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Abstract

The main focus of this paper is twofold. First is to design Pigouvian taxes for different kinds of environmental external effects on the market economy in order to be possible to say what is the relevant information in each situation. Second is to address the question of the welfare effects of using close to Pigouvian taxes in real economies. This is done by considering more general conditions than those used by Aronsson et al. (2003). In particular, we take the ecosystem dynamics as being density dependent, and we assume that there are errors in measuring both the consumer's marginal utility of nature and its regeneration rate. Additionally, we derive cost benefit rules for a green tax reform on the context of a two sector economy, in particular, a final good sector and a knowledge sector, considering two externalities - one environmental externality and one technology externality.

Keywords: Environmental externalities, Non-Pigouvian taxes, social cost benefit analysis

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1 Introduction

The literature on green accounting has grown enormously over the last few decades. Recent times have witnessed a growing awareness of the interactions between economic, social and environmental issues (Weitzman, 2003). There has appeared a widespread interest in the idea of extending the concepts and measurement of national income to include important nonmarket activities in related areas that affect welfare and productivity - in particular, environmental goods and services.

Many questions have been raised about augmented (or comprehensive) national income accounting ranging from the broad concerns about its welfare foundations, to the basic issues of the design of green national income accounts. As Weitzman (2003) puts it, at the core of this branch of economic analysis runs a common strand attempting to connect a currently observable index of comprehensive net national income or product (NNP) with some appropriate but not observable welfare measure of future power to consume, which typically has a "sustainability-like" flavor. These ideas are close to the ideas of classic economists such as Fisher, Lindahl, Hicks, Pigou, Hayek and many others interested in the concept of income, and in linking this concept to other important concepts in capital theory (Weitzman, 2003).

Empirical attemps have also been made to improve the accounting practices by adding imputed values for the depletion of natural capital to the net national product and consumers' valuation of environmental services (Hartwick, 1990; Atkinson et al.1999; Perrings and Vincent, 2003; Pezzey et al., 2005). Most studies are based on the paper by Weitzman (1976), which showed that for the case of an economy with no technological progress and with utility being equal to consumption, if this economy follows the optimal path, an augmented net national product measure is directly proportional to the present value of future utility of the representative consumer. This framework was then generalized to include situations where natural resources are important parts of the economic system (Hartwick, 1990) or to include non-linear utility functions and consequently accounting for the consumer surplus (Aronsson et al., 2003).

There are two main critiques made to green accounting and its relation to the sustainability of dynamic economies. One relates to the theoretical results that state that (changes in) hamiltonian measures are an indicator of (non-sustainability) welfare but are derived in utility units. Aronsson et al. (2003), Asheim and Weitzman (2001) among others contribute to this topic. The other critique relates to the first best characteristic of these results. The data for the calculation of green NNP and genuine savings is obtained from an imperfectly functioning economy. Hence, the mismatch in Pezzey et al. (2005). Following a series of papers from Aronsson and Lofgrën (1998, 1999) and Aronsson et al., (2003) it is clear how the impact of external effects should be taken into account in welfare measures and sustainability concerns.

At a theoretical level one can use dynamic Pigouvian taxes to fully internalize external effects. Since dynamic Pigouvian taxes values the depletion of natural resources and effects of the representative consumer, they can also be usefull for green accounting. Usually it has been suggested the use of willingness to pay techniques to implement Pigouvian taxes (Aronsson and Lofgrën, 1999; Aronsson et al, 2003). This brings to discussion the problem of not being able to desing and implement the theoretical Pigouvian tax. The problem of small errors in the information collected through willingness to pay methods with the objective of designing Pigouvian taxes was adressed by Aronsson et al. (2003) in the context of the Brock model. The question is under what conditions the close to Pigouvian taxes are welfare improving.

The main focus of this paper is twofold. First is to design Pigouvian taxes for different kinds of environmental external effects on the market economy in order to be possible to say what is the relevant information in each situation. Second is to address the question of the welfare effects of using close to Pigouvian taxes in real economies. This is done by considering more general conditions than those used by Aronsson et al. (2003). In particular, we take the ecosystem dynamics as being density dependent, and not a constant as in Aronsson et al. (2003) and we assume that there are errors in measuring both the consumer's marginal utility of nature and its regeneration rate. Additionally, we derive cost benefit rules for a green tax reform on the context of a two sector economy, in particular, a final good sector and a knowledge sector, considering two externalities - one environmental externality and one technology externality.

Section 2 presents the one sector model, derives the Pigouvian taxes in different kinds of externalities, derives the cost benefit rules for a green tax reform and presents the welfare effects of using close to Pigouvian taxes. Section 3 presents the two sector dynamic economy and its market economy version with two externalities, derives the Pigouvian taxes, the cost benefit rules for a green tax reform and the welfare effects of using close to Pigouvian taxes.

2 General Equilibrium with the Environment - One Sector Economy

2.1 Central planner's perfect economy

The economy produces one homogenous investment/consumption good, $y(t)$, that can either be consumed at the rate, $c(t)$, or invested in physical capital at the rate, $\dot{k}(t)$. Assume no investment costs. The final good is produced (net of depreciation) by using up physical capital, labour $l(t)$ (in this case we normalize the model to get $l(t) = 1$) and natural resources harvested from the environment at the rate, $q(t)$, i.e., $y(t) = f(k(t), q(t))$. The unperturbed environment regenerates at the rate $R(N(t))$, where $N(t)$ is the stock of natural capital available to the economy. Hence, the dynamics of natural capital is $\dot{N}(t) = R(N(t)) - q(t)$. The action $q(t)$ on the environment can also be understood as pollution, or in a more general scope as an economic impact on the environment which we choosed to name as harvest.

Assume that it is possible to define an aggregate utility function for the representative consumer that exhibits preferences over the consumption rate $c(t)$ and the stock of natural capital $N(t)$, i.e., $U(c(t), N(t))$.

Assume also that there is a benevolent dictator (central planner) who whishes to maximize the discounted sum (using the utility discount rate, ρ) of $U(c(t), N(t))$ for all the planning horizon (which is taken to be $[0, \infty] \in \mathbb{R}$), by choosing the consumption and extraction rates. The optimal control problem is thus,

$$\begin{aligned} V(0; K_0, N_0) &= \max_{c,q} \int_0^{\infty} U(c(t), N(t))e^{-\rho t} dt \text{ s.t.} && (1) \\ \dot{k}(t) &= f(k(t), q(t)) - c(t), && (2) \\ \dot{N}(t) &= R(N(t)) - q(t), && (3) \\ k(0) &= k_0; N(0) = N_0;\ k_0,\ N_0 \text{ given.} && (4) \end{aligned}$$

From the Pontryagin's Maximum Principle we define the following present value hamiltonian,

$$H = U(c, N)e^{-\rho t} + \lambda^p \dot{k} + \mu^p \dot{N}. \quad (5)$$

The first order necessary conditions for this problem are[1],

$$\begin{aligned} H_c &= 0 \Leftrightarrow U_c^* = \lambda^p e^{\rho t} \equiv \lambda, && (6) \\ H_q &= 0 \Leftrightarrow f_q^* = \frac{\mu^p}{\lambda^p}, && (7) \\ \frac{\dot{\lambda}^p}{\lambda^p} &= -f_k^* \Leftrightarrow \frac{\dot{\lambda}}{\lambda} = \rho - f_k^*, && (8) \\ \frac{\dot{\mu}^p}{\mu^p} &= -\frac{U_N^* e^{-\rho t}}{\mu^p} - R^{*\prime} \Leftrightarrow \frac{\dot{\mu}}{\mu} = \rho - R^{*\prime} - \frac{U_N^*}{\mu}. && (9) \end{aligned}$$

along the usual transversality conditions. This set of equations implies the following system of necessary differential equations for the controls,

$$\begin{aligned} \frac{\dot{U}_c^*}{U_c^*} &= \rho - f_k^* && (10) \\ \frac{\dot{f}_q^*}{f_q^*} &= f_k^* - R^{*\prime} - \frac{1}{f_q^*}\frac{U_N^*}{U_c^*}. && (11) \end{aligned}$$

[1] $R^{*\prime} \equiv \frac{dR^*}{dN}$

that, with equations (2) and (3), characterizes the optimal trajectory. The decisions concerning the harvest rate are taken from equation (11). Note that firms take into account the consumers' valuation of nature, valued at the marginal cost of harvesting or in a pollution context, the marginal abatement costs (Hamilton, 1996; Pezzey et al, 2005).

2.2 Decentralized market economy

In a market economy there is no central planner and both the representative firm and consumer have distinct optimization problems. If the set of necessary conditions characterizing the solution of the general equilibrium of the market economy coincides with the set of necessary conditions characterizing solution of the central planner's economy then the market economy's equilibrium is said to be Pareto optimal.

If there are externalities it is most likely that the market economy does not have an optimal path that is Pareto optimal. However, it is possible to control the market economy by introducing taxes that are defined in order to the second best optimal path to be Pareto optimal. These taxes are called dynamic Pigouvian taxes.

The market economy, to be Pareto optimal may need the introduction of government intervention in the form of a tax and a lump-sum transfer. This is usually due to the fact that firms when making marginal choices of the extraction rates ignore that consumers value nature ($U_N \neq 0$). If $U_N = 0$ is the case, then there is no need for government intervention to get the market economy on the Pareto optimal trajectory.

2.2.1 Representative consumer's problem

There is now the question of what kind of tax is needed so that the market economy is Pareto optimal. For instance, Pezzey (2004) considers seven tax instruments in his model: taxes on consumption, capital, investment, resource stock and emissions. Aronsson et al. (2004) show that for the externality referred above this could be done with an extraction tax rate[2]. We consider taxes on consumption, τ_c, on extraction of natural resources, τ_q, and on the natural capital stock, τ_N. The problem for the representative consumer is,

$$\begin{aligned} & \max_c \int_0^\infty U(c(t), N(t)) e^{-\rho t} dt \text{ s.t.} \\ \dot{a} &= wl + da - (1+\tau_c)c + T \end{aligned}$$

where a is the net assets owned by the representative consumer in the form of ownership of claims on capital or loans (Barro and Sala-i-Martin, 2003), w is the market wage rate, l is labour rate (we assume that the consumer offers an inelastic quantity of labour equal to one), T is a lump-sum transfer from the government, d is the market interest rate (market capital rate of return). The set of necessary conditions for the solution of this problem is,

$$\begin{aligned} U_c^\circ &= (1+\tau_c)\lambda^p e^{\rho t} \\ \frac{\dot{\lambda}^p}{\lambda^p} &= -d \Leftrightarrow \frac{\dot{\lambda}}{\lambda} = \rho - d \end{aligned}$$

implying

$$\frac{\dot{U}_c^\circ}{U_c^\circ} = \rho - d - \frac{\dot{\tau}_c}{1+\tau_c}.$$

[2] The model they use is based on the Brock's model and uses the stock of pollution instead of the stock of natural capital, and a tax on pollution emissions. Of course their model and our model have simetrical approaches to the humans' impact on nature but the conclusions are equivalent.

2.2.2 Representative firm's problem

The firm chooses the investment rate, $I(t)$, and the extraction rate, $q(t)$, in order to maximize the discounted sum of its instantaneous profit

$$\max_{q,I} \int_0^{\infty} (f(k,q) - I - \tau_N N - \tau_q q)e^{-rt}dt,$$

subject to,

$$\begin{aligned} \dot{k} &= I \\ \dot{N} &= R(N) - q, \end{aligned}$$

where r is the market interest rate. The set of necessary conditions for the solution of this problem is[3],

$$\begin{aligned} r &= f_k^{\circ} \\ f_q^{\circ} - \tau_q^{\circ} &= \mu^p e^{rt} \\ \dot{\mu}^p &= -\tau_N e^{-rt} - \mu^p R^{\circ\prime}. \end{aligned}$$

Using these conditions we get,

$$\frac{\dot{f}_q^{\circ}}{f_q^{\circ}} = r - R^{\circ\prime} - \frac{1}{f_q^{\circ}}(\tau_q(r - R^{\circ\prime}) + \tau_N - \dot{\tau}_q).$$

These equation rules the choice of harvesting by the representative firm. Note that both taxes affect the firm's decision to harvest while the representative consumer's valuation of nature takes no part on these decisions.

2.2.3 General equilibrium

We are assuming micro-macro consistency, i.e., $c_i^{\circ}(t) = c^{\circ}(t)$, $a_i^{\circ}(t) = a^{\circ}(t)$ for each consumer $i = 0, ..., S$ and $I_j^{\circ}(t) = I^{\circ}(t)$, $k_i^{\circ}(t) = k^{\circ}(t)$, $q_i^{\circ}(t) = q^{\circ}(t)$ for each firm $j = 0, ...J$. The variables without indexes represent the aggregated values. Since the consumers hold the capital of the firms as loans, and assuming no adjustment costs and imperfections in the capital market then $k(t) = a(t)$ and $d(t) = r(t)$.

Proposition 1 *The general equilibrium is given by the trajectories* $\{c^{\circ}(t), q^{\circ}(t), k^{\circ}(t), N^{\circ}(t)\}_{t=0}^{\infty}$ *that solve the system,*

$$\dot{k}^{\circ} = f(k^{\circ}, q^{\circ}) - c^{\circ} \tag{12}$$

$$\dot{N}^{\circ} = R(N^{\circ}) - q^{\circ} \tag{13}$$

$$\frac{\dot{U}_c^{\circ}}{U_c^{\circ}} = \rho - f_k^{\circ} - \frac{\dot{\tau}_c}{1 + \tau_c} \tag{14}$$

$$\frac{\dot{f}_q^{\circ}}{f_q^{\circ}} = f_k^{\circ} - R^{\circ\prime} - \frac{1}{f_q^{\circ}}(\tau_q(f_k^{\circ} - R^{\circ\prime}) + \tau_N - \dot{\tau}_q) \tag{15}$$

[3] The hamiltonian is $H = f(k,q) - I - \tau_N N - \tau_q q + \lambda^p I + \mu^p \dot{N}$.

$$\begin{aligned} H_I &= 0 \Leftrightarrow \lambda^p = e^{-rt} \\ \dot{\lambda}^p &= -e^{-rt} f_K^{\circ}. \end{aligned}$$

These conditions imply $r = f_k^{\circ}$.

and for the second best optimal trajectory to be Pareto optimal it is sufficient that, the production function $y(t) = f(k(t), q(t))$ is homogenous of degree one and,

$$\begin{aligned} T &= f_q^\circ q^\circ, \\ f_l^\circ &= w^\circ, \\ \dot{\tau}_c &= 0, \\ \dot{\tau}_q &= \tau_q(f_k^\circ - R'(N^\circ) + \tau_N - \frac{U_N^\circ}{U_c^\circ}. \end{aligned}$$

Proof. For equation (14) to be equivalent to equation (10) it is sufficient that $\dot{\tau}_c = 0$. In particular the tax could be zero. Since the production function is homogenous of degree one then, $f(k^\circ, q^\circ, l^\circ) = f_k k + f_q q + f_l l$, meaning that all the revenue from the firms is distributed to the representative consumer in the form of wage $w^\circ l^\circ$, and profits $r^\circ k^\circ + f_q^\circ q^\circ$ using $r = f_k^\circ$ and $f_l^\circ = w^\circ$. Substituting this in the equation for $\dot{a}$, we obtain (12) with $\tau_c = 0$ and $T = f_q^\circ q^\circ$. Comparing equation (15) with (11) we get the equation for $\dot{\tau}_q$. ■

Since the Pigouvian tax on resource stock is defined in relation to the tax on extraction it also suffient, for Pareto optimality, that $\tau_N = 0$. From proposition 1 it can be concluded that it is possible to internalize the effect of the consumer's valuation of nature on the decision to harvest by using either a tax rate on harvesting or on natural capital stock. For this, the Pigouvian tax, in particular requires information about consumer's valuation of nature in units of final good.

In Aronsson et al. (2003) the firms optimization problem is static and firms ignore $\dot{N}$ ($\mu = 0$). The tax in this case is, $\tau_q^\circ = f_q^\circ$, meaning that the lump-sum transfer is, equal to $f_q^\circ q^\circ$.

We want to compare three types of externalities usually present in environmental and resource economic models:

Case 2 (C1) *There is an externality $U_N (= 0)$. Meaning that firms when choosing the optimal extraction rate ignore the consumer's valuation of natural capital services.*

Case 3 (C2) *The firms take the shadow value of natural capital equal to zero ($\mu = 0$) (Adapted from Aronsson et al. (2003)). This means that firms ignore natural capital dynamics and consumer's valuation of nature when making marginal decisions about the extraction rate.*

Case 4 (C3) *There is an externality U_N and additionally firms take $R'(N) = 0$ (adapted from Pezzey (1995)). This means that firms take nature's unperturbed dynamics as constant (unrenewable) in the optimal decisions to extract.*

Let us now determine the dynamic Pigouvian taxes in these three cases. Starting with C1, the market economy optimization problem can be written as

$$V(0; K_0, N_0) = \max_{c,q} \int_0^\infty U(c, \bar{N}) e^{-\rho t} dt \text{ s.t.} \tag{16}$$

$$\dot{k} = f(k, q) - c - \tau_q q + T, \tag{17}$$

$$\dot{N} = R(N) - q, \tag{18}$$

where $\bar{N}$ is taken as constant. From the maximum principle we have that the second best optimal path is characterized by equations in proposition 1. So, the dynamic Pigouvian tax is defined in proposition 1 with $\tau_N = 0$. This can be solved to give

$$\tau_q(t) = \int_t^{\infty} \frac{U_N^{\circ}(s)}{U_c^{\circ}(s)} e^{-\int_t^s f_k^{\circ}(\xi) - R^{\circ\prime}(\xi) d\xi} ds. \tag{19}$$

So, the tax at each instant is the discounted sum, in the planning horizon, of consumer's marginal preferences towards nature in monetary (consumption) units, where the discount term depends negatively (and exponentially) on $f_k^{\circ} - R^{\circ\prime}$. If $f_k^{\circ} > R^{\circ\prime}$ then the tax goes exponentially to zero following the second best path; on the other hand, if $f_k^{\circ} < R^{\circ\prime}$ then, along the second best path, the tax increases exponentially. The information requirements to implement this tax are infinite! Information from future marginal utility of ecosystem services in final goods units (note that the marginal utility of consumption is also changing) is needed, along with information on future natural capital's marginal rate of regeneration and future capital rate of return.

Considering case C3, the market economy optimization problem is given by equations (16), (17) and

$$\dot{N} = R(\bar{N}) - q. \tag{20}$$

Following the Pontryagin's maximum principle the second best optimal path is given by equations, (12), (14), (20) and

$$\frac{\dot{f}_q^{\circ}(\cdot)}{f_q^{\circ}(\cdot)} = f_k^{\circ}(\cdot) - \frac{\tau_q f_k^{\circ}(\cdot) - \dot{\tau}_q}{f_q^{\circ}(\cdot)}. \tag{21}$$

Thus, for the second best path to be Pareto optimal it is sufficient that

$$\dot{\tau}_q = \tau_q f_k^{\circ}(\cdot) - \left(f_q^{\circ}(\cdot) R^{\circ\prime}(\cdot) + \frac{U_N^{\circ}(\cdot)}{U_c^{\circ}(\cdot)} \right), \tag{22}$$

which can be solved, yielding

$$\begin{aligned}
\tau_q(t) &= \int_t^{\infty} \left(\frac{U_N(s)}{U_c(s)} + f_q^{\circ}(s) R^{\circ\prime}(s) \right) e^{-\int_t^s f_k^{\circ}(\xi) d\xi} ds, \text{ or} \\
\tau_q(t) &= \int_t^{\infty} \frac{U_N(s)}{U_c(s)} e^{-\int_t^s f_k^{\circ}(\xi) d\xi} ds + \int_t^{\infty} f_q^{\circ}(s) R^{\circ\prime}(s) e^{-\int_t^s f_k^{\circ}(\xi) d\xi} ds.
\end{aligned}$$

In this case, the Pigouvian resource extraction tax is the sum of two "taxes", one that equals the Pigouvian tax of an exhaustible resource externality (equation (22) with $R^{\circ\prime} = 0$) and the other accounts for the external indirect effect of impacts on marginal production of harvested resources due to changes in the marginal growth rate of natural capital. The discount term depends only on the marginal productivity of physical capital.

The optimal control problem of the market economy with an externality like C2 is the optimization problem given by equations (16) and (17)[4]. The necessary conditions defining the second best path for this market economy are $\dot{U}_c^{\circ} = U_c^{\circ}(\rho - f_k^{\circ})$ and $f_q^{\circ} = \tau_q$. This implies that $\dot{f}_q^{\circ} = \dot{\tau}_q$, so, for the second best path to be Pareto optimal it is sufficient that

$$\dot{\tau}_q = \tau_q \left(f_k^{\circ} - R^{\circ\prime} \right) - \frac{U_N^{\circ}}{U_c^{\circ}}, \tag{23}$$

[4] This is equivalent to the market economy in Aronsson et al. (2003) where the firm faces a static profit optimization problem.

which gives the Pigouvian tax defined in equation (19). In another way, following Aronsson et al. (2003), for second best path to be Pareto optimal it is sufficient that $\tau_q = \mu^p/U_c$, where μ^p follows equation (9) and N^* follows equation (3). This is implies that the Pigouvian tax is,

$$\tau_q(t) = \frac{1}{U_c^\circ(t)} \int_t^\infty U_N^\circ(s) e^{-\rho s} e^{\int_t^s R^{\circ\prime}(\xi) d\xi} ds. \tag{24}$$

which is equivalent to the Pigouvian tax for the case C1. This can be noted by solving equation (10) and subsitute the solution in equation (19). The tax in equation (19) is the Pigouvian tax for externalities in case C1 and C2. From this we can conclude that externalities C1 and C2 are equivalent.

For these three environmental external effects, the informational requirements are identical and not easily solved. It is necessary to obtain information about future marginal utilities and marginal regeneration rate of natural capital, along with information about man-made capital rates of return. As discussed in the introduction, this is an interesting link between contingent valuation methods or willingness to pay techniques and theoretical models of dynamic economies. From dynamic economies it is easy to design Pigouvian taxes that state the information requirements very clearly. This gives solid foundation for the use of such empirical valuation studies. However, as stated before, this information is impossible to obtain in real economies, and tax designers have to deal with defective information, either because when applying contingent valuation methods the consumer's do not state the correct value for any reason, or because the Pigouvian taxes require continuous information. The problem of not having information continuously was addressed in Aronsson et al. (2003) and they concluded that an approximation to Pigouvian taxes could be used by collecting information from time to time, depending on the interval between willingness-to-pay information. Here we address the problem of misestimation errors in a way that generalizes Aronsson et al. (2003) findings. Before this, a cost benefit rule for a green tax reform is needed.

2.3 Cost benefit rule for a tax change

In this section we examine the welfare implications of using a non-Pigouvian tax to control the economy. As discussed above we have informational problems calculating Pigouvian taxes, whether from not knowing the real future marginal utility of ecosystem, or the ecosystem dynamics. Hence, in this section we derive the cost-benefit rule for a small change in the ecosystem harvest tax and analyze its welfare improving compared with the uncontrolled market by using a close to Pigouvian tax where the used marginal utility from ecosystem stock and its marginal rate of regeneration have a time dependent bias to the real values.

Consider here that the external effect is such that firms take the shadow price of natural capital as zero. Assume that there is an increase by a small positive constant α, so the tax is now $\tau_q(t) + \alpha$. In this case, we consider the optimal value function to be,

$$V(0,\zeta) = \int_0^\infty U(c^\circ(t,\zeta), N^\circ(t,\zeta)) e^{-\rho t} dt$$

where ζ is a vector of parameters including α. The welfare effect of the tax change is given by $\partial V(0,\zeta)/\partial\alpha$. This expression is a dynamic cost benefit rule for a tax change (Aronsson et al., 2003) and it can be obtained by using the dynamic envelope theorem (LaFrance and Barney, 1991). The cost benefit rule is obtained by differentianting $V(0,\zeta)$ w.r.t. α

and evaluate the resulting derivative along the initial (second best) optimal path using the necessary conditions and $\alpha(t) = 0$ (Aronsson et al., 2003).

Proposition 5 *The dynamic cost benefit rule for a tax change $\tau_q(t) + \alpha$, with α, a small positive constant, is*

$$\frac{\partial V(0,\zeta)}{\partial \alpha} = \int_0^\infty U_N^\circ(\cdot) N_\alpha^\circ e^{-\rho t} + \lambda^p f_q^\circ(\cdot) q_\alpha^\circ dt.$$

Proof. Define the present value hamiltonian-like function, $\bar{H}^\circ = U^\circ(\cdot)e^{-\rho t} + \lambda^p \dot{k}^\circ$, where λ^p does not depend on α. With this, the value function can be written as

$$V(0,\zeta) = \int_0^\infty \bar{H}^\circ - \lambda^p \dot{k}^\circ dt.$$

Integrating by parts the second term we obtain,

$$V(0,\zeta) = \int_0^\infty \bar{H}^\circ + \dot{\lambda}^p k^\circ dt - \lambda^p k^\circ \mid_0^\infty$$

Taking the derivative w.r.t. α, the term $\partial \lambda^p k^\circ \mid_0^\infty / \partial \alpha$ is zero since, $\lim_{t\to\infty} \lambda^p(t) = 0$, k_0 is fixed and λ^p does not depend on α. Hence,

$$\frac{\partial V(0,\zeta)}{\partial \alpha} = \int_0^\infty \frac{\partial \bar{H}^\circ}{\partial \alpha} + \dot{\lambda}^p \frac{\partial k^\circ}{\partial \alpha} dt.$$

$$\frac{\partial \bar{H}^\circ}{\partial \alpha} = \left(U_c^\circ(\cdot) c_\alpha^\circ + U_N^\circ(\cdot) N_\alpha^\circ\right) e^{-\rho t} + \lambda^p \left(f_k^\circ k_\alpha^\circ + f_q^\circ q_\alpha^\circ - c_\alpha^\circ\right),$$

where we make use of equation (12). For simplicity, we use the notation where the subscript α in c_α° means $\partial c^\circ / \partial \alpha$.

$$\bar{H}_\alpha^\circ + \dot{\lambda}^p k_\alpha^\circ = U_N^\circ(\cdot) N_\alpha^\circ e^{-\rho t} + \lambda^p \left(f_k^\circ(\cdot) k_\alpha^\circ + f_q^\circ(\cdot) q_\alpha^\circ\right) + \dot{\lambda}^p k_\alpha^\circ = U_N^\circ(\cdot) N_\alpha^\circ e^{-\rho t} + \lambda^p f_q^\circ(\cdot) q_\alpha^\circ$$

where the first equality comes from using $U_c^\circ = \lambda^p e^{\rho t}$, and the second equality from using $\dot{\lambda}^p = -\lambda^p f_k^\circ$. Hence we obtain the dynamic cost benefit rule. ■

This is equivalent to the cost benefit rule derived in Aronsson and Löfgren (1999) and Aronsson et al. (2004) but now applied to an ecosystem harvesting tax and generalized to include the natural capital's density dependent dynamics. The second term of the cost benefit rule represents the cost of the loss of consumption due to a decrease in the ecosystem resources harvested (due to an increase in the harvest tax), and the first term the utility gains due to an increase in natural capital. Since $f_q^\circ = \tau_q$ and since it can be easily proved that $N_\alpha^\circ > 0$ then if the tax is zero the welfare effect is positive. This means that introducing a small tax in the market economy is welfare improving. Note that, since $q_\alpha^\circ < 0$ if the tax rate is too high then decreasing the tax rate is welfare improving, implying that there is an interval for welfare improving tax rates.

What happens to the cost benefit rule if the Pigouvian tax is wrongly implemented? Extending Aronsson et al. (2003) case, we consider measurement errors both in the marginal utility of natural capital as, $U_N(c(t), N(t)) + \beta(t)$ and in its marginal rate of natural regeneration as, $R'(N(t)) + \gamma(t)$.

Proposition 6 *Consider measurement errors $\beta(t)$ and $\gamma(t)$ in the marginal utiliy of natural capital $U_N(c(t), N(t))$ and in the natural rate of regeneration of natural capital $R'(N(t))$, respectively. Then, the dynamic cost benefit rule can be written as*

$$\frac{\partial V(0,\zeta)}{\partial \alpha} = -\int_0^\infty N_\alpha^\circ(t)\left[\lambda^p(t)\tau_q(t)\gamma(t) + \beta(t)e^{-\rho t}\right]dt.$$

Proof. Deriving w.r.t. α the equation for the dynamics of the natural capital in cases C1 or C2 implies $q_\alpha^\circ = R^{\circ\prime}(\cdot)N_\alpha^\circ - \dot{N}_\alpha^\circ$, which, substituting in the cost benefit rule (CBR) in proposition 5 gives,

$$\frac{\partial V(0,\zeta)}{\partial \alpha} = \int_0^\infty U_N^\circ(\cdot)N_\alpha^\circ e^{-\rho t} + \lambda^p f_q^\circ(\cdot)\left(R^{\circ\prime}(\cdot)N_\alpha^\circ - \dot{N}_\alpha^\circ\right)dt.$$

Now, integrating the term $-\int_0^\infty \lambda^p f_q^\circ(\cdot)\dot{N}_\alpha^\circ dt$ by parts we obtain

$$\int_0^\infty \left(\dot{\lambda}^p f_q^\circ(\cdot) + \lambda^p \dot{f}_q^\circ(\cdot)\right)N_\alpha^\circ dt - \lambda^p f_q^\circ(\cdot)N_\alpha^\circ \mid_0^\infty .$$

Since $\lim_{t\to\infty}\lambda^p(t) = 0$ and N_0 is fixed then the last term in the last expression is zero. With $\dot{\lambda}^p = -\lambda^p f_k^\circ$ this expression then becomes

$$\int_0^\infty \lambda^p N_\alpha^\circ\left(\dot{f}_q^\circ(\cdot) - f_k^\circ(\cdot)f_q^\circ(\cdot)\right)dt.$$

Using this, the CBR can be written as

$$\frac{\partial V(0,\zeta)}{\partial \alpha} = \int_0^\infty U_N^\circ(\cdot)N_\alpha^\circ e^{-\rho t} + \lambda^p N_\alpha^\circ f_q^\circ(\cdot)\left[\frac{\dot{f}_q^\circ(\cdot)}{f_q^\circ(\cdot)} - \left(f_k^\circ(\cdot) - R^{\circ\prime}(\cdot)\right)\right]dt.$$

Now using equation (15) the CBR is

$$\frac{\partial V(0,\zeta)}{\partial \alpha} = \int_0^\infty U_N^\circ(\cdot)N_\alpha^\circ e^{-\rho t} + \lambda^p N_\alpha^\circ\left[\tau_q\left(R^{\circ\prime}(\cdot) - f_k^\circ(\cdot)\right) + \dot{\tau}_q\right]dt.$$

The non-Pigouvian tax here considered solves

$$\dot{\tau}_q = \tau_q\left(f_k^\circ(\cdot) - R^{\circ\prime}(\cdot) - \gamma\right) - \frac{U_N^\circ(\cdot) + \beta}{\lambda^p}e^{-\rho t}$$

using equation (23) and $U_c^\circ(\cdot) = \lambda^p e^{\rho t}$. Substituting this in the CBR we obtain the cost benefit rule desired. ■

Note that if measurement errors were both zero, then the tax change produces no welfare effects, meaning that the tax internalises the environmental externality. If both the measured marginal utility from the ecosystem and of marginal ecosystem productivity overestimate the true respective values, then a permanent small increase in the harvest tax reduces the welfare level. By the same token, reducing the harvest tax is welfare improving in this case. On the other hand, if we underestimate the marginal ecosystem productivity and we have $-u_N < \beta < -\lambda^p\tau_q\gamma^5$, then increasing the harvest tax is always welfare improving.

[5] This means that we have a tax and not a subsidy.

An interesting effect is that there is the possibility of tradeoffs between the estimates to generate welfare improving increases in the harvest tax. If, for instance, we are overestimating the ecosystem marginal productivity, for the controlled economy to be welfare superior to the uncontrolled we need to underestimate the marginal utility from the ecosystem. From this it can be concluded that it is not relevant to put all the effort on obtaining measures of marginal utility when no effort is put in understanding the way ecosystems work, in particular the marginal rate of regeneration is needed. Additionally, note the term related to mis-estimations of the marginal utility from ecosystem services is being discounted according to the representative consumer's discount rate. This implies that this term looses 'power' in comparing to the terms affected by errors in the mis-estimation of marginal rates of ecosystem regeneration when computing welfare effects of a tax change. This means that in a sense the errors made in measuring ecosystem growth rates have persistent welfare effects, and ought to be taken more seriously than marginal utilities from ecosystem services.

3 General Equilibrium with the Environment - Two Sector Economy

In this section we introduce knowledge formation in the model analysed in the previous section. The objective is, to understand the effects and interactions among two externalities and two close to Pigouvian taxes.

The dynamic economy is composed of two sectors. One is the final good sector and the other is a pure investment R&D sector that produces knowledge about the efficient allocation of harvest resources to production. As in the above sections, we derive the cost benefit rules for the case of a green tax reform with the implementation of a close-to-Pigouvian tax with measurement errors. We also explore the relations of the Pigouvian taxes with welfare measures in the market economy.

3.1 The perfect economy with knowledge formation

The environment is modeled as a renewable resource and the economic actions on the environment are taken to be a negative effect on the environment dynamics, interpreted either as resources harvested or as polluting, using equation (3). The final goods sector produces a final output that can either be consumed at rate $c(t)$ or invested for physical capital accumulation. The final good $y(t)$ is produced by using the fraction, $u(t)$, of man-made capital allocated to production, $u(t)k(t) \equiv K_y$ (where $k(t)$ includes physical and human capital excluding knowledge capital related to harvest), and by using the share $\upsilon(t)$ of effective input of harvested resources, $\upsilon(t)h(t)q(t) \equiv Q_y$, i.e., $y(t) = f(K_y, Q_y)$. The amount of harvesting necessary for the production of the final good depends on the state of the knowledge of harvesting-augmenting techniques, depicted by $h(t)$.

The knowledge sector is a pure investment sector that produces knowledge about techniques related to the allocation of harvested resources. To this end, the knowledge sector uses as inputs, the physical capital, $K_H \equiv (1 - u(t))k(t)$, and effective harvested resources, $Q_H \equiv (1 - \upsilon(t))\, h(t)q(t)$, i.e. the dynamics of the technical knowledge stock is $\dot{h} = H(K_H, Q_H)$.

The economy encompasses a usual accumulation equation that links the evolution of the stock of economy-wide physical capital to investment, $y - c$,

$$\dot{k} = y - c.$$

Additionally the economy obeys the resource constraints, $k = K_y + K_h$ with $u \equiv K_y/k$ and $Q = Q_y + Q_H$,with $\upsilon = Q_y/Q$. The representative consumer exhibits preferences over consumption goods and environmental stock, so that the central planner chooses the consumption rate, the harvest rate and the shares of man-made capital and pollution allocated to the final good's and knowldge sectors, i.e.

$$\begin{aligned}
V(0; k_0, N_0) &= \max_{c,q,u,\upsilon} \int_0^\infty U(c, N)e^{-\rho t} dt \text{ s.t.} && (25)\\
\dot{k} &= f(uk, \upsilon hq) - c, && (26)\\
\dot{N} &= R(N) - q, && (27)\\
\dot{h} &= H((1-u)k, (1-\upsilon)hq), && (28)\\
k(0) &= k_0;\ N(0) = N_0;\ h(0) = h_0;\ k_0,\ N_0,\ h_0 \text{ given.} && (29)
\end{aligned}$$

where the time is supressed for clarity. This model is a particular case of the model analised in Bovenberg and Smulders (1995). The present value hamiltonian for this problem is $H = U(c, N)e^{-\rho t} + \lambda^p \dot{k} + \mu^p \dot{N} + \sigma^p \dot{h}$.

The conditions for the static optimal allocation are,

$$\begin{aligned}
\lambda^p f_{K_y} &= \sigma^p H_{K_H}, && (30)\\
\lambda^p f_{Q_y} &= \sigma^p H_{Q_H}, && (31)\\
f_{Q_y} h &= \frac{\mu^p}{\lambda^p} \Leftrightarrow H_{Q_H} h = \frac{\mu^p}{\sigma^p}. && (32)
\end{aligned}$$

The firts (second) condition states that the marginal product of physical capital (effective harvest) should be the same in both sectors measured in units of physical capital. The third equation states that for both sectors, the product of one additional unit of harvest resources equals its marginal cost in terms of deterioration of the environmental stock (Bovenberg and Smulders, 1995).

The conditions for the optimal dynamic allocation are,

$$\begin{aligned}
\frac{\dot{\lambda}^p}{\lambda^p} &= -f_{K_y} = -\frac{\sigma^p}{\lambda^p} H_{K_H}, && (33)\\
\frac{\dot{\mu}^p}{\mu^p} &= -\frac{U_N}{\mu^p e^{\rho t}} - R', && (34)\\
\frac{\dot{\sigma}^p}{\sigma^p} &= -H_{Q_H} q = -\frac{\lambda^p}{\sigma^p} f_{Q_y}. && (35)
\end{aligned}$$

The first equation along with the first order condition for the static optimization of the consumption rate gives the usual Ramsey rule for dynamic efficiency in the trade-off between consumption and investment in each sector as in equation (10). The other two equations for the shadow prices of natural capital and knowledge stock give arbitrage conditions implying that the h and N should yield the same return (Bovenberg and Smulders, 1995). Rewriting this equations using the current value shadow prices it is obviously seen that, as usual in dynamic growth models, the global rate of return of the assets considered, which includes capital gains (change in the shadow price) and the asset specific rate of return, should be equal to the representative consumer's discount rate.

3.2 Externalities in the decentralized market economy

The conditions presented in the last subsection serve as a benchmark in order to obtain the Pigouvian taxes in a second best world. Here we shall consider two externalities - one environmental externality and one technological externality. The environmental externality considered here is the same as case C2 meaning that firms take the shadow price of nature as being zero and overpollute (Aronsson et al., 2003). The external effect of the knowledge sector considered here is similar in the sense that the final goods sector is assumed to ignore the dynamics of the knowledge sector, which affects the optimal allocation of physical capital between sectors. So in the imperfect market economy the shadow price of the knowledge stock h is taken as zero. We may interpret this as being an external benefit to the final goods sector that is freely using green technology.

In order to internalize these two external effects, various sets of taxes could be used and here we choose the usual tax rate on harvest resources τ_q, and to internalize the technological externality we consider a tax (or subsidy) on the fraction of man-made capital that should be devoted to the knowledge sector, τ_u. Other taxes could be considered though they do not add to the cost benefit rules and welfare effects derived below.

Assume that the technical knowledge is produced by using only man-made capital, so that $\dot{h} = H\left((1-u)k\right)$. This simplifies the analysis and does not changes the main conclusions for the welfare effects of tax changes. In a market economy with these two external effects, the conditions for the second-best optimal path can be derived from the present value Hamiltonian

$$\hat{H} = U(\cdot)e^{-\rho t} + \lambda^p \left(f\left(uk, hq\right) - c -_{\tau_q} q - \tau_u u + T\right)$$

As seen above, the representative consumer chooses the consumption rate of final goods in order to maximize its discounted sum of utilities and the firms of the final good sector maximizes their profits in an environment of perfect competition. Hence, the conditions for the second-best optimum include,

$$\begin{aligned} U_c &= \lambda^p e^{\rho t}, && (36)\\ f_{K_y} k &= \tau_u, && (37)\\ f_{Q_y} h &= \tau_q. && (38) \end{aligned}$$

for the static optimal allocation, and $\dot{\lambda}^p = -\lambda^p f_{K_y}$ for the optimal dynamic allocation. In this imperfect market economy the decisions to extract are taken from equation (38)

Proposition 7 *For the second best optimal trajectory to be Pareto optimal it is sufficient that the taxes are designed as follows*

$$\begin{aligned} \dot{\tau}_q &= \tau_q \left(f_{K_y} - R'\right) - \frac{U_N^\circ}{U_c^\circ}, && (39)\\ \dot{\tau}_u &= \tau_u \left(f_{K_y} + \frac{\dot{H}_{K_H}}{H_{K_H}} + \frac{\dot{k}}{k}\right) - f_{Q_y} q H_{K_H} k. && (40) \end{aligned}$$

Proof. As in the one sector models, taking the time derivative of the second best static optimal conditions and using the conditions for the optimal dynamic allocation of the first best solution we obtain the expressions for the Pigouvian taxes. ■

3.3 Cost benefit rule for a tax change

The pupose of this section is to examine , close to Pigouvian taxes improve the welfare level when comparing with the uncontrolled market economy as done in Aronsson et al. (2003). For this matter consider a green tax reform as in the one sector model, so that the harvest tax rate is now, $\tau_q(t) + \alpha$. With this, a cost benefit rule for the tax change is derived and from this cost benefit rule the conditions under which the close to Pigouvian taxes improve welfare are derived. The method followed is the same for the one sector model and the cost benefit rule is

Proposition 8 *The dynamic cost benefit rule for an infinitesimal tax change in the two sector economy is*

$$\frac{\partial V(0,\zeta)}{\partial \alpha} = \int_0^{\infty} U_N N_\alpha e^{-\rho t} + \lambda^p[\tau_u u_\alpha + \tau_q q_\alpha] - \lambda^p \left(f_{K_y} (1-u) k_\alpha + f_{Q_y} h_\alpha q \right) dt. \tag{41}$$

Proof. The proof follows that of proposition 5 applied to the two sector market economy. See appendix A. ■

The first term of the integral is the utility value of an increase of natural capital stock due to an increase in the harvest tax rate. The term inside square brackets can be understood as an effect on the preexisting tax system. This accounts for the erosion of the tax base, meaning the effects of the tax reform (α) on q and u. The term in curve brackets represent the additional effects of the tax reform in terms of forgone consumption of goods due to a decrease in the harvested resources (second term inside square brackets) and due to the indirect effect of affecting the marginal productivity of physical capital due to the technological externality. This is the effect of α on physical capital and knowledge along the general equilibrium path.

Note that increasing the harvest tax increases the cost of a unit of harvest, hence given the assumptions of the underlying model $q_\alpha < 0$ for all t. From this, $N_\alpha > 0$, implying that the first term is benefit due to better environmental quality. The signs of the terms involving the equilibrium responses in u, h and k cannot be determined a priori. If $k_\alpha < 0$ for all t, then the increase in the environmental tax produces, as opposite to Aronsson (1999), a welfare benefit via the additional distortion of the capital stock, and this effect is more pronounced the higher the share of physical capital devoted to the knowledge (green technology) sector along the general equilibrium. The last term is explicitly tracking the effect of an alteration of the technology in the marginal productivity of harvested resources and ultimately in the harvest rate, to the green tax reform. If $h_\alpha > 0$ then this term is a welfare cost due in terms of the productivity of a unit of resource harvested and consenquently in terms of foregone consumption.

Now to answer the question of what are the welfare effects of using close to Pigouvian taxes consider that there are measurement errors in the marginal utility of natural capital and in its marginal regeneration rate (as in the one sector model) and additionally assume measurement errors in implementing the knowledge tax rate, specifically in the measurement of the marginal productivity of harvested resources $f_{Q_y} + \varepsilon$ (since the technology is considered an externality).

Proposition 9 *Considering measurement errors as in proposition 6, and additionally that $f_{Q_y} + \varepsilon$ in the tax rate associated to the tax τ_u, the dynamic cost benefit rule can be written as*

$$\frac{\partial V(0,\zeta)}{\partial \alpha} = -\int_0^{\infty} \lambda^p(t)\{h_\alpha(t)q(t)\varepsilon(t) + N_\alpha(t)\tau_q(t)\gamma(t)\} + N_\alpha(t)\beta(t)e^{-\rho t} dt \tag{42}$$

Proof. This proof follows very closely the proof of proposition 6. Derive w.r.t. α the equations for $\dot{h}$ and $\dot{N}$ and substitute these in the above cost benefit rule. Integrate by parts the terms having $\dot{h}_\alpha$ and $\dot{N}_\alpha$ and use the first order conditions for the two sector market economy. Now, since the close to Pigouvian taxes obey

$$\begin{aligned} \dot{\tau}_q &= \tau_q \left(f_{K_y} - R' \right) - \frac{U_N}{U_c} - \tau_q \gamma - \frac{\beta e^{-\rho t}}{\lambda^p}, \\ \dot{\tau}_u &= \tau_u \left(f_{K_y} + \frac{\dot{H}_{K_H}}{H_{K_H}} + \frac{\dot{k}}{k} \right) - \left(f_{Q_y} + \varepsilon \right) q H_{K_H} k, \end{aligned}$$

substituting them in the resulting cost benefit rule and obtain the result of this proposition. ■

This approach allows for the inclusion of various kinds of errors when implementing Pigouvian taxes. This proposition is a generalization of the cost benefit analysis performed in Aronsson et al. (2003) to include measurement errors in both the Pigouvian harvest tax rate and the technology tax rate. Note that, since $N_\alpha > 0$ it is known that the last term increases welfare (in case of a green tax reform) if we underestimate the true value of the marginal utility of ecosystem services for the representative consumer, as compared to the uncontrolled market economy. Again, note that this term is being discounted. Most likely the effects of misestimating future marginal utilities are smaller than the effects of mis-estimating ecosystem regeneration rates and the marginal productivity of harvested resources, which include the effects of using green technology h.

Again, there is a tradeoff between errors in the measurement of specific terms in the implementation of the Pigouvian taxes but now including welfare effects via the use of green technology in handling the harvested resources when producing the consumption good. However, as it can be seen from the cost benefit rule in proposition 9 it is always possible to design welfare improving taxes.

Moreover, this cost benefit rules gives interesting indications of how to proceed in designing Pigouvian taxes in the real world. For instance, it can be seen that if we have data on some term of the Pigouvian tax that we know is an under(over)-estimation of the true value, according to propositions 9 and 6 we could assert what the impact of data on other variables would be, and the relevance of it in terms of welfare, thus yielding guidelines for the practical implementation of theoretically defined Pigouvian taxes.

4 Conclusions

In this paper we designed Pigouvian taxes for different kinds of externalities in one and two sector dynamic economies. Moreover, we analized the cost benefit rules of a (infinitesimal) green tax reform in a more general context than Aronsson et al. (2003). From the Pigouvian tax derived it is possible to know exactly what are the information requirements for the implementation in real economies. Since the data collected to implement these taxes is not perfect for a variety of reasons, we analized the welfare effects of errors in estimating certain quantities used in the Pigouvian taxes. We found conditions for which the definition of non-Pigouvian taxes is welfare improving for the controlled economy in comparison to the uncontrolled. We focused particularly on the effects of mis-estimating marginal utility from ecosystem services and the marginal ecosystem productivity for the one sector model and additionally the marginal productivity of harvested resources when using green technology in the two sector dynamic economy. The two sector model is an endogenous growth model where we considered the interaction of two externalities - one environmental and another related to the available technology.

Our analysis of non-Pigouvian taxes generalized Aronsson et al. (2004) work in various ways. First we considered a more general ecosystem dynamics, second we considered wrong estimates for the marginal ecosystem productivity, third we considered cost benefit rules in a context of a two sector dynamic economy.

The main conclusion is that even when we consider misestimates of the marginal ecosystem productivity it is always possible to design welfare improving tax changes, in comparison to the uncontrolled market economy. This is true for both the one sector and two sector market economies. Another interesting conclusion is that, in a sense, we derived guidelines for the implementation of Pigouvian taxes, particularly concerning the approach to the obtained data from real imperfect economies. This approach provides useful tools for the design of Pigouvian taxes in real economies.

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6 Appendix A

Proof of proposition 8. We have assumed that the externalities on these model imply that $\mu = \sigma = 0$ in the economy composed of equations (25) - (29). Thus, assume that

$\bar{H}^{\circ} = U^{\circ}(\cdot)e^{-\rho t} + \lambda^p \dot{k}^{\circ}$. With this, the value function rewrites as

$$V(0,\zeta) = \int_0^{\infty} \bar{H}^{\circ} - \lambda^p \dot{k}^{\circ} dt.$$

Taking the derivative w.r.t. α, integrating by parts and noting that $\lim_{t\to\infty} \lambda^p(t) = 0$, k_0 is fixed and λ^p does not depend on α we get

$$\frac{\partial V(0,\zeta)}{\partial \alpha} = \int_0^{\infty} \bar{H}_{\alpha}^{\circ} + \dot{\lambda}^p k_{\alpha}^{\circ} dt.$$

$$\bar{H}_{\alpha}^{\circ} = U_N^{\circ} N_{\alpha}^{\circ} e^{-\rho t} + \lambda^p \left[f_{K_y} \left(u_{\alpha} k + u k_{\alpha} \right) + f_{Q_y} \left(h_{\alpha} q + h q_{\alpha} \right) \right],$$

where we make use of $\lambda^p = U_c e^{-\rho t}$ and the equation for $\dot{k}$.

$$\bar{H}_{\alpha}^{\circ} + \dot{\lambda}^p k_{\alpha}^{\circ} = U_N^{\circ} N_{\alpha}^{\circ} e^{-\rho t} + \lambda^p \left[f_{K_y} \left(u_{\alpha} k - (1-u) k_{\alpha} \right) + f_{Q_y} \left(h_{\alpha} q + h q_{\alpha} \right) \right],$$

making use of use $\dot{\lambda}^p = -\lambda^p f_k^{\circ}$. Substituting in the integral,

$$\frac{\partial V(0,\zeta)}{\partial \alpha} = \int_0^{\infty} U_N N_{\alpha} e^{-\rho t} + \lambda^p \left[f_{K_y} \left(u_{\alpha} k - (1-u) k_{\alpha} \right) + f_{Q_y} \left(h_{\alpha} q + h q_{\alpha} \right) \right] dt, \tag{A1}$$

which rearranging and using equations (37) and (38) gives the CBR desired. ■

Proof of proposition 9. Deriving w.r.t. α the equation for the dynamics of the natural capital and of the knowledge stock implies $q_{\alpha}^{\circ} = R^{\circ\prime} N_{\alpha}^{\circ} - \dot{N}_{\alpha}^{\circ}$ and $\left(u_{\alpha} k - (1-u) k_{\alpha} \right) = -\frac{\dot{h}_{\alpha}}{H_{K_H}}$, respectively. Substituting in the CBR we obtain,

$$\frac{\partial V(0,\zeta)}{\partial \alpha} = \int_0^{\infty} U_N N_{\alpha} e^{-\rho t} + \lambda^p \left[f_{K_y} \left(-\frac{\dot{h}_{\alpha}}{H_{K_H}} \right) + f_{Q_y} \left(h_{\alpha} q + h \left(R^{\circ\prime} N_{\alpha}^{\circ} - \dot{N}_{\alpha} \right) \right) \right] dt.$$

Integrating by parts the term $-\lambda^p f_{K_y} \left(-\frac{\dot{h}_{\alpha}}{H_{K_H}} \right)$, noting that $\lim_{t\to\infty} \lambda^p(t) = 0$, k_0 and h_0 is fixed and λ^p does not depend on α, and using the second best conditions for the shadow price of capital, then,

$$-\int_0^{\infty} \lambda^p f_{K_y} \left(-\frac{\dot{h}_{\alpha}}{H_{K_H}} \right) dt = \int_0^{\infty} \frac{h_{\alpha} \lambda^p}{H_{K_H} k} \left[\dot{\tau}_u - \tau_u \left(f_{K_y} + \frac{\dot{H}_{K_H}}{H_{K_H}} + \frac{\dot{k}}{k} \right) \right] dt.$$

By the same token the following holds,

$$-\int_0^{\infty} \lambda^p f_{Q_y} h \dot{N}_{\alpha} dt = \int_0^{\infty} \lambda^p N_{\alpha} \left[\dot{\tau}_q - f_{K_y} \tau_q \right] dt.$$

Substituting these expressions in the CBR we obtain,

$$\begin{aligned} \frac{\partial V(0,\zeta)}{\partial \alpha} \quad = \quad & \int_0^{\infty} U_N N_{\alpha} e^{-\rho t} + \frac{h_{\alpha} \lambda^p}{H_{K_H} k} \left[\dot{\tau}_u - \tau_u \left(f_{K_y} + \frac{\dot{H}_{K_H}}{H_{K_H}} + \frac{\dot{k}}{k} \right) + f_{Q_y} q H_{K_H} k \right] + \\ & + \lambda^p N_{\alpha} \left[\tau_q \left(R' - f_{K_y} \right) + \dot{\tau}_q \right] dt. \end{aligned}$$

Now substituting the close to Pigouvian taxes in this expression we prove our result. ■