Ordered Search in Differentiated Markets

Zhou, Jidong

Department of Economics, University College London

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Jidong Zhou

Department of Economics, University College London

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Abstract

This paper presents an ordered search model in which consumers search both for price and product fitness. We show that there is price dispersion in equilibrium and prices rise in the order of search. The top firms in consumer search order, though charge lower prices, earn higher profits due to their larger market shares.

Keywords: non-random search, price dispersion, product differentiation
JEL classification: D43, D83, L13

1 Introduction

In a variety of circumstances, consumers need to search to find a satisfactory product. However, not as most of the search literature assumes, the order in which consumers search through alternatives is often not random. For example, when facing options in a list such as links on a search engine webpage, dishes on a menu, or candidates on a ballot paper, people often consider them from the top down; when shopping in a high street, a bazaar, or a supermarket, consumers' search order is restricted by the spatial locations of sellers or products; when we go to a travel agent to buy airline tickets or a financial advisor to buy a savings product, the advisor may tell us the options one by one in a predetermined order.

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In a costly search environment, the order in which consumers sample products is an important determinant of their choices, and conceivably firms can increase their sales by occupying top positions in consumer search process. In effect it is a common practice for firms to pay for their products to be displayed in favorable positions. For example, manufacturers pay supermarkets for access to prominent positions; firms bid for sponsored links on search engines; sellers pay more for salient advert slots in yellow page directories. What is less clear is how non-random consumer search (which may be itself a consequence of marketing competition) affects firms' pricing behavior in the market.

Arbatskaya (2007) has studied an ordered search model where firms supply a homogeneous product and all consumers search sequentially in a predetermined order. Since consumers only care about price, in equilibrium the price should decline with the rank of products, otherwise no rational consumer would have an incentive to sample products in unfavorable positions.[1] We will consider an ordered search model with horizontally differentiated products where consumers search both for price and product fitness. Our model is not only more realistic, especially for markets where consumers have diverse tastes, but also provides new insights into firms' pricing incentive in an ordered-search environment. In particular, we will show that, with product differentiation, the price will *rise* with the rank of products. This is essentially because a firm positioned earlier in consumer search process has a more price sensitive demand. Therefore, introducing product differentiation may significantly change the price prediction in an ordered search model. Our prediction is consistent with the observation that a product displayed in a favorable position—such as a book at the entrance to a bookshop—is often sold at a discount. The top firms in consumer search order, though charge lower prices, still earn higher profits due to their larger market shares, which also supports the fact that firms are willing to pay for top positions.

The search model with differentiated products is initiated by Wolinsky (1986) and further developed by Anderson and Renault (1999). Both papers consider random consumer search. More recently, Armstrong, Vickers, and Zhou (2007) (AVZ thereafter) use that framework to model prominence by assuming that all consumers

[1] An earlier paper on ordered consumer search is Perry and Wigderson (1986). There is two-sided asymmetric information in their model: products are homogenous but each seller has uncertain costs, and consumers differ in their willingness-to-pay for the product. They also assume no scope for going back to a previous offer. They argue that in equilibrium the observed prices, on average, could be non-monotonic in the order of sellers.

will sample a prominent product first and, if it is not satisfactory, they will continue to search randomly among other products.[2] One result in AVZ is that in equilibrium the prominent product is cheaper than others.[3] This paper considers a completely ordered search model and generalizes the price result in AVZ. Another important difference emerged in our ordered search model is, when there are more than two firms, the form of consumers' optimal stopping rule crucially depends on whether they expect a rising or declining price sequence. Hence, we need to deal with the issue of multiple equilibria, which is absent is AVZ.

The remainder of this paper is organized as follows. Section 2 presents the model, and it is analyzed in section 3. Section 4 concludes and discusses possible extensions. Technical proofs are included in the Appendix.

2 A Model of Ordered Search

Our underlying model of consumer choice is based on the framework developed by Wolinsky (1986). There are $n \geq 2$ firms indexed by $1, 2, \cdots, n$, each of which supplies a single product at a constant unit cost which we normalize to zero. There are a large number of consumers with measure of one, and each consumer has a unit demand for one product. The value of a firm's product is idiosyncratic to consumers. Specifically, $(u_1, u_{2,} \cdots, u_n)$ are the values attached by a consumer to different products, and u_k is assumed to be independently drawn from a common distribution $F(u)$ on $[u_{\min}, u_{\max}]$ which has a positive and differentiable density function $f(u)$. Therefore, there are no systematic quality differences among products. We also assume that all match utilities are realized independently across consumers. The surplus from buying one unit of firm k's product at price p_k is $u_k - p_k$. If all match utilities and prices are known, a consumer will choose the product providing the highest surplus. If $u_k - p_k < 0$ for all k, she will leave the market without buying anything.

Initially, however, we assume consumers have imperfect information about prices

[2] Hortaçsu and Syverson (2004) construct a related empirical non-random search model, where investors sample differentiated mutual funds with unequal probabilities. But they did not explore theoretical predictions of their model, and there is also no empirical conclusion about the relationship between sampling probability and price.

[3] AVZ also examine the welfare impact of introducing a prominent product. They show that making a product prominent will usually increase industry profit but decrease consumer surplus and total welfare.

and match values. They must gather information through a sequential search process. By incurring a search cost $s > 0$, a consumer can find out any product's price and match utility. We assume that the search process is without replacement and there is costless recall (i.e., a consumer can return to any product she has sampled without extra cost). Departing from the traditional search literature, we suppose that all consumers sample firms in an exogenously specified order. Without loss of generality, firm k will be sampled before firm $k+1$.

Firms know their own positions in consumers' search process, and they simultaneously set prices p_k $(k = 1, 2, \cdots, n)$ to maximize profit.

3 Analysis

3.1 Demand

We first analyze consumers' search behavior. Let a solve

$$\int_a^{u_{\max}} (u - a) dF(u) = s. \tag{1}$$

Thus, if there is no price difference among products and if a consumer has found a product with match value a, she is indifferent between buying this product and sampling one more product. As long as the search cost is not too high, a exists uniquely and decreases with s. Throughout this paper, we assume the search cost is relatively small such that in equilibrium $p_k < a$ for all k and so each firm has a chance to be sampled by consumers.[4]

The form of consumers' optimal stopping rule depends on whether they expect a rising or declining price sequence. Since we aim to show that $p_1 < p_2 < \cdots < p_n < a$ is an equilibrium, we first assume that consumers hold such an expectation of increasing prices. We will discuss the issue of multiple equilibria in Section 3.3,

[4] When a consumer expects $p_1 < a$, her expected surplus from sampling product 1 is $\int_{p_1}^{u_{\max}} (u - p_1) dF(u) - s > 0$ and so she is willing to participate in the market. Similarly, when a consumer expects $p_{k+1} < a$, there is a positive probability that she will further sample product $k+1$ after sampling the first k products. Therefore, $p_k < a$ for all k ensures that every firm is active in the market. However, as usual in search models, there are uninteresting equilibria where consumers are only willing to sample the first $k \leq n-1$ products, because they expect that all other firms are charging very high prices such that visiting them is not worthwhile at all. Since they do not expect consumers to visit them, those firms have no incentive to lower their prices. We do not consider these equilibria further.

and there we will exclude the possibility of an equilibrium with declining prices.[5]

The Optimal Stopping Rule *Suppose consumers expect an increasing price sequence* $p_1^e < p_2^e < \cdots < p_n^e < a$*. Then the optimal stopping rule is characterized by a sequence of decreasing cutoff reservation surplus levels* $z_1 > z_2 > \cdots > z_{n-1}$*, where* $z_k \equiv a - p_{k+1}^e$*. That is, if a consumer has already sampled* k firms (with $k \leq n-1$)*, she will search on if the maximum surplus so far* $v_k \equiv \max\{0, u_1 - p_1, \cdots, u_k - p_k\}$ *is less than* z_k*; otherwise she will stop searching and buy the best product so far. After sampling all products, she will either buy the best one if it provides positive surplus, or leave the market without buying anything.*

With such a stopping rule, consumers seem to be making "myopic" search decisions because at each firm $k \leq n-1$ they behave as if there were only one firm left. But it proves to be optimal by means of backward induction. When a consumer has already sampled product $n-1$, it is clear that she should sample product n further if and only if the maximum surplus so far is less than $z_n = a - p_n^e$ according to the definition of a in (1). Now make the inductive assumption and consider the situation when she has already sampled product $k \leq n-2$. If v_k is less than $z_k = a - p_{k+1}^e$, then sampling product $k+1$ is always worthwhile. If v_k is greater than z_k, expecting that she will stop searching whatever surplus she discovers at the next firm (because of $v_{k+1} \geq v_k > z_k > z_{k+1}$ and the inductive assumption), she should actually cease her search now.

We now derive demand functions. We claim that firm k's demand when it charges p_k is

$$q_k(p_k) = h_k \left[1 - F(z_{k-1} + p_k)\right] + r_k, \tag{2}$$

where

$$h_k = \prod_{j \leq k-1} F(z_{k-1} + p_j)$$

is the number of consumers who visit firm k, and

$$r_k = \sum_{i=k}^{n} \int_{z_i}^{z_{i-1}} f(u + p_k) \prod_{j \leq i, \neq k} F(u + p_j) du.$$

For this expression to be valid for every k, we use $z_0 = a - p_1^e$, $z_n = 0$, and $\prod_{j \leq 0} = 1$.

[5] There is no such an issue when $n = 2$ or when consumers sample randomly among all other firms after visiting firm 1 as in AVZ.

This demand function can be understood as follows. A consumer will come to firm k if she does not stop at any of the first $k-1$ firms (i.e., if $v_i < z_i$ for all $i \leq k-1$). This condition is equivalent to $v_{k-1} < z_{k-1}$, since v_i increases weakly while the cutoff reservation surplus level z_i decreases. So the probability of this event is $\Pr(v_{k-1} < z_{k-1}) = h_k$. (In particular, $h_1 = 1$ since all consumers sample product 1 first.) This consumer will immediately buy at firm k if she finds out $u_k - p_k$ is greater than z_{k-1}. This is because now $v_k > z_{k-1} > z_k$ and so she will not search on, and at the same time all previous products have surplus lower than z_{k-1}. The probability of that is $1 - F(z_{k-1} + p_k)$. This explains the first term in (2), and we illustrate it as (a) in Figure 1 below.

If this consumer finds out $u_k - p_k$ is less than z_{k-1} but greater than z_k, she will not search on either, but she will buy at firm k only if product k is better than each previous product. (That is, firm k is now competing with all firms positioned before it but none of those positioned after it.) The (unconditional) probability of this whole event is

$$
\begin{aligned}
& \Pr(\max\{z_k, v_{k-1}\} < u_k - p_k < z_{k-1}) \\
= & \int_{z_k+p_k}^{z_{k-1}+p_k} \prod_{j\leq k-1} F(u_k - p_k + p_j)\, dF(u_k) \\
= & \int_{z_k}^{z_{k-1}} f(u+p_k) \prod_{j\leq k-1} F(u+p_j) du,
\end{aligned}
$$

where the second equality is from changing the integral variable from u_k to $u = u_k - p_k$. This explains the first term in r_k, and we illustrate it as (b) in Figure 1.

If this consumer finds out $u_k - p_k$ is less than z_k but greater than z_{k+1}, then the only possibility that she will patronize firm k eventually is when v_{k-1} is also less than z_k (which means that she will continue to sample product $k+1$ but none of further ones), and product k is the best one among the first $k+1$ products. (Now firm k is competing not only with all firms positioned before it but also with one positioned after it). The (unconditional) probability of this whole event is

$$
\begin{aligned}
& \Pr(\max\{z_{k+1}, v_{k-1}, u_{k+1} - p_{k+1}\} < u_k - p_k < z_k) \\
= & \int_{z_{k+1}+p_k}^{z_k+p_k} \prod_{j\leq k+1, \neq k} F(u_k - p_k + p_j)\, dF(u_k) \\
= & \int_{z_{k+1}}^{z_k} f(u+p_k) \prod_{j\leq k+1, \neq k} F(u+p_j) du,
\end{aligned}
$$

where the second equality is again from changing the integral variable. This explains the second term in r_k, and we illustrate it as (c) in Figure 1. In general, the term indexed by i in r_k is firm k's demand when $u_k - p_k \in (z_i, z_{i-1})$.

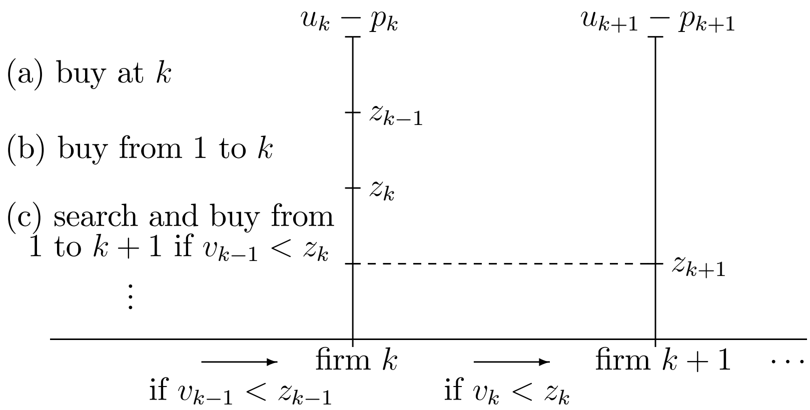

Figure 1: Consumer Decision at firm k

Using the terminology in AVZ, we call the first portion of demand in (2) the "fresh demand" and the second portion of demand (i.e., r_k) the "returning demand".[6] Notice that h_k is independent of firm k's actual price. This is because whether a consumer will come to firm k is not affected by its actual price but by this consumer's expectation of p_k. Also notice that how a firm's returning demand varies with its actual price only depends on the density function f. In particular, for the uniform distribution, a firm's returning demand is independent of its actual price. When a firm increases its price, more consumes will search on (which implies a larger number of potential returning consumers), but they will be less likely to return to this firm. In the uniform setting, these two effects just cancel out each other. As a result, in the uniform setting, returning demand is less price responsive than fresh demand.[7]

3.2 Equilibrium prices and profits

The above analysis of demand is predicated on that consumers hold an expectation of increasing prices, and now we want to verify that $p_1 < p_2 < \cdots < p_n < a$

[6] Notice that the first term in r_k is derived when the consumer does not search on, so the name of "returning demand" is not appropriate for it. We include it in r_k simply because it shares the similar price elasticity with all other terms in r_k.

[7] More generally, when a firm raises its price, its fresh demand will decrease for sure since more consumers will then search on. But part of these consumers will become returning consumers, which is a positive effect of raising a firm's price on its returning demand. Therefore, we expect that returning demand is less price sensitive than fresh demand even for more general distributions. From the expression for r_k in (2), we can see that this happens at least when the density function increases or does not decrease too fast.

is indeed an equilibrium. For simplicity, from now on we focus on the case with uniform distribution on $[0,1]$ (i.e., $F(u) = u$). Then a defined in (1) equals $1-\sqrt{2s}$. We keep the following condition to ensure $p_k < a$ in equilibrium:

$$s \in (0, 1/8), \text{ or } a \in (1/2, 1). \tag{3}$$

Since in the uniform case both h_k and r_k are independent of firm k's *actual* price p_k, profit maximization yields the first-order condition:[8]

$$h_k (1 - z_{k-1} - 2p_k) + r_k = 0, \tag{4}$$

where

$$h_k = \prod_{j \le k-1} (z_{k-1} + p_j); \quad r_k = \sum_{i=k}^{n} \int_{z_i}^{z_{i-1}} \prod_{j \le i, \ne k} (u + p_j) du.$$

Using the fact that consumers' expectation is fulfilled in equilibrium (i.e., $z_{k-1} = a - p_k$), we have

$$p_k = 1 - a + \frac{r_k}{h_k}. \tag{5}$$

Since $h_k (1-a)$ is firm k's fresh demand in equilibrium, r_k/h_k is proportional to the ratio of returning demand to fresh demand. Then the economic meaning of (5) is that a firm whose demand consists of more returning demand proportionally will charge a higher price.

Although it is infeasible to solve equilibrium prices analytically, they exist and have the following property (all omitted proofs are included in the Appendix):[9]

Proposition 1 *Under condition (3), our ordered search model has an equilibrium with*

$$1 - a < p_1 < p_2 < \cdots < p_n < 1/2.$$

The intuition of this result is as follows: the restricted consumer search order tends to make firm k have more fresh demand proportionally than firm $k+1$, while

[8] In the uniform-distribution setting, the first-order condition is sufficient for no local profitable deviation. However, if firm k deviates to a too high price ($p_k > 1 - z_k$), its fresh demand will become zero and its returning demand will become price dependent. This may make the profit function no longer globally concave. However, as in AVZ, we can show that, given other firms are charging their equilibrium prices, firm k's profit function is decreasing at any $p_k > 1 - z_k$. Thus, the equilibrium price we derived below is still valid.

[9] We have not developed a proof for uniqueness, but numerical simulations suggest that, under condition (3), within the region of $[0,1]^n$ the system of first-order conditions (5) has a unique solution in $(1-a, 1/2)^n$.

fresh demand is more price sensitive than returning demand. Therefore, firm k has an incentive to charge a lower price than firm $k+1$. (We also expect our result to hold even for more general distributions so long as fresh demand is more price sensitive than returning demand (see the discussion in footnote 7).) The graph below depicts equilibrium prices when there are three firms, where the three curves from the bottom up represent p_1, p_2 and p_3, respectively.

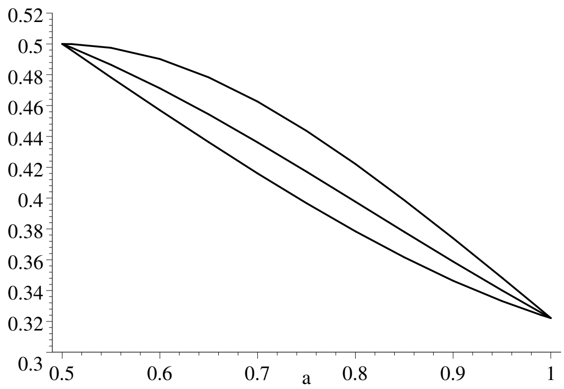

Figure 2: Equilibrium Prices With a When $n = 3$

Two polar cases deserve mention: First, when the search cost is sufficiently small ($a \approx 1$), consumers tend to sample all firms before they purchase, and so non-random search order has no impact and all prices will converge to the full-information equilibrium price $\bar{p}$, say, which satisfies $n\bar{p} = 1 - \bar{p}^n$. (This formula for $\bar{p}$ is obtained from (5) by letting $a \to 1$.) Second, when the search cost is sufficiently large ($a \approx 1/2$), all prices will converge to the monopoly price 1/2. Here, the high search cost makes a consumer willing to stop searching whenever she finds a product which yields her positive surplus, and so each firm acts as a monopolist. Therefore, the price dispersion caused by non-random consumer search is most pronounced when the search cost is at an intermediate level.[10]

In equilibrium firm k has a larger demand than firm $k+1$ (since both $h_k > h_{k+1}$ and $r_k > r_{k+1}$ hold), but it charges a lower price. Hence, it is *a priori* unclear whether firm k earns more or less than firm $k+1$. Let π_k be firm k's equilibrium profit. The following result indicates that the demand effect dominates.

[10] Another polar case is when there are a large number of firms in the market ($n \to \infty$). In that case, all prices will converge to $1 - a$. This is because $\frac{r_n}{h_n} < \int_{p_n}^{a} \left(\frac{u}{a}\right)^{n-1} du$, and the latter tends to zero as $n \to \infty$.

Proposition 2 *Firm 1 earns more than firm 2. For $k \geq 2$, firm k earns more than firm $k+1$ at least when $a < n/(n+1)$.*

Though we only derive a sufficient condition for $\pi_k > \pi_{k+1}$ with $k \geq 2$, numerical simulations suggest that it is true for any $a \in (1/2, 1)$. The following graph is an example with three firms, where the curves from the top down represent π_1, π_2 and π_3, respectively.[11]

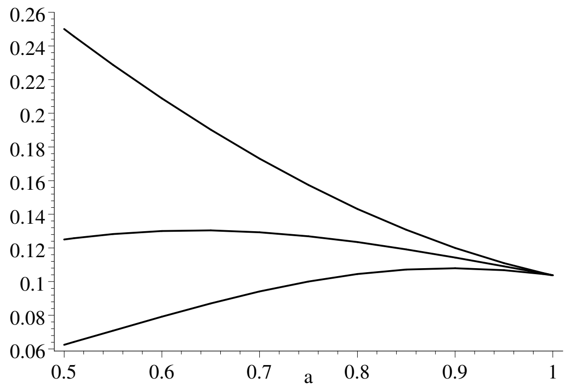

Figure 3: Equilibrium Profits With a When $n = 3$

3.3 Are there other equilibria?

Our analysis so far has shown that a rising price sequence is an equilibrium outcome. Nevertheless, we have not yet discussed other possible equilibria. Particularly, we want to know whether a decreasing price sequence (as in Arbatskaya (2007)) could also be an equilibrium outcome.[12]

Suppose consumers hold an expectation of $a > p_1^e \geq p_2^e \geq \cdots \geq p_n^e$ (but their search order is still restricted). According to Kohn and Shavell (1974), their optimal stopping rule is well defined and is characterized by a sequence of cutoff reservation surplus levels $(z_1, \cdots, z_{n-1})$. That is, a consumer at firm $k \leq n-1$ will continue to search if and only if the maximum surplus so far is less than z_k. One can further

[11] This example also shows that in an ordered search market firms positioned relatively down in consumer search process can benefit from the reduction of search cost. When the search cost becomes smaller (i.e., when a increases), the market share redistribution effect due to the restricted search order is weakened, which tends to harm firm 1 but benefit firms 2 and 3. At the same time, a smaller search cost implies more intense price competition, which harms all firms. The combination of these two effects explains why π_1 decreases while π_2 and π_3 vary non-monotonically with a.

[12] It is still an open question whether our ordered search model has equilibria with non-monotonic price sequences.

show that $z_1 \leq \cdots \leq z_{n-1} = a - p_n^e$ and $z_k \geq a - p_{k+1}^e$ for all $k \leq n-1$.[13] (To have a unified expression for demand functions, we use $z_n = a - p_n^e$ though at firm n there is no need to set a cutoff reservation surplus level at all.)

Now we derive demand functions. For firm k, a consumer will come to it if $u_i - p_i < z_i$ for all $i \leq k-1$. If she finds out $u_k - p_k \geq z_k$, she will buy immediately since she will not search on and product k is better than all previous ones (because of increasing z_i); if she finds out $u_k - p_k < z_k$, then she will continue to search. Once she leaves, she will eventually return and buy at firm k only when she has sampled *all* products (again because of increasing z_i) and product k has the highest positive surplus. Hence, firm k's demand is

$$\begin{aligned} q_k(p_k) &= \Pr(u_i - p_i < z_i \text{ for } i \leq k-1 \text{ and } u_k - p_k \geq z_k) \\ &\quad + \Pr(u_i - p_i < z_i \text{ for } i \leq k-1 \text{ and } \max\{v_{k-1}, u_j - p_j\}_{j \geq k+1} < u_k - p_k < z_k) \\ &= h_k[1 - (p_k + z_k)] + r_k, \end{aligned}$$

where $h_k = \prod_{i \leq k-1}(z_i + p_i)$ is the number of consumers who come to firm k, and r_k represents the number of returning consumers. In our uniform setting, r_k is again independent of firm k's actual price p_k. Moreover, we have $r_k \leq r_{k+1}$. This is because, if a consumer has left both firm k and firm $k+1$, the former's product must on average have a lower net surplus given $z_k \leq z_{k+1}$, and so it can win this consumer back less likely.

Due to the restricted search order, firm k tends to have more fresh demand than firm $k+1$. At the same time, firm k has less returning demand than firm $k+1$ as we have pointed out. Since fresh demand is again more price sensitive than returning demand, firm k will actually have an incentive to charge a lower price. This leads to a contradiction. We formalize this argument in the Appendix.

Proposition 3 *Our ordered search model has no equilibrium with $a > p_1 \geq p_2 \geq \cdots \geq p_n$.*

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[13] Keep the notation $v_k \equiv \max\{0, u_1 - p_1, \cdots, u_k - p_k\}$. First of all, at firm $n-1$, $z_{n-1} = a - p_n^e$ is simply from the definition of a. Now consider a consumer at firm $n-2$. If $v_{n-2} < a - p_{n-1}^e$, then sampling product $n-1$ is always worthwhile. If $v_{n-2} \geq z_{n-1}$, this consumer will never sample firm n no matter what surplus she discovers at firm $n-1$, and so she has no incentive to just visit firm $n-1$ given $v_{n-2} \geq z_{n-1} = a - p_n^e \geq a - p_{n-1}^e$. Thus, we have $a - p_{n-1}^e \leq z_{n-2} \leq z_{n-1}$. The same logic can go backward further to explain the property of the stopping rule.

4 Conclusion

This paper has presented an ordered search model with differentiated products in which consumers search both for price and product fitness. We have shown that in equilibrium there is price dispersion and prices rise in the order of search. The top firms in consumer search process, though charge lower prices, earn higher profits due to their larger market shares.

We have focused on the case where all consumers have the same search cost. When consumers have heterogenous search costs, those with higher search costs are more likely to buy at the top firms, which provides the top firms an incentive to charge higher prices. The final price prediction will then depend on the relative importance of the effect from product differentiation as this paper has identified and the effect from heterogeneous search costs.

We have also restricted our attention to the case with exogenous search order.[14] It would be interesting to endogenize consumer search order through, for example, advertising competition or bidding for online paid placements. Hann and Moraga-Gonzalez (2007) consider a similar search model with differentiated products in which a consumer's likelihood of sampling a firm is proportional to that firm's advertising intensity. But in symmetric equilibrium, all firms set the same price and advertise with the same intensity, and consumers end up searching randomly. Chen and He (2006), and Athey and Ellison (2007) present two auction models in which advertisers bid for sponsor-link positions on a search engine. Distinct from other papers on position auctions, they have a formal search model in the consumer side. In equilibrium, consumers search through the sponsor links in the order presented since they anticipate that high-quality links will be placed higher up the listing, and higher-quality firms do have a greater incentive to buy top positions given consumers' search order. But there is no effective price competition between sellers in both papers, and so no role for non-random consumer search to affect market prices.[15]

[14] Given our equilibrium price result, consumers' search order is actually rational since the top firms are charging lower prices. But if we allow consumers to choose their search order freely, then random search with a uniform price in the market will also be an equilibrium outcome.

[15] Chen and He (2006) do have prices charged by advertisers, but the structure of consumer demand in their model means that the Diamond Paradox is present, and all firms set monopoly prices.

A Appendix

A.1 Proof of Proposition 1

We will first show that, in the uniform setting, under the condition $a \in (1/2, 1)$ our ordered search model has an equilibrium with $1 - a \leq p_1 \leq \cdots \leq p_n \leq 1/2$. We will then exclude the possibility of equalities.

Suppose consumers hold an expectation of $\mathbf{p}^e = (p_1^e, \cdots, p_n^e)$ with $1 - a \leq p_1^e \leq \cdots \leq p_n^e \leq 1/2$. Given other firms' prices p_{-k}, the demand function of firm k is

$$q_k(p_k) = h_k\left(1 - z_{k-1} - p_k\right) + r_k,$$

where

$$h_k = \prod_{j \leq k-1} (z_{k-1} + p_j); \quad r_k = \sum_{i=k}^{n} \int_{z_i}^{z_{i-1}} \prod_{j \leq i, \neq k} (u + p_j) du.$$

More precisely, since we are using uniform distribution on $[0, 1]$, every term $(x + p_j)$ in h_k and r_k should be replaced by $\min\{1, x + p_j\}$. Notice that both h_k and r_k are independent of firm k's actual price p_k, and so we can write the first-order condition as

$$2p_k = 1 - z_{k-1} + \frac{r_k}{h_k}. \tag{6}$$

Step 1: Given $\mathbf{p}^e$, the system of (6) for $k = 1, \cdots, n$ has a solution with

$$p_k = \gamma_k\left(\mathbf{p}^e\right) \in \left[1 - a, 1/2\right].$$

Equation (6) defines the best response of p_k to other prices p_{-k}, which we denote $p_k = b_k\left(p_{-k}; \mathbf{p}^e\right)$. First, from $2p_k \geq 1 - z_{k-1} = 1 - a + p_k^e \geq 2\left(1 - a\right)$, we have $p_k \geq 1 - a$. Second, since z_i decreases and $z_n = 0$, we have

$$r_k \leq h_k \sum_{i=k}^{n} \left(z_{i-1} - z_i\right) = h_k z_{k-1},$$

and so (6) implies $p_k \leq 1/2$. Hence, given $\mathbf{p}^e$ we can now construct a continuous mapping

$$\mathbf{b}\left(\mathbf{p}; \mathbf{p}^e\right) = \left[b_1\left(p_{-1}; \mathbf{p}^e\right), \cdots, b_n\left(p_{-n}; \mathbf{p}^e\right)\right]$$

from $\left[1 - a, 1/2\right]^n$ to itself. A fixed-point argument yields our result.

Step 2: Given $\mathbf{p}^e$, we have $\gamma_{k+1}\left(\mathbf{p}^e\right) \geq \gamma_k\left(\mathbf{p}^e\right)$.

From (6), we have

$$2\left(p_{k+1} - p_k\right) = z_{k-1} - z_k + \frac{r_{k+1}}{h_{k+1}} - \frac{r_k}{h_k} \geq z_{k-1} - z_k + \frac{1}{h_k}\left(r_{k+1} - r_k\right),$$

where the inequality is because $h_k \geq h_{k+1}$. (The equality holds if both of them equal to one.) On the other hand, if we let

$$A = \sum_{i=k+1}^{n} \int_{z_i}^{z_{i-1}} \prod_{j \leq i, \neq k, k+1} (u + p_j) du,$$

then

$$\begin{aligned} r_{k+1} - r_k &= (p_k - p_{k+1}) A - \int_{z_k}^{z_{k-1}} \prod_{j \leq k-1} (u + p_j) \, du \\ &\geq (p_k - p_{k+1}) A - (z_{k-1} - z_k) h_k. \end{aligned}$$

Therefore,

$$2 (p_{k+1} - p_k) \geq \frac{A}{h_k} (p_k - p_{k+1}),$$

which implies $p_{k+1} - p_k \geq 0$.

Step 3: The above analysis implies that, for any consumer expectation $\mathbf{p}^e$ in the domain of $\Omega = \{\mathbf{p} \in [1-a, 1/2]^n : p_1 \leq \cdots \leq p_n\}$ (which is compact and convex), the price competition has an equilibrium $\boldsymbol{\gamma}(\mathbf{p}^e) = [\gamma_1(\mathbf{p}^e), \cdots, \gamma_n(\mathbf{p}^e)]$ which also lies in Ω. Thus, a fixed-point argument implies that our ordered search model has an equilibrium in Ω.

Step 4: We now exclude the equality possibility. First, given $p_k \leq 1/2$, in equilibrium $z_{k-1} = a - p_k > 0$ under the condition $a \in (1/2, 1)$. Also recall that we use $z_n = 0$. Thus, $r_k > 0$ and so equation (5) in the main text implies $p_k > 1 - a$. Second, notice that in equilibrium

$$r_n = \int_0^{a-p_n} \prod_{j \leq n-1} (u + p_j) \, du < h_n (a - p_n).$$

So equation (5) for $k = n$ implies $p_n < 1/2$. Finally, given the equilibrium price $\mathbf{p} \in \Omega$ and the condition $a \in (1/2, 1)$, h_k is *strictly* greater than h_{k+1}. Then, a similar argument as in Step 2 implies $p_{k+1} > p_k$.

A.2 Proof of Proposition 2

Notice that $\pi_k > p_{k+1} q_k(p_{k+1})$ since firm k can at least charge the same price as firm $k+1$. Thus, it suffices to show $q_k(p_{k+1}) > q_{k+1}(p_{k+1})$, or

$$h_k(1 - a + p_k - p_{k+1}) + r_k > h_{k+1}(1-a) + r_{k+1}. \tag{7}$$

(Due to the higher cutoff reservation surplus level at firm k, it may now have less fresh demand than firm $k+1$.) Decompose r_k into two parts: $r_k = A_k + B_k$, where

$$A_k = \int_{a-p_{k+1}}^{a-p_k} \prod_{j \leq k-1} (u + p_j) du$$

is the first term in r_k and B_k includes all other terms. It is ready to see $B_k > r_{k+1}$ since $p_k < p_{k+1}$.

For $k = 1$, we further have $A_k = h_k(p_{k+1} - p_k)$, and so (7) holds since $h_k > h_{k+1}$. That is, firm 1 must earn more than firm 2.

For $k \geq 2$, we have

$$A_k > (p_{k+1} - p_k) \prod_{j \leq k-1} (a - p_{k+1} + p_j) > (p_{k+1} - p_k) h_{k+1},$$

and so (7) holds if $(h_k - h_{k+1}) [1 - a - (p_{k+1} - p_k)] > 0$, or equivalently $p_{k+1} - p_k < 1 - a$. A looser sufficient condition is $r_{k+1}/h_{k+1} < 1 - a$ by using (5). Moreover, we know that r_k/h_k increases with k, and so it suffices to show $r_n/h_n < 1 - a$. From

$$\frac{r_n}{h_n} < \int_{p_n}^{a} \left(\frac{u}{a}\right)^{n-1} du = \frac{a^n - p_n^n}{na^{n-1}} < \frac{a}{n}, \tag{8}$$

we obtain the sufficient condition $a < n/(n+1)$.

A.3 Proof of Proposition 3

We continue our argument in Section 3.3. Given the demand functions derived under consumers' expectation of a declining price sequence, profit maximization yields the first-order conditions:

$$h_k (1 - z_k - 2p_k) + r_k = 0$$

for all k. In particular, for firm n we have

$$h_n (1 - a - p_n) + r_n = 0 \tag{9}$$

by using $z_n = a - p_n$ in equilibrium, and so $1 - a - p_n < 0$. If $p_1 \geq \cdots \geq p_n$, then the first-order condition for any firm $k \leq n - 1$ implies

$$0 = h_k (1 - z_k - 2p_k) + r_k \leq h_k (1 - a - p_k) + r_k < h_n (1 - a - p_n) + r_n,$$

where the first inequality is because $z_k \geq a - p_{k+1} \geq a - p_k$, and the second one is because $p_k \geq p_n$, $h_k > h_n$ and $r_k \leq r_n$. This, however, contradicts to (9).

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