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Optimal Ecosystem Management with Structural Dynamics

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Abstract

We address the problem of optimal management of a self-organizing ecosystem along ecological succession. A dynamic carrying capacity is interpreted as depicting the dynamics of habitat creation and occupation along ecological succession. The ecosystem may have three growth modes: pure compensation (concave ecosystem regeneration function), depensation (convex-concave regeneration function) and critical depensation (additionally having negative growth rates for low biomass). We analyse the optimal policies for the management of the ecosystem for the three growth modes. Accordingly, we prove the existence of a Skiba points for certain types of ecosystems. Further, we compare usual golden rule paths with the derived optimal policies near the Skiba points.

Keywords: Ecosystem management, habitat creation, optimal policies, Skiba point.

JEL Classification: Q20, C61, C62

1. Introduction

In this work we follow a theoretical approach in the context of ecosystem management to deal with the ecosystem dynamics along ecological succession. Ecosystem succession is the process of species change over time. It includes the development of complex systems from simple biotic and abiotic components. Effective manipulations of our environment to improve fertility, productivity or diversity and an ability to mitigate undesirable conditions all depend on our ability to understand ecological succession. In this paper we bring this subject into the realm of economic analysis.

We take the perspective of ecosystem management. An ecosystem consists of so many interacting components that it is impossible ever to be able to separate and examine all these relationships (Kay and Schneider, 1994; Jorgensen, 1992). Scientists, land managers, and others are proposing ecosystem management as the best way to manage natural resources. We must, then, explore how this approach differs from the theoretical assumptions of current natural resource management.

For ecosystem management, the ecosystem production function is the most useful feature of an ecological model. The ecosystem production function or ecological productivity is simply the graph of dN/dt versus N , being N the ecosystem biomass. It is a highly aggregated representation of an ecosystem's dynamics that depends on innumerable biological and spatial details (Roughgarden, 1997). The ecosystem production function can be thought of as indicating the interest or return on biomass products and services per unit time (Roughgarden, 1997). We argue that ecosystem production functions should include the dynamics of the structural organization of ecosystems during ecological succession.

In order to derive optimal policies along ecological succession we devise a model of an extractive economy using an ecosystem with a dynamics proposed by Rodrigues et al. (2002) and further analyzed in Mota et al. (2004). These kinds of behavior of optimal policies in the presence of convex-concave ecological regeneration functions have also been seen for lake management problems (e.g., Brock and Starrett, 1999; Mäler et al. 2003), for fishing models (Dasgupta and Mäler, 2003), or for multi-species models (Scholes, 2003; Crépin, 2003). Here we have somewhat similar results applied to the problem of managing an ecosystem along ecological succession.

2. Ecosystem dynamics

In this section we briefly present the main behaviours of an ecosystem whose structure is changing along ecological succession. The following results and figures have been derived in Mota et al. (2004).

We interpret the dynamics of biomass as following the dynamics of ecological succession. Odum (1969) notes that ecological succession drives ecosystems' biomass along a logistic curve and net production along a U-inverted curve. Following this, we assume that the regeneration function for biomass is given by the logistic curve,

$$\frac{dN}{dt} = rN(CC - N), \quad (1)$$

where N is ecosystem biomass, CC is the ecosystem carrying capacity and r is a positive constant. This functional form is not the most common form of the logistic but it is more convenient for our purposes.

The important feature of ecological succession depicted by the logistic function is the existence of a “reasonably directional development, which culminates in a stabilized ecosystem climax” (Odum, 1969). Ecosystem carrying capacity is the value of biomass attained at this state. It reflects the ecosystem’s organization that best allocates the available resources. However, as Walker and Moral (2003) argue, ‘the more common held view now is that succession is a process of change that is not always linear and rarely reaches equilibrium’. Disturbance often redirects successional trajectories, leading to the observation that stable endpoints are rarely achieved. Walker and Moral citing Cowles (1901) suggest that succession was a ‘variable approaching a variable rather than a constant’.

If a new unit of ecosystem biomass can alter the ecosystem carrying capacity, this may be understood as a structural change, since an alteration of the efficiency of the allocation of resources has occurred. This interpretation relates the ecosystem carrying capacity to structural properties of the system. The activities of organisms can result in significant, consistent (non-random), and directed (predictable) changes in their local environment. All living creatures through their metabolism, their activities and their choices, part create and partly destroy their own niches, on scales ranging from extremely local to the global (Odling-Smee, et al., 2003; Jones and Lawton; 1995).

Most plants modify their immediate environment in some way that can impact establishment and growth of both other species and other individuals of the same species. Plants selectively concentrate soil nutrients, transport water from the soil to the atmosphere and add organic matter when they decay. Plants also remove resources and impact their neighbors that need the same, often limiting resources. Animals alter soils by burrowing, feeding, defecating and dying (Walker and Moral, 2003, p.189).

To take niche construction and specialization along ecological succession into account, Rodrigues et al. (2002) proposed to consider carrying capacity as a dynamic state variable of the ecosystem. The dynamics proposed is:

$$\frac{dCC}{dt} = \frac{l}{N+h} \frac{dN}{dt}, \quad (2)$$

and was adapted from a model devised by Cohen (1995) for human populations. The parameter l is the self-organizing capability of the ecosystem and h is the feedback saturation, since it imposes an upper limit to the intensity of feedbacks between ecosystem biomass and ecosystem carrying capacity (Mota et al., 2004). Moreover, if l is high, it means that biomass has strong feedbacks with carrying capacity and small changes in population may have large effects on niche availability. This way, we are endowing the

ecosystem with autocatalytic behaviour, since a rise in ecosystem biomass inflicts a rise in ecosystem carrying capacity, which creates a greater potential for biomass to grow (provided that $N < CC$).

Assuming that the dynamics of the ecosystem is governed by (1) and (2), this implies the following ecosystem production function (Mota et al., 2004):

$$\frac{dN}{dt} = rN \left(CC_l + l \ln \left(\frac{N+h}{b} \right) - N \right) \equiv R(N). \quad (3)$$

A simple conclusion drawn from the analogy with the logistic model is that this ecosystem has a carrying capacity given by $CC_l + l \ln((N+h)/b)$ which is constant for $l=0$. That is, for zero self-organizing capability, we have the conventional one-population dynamics. This way, the optimum operating point of the ecosystem is not imposed on the system by exogenous conditions but depends on the historical information “remembered” by the system. Feedbacks on carrying capacity may be positive or negative depending on the ecosystem biomass for a fixed l and h .

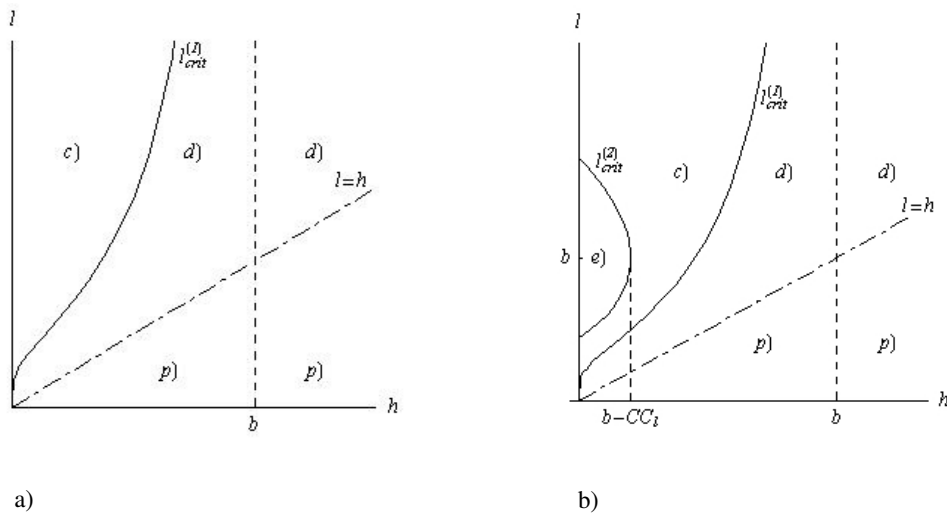


Figure 3.1 – **a)** Bifurcation diagram for the ecosystem dynamics for the case where $CC_l \geq b$. *p)* represents pure compensation; *d)* non-critical depensation and *c)* critical depensation. The line $h = b$ is the asymptote of l_{crit}^1 . **b)** Bifurcation diagram for the case where $b > CC_l$. *e)* represents unconditional extinction (Mota et al., 2004).

Depending on the feedbacks on carrying capacity, the ecosystem may have different kinds of behavior. The main conclusions are depicted in bifurcation diagrams in the $l-h$ plane, for fixed values of all the other parameters (Figure 3.1).

The line $l = h$ separates pure compensation from depensation behaviours. In pure compensation, for all biomass values, each unit added to the existing stock causes a decrease in the per capita growth rate. In other words we have a concave ecosystem production function. In depensation, for sufficiently low N ,

each unit added increases the per capita growth rate (convex-concave production function). If an ecosystem has depensation behaviour with the additional property of negative growth rates for some biomass levels, it is said to have critical depensation (Clark, 1976, p. 16). In this case, there is a certain threshold biomass stock level below which the ecosystem will become extinct.

We conclude that if feedbacks on carrying capacity are low enough (small l and high h) the ecosystem exhibits pure compensation. On the other hand, if feedbacks on carrying capacity are sufficiently high the ecosystem exhibits depensation and, for even higher feedbacks on carrying capacity, critical depensation. This reflects the greater intensity of the relations among populations of the ecosystem: at equal low biomass levels, ecosystems with higher self-organization ability have lower growth rates.

3. The bioeconomic system

We consider the economic system composed by the aggregate of all firms that directly extract resources from the self-organizing ecosystem and export a resource flow to the wider economy. We call this economic system the extractive economy. This is an open economy that trades resources and investment flows with the wider economy for investment rents and consumption goods.

The state of the extractive economy is described by its state variables: ecosystem biomass, $N(t)$, and capital, $K(t)$. Decisions can be made at different levels, through the control variables: consumption rate, $c(t)$, and rate of extraction of environmental products or harvesting rate, $q(t)$. Once we choose to perform the optimisation at the macroeconomic level, it makes sense to consider that the analysis is performed in continuous time and all variables are continuous. This is because the discontinuities that result from individual decisions of harvesting or consumption overlap, being imperceptible at this scale.

The unperturbed ecosystem has a dynamics described in the previous section. Hence, the growth rate of the ecosystem is equal to the undisturbed regeneration function minus the extraction rate:

$$\frac{dN}{dt} = R(N) - q. \quad (4)$$

The extractive economy exports a flow of natural resources to the global economy with net economic revenue $V(q)$. This revenue depends on the supply of the ecosystem's resources, $q(t)$, since it is assumed that this flow is sufficiently large to alter the price of the resources. The net economic revenue function, $V(q)$, has the following properties: $V \in C^2$, $V(0) = 0$, $V' > 0$, $V'' < 0$, which means that the net economic revenue function has positive but decreasing marginal benefits.

This economy has two sources of income, one resulting from the exported natural resources flow, $V(q)$, and the other resulting from the interest obtained from the capital investments in the global economy, δK , where δ is the constant exogenous interest rate. This means that the extractive economy is small enough so that its investments in the global economy have no effect on the interest rate.

The proceeds from these two sources of income can either be directed to consumption or invested in accumulation of productive capital belonging to the extractive economy, dK/dt . Hence we have

$$\frac{dK}{dt} = \delta K + V(q) - c. \quad (5)$$

We assume that it is possible to define a utility function for this economy – citizens' preferences are identical and depicted by the representative consumer's utility function, $U(c)$ ¹ – and that a continuous and infinite overlapping succession of individuals will behave rationally so as to maximise their utility function throughout their lives. Hence, we are considering that future decisions feedback into the present to give an overall maximum discounted sum of the consumers' utility flow, $\int_0^{\infty} e^{-\rho t} U(c) dt$, where ρ is the discount or haste rate. Population is assumed to be constant, so the problem is solved on the behalf of a representative agent.

4. Optimal management with structural dynamics

After briefly presenting the different kinds of behaviour the ecosystem here considered may exhibit, we discuss the optimal management policies of an ecosystem with structural dynamics. It is important to stress that we have given a special treatment to the solutions where the extraction rate is zero. Consequently, we have found some special cases of optimal policies that use zero extraction rates for some period of time. This kind of management policies is seldom found in the body of work in this area.

4.1. The optimal management problem

The problem that this economy faces consists in choosing the temporal trajectories for the consumption $c(t)$ and the extraction rates $q(t)$ in a context of interaction with the environment, in order to achieve the maximum inter-temporal utility benefit. Formally:

$$\max_{c,q} \int_0^{\infty} U(c) e^{-\rho t} dt, \text{ s.t.}$$

$$\frac{dN}{dt} = R(N) - q, \quad (6)$$

$$\frac{dK}{dt} = \delta K + V(q) - c, \quad (7)$$

$$N(t), K(t) \geq 0, c(t), q(t) \geq 0. \quad (8)$$

¹ Utility is a continuous, twice differentiable, increasing, and strictly concave function of consumption rate.

Assume that the extractive economy has fully known and fixed initial stocks $N(0) = N_0$, $K(0) = K_0$.

We use optimal control theory to characterize optimal solutions. Here the maximum is taken over the set of piecewise continuously differentiable functions $c(t), q(t) \geq 0$. In agreement with Pontryagin's maximum principle (Tu, 1994), the optimal candidate solutions maximise the current value Hamiltonian:

$$H(N, K, q, c, \lambda^N, \lambda^K) = U(c) + \lambda^N [R(N) - q] + \lambda^K [\delta K + V(q) - c], \quad (9)$$

where the λ^i , are the co-state variables or shadow prices of capital $i = N, K$. The shadow prices are equal to the marginal value of the capital stock i at time t , i.e., if the stock level is reduced by one unit, its value at time t will be reduced by $\lambda^i(t)$. It is the price that produces the optimal exploitation of the resource if used in real markets. The Hamiltonian is the total rate of increase of total assets in units of the objective functional (utility units in this case), and the sum of the terms that have shadow prices is the value flow from investment in capital assets (Clark, 1976, p.104). Therefore, the shadow prices are multiplier factors that alter the units from capital units to units of the objective functional.

The optimal consumption and extraction rates must obey the first order conditions that optimise the Hamiltonian,

$$H_c = 0 \Leftrightarrow U' = \lambda^K \text{ or } c = 0, \quad (10)$$

$$H_q = 0 \Leftrightarrow \lambda^K V' = \lambda^N \text{ or } q = 0. \quad (11)$$

In this optimisation we are also interested in the possibility of corner solutions, hence we have to consider the maximisation of the Hamiltonian at the boundaries of the controllability domain, $q = 0$ and $c = 0$.

The first order conditions and the assumptions made on the utility and the net economic revenue imply that the shadow prices of natural capital and financial capital are both positive. The second order conditions for a local maximum require that all the pivots of the Hamiltonian Hessian matrix are negative. The Hessian for this problem is a 2×2 diagonal matrix with $U''(c)$ and $\lambda^K V''(q)$ as elements. Since utility and net economic revenue are both assumed to have decreasing marginal gains, the pivots of the Hessian matrix are negative and conditions (10) and (11) are maximising conditions for the control variables.

Condition (10) is the usual optimal condition in economic growth models, and simply states that in the optimal strategy marginal benefits from consumption should equal marginal costs from the reduction in savings. To have the marginal benefit, $U'(c)$, economic capital value must be reduced by λ^K . In other words, the marginal benefit from consumption must equal the capital value imputed by its future

productivity; hence this condition depicts the inter-temporal effect associated with present or future consumption.

Condition (11) refers to the same effect, but now applied to the ecosystem – the marginal revenue from extraction of biological resources in utility units, should equal the marginal cost of depleting the resource, measured in loss of future productivity of the ecosystem. To have a marginal revenue in utility units², $\lambda^K V_q$, biological resource capital value should be reduced by λ^N .

Following Pontryagin's maximum principle the shadow prices for the optimal solution will evolve according to the following non-linear autonomous dynamical system:

$$\frac{1}{\lambda^N} \frac{d\lambda^N}{dt} + R'(N) = \rho, \quad (12)$$

$$\frac{1}{\lambda^K} \frac{d\lambda^K}{dt} + \delta = \rho, \quad (13)$$

with the usual transversality conditions,

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda^K K = \lim_{t \rightarrow \infty} e^{-\rho t} \lambda^N N = 0. \quad (14)$$

Equations (12) and (13) describe the dynamics of the marginal value associated with the accumulation process of the assets. As usual, in economic growth theory, these equations show that the global remuneration rate (income rate) from the assets must equal society's haste rate. Equation (13) shows the effect of investment in the global economy.

Equation (12) is the Hotelling rule for a renewable resource. The influence on the remuneration rate of changes in ecosystem's organisation is implicit in the term $R'(N)$. This model predicts that the variation rate of the marginal value of the asset is now higher or lower than in traditional models with constant carrying capacity according to whether feedbacks on carrying capacity are positive or negative.

The optimal dynamics of consumption for interior solutions of (10) can be described by differentiating equation (10) with respect to time and using equation (13) to show that:

$$\frac{1}{c} \frac{dc}{dt} = \frac{\delta - \rho}{\sigma(c)}, \quad (15)$$

² Or a marginal benefit from consumption due to the proceeds of extraction. So, this term shows how much the utility changes by changing one unit of the extraction rate.

where $\sigma(c) = -(c/U_c)(dU_c/dc)$ is the elasticity of marginal utility relative to consumption.

The optimal dynamics of biomass extraction for interior solutions of (11), $q(t)$, is obtained by differentiating (11) with respect to time and substituting (12) and (13), yielding:

$$\frac{1}{V'(q)} \frac{dV'(q)}{dt} = \delta - R'(N). \quad (16)$$

This condition implies that any positive disparity between the interest rate of the global economy and the marginal productivity of natural capital must be compensated by an increase in the value of the stock, which is done by increasing the price of the resource or reducing the flow of natural products to the global economy.

The dynamical system consisting of necessary conditions for the controls, $c(t)$ and $q(t)$, and the responses, $K(t)$ and $N(t)$, that maximises the Hamiltonian is composed of equations (6), (7), (15) and

$$\frac{1}{q} \frac{dq}{dt} = \frac{V'(q)}{V''(q)} (\delta - R'(N)), \quad 20(17)$$

for interior solutions. For the optimal policy we must find the functions $c(t)$ and $q(t)$ such that these equations are satisfied for all times.

This system has 4 equations for 4 variables. However, the solution of (7) can be obtained after solving the remaining equations. Additionally, it is possible to solve separately equation (15) and the system of equations (6) and (17). Conceptually, this means that for this society the problem of optimal management of natural capital is independent of the optimal consumption problem. In Appendix I we present a demonstration of the separation of the optimisation problem.

This occurs because we have not considered nature services other than supply of resources, and assumed that the extractive economy does not affect the global economy's interest rate. In other words, this independent maximisation of utility and of extraction revenues is due to the fact that ecosystem resources and services only enter the economy (contributing to utility) through the revenue function. As the optimal consumption problem for this society is independent of the natural resource management problem we may focus on the latter without loss of generality of the optimal policies derived.

Moreover, to clearly depict the corner solutions of the extraction problem it is necessary to study the system composed of the equations for the shadow prices and the state equation for the ecosystem. To do this, a new shadow price is defined as $\lambda \equiv \lambda^K / \lambda^N$, which comes naturally from the optimisation of the ecosystem's revenue as shown in Appendix I.

Lemma 1

For every fixed N , the function

$$V'(q) = \lambda \equiv \varphi^{-1}(q)$$

is a differentiable and strictly increasing function of q . Hence it has a differentiable inverse $\varphi(\lambda) = q$.

Proof: From the first order condition (11) we get $V'(q) = \lambda$, using $\lambda \equiv \lambda^K / \lambda^N$. From the assumptions we have made on the net revenue function $V(q)$, it is obvious that it has a differentiable inverse. This implies that there exists a continuous function, $\varphi(\cdot)$, with the property $\varphi'(\lambda) < 0$, such that it is possible to rewrite condition (11) as $q = \varphi(\lambda)$. Q.E.D.

This lemma provides a way to exchange the state variable with the controls, making it possible to write the system (see Appendix I)

$$\frac{dN}{dt} = R(N) - \varphi(\lambda), \quad (21)$$

$$\frac{1}{\lambda} \frac{d\lambda}{dt} = \delta - R'(N). \quad (22)$$

The corner solution is now obvious ($\varphi(\lambda) = 0$) and can be depicted in $N - \lambda$ phase space.

In the following sections we describe the optimal policies for the management of an ecosystem considering three different scenarios for ecosystem dynamics: pure compensation, depensation and critical depensation.

We shall present the optimal policies using geometrical devices like phase diagrams, where the axes are the ecosystem biomass, N , and its modified shadow price, λ , which is now in capital units. Note that, the shadow price $\lambda(t)$ is inversely related to the extraction rate $q(t)$. We have decreasing strictly positive extraction rates for increasing values of the shadow price, reaching a zero extraction rate for $\lambda \geq V'(0)$. Moreover note that we have imposed the restriction $N \geq 0$ on the phase space. So, the sections of optimal trajectories that lie entirely in the interior of our phase space region, i.e. $N > 0$, satisfy the usual maximum principle. The sections of optimal trajectories that lie entirely on the boundary of this region, $N = 0$, satisfy Theorem 22 of Pontryagin et al. (1962). Finally, every pair of adjoining sections of an optimal trajectory, one of which lies in the interior and the other lies on the boundary, satisfy the junction condition in Pontryagin et al. (1962). We use with this in Appendix III.

4.2. Pure compensation

In figure 4.1 we have the case of an ecosystem with pure compensation. The bioeconomic system possesses one optimal equilibrium solution $N = N^*$ which is a saddle point (Appendix II). As shown in

Appendix III the only possible optimal solutions are those that have initial conditions on the one-dimensional stable manifold that extends for all values of N (global maximum), converging to the steady state, as depicted in figure 4.1.

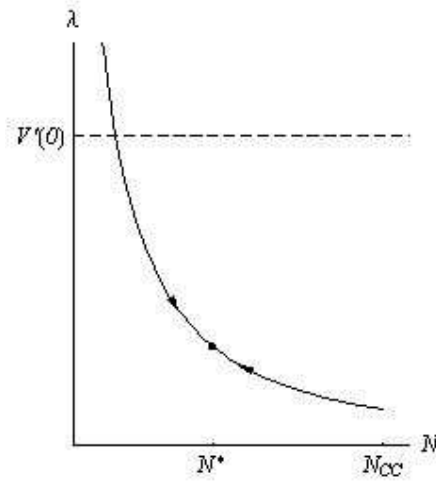


Figure 4.1 – Phase diagram with the optimal trajectories for the extraction revenues maximisation problem in the case of an ecosystem with pure compensation behaviour.

So, the optimal policy implies that the ecosystem will be harvested up to the point, at which its marginal net productivity –which is the rate of return from holding the ecosystem – is equal to the interest rate of the economy, $\delta = R'(N^*)$. This is the usual golden rule of marginal productivity used to derive simple management policies for renewable resources (Clark, 1976). As we will see below, the policies derived in order to achieve the capital level of the golden rule are not optimal in the case of ecosystems with depensation and critical depensation.

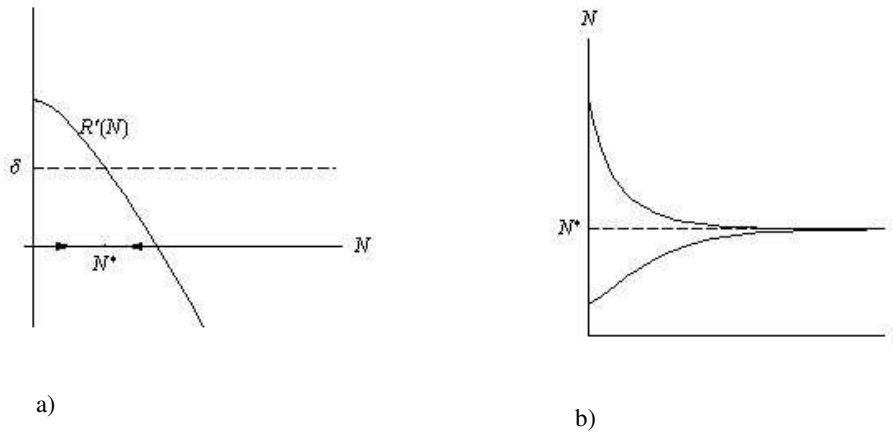


Figure 4.2 – a) Graphical representation of the solution of $\delta = R'(N)$ for an ecosystem with pure compensation behaviour. b) Optimal stock paths $N(t)$ for two different initial conditions.

In terms of management policies, the marginal productivity rule states that, if the interest rate exceeds the rate of return from holding the biomass constant, it is more profitable to harvest the resource at a rate higher than the regeneration rate and invest the proceeds in the global economy. In particular, if $\delta \geq R'(0)$ it follows that extinction is the optimal policy (Clark, 1976). On the other hand, the ecosystem will continue to grow or regenerate if that choice yields a greater net present value than immediate harvesting without regeneration. Hence, a renewable resource is a relatively poor investment if its natural regeneration rate is less than the rate of return of investing in capital in the global economy.

Figure 4.2 shows the approach to the bioeconomic equilibrium N^* . This solution corresponds to following the one-dimensional stable manifold in figure 4.1, and has the stock-level path depicted in figure 4.2 b). Numerically computing the separatrices show in figure 4.1 we have the optimal feedback control law for the extraction of ecosystem products. This, as seen in figure 4.2 b) is an optimal approach path that is a gradual approach, engendered by market reactions to the harvest rate (Clark, 1976).

4.3. Depensation

If we are exploiting an ecosystem with depensation behavior, we expect that the optimal policy should be different from that of an ecosystem with less self-organization (pure compensation). In particular, we expect this to happen for low levels of the ecosystem's biomass, since it is for low biomass levels that the main qualitative differences appear in the marginal growth rates. This can be seen by comparing figure 4.2 a) with figure 4.3.

In this case, there are now two steady states, one stable and another unstable. In appendix II we show that one is a saddle-point and the other may be an unstable focus or an unstable node depending on the interest rate.

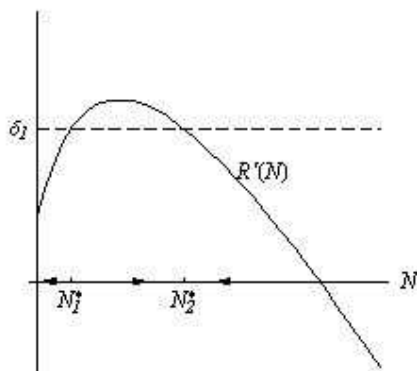


Figure 4.3 – Graphical representation of the solutions of $\delta_1 = R'(N)$ for an ecosystem with depensation behaviour. In this case there is a zone of optimal extinction of the ecosystem.

From figure 4.3, using the golden rule of marginal productivity we can still draw some general ideas about the optimal policy for the management of an ecosystem with depensation behavior. If $N_0 > N_2^*$

then the optimal solution would be, as for an ecosystem with pure compensation, to harvest it at a rate higher than its regeneration. Consequently the ecosystem biomass would decrease, approaching the equilibrium biomass N_2^* . If the ecosystem has a higher marginal growth than the interest rate it is more profitable to harvest the ecosystem at a rate lower than its regeneration, so that its biomass approaches the equilibrium biomass. In this case, this leads to the prediction of optimal extinction policies for low ecosystem biomass levels. However, when the initial state of the ecosystem is near the unstable steady state, Pontryagin's maximum principle does not provide a unique optimal policy, as we will show, and the application of the golden rule fails to yield optimal policies.

In appendix III we prove that the optimal candidate solutions are those depicted in figure 4.4. Near the focus, the optimal trajectories oscillate, making it possible, for a given, the existence of multiple optimal initial extraction rates. Each of these is a local maximum of the value function subject to $N(0) = N_0$.

The decision of what extraction rate to choose should then be made in terms of revenue gains (i.e. choosing the global maximum from a set of local maxima). To choose global optimal candidates among these we use the criterion devised by Skiba (1978) and generalized in Brock and Starrett (1999). Hence, even though the golden rule of capital accumulation is widely accepted and used for renewable resource management, here we see that it cannot be generalized to an ecosystem with depensation and critical depensation behavior. The optimal feedback control law must be derived by solving equations (21) and (22), with $q = \varphi(\lambda)$.

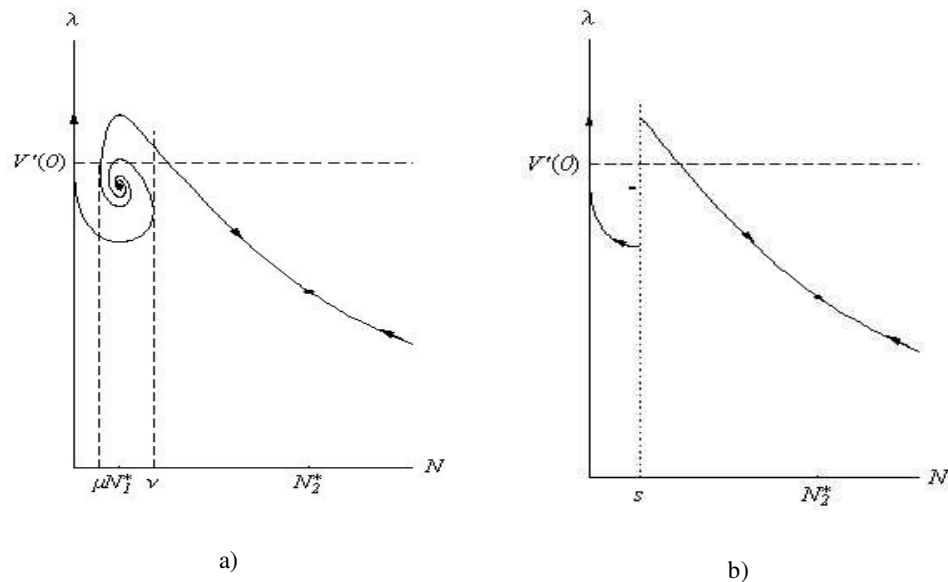


Figure 4.4 – a) Phase diagram with the optimal candidate solutions for an ecosystem with depensation, showing the interval $[\mu, \nu]$ containing the Skiba point. **b)** Phase space depicting the optimal trajectories after applying Lemma 3.

There exists, now, more than one locally stable manifold satisfying all transversality and necessary conditions. Our strategy is, as in Brock and Starrett (1999) or Wagener (2001), to use the Hamilton-Jacobi-Bellman (HJB) equation to compute a collection of ‘candidate’ value functions $W_i(q^*; N_0)$ for every i . The following lemma is taken from Skiba (1978) (proposition 2).

Lemma 2:

If (N, q^) is an optimal trajectory starting at (N_0, q_0^*) , then the following equality holds:*

$$W(q^*; N_0) \equiv \int_0^\infty V(q^*)e^{-\delta t} dt = \frac{1}{\delta} H(N_0, q_0^*, \lambda_0^*), \quad (23)$$

where $H(N_0, q_0^*, \lambda_0^*) = V(q_0^*) + \lambda_0^*[R(N_0) - q_0^*]$, and $q_0^* = \varphi(\lambda_0^*; N_0)$.

Note that, as stressed by Wagener (2001) in the context of lake management, the value function W is defined for all starting points on optimal trajectories.

Given this, the globally optimal value function is given by

$$W(q^*; N_0) = \max_i [W(q_i^*; N_0)], \quad (24)$$

where the index i represents different locally optimal trajectories, and the criterion for the comparison of the different value functions depending on q_i^* is the following.

Lemma 3: (Candidate value function comparison)

Consider two optimal extraction rates q_1^ , q_2^* implying two optimal candidate value functions $W(q_1^*)$ and $W(q_2^*)$, evaluated at any chosen N_0 . If*

$$\lambda_2^* dN_1/dt \geq \lambda_1^* dN_1/dt \quad (25)$$

where $dN_i/dt = R(N_0) - q_i^*$; then $W(q_2^*) \geq W(q_1^*)$.

Proof: Applying lemma 2 for the comparison of the two value functions we have

$$\delta[W(q_2^*) - W(q_1^*)] = V(q_2^*) - V(q_1^*) + \lambda_2^* dN_2/dt - \lambda_1^* dN_1/dt$$

Using inequation (25), we get $\delta[W(q_2^*) - W(q_1^*)] \geq V(q_2^*) - V(q_1^*) + \lambda_2^* [dN_2/dt - dN_1/dt]$. From equation (6) the right hand side of the former inequality becomes $V(q_2^*) - V(q_1^*) + \lambda_2^* [q_2^* - q_1^*]$.

Knowing that $V(q)$ is concave and $\lambda_2^* = V'(q_2^*)$, we have $V(q_2^*) - V(q_1^*) + \lambda_2^*[q_2^* - q_1^*] \geq 0$.
Q.E.D.

This lemma is equivalent to proposition 5 of Brock and Starrett (1999) or Lemma 4 of Wagener (2001).

From figure 4.4 we obtain the same management policies as the ones derived from figure 4.3 for $N_0 > \nu$. The same is true for the discussion of the existence of optimal extinction using figure 4.3, i.e., for $N_0 < \mu$, the optimal candidate management policy is to follow the unstable one-dimensional manifold that approaches $N = 0$ asymptotically. The existence of optimal trajectories towards the saddle point or towards $N = 0$ implies the existence of a Skiba point (Brock, 1999; Wagener, 2001). This is the initial state where the long run steady states for the optimal programs change.

So, applying lemma 3 to find the optimal candidate solutions for all values of ecosystem biomass in the case of convex-concave ecosystem production functions we have the following lemma.

Lemma 4:

For the exploitation of an ecosystem with a convex-concave production function, there is a unique Skiba point s in the interval $[\mu, \nu]$ such that the saddle point is the long run optimal candidate solution for $N_0 > s$ and the optimal extinction of the ecosystem is the long run optimal solution for $N_0 < s$.

Proof: This lemma is obvious since for $N_0 > \nu$ the saddle manifold must be optimal, the extinction manifold must be optimal for $N_0 < \mu$ and by continuity of the solution there must be a switch point inside $[\mu, \nu]$. Q.E.D.

By using lemma 1, the sections of the optimal trajectory on the boundary of the controllability domain appear naturally. For instance, if $s < N_0 < \nu$ the optimal trajectory has a period of zero extraction rates followed by the gradual approach to the saddle point extraction rate.

A direct implication of lemma 4 is the following generalization of the golden rule of marginal productivity for convex-concave ecosystem production functions.

Lemma 5: (Generalized golden rule of marginal productivity)

Consider the conditions of lemma 4.

If $s < N_1^$ then the golden rule is applied for every N except in the interval $[s, N_1^*]$, where the optimal policy is to let the ecosystem regenerate.*

If $s > N_1^$ then the golden rule is applied for every N except in the interval $[N_1^*, s]$, where the optimal policy is to harvest at a rate higher than its regeneration.*

If $s = N_1^$ then the golden rule is applied for every N .*

This Lemma is an important remark due to the wide acceptability of golden rule derived policies, and the high probability of encountering a renewable resource with a convex-concave regeneration function in management problems.

The existence of a Skiba point indicates that the long-term behavior of the bioeconomic system will be history dependent: in our case, this means that according to the conditions prevailing in the first phases of development, the ecosystem is led to the saddle point or towards extinction. Small changes of initial values near the Skiba point have dramatically different responses, one leading to the sustainable conservation of the ecosystem and the other leading to its extinction.

An ecosystem needs to be a high productivity system in order to be maintained by the owners. It is important, however, to stress that the optimal extinction of an ecosystem involves costs that are not considered in this model, such as increasing cost of extraction due to scarcity of resources, social costs associated with environmental functions and other externalities. Still, optimal extinction may be a real policy option for the management of natural resources.

Moreover, this only depends on the productivity of the ecosystem relative to the interest rate and the initial level of the ecosystem biomass. If the interest rate has a sudden increase (more economic pressure on the ecosystem), the realm of optimal extinction increases, as can be seen from figure 4.3 and 4.4. However, both, pure compensation and depensation ecosystems may be prevented from optimal extinction if the interest rate becomes sufficiently low. Accordingly, if the ecosystem has depensation behavior and if the interest rate is lower than $R'(0)$, the optimal management policies will be the same as in the case of pure compensation behavior.

4.4. Critical depensation

The candidate optimal trajectories for the management problem with critical depensation are depicted in figure 4.7. The ecosystem threshold biomass N_{cr} is where the dashed curve crosses the line $\lambda = V'(0)$. The Skiba point, s , for this case is depicted in the picture.

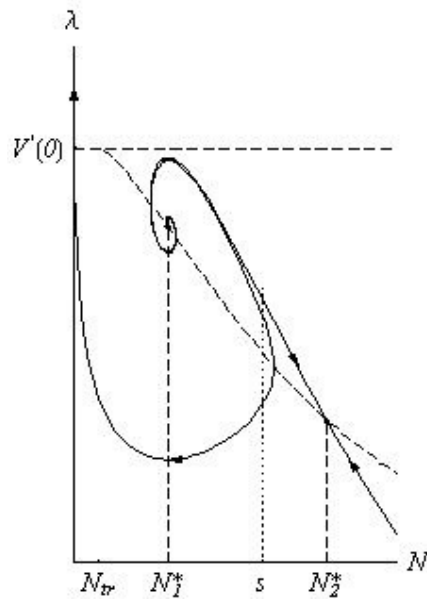


Figure 4.7 – Phase diagram with the optimal trajectories for an ecosystem exhibiting critical depensation.

If the ecosystem has critical depensation behavior, the optimal system exhibits all the phenomena encountered in the depensation case, as well as an additional phenomenon: irreversibility. All the lemmas used in the previous section hold for critical depensation. However, for pure compensation and depensation there are certain interest rates that prevent the existence of history dependence, whereas in critical depensation there is a Skiba point in the interval $[\mu, V]$ for any interest rate for which $\delta = R'(N)$ has solutions. In general, in the case of critical depensation there are always initial conditions for which it is optimal to extinguish the ecosystem for any interest rate. This can be seen in figure 4.8. This shows that it is only in special cases that management policies are the same for ecosystems with different self-organizing capability.

Now, comparing two ecosystems with the same biomass, the conclusion, as expected, is that extracting and exporting is still a good investment for higher interest rates or, for the same interest rate, the equilibrium population corresponding to the saddle point is greater. This can be seen from figure 4.9, where we have plotted the marginal growth curves for two ecosystems, one with pure compensation and the other with depensation behaviour. This means that when the system loses organization it also loses productivity and competitiveness for the global economy. The ecosystem's self-organization potentiates the creation of new ecosystem biomass, increasing the surplus from holding the resources, and consequently augmenting the range for economic profitability. On the other hand, for low biomass levels the ecosystem with pure compensation behaviour has a greater surplus.

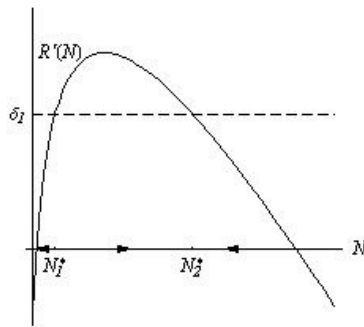


Figure 4.8 – Bioeconomic equilibria with an ecosystem with critical depensation behaviour. In this case there is always a zone of optimal extinction ecosystem.

The autocatalytic relations between the actual biomass and ecosystem carrying capacity creates extreme states of productivity for the ecosystem, whether they are of very high productivity or very low.

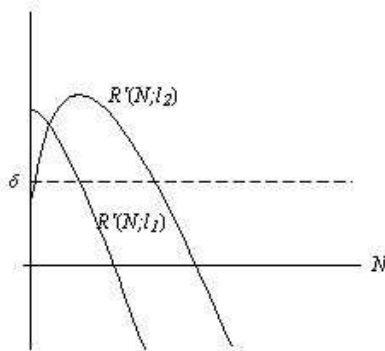


Figure 4.9 – Comparison of ecosystems with different self-organization capabilities. In this case, according to figure 3.1a) we have $l_1 < l_{crit}^{(1)} < l_2$.

5. Optimal policies bifurcation diagram

As seen above, depending on the relative values of the parameters of the model, the optimal policies for the ecosystem management are different. We sketch this changes in what we call optimal policies bifurcation diagrams (Fig. 5.1). These diagrams represent different optimal policies in various l - h diagrams for different relative values of CC_l , as in accordance with the ecological bifurcation diagrams. The line separating SP from E - SP is a line of saddle-node bifurcations points, and the line separating E from E - SP is a line of heteroclinic bifurcations.

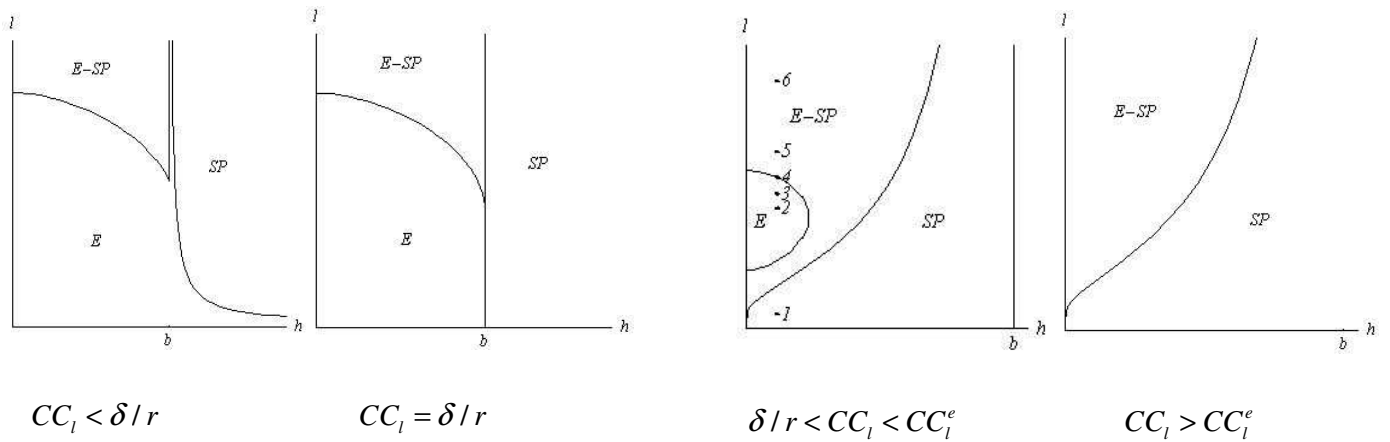


Figure 5.1 – Optimal policies bifurcation diagrams. **a)** *SP* means Saddle point directed policies. *E* means optimal extinction policies. *E-SP* means that the system has a both optimal policies depending on the initial ecosystem biomass level.

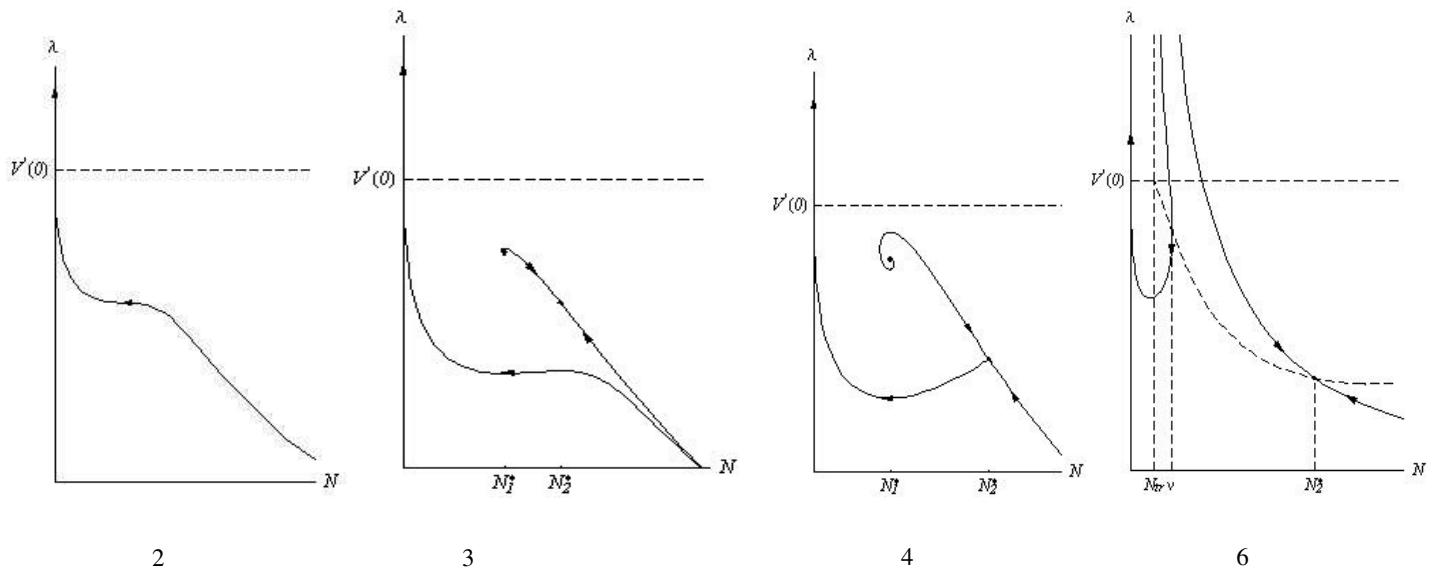


Figure 5.2 – Phase portraits for the different parameter points in figure 5.1.

The phase portrait of point 1 in figure 5.1 is figure 4.1, and for point 5 is figure 4.7. For the phase portrait 3 we have represented trajectories that are not optimal. The only optimal trajectory is the optimal extinction path. This is known by applying Lemma 3 to any point higher than N_2^* . The phase portrait 4 represents a heteroclinic bifurcation. Finally, phase portrait 6 represents the case where the equilibrium N_1^* vanishes by crossing the line $\lambda = V'(0)$. This happens for high values of l . Nevertheless, applying

Lemma 3 to the interval $[N_{ir}, V]$ we know that the optimal trajectory is the saddle path. So, for $N < N_{ir}$ the optimal policy leads the ecosystem to extinction, and the optimal policy leads to the saddle point for $N > N_{ir}$.

6. Acknowledgments

This work is integrated and has been supported by the project “General Theory of Sustainability and Application to the Agricultural Sector” financed by POCTI/MGS/47731/2002.

7. Concluding remarks

In this work we have characterized and discussed the optimal management policies for an ecosystem with self-organization along ecological succession. For this, we have used a phenomenological model for the ecosystem dynamics that grasps the dependence of the ecosystem productivity on its self-organisation. The (variable) ecosystem carrying capacity is assumed to be a measure of the self-organisation of the system. This has a sound ecological interpretation, related to niche specialisation along ecological succession.

We performed a dynamical optimisation of an extractive economy exploiting this ecosystem. The main conclusion is that the optimal policies will, in general, depend on the organization of the ecosystem. Only in the case of the interest rate being low enough are pure compensation and depensation ecosystems managed with the same type of optimal policies. Moreover, the usual golden rule of marginal productivity of renewable resources has been generalised to derive optimal policies for convex-concave ecosystem production functions. For ecosystems with depensation or critical depensation behaviours we show the existence of a Skiba point. Hence, there is history dependence in this bioeconomic model.

Generally, we can conclude that an ecosystem with high self-organization capability has a wider range of profitability than an ecosystem with lower self-organization capability. On the other hand, it also has a higher realm of optimal extinction. So, managers must balance the need for high productivity of the ecosystem with its high risk of extinction.

8. References

- Brock W., A., Starrett D., 2003 Managing systems with non-convex positive feedback. *Environmental and Resource Economics* 26:575-602.
- Clark, C., W., 1976. *Mathematical bioeconomics: The optimal management of renewable resources*. John Wiley and Sons, New York
- Cohen, J. E., 1995. Population Growth and Earth's Human Carrying Capacity. *Science* 269: 341-348.
- Crépin, A. S., 2003. Multiple species boreal forests – What Faustmann missed. *Environmental and Resource Economics* 26:625-646.

- Dasgupta, P., Maller, K. G., 2003. The economics of non-convex ecosystems: Introduction. *Environmental and Resource Economics* 26:499-525.
- Jones, C. G., Lawton, J. H., 1995. *Linking Species & Ecosystems*. Chapman & Hall, New York.
- Jorgensen, S. E., 1992. *Integration of Ecosystem Theories: A Pattern*. Kluwer Academic Publishers, Copenhagen.
- Jorgensen, S., E., Patten, B., C., Straskrba, M., 1999. Ecosystems emerging: 3. Openness. *Ecological Modelling* 117:41-64.
- Kay, J. J., Schneider, E., 1994. Embracing Complexity: The Challenge of the Ecosystem Approach. *Alternatives* 20(3): 32-38
- Maler, K. G., Xepapadeas, A., Zeeuw, A., 2003 The economics of shallow lakes. *Environmental and Resource Economics* 26:603-624.
- Mota, R. P., Domingos, T., J. Rodrigues, 2004. Derivation and analysis of an ecological production function with structural dynamics. Submitted to *Ecological Modelling*.
- Odling-Smee, F. J., Laland, K. N., Feldman, M. W., 2003. *Niche Construction. The neglected process in evolution*. Monographs in population biology: 34. Princeton University Press, New Jersey.
- Odum, E. P., 1969. The Strategy of Ecosystem Development. *Science*, 164: 262-270.
- Pontryagin, L., S., Boltyanskii, V., G., Gamkrelidze, R., V., Mishchenko, E., F., 1962. *The mathematical theory of optimal processes*. John Wiley and Sons, New York.
- Rodrigues, J., T. Domingos, P. Conceio, 2002. Human-Nature Interactions – The problem of structural interference. Unpublished manuscript.
- Roughgarden, J., 1997. Production Functions from Ecological Populations: A Survey with Emphasis on Spatially Implicit Models. In Tilman and Kareiva (eds.), 1997 Princeton University Press, New Jersey.
- Scholes, R. J., 2003. Convex relationships in ecosystems containing mixtures of trees and grass. *Environmental and Resource Economics* 26:559-574.
- Skiba, A., K., 1978. Optimal growth with a convex-concave production function. *Econometrica*, 46 N3 527-539.
- Tu, P., N., V., 1994. *Dynamical Systems: An Introduction with Applications in Economics and Biology*. Springer-Verlag, New York.
- Wagener, F., O., O., 2003. Skiba points and heteroclinic bifurcations, with applications to the shallow lake system. *Journal of economic dynamics and control*, 27(9): 1533-1561.
- Walker, L. R., 2003. *Primary Succession and Ecosystem Rehabilitation*. Cambridge University Press, Cambridge.

Appendix I – Corner solutions and separation of problems

As mentioned in the text, the system we obtained can be viewed as maximizing two separate problems. One is the problem of maximization of utility using the consumption rate and the other is the maximization of economic revenue using the natural resource extraction rate. In a more formal perspective, if we could define the problems that give rise to the solutions here presented we would have demonstrated the separability of the optimization problems.

Formalizing the two problems, we have for the utility maximization,

$$\max_c \int_0^{\infty} U(c)e^{-\rho t} dt, \text{ s.t.}$$
$$dK/dt = \delta K + V(q) - c. \quad (\text{A1.1})$$

The Hamiltonian is

$$H(K, c, \lambda^K) = U(c) + \lambda^K [\delta K + V(q) - c], \quad (\text{A1.2})$$

and the shadow price equation is

$$\frac{1}{\lambda^K} \frac{d\lambda^K}{dt} + \delta = \rho, \quad (\text{A1.3})$$

with the transversality condition

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda^K K = 0,$$

which are the same as the equations derived from the original optimization problem. The first order condition is $H_c = 0 \Leftrightarrow U'(c) = \lambda^K$, from which equation (19) can be derived.

Now, for the maximization of the ecosystem's rent we have,

$$\max_q \int_0^{\infty} V(q)e^{-\delta t} dt, \text{ s.t.}$$
$$dN/dt = R(N) - q. \quad (\text{A1.4})$$

The functional objective now represents a sum of financial capital flows discounted using the rate of interest of the global economy. The Hamiltonian function is

$$H(N, q, \lambda) = V(q) + \lambda[R(N) - q], \quad (\text{A1.5})$$

where the shadow price, λ , is the marginal value of the ecosystem in financial capital units, so it must have units of financial capital per 'ecosystem capital units'. The shadow price equation is

$$\frac{1}{\lambda} \frac{d\lambda}{dt} = \delta - R'(N). \quad (\text{A1.6})$$

with the transversality condition given by

$$\lim_{t \rightarrow \infty} e^{-\delta t} \lambda N = 0.$$

Defining $\lambda \equiv \lambda^K / \lambda^N$, we have

$$\frac{d\lambda}{dt} = \frac{1}{\lambda^K} \frac{d\lambda^K}{dt} - \frac{\lambda^N}{\lambda^K} \frac{1}{\lambda^K} \frac{d\lambda^N}{dt}. \quad (\text{A1.7})$$

Substituting this and (A1.3) in (A1.6), we obtain

$$\frac{1}{\lambda^N} \frac{d\lambda^N}{dt} + R'(N) = \rho, \quad (\text{A1.8})$$

which corresponds to the shadow price equation derived from the original optimization problem. The first order condition is $V'(q) = \lambda$, which leads to equation (20), proving that the optimal system for the original problem is equivalent to having the two above-mentioned dynamical systems.

To make the corner solutions appear naturally in phase space we have to use lemma 1 with the state equation (17) and equation (A1.6). So, the system to be analyzed is:

$$\frac{dN}{dt} = R(N) - \varphi(\lambda), \quad (\text{A1.9})$$

$$\frac{1}{\lambda} \frac{d\lambda}{dt} = \delta - R'. \quad (\text{A1.10})$$

To obtain the system corresponding to the corner solutions we simply have to consider a zero extraction rate, or $\varphi(\lambda) = 0$ in equation (A1.9). So, if we are considering the phase space N - λ the separation of the interior from the corner solutions is when the extraction rate is zero, which corresponds to $\lambda = V'(0)$, from Lemma 1. So, the next question is whether the interior and corner solutions verify a junction condition imposing that they are continuously differentiable when $\lambda = V'(0)$.

To study the differentiability of solutions we have to look at the derivative of the trajectories at the junction line,

$$\frac{d\lambda}{dN} = \frac{\delta - R'(N)}{R(N) - \varphi(\lambda)}. \quad (\text{A1.11})$$

Observing that, either, when $\varphi(\lambda) \rightarrow 0^+$ or $\varphi(\lambda) \rightarrow 0^-$, $d\lambda/dN \rightarrow (\delta - R')/R$ we conclude that the solutions are continuous and differentiable at $\lambda = V'(0)$.

Appendix II – Qualitative characterization of equilibria

Consider the nonlinear system $dx/dt = f(x)$. Using Taylor expansion about some equilibrium x^* , we have

$dx/dt = Ax + \zeta(x)$, where

$$A \equiv Df(x)_{x=x^*} \equiv \begin{pmatrix} \partial f_1 / \partial x_1 & \dots & \partial f_1 / \partial x_n \\ \vdots & \ddots & \vdots \\ \partial f_n / \partial x_1 & \dots & \partial f_n / \partial x_n \end{pmatrix}_{x=x^*},$$

and $\zeta(x)$ is such that $\lim_{x \rightarrow x^*} \zeta(x) = 0$. The system $dx/dt = Ax$ is the linearized system.

Theorem A4.1 (Linearization theorem of Hartman & Grobman, in Tu, 1994). Let the nonlinear dynamic system $dx/dt = f(x)$ have a hyperbolic equilibrium point x^* . Then, in the neighborhood of $x^* \in \mathbb{R}^n$ the phase portraits of the nonlinear system and its linearization are equivalent.

This theorem simply states that provided $\text{Re}(\alpha_i) \neq 0$ then the nonlinear and linear systems are qualitatively equivalent near the equilibrium point.

Applying the linearization theorem to our system,

$$dN/dt = R(N) - \varphi(\lambda), \quad (\text{A2.1})$$

$$\frac{1}{\lambda} \frac{d\lambda}{dt} = \delta - R'(N). \quad (\text{A2.2})$$

we have that

$$A = \begin{pmatrix} R'(N) & -\varphi'(\lambda) \\ -\lambda R''(N) & \delta - R'(N) \end{pmatrix}_{(N^*, \lambda^*)} = \begin{pmatrix} \delta & -\varphi'(\lambda^*) \\ -\lambda^* R''(N^*) & 0 \end{pmatrix}. \text{ Now, to find the eigenvalues}$$

α_i of A we solve $\det(A - \alpha I) = 0$. This is equivalent to solving the characteristic polynomial

$$P(\alpha; N^*) \equiv \alpha^2 - \delta\alpha + (R(N) + 1)R''(N) = 0. \quad (\text{A2.3})$$

We will now make two propositions concerning the types of steady states of our system. We use geometrical devices for the proofs in order for it to be more intuitive. However, the detailed derivation of the figures used is made in Mota and Domingos (2004).

Proposition A2.2 – Let the system comprised of equations (21) and (22) have one equilibrium point N^* . Then N^* is a saddle point.

Proof. N^* being a saddle point is equivalent to having $\alpha_1 < 0 < \alpha_2$ (Tu, 1994). From $P(\alpha; N^*)$ we know that the coordinate of its vertex is $(\delta/2, \Delta - \delta^2/4)$, where $\Delta \equiv (R + 1)R''$. So, for different equilibria, Δ is the only variable expression, and its sign is controlled by R'' . The solutions of $\delta - R' = 0$ are depicted in figure A2.1 for the case of depensation and compensation and for different interest rates. For critical depensation there are always two steady states (see figure 4.8), so we may focus on the depensation case without loss of generality. We can see from figure A2.1 that in the case of one equilibrium, $\delta = \delta_2$, we have $R''(N^*) < 0$. So $\Delta < 0$ or $P(0; N^*) < 0$, which, combined with the position of the vertex of the characteristic polynomial, implies that there is one positive and one negative solution of $P(\alpha; N^*) = 0$. This is the case of a saddle point.

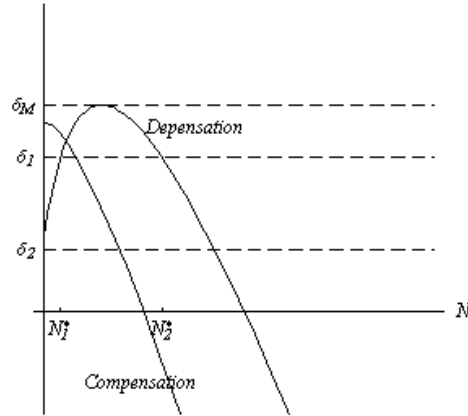


Figure A2.1 – Graphical solution of $\delta = R'(N)$ in the case of compensation and depensation, for several interest rates.

Proposition A2.3 – Let the system comprised of equations (A2.1) and (A2.2) have two equilibrium points $N_1^* \neq N_2^*$. Then one is a saddle point and the other is an unstable focus or an unstable node. Furthermore, there is a saddle-node bifurcation for $\delta = \delta_M$ (figure A2.1).

Proof. If there are two equilibrium points, $\delta = \delta_1$ for instance, without loss of generality, then (as seen from figure A2.1) we know that $R''(N_1^*) > 0 > R''(N_2^*)$. This, as the proof of the last theorem states, suffices to show that the equilibrium at N_2^* is a saddle point. Using the same argument for N_1^* , we can conclude that N_1^* is an unstable focus for low δ becomes an unstable node for increasing δ and later a saddle-node bifurcation occurs for $\delta = \delta_M$. The characteristic polynomial for the unstable node and unstable focus cases are represented in figure A2.2. We know that they are unstable since the real part of the eigenvalues is given by $\delta/2 > 0$. When $\delta = \delta_M$, the linearization of the system is not valid since there is an eigenvalue that is zero (bifurcation point).

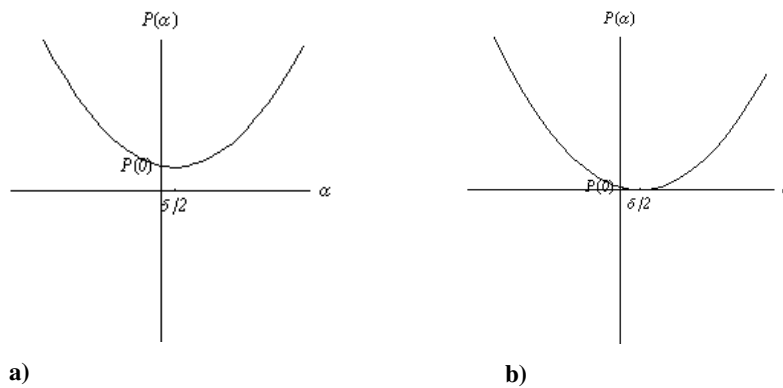


Figure A2.2 – Characteristic polynomial for the unstable focus and the unstable node. In the cases depicted we have the eigenvalues of A: a) $\alpha_1 = 0.1 - i0.393$ and $\alpha_2 = 0.1 + i0.393$; b) $\alpha_1 = 0.124$ and $\alpha_2 = 0.141$.

Appendix III – Using transversality conditions

Since each phase plane diagram gives necessary conditions for the trajectories, not all of them are optimal; hence we have to decide what trajectories should be considered a solution of the optimisation.

Pure compensation

The phase portrait of the system composed of equations (21) and (22) for pure compensation growth is depicted in figure A3.1. The bioeconomic system possesses one steady state $N = N^*$ which is a saddle point (Appendix II). In the interior of the phase space there are four kinds of trajectories: the equilibrium; the stable and the unstable separatrices of the saddle point; the trajectories that lead the ecosystem to extinction; and the trajectories that approach N_{CC} for $t \rightarrow +\infty$.

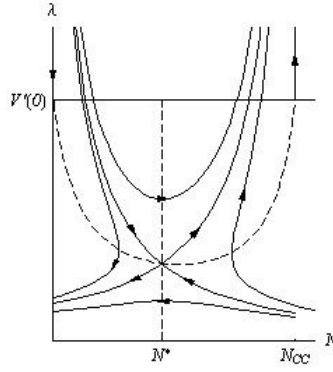


Figure A3.1 – Phase portrait of the system of equations (21) and (22) for a pure compensation ecosystem production function.

In our phase space, the boundary for the ecosystem biomass is $N = 0$. The jump condition in Pontryagin et al. (1962) implies that the optimal trajectory remains smooth at the junction point. So, the trajectories that lead the ecosystem to extinction in the case of compensation are not optimal since they are not smooth at $N = 0$. This also holds in the cases of depensation and critical depensation.

In the case of trajectories that are increasing in λ for $t \rightarrow +\infty$, if they do not verify the transversality condition,

$$\lim_{t \rightarrow \infty} e^{-\delta t} \lambda N = 0, \quad (\text{A3.1})$$

they are not solutions of the optimisation problem. For t near infinity, we know that $N \approx N_{CC}$ from the dynamics of the ecosystem biomass equation (21), and $d\lambda/dt \approx \lambda(\delta + \xi)$, where $\xi = -R'(N_{CC}) > 0$, as we know from equation (3) and can be seen in figure 4.2 a). Integrating, we have $\lambda \approx e^{(\delta + \xi)t}$, and substituting in the condition (A3.1) we obtain,

$$\lim_{t \rightarrow \infty} e^{\xi t} N_{CC} = +\infty, \quad (\text{A3.2})$$

which violates the transversality condition. Therefore the candidate optimal trajectories are depicted in figure 4.1.

Depensation

The phase portrait for an ecosystem with depensation growth is showed in figure A3.2. The results of the previous section for pure compensation growth still hold for depensation growth except for the trajectory that approaches $N = 0$ asymptotically.

Applying transversality condition (20) we know from equation (18), and can be seen in figure A.3.2, that if t' is higher than some t^* , for which we have $N(t^*) = 0$, then $N(t') = 0$. This implies that the transversality

condition is verified, and the trajectory is a candidate optimal trajectory. So, the optimal candidate solutions for the depensation case are depicted in figure 4.4.

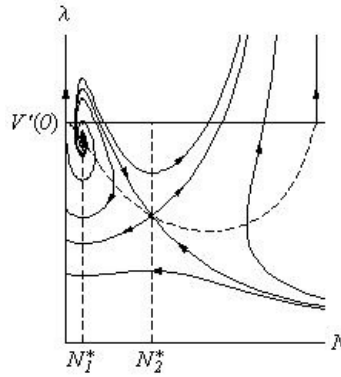


Figure A3.2 – Phase portrait of the system of equations (21) and (22) for a depensation ecosystem production function.

Critical depensation

The phase portrait for an ecosystem with depensation growth is showed in figure A3.3. In order to obtain the candidate optimal trajectories for the exploitation of an ecosystem with critical depensation we can use the results obtained for compensation and depensation. So, we need only to prove whether the trajectories that asymptotically tend to zero with no extraction rate are candidate optimal trajectories or not. Following these trajectories, for t near infinity, $N(t)$ approaches zero and $d\lambda/dt \approx \lambda(\delta + \zeta)$, where λ , as know from equation (2) and depicted in figure A.3.3. Integrating, we have $\lambda \approx e^{(\delta+\zeta)t}$. We may, now, use an approximation of the trajectories of interest near zero ecosystem biomass. For this purpose we linearize the function in the neighborhood of zero as,

$$R(N) = R(0) + R'(0)N + o(N^2). \quad (\text{A3.3})$$

This can be rewritten as $\lambda = e^{-\zeta t}$, from which we know that $N(t) = e^{-\zeta t}$. Substituting this in the transversality condition we get,

$$\lim_{t \rightarrow \infty} e^{-\zeta t} e^{\zeta t} = 1, \quad (\text{A3.4})$$

implying that the trajectories are not candidate optimal policies. This makes sense, since it would not be optimal to stop the extraction rate for some positive value of ecosystem biomass, for if the ecosystem is going to extinction then it is better to extinguish it by using some positive extraction rate and obtain revenues from it. Thus, the optimal candidate solutions for the critical depensation case are those depicted in figure (4.7).

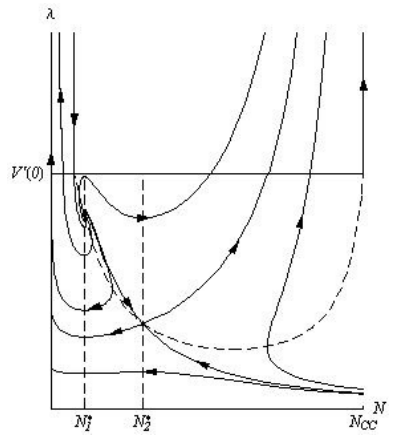


Figure A3.3 – Phase portrait of the system of equations (21) and (22) for a critical depensation ecosystem production function.