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Economies of Scale in Banking, Confidence Shocks, and Business Cycles*

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Abstract

This paper quantitatively investigates equilibrium indeterminacy due to economies of scale (ES) in financial intermediation. Financial intermediation provides deposits (inside money) which can substitute with currency to purchase consumption, and depositing decisions are susceptible to non-fundamental confidence (sunspot) shocks. With the intermediation sector calibrated to match US data: (i) indeterminacy arises for small degrees of ES; (ii) sunspot shocks qualitatively resemble monetary shocks; and (iii) monetary policies can stabilize the real impact of sunspot shocks, but only under complete information. The analysis also assesses the removal of these shocks on the volatility decline observed during the US Great Moderation.

Keywords: Financial Intermediation, Inside Money, Indeterminacy, Business Cycles

JEL: C68, E32, E44

1. Introduction

What are the conditions through which economies of scale (ES) in financial intermediation gives rise to equilibrium indeterminacy? When these conditions are satisfied, what is the quantitative importance of non-fundamental shocks to confidence in the financial sector on economic volatility? These questions are motivated by two distinct literatures: the literature on increasing returns to scale in production where indeterminacy delivers belief-induced business cycles (e.g. Benhabib and Farmer, 1994), and the literature on banking crises where a strategic complementarity in intermediation delivers multiple (steady state) equilibria (e.g. Cooper and Corbae, 2002). In contrast to the literature on indeterminacy from the production sector, this analysis builds on the empirical evidence of Hughes and Mester (1998) that banks of all sizes exhibit significant economies of scale.¹ In contrast to the literature on banking crises, the potential for indeterminacy around a single steady state is quantitatively assessed in an otherwise standard, business-cycle environment. By combining elements of both literatures, this paper is a first attempt at quantitatively examining the significance of banking as an avenue through which agents' beliefs (or *animal spirits*) can be a source of business-cycle fluctuations.

To illustrate how ES can deliver equilibrium indeterminacy, suppose intermediaries possess decreasing marginal costs of managing deposits and pass these costs onto depositors. ES will distort the depositing decisions of the household and provide the basis for a strategic complementarity in the households' depositing decisions. If a single household believes that the deposit holdings of other households will decrease (increase), then her anticipated cost of holding deposits increase (decrease) and she will hold less (more) deposits. The sunspot shocks resulting from this strategic complementarity, which can be interpreted as self-fulfilling, belief-driven shocks to confidence in the intermediary, will influence the composition of inside (deposit) and outside (currency) money holdings, aggregate prices, and

¹See Berger and Mester (1999) for references both in support and in contrast to this result. More recently, Wang (2003) and Allen and Liu (2007) have uncovered modest ES in intermediation.

potentially real quantities.

The intermediation technology described above is explored in an environment featuring multiple mediums of exchange (currency and deposits) as in Freeman and Kydland (2000), financial intermediaries as in Corbae and Dressler (2009), and nominal wage rigidity. Both currency and deposits can be used to purchase consumption, and financial intermediaries provide the necessary inside monetary component. When banks give rise to equilibrium indeterminacy, sunspot shocks have a nominal impact because they influence broad monetary aggregates which in turn influence aggregate prices. Nominal wage rigidity links these shocks to the real economy and delivers business-cycle fluctuations.

The results of the analysis are as follows. First, with the size of the intermediary sector calibrated to US data (a value added of approximately 1 percent of output), indeterminacy arises for small degrees of ES in the intermediation sector without the need for multiple productive sectors or unusual parameter values.² Second, since sunspot shocks and monetary shocks both influence the trade off between inside and outside money balances, the real response of these shocks are qualitatively similar. Third, monetary policies are able to assuage the real (but not nominal) impact of sunspot shocks, but only when the monetary authority has complete information. In other words, the monetary authority can offset the real impact of a confidence shock when the shock can be fully observed. When the confidence shock is observed with a lag, the ability of the monetary authority to stabilize the real economy is severely hindered.

The results presented in this paper can be considered a deeper, quantitative analysis of the qualitative conclusions proved in Dressler (2009a). Using a simple monetary environment along the lines of Carlstrom and Feurst (2001), the main results of Dressler (2009a) was that equilibrium indeterminacy does not depend on a large degree of ES in intermediation nor a large intermediation sector. What indeterminacy does depend on is monetary policy and

²The degree of increasing returns needed for indeterminacy in environments with one sector of production [e.g. Farmer and Guo (1994) and Gali (1994)] far exceeds the estimates of Basu and Fernald (1997). Furthermore, Farmer (1993) shows that indeterminacy arises in cash-in-advance economies only for weak degrees of intertemporal substitution.

the determination of nominal interest rates. In other words, models with multiple mediums of exchange must have equal margin costs of using each medium in equilibrium. The cost of deposit use is the potential source of indeterminacy in the environment, and the cost of currency is the nominal interest rate. Therefore, the determination of nominal interest rates becomes a crucial ingredient in the environment. If the monetary authority chooses to not target nominal interest rates and allows them to be influenced by changes in deposit costs, then indeterminacy from financial intermediation manifests itself as an indeterminacy in the nominal interest rate path. When the monetary authority targets the nominal rate, as in following a backward-looking Taylor (1993)-type rule, the nominal rate (and the cost of deposits) are realized and indeterminacy fails to arise for any degree of ES.

Given these previously established results, the present analysis restricts attention to monetary policies which are either exogenous or target money growth rates similar to McCallum (1993)-type rules, and quantitatively explores the degree of ES necessary for indeterminacy and the quantitative importance of the resulting non-fundamental shocks. The impact of these shocks are quantified through a calibration exercise using the 1982 adoption of a nominal interest rate targeting rule by the US as a natural experiment (see Meulendyke, 1989, ch 2). Prior to 1982, indeterminacy from the financial intermediation sector could impact economic volatility, but not after. Since this policy change occurred around the time the US observed a large decline in economic volatility (termed the Great Moderation), the exercise can assess a consequence of this policy change that has not been previously considered. Depending on the degree of nominal wage rigidity, the model accounts for up to 10 percent of the decline in output volatility that Stock and Watson (2003) were unable to explicitly account for in their assessment of the Great Moderation and attributed to “other unknown forms of good luck”.

The paper is organized as follows. Section 2 presents the model. Section 3 analyzes the model dynamics. Section 4 concludes.

2. The Model and Equilibrium

2.1. The Model

Time is discrete and the horizon is infinite. The economy is populated by a continuum of households indexed by $i \in [0, 1]$ which supply differentiated labor, a continuum of industries which produce differentiated goods indexed by $j \in [0, 1]$ with a large number of perfectly-competitive firms operating in each industry, a financial intermediary, and a monetary authority. Each of these agents are described in turn.

Households

The preferences of household i are given by

$$(1) \quad E_0 \sum_{t=0}^{\infty} \beta^t u [c^i(t), h^i(t)],$$

where $c^i(t)$ is a composite consumption good, $h^i(t) = \int_0^1 h_j^i(t) dj$ is labor supply across industries, and $\beta \in (0, 1)$ is the discount rate. Composite consumption of household i is an (Armington) aggregate of differentiated goods given by the CES specification

$$(2) \quad c^i(t) = \left[\int_0^1 \varphi_j^{\frac{1}{\varpi}} c_j^i(t)^{\frac{\varpi-1}{\varpi}} dj \right]^{\frac{\varpi}{\varpi-1}},$$

where $c_j^i(t)$ denotes household i 's consumption of good j , φ_j denotes the weight associated with good j , and ϖ denotes the elasticity of substitution across consumption goods.

Household i begins period t with amounts of physical capital $k^i(t)$ and nominal currency $m^i(t)$. Every household receives an identical lump-sum transfer $T(t)$ of currency from the monetary authority, and buys and sells nominal bonds $B^i(t)$ which are zero in net supply (across locations) and earn a gross nominal return $1 + R(t)$. The household then deposits $d^i(t)$ of its capital into a financial intermediary earning a gross real return $r_d(t)$, and lends $a^i(t)$ directly to firms earning a gross real return $r(t)$. Therefore, $k^i(t) = a^i(t) + d^i(t)$.

Both deposits and currency can be used to purchase consumption. As in the standard cash-in-advance model, previously held currency balances can costlessly purchase consumption goods. Deposits are chosen at the beginning of the period and pay interest, but bear a fixed real cost γ for each consumption good purchased. This cost can be interpreted as a per-check processing cost.

The use of money balances deliver the conditions

$$(3) \quad m^i(t) + T(t) - B^i(t) \geq \int_0^1 \mathbf{1}_{Jm}(j) P_j(t) c_j^i(t) dj,$$

$$(4) \quad P_k(t) d^i(t) \geq \int_0^1 \mathbf{1}_{Jd}(j) P_j(t) c_j^i(t) dj,$$

where $P_j(t)$ denotes the price of consumption good j , $P_k(t)$ is the price of capital (and capital deposits), and Jm and Jd are subsets of $[0, 1]$ which denote the good types purchased with currency and deposits, respectively. The indicator function $\mathbf{1}_{Jm}(j)$ ($\mathbf{1}_{Jd}(j)$) equals one if a particular good of type j is a member of Jm (Jd), and equal zero otherwise.

Household i is a monopoly supplier of type- i labor which is sold to all firms. Since households substitute imperfectly for one another in production, each household sells its labor in a monopolistically-competitive market: household i sets the nominal wage $W_j^i(t)$ to a representative firm from industry j (henceforth, firm j) such that it satisfies firm j 's demand taking all prices as given. It is assumed that the household faces a quadratic cost of adjusting its nominal wage as in Rotemberg (1982),

$$\frac{\phi}{2} \left[\frac{W_j^i(t)}{\pi W_j^i(t-1)} - 1 \right]^2,$$

where ϕ governs the size of the real adjustment cost and π denotes the gross, long-run inflation rate.

The flow budget constraint of household i is given by

$$(5) \quad \int_0^1 P_j(t) c_j^i(t) dj + m^i(t+1) + P_k(t) k^i(t+1) \leq \\ \int_0^1 W_j^i(t) h_j^i(t) dj + r(t) P_k(t) a^i(t) + r_d(t) P_k(t) d^i(t) + R(t) B^i(t) + m^i(t) + T(t) \\ - P_k(t) \gamma \left(\int_0^1 \mathbf{1}_{Jd}(j) dj \right) - P_k(t) \int_0^1 \frac{\phi}{2} \left[\frac{W_j^i(t)}{\pi W_j^i(t-1)} - 1 \right]^2 dj$$

where $\gamma \left(\int_0^1 \mathbf{1}_{Jd}(j) dj \right)$ denotes the total cost of using deposits.

Production

There are j types of output produced, and there exists a large number of perfectly-competitive firms producing each type. A representative type j firm hires differentiated labor from every household i and aggregates these labor services into a homogeneous labor input $H_j(t)$ using the CES technology:

$$(6) \quad H_j(t) = \left(\int_0^1 h_j^i(t)^{\frac{\xi-1}{\xi}} di \right)^{\frac{\xi}{\xi-1}},$$

where ξ denotes the elasticity of substitution between labor types.³

The production technology for type j output is a CRS function of capital and aggregate labor: $y_j(t) = f(z(t), K_j(t), H_j(t))$, where $z(t)$ denotes the exogenous level of total factor productivity identical across firms and evolves according to $z(t) = \kappa_z + \rho_z z(t-1) + \varepsilon_z(t)$ with $\varepsilon_z(t) \sim N(0, \sigma_z^2)$. Profits of a representative type j firm are given by

$$(7) \quad P_j(t) y_j(t) + (1 - \delta - r(t)) P_k(t) K_j(t) - \int_0^1 W_j^i(t) h_j^i(t) di,$$

where $P_j(t)$ is taken as given.

³One could easily establish an equivalent environment where an additional production sector aggregates heterogeneous labor units and sells homogeneous labor units to good producing firms as in Erceg et al. (2000). Allowing each firm to hire heterogeneous labor is employed here simply to streamline the environment.

Financial Intermediaries

The financial intermediary accepts capital deposits from households and grants capital loans to firms. As discussed above, the benefit to saving in the form of deposits rather than currency is the earned interest r_d , while the benefit to deposits relative to direct capital investment is that deposits can purchase consumption (for a processing cost γ).

It is assumed that intermediaries have no minimum reserve requirements and therefore lend all of their deposits.⁴ The profit function of an intermediary can be expressed as

$$(8) \quad r(t) D(t) - r_d(t) D(t) - C(D(t)),$$

where $D(t)$ denotes aggregate real deposits, and $C(D(t))$ denotes real operating costs.⁵ If these costs are marginally decreasing, the intermediary will exhibit ES. Assuming $C(D(t)) = \Gamma D(t)^{1+\theta}$, ES arises for any $\theta \in [-1, 0)$.

There is free entry into intermediation so that if the incumbent intermediary receives positive profits, another intermediary can enter and capture the market. Therefore, zero profits in (8) generates the following average-cost pricing rule

$$(9) \quad r_d(t) = r(t) - \tau(t),$$

where $\tau(t) = C(D(t))/D(t) = \Gamma D(t)^\theta$ denotes average (per deposit) operating costs.⁶

The Monetary Authority

The budget constraint of the monetary authority is given by $T(t) = M(t+1) - M(t)$, where $M(t+1)$ denotes the aggregate stock of currency (the monetary base) available at

⁴The results presented below are relatively unchanged if the intermediary is required to keep a minimum, fixed percentage of deposits in reserves.

⁵Since the processing costs γ are passed on to the households [see equation (5)] they do not appear in (8).

⁶Of course, other decentralizations exist such as nonlinear pricing as in Cooper and Corbae (2002). This would greatly complicate the analysis.

each location at the end of period t . The currency base evolves according to $M(t+1) = \mu(t)M(t)$ where $\mu(t)$ denotes the gross growth rate.

The analysis considers several cases of how the monetary authority chooses $\mu(t)$. The benchmark case considers exogenous monetary policy where money growth evolves according to

$$(10) \quad \mu(t) = \kappa_\mu + \rho_\mu \mu(t-1) + \varepsilon_\mu(t),$$

where $\varepsilon_\mu(t) \sim N(0, \sigma_\mu^2)$. Additional cases consider variations of a money growth rule,

$$(11) \quad \mu(t) = E \left[\mu \left(\frac{\pi(t)}{\pi} \right)^\nu \mid \Omega(t) \right],$$

where $\pi(t) = P(t)/P(t-1)$, $P(t)$ denotes the aggregate price level (defined below), and variables without a time subscript denote their long-run (steady state) values. By considering alternative values for the inflation elasticity, as well as changing the information available to the monetary authority ($\Omega(t)$), the model can examine the full interaction of monetary policy and indeterminacy.

2.2. Equilibrium

Firm j 's Problem

Since households choose $W_j^i(t)$ taking firm j 's demand as given, it is beneficial to establish the model equilibrium by first outlining the firm's problem.

A representative type j firm chooses $K_j(t)$ and $h_j^i(t) \forall i$ in order to maximize profits (7) subject to (6). A profit-maximizing firm equates the marginal product of each input with

its marginal cost.

$$(12) \quad f_{K_j}(z(t), K_j(t), H_j(t)) = (r(t) - 1 + \delta) \frac{P_k(t)}{P_j(t)}$$

$$(13) \quad f_{H_j}(z(t), K_j(t), H_j(t)) P_j(t) = \left(\frac{h_j^i(t)}{H_j(t)} \right)^{\frac{1}{\xi}} W_j^i(t), \quad \forall i$$

Defining the left-hand side of (13) to be firm j 's nominal wage index $W_j(t)$ illustrates firm j 's demand for type i 's labor,

$$(14) \quad h_j^i(t) = \left(\frac{W_j(t)}{W_j^i(t)} \right)^{\xi} H_j(t),$$

which is a typical result of models featuring nominal-wage rigidity (e.g. Erceg et al., 2000).

Household i 's Problem

Household i 's problem is to maximize (1) subject to (3), (4), (5) and (14) by choosing $c_j^i(t) \forall j \in Jm$, $c_{j'}^i(t) \forall j' \in Jd$, $m^i(t)$, $a^i(t)$, $d^i(t)$, $B^i(t)$, and $W_j^i(t) \forall j$ taking all prices as given. This problem can be simplified by solving an equivalent problem where household i chooses composite consumption $c^i(t)$ taking as given a composite price (or price index) $P(t)$. Explicitly, household i 's problem is equivalent to maximizing the expected discounted value of (1) subject to

$$(15) \quad m^i(t) + T(t) - B^i(t) \geq P(t) c^i(t) \int_0^1 \mathbf{1}_{Jm}(j) \varphi_j dj,$$

$$(16) \quad P_k(t) d^i(t) \geq P(t) c^i(t) \int_0^1 \mathbf{1}_{Jd}(j) \varphi_j dj,$$

and (5) with $\int_0^1 P_j(t) c_j^i(t) dj$ replaced by $P(t) c^i(t)$. The indicator functions and weights φ_j in (15) and (16) keep track of consumption purchased with currency and deposits. While leaving the details to an appendix, the first order conditions of this equivalent problem can be used to define the price index as a function of weighted good prices, as well as a demand

for type j consumption given the elasticity of substitution ϖ .

$$(17) \quad P(t) = \left[\int_0^1 \varphi_j P_j(t)^{1-\varpi} \right]^{\frac{1}{1-\varpi}}$$

$$(18) \quad c_j^i(t) = \left(\frac{P(t)}{P_j(t)} \right)^{\varpi} \varphi_j c^i(t)$$

In order to simplify the problem, it is assumed that the differentiated consumption goods are perfect complements (i.e. $\omega \rightarrow 0$) and the consumption weights φ_j are chosen to deliver an ordinal ranking of consumption good types. Letting $\varphi_j = 2j$ so $\int_0^1 \varphi_j dj = 1$, equations (17) and (18) become

$$(19) \quad P(t) = \int_0^1 (2j) P_j(t) dj,$$

$$(20) \quad c_j^i(t) = (2j) c^i(t).$$

These assumptions result in the price index being a weighting of differentiated prices, and the demand for each good is its weighted contribution to total consumption. Note the smaller the value of j , the smaller the contribution good $c_j^i(t)$ is to composite consumption $c^i(t)$.

The final step in characterizing household i 's problem is to address the following question: is it more attractive to purchase $c_j^i(t)$ with currency or deposits? If a household purchases $c_j^i(t)$ with deposits, the real end-of-period cost is given by

$$[(1 + r(t) - r_d(t)) c_j^i(t) + \gamma] / r(t),$$

which illustrates that in addition to purchasing the good, the household gives up the interest spread $(r(t) - r_d(t))$ by depositing (instead of directly investing) capital and pays the check processing cost. If a household purchases $c_j^i(t)$ with currency, the real cost is given by

$$P_j(t) c_j^i(t) / P_j(t-1),$$

which illustrates that the needed currency was acquired last period. Substitution of (9) and (20) into these costs and comparing them results in

$$(21) \quad \left[1 + \tau(t) + \frac{\gamma}{2j c^i(t)} \right] r(t)^{-1} \begin{array}{l} \geq \\ \leq \end{array} \frac{P(t)}{P(t-1)}, \quad \forall j \in [0, 1],$$

where $P_j(t)$ is replaced with $P(t)$. Equation (21) illustrates that the cost to using deposits approaches infinity as j approaches zero (all else equal). In other words, some consumption goods are purchased in quantities small enough such that the cost to using deposits (γ) outweighs the return. Therefore, define $j^{*i}(t)$ to be a critical good type such that household i is indifferent to purchasing $c_{j^*}^i(t)$ with either currency or deposits because they share the same cost,

$$(22) \quad \left[1 + \tau(t) + \frac{\gamma}{2j^{*i}(t) c^i(t)} \right] r(t)^{-1} = \frac{P(t)}{P(t-1)},$$

and all goods j above (below) $j^{*i}(t)$ will be purchased with deposits (currency). This delivers the subsets $Jm = [0, j^{*i}(t)]$ and $Jd = [j^{*i}(t), 1]$.

Household i 's constraint set can now be stated in terms of composite consumption $c^i(t)$, the price index $P(t)$ and the critical good index $j^{*i}(t)$,

$$(23) \quad \frac{m^i(t) + T(t) - B^i(t)}{P(t)} \geq j^{*i}(t)^2 c^i(t),$$

$$(24) \quad d^i(t) \geq (1 - j^{*i}(t)^2) c^i(t),$$

and

$$(25) \quad c^i(t) + \frac{m^i(t+1)}{P(t)} + k^i(t+1) \leq \int_0^1 \frac{W_j^i(t)}{P(t)} h_j^i(t) dj + r(t) [k^i(t) - d^i(t)] + r_d(t) d^i(t) + \frac{m^i(t) + T(t) + R(t) B^i(t)}{P(t)} - \gamma(1 - j^{*i}(t)) - \int_0^1 \frac{\phi}{2} \left[\frac{W_j^i(t)}{\pi W_j^i(t-1)} - 1 \right]^2 dj$$

where $d^i(t) = k^i(t) - d^i(t)$. Since composite consumption can be transformed into units of investment, $P_k(t) = P(t)$.

Household i chooses $c^i(t)$, $j^{*i}(t)$, $B^i(t)$, $d^i(t)$, $m^i(t+1)$, $k^i(t+1)$, and $W_j^i(t)$, $\forall j$ in order to maximize (1) subject to (14), (23), (24), and (25) taking all prices and the state of the economy as given. The transformation of the generalized problem to the simplified problem, as well as illustrating that (22) can be derived from the first-order conditions of the household's optimization problem are detailed in an appendix.

Market Clearing and Definition of Equilibrium

Given that all households face the same elasticity regarding their labor demand (ξ), and all firms are perfectly competitive within their respective industry, we can restrict attention to symmetric labor and goods market equilibria and treat household i as a representative household and firm j as a representative firm. Therefore, $W_j^i(t) = W_j(t) = W(t)$, $h_j^i(t) = h_j(t) = h(t)$, and $c^i(t) = c(t)$.

Goods market clearing is given by

$$(26) \quad Y(t) = C(t) + I(t) + \Gamma + \gamma(1 - J^*(t)) + \frac{\phi}{2} \left[\frac{W(t)}{\pi W(t-1)} - 1 \right]^2,$$

where $Y(t) = \int_0^1 (2j) y_j(t) dj$ conforms with composite consumption, and $I(t) = K(t+1) - (1 - \delta)K(t)$ denotes aggregate investment. Equation (26) states that aggregate output is distributed amongst consumption, investment, and aggregate financial and wage adjustment

costs. Capital market clearing is given by $K(t) = k(t)$.

Currency market clearing is given by $M(t) = m(t)$. A broader monetary aggregate ($M1(t)$) is defined as the nominal sum of currency and deposits,

$$(27) \quad M1(t) = M(t) + P(t)D(t) = M(t) \left(1 + \frac{P(t)D(t)}{M(t)} \right),$$

where the third equality defines M1 as the product of the currency base and the endogenously determined money multiplier. Zero-net supply in the bond market results in $B(t) = 0$.

The decision rules of the households, firms, and pricing functions can now be defined as functions of $k(t)$, $W(t-1)$, $\mu(t)$ (exogenous or endogenous), and the fundamental shock $z(t)$. When ES in the financial intermediary sector delivers equilibrium indeterminacy, it is assumed that agents also base their decisions upon observing a non-fundamental sunspot or confidence shock $\zeta(t)$. Therefore, for all $\{k(t), W(t-1), \mu(t), z(t), \zeta(t)\}$, an equilibrium is defined as a list of prices $\{P(t), r(t), r_d(t), W(t), R(t)\}$ and allocations $\{k(t+1), m(t+1), h(t), c(t), j^*(t), d(t), B(t)\}$ such that: (i) households maximize (1) subject to (23), (24), and (25), (ii) firms maximize profits, (iii) labor demand is determined by (14), (iv) the markets for goods (26), currency, bonds, and deposits ($D(t) = d(t)$) clear, and (v) $\tau(t) = \Gamma d(t)^\theta$.

3. Quantitative Analysis

The quantitative analysis begins with stating the functional form assumptions, model calibration, and alternative monetary policies used throughout the analysis. A search is then conducted over a subset of the parameter space for zones where the model dynamics are either determinate (unique) or indeterminate, and the dynamic properties of the model under indeterminacy are analyzed. The section concludes with a calibration exercise to assess the elimination of the non-fundamental shocks on the observed decline in US volatility during the 1980s (the Great Moderation).

3.1. Functional Forms and Calibration

The functional forms and parameter values are determined according to the business-cycle literature (e.g. Cooley and Hansen, 1989) and so the resulting steady state of the model matches particular long-run properties of the US economy.

The money growth rate $(\mu - 1)$ is set to 3 percent annually, and the discount parameter β is calibrated to 0.99 so the annual real interest rate is roughly 4 percent.

Investment is one quarter of steady state output. With a 10 percent depreciation rate, the capital stock to annual output ratio is 2.5. The production function is assumed to be $y = zk^\alpha h^{1-\alpha}$, and α is calibrated so labor's share of national income is roughly two-thirds. The parameters governing the evolution of technology shocks (ρ_z, σ_z) are respectively set to 0.95 and 0.0076 as in Prescott (1986).

The utility function is assumed to be $[c^\eta (1 - h)^{1-\eta}]^{1-V} / (1 - V)$. The parameter η is calibrated so a household's average allocation of time to market activity (net of sleep and personal care) is one-third which is in line with estimates of Ghez and Becker (1975). V is set to 2 which is within the range of results reported by Neely et al. (2001).

The parameter ξ is calibrated so the average mark-up of type i labor is five percent as in post-war US data (see Christiano et al., 2005). The cost parameter governing nominal wage changes (ϕ) corresponds to an average wage duration of 3 quarters.⁷

The benchmark model assumes exogenous monetary policy given by (10) with ρ_μ and σ_μ respectively set to 0.32 and 0.0038 as in Christiano (1991) and Fuerst (1992). Under endogenous monetary policy (11), two cases are considered where the elasticity of the money growth rate to observed changes in inflation $(\frac{\nu}{1-\nu})$ is set to -0.5 and -0.999 , respectively.⁸ A third case sets elasticity at -0.5 and assumes that the monetary authority does not initially observe the sunspot shock $(\zeta(t) \notin \Omega(t))$.

Three parameters remaining to be determined are the check-writing cost (γ) , and the

⁷See Chugh (2006) for a mapping from Rotemberg-style costs to Calvo-style rigidity.

⁸The normalized monetary policy rule is $\hat{\mu}(t) = \frac{\nu}{1-\nu} \hat{\pi}(t)$, where a hat refers to percentage changes and the elasticity is given by $\frac{\nu}{1-\nu}$. Therefore, an elasticity of -0.5 (-0.999) results in $\nu = -1$ (-1000).

parameters defining the cost of managing deposits (Γ and θ). Since θ is central to indeterminacy of equilibria in the model, it is treated as a free parameter and analyzed in the following section. The remaining two parameters are pinned down so the model's steady state matches the US deposit-currency ratio and the value added of the financial intermediation sector. The deposit-currency ratio is defined as dP/m and set to 7. This ratio is close to the post-war minimum considering that two-thirds to three-quarters of the US currency base is held abroad (see Porter and Judson, 1996).⁹ Value added is defined as total banking costs per unit of output ($[\tau d + \gamma(1 - j^*)]/y$), and serves as a proxy for the size of the intermediation sector. Diaz-Gimenez et al. (1992) compute the value added from 'banking and credit agencies other than banks' to be 1.8 to 2.7 percent of GNP for the years 1970 to 1989 (Table 3a). While this value added has undoubtedly changed over the last 18 years, it is not clear how much of this measure is represented by the simple structure of financial intermediaries in the model. Still, this information serves as an upper bound for the size of the financial intermediary sector considered here.

3.2. Economies of Scale and Indeterminacy

While a concave cost function is sufficient for banks to exhibit ES in textbook models of banking (e.g. Freixas and Rochet, 1997), it may not be sufficient for equilibrium indeterminacy in the model because the banking sector is small relative to the aggregate economy by construction. The equilibrium properties of the model over values of θ and the value added of the intermediary sector are illustrated in Figure 1. The shaded and non-shaded regions correspond to parameter values which deliver determinate and indeterminate equilibria, respectively.¹⁰ The figure illustrates that slight changes to the value added of the intermediation sector effects the minimum (absolute value) of θ required for indeterminacy.

⁹A similar measure was considered by Freeman and Kydland (2000) and Dressler (2007).

¹⁰This exercise begins with values of θ and value added (used to calibrate Γ) distributed over a fine grid. For each point in this space, the model is solved and the eigenvalues of the system are counted to determine whether the resulting dynamics of the model are either determinate or indeterminate.

Table 1: Calibrated Parameter Values

Symbol	Description	Value
α	capital's share	0.3421
β	discount factor	0.9900
δ	depreciation rate	0.0241
ς	consumption's share	0.3783
V	risk aversion	2
ξ	labor elasticity	20
ϕ	wage cost parameter	6.03
γ	check-clearing cost	$8.1481e^{-6}$
ρ_z	AR coefficient (z)	0.95
σ_z	standard deviation (z)	0.0076
ρ_μ	exog. AR coefficient (μ)	0.32^a
σ_μ	exog. standard deviation (μ)	0.0038^a
ν	endog. policy parameter	$-1, -1000^b$
θ	banking cost parameter	-0.01
Γ	banking cost parameter	$1.75e^{-2}$

Notes: ^aValues under benchmark model
^bValues under endogenous monetary policy (see text)

For value added roughly between 1.0290 and 1.1504 percent, indeterminacy arises for values of θ where marginal costs are positive and decreasing.¹¹ While the indeterminacy zone illustrated here is for the benchmark (exogenous monetary policy) case, it is identical to the models featuring endogenous monetary policy.

The quantitative analysis proceeds with a conservative degree of ES and sets $\theta = -0.01$. The minimum value added delivering indeterminacy under this degree of ES is approximately 1.15 percent, which is below the range determined by Diaz-Gimenez et al. (1992). A sensitivity analysis below considers several points within the indeterminacy zone to confirm the robustness of the model predictions.

¹¹For value added below 1.0290 percent, indeterminacy requires $\theta < -1$ and delivers negative marginal costs. Value added greater than 1.1504 percent delivers negative values for either γ or Γ . Since the deposit-currency ratio is determined in the previous section, there exists a negative relationship between the size of the bank and the parameters delivering value added.

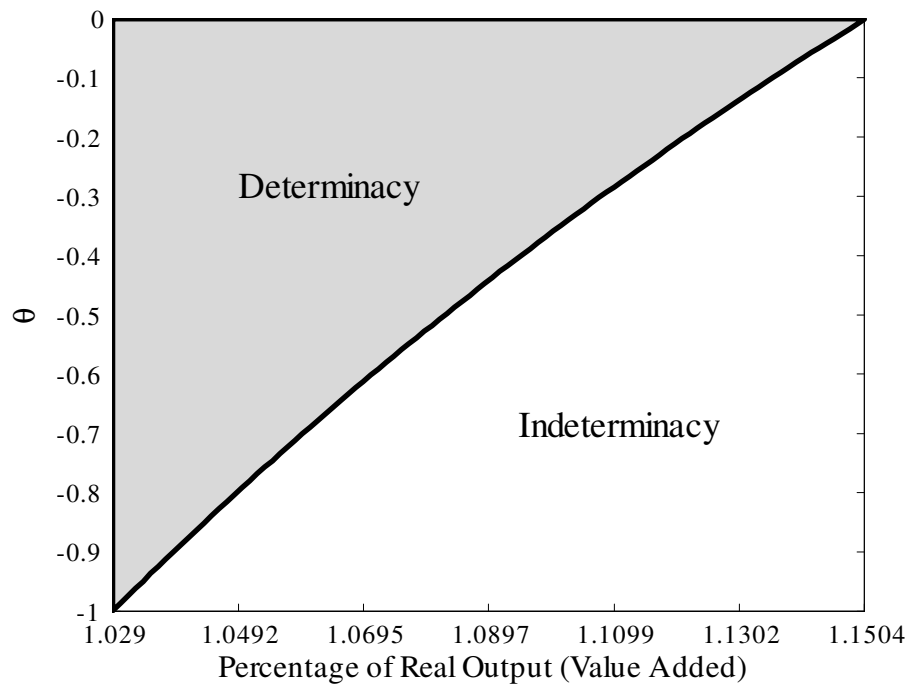


Figure 1: Pairs of value added to financial intermediation and θ which deliver indeterminate and determinate equilibria for the benchmark model.

3.3. Model Results

All versions of the model presented above are solved following the solution algorithm proposed by Lubik and Schorfheide (2003). They show that when a model exhibits indeterminacy, the rational expectations forecast errors of the economic agents can be decomposed into influences from the fundamental and non-fundamental shocks. However, while the non-fundamental shock can be interpreted as a reduced-form sunspot shock, one needs to make an additional assumption in order to uniquely identify the transmission of the fundamental shocks on the forecast errors. The analysis therefore considers both identification schemes proposed by Lubik and Schorfheide: *orthogonality* and *continuity*. Under orthogonality, the influences of the fundamental and non-fundamental shocks are uniquely identified by assuming that they are orthogonal to each other. Under continuity, the fundamental shocks are identified by imposing that their influence on the endogenous forecast errors do not abruptly change when the economy transitions from regions of determinacy to indeterminacy. The benefit of considering the continuity assumption is that the dynamics of the model in response to the fundamental shocks under indeterminacy preserves properties that the model dynamics exhibit under determinacy. Therefore, considering both identification assumptions allows the analysis to assess not only the effect of the sunspot shock on the economy, but how ES in banking influences the impact of the fundamental shocks. The solution algorithm and the use of both identification assumptions are detailed in an appendix.

Benchmark Model: Exogenous Monetary Policy

The response of the model to positive (one-percent) monetary and sunspot shocks are illustrated in Figure 2. First consider the events following a monetary shock under the continuity assumption. An injection of currency immediately increases the price level and leads to an increase in the inflation rate. The increase in inflation makes deposits more attractive than currency (i.e. $j^*(t)$ decreases), and the increase in deposit holdings results in a further increase in prices because more currency is used to purchase a smaller portion of

consumption. Nominal wage rigidity makes it costly to adjust wages as prices increase, and the decline in real wages results in increases in labor demand and all other real aggregates. In the period following the shock, prices remain above steady state along with the portion of consumption purchased with deposits (i.e. $j^*(t)$ remains below steady state). Real wages remain below steady state, so real aggregates remain above. Eventually, an increase in the demand for currency returns all nominal variables to their steady state values. Once the paths of prices and nominal wages align, the real wage and all other real aggregates return to steady state.

Under the orthogonality assumption, the initial impact to a monetary shock is qualitatively similar to the impact under continuity. Prices, M1, and $j^*(t)$ illustrate that deposits become more attractive. ES in the intermediary implies that the shift towards deposits influences the net deposit rate $(r(t) - \tau(t))$, and the initial impact of a monetary shock is diminished. In the following period, prices decline below steady state resulting in currency becoming more attractive. As households choose to hold less deposits, the net return to deposits declines. This results in a persistent shift away from deposits, illustrated by the persistent increase in $j^*(t)$ and the persistent decrease in M1. This persistence is only in nominal variables; the real economy again returns to steady state once nominal wages and prices align.

The final set of impulse responses in Figure 2 illustrates the impact of a positive one-percent sunspot shock. Quantitatively speaking, the real impact of a sunspot shock is approximately one-half the size of a monetary shock calculated under continuity and three-quarters the size under orthogonality. The reason for these similar predictions stems from the fact that both monetary and sunspot shocks impact the economy through the households' liquidity preference and the portfolio choice of cash and deposit holdings. A sunspot shock induces agents to increase deposit holdings due to a perceived decrease in the cost to intermediating assets, resulting in deposits dominating currency for a larger portion of total consumption purchases. The increase in deposit holdings results in an immediate increase in M1 and

prices, while the nominal interest rate declines. The decline in real wages again increases the demand for labor and real aggregates. In the following period, the increase in deposits keeps the net return high and delivers the persistence in Prices, M1, and $j^*(t)$. Although nominal aggregates continue to remain far from steady state, nominal wages and prices eventually align so the real economy converges to its pre-shock state.

Figure 3 illustrates the impulse responses of the economy to a positive, one-percent productivity shock under both the continuity and orthogonality assumptions. As is common in business-cycle analyses, a positive productivity shock increases real output as well as real and nominal interest rates. Deposits become more attractive than currency, so there is an endogenous increase in broad monetary aggregates. The real responses of the model under the orthogonality and continuity assumptions are roughly identical. This supports the fact that although indeterminacy is introduced through a real cost, the impact of this mechanism lies in the nominal side of the economy. For nominal variables (the bottom six panels), the continuity and orthogonality assumptions result in different impulse responses. Under orthogonality, ES in banking implies a larger increase in deposits due to decreasing marginal costs. A larger portion of total consumption is purchased with these deposits, resulting in a smaller decline in prices because the household's previously held currency balances are used to purchase a smaller portion of total consumption. Nonetheless, the nominal movements have no noticeable impact on the real aggregates under either assumption. Since the size of the financial sector is small relative to the rest of the real economy, ES in the banking sector does not add much to real shocks.

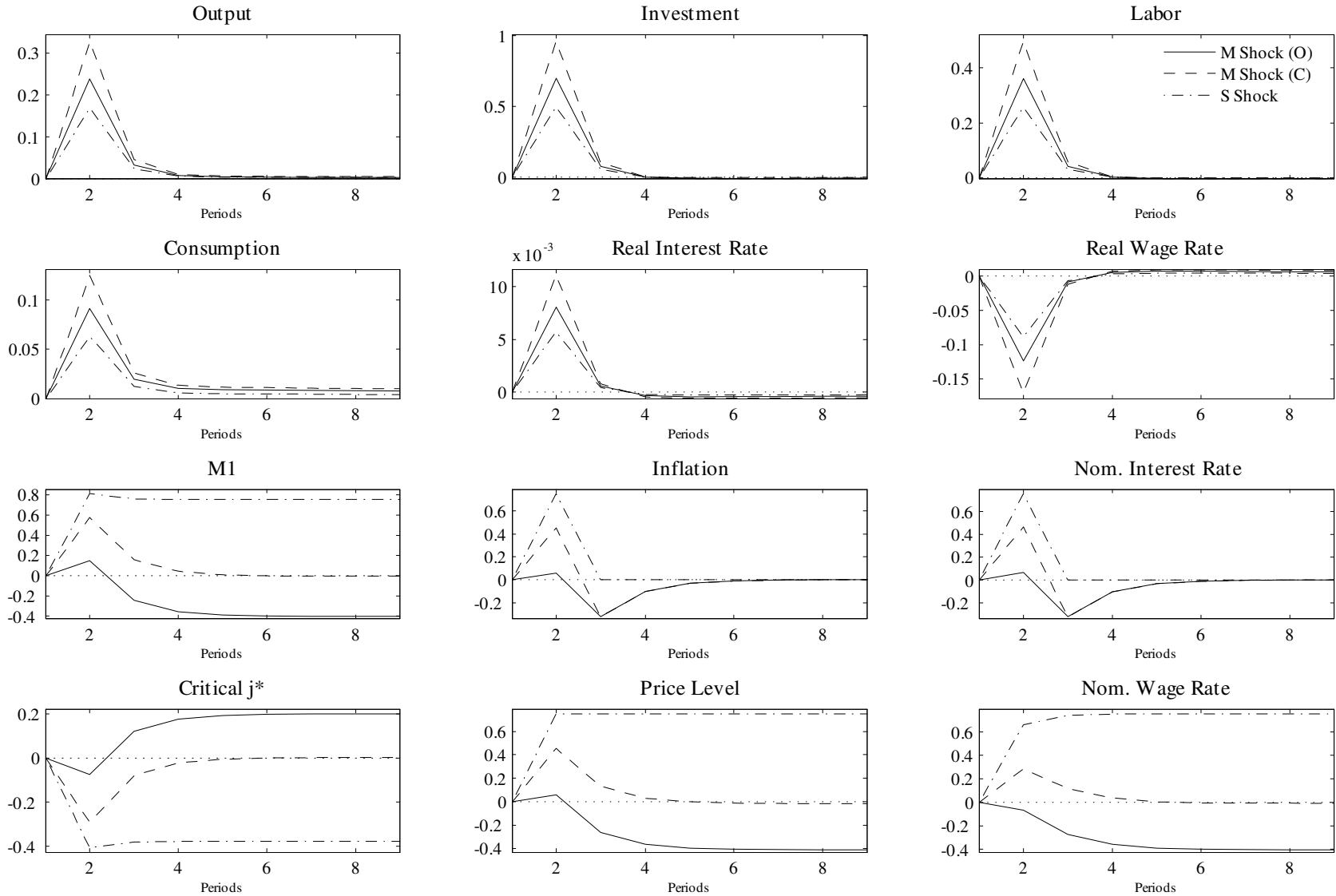


Figure 2: Impulse responses to a one percent increase in the monetary base (M Shock) and the reduced-form sunspot shock (S Shock). The Y-axes denote percentage changes from steady state. Impulse responses calculated under the orthogonality (continuity) assumption are denoted with O (C).

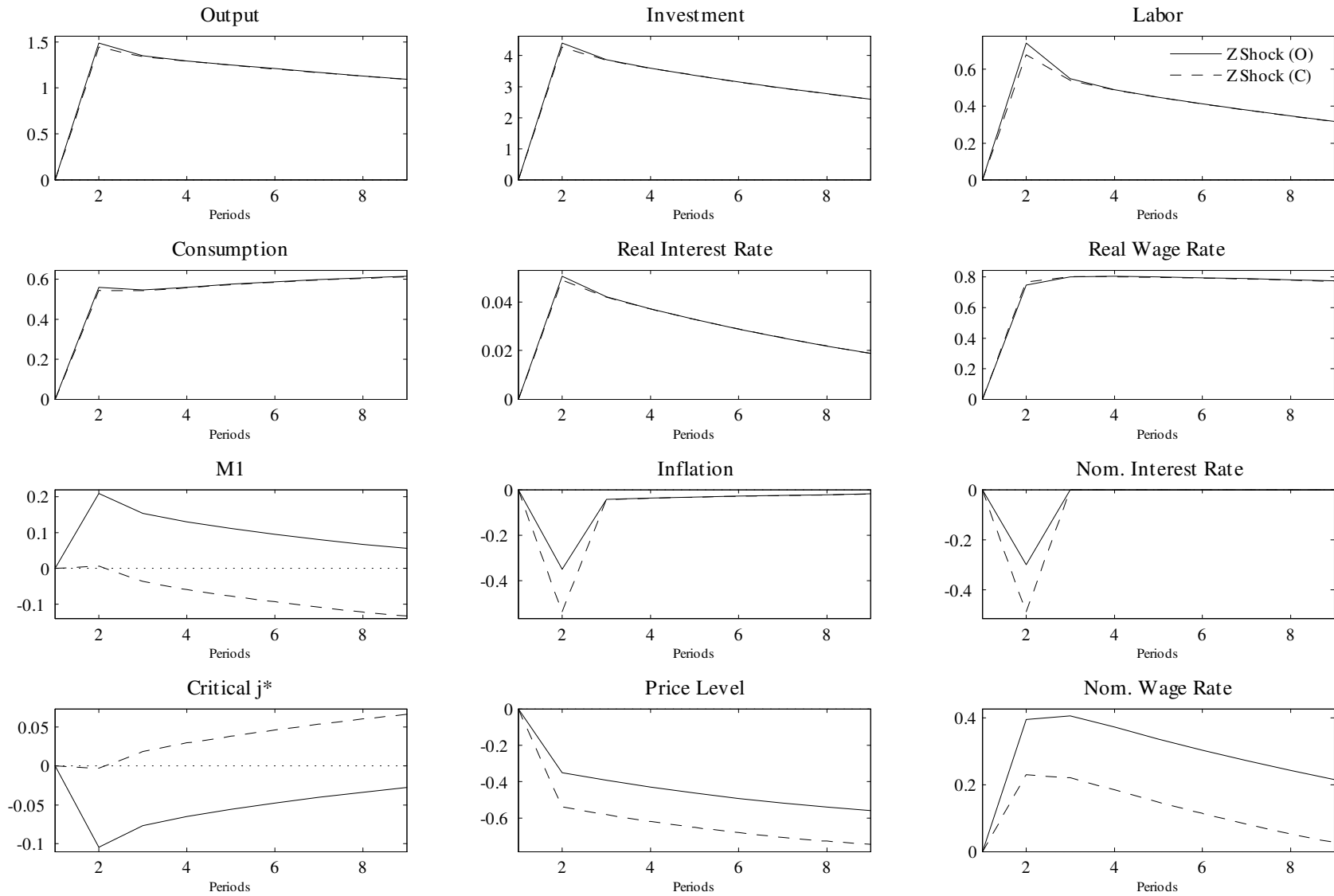


Figure 3: Impulse responses to a one percent increase in technology (Z Shock). The Y-axes denote percentage changes from steady state. Impulse responses calculated under the orthogonality (continuity) assumption are denoted with O (C).

Endogenous Monetary Policy

The dynamic responses to a sunspot shock under endogenous monetary policy are compared with the benchmark model in Figure 4.¹²

The intuition behind the policy rule (11) is straightforward: with an inflation elasticity ($\omega = \frac{\nu}{1-\nu}$) of -0.5 (-0.999), the contractionary response of the monetary authority to an observed increase in inflation is half (roughly equal to) the size of the observed increase in inflation. As the figure illustrates, an elasticity of -0.5 results in the real response of a sunspot shock to be roughly half the size of the benchmark model, while an elasticity of -0.999 results in the real response to be almost entirely diminished. These real responses can be explained by comparing the nominal responses. In the benchmark model, the real impact of a sunspot shock occurs when movements in prices and nominal wages result in a movement in real wages. When the monetary authority observes the response of prices to the sunspot shock, it alters the discrepancy between prices and nominal wages and effectively influences real wages. With an elasticity of -0.999 , the monetary authority can almost entirely stabilize real wages and eliminate the real effects of the shock.

The analysis also considers the case when the monetary authority cannot observe the sunspot shock ($\zeta(t) \notin \Omega(t)$). This case is difficult to see in Figure 4 because the response is *exactly the same* as the response under exogenous monetary policy. Since the sunspot shock has zero persistence, the inflation response is immediate and dies out in the period after the shock. Therefore, the monetary authority has nothing to observe in the period following the shock because inflation has already returned to its long-run level. In other words, the real impact has already been felt and any response would no longer be warranted.

¹²Restricting attention to sunspot shocks implies no distinction between the orthogonality or continuity assumptions - the assumption only identifies the effect of fundamental shocks.

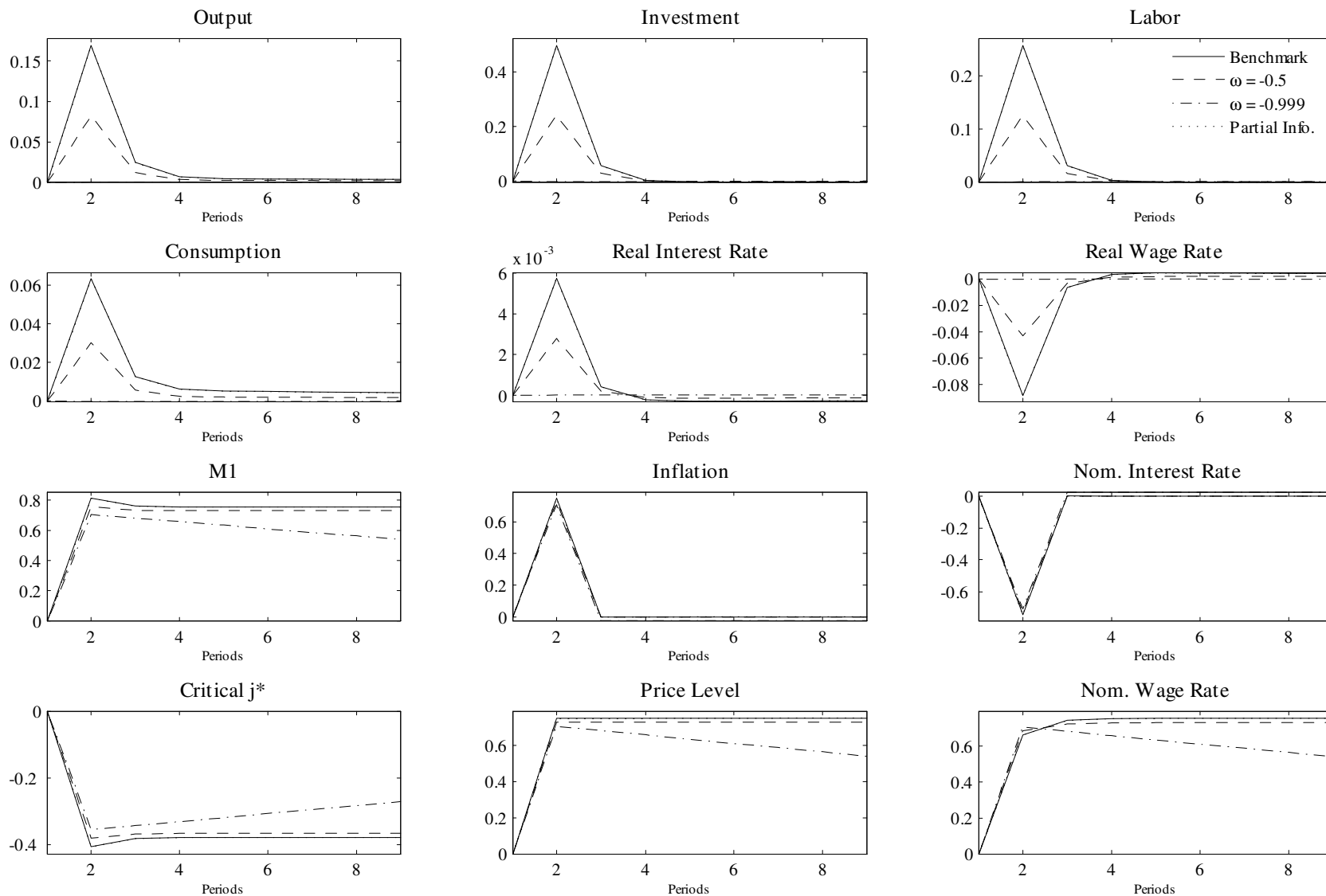


Figure 4: Impulse responses to a one percent increase in the reduced-form sunspot shock under different specifications of monetary policy. The Y-axes denote percentage changes from steady state. Benchmark refers to the model with exogenous monetary policy.

Sensitivity Analysis

This section briefly assesses the robustness of the above results to two key assumptions: the degree of ES in the intermediary (θ), and the size of the intermediation sector (quantified by value added).

To get some sense of θ from the data, taking the log of (9) delivers a regression equation,

$$(28) \quad \log(r(t) - r_d(t)) = \log(\Gamma) + \theta \log(D(t)),$$

where the left-hand side is the logged spread between real lending and deposit rates, while the right-hand side is the log-linearized version of $\tau(t)$. The results for estimating (28) over the full US post-war data and two subsamples are presented in Table 2.¹³ Considering up to two lagged dependent variables was sufficient to render white noise residuals for all cases. For the full data sample, θ is estimated to be -0.87 and significantly less than zero. The point estimate is lower in the earlier subsample (-5.66), but not significantly different than the full-sample estimate at the 95 percent confidence level. The estimate in the later subsample is significantly higher than the full-sample estimate (-0.30), but still significantly less than zero at the 90 percent confidence level. While this simple exercise is far from concrete evidence supporting ES in the financial intermediary sector, it provides alternative values of θ to assess the sensitivity of the model.

The model was analyzed under two additional points within the indeterminacy zone of Figure 1: (Case 1) the degree of ES estimated for the post-war data sample ($\theta = -0.8666$) with the benchmark value added of 1.15 percent, and (Case 2) $\theta = -0.8666$ and a value added equal to 1.05 percent. Together with the benchmark result, these three cases roughly

¹³The spread between lending and deposit rates was taken to be the spread between the prime lending rate (series name: MPRIME) and the 3 month Tbill rate (series name: TB3MS), while real deposits were defined as the sum of M1: demand deposits and M1: other checkable deposits (series names: DD.US and OCD.US) deflated by the GDP deflator (series name: GDPDEF). The annualized interest rate data was transformed into gross, monthly rates, and trends were removed from all variables using the HP filter. All monthly data was transformed to quarterly by taking three-month averages. The data sample from 1959:1 to 2006:4 is available from the Board of Governors of the Federal Reserve System.

Table 2: Increasing Return Estimates

Data	θ	R^2
1959:1-2006:4	-0.8666 (0.3139)	0.42
1959:1-1979:1	-5.6641 (2.5143)	0.57
1986:1-2006:4	-0.3024 (0.1574)	0.36

Notes: Standard errors are in parentheses.

span the indeterminacy zone.

Figure 5 compares the (orthogonal) impulse responses of several variables to a technology shock (left column), monetary shock (middle column), and sunspot shock (right column). The nominal variables (M1, aggregate prices, and nominal wages) were illustrated because these variables change the most across the three shocks. The figure indicates that there is a large degree of short-run stability in the predicted responses. Across the three cases, there are no noticeable differences in the responses to a monetary or sunspot shock. When looking at the model's response to a productivity shock, there are slight changes in nominal variables in later periods. However, for cases which consider a large degree of ES in financial intermediation, the reduced cost to intermediation results in a continued decline in aggregate prices. The decline in prices is larger than the rise in deposits, which results in the decline in M1 (see (27)). The real effect of the persistent decline in prices is offset by an equivalent decline in nominal wages, which explains the equivalence in real output.

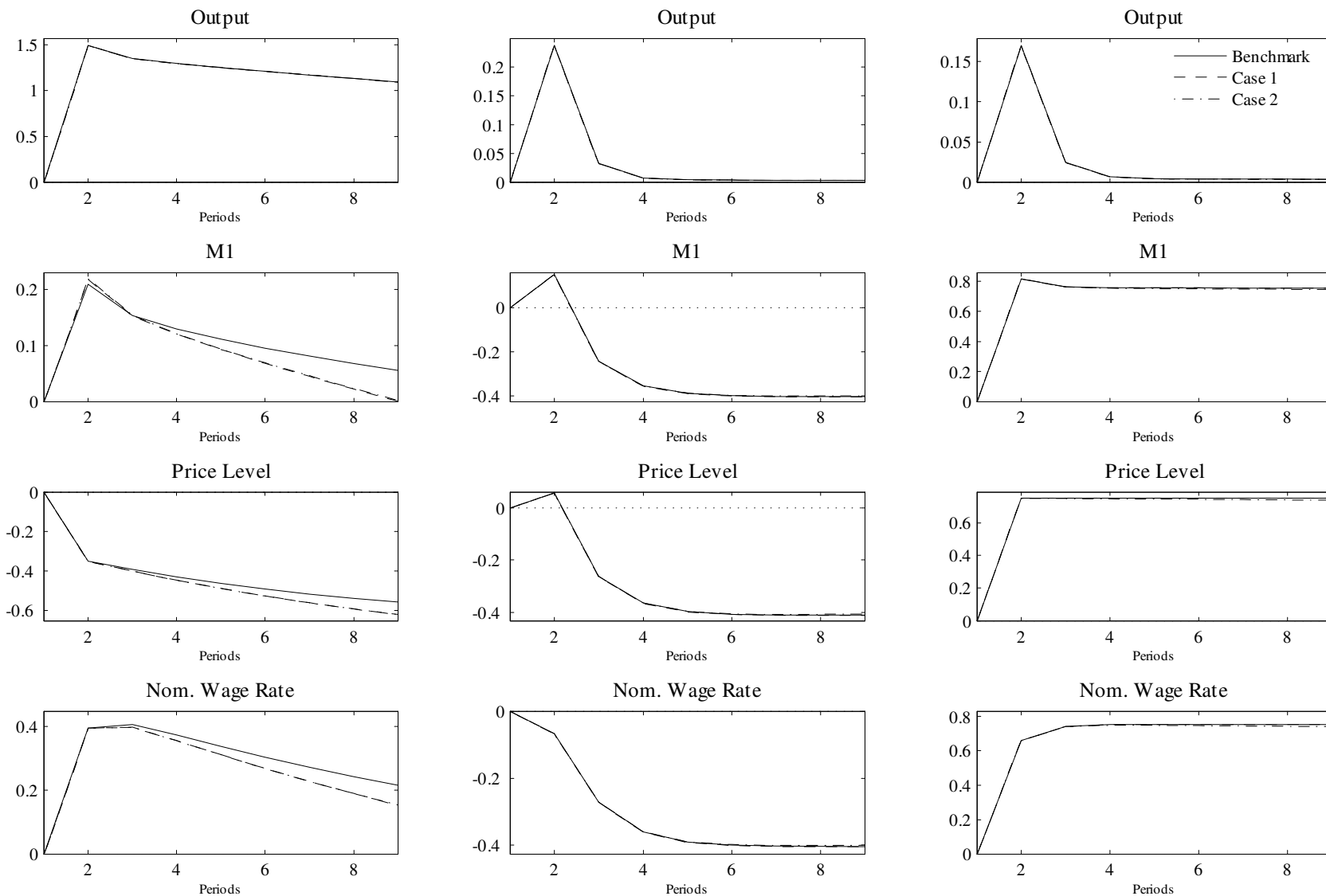


Figure 5: Impulse responses to a one percent increase in technology (left column), monetary base (middle column), and reduced-form sunspot shock (right column). Y-axes denote percentage changes from steady state. The benchmark model uses $\theta = -0.01$ and value added (VA) = 1.15 percent. Case 1: $\theta = -0.8666$ and VA = 1.15 percent. Case 2: $\theta = -0.8666$ and VA = 1.05 percent.

3.4. The Great Moderation: A Calibration Exercise

While the above analysis compares the impact of fundamental and nonfundamental shocks of equal size, it fails to consider the relative sizes of these shocks. This issue is addressed here through a calibration exercise concerning the quantitative importance of removing these sunspot shocks on the observed decline in economic volatility observed in the US during the 1980s (termed the Great Moderation).

A brief background of the Great Moderation begins with the observation by Blanchard and Simon (2001) that the variability (standard deviation) of quarterly real output growth and inflation since the mid-1980s has declined by one-half and two-thirds, respectively. While it would be impossible to adequately summarize the various explanations to this unprecedented observation, Stock and Watson (2003) determined that the Great Moderation was attributable to a combination of 10-25 percent improved policy, 20-30 percent identifiable good luck in the form of productivity and commodity price shocks, and 40-60 percent *unknown forms of good luck that manifest themselves as smaller reduced-form forecast errors*.

As discussed in the introduction, Dressler (2008) explicitly shows that the equilibrium indeterminacy examined above ceases to exist when the monetary authority follows an explicit interest rate rule. Since monetary policies other than interest rate rules were in use before 1982, this suggests that non-fundamental shocks might have had an impact before 1982, but not after. The present exercise therefore uses the policy adoption as a natural experiment to identify the relative sizes of the fundamental and non-fundamental shocks, and assess their quantitative importance on economic volatility.

The exercise first sets out by identifying the reduction in economic volatility that a model without durable goods, fiscal policies, and international sectors can hope to explain. The first row of Table 3 presents the percentage change in the standard deviation of output (defined as the sum of nondurable consumption, services, and investment) and prices (defined as the CPI) in US data before and after the identified break of the first quarter of 1984.¹⁴ Although

¹⁴The data for output was constructed as the sum of (i) Real Personal Consumption Expenditures: Non-

the data definitions differ from earlier analyses of the Great Moderation, they indicate a 38 percent decline in output volatility and a 63.5 percent decline in price volatility.

The next step is to use the pre-1984:1 data to calibrate the standard deviations of the three exogenous shocks $\{\sigma_z, \sigma_\mu, \sigma_\zeta\}$ in the benchmark model. These parameters are unidentified in steady state, so they are chosen to minimize the distance between the standard deviations of output (1.9287), the monetary base (0.8315), and M1 (1.6176) observed in the pre-1984:1 data with simulations of the model.¹⁵ The calibration exercise was performed for two degrees of nominal wage rigidity: the benchmark case ($\phi = 6.03$), and a higher value ($\phi = 11.65$) corresponding to a 4 quarter average wage duration. Under the benchmark case, the standard deviations were calibrated to $\sigma_z = 0.0115$, $\sigma_\mu = 0.0048$, and $\sigma_\zeta = 0.0147$. Under the higher ϕ , the calibration resulted in $\sigma_z = 0.0110$, $\sigma_\mu = 0.0048$, and $\sigma_\zeta = 0.0136$. It is interesting to note that the standard deviations for the fundamental shocks are rather close to the benchmark values taken from the literature, and are both smaller than the sunspot shock. Both calibrations achieved the targeted moments within 0.0002.

Using the calibrated standard deviations of the fundamental and non-fundamental shocks, simulations of the model with and without sunspot shocks can be compared in order to isolate the quantitative importance this indeterminacy on the Great Moderation. The results of the exercise are presented in the final two rows of Table 3. Under the benchmark degree of wage rigidity, removing the nonfundamental shocks results in over one half of a percent reduction in the standard deviation in output and almost a 43 percent reduction in the standard deviation of prices. Under the higher wage rigidity, the reduction in output volatility increases to over 1.5 percent. The reduction in price volatility between the two degrees of wage rigidity is virtually unchanged.

urable Goods (PCNDGC96) and Services (PCESVC96), and (ii) Real Gross Private Domestic Investment (GDPIC96). The data for prices is the Consumer Price Index for all items (CPIAUCSL). The data for the monetary base was the currency component of M1 (CURRSL), while M1 was defined to be the currency component of M1 plus demand deposits (CURRDD). All data is seasonally adjusted and available from the Federal Reserve of St. Louis database for the range 1959:1 to 2007:4. Monthly data was made quarterly by taking monthly averages, and trends were removed using the HP filter.

¹⁵These three moments were chosen to best identify the standard deviations of the shocks. The details of the calibration exercise are described in an appendix.

Table 3: Volatility of Output and Prices

	Output		Prices	
	st. dev. ^a	% Δ	st. dev. ^a	% Δ
Data	1.9287/1.1937	-38.12	1.0194/0.3720	-63.51
Model ($\phi = 6.03$)	1.9289/1.9182	-0.55	1.7092/0.9750	-42.95
Model ($\phi = 11.65$)	1.9288/1.8993	-1.53	1.6507/0.9499	-42.46

Notes: ^aFirst (second) entry uses pre (post) 1984:1 data.

While the calculated decline in output volatility appears small, this simple exercise accounts for 2.5 to 10 percent of the observed decline that Stock and Watson (2003) were unable to explicitly account for (depending on the degree of nominal rigidity). This impact is rather significant considering that the sector giving rise to these sunspot shocks makes up a small part of the economy.

4. Conclusion

The goal of this paper was to quantitatively assess the economic effects of indeterminacy resulting from ES in the financial intermediation sector. A monetary model with multiple mediums of exchange is extended to feature financial intermediaries which exhibit economies of scale through decreasing marginal costs to managing deposits. Although the size of the financial intermediary sector is calibrated to match US data, the model illustrates: (i) indeterminacy arises for small degrees of ES and standard parameter assumptions; (ii) confidence shocks have significant effects due to the money multiplier, and qualitatively resemble exogenous monetary shocks; and (iii) endogenous monetary policy can stabilize the real effects of sunspot shocks, but only under complete information. Furthermore, the quantitative importance of these confidence shocks are assessed by asking how much of the decline in economic volatility observed in the 1980s can be accounted for by the removal of these sunspot shocks. Conditional on the degree of wage rigidity, the exercise suggests that the removal of belief shocks by the adoption of an interest-rate targeting monetary policy can explain up to 10 percent of the Great Moderation that Stock and Watson (2003) categorize as “other sources

of good luck”.

These results warrant some discussion. While not directly adding to the controversy in the empirical literature on ES in intermediation, the analysis suggests that the degree of ES required to give rise to equilibrium indeterminacy can be small and therefore may be difficult to empirically estimate. Unfortunately, the stability of the quantitative results with respect to the degree of ES (i.e. the value of θ) - which is desirable for the analysis presented here - makes this model an unsuitable tool for actually estimating θ . Nonetheless, the results presented here do suggest that belief-induced shocks to financial intermediation can have large effects under some circumstances. For example, this framework may be useful in explaining the large amount of economic volatility observed in developing nations where monetary policies presently allow intermediaries to give rise to indeterminacy. This application is left for future work.

Appendices

Household i 's Generalized and Aggregated Problems

This appendix outlines both the generalized and aggregated problems of the household, and shows that given the assumptions of zero elasticity of substitution between good types and the specific weighting scheme, they are equivalent.

The generalized problem of household i can be stated as

$$\begin{aligned} & \max \sum_{t=1}^{\infty} \beta^t \{u [c^i(t), h^i(t)] \\ & + \lambda_1^i(t) \left[m^i(t) + T(t) - \int_0^1 \mathbf{1}_{Jm}(j) P_j(t) c_j^i(t) dj \right] \\ & + \lambda_2^i(t) \left[P_k(t) d^i(t) - \int_0^1 \mathbf{1}_{Jd}(j) P_j(t) c_j^i(t) dj \right] \\ & + \lambda_3^i(t) \left[\begin{aligned} & \int_0^1 W_j^i(t) h_j^i(t) dj + r(t) P_k(t) [k^i(t) - d^i(t)] + r_d(t) P_k(t) d^i(t) \\ & + m^i(t) + T(t) + R(t) B^i(t) - P_k(t) \gamma \left(\int_0^1 \mathbf{1}_{Jd}(j) dj \right) - P_k(t) \int_0^1 \frac{\phi}{2} \left[\frac{W_j^i(t)}{\pi W_j^i(t-1)} - 1 \right]^2 dj \\ & - \int_0^1 P_j(t) c_j^i(t) dj - m^i(t+1) - P_k(t) k^i(t+1) \end{aligned} \right] \right\}, \end{aligned}$$

where $c^i(t) = \left[\int \varphi_j^{\frac{1}{\varpi}} c_j^i(t)^{\frac{\varpi-1}{\varpi}} dj \right]^{\frac{\varpi}{\varpi-1}}$. The first order conditions for choices of $c_j^i(t) \forall j \in Jm$, $c_{j'}^i(t) \forall j' \in Jd$, $d^i(t)$, $B^i(t)$, $m^i(t+1)$, $k^i(t+1)$ and $W_j^i(t) \forall j$ are given by

$$(29) \quad u_{c_j^i}(t) (c^i(t) \varphi_j)^{\frac{1}{\varpi}} = c_j^i(t)^{\frac{1}{\varpi}} P_j(t) [\lambda_1^i(t) + \lambda_3^i(t)], \quad \forall j \in Jm,$$

$$(30) \quad u_{c_{j'}^i}(t) (c^i(t) \varphi_{j'})^{\frac{1}{\varpi}} = c_{j'}^i(t)^{\frac{1}{\varpi}} P_{j'}(t) [\lambda_2^i(t) + \lambda_3^i(t)], \quad \forall j' \in Jd,$$

$$(31) \quad \lambda_2^i(t) = \lambda_3^i(t) [r(t) - r_d(t)],$$

$$(32) \quad \lambda_1^i(t) = \lambda_3^i(t) R(t),$$

$$(33) \quad \lambda_3^i(t) = \beta E(t) [\lambda_1^i(t+1) + \lambda_3^i(t+1)],$$

$$(34) \quad \lambda_3^i(t) = \beta E(t) r(t+1) \lambda_3^i(t+1),$$

and

$$\begin{aligned}
(35) \quad & u_{h_j^i}(t) \xi H_j(t) \left(\frac{W_j(t)}{W_j^i(t)} \right)^\xi - \lambda_3^i(t) \left[\begin{array}{l} (1 - \xi) H_j(t) W_j^i(t) \left(\frac{W_j(t)}{W_j^i(t)} \right)^\xi \\ - P_k(t) \phi \left[\frac{W_j^i(t)}{\pi W_j^i(t-1)} \right] \frac{W_j^i(t)}{\pi W_j^i(t-1)} \end{array} \right] \\
& = \beta E(t) \lambda_3^i(t+1) \left[P_k(t+1) \phi \left[\frac{W_j^i(t+1)}{\pi W_j^i(t)} \right] \frac{W_j^i(t+1)}{\pi W_j^i(t)} \right], \quad \forall j.
\end{aligned}$$

The aggregated problem of household i can be stated as

$$\begin{aligned}
& \max \sum_{t=1}^{\infty} \beta^t \{ u [c^i(t), h^i(t)] \\
& + \hat{\lambda}_1^i(t) \left[m^i(s_{t-1}) + T(t) - P(t) c^i(t) \int_0^1 \mathbf{1}_{Jm}(j) \varphi_j dj \right] \\
& + \hat{\lambda}_2^i(t) \left[P_k(t) d^i(t) - P(t) c^i(t) \int_0^1 \mathbf{1}_{Jd}(j) \varphi_j dj \right] \\
& + \hat{\lambda}_3^i(t) \left[\begin{array}{l} \int_0^1 W_j^i(t) h_j^i(t) dj + r(t) P_k(t) [k^i(t) - d^i(t)] + r_d(t) P_k(t) d^i(t) \\ + m^i(t) + T(t) + R(t) B^i(t) - P_k(t) \gamma \left(\int_0^1 \mathbf{1}_{Jd}(j) dj \right) - P_k(t) \int_0^1 \frac{\phi}{2} \left[\frac{W_j^i(t)}{\pi W_j^i(t-1)} - 1 \right]^2 dj \\ - P(t) c^i(t) - m^i(t+1) - P_k(t) k^i(t+1) \end{array} \right] \Bigg\},
\end{aligned}$$

and the first order condition for the choice of $c^i(t)$ is given by

$$(36) \quad u_{c^i}(t) = P(t) \left[\hat{\lambda}_3^i(t) + \hat{\lambda}_1^i(t) \int_0^1 \mathbf{1}_{Jm}(j) \varphi_j dj + \hat{\lambda}_2^i(t) \int_0^1 \mathbf{1}_{Jd}(j) \varphi_j dj \right].$$

The remaining first order conditions (with the exception of the multipliers) are identical to the generalized problem.

Deriving the aggregate price and consumption demand equations begins with the claim (and verification) that the problems above are equivalent. This claim implies that the multipliers are equivalent (e.g. $\hat{\lambda}_3^i = \lambda_3^i$). Use (29) and (30) to solve for $\lambda_1^i(t)$ and $\lambda_2^i(t)$. This

requires repeated use of (2) and integrating both sides with respect to j .

$$(37) \quad \lambda_1^i(t) = u_{c_j^i}(t) \left[\int_0^1 \varphi_j P_j(t)^{1-\varpi} dj \right]^{\frac{1}{\varpi-1}} - \lambda_3^i(t), \quad \forall j \in Jm$$

$$(38) \quad \lambda_2^i(t) = u_{c_{j'}^i}(t) \left[\int_0^1 \varphi_{j'} P_{j'}(t)^{1-\varpi} dj \right]^{\frac{1}{\varpi-1}} - \lambda_3^i(t), \quad \forall j' \in Jd$$

Substitution of these multipliers into (36) results in

$$(39) \quad u_{c^i}(t) = P(t) \left[\begin{aligned} & \lambda_3^i(t) + \left[u_{c_j^i}(t) \left[\int_0^1 \varphi_j P_j(t)^{1-\varpi} dj \right]^{\frac{1}{\varpi-1}} - \lambda_3^i(t) \right] \int_0^1 \mathbf{1}_{Jm}(j) \varphi_j dj \\ & + \left[u_{c_{j'}^i}(t) \left[\int_0^1 \varphi_{j'} P_{j'}(t)^{1-\varpi} dj \right]^{\frac{1}{\varpi-1}} - \lambda_3^i(t) \right] \int_0^1 \mathbf{1}_{Jd}(j) \varphi_j dj \end{aligned} \right].$$

Since Jm and Jd span the set of goods, $\lambda_3^i(t)$ and $u_{c^i}(t)$ drops out leaving

$$(40) \quad P(t) = \left[\int_0^1 \varphi_j P_j(t)^{1-\varpi} dj \right]^{\frac{1}{1-\varpi}}.$$

Verifying that the multipliers are equal (and the problems are equivalent) can be done by verifying that $P(t) c^i(t) = \int_0^1 P_j(t) c_j^i(t) dj$. Using only the generalized problem, replacing either $\lambda_1^i(t)$ in (29) with its expression in (37) or $\lambda_2^i(t)$ in (30) with its expression in (38) results in

$$(c^i(t) \varphi_j)^{\frac{1}{\varpi}} = c_j^i(t)^{\frac{1}{\varpi}} P_j(t) \left[\int_0^1 \varphi_j P_j(t)^{1-\varpi} dj \right]^{\frac{1}{\varpi-1}}.$$

Raising both sides to the power ϖ , rearranging terms, integrating both sides with respect to j , and using (40) verifies the result and delivers (18).

Under the aggregated problem, household optimization is characterized by the binding

constraint set and the Euler equations

$$(41) \quad u_{c^i}(t) \Psi^i(t) = \beta E(t) r(t+1) u_{c^i}(t+1) \Psi^i(t+1),$$

$$(42) \quad u_{c^i}(t) \Psi^i(t) = \beta E(t) \frac{1+R(t+1)}{\pi(t+1)} u_{c^i}(t+1) \Psi^i(t+1),$$

$$(43) \quad u_{h_j^i}(s_t) \xi H_j(s_t) \left(\frac{W_j(s_t)}{W_j^i(s_t)} \right)^\xi = u_{c^i}(t) \Psi^i(t) \left[\begin{array}{c} (1-\xi) H_j(s_t) \frac{W_j^i(s_t)}{P(s_t)} \left(\frac{W_j(s_t)}{W_j^i(s_t)} \right)^\xi \\ -\phi \left[\frac{W_j^i(s_t)}{\pi W_j^i(s_{t-1})} \right] \frac{W_j^i(s_t)}{\pi W_j^i(s_{t-1})} \end{array} \right] \\ + \beta E_t u_{c^i}(t+1) \Psi^i(t+1) \left[\phi \left[\frac{W_j^i(s_{t+1})}{\pi W_j^i(s_t)} \right] \frac{W_j^i(s_{t+1})}{\pi W_j^i(s_t)} \right], \quad \forall j$$

and

$$(44) \quad R(t) = (r(t) - r_d(t)) + \frac{\gamma}{2j^{*i}(t) c^i(t)},$$

where

$$(45) \quad \Psi^i(t) = [1 + j^{*i}(t)^2 R(t) + (1 - j^{*i}(t)^2) (r(t) - r^d(t))]^{-1}.$$

Using (9) and $r(t) \pi(t) = 1 + R(t)$, it is easy to show that the first-order condition for the household's choice of $j^{*i}(t)$ is equivalent to (22), suggesting that the optimal choice for the composition of money balances is chosen such that their costs of use are equated.

Model Solution

The solution methodology described in this appendix follows Lubik and Schorfheide (2003) and their extension of Sims (2001). After removing all multipliers from the household's first-order conditions and imposing symmetry, the normalized system of equations comprising

the dynamic solution are given by

$$\begin{aligned}
& u_c(t) \Psi(t) - \\
& \beta E(t) \frac{P(t)}{P(t+1) \mu(t+1)} u_c(t) \left(1 + \frac{\gamma}{2j^*(t+1) c(t+1)} + \Gamma d(t+1)^\theta \right) \Psi(t+1) = 0 \\
& u_c(t) \Psi(t) - \beta E(t) r(t+1) u_c(t+1) \Psi(t+1) = 0 \\
& u_h(t) \xi h(t) + u_c(t) \Psi(t) \left[(1 - \xi) \frac{W(t) h(t)}{P(t)} - \phi \left(\frac{\mu(t) W(t)}{\pi W(t-1)} - 1 \right) \frac{\mu(t) W(t)}{\pi W(t-1)} \right] - \dots \\
& \beta E(t) \Psi(t+1) \phi \left(\frac{\mu(t+1) W(t+1)}{\pi W(t)} - 1 \right) \frac{\mu(t+1) W(t+1)}{\pi W(t)} = 0 \\
& z(t) = \kappa_z + \rho_z z(t-1) + \varepsilon_z(t) \\
& \mu(t) = \kappa_\mu + \rho_\mu \mu(t-1) + \varepsilon_\mu(t) \\
& z(t) k^\alpha(t) h^{1-\alpha}(t) + (1 - \delta) k(t) = \\
& c(t) + k(t+1) + \phi \left(\frac{\mu(t) W(t)}{\pi W(t-1)} - 1 \right)^2 + \Gamma d(t)^{1+\theta} + \gamma(1 - j^*(t)) \\
& \frac{1}{P(t)} = j^*(t)^2 c(t) \\
& d(t) = (1 - j^*(t)^2) c(t) \\
& r(t) = \alpha z(t) \left(\frac{h(t)}{k(t)} \right)^{1-\alpha} + 1 - \delta \\
& \frac{W(t)}{P(t)} = (1 - \alpha) z(t) \left(\frac{k(t)}{h(t)} \right)^\alpha
\end{aligned}$$

where $\Psi(t) = \left[1 + \frac{\gamma j^*(t)}{2c(t)} + \Gamma d(t)^\theta \right]^{-1}$. After the above system is log-linearized around the model's steady state, the dimension of the system is reduced by using the bottom five equations to remove $\{c(t), h(t), j^*(t), r(t), d(t)\}$. The remaining five equations (and six identities) comprise the linear rational expectations model and can be represented in the canonical form:

$$(46) \quad \Xi_0 s(t) = \Xi_1 s(t-1) + \Upsilon \varepsilon(t) + \Pi \vartheta(t)$$

where

$$\begin{aligned}
s(t) &= [k(t+1), W(t), P(t), z(t), \mu(t), E(t)k(t+2), E(t)W(t+1), E(t)P(t+1)]' \\
\varepsilon(t) &= [\varepsilon_z(t), \varepsilon_\mu(t)]' \\
\vartheta(t) &= [k(t+1) - E(t-1)k(t+1), W(t) - E(t-1)W(t), P(t) - E(t-1)P(t)]'
\end{aligned}$$

Solving the model requires the use of the generalized Schur decomposition (QZ) of Ξ_0 and Ξ_1 . This results in matrices Q , Z , Λ and Ω such that $QQ' = ZZ' = I_n$, Λ and Ω are upper triangular, and $\Xi_0 = Q'\Lambda Z$ and $\Xi_1 = Q'\Omega Z$. Defining $\varpi_t = Z's(t)$, premultiplying (46) by Q results in

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{22} \end{bmatrix} \begin{bmatrix} \varpi_{1t} \\ \varpi_{2t} \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{bmatrix} \begin{bmatrix} \varpi_{1t-1} \\ \varpi_{2t-1} \end{bmatrix} + \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} (\Upsilon\varepsilon(t) + \Pi\vartheta(t))$$

where, without loss of generality, the system has been partitioned such that the lower blocks of Λ , Ω and Q correspond to the portion of the system delivering unstable eigenvalues. In other words, the lower block contains all equations in which the ratio between the diagonal elements of Ω and Λ are greater than unity.

This ‘explosive’ block is written as

$$\varpi_2(t) = \Lambda_{22}^{-1}\Omega_{22}\varpi_{2t-1} + \Lambda_{22}^{-1}Q_2(\Upsilon\varepsilon(t) + \Pi\vartheta(t)).$$

A non-explosive solution of the model requires $\varpi_2(t) = 0 \forall t \geq 0$. This is accomplished by choosing $\varpi_2(0) = 0$ and for every vector $\varepsilon(t)$ the endogenous forecast error $\vartheta(t)$ that satisfies

$$(47) \quad \Upsilon^*\varepsilon(t) + \Pi^*\vartheta(t) = 0$$

where $\Upsilon^* = Q_2\Upsilon$ and $\Pi^* = Q_2\Pi$. If the number of endogenous forecast errors is equal to the number of unstable eigenvalues, then (47) uniquely determines $\vartheta(t)$. If the number of

endogenous forecast errors exceeds the number of unstable eigenvalues, then the system is undetermined and sunspot fluctuations can arise.

Using the singular value decomposition $\Pi^* = UDV'$, a general solution for the endogenous forecast errors is given by

$$\vartheta(t) = (-V_1 D_{11}^{-1} U'_1 \Upsilon^* + V_2 M_1) \varepsilon(t) + V_2 M_2 \zeta(t)$$

where M_1 and M_2 govern the influence of the sunspot shock.

Assuming Ξ_0^{-1} exists, the solution of the model takes the form of a law of motion for the endogenous variables

(48)

$$s(t) = \Xi_0^{-1} \Xi_1 s(t-1) + [\Xi_0^{-1} \Upsilon^* - \Xi_0^{-1} \Pi^* V_1 D_{11}^{-1} U'_1 \Upsilon^*] \varepsilon(t) + \Xi_0^{-1} \Pi^* V_2 (M_1 \varepsilon(t) + M_2 \zeta(t)).$$

Setting $M_2 = 1$ results in the interpretation of $\zeta(t)$ as a reduced-form sunspot shock. Determining the value for M_1 requires choosing one of two alternative identification schemes. If one assumes that the effects of fundamental and non-fundamental shocks on the forecast error are *orthogonal* to each other, then $M_1 = 0$. Otherwise, M_1 is chosen such that the impulse responses of the model $(\partial s(t) / \partial \varepsilon(t))$ are *continuous* at the boundary between the determinacy and indeterminacy regions. Under indeterminacy, the impulse response is given by

$$B_1 + B_2 M_1 = (\Xi_0^{-1} \Upsilon^* - \Xi_0^{-1} \Pi^* V_1 D_{11}^{-1} U'_1 \Upsilon^*) + \Xi_0^{-1} \Pi^* V_2 M_1.$$

For a corresponding determinacy solution, the impulse response is given by

$$\tilde{B}_1 = \tilde{\Xi}_0^{-1} \tilde{\Upsilon}^* - \tilde{\Xi}_0^{-1} \tilde{\Pi}^* \tilde{V}_1 \tilde{D}_{11}^{-1} \tilde{U}'_1 \tilde{\Upsilon}^*$$

where a tilde denotes that a different point in the parameter space is needed to alter the model dynamics. To get the indeterminate impulse responses as close as possible to the

determinate ones, M_1 is computed by applying the least squares criterion

$$M_1 = [B_2' B_2]^{-1} B_2' [\tilde{B}_1 - B_1].$$

This result is substituted in (48) while maintaining $M_2 = 1$.

Calibration Exercise

Let Φ denote a vector of standard deviations calculated from data, and $\Phi(\Theta)$ denote the corresponding calculations from a simulation of the model where Θ denotes the vector of parameters to be calibrated. The parameter vector delivered by the calibration exercise is that which minimizes

$$(\Phi(\Theta) - \Phi)' \Sigma (\Phi(\Theta) - \Phi),$$

where Σ is an identity matrix.

The calibration exercise chooses Φ to be a 3×1 vector consisting of the pre-1984:1 standard deviations of real output, the monetary base and M1 (the data), and Θ is a 3×1 vector of standard deviations of the exogenous shocks of the model (the parameters). Note that minimizing the above expression would be equivalent to a simulated method of moments exercise if Σ were replaced by a weighting matrix that corresponds to the inverse of the variance-covariance matrix of Φ .

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