The Undervaluation of Distressed Company's Equity

Schmidt, Frederik

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Frederik Schmidt*

Ernst & Young

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Abstract

In a simple firm value model we consider the impact of the insolvency probability on the valuation of equity and debt, which are assumed to be not publicly traded. For the case of a distressed company, which usually has high debt and low equity, we can show that the impact becomes increasingly important. Disregarding this yields an overvaluation of debt and an undervaluation of equity. We calculate the sensitivity of equity with regard to debt, which is isomorphic to the sensitivity of a call option with regard to the strike price, and show that this sensitivity rises with increasing debt. Furthermore, we provide a numerical example of this effect.

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*Ernst & Young AG, Transaction Advisory Services - Restructuring, Mergenthalerallee 10-12, 65760 Eschborn, Germany; Tel.: +49 174 702 702 3, Fax: +49 181 3943 18592, e-mail: frederik.schmidt@de.ey.com, fs.schmidt@gmx.de

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1 Introduction

The valuation of companies is comparatively simple if we know the market value of equity and/or the market value of debt. This is the case for many big listed companies. But if we consider small and medium sized firms we have to approximate these values. In practice, there are several well-known methods to determine the enterprise value of a firm, e.g. the Discounted Cash Flow method or the Multiple method, with each of them having various extensions and refinements. As the enterprise value is the sum of the market value of equity and the market value of debt, we only need one of these two values to calculate the other if we take the enterprise value as given. Unfortunately, we usually have none of these market values considering a small or medium sized firm. Therefore, in order to arrive at an estimation of the market value of equity, usually the balance sheet value of debt is deducted from the enterprise value.

We want to sway the focus from the valuation difficulties regarding the enterprise value to the problems which arise in the context of debt valuation. In a simple model, we can show that the deduction of debt's balance sheet value does not take the insolvency probability adequately into account, in doing so we use the discounted face value of the debt as a proxy for the balance sheet value of debt. The described error results in an overvaluation of debt and, therefore, an undervaluation of equity. We can show that the undervaluation of equity rises with higher insolvency probability, i.e. higher debt. As distressed companies are usually financed with low equity and much debt, this issue has especially to be considered regarding the valuation of distressed companies.

In the following, we use an option approach for the valuation of equity and debt. As Black and Scholes (1973) and Merton (1974) have shown, there is an isomorphism between the valuation of securities and options. Equity and debt can be seen as derivatives of the total enterprise value. Taking the enterprise value as underlying, this allows to investigate the impact of the different measures of influence on the market value of equity or debt.

We are especially interested in the valuation of distressed company's equity. Usually, these companies have high debt financing and low equity. In these circumstances, an incorrect calculation of the large portion of debt yields high inaccuracies regarding the low portion of equity. This is because higher debt causes a higher insolvency probability which has an increasing negative impact on the valuation of debt. Therefore, there is a positive impact on the valuation of equity due to the fact that the company does not have to pay back the liabilities in each state of the world.

Summarizing, rising debt has two impacts on equity, the first being the direct impact due to the higher debt which lowers the equity value.[1] The second and indirect impact is due to a higher insolvency probability of a firm with higher debt. This effect lowers the value of debt and increases the value of equity as it is an advantage of the equity holders to have the valuable option to file for insolvency and not have to pay back the entire debt. As the impact of the direct effect is linear but the impact of the indirect effect is increasing with higher debt, considering the second effect is especially necessary regarding valuations of distressed companies. Thus, a misstatement of debt's market value will cause misstatements in equity value, which is more severe in the context of distressed companies.

The option approach to firm valuation is discussed in detail by Hull (2006) as well as Schönbucher (2003). Hull (2006) treats multiple different derivatives and considers firm value models in chapter 20.6 and 31.4. Otherwise, Schönbucher (2003) shows a rigorous treatment of credit derivatives and their pricing, where the firm value is used to consider the risk of credit derivatives. He develops firm value and share price-based models in chapter 9, where also a discussion of the practical implementation within the KMV approach can be found.[2] A combination of the options approach used here and the widely used Discounted Cash Flow method is delivered by Schwartz and Moon (2000), which show the impact of the volatility on the discounted cash flow. In this model, the volatility of the enterprise value is incorporated into the cash flow of the company.

2 The model

Consider a firm with assets which have the market value V. Furthermore, the firm has debt which equals a zero-coupon bond of maturity T and a face value of $\overline{D}$. The equity value of the company is denoted by E.

A usual problem concerning the valuation of distressed companies is the fact that these firms are often small and thus not actively traded, either because they have never been traded due to their smallness or because the market has dried out because of the distressed situation. Thus, we assume that the market value of the debt, which will be denoted by D, as well as the market value of equity, E, are unobservable. In this situation it is obvious that the usual accounting identity of the balance sheet that the value of the firm's assets has to equal the sum of the value of equity and

[1] We assume a constant enterprise value, i.e. a constant value of the sum of equity and debt.

[2] Furthermore, Schönbucher (2003) provides different extensions of the model as well as further literature.

liabilities

$$V_t = D_t + E_t \quad \forall t \tag{1}$$

can not be applied directly due to a lack of information.
The standard treatment of this situation is as follows: The enterprise value V is calculated using a Discounted Cash Flow or a Multiple approach in order to get a proxy for the market value of firm's assets. Thereafter, the balance sheet value of the debt is deducted to reach the firm's 'equity value'. This approach does not take into account that equity and debt can be viewed as derivative securities on the value V of the firm's assets.
The payoffs of these derivatives at time T are:

$$D(V,T) = \min(\overline{D}, V) \tag{2}$$

$$E(V,T) = \max(V - \overline{D}, 0). \tag{3}$$

Thus, equity has the same payoff as a European call option on the firm's value. The payoff of the debt is either its face value if the firm's value is high enough, or whatever is left of the company value V if the company value is below the face value of the debt.
To eliminate any arbitrage opportunities, the sum of equity and debt has to equal the total value of the firm:

$$V_t = D(V,t) + E(V,t) \quad \Longleftrightarrow \quad E(V,t) = V_t - D(V,t). \tag{4}$$

Here, $D(V,t)$ would be the market value of the debt at time t which is assumed to be non-observable and $E(V,t)$ represents the non-observable market value of equity. To arrive at the market value of equity, we have to take the company value and deduct the market value of debt.
It is assumed that the value of the assets V follows a geometric Brownian motion under the martingale measure Q

$$dV = \mu V\,dt + \sigma V\,dW, \tag{5}$$

where μ is the drift of the process, σ is the volatility of firm's value[3], and dW is a standard Wiener process.

[3] In general, it is difficult to observe the volatility of firm's value. Schönbucher (2003), chapter 9.4.3, provides an approach where the volatility of firm's value can be obtained from the volatility of its shares.

As the only source of uncertainty is firm's value and we have two securities, the bonds and the shares, it is possible to hedge these two securities with each other. Then, it is possible to eliminate the stochastic dV from the portfolio to gain the Black-Scholes partial differential equation. Using this it is possible to directly price the share.
The value of the share is given by the Black-Scholes formula for an European call option

$$E(V,t) = C^{BS}(V,t;T,\overline{D},\sigma,r), \tag{6}$$

where $C^{BS}(\cdot)$ denotes the Black-Scholes price of an European call option with underlying V which has the volatility σ, the exercise price $\overline{D}$, expiry date T, and risk-free interest rate r.
Thus, it is possible to calculate its value[4]:

$$E(V,0) = V_0\, N(d_1) - e^{-r\,T}\,\overline{D}\,N(d_2), \tag{7}$$

where

$$d_1 = \frac{\ln\frac{V_0}{\overline{D}} + (r+\frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \tag{8}$$

and

$$d_2 = d_1 - \sigma\sqrt{T}. \tag{9}$$

Today's value of debt can be calculated by $V_0 - E(V,0)$.
As we are interested in valuation of distressed companies with high debt positions, we consider the effect of rising debt within this model. Thus, we calculate the sensitivity of equity with regard to debt:[5]

$$\Omega = \frac{\partial\, E(\cdot)}{\partial\,\overline{D}}, \tag{10}$$

which is

$$\Omega = V_0\, N'(d_1)\,\frac{\partial\, d_1}{\partial\,\overline{D}} - \overline{D}\,e^{-r\,T}\,N'(d_2)\,\frac{\partial\, d_2}{\partial\,\overline{D}} - e^{-r\,T}\,N(d_2). \tag{11}$$

Using $V_0\,N'(d_1) = \overline{D}\,e^{-r\,T}\,N'(d_2)$ and $\frac{\partial\, d_1}{\partial\,\overline{D}} = \frac{\partial\, d_2}{\partial\,\overline{D}}$ we can simplify the sensitivity and obtain

$$\Omega = -e^{-r\,T}\,N(d_2) \geq -e^{-r\,T}. \tag{12}$$

[4]Without loss of generality we set today's time at zero.
[5]In option theory the sensitivities of the option price are denoted with Greek letters. Here, we consider the sensitivity of the option price with regard to the strike price which is not a common key figure in option theory, so that we introduce the letter Ω.

Thus, if the debt is rising, the valuation of company's equity is lowered by an amount which is equal or smaller than the discounted debt. The amount is smaller than the discounted debt for $N(d_2) < 1$, i.e. $d_2 < \infty$.
In the following, we consider the amount of debt where the difference between the discounted face value and the market value of the debt is smallest.
Due to $\lim_{\overline{D}\to 0} d_2 = \infty$ we have

$$\lim_{\overline{D}\to 0} N(d_2) = 1, \tag{13}$$

which shows that in the case of a wholly equity-financed firm the market value and the discounted face value of debt coincide. The difference in a dexter neighborhood of $\overline{D} = 0$ is small due to continuity of the functions involved.
In the next step we start at this point and increase the amount of debt. Considering

$$\begin{aligned} \frac{\partial N(d_2)}{\partial \overline{D}} &= \frac{\partial}{\partial d_2}\left[\frac{1}{\sqrt{2\,\pi}}\int_{-\infty}^{d_2} e^{-\frac{\kappa^2}{2}}\,d\kappa\right]\frac{\partial d_2}{\partial \overline{D}} \\ &= -\,\frac{1}{\sqrt{2\,\pi}}\,e^{-\frac{d_2^2}{2}}\,\frac{1}{\sigma\sqrt{T}}\,\frac{1}{\overline{D}} \;<\; 0 \end{aligned} \tag{14}$$

we gain the result that the difference between discounted face value and market value grows with rising debt. This is in line with intuition regarding a higher insolvency probability[6] of a firm which is financed with a higher leverage.
We summarize our results in the following proposition.

Proposition 1
The sensitivity of equity with regard to debt is

$$\Omega = -e^{-r\,T}\,N\left(\frac{\ln\frac{V_0}{\overline{D}} + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\right)$$

which equals the negative discount factor for zero debt and rises with increasing debt.

Thus, if the equity value is calculated using discounted face value, the error is small for low debt and is rising with higher debt. As distressed companies usually have high debt positions the error becomes significant in these cases. Due to the fact that equity is a call option on firm's value the rising insolvency probability works in favor of equity holders who own a company with high debt financing. Therefore,

[6] The risk-neutral probability that the firm becomes insolvent and is not able to pay back its debt is $N(-d_2)$.

the equity value of distressed companies has to be adjusted for this effect and will end up at a higher amount.
As the value of a call option is always positive[7] it is even possible that the option is 'out-of-the-money', i.e. that the firm has higher debt than the total value of the company (over-indebtedness), and a positive equity value is obtained. In these cases the difference is very high and adoption of this model instead of the above described changes the equity value from a negative to a positive amount.

A numerical example We provide a numerical example to show the impact of this effect. Consider a company with total enterprise value of $V_0 = 100$ which has a volatility of $\sigma = 0.3$. As the debt should be high we assume a zero-coupon debt with face value $\overline{D} = 95$ and maturity $T = 8$ years. The risk-free interest rate is assumed to be $r = 0.03$.
With this assumptions we can calculate

$$E_0 = 42.9, \tag{15}$$

$$D_0 = 57.1, \tag{16}$$

$$\overline{D}\, e^{-r\,T} = 74.7. \tag{17}$$

The discounted face value of the debt is 74.7, whereas the calculated market value of the debt is 57.1. The difference between these values is 17.6, which is nearly 18 % of the total enterprise value.
Considering the difference in equity value, the impact becomes even more obvious. The calculated market value in this example is 42.9. In the case where the equity value is calculated using the enterprise value minus the discounted face value of debt we get a equity value of 25.3. Using this value would result in an error of 69.6 %.

3 Conclusion

We have considered a firm value model within the context of a distressed company. It could be shown that with low enough debt the difference between the calculated market value of debt and the discounted face value becomes zero. Investigating the case of rising debt the result was that this difference increased. As we used discounted face value of the zero-coupon debt as a proxy for the balance sheet value of debt, the error of deducting the balance sheet value of the enterprise value in

[7] We do not consider the boundaries where the value becomes zero.

order to calculate equity value became larger with higher debt financing. Thus, the balance sheet value of debt has to be adjusted for an increasing insolvency probability which is especially true for distressed companies.

The rising insolvency probability with higher debt financing works in favor of the equity holders. As this effect is even more severe with high debt, the comparative advantage of equity holders over debt holders is especially arising in circumstances like distressed companies. Here, the equity holders have the possibility to let the firm go into bankruptcy which is a valuable advantage.

Future research concerning distressed companies within these models should focus on firm's volatility σ, which may differ from the volatility of usual companies, as well as the extension of the model regarding imperfect capital markets with different interest rates. Furthermore, the results of this simple model should be challenged by empirical studies.

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