon request. However, we provide the Appendix B that will not be included in the main paper. The Appendix B is is only available for the referees.

Since the minimum value is attained from $32 + 24n > 0$, there can exist a critical value such that for all $a < a^*$, we obtain the derivative as $\frac{\partial W^*}{\partial n} < 0$, and for all $a > a^*$, as $\frac{\partial W^*}{\partial n} > 0$[10].

The intuition for the case $n > 1$ is as follows. First, consider the condition (17) for the special case where $a = 0$. In this case, the condition is given by $\frac{\partial W^*}{\partial n} = \frac{-6(2+3n)^2}{(3n+4n)^4} < 0$. It follows that given the same objective function for the public firm and the government, social welfare can decrease with an increase in the number of firms. Moreover, since

$$\frac{\partial W^*}{\partial a} = \frac{-6(1+a)(2+3n)(2+n)^2}{[3n+2+2a(3n+4)]^3} < 0,$$

which indicates a negative value of the derivative, $\frac{\partial W^*}{\partial n}$ is initially decreases in n given a. However, as n increases, a increases and then reaches the critical value of a^*, following which the sign of the derivative $\frac{\partial W^*}{\partial n}$ becomes positive[11]. Thus, an increase in the number of firms induces a to reach the value of a^*, after which the critical value $n > n^*$ can exist such that for all $a > a^*$ and $n > n^*$, we obtain the derivative as $\frac{\partial W^*}{\partial n} > 0$. Consequently, in terms of the overall effect of the number of firms on the social welfare W^*, we observe that $\frac{\partial W^*}{\partial n}$ is initially negative given a sufficient small $a < a^*$ but becomes positive as a and n increase.

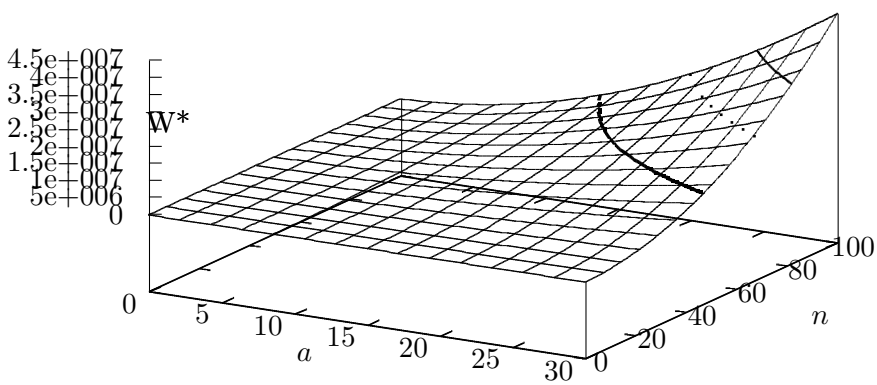

Figure 1. The Unionized Mixed Oligopoly: $a \in (0, 30]$ and $n \in [1, 100]$

Figure 1 depicts over the parameter space the different values of social welfare in a bargaining equilibrium within a monopoly. Thus, the comparative statics of the social welfare are summarized by the following.

Proposition 1: *Suppose that each union is allowed to bargain collectively. Then, the social welfare initially decreases in n given sufficiently small values of a and subsequently increases with n given sufficiently large values of a.*

[10] We also obtain $a^{**} = 35.5$ with $n = 1$. Hence, there can exist a critical value such that for all $a^{**} = 35.5 > a$, we obtain the derivative as $\frac{\partial W^*}{\partial n} |_{n=1} < 0$, and for all $a > a^{**} = 35.5$, as $\frac{\partial W^*}{\partial n} |_{n=1} > 0$.

[11] Nalyor (2002) showed that in a unionized bilateral oligopoly, industry profits are initially increasing in the number of firms if unions have sufficient bargaining power. Ishida *et al.*(2008) showed that when the effect of increase in the number of firms increases the dominant firm's incentive for R & D investment, industry profits are increasing in the number of firms.

The intuition underlying Proposition 1 is straightforward. Initially, for a small values of $a < a^*$, the social welfare-reducing effect operates because it dominates the social welfare-enhancing effect given a sufficiently small n and vice versa. Thus, contradicting to the standard Cournot-Nash oligopoly model, social welfare can decrease with n given sufficiently small levels of a. In other words, in a mixed oligopoly, wages are determined through endogenous bargaining. However, the social welfare-reducing effect is offset by the welfare-enhancing effect within a labor market that arises from an endogenous wage bargaining process. This is because at sufficiently large levels of n, the increase in social welfare outweighs the impact of the decrease in a, as is evident from the derivation $\frac{\partial W^*}{\partial a} < 0$. Therefore, the decrease in welfare with a is due to the larger weight that the government assigns to tax revenue in comparison with social welfare.

On the other hand, we find that the sign of

$$\frac{\partial G^*}{\partial a} = \frac{(1+a)(3n+2)[4a+3an-2]}{[3n+2+2a(3n+4)]^2} \tag{18}$$

changes in accordance with $a = \frac{2}{4+3n}$. Thus, the government's payoff is U-shaped with respect to a, for a given number of firms, n. In addition,

$$\frac{\partial G^*}{\partial n} = \frac{6a(1+a)^2(3n+1)}{[3n+2+2a(3n+4)]^2} > 0, \tag{19}$$

which shows that the government's payoff always increases in n. This is because when the number of existing private firms increases, the negative effect of the government's preference for tax revenue is dominated by the positive effect of the rising tax rate derived from $\partial t^*/\partial n > 0$.

3.2 The Unionized Privatized Oligopoly

The previous subsection examined at the impact of unionized mixed oligopoly given the bargaining case. This subsection compares the equilibrium of a unionized mixed oligopoly which would be established in a unionized privatized oligopoly case with unions' decentralized bargaining process. As discussion in the basic model, consider a unionized privatized oligopoly situation for a homogeneous good supplied by firm $l = 1, ..., n+1$. Firm l is a profit-maximizing private firm.

In the third stage, given w_l and t, the firm l's maximization problem is to maximize $\pi_l = (p^c - w_l - t)x_l$ where $p^c = 1 - \sum_{l=1}^{n+1} x_l : n \geq 1$. Hence, solving the first-order condition yields

$$x_l = \frac{1 - w_l - nx_m - t}{2}, \quad l \neq m.$$

Thus, the output levels are given by

$$x_l = \frac{(2-n)(1-t) - 2w_l + nw_m}{4-n^2}, \quad l \neq m. \tag{20}$$

Turning to the second stage, we consider a case where each union's wage is determined as a result of collective bargaining between the firm and the union. Thus, problem for union l is defined as

$$\max_{w_l} \; u_l = w_l x_l = \frac{w_l[(2-n)(1-t) - 2w_l + nw_m]}{4-n^2}.$$

Straightforward computation and symmetry among private firms yield each firm's wage;

$$w_l = \frac{(2-n)(1-t) + nw_m}{4}, \quad l \neq m. \tag{21}$$

Therefore, an equilibrium wage, denoted as w_i^c is obtained by solving (21), and substituting each (21) into (20) yield the equilibrium output x_i^c. Thus, we have the following result:

$$w_l^c = \frac{(2-n)(1-t)}{4-n}, \quad x_l^c = \frac{2(1-t)}{(4-n)(2+n)}. \tag{22}$$

Turning to the first stage and using the equilibrium output and wage, the government's payoff G^c in unionized privatized oligopoly can be rewritten as follows:

$$\max_t \; G^c = \frac{2(n+1)(1-t)[7+n-n^2+nt+8at+2ant-an^2t+t]}{[(4-n)(2+n)]^2}.$$

Straightforward computation yields optimal tax rate in the unionized privatized oligopoly as follows:

$$t^c = \frac{a(4-n)(2+n)-6+n^2}{2[(1+n)+a(4-n)(2+n)]}. \tag{23}$$

If the weight of the government preference for the tax revenue is sufficiently large (in the case of $a > \frac{6-n^2}{(n+2)(4-n)}$), the optimal tax rate becomes positive[12]. Conversely, when it is small (in the case of $0 \leq a \leq \frac{6-n^2}{(n+2)(4-n)}$), the optimal tax rate becomes negative; in the case of $a = \frac{6-n^2}{(n+2)(4-n)}$, the optimal tax rate is zero. As in the previous analysis, we also find that the greater the weight of the government preference for the tax revenue, the higher the tax rate the government imposes. Similar to previous subsection, we have the following result.

Lemma 2: *Suppose that the all private firms' union is allowed to bargain collectively. Then, the equilibrium wages and output levels are given by*

$$w_l^c = \frac{(4-n^2)(1+a)}{2[(1+n)+a(4-n)(2+n)]}, \quad x_l^c = \frac{(1+a)}{(1+n)+a(4-n)(2+n)}.$$

Similar to previous subsection, using lemma 2, and noting that $G^c = W^c + (1+a)T^c$ and $W^c = U^c - T^c$, we can compute the government's payoff G^c and social welfare W^c as follows;

$$G^c = \frac{(1+a)^2(1+n)}{2[(1+n)+a(4-n)(2+n)]}, \tag{24}$$

$$W^c = \frac{(1+a)^2(1+n)[7+n(1-n)]}{2[1+n+a(4-n)(2+n)]^2}. \tag{25}$$

As mentioned in the case of unionized mixed oligopoly, we now investigate how social welfare varies with the number of firms in the unionized privatized oligopoly. Differentiating (25) with respect to n, we obtain

$$\begin{aligned}\frac{\partial W^c}{\partial n} = \frac{(1+a)^2}{2[1+n+a(4-n)(2+n)]^4}\{&a^2(288+168n-12n^2-28n^3-12n^4+n^6) \\ &-a(12+28n+22n^2+8n^3+2n^4)-(6+14n+11n^2-4n^3-n^4)\}\end{aligned} \tag{26}$$

for which the sign changes with the numerator of (26). Similar to $\frac{\partial W^*}{\partial n}$ in the previous subsection, let the numerator of (26) denote $g(a)$. Again we can determine the sign of $g(a)$ by applying $g(a)$

[12] Differentiating (23) with respect to n, we also obtain $\frac{\partial t^c}{\partial n} = \frac{2(4-n)^2(3+n)}{4[(1+n)+a(4-n)(2+n)]^2} > 0$.

to a discriminant. Thus, ignoring the negative solution for a^c by the assumption $a > 0$, we have the solution a^c is as follows[13].

$$a^c = \frac{(12 + 28n + 22n^2 + 8n^3 + 2n^4) \pm \sqrt{Q}}{2(288 + 168n - 12n^2 - 28n^3 - 12n^4 + n^6)} \quad \text{where}$$
$$Q = (12 + 28n + 22n^2 + 8n^3 + 2n^4)^2$$
$$+ 4(288 + 168n - 12n^2 - 28n^3 - 12n^4 + n^6)(6 + 14n + 11n^2 - 4n^3 - n^4)$$

Since the minimum value is attained from $288 + 168n - 12n^2 - 28n^3 - 12n^4 + n^6 > 0$ with $n > 1 (n \neq 3$ and $n \neq 4)$[14], there can exist a critical value such that for all $a < a^c$, we obtain the derivative as $\frac{\partial W^c}{\partial n} < 0$, and for all $a > a^c$, as $\frac{\partial W^c}{\partial n} > 0$.

Therefore, the social welfare in the monopoly bargaining equilibrium over the parameter space $\{a, n\}$ is drawn in Figure 2.

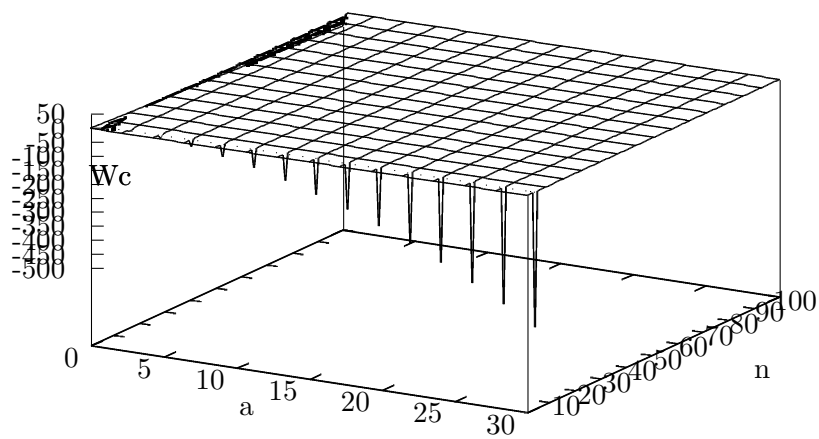

Figure 2: The Unionized Privatized Oligopoly: $a \in (0, 30]$ and $n \in [1, 100]$

Intuition is similar to the mixed oligopoly case. Depending on the critical level of a^c and n^c, the social welfare tends to increase as the number of firm increases. Thus, the comparative statics of the social welfare are summarized by

Proposition 2: *Suppose that each union is allowed to bargain collectively. Then, except for $n = 3$, the social welfare in unionized privatized oligopoly initially decreases in n given sufficiently small values of a and subsequently increases with n given sufficiently large values of a.*

From Proposition 2, we observe that $\frac{\partial W^c}{\partial n}$ is initially negative with sufficiently small $a < a^c$ and then positive as a and n increase.

[13]Similar to the solutions of a^*, for tedious calculations, the formal solutions of these results are available from author upon request. However, we provide the Appendix B that will not be included in the main paper. The Appendix B is is only available for the referees.

[14]Note that there are exceptions when $n = 3$ and $n = 4$. Since the maximum value is attained from $288 + 168n - 12n^2 - 28n^3 - 12n^4 + n^6 = -315$ with $n = 3$. Thus, there can exist a critical value such that for all $a < a^p \fallingdotseq 0.736$, we obtain the derivative as $\frac{\partial W^c}{\partial n} > 0 \mid_{n=3}$, and for all $a > a^p$, as $\frac{\partial W^c}{\partial n} \mid_{n=3} < 0$. Furthermore, $g(a)$ becomes linear function when $n = 4$. Thus, we obtain the derivative as $\frac{\partial W^c}{\partial n} < 0$ for $n \leq 2$. Otherwise, $\frac{\partial W^c}{\partial n} > 0$ for $n > 2$.

In addition,

$$\frac{\partial W^c}{\partial a} = \frac{(1+a)(1+n)(7-n-n^2)[n^2-6+a(4-n)(2+n)]}{2[1+n+a(4-n)(2+n)]^4}, \tag{27}$$

$$\frac{\partial G^c}{\partial a} = \frac{(1+a)(1+n)[a(4-n)(2+n)-6+n^2]}{2[1+n+a(4-n)(2+n)]^2} \tag{28}$$

whose signs change along $a = \frac{(7-n-n^2)(n^2-6)}{(4-n)(2+n)}$ and $a = \frac{n^2-6}{(4-n)(2+n)}$, respectively. Furthermore, we obtain

$$\frac{\partial G^c}{\partial n} = \frac{a(1+a)^2(6+2n+n^2)}{4[1+n+a(4-n)(2+n)]^2} > 0, \tag{29}$$

which this feature is similar to the mixed oligopoly case.

4 Comparative Statics

Once the equilibria for all firms and the government are derived as discussed in the previous section, the mixed and privatized oligopolies can be endogenously determined by taking the level of social welfare, each private firm's profit, and the government's payoff as given. Thus, each difference in the optimal tax rate, outputs ($x_0+nx_i = X^*, (n+1)x_l = X^c$), and the government's payoff are given by

$$G^* - G^c = \frac{a(1+a)^2(12+21n+n^2-3n^3)}{4[(1+n)+a(4-n)(2+n)][3n+2+2a(3n+4)]} > 0 \quad \text{if} \quad 3 \geq n. \tag{30a}$$

$$\text{Otherwise,} \quad G^* - G^c < 0. \tag{30b}$$

$$X^* - X^c = \frac{(1+a)a[8+30n-2n^2-3n^3]}{[(3n+2)+2a(3n+4)][(1+n)+a(4-n)(2+n)]} > 0 \quad \text{if} \quad 3 \geq n. \tag{31a}$$

$$\text{Otherwise,} \quad X^* - X^c < 0. \tag{31b}$$

$$t^* - t^c = \frac{(1+a)[8+14n-2n^2-3n^3]}{2[(3n+2)+2a(3n+4)][(1+n)+a(4-n)(2+n)]} > 0 \quad \text{if} \quad 2 \geq n. \tag{32a}$$

$$\text{Otherwise,} \quad t^* - t^c < 0. \tag{32b}$$

From (30), (31) and (32), we show that if $n \leq 3$, both the total output and the government's payoff are always larger than those when the public firm is privatized. In this case where $n \leq 3$, the government does not have an incentive to privatize the public firm regardless of the government's preference for tax revenue. That is, the privatization of the public firm is not desirable in terms of the government's payoff when $n \leq 3$ and the unions of all the firms, including the public firm, are allowed to bargain collectively. On the other hand, if $n > 4$, both the total output and government's payoff in the mixed oligopoly are smaller than those in the privatized oligopoly: therefore, the government has an incentive to privatize the public firm. This is because the effect of the government's preference for tax revenue dominates the effect of the welfare-enhancing effect. That is, the positive value of $\frac{\partial G^c}{\partial n}$ has a larger effect on the government's payoff than the positive value of $\frac{\partial G^*}{\partial n}$ when the number of existing firm is sufficiently large and vice versa. The results of this comparison are summarized in the following proposition:

Proposition 3: *Suppose that firms' unions in the mixed and privatized oligopolies are allowed to bargain collectively. Then, the government's privatization of the public firm depends on the number of existing firms, and the difference in the optimal tax rate between the unionized mixed*

and unionized privatized oligopolies also depends on the number of firms.

Regardless of the government's preference for tax revenue, the Proposition 3 suggests that if the number of existing private firms is sufficiently large, both the total output and the government's payoff is larger in a unionized mixed oligopoly than in a unionized privatized oligopoly. Thus, we find that regardless of the government's preference for tax revenue, the privatization of a public firm is not desirable in terms of the government's payoff if the number of firms is sufficiently small and vice versa.

This proposition 3 differs from that of Kato (2008), which focused on comparing a mixed duopoly with a privatized duopoly when there are no trade unions. Furthermore, Kato (2008) demonstrated that if the government sufficiently prefers tax revenues, it does not privatize the public firm, while our paper shows that regardless of the government's preference for tax revenues, the government has an incentive to privatize the public firm, which depends on the number of private firms when all firms, including the public firm, have incentives to opt for decentralized bargaining.

Next, we consider the case where social welfare and the government's payoff are compared to examine the incentives to privatize from the government's and public firm's perspectives. Comparing W^* with W^c, we obtain

$$\begin{aligned} W^* - W^c &= a^2(320 + 736n + 180n^2 - 272n^3 - 96n^4 + 24n^5 + 9n^6) \\ &- a(32 + 136n + 180n^2 + 52n^3 - 42n^4 - 18n^5) - (16 + 18n + 90n^2 - 26n^3 - 21n^4 - 9n^5). \end{aligned}$$

Applying directly above equation to a discriminant, we have the solution a is as follows:

$$\begin{aligned} a^C &= \frac{(32 + 136n + 180n^2 + 52n^3 - 42n^4 - 18n^5) \pm \sqrt{Y}}{2(320 + 736n + 180n^2 - 272n^3 - 96n^4 + 24n^5 + 9n^6)} \\ \text{where} \quad Y &= (32 + 136n + 180n^2 + 52n^3 - 42n^4 - 18n^5)^2 \\ &+ 4(320 + 736n + 180n^2 - 272n^3 - 96n^4 + 24n^5 + 9n^6)(16 + 18n + 90n^2 - 26n^3 - 21n^4 - 9n^5). \end{aligned}$$

We have the solution a with $n = 1, n = 2$ and $n = 3$ is as follows.

$$\begin{aligned} &a^{C1} \fallingdotseq -0.662260392 \quad \text{or} \quad a \fallingdotseq 0.284901902 \quad \text{when} \quad n = 1, \\ &a^{C2} \fallingdotseq -1.72075922 \quad \text{or} \quad a \fallingdotseq 0.387425887 \quad \text{when} \quad n = 2 \\ &a \fallingdotseq 2.403119488 \quad \text{or} \quad a \fallingdotseq 0.63136327 \quad \text{when} \quad n = 3 \end{aligned}$$

Since the minimum value is attained from $(320+736n+180n^2-272n^3-96n^4+24n^5+9n^6) > 0$ with $n = 1, n = 2$ and $n = 3$, ignoring the negative solution for a by the assumption $a > 0$, there can exist a critical value such that for all $a < a^{C1}$(respectively $a < a^{C2}$) when $n = 1$ (respectively $n = 2$), we obtain $W^* - W^c < 0$, and for all $a > a^{C1}$(respectively $a > a^{C2}$), we obtain $W^* - W^c > 0$. However, since there always exist negative solutions for a when $n = 3$, we obtain $W^* > W^c$ regardless of the critical value. Hence the critical level is $a^{C1} \fallingdotseq 0.662260392$ (respectively, $a^{C2} = 1.72075922$) whereby $W^* = W^c$ when $n = 1$ (respectively, $n = 2$).

However, since a comparison between the levels of social welfare with $n > 4$, we have a critical value with imaginary number. Thus, W^* and W^c becomes complicated as a result of the simultaneous variation in the number of private firms and the preference level of the government for tax revenues when $n > 4$, it is necessary to use numerical examples to illustrate the impact of privatization and degree of social welfare. The table 1 in the Appendix A illustrates the this case. The exogenous parameters are a and n. Starting from a given $n \geq 3$, the social welfare, W^* is always larger than W^c. It follows that an increase in the number of private firms in the

unionized mixed oligopoly improves social welfare more than a corresponding increase in the unionized privatized oligopoly.

Hence, the comparison between social welfare in the unionized mixed oligopoly and unionized privatized oligopoly can be interpreted as follows: if both the number of private firms and the government's preference for tax revenue are sufficiently small, the public firm has an incentive to privatize while the government does not. However, if $n = 3$, then both the social welfare and the government's payoff in the mixed oligopoly are larger than those in the privatized oligopoly, i.e., $W^* > W^c$ and $G^* > G^c$, irrespective of the government's preference for tax revenues. Therefore, the government never privatizes the public firm and there is no conflict of interest between the public firm and the government if $n = 3$. Finally, if $n > 4$, then the government always has an incentive to privatize while the public firm does not (i.e., $W^* > W^c$ and $G^* < G^c$). The results of this comparison are summarized in the following proposition.

Proposition 4: *There are no conflicts of interest with respect to privatization between the public firm and the government if the government has the perfect authority to privatize the public firm and $n = 3$. However, when the public firm can intervene in the policy of privatization, conflicts of interest with respect to privatization can arise between the public firm and the government if one of the following holds: $n > 4$; $a \in (0, 0.662260392)$ with $n = 1$; and $a \in (0, 1.72075922)$ with $n = 2$.*

Proposition 4 suggests that differences in the implementation of privatization depend on the political power structure between the public firm and the government. Contrary to De Fraja and Delbono (1989), we have shown that the privatization of a public firm with the firm's bargaining process is desirable in terms of social welfare when the number of existing private firms and the government's preference for tax revenues are sufficiently small. These two contrasting views of objective functions clearly reflect profoundly different perceptions of policy-making (i.e., the privatization in the present paper). In other words the government can be a benevolent maximizer when the number of firms is sufficiently small and the parameter relating to the government's preference for tax revenues is relatively large, and vice versa. Perhaps not surprisingly, the conflict between these two views of objective functions typically induces a conflict of the privatization.

5 Concluding Remarks

By introducing the government's preference for tax revenues into the theoretical framework of unionized mixed oligopolies, this study provides new insight into the trade-off between social welfare and the government's payoff in a government's optimal policy of privatization. Unlike extant literature on mixed oligopolies that is based on the assumption of a monolithic entity that involves the government and the public firm and that seeks to maximize social welfare, we have found that the optimal privatization policies potentially differ from Kato (2008), which focused on the government's payoff for comparing mixed and privatized duopolies.

We have found that if both the number of private firm and the government's preference for tax revenues are sufficiently small, then the social welfare under a unionized privatized oligopoly is greater than under a unionized mixed oligopoly, while the government has an incentive not to privatize the public firm. Moreover, social welfare can decrease with an increase in the number of firms depending on the level of government preference for tax revenue. On the other hand, if the number of private firms is sufficiently large, the government always has an incentive to privatize the public firm, regardless of its preference for tax revenues, while the level of social welfare under the unionized mixed oligopoly is higher than that under the unionized privatized

oligopoly. These results may indicate that differences in the implementation of privatization depend on the political power structure between the public firm and the government.

Finally, we did not extend our results by considering a model where the public firm competes with both domestic and foreign private firms, wherein the government seeks to maximize tax revenues and social welfare at the same time. Also, in this paper, we have limited the policy analysis to privatization. However, a richer policy, such as a lump-sum, ad valorem tax and subsidy policies towards both the domestic and international mixed oligopolies are worth considering. There could be important economic implications if the analysis is expanded to include the different bargaining motives among firms in the framework of the existing mixed oligopolistic market. The extension of our model in these directions remains an agendum for future research.

References

[1] Barcena-Ruiz, J.C. 2007,"Endogenous Timing in a Mixed Duopoly: Price Competition," *Journal of Economics* 91, 263-272.

[2] Bös, D., 1991, *Privatization: A Theoretical Treatment*, Oxford University Press.

[3] Brennan, G, and J, Buchanan, 1980, *The Power to Tax: Analytical Foundations of a Fiscal Constitution*, Cambridge University Press.

[4] Brülhart, M, and M, Jametti, 2006, "Vertical versus Horizontal Tax Externalities: An Empirical Test," *Journal of Public Economics,* 90, 2027-2062.

[5] Brülhart, M, and M, Jametti, 2007, "Does Tax Competition Tame the Leviathan?," CEPR Discussion Papers, No 6512.

[6] De Fraja, G. 1993, "Union and Wages in Public and Private Firms: A Game Theoretic Analysis," *Oxford Economic Papers*, 45, 457-469.

[7] De Fraja, G. and Delbono F., 1989, "Alternative Strategies of a Public Enterprise in Oligopoly," *Oxford Economic Papers*, 41, 302-311.

[8] De Fraja, G. and Delbono F., 1990, "Game Theoretic Models of Mixed Oligopoly," *Journal of Economic Surveys*, 4, 1-17.

[9] Edwards, J. and M. J, Keen, 1996, "Tax Competition and Leviathan," *European Economic Review*, 40, 113-140.

[10] Fjell, K. and J, Heywood, 2004, "Mixed Oligopoly, Subsidization and the Order of Firm's Moves: The Relevance of Privatization," *Economics Letters*, 83, 411-416.

[11] Forbes, k, F., and E, M, Zampelli, 1989, "Is Leviathan a Mythical Beast?," *American Economic Review*, 79, 587-596.

[12] Haucap, J. and C. Wey, 2004,"Unionisation Structures and Innovation Incentives," *Economic Journal*, 114, C149-C465.

[13] Ishida, J., T, Matsumura and N, Matsushima, 2008, "When Market Competition Benefits Firms," Osaka University, OSIPP Discussion Paper 2008-E-011.

[14] Ishida, J, and N., Matsushima, 2008, "Should Civil Servants be Restricted in Wage Bargaining? A Mixed-Duopoly Approach," Kobe University, GSBA Discussion Paper Series 2008-49, forthcoming in *Journal of Public Economics.*

[15] Kato, H, 2008, "Privatization and Government Prefernce," *Economics Bulletin*, 12, 1-7.

[16] Keen, M, J. and C, Kotsogiannis, 2002,"Does Federalism Lead to Excessively High Taxes?" *American Economic Review*, 92, 363-370.

[17] Leahy, D, and C, Montagna, 2000, "Unionisation and Foreign Direct Investment: Challenging Conventional Wisdom?" *Economic Journal*, 110, C80-C92.

[18] Lommerud K, E., F, Meland, and L, Sørgard 2003,"Unionised Oligopoly, Trade Liberalisation and Location Choice," *Economic Journal*, 113, 782-800.

[19] Matsumura. T, 1998, "Partial Privatization in Mixed Duopoly," *Journal of Public Economics.* 70, 473-483.

[20] Matsumura, T., and N. Matsushima, 2004, "Endogenous Cost Differentials between Public and Private Enterprises: A Mixed Duopoly Approach," *Economica*, 71, 671-688.

[21] Mujumdar, S. and D. Pal, 1998, "Effects of Indirect Taxation in a Mixed Ooligopoly," *Economics Letters*, 58, 199-204.

[22] Myles, G., 2002, "Mixed Oligopoly, Subsidization and the Order of Firms' Moves: An Irrelevance Result for the General Case," *Economics Bulletin,* 12, 1-6.

[23] Naylor, R, A, 1998,"International Trade and Economic Integration when Labour Markets are Generally Unionised," *European Economic Review,* 42, 1251-1267.

[24] Naylor, R, A, 1999,"Wage Union Strategies and International Trade," *Economic Journal,* 109, 102-125.

[25] Naylor, R, A, 2002,"Industry Profits and Competition under Bilateral Oligopoly," *Economics Letters,* 77, 169-175.

[26] Nett, L., 1993, "Mixed Oligopoly with Homogeneous Goods," *Annals of Public and Cooperative Economics,* 64, 367-393.

[27] Oates, W, E., 1985, "Searching for Leviathan: An Empirical Study," *American Economic Review*, 75, 748-757.

[28] Oates, W, E., 1989, "Searching for Leviathan: A Reply and Some Further Reflection," *American Economic Review*, 79, 578-583.

[29] Oswald, A, J., and P., Turnbull, 1995, "Pay and Employment Determination in Britain: What are Labour Contracts Really Like," *Oxford Review of Economic Policy*, 1, 80-97.

[30] Poyago-Theotoky, J., 2001, "Mixed Oligopoly, Subsidization and the Order of Firms' Moves: An Irrelevance Result," *Economics Bulletin,* 12, 1-5.

[31] Rauscher, M., 2000, "Interjurisdictional Competition and Public-Sector Prodigality: The Triumph of the Market over the State?," *Finanzarchiv*, 57, 89-105.

[32] Saha, B, and R, Sensarma, 2008, "The Distributive Role of Managerial Incentives in a Mixed Duopoly," *Economics Bulletin,* 12, 1-10.

[33] Skaksen, J. R., 2004, "International Outsourcing when Labour Markets are Unionized," *Canadinan Journal of Economics,* 37, 78-94.

[34] Tirole, J., 1994, "The Internal Organization of Government," *Oxford Economic Review*, 46, 1-29.

[35] White, M., 1996, "Mixed Oligopoly, Privatization and Subsidization," *Economics Letters*, 53, 189-195.

[36] Wilson, J, Q., 1989, *Bureaucracy: What Government Agencies Do and Why They Do It,* New York, Basic Books.

[37] Zax, J, S., 1989, "Is There a Leviathan in Your Neighborhood?." *American Economic Review*, 79, 560-567.

6 Appendix A

In this case where we have been abbreviated, we present on separate page[15].

Table 1: Numerical Examples: When $n \geq 3$

	$a = 0.1$		$a = 0.5$		$a = 1$		$a = 11$	
	W^*	W^c	W^*	W^c	W^*	W^c	W^*	W^c
$n = 3$	0.5397	0.1195	0.3222	0.1065	0.2410	0.0988	0.1347	0.0827
$n = 5$	0.4992	-1.6799	0.3099	-14.04	0.2360	-156	0.1358	-1.1140
$n = 10$	0.4630	-38.2524	0.2975	-1.6434	0.2304	-0.4907	0.1363	-0.1078
$n = 15$	0.4495	-269.5528	0.29260	-0.6084	0.2280	-0.2221	0.1364	-0.0561
$n = 20$	0.4425	-23.5021	0.2900	-0.3668	0.2267	-0.1430	0.1363	-0.0380
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

	$a = 15$		$a = 20$		$a = 25$		$a = \cdots$	
	W^*	W^c	W^*	W^c	W^*	W^c	W^*	W^c
$n = 3$	0.131342467	0.082038135	0.129033803	0.081545858	0.127643212	0.081245117	$\cdots$	$\cdots$
$n = 5$	0.13261785	-1.018671564	0.130387144	-0.95784139	0.129042262	-0.923076923	$\cdots$	$\cdots$
$n = 10$	0.133238879	-0.102264603	0.131093936	-0.098585925	0.129799615	-0.096419927	$\cdots$	$\cdots$
$n = 15$	0.133323542	-0.053447693	0.131214516	-0.051642264	0.1299258	-0.055871375	$\cdots$	$\cdots$
$n = 20$	0.133333198	-0.036251982	0.131243814	-0.035057927	0.129982358	-0.034352214	$\cdots$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

[15] Table 1 is obtained using Microsoft Office Excel.

7 Appendix B

This appendix will not be included in the main paper. However, this is only available for the referees. In this case where we have been abbreviated, we present on separate page[16].

Table $\star$: Solution of a under the Mixed Privatized Oligopoly

The number of firm n	a^*	a^*
1	60.5	-35.5
2	-1.683823529	0.742647059
3	-0.976439791	0.342931937
4	-0.751358696	0.21875
5	-0.641903172	0.159432387
$\vdots$	$\vdots$	$\vdots$
20	-0.394963961	0.030396434
21	-0.391829195	0.0288255
22	-0.388997498	0.027408508
$\vdots$	$\vdots$	$\vdots$

Table $\star\star$: Solution of a under the Unionized Privatized Oligopoly

The number of firm n	a^c	a^c
1	0.357693228	-0.179475406
2	1.643453416	-0.068453416
3	-0.73644954	0.092771383
4	N/A	N/A
5	-0.607736499	0.317528083
$\vdots$	$\vdots$	$\vdots$
20	-1.735125404	1.551748189
21	-1.817403863	1.6349634
22	-1.899811748	1.718209454
$\vdots$	$\vdots$	$\vdots$

[16] Table $\star$ and $\star\star$ are obtained using Microsoft Office Excel.