RESWITCHING AND DECREASING DEMAND FOR CAPITAL

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ABSTRACT

We consider a model of production with a continuum of linear techniques and examine the related choice of technique and shape of the demand for capital schedule. The primary conclusion regards the possibility of a decreasing demand for capital schedule combined with reswitching and reverse capital deepening.

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1. INTRODUCTION

The phenomena of reswitching and reverse capital deepening – understood respectively as the case in which a technique is in use for interest rates $\hat{r}$ and $\breve{r}$, but not for some r between the two and the case in which a rise in the interest rate entails the use of techniques with higher net product per worker[1] – are generally associated with an increasing demand for capital schedule (at least over a certain stretch). This is in fact always the case when it is assumed that the number of alternative production techniques is finite (cf. Pasinetti 1969, p. 527).

We shall instead consider a case with a continuum of linear production techniques and present a simple model to show the possibility of a monotonically decreasing demand for capital schedule despite the presence of reswitching and reverse capital deepening.

We start by describing the model (sec. 2) and examining the general rules for the choice of technique (sec. 3). Section 4 then introduces a sufficient condition for reswitching and reverse capital deepening.

The characteristics of the demand for capital schedule are discussed in section 5, where the possibility of its decreasing shape associated with a direct relationship between rate of interest and net product per worker is also shown. Finally, section 6 presents an example with a monotonically decreasing demand for capital schedule even though the sufficient condition for reswitching is satisfied and the behaviour of the net product per worker schedule is therefore non-monotonic.

2. ASSUMPTIONS ON TECHNIQUES

We consider an economy with n products: a pure consumption good, labelled commodity [1], and n – 1 pure capital goods, labelled commodities [2], [3], ..., [n].

A continuum of possible techniques of production is available. Each technique is characterized by a n×n matrix $A(\theta)$ and a n-vector $\ell(\theta)$, for every $\theta \in \Theta$, with $\Theta = \{\theta \in R : 0 \le \theta \le 1\}$, such that $a_{ij}(\theta) \ge 0$ and $\ell_i(\theta) > 0$ are, respectively, the quantity of the j-th commodity and the amount of labour employed in the production of one unit of the i-th commodity.[2]

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[1] Once a technique with greater net product per worker is regarded as more capital-intensive, the definition of "reverse capital deepening" adopted here corresponds to the standard one (cf. in particular Scazzieri 1987, p. 172).

[2] It is worth pointing out that our way of representing a continuum of possible production techniques, though not very familiar to economists, is general in character whereas the more common C.E.S. of Cobb-Douglas production functions are not. See Schefold (2008).

Assumption 1. *The functions* $a_{ij}(\theta)$ *and* $\ell_i(\theta)$ *are continuous and at least twice differentiable on the open set S, with* $S \supset \Theta$*.*

Given a (row) vector of activity levels (gross products) $b \geqq 0$, $b \cdot A(\theta)$ is the corresponding vector of demand for capital with technique θ, $b \cdot [I - A(\theta)]$ is the vector of net products and $b \cdot \ell(\theta)$ is the demand for labour.

For each technique θ, there exist a scalar y(θ) and a vector q(θ) that are respectively the net product of commodity [1] per worker and the vector of activity levels generating it. These can be obtained by solving the following equations[3]:

(1) $$y(\theta) \cdot e_1 = q(\theta) \cdot [I - A(\theta)]$$

(2) $$1 = q(\theta) \cdot \ell(\theta).$$

From equation (1) it is easy to obtain:

(3) $$y(\theta) \cdot e_1 \cdot [I - A(\theta)]^{-1} = q(\theta)$$

and the substitution of equation (3) in equation (2) gives:

(4) $$1 = y(\theta) \cdot e_1 \cdot [I - A(\theta)]^{-1} \cdot \ell(\theta)$$

which implies:

(5) $$y(\theta) = \frac{1}{e_1 \cdot [I - A(\theta)]^{-1} \cdot \ell(\theta)}$$

Since A(θ) is a matrix with non-negative entries only, because of a well-known linear algebra theorem[4] we have:

[3] The (row) vector e_1 is [1, 0, ..., 0].

[4] Cf. Dorfman, Samuelson and Solow (1958) pp. 500, 501; Pasinetti (1977) pp. 265, 266; Kurz and Salvadori (1995), theorem A.3.3 p. 513. Note that it is taken for granted here that A(θ) is "productive" – i.e. the technology is such that activity levels exist making it possible to obtain a strictly positive net output of each product – or equivalently that matrix A(θ) has no eigenvalue λ such that $|\lambda| \geq 1$.

(6) $$[I - A(\theta)]^{-1} = I + \sum_{i=1}^{\infty}[A(\theta)]^i$$

Assumption 2. *Production is not circular, which means that no capital good enters directly or indirectly into its own production.*

The assumption of non-circularity means that $[A(\theta)]^i$ is the n×n null matrix[5] for every i greater than a certain integer number $\eta \leq n$. In particular, for the sake of simplicity, we shall adopt the following assumption:

Assumption 3. $[A(\theta)]^i$ *is the n×n null matrix for every i > 2. In other terms, we assume:* $[I - A(\theta)]^{-1} = I + A(\theta) + [A(\theta)]^2$.

Because of the above assumption, equation (5) becomes:

(7) $$y(\theta) = \frac{1}{u_0(\theta) + u_1(\theta) + u_2(\theta)}$$

where $u_0(\theta) = e_1 \cdot \ell(\theta)$, $u_1(\theta) = e_1 \cdot A(\theta) \cdot \ell(\theta)$ and $u_2(\theta) = e_1 \cdot [A(\theta)]^2 \cdot \ell(\theta)$.

The sum $u_0(\theta) + u_1(\theta) + u_2(\theta)$ represents the labour that is embodied in one unit of commodity [1] obtained as a net product by technique θ. $u_0(\theta) = e_1 \cdot \ell(\theta)$ is the amount of labour directly employed in the production of one unit of commodity [1], $u_1(\theta) = e_1 \cdot A(\theta) \cdot \ell(\theta)$ the labour employed in the production of capital goods directly employed in the production of one unit of commodity [1], and $u_2(\theta) = e_1 \cdot [A(\theta)]^2 \cdot \ell(\theta)$ the labour employed in the production of capital goods indirectly employed in the production of one unit of commodity [1].[6]

Assumption 4. *No technique in Θ is dominated. In other words, for every technique $\theta \in \Theta$, there exists no other technique $\hat{\theta}$ such that* $u_0(\theta) \geq u_0(\hat{\theta})$, $u_1(\theta) \geq u_1(\hat{\theta})$ *and* $u_2(\theta) \geq u_2(\hat{\theta})$.

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[5] Since no capital good can enter directly or indirectly into its own production, the commodities can be labelled in such a way that A(θ) is a n×n triangular matrix with all zero entries on the main diagonal, which implies that $[A(\theta)]^n$ is necessarily the n×n null matrix. It is, however, clear that $[A(\theta)]^i$ might become a null matrix even for i smaller than n.

[6] Cf. Burmeister (1980) pp. 135, 136; Pasinetti (1981) p. 113; Kurz and Salvadori (1995) pp. 111, 112.

Assumption 5. *The techniques in* Θ *are labelled in such a way that* $y(\hat{\theta}) > y(\breve{\theta})$ *whenever* $\hat{\theta} > \breve{\theta}$. *That is,* $u_0(\hat{\theta}) + u_1(\hat{\theta}) + u_2(\hat{\theta}) < u_0(\breve{\theta}) + u_1(\breve{\theta}) + u_2(\breve{\theta})$ *whenever* $\hat{\theta} > \breve{\theta}$.

It should be noted that assumption 5 implies $dy/d\theta > 0$, and so, because of equation (7), we have: $u_0'(\theta) + u_1'(\theta) + u_2'(\theta) < 0$, $\forall \theta \in (0,1)$.

3. THE CHOICE OF TECHNIQUES

For an interest rate r, a wage rate w and a price vector $p \in R_+^n$, the unit full cost $c \in R_+^n$ of the n products with technique θ is defined by the following equation:

(8) $$c = (1+r) \cdot A(\theta) \cdot p + \ell(\theta) \cdot w$$

Under the hypothesis of free competition, we can focus the attention on price vectors that leave no extra profit above the unit full cost of production. In so doing, we have[7]:

(9) $$c = p = [I - (1+r) \cdot A(\theta)]^{-1} \cdot \ell(\theta) \cdot w$$

The full cost of production of one unit of commodity [1] obtained as a net product from technique θ is therefore:

(10) $$e_1 \cdot c = e_1 \cdot [I - (1+r) \cdot A(\theta)]^{-1} \cdot \ell(\theta) \cdot w = [u_0(\theta) + (1+r) \cdot u_1(\theta) + (1+r)^2 \cdot u_2(\theta)] \cdot w$$

According to customary procedure, for every given interest rate and wage rate, the optimal technique is the one that minimises the unit production cost of net output.[8] The techniques in use will in fact constitute the solution to the following minimisation problem:

(11) $$\begin{cases} \min\limits_{\theta} u_0(\theta) + (1+r) \cdot u_1(\theta) + (1+r)^2 \cdot u_2(\theta) \\ \text{s.t.}: \theta \in \Theta \end{cases}$$

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[7] Cf. also Burmeister (1980) p. 152.

[8] As is known, under constant returns to scale, which is the case considered here, the typical problem of minimising the cost of production for a given level of net output can be solved by minimising the unit production cost of net output.

It can be assumed for our purposes that the function to be minimised is convex on S (concave from above), so that the following FOC is necessary for internal minima:

$$(12) \qquad u_0'(\theta) + (1+r)\cdot u_1'(\theta) + (1+r)^2 \cdot u_2'(\theta) = 0$$

We have corner solutions when the FOC determines a θ outside the set Θ, i.e. greater than 1 or smaller than 0. In any case, from the solution of the minimum problem (11) we obtain the function $\theta(r)$, which gives us the technique in use at the interest rate r.

Proposition 1. $\theta(0) = 1$.

Proof: when r = 0, the function to minimise is $u_0(\theta) + u_1(\theta) + u_2(\theta)$ and, because of assumption 5, we have $u_0(1) + u_1(1) + u_2(1) < u_0(\theta) + u_1(\theta) + u_2(\theta) \ \forall\ \theta \in \Theta \setminus \{1\}$. ■

Proposition 2. *If commodity [1] is the numeraire,* $\theta(r)$ *is the technique making it possible to pay the highest wage rate when the interest rate is r.*[9]

Proof: if we posit $p_1 = 1$, then clearly $e_1 \cdot p = 1$. Equation (10) therefore implies:

$$(13) \qquad w(\theta, r) = \frac{1}{u_0(\theta) + (1+r)\cdot u_1(\theta) + (1+r)^2 \cdot u_2(\theta)}$$

where w(θ, r) is the wage rate that can be paid using technique θ at the interest rate r.

For this interest rate level, $\theta(r)$ is by definition the technique that minimises $u_0(\theta) + (1+r)\cdot u_1(\theta) + (1+r)^2 \cdot u_2(\theta)$ and therefore the technique with the highest w(θ, r). ■

Because of equation (13), negative interest rates imply wage rates greater than the net product per worker and can therefore be regarded as economically inadmissible.

For a zero interest rate, proposition 1 – the "golden rule" – tells us that θ = 1 is the technique in use. If we exclude the trivial case in which $\theta(r) = 1$ ∀r, there are interest rate levels r > 0 such that $\theta(r) < 1$. As a result, for interest rates sufficiently close to zero, the function $\theta(r)$, which is continuous under our assumption, must be non-increasing (cf. also Mas-Colell 1989).

The function may remain non-increasing for every non-negative interest rate, which means that if $\theta(\hat{r}) \neq \theta(\breve{r})$ with $\hat{r} < \breve{r}$, then $\theta(\hat{r}) > \theta(\breve{r})$. This is in fact the typical neoclassical case where a

[9] A similar result, for a model with a finite number of linear techniques, is proved in Garegnani (1970) p. 411, footnote 1.

rise in the rate of interest entails the use of a technique with a lower net product per worker and thus a less capital-intensive technique.

The function $\theta(r)$ may also be non-monotonic, however, and therefore become increasing after an initial decreasing stretch. This case will be considered in the following section.

4. RESWITCHING

If the function $\theta(r)$ is non-monotonic, reverse capital deepening occurs. In other words, a rise in the rate of interest entails the use of a technique with a higher net product per worker.

Non-restrictive sufficient[10] conditions for this phenomenon can easily be found.

Proposition 3. *In the economy we are considering, if* $u_0'(1) < u_2'(1) < 0$, $u_1'(1) > 0$ *and* $u_1'(1) > 2\sqrt{u_0'(1) \cdot u_2'(1)}$, *then* $\theta(r)$ *is non-monotonic.*

Proof: for every given θ, we can use the FOC in order to determine the interest rates at which that technique is in use. Since the FOC is second-degree with respect to (1 + r), every given θ might be in use at two different interest rate levels:

$$(14) \qquad r_{1,2}(\theta) = \frac{-u_1'(\theta) \pm \sqrt{[u_1'(\theta)]^2 - 4u_0'(\theta) \cdot u_2'(\theta)}}{2u_2'(\theta)} - 1.$$

Solutions of (14) are considered economically admissible if they are non-negative real numbers.

Let us now focus attention on the case with θ = 1. Since $u_1'(1) > 2\sqrt{u_0'(1) \cdot u_2'(1)}$, it is certain that for θ = 1 the solutions of equation (14) are real numbers. Moreover, because of $u_1'(1) > 0$ and $u_2'(1) < 0$, we have:

$$r_2(1) = \frac{-u_1'(1) - \sqrt{[u_1'(1)]^2 - 4u_0'(1) \cdot u_2'(1)}}{2u_2'(1)} - 1 > r_1(1) = \frac{-u_1'(1) + \sqrt{[u_1'(1)]^2 - 4u_0'(1) \cdot u_2'(1)}}{2u_2'(1)} - 1$$

and therefore, in order to prove that both $r_1(1)$ and $r_2(1)$ are strictly positive, it is sufficient to prove that $r_1(1) > 0$.

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[10] A necessary condition is given in Hatta (1976) pp. 130, 131.

Because of assumption 5, we know that $-u_0'(1)-u_2'(1)>u_1'(1)$. Indeed, since $u_2'(1)<0$, we have: $-4u_2'(1)\cdot u_0'(1)<4u_2'(1)\cdot u_1'(1)+4[u_2'(1)]^2$. The addition of $[u_1'(1)]^2$ to both sides gives us $[u_1'(1)]^2-4u_2'(1)\cdot u_0'(1)<[u_1'(1)]^2+4u_2'(1)\cdot u_1'(1)+4[u_2'(1)]^2$, which implies[11]:

$$\sqrt{[u_1'(1)]^2-4u_0'(1)\cdot u_2'(1)}<u_1'(1)+2u_2'(1).$$

Therefore:

$$\frac{-u_1'(1)+\sqrt{[u_1'(1)]^2-4u_0'(1)\cdot u_2'(1)}}{2u_2'(1)}>1.$$

There are thus two (disjointed) strictly positive interest rates at which the FOC is satisfied with $\theta = 1$. Since $\theta(r)$ is a continuous function, it is non-monotonic. ■

The conditions used in proposition 3 are non-restrictive in two senses. First, there is no general economic principle or hypothesis in conflict with them. Second, they are generic or robust, in that they are not affected by arbitrarily small changes in the underlying functions.

5. DEMAND FOR CAPITAL

For the economy addressed here, the vector of demand for capital per worker with technique θ is[12]:

$$k(\theta)=q(\theta)\cdot A(\theta) \tag{15}$$

and the demand for capital in value terms at prices p is therefore:

$$v=k(\theta)\cdot p=q(\theta)\cdot A(\theta)\cdot p. \tag{16}$$

Now, for an interest rate r, if the technique in use is $\theta = \theta(r)$ and there are zero extra-profits for every activity, the prices p must satisfy the following condition:

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[11] Note that $u_0'(1)<u_2'(1)<0$ and $u_1'(1)>2\sqrt{u_0'(1)\cdot u_2'(1)}$ imply $u_1'(1)+2u_2'(1)>0$.

[12] Since commodity [1] is a pure consumption good, we have $k_1(\theta)=0\ \forall\ \theta\in\Theta$.

(17) $$p = (1+r) \cdot A(\theta) \cdot p + \ell(\theta) \cdot w(\theta, r)$$

which implies:

(18) $$q(\theta) \cdot p = (1+r) \cdot q(\theta) \cdot A(\theta) \cdot p + q(\theta) \cdot \ell(\theta) \cdot w(\theta, r).$$

Assuming commodity [1] as the numeraire, from equation (1) we have:

(19) $$q(\theta) \cdot p = y(\theta) + q(\theta) \cdot A(\theta) \cdot p$$

so that, combining equations (18) and (19) and remembering that $q(\theta) \cdot \ell(\theta) = 1$ because of equation (2), we have:

(20) $$y(\theta) = r \cdot q(\theta) \cdot A(\theta) \cdot p + w(\theta, r)$$

which allows us to obtain another expression for the demand for capital in value terms:

(21) $$v = \frac{y(\theta) - w(\theta, r)}{r}.$$

We can now define the functions $y(r) = y(\theta) \circ \theta(r)$ and $w(r) = w(\theta, r) \circ \theta(r)$, respectively expressing the net product (per worker) and the wage rate as functions of the interest rate. In this way, equation (21) implies:

(22) $$v'(r) = \frac{1}{r} \cdot [y'(r) - w'(r) - v(r)].$$

with:

(23) $$y'(r) = y'(\theta) \cdot \theta'(r)$$

and, remembering that $w'_{\theta}(\theta, r) = 0$ when $\theta = \theta(r)$ (proposition 2), then:

(24) $$w'(r) = w'_{\theta}(\theta, r) \cdot \theta'(r) + w'_{r}(\theta, r) = w'_{r}(\theta, r).$$

Moreover, it can be easily verified that in the model considered here, $w'(r) < 0$ and $w''(r) > 0$. The following proposition can therefore be stated:

Proposition 4. *If* $w'(r) < 0$ *and* $w''(r) > 0$*, then* $v(r) > -w'(r)$.

Proof: from equations (7) and (13) we know that $y(\theta) = w(\theta,0)$ $\forall$ $\theta \in \Theta$. Therefore, given a technique $\theta \in \Theta$, the demand for capital in value terms is $v = [w(\theta,0) - w(\theta,r)]/r$.

According to a well-known result of analysis (the Taylor formula[13]), if $w'_r(\theta,r) < 0$ and $w''_{rr}(\theta,r) > 0$, then $[w(\theta,0) - w(\theta,r)]/r > -w'_r(\theta,r)$.

When $\theta = \theta(r)$, $[w(\theta,0) - w(\theta,r)]/r = v(r)$. Therefore, because of equation (24), we have: $v(r) > -w'(r)$. ■

Since $y'(\theta)$ is always positive because of assumption 5, proposition 4 implies that $v'(r)$ is certainly negative for every interest rate r such that $\theta'(r) < 0$. Moreover, if $\theta(r)$ is non-monotonic, there must be at least one interest rate $\bar{r}$ such that $\theta'(\bar{r}) = 0$. In this case, because of the continuity of the functions in question here, $v'(r)$ is certainly negative for interest rates in a neighbourhood of $\bar{r}$, regardless of the sign of $\theta'(r)$.

We can therefore have $v'(r) < 0$ even for interest rates such that $\theta'(r) > 0$ (and $y'(r) > 0$). In particular, as we will prove by means of the numerical example in the next section, we can even have a monotonically decreasing demand for capital in value despite reswitching of technique and reverse capital deepening.

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[13] Let us take a twice differentiable function $f(\cdot)$. Because of the Taylor formula we have:

$$f(\alpha) = f(\beta) + f'(\beta)\cdot(\alpha - \beta) + \frac{1}{2} f''(x)\cdot(\alpha-\beta)^2, \text{ with } x \in (\alpha,\beta).$$

This formula implies:

$$\frac{f(\alpha) - f(\beta)}{\alpha - \beta} = f'(\beta) + \frac{1}{2} f''(x)\cdot(\alpha - \beta).$$

Therefore, if $f''(x) > 0$ and $\alpha - \beta < 0$, then:

$$\frac{f(\alpha) - f(\beta)}{\alpha - \beta} < f'(\beta) \text{ or } \frac{f(\alpha) - f(\beta)}{\beta - \alpha} > -f'(\beta).$$

The result in the text is obtained on positing $\alpha = 0$, $\beta = r$ and $f(\cdot) = w(\theta,\cdot)$.

6. A NUMERICAL EXAMPLE

We shall consider a very simple numerical example. Let us assume $u_0 = \frac{a}{b+\theta}$; $u_1 = \theta$ and $u_2 = \frac{c}{b+\theta}$, with a, b and c taken as given parameters. In this case, equation (7) implies $y = \frac{b+\theta}{a+\theta b+\theta^2+c}$ and equation (13) $w = \frac{b+\theta}{a+(\theta b+\theta^2)(1+r)+c(1+r)^2}$.

Moreover, from the cost minimisation problem (11) and the first-order condition (12), we obtain $\theta = \min\left\{1; \max\left[0; \sqrt{\frac{a+c(1+r)^2}{(1+r)}} - b\right]\right\}$.

On positing a = 3, b = 1 and c = 1, we therefore obtain the results presented in table 1 and represented graphically in figures 1 and 2, which show respectively the demand for capital schedule v(r) and the net product schedule y(r).

Tab. 1

interest rate	tecnique in use	wage rate	net product	demand for capital
0,0	1,00000	0,33333	0,33333	0,22222
0,1	0,95634	0,31211	0,33323	0,21111
0,2	0,92354	0,29270	0,33300	0,20149
0,3	0,89939	0,27484	0,33274	0,19299
0,4	0,88225	0,25838	0,33252	0,18535
0,5	0,87083	0,24316	0,33235	0,17837
0,6	0,86414	0,22908	0,33224	0,17192
0,7	0,86137	0,21605	0,33219	0,16592
0,8	0,86190	0,20396	0,33220	0,16029
0,9	0,86519	0,19276	0,33225	0,15499
1,0	0,87083	0,18237	0,33235	0,14997
1,1	0,87845	0,17273	0,33246	0,14521
1,2	0,88776	0,16377	0,33259	0,14069
1,3	0,89851	0,15544	0,33273	0,13637
1,4	0,91050	0,14770	0,33287	0,13226
1,5	0,92354	0,14049	0,33300	0,12833
1,6	0,93748	0,13378	0,33311	0,12458
1,7	0,95221	0,12752	0,33320	0,12099
1,8	0,96759	0,12168	0,33327	0,11755
1,9	0,98355	0,11622	0,33332	0,11426
2,0	1,00000	0,11111	0,33333	0,11111
2,1	1,00000	0,10633	0,33333	0,10810
2,2	1,00000	0,10183	0,33333	0,10523
2,3	1,00000	0,09761	0,33333	0,10249
2,4	1,00000	0,09363	0,33333	0,09988

Fig. 1

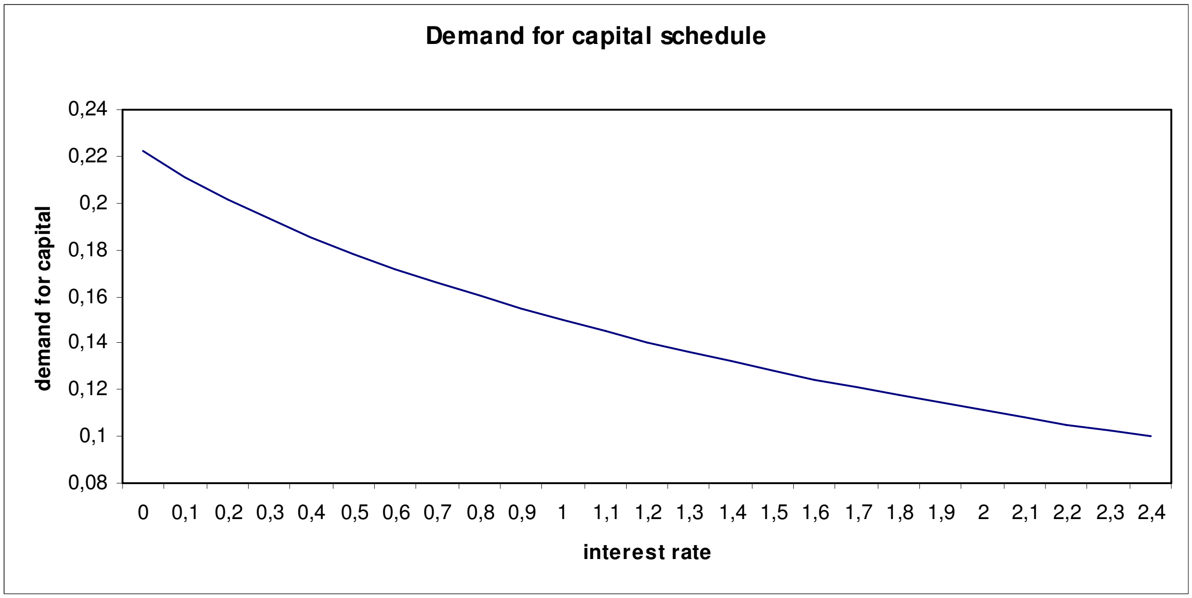
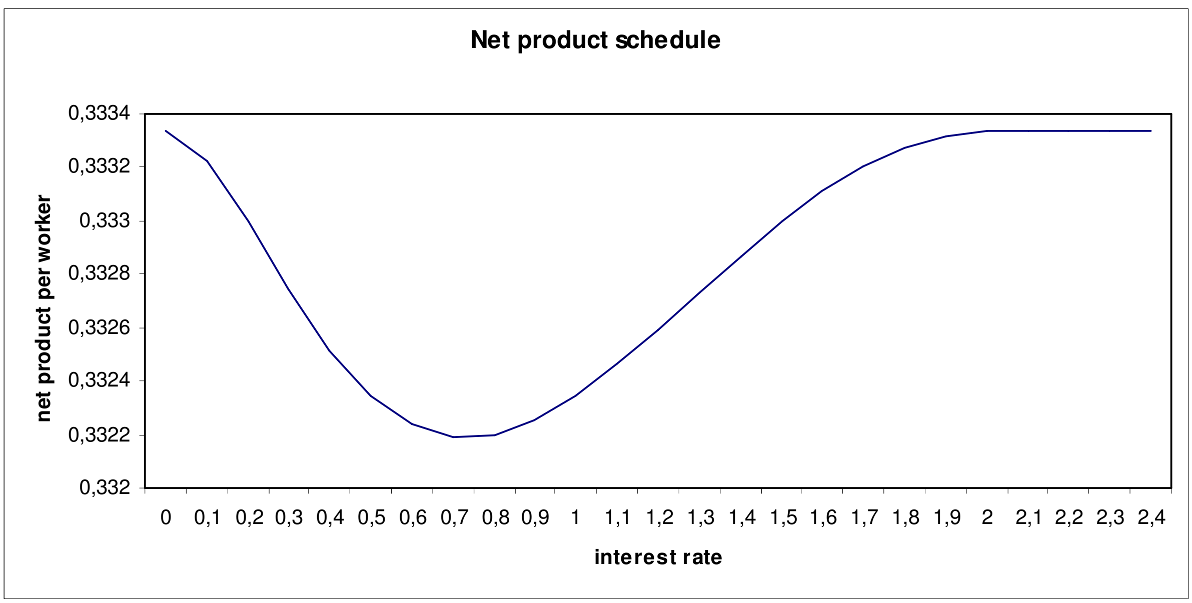


Fig. 2



For the numerical example considered here, the cost minimisation problem (11) has an interior solution for every $0 \le r \le 2$ and, as predicted by proposition 3, the technique $\theta = 1$ satisfies the first-order condition for two disjointed interest rates r = 0 and r = 2. (The data of our example were chosen precisely in order to obtain this result.)

As a result, reswitching occurs and the net product schedule has a non-monotonic shape. Despite this, however, the demand for capital schedule is monotonically decreasing.

7. CONCLUSIONS

The influence of a change in the interest rate on the demand for capital (in value terms) can be broken down into a "real effect" and a "price effect" (cf. in particular Bhaduri 1966, Burmeister 1976 and Garegnani1984). The first is the effect of a change in interest rate on the technique in use, as well as the physical capital employed and the net product per worker obtained. The second is simply the effect on the value of the same physical capital caused by the change in relative prices of capital goods associated with a variation in interest rate.

As defined at the outset, reswitching and reverse capital deepening exclusively regard the real effect and can in this connection affect the shape of the demand for capital schedule. This shape also depends, however, on the price effect, which may be opposite in sign to the real effect and may prevail.

It is therefore possible to have a monotonically increasing demand for capital even in the case of a well-behaved choice of techniques and indeed a negative real effect. This possibility is well known and the simplest example is the case with a capital good whose production is more capital-intensive than that of a consumption good. Much less known is the possibility of a monotonically decreasing demand for capital associated with reswitching, reverse capital deepening and indeed a positive real effect. This is the case examined here.

More precisely, a model with a continuum of linear techniques has been used to show two things. First, the demand for capital schedule may be decreasing even when a rise in the interest rate entails an increase of the net product per worker (section 5). Second, there may be a monotonically decreasing demand for capital schedule even though a sufficient condition for reswitching and reverse capital deepening is satisfied (section 6). Both those results are due to a negative price effect prevailing over the positive real effect.

Finally, the results obtained here suggest that the shape of the demand for capital schedule is not a good indicator for the phenomena of reswitching and reverse capital deepening. In particular, even when the demand for capital is monotonically decreasing, it is not possible to conclude that there are no problems arising from the choice of technique, just as an increasing demand for capital does not entail their presence.

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