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The Role of Trends and Detrending in DSGE Models

– Emerging Countries Need “Trendy” Models

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Abstract

The paper discusses the role of stochastic trends in DSGE models and effects of stochastic detrending. We argue that explicit structural assumptions on trend behavior is convenient, namely for emerging countries. In emerging countries permanent shocks are an important part of business cycle dynamics. The reason is that permanent shocks spill over the whole frequency range, potentially, including business cycle frequencies. Applying high- or band-pass filter to obtain business cycle dynamics, however, does not eliminate the influence of permanent shocks on comovements of time series. The contribution of the paper is to provide a way how to calculate the role of permanent shocks on the detrended/filtered business cycle population dynamics in a DSGE model laboratory using the frequency domain methods. Since the effects of permanent shocks pervade the cyclical part of a time series, a stationary ‘gap’ versions of DSGE model must have hard times to explain the comovement of the data. For a special case of Hodrick-Prescott and band-pass filter we provide analytical results, reinterpreting some of their features. We also give a guidance for model-builders why detrending may complicate the policy analysis with DSGE models and how to avoid the need for detrending.

JEL Codes: D58, E32, C53.

Keywords: detrending, band-pass filter, spectral density, DSGE.

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Introduction

The main idea of the paper is simple, indeed. It is based on the fact that *trend-cycle interactions* are important part of economic dynamics. Specifically, there are potentially large differences between effects of a temporary and permanent shock in the economy. Permanent shocks create not only “trending” behavior, but also business cycle dynamics. Thus, *permanent shocks affect the whole frequency range*, including business cycle frequencies and affect detrended series.

Obviously then, there is no trend-cycle dichotomy, but there are important trend-cycle interactions. Eliminating or amplifying dynamics over certain frequency range, i.e. filtering, thus leaves potentially non-negligible influence of permanent shocks in the stationary detrended data. The problem is that the notion of “detrending” in the econometric practice is different than the mechanisms of trending behavior in many DSGE economic models. Importantly, DSGE structure suggests that the “trend” and “cyclical” parts are nontrivially correlated.

The point is that the stationary DSGE model then may be unable to explain comovements of filtered time series since it cannot explain the dynamics induced by permanent shocks in the detrended variables. To make our point we provide a method that assess how general linear time invariant filters (LTI) used for detrending affect the dynamics of the laboratory DSGE economy. This allows us to specify a DSGE model with stochastic trends and judge to what extent the stochastic trends affect the population moments of detrended series, used potentially to build a stationary ‘gap’ version of a DSGE model.

Using frequency domain approach, we are able to quantify for any particular linearized model and for any particular filtering procedure what part of the population covariance structure of filtered stationary variables is due to permanent shocks. Analytic results for univariate Hodrick-Prescott and band-pass filters are provided. We demonstrate that the factorisation of HP filter’s transfer function by Cogley and Nason (1995), criticised by Pedersen (2001) or Valle e Azavedo (2007), is the transfer function of the filter that takes us from first-differenced time series to cyclical HP filter part of the level of the series.

We do not analyze that different filtering methods have different impact on the covariance structure of the actual data as in Canova (1998) or Cogley and Nason (1995), inter alia. It is clear, that different filtering affects the covariance structure in different ways. Instead, we focus on the interaction of a particular filtering method with a “true” economic structure of the model economy. The question is not whether a particular filter sharply dissects certain frequency ranges, but what portion of a spectral density of a series due to permanent shocks is left in the cyclical component of the series. In principle, we could use our results for optimal filter design. We condition on an economic structure of any chosen model and assume out trend-stationary processes for simplicity.

We argue that when one is building a structural economic model (DSGE model, for instance) she should take a stand on the way how trending behavior of certain macroeconomic variables is modelled. Our view is that it is more convenient to make explicit assumptions on trending behaviour rather than less obvious or implicit assumptions in case of ad-hoc filtering.

Some may argue whether the issue is relevant for them in case they are interested in the business cycle dynamics only and thus have no intentions to use the model for forecasting or filtering with the actual data. Is the ad-hoc detrending suitable then?

We argue that even in case one is interested in the business cycle dynamics only, understanding of the trending behavior is important, especially when stochastic trends are implicitly (or explicitly) assumed in the model. First, a careful inspection of original data is crucial in order to discover important trend-cycle stylised facts that need to be captured by the model. Second,

the notion of the trend usually understood by the detrending procedure is not necessarily compatible with the notion of the trend in your structural model. Third, not respecting common trend restrictions in the filtering problem complicates real-world forecasting with the model. The problem is that each series, given an economic structure of the model economy, is affected by permanent shocks differently along its frequency range.

The importance of explicit assumptions about trending mechanisms of the model are then quite clear. Moreover, the role of trend-cycle interactions is expected to be larger in emerging market economies, that are buffeted by many permanently-viewed structural or institutional shocks.

The first part of the paper discusses the intuition behind trend-cycle interactions. The second part illustrates the effects of filtering on trend-cycle dynamics and its consequences for forecasting and policy analysis.

1. Where Do Trend-Cycle Interactions Come from?

The question of whether one should detrend or not is important mainly when the focus is on the business cycle dynamics. We argue that one should not detrend and rather make explicit assumptions about the trending behavior in the business cycle model. The reason is that the long-run behavior interacts with cyclical dynamics, i.e. there are *trend-cycle interactions*.

The definition of trends and a cycles in economics is rather vague and unclear. Usually, cycle is understood as the dynamics of the series around its trend, often adjusted for noise. The most common approach to define business cycle dynamics today is a frequency-domain definition, where movements in a certain frequency range, e.g. from 8 to 32 quarters, are declared as the business cycle dynamics.

Once researchers employ detrending methods other than deterministic detrending by time polynomials, trends are assumed to be stochastic. In economics the trending behavior in structural and/or econometric models is usually understood as either deterministic or stochastic, or a mixture of these. We shall mainly focus on stochastic trends, since these are based on integrated processes and imply permanent effects.

In structural (DSGE) models the presence of stochastic trends implies a non-trivial dynamics for the (model) economy. Due to both real and nominal rigidities of any kind a reaction to economic shocks is not a one-shot adjustment, but often a gradual reaction. More importantly, as it is notoriously known, reactions to transitory and permanent shocks may be strikingly different. For instance, permanently-viewed changes to income streams, either due to a policy or a technology innovation, induce much stronger *wealth effects* than transitory yet persistent shocks do.

As an interesting and important example of *wealth effects* linked to permanent shocks, one can take the consumption and current account comovement in small open economies, see e.g. Obstfeld and Rogoff (1996). In a simple model with forward-looking households and firms, an unexpected transitory innovation to income leads to *current account surplus*, since people intertemporally smooth their consumption path. Now assume that the the economy enjoys an innovation to the growth rate of output, hence the expected future level of output is viewed as gradually reaching permanently higher levels. Then, the permanent output is higher than the current output and associated wealth effects lead to a higher increase in consumption resulting in a *current account deficit*.

What are then the consequences of the different reaction to transient innovation to level or to growth rates for the business cycle dynamics? Again, it depends what is meant by the business cycle dynamics. For some, it might be any deviation from a *balanced growth path* (BGP) of

the model. But then all dynamics, including dynamics due to permanent shocks is a business cycle dynamics. But let us adhere to the frequency notion of the business cycle and explore the dynamics at *business cycle frequencies* only. This amounts to the investigation of filtered series dynamics, after applying e.g. the Hodrick-Prescott, band-pass or some other filter to the levels or logarithm of levels of economic variables in the model economy.

Not surprisingly, we find that in case of the frequency notion of the business cycle, permanent shocks may form an important part of the business cycle dynamics and correlation structure among filtered variables. We label the result as *trend-cycle interactions*. We thus see that often assumed orthogonality between trend and cyclical component of time series may lead researchers to investigate flawed dynamics.

What is needed is the way how to assess what portion of permanent shocks induced dynamics spills-over into filtered cyclical dynamics. A simple experiment with a model featuring permanent shock is to simulate the model conditional on a group of permanent shocks and apply detrending filter (e.g. HP filter) to (log) levels of relevant variables. However, a more illuminating and intuition enhancing way of the analysis is to rely on the population dynamics and the use of frequency-domain methods we discuss more deeply below.

1.1 Emerging Countries' View

We conjecture that for emerging countries trend-cycle interactions are even more important than for developed countries. That is not to say that for developed ones trend-cycle interactions are unimportant. But most emerging countries' economies are buffeted by pronounced permanently-viewed structural shocks to productivity and technology, not mentioning the changes in business environment.

Our own results with the model introduced in Andrle et al. (2007) in case of the Czech Republic strongly support the view that permanent shocks are an important driving force of the economic dynamics. This finding squares also with our intuition about the economy during the catch-up process within the European Union.

Our view seems to be supported also by a splendid paper by Aguiar and Gopinath (2004). They argue that "cycle is the trend" for small open emerging economies, when analyzing empirical regularities of a group of countries and applying a simple open economy real business cycle model with shocks to growth rates of technology. They also stress the importance of higher GDP components volatility, strong pro-cyclicality of current accounts or decompositions of time series using the method of King et al. (1991).

The model in Andrle et al. (2007) is much larger and complex than the one in Aguiar and Gopinath (2004), but the results point in the same direction. Trend-cyclical interactions seem to be a very important part of the dynamics in emerging economies and ad-hoc detrending is potentially a very harmful practice.

Yet, there are some caveats. Arguments we make in favor of our view on detrending are conditioned on theoretical economic models of general equilibrium. In these models permanent shocks are needed in order to generate certain kind of dynamics and cross-correlations. However, we do not exclude the possibility that the reality may be different and simply condition the analysis on structural DGE theory.

2. What Are the Effects of Detrending?

We believe that structural economic models (DSGE models) are a very convenient tool to address the issues of business cycle dynamics without previous ad-hoc detrending or pre-filtering. We build these structural models to mimic the working of our economies to explore various

hypotheses. As it turns out, deep principles of *DSGE models do not need a definition of trend and cycle dynamics*.

Our suggestion is that one should not haste to pre-filter the data unless there are compelling reasons to do so. Various notions of trend and cycle definitions preceded the recent developments in DSGE modelling and are useful for many purposes. However these trend/cycle notions are not always useful in the area of validating DSGE models and in forecasting and policy analysis with these models. Structural model without permanent shocks is a perfectly plausible tool, yet its use with ad-hoc detrended data is potentially very misleading due to dynamics caused by permanent shocks that is left in the filtered data.

We provided the intuition why trend-cycle interactions in structural economic models are important.¹ We continue to explore how the effects of filtering methods affects the analysis using DSGE models. First, we show how to measure the role of permanent shocks on filtered gap variables. Then, we explore consequences of family of linear time-invariant filters on the economic analysis with DSGE models.

2.1 Measuring How Permanent Shocks Influence Filtered Data

A natural question is to what extent the trend-cycle interactions are important in the reality and whether one should care about them. Economists attempt to bring DSGE models closer to data. We should then pay attention to a specification of structural shocks, both transient and permanent. Fully specified DSGE model may serve as a laboratory for exploration to which extent the detrending corrupts information in the data.

We demonstrate how it is possible to quantify effects of stochastic detrending within a framework of a linear(ized) DSGE model. In addition, the frequency-domain chosen approach enhances intuition and allows us to work with population moments of the data.

We assume a linear(ized) version of a DSGE model featuring both permanent and transitory shocks. We assume out trend stationary growth and specify permanent shocks as innovations to growth rates, i.e. in logs

$$g_{i,t} = A_{i,t} - A_{i,t-1} \tag{2.1}$$

$$g_{i,t} = \rho_i \bar{g}_{i,SS} + (1 - \rho_i)g_{i,t-1} + \varepsilon_{i,t}, \tag{2.2}$$

where $A_{i,t}$ is the level of i -th permanent shock, $g_{i,t}$ denotes its growth rate. Transitory persistent structural shocks are defined analogously via AR(p) processes in levels. The specification above is quite standard and we used it also in Andrlle et al. (2007) for labor-augmenting and sector-specific productivities.

The model is assumed to be in a stationary form, hence there are not levels of non-stationary variables.² Stationary variables appear untransformed, stationarised counterparts to non-stationary variables are linked to growth rates of these variables, consumption growth being an example. We could use ratios as well, the adaptation of our calculations is straightforward then. This link will be exploited later to quantify influence of permanent shocks on covariance structure of filtered variables.

¹ Note, however, that in our analysis the economic growth is exogenous by definition. This may be useful simplifying assumption, but in reality or other in models there may be link also from cycle to growth, not just from growth to cycle. We investigate only the later, but the core of our discussion would remain.

² Although we can proceed without stationarisation of the model and solve it, as we sometimes do, it is convenient as a check for properly defined dynamics and to guarantee well-defined moments of all variables in the model.

To show how permanent shocks influence the model we make use of the *population spectrum of the model*, defined as

$$f_y(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \Gamma_k e^{-i\omega k}, \quad (2.3)$$

where Γ_k is the k -th order population covariance matrix of the model, while ω denotes a frequency $\omega \in [-\pi, \pi]$. As is well known, there is an equivalence in the time- and frequency-domain analysis of time series. There is also an inverse transformation to (2.3) which allows to retrieve covariance matrices from the spectral density, i.e.

$$\Gamma_k = \int_{-\pi}^{\pi} f_y(\omega) e^{i\omega k} d\omega. \quad (2.4)$$

For $k = 0$ we can see that the area under the population spectrum represents the population unconditional variance of the model.

Plotting the spectrum of a variable can yield uncovers information on the distribution of variance among the individual frequencies. This is very intuitive and useful. We can inspect distribution of variance for all variables in the model, e.g. interest rates, consumption growth, etc.³

How can we assess the importance of permanent shocks? Let us divide all shocks in the model into non-overlapping groups – say, stationary and permanent ones. Due to linearity of the model and orthogonality of structural shocks we can easily calculate the portion of variance due to each group of shocks and plot the portion of spectral density due to each groups. Thus we have

$$f_y(\omega) = f_y^S(\omega) + f_y^P(\omega), \quad (2.5)$$

where the superscripts S , P denote stationary and permanent shocks' group, respectively.

We can then inspect in a very simple and intuitive way the contribution of permanent shocks to dynamics of individual variables and at what frequencies.

Still, we are interested in filtered variables, not just in variables present in the model – these are stationary due to stationarity-inducing transformations used. *We need to obtain the levels of the series of interest and apply a filter desired*, multivariate or univariate. It can be done by simulation, but our point is that we can calculate population characteristics using filters, with no need to simulate.

In order to assess how the filtering affects the dynamics, we it may be useful to remind how filters operate.

Detrending via linear filters To enhance the intuition we first focus on an *ideal band-pass filters* and postpone discussion of other filters to next section. As it turns out majority of detrending methods that assume stochastic trends can be rewritten as linear filters, either univariate or multivariate, in the form

$$H(L) = \sum_{j=-\infty}^{\infty} H_j L_j, \quad h(L) = \sum_{j=-\infty}^{\infty} h_j L_j \quad (2.6)$$

where H_j is the matrix of weights and L is the standard lag operator. We can rewrite band- and high-pass filters, multivariate state-space filters solved by Kalman filtering and/or smoothing, etc.

³ See appendix for more details on spectral analysis, linear filters and references.

It is easy to show that in the univariate case applying the linear filter to a series $\{y\}$ changes the spectral density of the new variable (filtered variable) x . The spectrum of $\{x_t\}$ is then

$$f_x(\omega) = |h(e^{-i\omega})|^2 f_y(\omega), \quad (2.7)$$

where $|h(e^{-i\omega})|$ is called *power transfer function* of the filter. Note that since the unconditional variance of the variable is the area under spectrum, the variance is obviously affected by the filtering procedure.

To obtain the business cycle component of time series the band-pass filter $C_{BP}(L)$ is often used. We use it because of a very intuitive form of its transfer function $C_{BP}(e^{-i\omega})$. Its power transfer function $|C_{BP}(e^{-i\omega})|^2 = 1$ for business cycle frequencies $\omega \in [\omega_L, \omega_U]$ and $|C_{BP}(e^{-i\omega})|^2 = 0$ otherwise.⁴

The transfer function of an ideal band-pass filter transfers fully part of the spectrum at selected frequency range and completely eliminates all other parts. Clearly, only part of the unconditional variance is left, the rest is cut off, which can be illustrated using plotting the spectrum of a variable and *cut-off frequencies* ω_L and ω_U .

Other filters, approximate BP or Hodrick-Prescott⁵ filters for instance, operate in a similar way, but they do not only cut-off completely some frequencies but re-weight other, since they are not able achieve the zero-one precision cut-off of an ideal band-pass filter.

A lot of literature discusses properties of linear filters and filter design as to obtain “sharp” filters as close as possible to ideal band-pass filters, limiting the leakage from other than selected frequencies. What we discuss in the paper is, however, completely different issue, since we argue that the cut-off leaves the effects of permanent shocks in the filtered data.

Spectrum and Covariance Structure of Filtered Data Yet, up to now the results above do not show the part of the covariance of filtered data due to permanent shocks. The problem is that for the series of interest the model features only growth rates of these variables, or some other stationarity-inducing transformation. What is needed is to apply detrending filter to *levels* of selected series.

What we do is that we apply the *integration filter* to growth rates of selected variables with a power transfer function $q(\omega) = 1/(2 - 2 \cos \omega)$. The integration filter is the inverse of the simple first-difference filter. By applying the integration filter we obtain pseudo-spectra for the variables; these are well-defined for all frequencies but $\omega = 0$. For the notion of pseudo-spectra see Bujosa et al. (2002), *inter alia*.⁶

In the sequel we need to apply selected detrending univariate of multivariate filter to levels of variables of interest, which amounts to applying the filter to pseudo-spectra of the series. Since all detrending linear filters attribute zero weight to frequency $\omega = 0$ the pseudo-spectrum poses no obstacles in the filtering problem.

The only issue left now is to calculate the covariance of the filtered variables and to report what part of covariance is due to permanent shocks in the model. Since we kept the track of the part of spectrum due to stationary and permanent shocks, $f_y^S(\omega)$, $f_y^P(\omega)$ in (2.5), after all filters are applied, the calculation of the two part of covariance matrices is a trivial application of the formula (2.4). Again, the partition of shocks into groups is arbitrary as far as they do not overlap.

⁴ We use only $\omega \in [0, \pi]$ due to symmetry of the analysis.

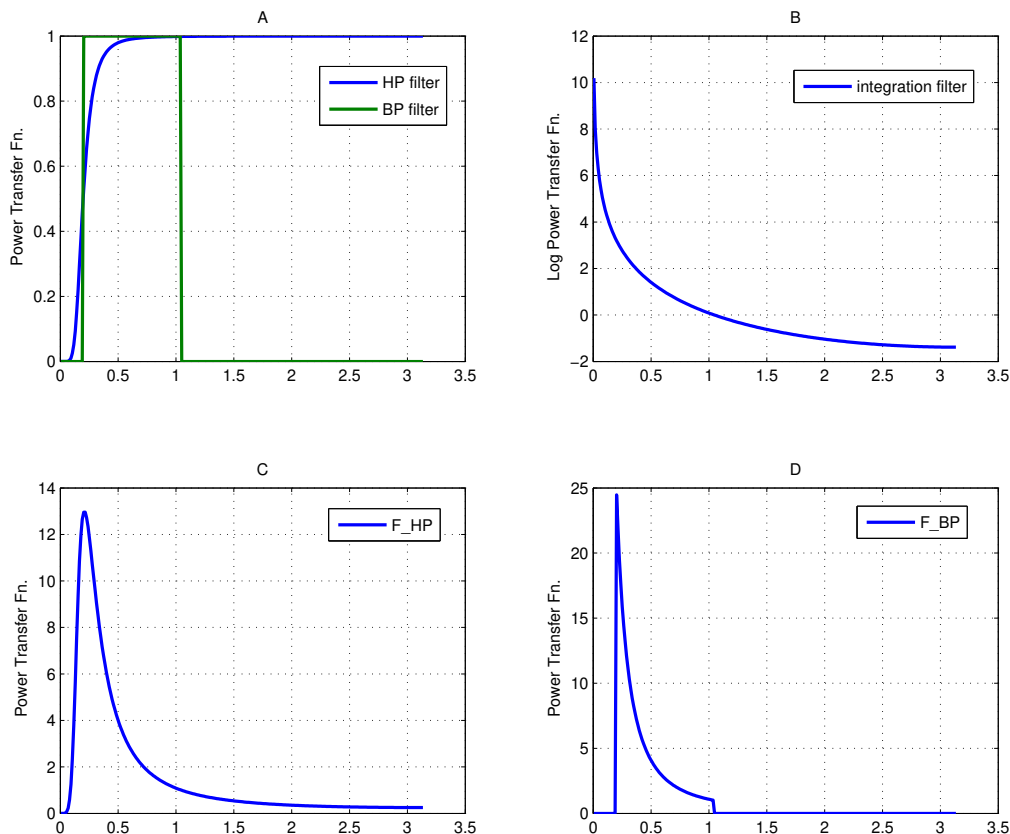
⁵ See appendix for more details on Hodrick-Prescott filter. Note that it is linear filter.

⁶ Note that the definition of pseudo-spectra is need always in case of analysis of linear filters to non-stationary series, e.g. it concerns HP filters or band-pass filters always.

2.1.1 Illustration and Special Cases

To illustrate our arguments, we provide some examples bellow and focus on Hodrick-Prescott and ideal-band pass filters. The appendix discusses some additional details and analytical expressions of power transfer functions of the integration-detrending filter for the Hodrick-Prescott and Band-pass filter.

Fig. 1: Power Transfer Functions



The power transfer function of the filter filter that transforms spectral density of growth of a variable in to a filtered gap variable is displayed in the panel C of the Fig. 1 (i.e. $F_{HP}(\omega)$) in case H-P detrending is used, the panel D depicts the power transfer function of the same problem in case of using band-pass filter, $F_{BP}(\omega)$. In the panel A of the figure we illustrate the power transfer function of HP ($\lambda = 1600$) and band-pass filter, panel B depicts the log of integration filter' power transfer function.

The interpretation of panels C and D is straightforward using the relation in (2.7). Given a spectral density of a growth-rate of a variable, say real exports, multiplying its value by associated value of the power transfer function in the panel C or D we obtain spectral density of (real exports) gap as if we HP or band-pass filtered the log level of the variable.

The variability at frequencies with non-zero weights will spill-over into filtered variables. Clearly, if the variance of variable's growth is affected by permanent shocks at business cycle

frequencies, the variance of associated gap variable will also be affected by these permanent shocks.

Using the model small open economy model in Andrieu et al. (2007) we provide a decomposition of exports as an illustration. We decompose the growth rate of exports into effect of permanent shocks and all others, we do the same for exports gap. Then we decompose the autocovariance of cyclical exports and imports into effect of permanent and other shocks. The illustration is based on HP filtering.

Fig. 2: Decomposition of Exports Spectra

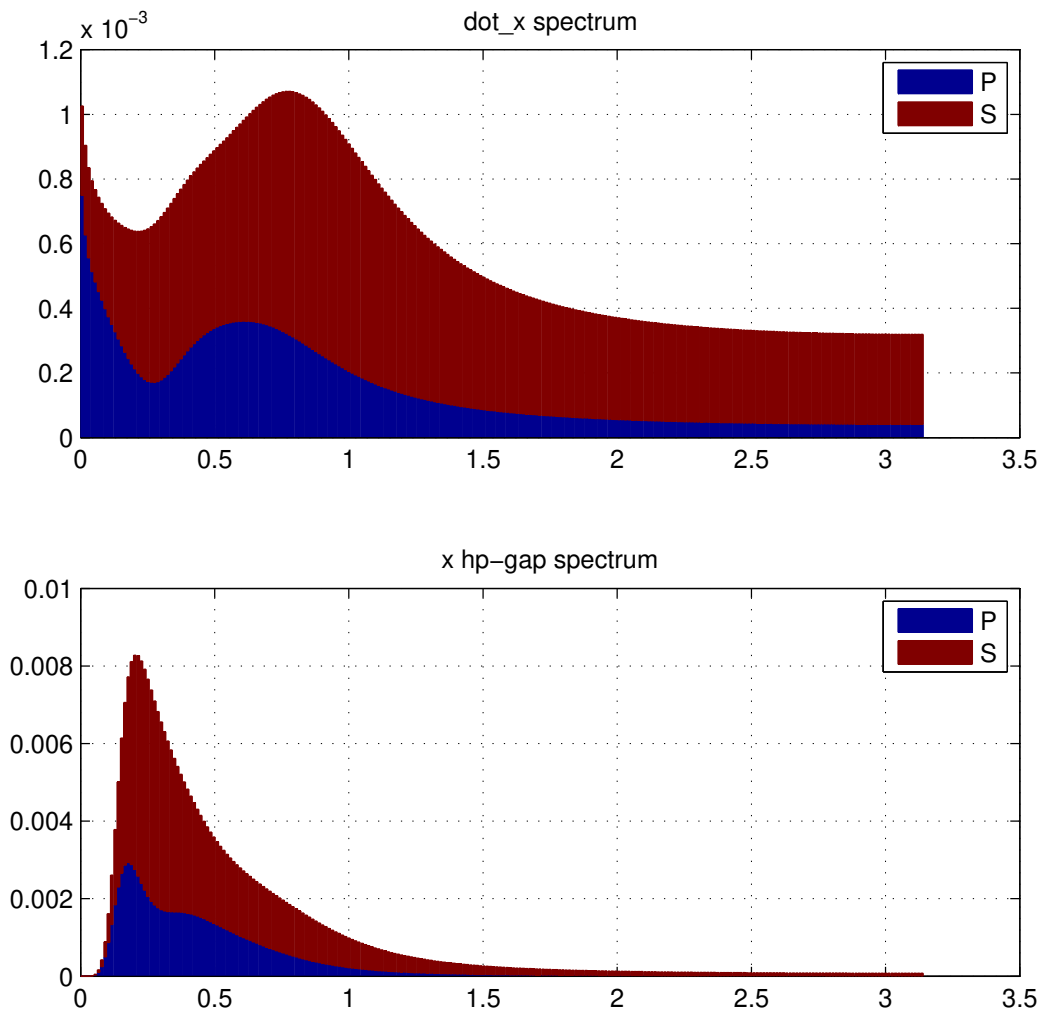
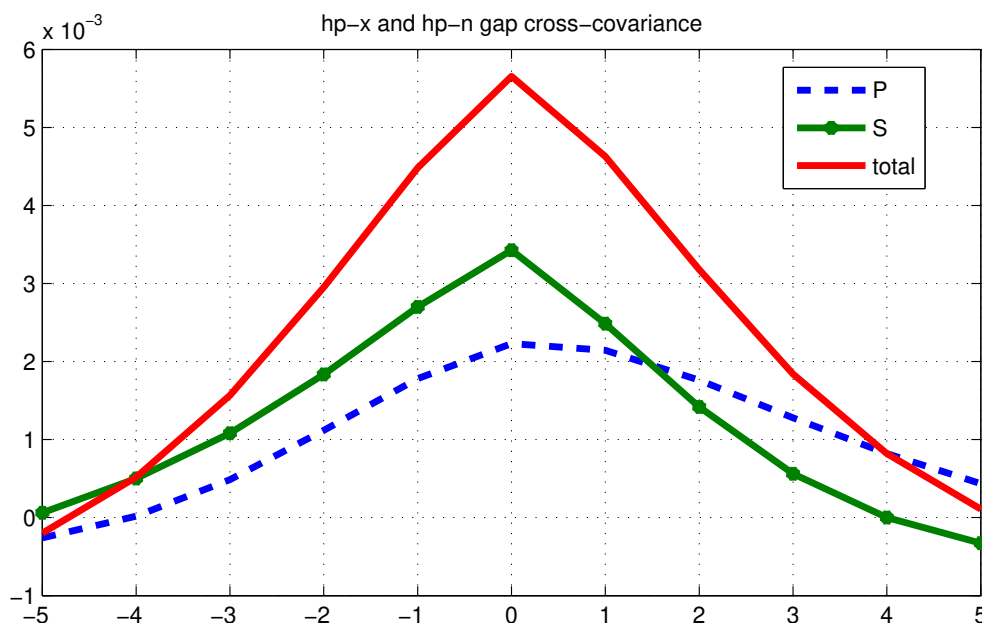


Fig. 2 decomposes spectral density of quarter-on-quarter export growth and corresponding cyclical component of HP filtered log of level of exports due to stationary (S) and permanent (P) groups of shocks. In the sequel, using the relationship between spectral density and covariance information of the process Fig. 3 illustrates the decomposition of covariance due to stationary and permanent shocks of the DSGE model.

Fig. 3: Exports and Imports HP Gaps Cross-Cov. Decomposition



One could easily check for many other relationships to judge the extent to what permanent shocks affect the covariance of the filtered variables, given a particular model and filtering framework. The only thing that seems quite general is that permanent shocks matter for the business cycle dynamics and the method introduced above is a convenient way how to analyse the issue.

2.1.2 Hodrick-Prescott Factorisation

More detailed analysis of Hodrick-Prescott filter is particularly appealing due to its simplicity and widespread use. Combining the integration filter with the H-P filter we get in our simple univariate example a transfer function $F_{HP}(\omega)$ that takes us from the stationary DSGE model spectral function $f_x(\omega)$ to spectral density of the HP filtered levels of desired variables $f_y(\omega)$

$$f_y(\omega) = |F_{HP}(\omega)|^2 f_x(\omega), \quad (2.8)$$

where the power transfer and transfer functions of the filter F are

$$|F_{HP}(\omega)|^2 = \frac{8\lambda^2 [1 - \cos(\omega)]^3}{[4\lambda [1 - \cos(\omega)]^2 + 1]^2} \quad F_{HP}(\omega) = \frac{\lambda e^{2i\omega} (1 - e^{-i\omega})^3}{\lambda e^{2i\omega} (1 - e^{-i\omega})^4 + 1}. \quad (2.9)$$

for $\omega \neq 0$.

As an interesting side comment we may note that the filter $F_{HP}(\omega)$ was already discussed in the literature, albeit *in a different context from ours*.

Cogley and Nason (1995) criticise the Hodrick-Prescott filter of creating spurious peaks at business cycle frequencies. Their conclusions are based on the argument that applying the H-P filter

to non-stationary I(1) series amounts to differencing of the series first and then applying the asymmetric filter $C_1(\omega)$ such that $C(\omega) = (1 - e^{-i\omega})C_1(\omega)$, where $C(\omega)$ is the HP filter transfer function.

Pedersen (2001) argues that the critique of Cogley and Nason (1995) is *inadequate* since their results are on “inadequate definition of the Slutsky-effect – a definition which has the unfortunate consequence that even an ideal high-pass filter induces a Slutsky effect”. The correct way to analyse effect of Hodrick-Prescott filter on I(1) processes is to make use of pseudo-spectrum definition. Not surprisingly then, the ideal high-pass or band-pass filter does not produce any peaks or Yule-Slutsky effects, but only eliminates a portion of frequencies, see Valle e Azavedo (2007) for an interesting discussion.

We give thus *a new interpretation* to the filter $C_1(\omega)$, since it turns out that $F_{HP}(\omega) = C_1(\omega)$. It is simply because the integration and difference filter cancel out and $C_1(\omega)$ remains. We can interpret the power transfer function as a link between a growth rate of a variable and the cyclical component of the level of this particular variable.

Further analysis of HP filter and band-pass filter is given in the appendix.

3. Consequences of Filtering for Forecasting and Policy Analysis

At this state we already have some idea to what extent the filtered dynamics is important for our structural model at hand. In general, our argument is that ad-hoc filtering corrupts the data dynamics and leaves effects of permanent shocks in business cycles. But there are other related issues.

First, we should inspect whether all filters are all alike, or whether there are important differences for instance between univariate and multivariate filters. Second, a related issue is whether the *common trends* restrictions implied by the data and/or the your structural model matter for detrending and how. That is, what happens when we detrend series-by-series and what are the consequences for policy analysis and forecasting.

We argue that series-by-series univariate detrending seems to be a very problematic procedure, since it may never respect individual properties of the series and leads to important end-point-bias problems in policy analysis and forecasting. Multivariate filters thus theoretically may be superior to univariate, when they respect the model structure to some extent.

What is on the list? Are all filters alike? No, they are not. Detrending filters differ in many aspects. For a broad and recent review of filters for business cycle analysis, see Proietti (2008), *inter alia*. In the paper we label as *ad-hoc filtering* every detrending filter, since it does not fit into a structural world of DSGE models.

What detrending methods can be encompassed into our analysis? The list includes all linear band-pass and high-pass filters both univariate and multivariate. Hence, we treat Hodrick-Prescott filter (high-pass), band-pass filters, exponential smoothing, constant-parameters structural time series (unobserved components) models (e.g. random-walk plus noise, local linear trend, structural Phillips-curves based models, etc.) or Beveridge-Nelson decomposition.⁷ *Simply put, any filter with well-defined transfer function is amenable for the analysis we carried out above.*

In the filtering literature ad-hoc filters are understood those filters that are invariant to properties of individual time series. Economists worry about inducing spurious cycles, or better, Slutsky-Yule effects. Yet, we agree with Proietti (2008) that “the issue of spuriousness is problematic, at

⁷ See for instance Morley et al. (2002) for a discussion of state-space representation of Beveridge-Nelson decomposition.

least, if not tautological”. The problem is again with the fundamental question what is *the* cycle in economies time series. The convenience of our departure from structural DSGE paradigm is that we do not need to answer this question.

The more important issue for us is under what circumstances detrending filters may come close to identify as “trends” the evolution of permanent shocks as defined in the model. Note, that this is implicit in analysis using detrended data and models linearized around a steady-state instead around a balanced growth path.

Different Trends In general, univariate filters that do not respect the nature of time series at hand are used most often (HP, band-pass). The problem is that even in case of infinite amount of observations series-by-series filters identify *different trends for all variables* even in case that from a structural model point of view there is just one driving force of trending behavior. Common trends restrictions so important in the model analysis are then corrupted.

How come one can extract different trends then, even if it is known that there are common trends restriction in the data? The answer is very simple. It is because the filter eliminates and modifies the same set of frequencies of spectral density. Intuitively, it eliminates or attenuates the same region of variance in all series. Since different series have possibly different distribution of variance into frequencies, the extracted trends may be different. The reason is again that permanent shocks pervade across the whole frequency range.

The shorter is the length of a series, the more acute is the problem of different trends. Due to internal propagation mechanisms in the (model) economy economic aggregates react differently to the same sequence of permanent and transitory shocks. For instance the consumption smoothing may create moderate dynamics as opposed to trade variables. In case of short time series the underlying common trend restriction may not be correctly identified in terms of the “slope” of the trend. The problem is general, but dependent on the trajectory of all structural shocks.

End-Point Problems of Univariate Filtering Majority of detrending filters are of two-sided nature, often symmetric (band-pass, Hodrick-Prescott). In case we apply univariate filters, we obtain potentially different trends, but also *end-point problems* that mutually are unrelated. The end-point problem concerns all two-sided problems and results in estimates revisions after new data are available.⁸

The problem is not at revision per se, but in the fact that there is no systematic link among these estimates for individual series. Thus, the structure of deviations from the “trend” might be completely wrong and not reflect the multivariate nature of the actual state of the economy. Even in case that filtered “trends” drift in a more-or-less same way, the end-point problem poses a problem for forecasting. Together with the problem that the deviations from trends (gaps) are most probably *mutually inconsistently estimated* and have spurious dynamics, the end-point complicates forecasting simulations.

The correct assessment of the *actual state of the economy* is crucial in the forecasting and policy analysis process, due to significant role of initial conditions on the forecast. First, deviations from ad-hoc trends may give wrong story about the economy, since these are not mutually related (unrestricted) as opposed to reality and model structure. Second, if real variables are to be forecast not just in gap-form, assumption on the “trends” development is needed. When these are different and unrelated, projecting trends is again inconsistent and fraught with hazards.

Recalling the voluminous literature on output gap measurement, the latest estimates of the gap are usually the most important. In case one would be tempted to forecast with a structural

⁸ To be precise, even in case of no detrending, end-point problems arise with DSGE models. But the nature of the problem is somewhat different, see Andrle (2008), than what we want to convey in this paper.

model with filtered gap variables, the problem is much more complex due to multivariate nature of initial state of the economy, not just the output gap itself.

To sum up, there are three related problems with detrending univariate filtering for forecasting and policy analysis: (i) *deviations from trends are still influenced by permanent shocks and the dynamics is inconsistent*, (ii) *end-point estimates of gaps are unrelated and inconsistent, conveying hardly any useful information* and (iii) *projection of identified trends is fraught with hazards and again inconsistent*.

Are Multivariate Filters of any Help? Partially, they may be. To some extent they can alleviate the problem of multiple trends, in case common trends (common features) restrictions are imposed. Still, they are of no help with the spurious dynamics due to spill-over of permanent shocks into filtered gap variables.

The structural specification of the filtering problem is crucial and there are many diverse setups. In our view, an important issue is the correlation of trend-cycle innovations, see Proietti (2006) or Morley et al. (2002), inter alia. This is because in a structural model innovations to permanent structural shocks feed both into “trend” and “cycle” in terms of the frequency-domain notion. In this respect, trend-cycle decomposition delivering potentially more volatile trend than the original series (e.g. Beveridge-Nelson decomposition) are more understandable in the DSGE framework, if we would interpret the trend as the evolution of the permanent shocks (e.g. technology).

3.1 Still, I Am Interested in Business Cycles, What Shoud I Do?

Having trends in a model does not mean you cannot inspect business cycle dynamics in terms of the frequency-definition. The most simple thing to do is to simulate the model, apply your preferred filters and calculate statistics of interest.

However, population statistics are easier often to obtain using frequency domain methods and calculating frequency-specific moments directly either using ideal band-pass filters or any other filter desired. Frequency-domain statistics also indicate the balance between the trend-cycle dynamics of your model and are valuable guides in selecting parameters either via calibration or more formal methods.

4. What If Great Ratios Seem Not That Great in My Country?

Although one might agree that stochastic detrending is a potentially harmful practice, still one may try to eliminate trending behavior from hers DSGE model since the data regularities of the country seem problematic. We know that many economist claim that in their countries *great ratios are not that great*.

Indeed, for some countries the “standard” stylised facts often seem to be broken. Our experience, however, is that these seemingly aberrant features of the data convey important economic information and should be closely inspected rather than mindlessly detrended. That was the lesson learned when we working on the new structural model for the Czech National Bank and the experience with the use of the model is described in Andrlé et al. (2007). There we focused on trend in relative prices, increase in trade openness, terms-of-trade shifts and other issues.

First, it is helpful to realize that the economy might be perhaps described with *multisectoral model*, or a model with multiple permanent shocks. Second, one should pay close attention to methodology of national accounts and data reporting to understand what might be possible causes of aberrant stylised facts.

4.1 Multisectoral Economy

Multisectoral (growth) model may be one of the solutions to consider when analysing the economy, moreover an emerging one. A well-known problem is the discussion of sector-specific, namely investment-specific shocks, as analysed, see e.g. Greenwood et al. (1997) or Ireland and Schuh (2006), for instance. Investment-specific technologies introduce a trend in a relative price of consumption and investment goods.

Under a hypothesis of a multisectoral economy the effects of filtering are even more disputable, moreover when the filtering is univariate. The information in the data is completely destroyed then. In case of appropriate multivariate detrending obeying the long-run structure of the model, at least some trend consistency is left, yet the dynamics remains spurious. Moreover, if one is willing to impose DSGE models' long-run structure on the filtering problem, then why to carry out ad-hoc detrending at all?

Identification of all structural shocks and those permanent is more interesting solution both from theory and policy perspective. The long-run behavior is then tailor-made to the economy under investigation and the fully-fledged dynamics of the data can be analysed.

4.2 National Accounts and Trends

The often heard argument of aberrant great ratios may be also blurred by potentially inappropriate treatment of the data, per se. Whelan (2003) or Whelan (2005), inter alia, provides very interesting arguments why multisectoral models may be feasible description of the economy.

Note that a "standard" single-good neoclassical growth model (or an RBC model) implies also single price for all components of GDP spendings. But in real world there are important changes in relative prices in many sectors. This is also one reason why many central statistical offices adopted a methodology of *chain-weighted GDP calculation*. In principle this means that to calculate the "real" GDP growth prices of previous period are always used, to alleviate the problem of changes in sectoral relative prices.

In the older methodology the structure of relative prices was usually held fixed for five consecutive years and constant-prices GDP calculated. When relative prices undergo large changes such calculations are problematic. Thus a recommendation of Eurostat to EU member countries is to adopt some version of chain-weighted method to calculate GDP.

The consequence of working with chain-weighted aggregates is that calculations using "real shares" are meaningless. In fact, real expenditure components of GDP are no more additive. One should look then at *nominal expenditure shares* and check to what extent these are stable. Hence, it is necessary to look for *nominal stylised facts*. This is maybe even more important in countries when chained-linked national account methodology is not followed yet.

In emerging countries chances are that nominal stylised facts are much better behaved than the "real ones", which is the case for the Czech Republic as well. Using nominal expenditure share to analyze resource allocation is also very intuitive. Recall that the concept of "real output" is an abstraction, since in real world nothing like this exists. So is the case in a multisectoral model, we can calculate only nominal GDP. This however poses no problems even in a forecasting and policy analysis oriented models in Andrle et al. (2007) or Edge et al. (2005).

Of course there may be cases where using some sort of detrending is unavoidable at certain state of analysis, yet our view is that if there is a chance to make structural assumptions about trending behavior, one should take that opportunity.

5. Conclusions

We argued that ad-hoc filtering of stochastic trends for use with structural economic (DSGE) models is fraught with hazards. The main problem is that in reality the permanent shocks resulting in trending behavior of economic time series affect also what one may view as business cycle dynamics. There are significant trend-cycle interactions, mainly in emerging economies but also in developed economies.

When structural DSGE models are used for hypothesis testing or policy analysis, structural assumptions on the nature of trending behavior is feasible to make, even when one is interested mainly in business cycle dynamics. Should we accept the definition of cycle in terms of a dynamics in a limited frequency band, e.g. 8 to 32 quarters, then such business cycle dynamics is non-trivially affected by permanent shocks, unless the framework of DSGE models is viewed conceptually flawed.

We demonstrated how it is possible to analyse consequences of univariate and multivariate filters on time series of a hypothetical (model) economy with trending behavior. The framework is general enough, does not rely on simulations and its frequency-domain flavor enhances intuition and understanding of the problem.

For many economies, especially those of emerging countries, the need for detrending might be attenuated by close inspection of the data and nominal great ratios with consideration of multisectoral model. If detrending is carried out eventually, at least caution about possible consequences of the procedure are better understood. Multivariate filtering is preferable than to univariate filtering, if plausible common features restrictions are imposed.

We hope to have provided enough arguments to discourage from ad-hoc hasty filtering of economic data. Yet, in no way we argue to avoid detrending at any cost – clearly, that would not be economical.

6. Appendix

The first part of the appendix serves to give some more technical details of the issues discussed and deepens understanding of the results. However, the economic intuition of the main text should be self-contained. Principles of spectral analysis of time series can be found in Hamilton (1994), Brockwell and Davis (1991) or Priestley (1981), *inter alia*.

The second part of the appendix contains some detailed results for a stylised small open economy RBC model with regard to role of permanent shocks on the dynamics of filtered gap variables.

6.1 Spectral Analysis & Linear Filters – A Reminder

Let g_Y denote the (pseudo)-autocovariance generating function of the multivariate ARIMA stochastic process $\{Y_t\}_{t=-\infty}^{\infty}$

$$G_Y(z) \equiv \sum_{k=-\infty}^{\infty} \Gamma_j z^k, \quad (6.10)$$

where z is a complex scalar. By normalisation and evaluating (6.10) at $z = e^{-i\omega}$, we obtain population (pseudo)-spectrum of the process

$$f_Y(\omega) = \frac{1}{2\pi} G_Y(e^{-i\omega}) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \Gamma_k e^{-i\omega k} \quad (6.11)$$

where $i = \sqrt{-1}$ and ω is the angular frequency measured in radians.

There is also an inverse transform going from population spectra to covariance structure of the process, thus we have

$$\Gamma_k = \int_{-\pi}^{\pi} f_Y(\omega) e^{i\omega k} d\omega. \quad (6.12)$$

The particular case is for $k = 0$, implying that the area under the spectrum is the population variance of the process.

In principle, these results are valid only for stationary stochastic processes. Following Bujosa et al. (2002) and Valle e Azavedo (2007) we work with the pseudo-spectrum of stochastic processes, that is amenable to use with non-stationary variables. The pseudo-spectrum is well-defined for all frequencies apart from $\omega = 0$, where it is undefined.

Linear Filters Let the *filter* be defined as an absolutely summable sequence of matrices and let the multivariate process Y_t be a result of applying the filter to a multivariate process X_t

$$H(z) = \sum_{k=-\infty}^{\infty} H_k z^k \quad Y_t = \sum_{k=-\infty}^{\infty} H_k X_{t-k}. \quad (6.13)$$

If we know the population spectrum of the process X_t it is easy to show that the population spectrum of the Y_t process is given by

$$f_Y(\omega) = H(e^{-i\omega}) f_X(\omega) H(e^{i\omega})^T. \quad (6.14)$$

This is the basic result needed to analyse the effects of multi- or univariate linear time invariant filters on stochastic processes. The *transfer function* of the filter is $H(e^{-i\omega})$.

Note that any linear time-invariant filter that can be put into the form in (6.13) is covered by the discussion in the main text and can be analysed in a very straightforward way. Possibilities are numerous. In principle any filter with a *linear time-invariant state-space* representation is valid for the analysis, since it can be put into the required form, see Whittle (1983) for instance.

Intuition for the Univariate Case Large part of our analysis can be cast into the univariate framework, which is useful for the intuition.

First, due to symmetry of the autocovariance generating function, we can rewrite the univariate spectral density as

$$f_y(\omega) = \frac{1}{2\pi} \left[\gamma_0 + 2 \sum_{k=1}^{\infty} \gamma_k \cos(\omega k) \right], \quad (6.15)$$

which shows that the spectrum is a weighted combination of sinusoids. For $k = 0$ the area under the spectrum is equal to variance of the process. Hence, we can view the spectrum as a distribution of the variance among the particular frequencies that are behind the variance. We can thus tell what part of variance is due to high-frequency movements in the data or cycles with longer periods.

Applying the univariate filter $h(z) = \sum_{k=-\infty}^{\infty} h_k z^k$ to the univariate process x_t results in a new process y_t and it is easy to see that

$$y_t = h(L)x_t \quad f_y(\omega) = |h(e^{-i\omega})|^2 f_x(\omega), \quad (6.16)$$

where $|h(e^{-i\omega})|^2$ denotes the *power transfer function* of the filter.

Thus, the power transfer function is the key category of our interest in the paper, since we investigate the relation of the spectral density of one variable –in our case the growth rate of a series, for instance– to the spectral density of another one, that is the result of applying linear filters – in our case the HP filtered level of the log series, for instance.

6.2 More Detailed Results for H-P and BP Filter

In our view there are two filters most often used for stochastic detrending in applied economics and DSGE models framework. These are the Hodrick-Prescott filter and a variant of a band-pass filter. Due to their popularity and their intuitive formulation we investigate in detail the effects of detrending with these two filters. In particular how the power transfer function looks like.

6.2.1 Effects of Hodrick-Prescott Filter Detrending

The Hodrick-Prescott filter is popular and well-known. The frequency-domain analysis is carried out e.g. in King and Rebelo (1993).

The Hodrick-Prescott filter is formulated as

$$\min_{\{\tau_t\}} \sum_{t=1}^T (y_t - \tau_t) + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2, \quad (6.17)$$

where τ_t is defined as the trend component of the y_t series and $c_t = y_t - \tau_t$ is defined as the cyclical component of the series. The parameter λ sets the trade-off between the goodness of fit criterion and the smoothness criterion of the H-P filter.

The first order conditions with respect to $\{\tau_t\}_{t=1}^T$ lead to system of linear equations that is easy to solve. Note that λ and T fully determine the weights of the filter. Thus, the filter is linear, but it is not time invariant, since its weights are dependent upon T .

Following King and Rebelo (1993) we can formulate the doubly-infinite time variant of Hodrick-Prescott filter, which is the linear time invariant filter. For large amount of the data the weights in the middle of the sample are identical or close to ideal doubly-infinite H-P filter.

The doubly-infinite time H-P filter can be represented as two-sided moving average filter of the form (6.13), i.e. $\tau_t = C^T(L)y_t$. Since we are interested in the cyclical component c_t we are interested in the filter $c_t = (1 - C^T(L))y_t = C(L)y_t$.

It can be shown that the power transfer function of the cyclical H-P filter is

$$|C(\omega)|^2 = \left| \frac{4\lambda [1 - \cos(\omega)]^2}{4\lambda [1 - \cos(\omega)]^2 + 1} \right|^2. \quad (6.18)$$

Suppose we apply the doubly-infinite H-P filter to series x_t with the pseudo-spectrum $f_x(\omega)$. It follows from (6.16) and (6.18) what the spectrum of the new process (series) y_t is. In particular (6.18) determines what portion of variance are to be eliminated or re-weighted.

Effects of Detrending Let us turn to the exercise behind arguments of the main text. Assume that from a well-specified model we know the spectrum of the process $x_t = (1 - L)u_t = \Delta u_t$, say u_t being the logarithm of private consumption in real terms. We are interested in properties of $y_t = C(L)u_t$, that is we are interested in cyclical component (gap) of the u_t obtained by applying the Hodrick-Prescott filter to u_t and calculating cyclical component.

Combining integration filter with the H-P filter we get in our simple univariate example

$$f_y(\omega) = |F_{HP}(\omega)|^2 f_x(\omega), \quad (6.19)$$

where the power transfer and transfer functions of the filter F are

$$|F_{HP}(\omega)|^2 = \frac{8\lambda^2 [1 - \cos(\omega)]^3}{[4\lambda [1 - \cos(\omega)]^2 + 1]^2} \quad F_{HP}(\omega) = \frac{\lambda e^{2i\omega} (1 - e^{-i\omega})^3}{\lambda e^{2i\omega} (1 - e^{-i\omega})^4 + 1}. \quad (6.20)$$

for $\omega \neq 0$.

As an interesting side comment we may note that the filter $F_{HP}(\omega)$ was already discussed in the literature, albeit *in a very different context from ours*.

Cogley and Nason (1995) criticise the Hodrick-Prescott filter of creating spurious peaks at business cycle frequencies. Their conclusions are based on the argument that applying the H-P filter to non-stationary I(1) series amounts to differencing of the series first and then applying the asymmetric filter $C_1(\omega)$ such that $C(\omega) = (1 - e^{-i\omega})C_1(\omega)$.

Since it is obvious that $F_{HP}(\omega) = C_1(\omega)$ the factorisation done by Cogley and Nason (1995) receives a new interpretation in our framework. See Pedersen (2001) or Valle e Azavedo (2007), inter alia, for criticism of the approach of Cogley and Nason (1995).

6.2.2 Effects of Band-Pass Filter Detrending

The discussion of the ideal band-pass filter detrending is similar to a previous discussion of the H-P filter.

The *ideal band-pass* filter is defined using the power transfer function $|F_{BP}(\omega)|$ such that for $\omega \in [0, \pi]$

$$|F_{BP}(\omega)| = \begin{cases} 1 & \omega \in [\omega_L, \omega_U] \\ 0 & \text{otherwise.} \end{cases} \quad (6.21)$$

The ideal band-pass filter can also be represented in the form of (6.16). The ideal band-pass filter requires a doubly-infinite amount of the data, hence in practice variations of the approximate band-pass filters are used.

Effects of Detrending Having the same setup as above, i.e. $x_t = (1 - L)u_t = \Delta u_t$, we are interested in properties of $y_t = C_{BP}(L)u_t$, where $C_{BP}(L)$ is time-domain representation of ideal band-pass filter.

The calculations yield

$$f_y(\omega) = |F_{BP}(\omega)|^2 f_x(\omega), \quad (6.22)$$

with

$$|F_{BP}(\omega)| = \begin{cases} 1/[2(1 - \cos(\omega))] & \omega \in [\pi_L, \pi_U] \\ 0 & \text{otherwise.} \end{cases} \quad (6.23)$$

6.2.3 How We Tested Our Calculations

To test the calculation of the contribution of a group of shocks to the spectral density and covariance structure of the filtered cyclical variables we carried out simple simulation exercises.

For a particular model we simulated a path of $N = 1000$ observations using random draw of structural innovations from their distributions. For variables of interest we constructed (log) levels using the inverse of the stationarity-inducing transform (i.e. differences or ratios). In the sequel we applied a detrending filter of interest (Hodrick-Prescott or truncated band-pass filter).

To alleviate the end-point bias of approximate filters we cut-off 50 periods from each side of the sample to yield results comparable with the population counterparts of the filters. Then we used parametric methods to calculate moments and spectral properties of the data. In particular, we fitted $AR(p)$ or $VAR(p)$ models for several values of p

The resulting moments and spectral densities were then compared to population results. This simple simulation exercise suggest that direct use of frequency-domain approach is more precise and less sensitive to the method of moments and spectral densities calculations. In the parametric case it is the choice of lag length p , in case of non-parametric calculation it is the choice of the type of smoothing kernel. *The simulation approach thus does not reject the direct calculations of the paper.*

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