Dual Poverty Trap: Intra- and Intergenerational Linkages in Frictional Labor Markets

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23. November 2008

Online at http://mpra.ub.uni-muenchen.de/13484/
MPRA Paper No. 13484, posted 20. February 2009 13:06 UTC
Dual Poverty Trap: Intra- and Intergenerational Linkages in Frictional Labor Markets

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November 23, 2008

Abstract

This paper constructs an overlapping generations model with a frictional labor market to explain persistent low education in developing countries. When parents are uneducated, their children often face difficulties in finishing school and therefore are likely to remain uneducated. Moreover, if children expect that other children of the same generation will not receive an education, they expect that firms will not create enough jobs for educated workers, and thus are further discouraged from schooling. These intergenerational and intragenerational mechanisms reinforce each other, creating a serious poverty trap. Escape from the trap requires the well-organized and combined implementation of a subsidy for schooling, the provision of free education, support for disadvantaged children, and public awareness programs.

JEL Classification Numbers: O11, J62, J23.

Keywords: overlapping generations model, education, poverty trap, job search, coordination failure.

*The authors are grateful to Ryoichi Imai, Noritaka Kudoh, Kazuo Mino, Yasuhiro Sato, Takashi Shimizu, seminar participants at Osaka University and Kansai University, and conference attendees at the 2006 Japanese Economic Association Meeting at Osaka City University and the Public Economic Theory 2008 Seoul Meeting at Hangyang University for their valuable comments and suggestions. This research is partially supported by the Japan Society for the Promotion of Science, Grant-in-Aid for Young Scientists No. 18730164. Any remaining errors are naturally ours.

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1 Introduction

In low-income countries with a GNI per capita of $580 or less, the gross school enrollment rate for secondary education was only 45% in 2004 (World Bank 2006). This low enrollment rate implies that the majority of the workforce works in less productive sectors and earns low incomes, and this keeps these countries in poverty. Why then has the enrollment rate remained low? This paper develops a theory of persistent poverty and low education, focusing on both the transmission of educational attainment between generations and on the strategic complementarity in schooling choice within the same generation.

When the parents in a family are relatively uneducated, the home environment may be less favorable to study than otherwise, and in that case, children find it difficult to obtain an education. Recent microeconomic empirical studies strongly support the argument that parents’ (in particular, the mother’s) education status is one of the most important determinants of school enrollment and child labor, whereas the wealth level of the household has a relatively minor effect (e.g., Kurosaki et al. 2006, Bratti 2007). Thus, differential home environments generate an intergenerational transmission mechanism through which low educational attainment is inherited from one generation to the next.

In contrast, frictions in labor markets can create a strategic complementarity in schooling choice among children of the same generation. To see this, suppose that the majority of other children in the same generation do not receive an education. In such an economy, a child expects that firms find it difficult to get appropriately educated workers and will not create many jobs that require education.2 On this basis, the child will be discouraged in seeking education because they can rationally anticipate that it will be difficult to find a high-paying job, even when they receive an education.3 This suggests that a low enrollment rate may be a result of an intragenerational coordination failure, where children do not receive an education because others do not.

This paper develops a simple overlapping generations model that incorporates both the intergenerational transmission mechanism and the intragenerational coordination

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1See also Behrman and Rosenzweig (2002) and Antonovics and Goldberger (2005).

2Empirical studies have found that foreign direct investment (FDI) is likely to be attracted by countries with higher average schooling years (e.g., Blonigen et al. 2007). This finding is consistent with our viewpoint, as in low-income countries, jobs that require skilled workers are often created by foreign firms in the form of FDI.

3For instance, Wakabayasi (1998) found that poor employment opportunities discouraged parents from sending their children to secondary school based on interviews conducted in poor communities of the Roi-Et province in Thailand.
problem, and shows that the interaction of those two mechanisms can create a serious poverty trap. In fact, when only one of these two mechanisms is present, the economy can escape the poverty with little policy intervention. Even among uneducated parents, some will maintain a good family environment and therefore the rate of educational attainment will increase gradually over generations. Even when the coordination failure problem is present, it does not rule out the possibility that children choose another good equilibrium. However, when these mechanisms are present simultaneously, they reinforce each other: low educational attainment in the previous generation necessarily causes coordination failure, and coordination failure makes the intergenerational mechanism more persistent. As a result, the economy is “dually” poverty trapped.

An economy can be saved from this trap only by well-organized policy packages that simultaneously solve the intergenerational and the intragenerational problems. We show that an appropriate policy package generally comprises several steps and each stage requires multiple policy instruments, including education subsidies, provision of free education, support for children in difficult home environments, and public awareness campaigns. Conversely, if the authority aims to solve only one of the problems, it will fail to even partially resolve the problem targeted. This could be one of the reasons why various forms of development assistance have been unsuccessful (see Easterly (2008) for a recent survey).

Our paper relates to a number of other papers. Many studies explain the low-education poverty trap in terms of credit market imperfections (e.g., Galor and Zeira 1993; Banerjee and Newman 1993). However, the limited success of student loan systems in low-income countries suggests that the credit market is not the only source of the problem. Complementing this body of work, our model shows the emergence of a persistent poverty trap under complete credit markets. Both in the economic development and growth literature, a number of studies analyzed the consequences of the intergenerational transmission of educational attainment and human capital. To name just a few, Azariadis and Drazen (1990) and Moav (2005) showed the possibility of a poverty trap by combining the intergenerational transmission mechanism with certain sorts of nonconvexities assumed in the model (e.g., threshold externalities), while we combine it with the frictional labor market. Our study also relates to Laing, Parivos, and Wang (1995), Takii (1997), Acemoglu (1997), and Burdett and Smith (2002), who showed that coordination failure problems in the frictional labor market may generate multiple equilibria. While the low-education equilibrium may be chosen by chance in these studies, our model explains why the economy necessarily falls into the low-education equilibrium when the intergenerational mechanism is present. This

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4See, for example, the textbook by De la Croix and Michel (2002, Section 5.2).
paper also differs in that we focus on the strategic complementarity between children of the same generation rather than the strategic complementarity between workers and firms.

The remainder of the paper is organized as follows. Section 2 presents a simple overlapping generations model and derives the schooling choices of children. Section 3 introduces a frictional labor market and derives the job creation decisions of firms. Section 4 investigates the equilibrium dynamics of the economy and explains why a “dual” poverty trap emerges. In Section 5, we consider three types of effective policy packages depending on the various situations. Section 6 concludes. The Appendix discusses an alternative labor market setting.

2 Behavior of Individuals

Consider an overlapping generations economy where each agent lives for two periods: childhood and adulthood. In adulthood, agents can be either educated or uneducated, depending on whether they received an education during their childhood. Each adult agent has exactly one child, so that the number of agents in each generation is constant. We normalize the number of agents in each generation to unity.

2.1 Childhood

The life of an agent born in period \( t \) is as follows. Let \( e_t \) denote the number (or equivalently, fraction) of educated adult agents (parents) in period \( t \). Then \( e_t \) children are born to educated parents, and \( 1 - e_t \) children are born to uneducated parents. In the first period of their life, agents choose whether to attend school. If agents want to become educated in adulthood, they must attend school and pay a certain amount of effort (education is indivisible). The amount of effort needed to finish schooling varies among agents, partly because of their own home environment. In particular, each child’s home environment depends on their parents’ educational attainment. When compared with educated parents, uneducated parents may be weaker in providing effective preschool training for their children, may be less concerned with maintaining a good environment for their children to study, and/or may be reluctant to send their children to school. Regardless, children of these parents find it difficult to obtain an education. We assume that among \( 1 - e_t \) agents born to uneducated parents, a fraction \( p \) \((0 < p < 1)\) faces such difficulties. Specifically, among the unit population of generation \( t \) children, \( p(1 - e_t) \) are “children in difficult environments” who face a very high effort cost of obtaining an education and therefore never receive an education.
The remaining $1 - p + pe_t$ agents in this generation, i.e., those born to educated parents plus the fraction $1 - p$ of those born to uneducated parents, are “children in good environments” who can obtain an education with little effort: we assume, for simplicity, this has zero disutility. However, there is still an opportunity cost of obtaining an education. Agents attending school will earn no income in their childhood, while those not attending school can do some simple job that pays $z > 0$ units of goods.

### 2.2 Adulthood

In the second period of their life (period $t+1$), agents can work either in the modern or the traditional sector. In the modern sector, production takes place when a firm and a worker meet and agree to a division of output. Each pair of a firm and an educated worker produces $\widehat{y}_e$ units of goods, whereas each pair of a firm and an uneducated worker produces a smaller amount, $\widehat{y}_u < \widehat{y}_e$. The labor market in the modern sector is frictional and adult agents must search for a vacant job in a firm. For simplicity, we do not consider separate labor markets for educated and uneducated workers—they search for a vacant job in the same pooled market and find one with the same probability.\(^5\) This assumption is relaxed in Appendix A. The probability of an adult agent successfully matching with a vacancy is denoted by $q_{t+1} \in [0, 1]$, which is endogenous (to be explained in Section 3), but taken as given by each agent. If an adult agent fails to match with any vacancy in the modern sector, they work in the traditional sector where an employment position is easy to find with no friction. In that case, they earn $x \leq \widehat{y}_u$.

When an adult agent matches with a vacancy in a firm, the agent and the firm negotiate the division of the output. If the matched pair fails to agree to the division of output, the agent must then work in the traditional sector, where they can earn $x$, whereas the firm gets nothing from the vacant job. We assume that multiple contacts between agents and firms are not allowed in the same period. Given this, and assuming that both parties have equal bargaining power, the matched pair reaches the Nash bargaining solution as follows. When the matched agent is educated, the firm obtains $y_e \equiv (\widehat{y}_e - x)/2$ and the educated agent’s income is $y_e + x$. If the matched agent is uneducated, the firm obtains $y_u \equiv (\widehat{y}_u - x)/2$ and the uneducated agent’s income is

\(^5\)This assumption can be justified if firms cannot distinguish between educated and uneducated agents until they match with a worker. In other words, the search process is *undirected*, as shown in Acemoglu (2001) Alternatively, we can consider a *directed* search process in which firms can distinguish between educated and uneducated agents before the match and therefore only search for educated workers. In Appendix A, we show that the latter setting yields the same outcome if it is more costly for firms to search for an educated worker in an economy with a smaller fraction of educated workers.
2.3 Schooling Decision

Recall that there are $1 - p + pe_t$ children in good environments, who can obtain an education without disutility. They choose to attend school if and only if it yields an expected lifetime income at least as high as what they would have obtained by not attending school. If an agent born in period $t$ chooses not to attend school, they will earn $z$ in the first period, and their second period income is $y_u + x$ with probability $q_{t+1}$ and $x$ with probability $1 - q_{t+1}$. Therefore, the expected lifetime income is $z + \beta [q_{t+1}(y_u + x) + (1 - q_{t+1})y_u]$, where $\beta \in (0, 1)$ denotes the discount factor. If an agent decides to become educated, they must abandon their first-period income but may earn a higher income in the second period with probability $q_{t+1}$; in this case, the expected lifetime income is $\beta [q_{t+1}(y_e + x) + (1 - q_{t+1})x]$. We assume that there is a significant productivity gap between educated and uneducated workers in the modern sector. Specifically, $y \equiv y_e - y_u$ is assumed to be larger than $z/\beta$, otherwise children choose not to attend school, even if the probability of matching equals one. The net benefit of receiving an education, i.e., the difference between the above two lifetime incomes, is $-z + \beta q_{t+1}y$.

Note that the agents born in period $t$ decide whether to attend school in their childhood, while the probability of finding a job in period $t+1$ ($q_{t+1}$) is yet to be determined. Therefore, their decision is based on their expectation of $q_{t+1}$ as of period $t$, which is denoted by $E_t[q_{t+1}]$. They receive an education if the expected net benefit $-z + \beta E_t[q_{t+1}]y$ is zero or higher, which holds when $E_t[q_{t+1}] \geq z/\beta y$.

The following summarizes the schooling decision. There are $1 - p + pe_t$ children in good environments who receive an education if and only if $E_t[q_{t+1}] \geq z/\beta y$. Children in difficult environments never receive an education. Therefore, the number of adult agents who become educated in period $t+1$ is

$$e_{t+1} = \begin{cases} 0 & \text{if } E_t[q_{t+1}] < z/\beta y, \\ 1 - p + pe_t & \text{if } E_t[q_{t+1}] \geq z/\beta y. \end{cases} \tag{1}$$

Equation (1) describes how the expected probability of finding a job determines the number of educated workers supplied to the period $t+1$ labor market. Given this, the following section examines how the actual probability of finding a job is determined in the frictional labor market.

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6For simplicity, we assume that the agents’ utility function is linear (i.e., agents are risk neutral) or the economy considered is an open economy. In either case, the discount factor is exogenous and constant. It is also assumed that agents receive an education when they are indifferent in doing so.
3 Matching Technology and Job Creation

As described in section 2.2, a search-matching friction exists in the modern sector. Let $\ell_{t+1}$ denote the number of workers who search for a vacant job and $v_{t+1}$ represent the number of vacant jobs in period $t+1$. The number of matches in a period is then given by a standard Cobb–Douglas matching function,

$$M(\ell_{t+1}, v_{t+1}) = \min \{ A\ell_{t+1}^{1-\eta}v_{t+1}^\eta, \ell_{t+1}, v_{t+1} \},$$

where $A, \eta \in (0,1)$ are the parameters of the matching technology. Observe that in equation (2), the number of matches is bounded above by the number of workers and the number of vacant jobs, because workers and firms can match each other at most once each period.

Recall from the previous section that all adult agents, with population one, search for vacancies in the modern sector because the income there (either $y_e + x$ or $y_u + x$) is higher than the income in the traditional sector ($x$). This means that $\ell_{t+1} = 1$. From this, the probability of a worker matching with a vacant job is given by

$$q_{t+1} = M(\ell_{t+1}, v_{t+1})/\ell_{t+1} = M(1, v_{t+1})/1 = \min \{ Av_{t+1}^\eta, 1, v_{t+1} \}. \quad (3)$$

Equation (3) shows that probability $q_{t+1}$ is determined by the number of vacant jobs $v_{t+1}$. In this economy, through free entry firms create vacant jobs with a constant cost of $k \in (y_u, y_e)$, and these, filled or not, deteriorate in one period. Recall that if a vacant job is matched with an educated worker, the firm’s revenue is $y_e$, whereas it is $y_u$ when matched with an uneducated worker. Because firms are randomly matched with workers, the conditional probability that the matched worker is educated (or uneducated) upon matching is $e_t$ (or $1-e_t$). Therefore, the expected revenue, conditional upon the vacant job being matched with a worker is $e_{t+1}y_e + (1 - e_{t+1})y_u = e_{t+1} y + y_u$, where $y \equiv y_e - y_u$.

If $e_{t+1} y + y_u < k$, or equivalently if $e_{t+1} < (k - y_u)/y$, the firm’s expected revenue from a matched pair falls short of the cost of creating a vacant job. Obviously, firms do not create any job in this case.

$$v_{t+1} = 0 \quad \text{whenever} \quad e_{t+1} < (k - y_u)/y. \quad \quad (4)$$

Conversely, if $e_{t+1} y + y_u \geq k$, or equivalently $e_{t+1} \geq (k - y_u)/y$, it is profitable for firms to create vacant jobs as long as they have a reasonably high probability of finding workers. As the probability of a vacant job meeting with a worker is given by $M(1, v_{t+1})/v_{t+1}$, the expected profit of creating a vacant job is $\pi(v_{t+1}, e_{t+1}) = \min \{ Av_{t+1}^{\eta-1}, 1/v_{t+1}, 1 \} (e_{t+1} y + y_u) - k$. For a given level of $e_{t+1} \geq (k - y_u)/y$,
\( \pi(v_{t+1}, e_{t+1}) \) is decreasing in the number of vacant jobs \( v_{t+1} \), and job creation occurs until \( \pi(v_{t+1}, e_{t+1}) = 0 \) holds. Solving equation \( \pi(v_{t+1}, e_{t+1}) = 0 \) gives

\[
v_{t+1} = \min \left\{ \left( \frac{A(e_{t+1}y + y_u)}{k} \right)^{1/(1-\eta)}, \frac{e_{t+1}y + y_u}{k} \right\} \quad \text{whenever} \quad e_{t+1} \geq \frac{(k - y_u)}{y}.
\]

(5)

To summarize, the fraction of educated workers \( e_{t+1} \) determines the number of vacant jobs created by firms \( v_{t+1} \), which in turn affects the probability of each worker finding a job. Substituting (4) and (5) into (3) gives

\[
q_{t+1} = Q(e_{t+1}) \equiv \begin{cases} 
0 & \text{for } e_{t+1} < \frac{(k - y_u)}{y}, \\
\min \{m[e_{t+1}y + y_u]^\alpha, 1\} & \text{for } e_{t+1} \geq \frac{(k - y_u)}{y},
\end{cases}
\]

(6)

where \( \alpha \equiv \eta/(1 - \eta) \) and \( m \equiv A^{\alpha+1}/k^\alpha \) are given constants. Observe from (6) that the matching probability \( q_{t+1} \) is increasing in \( e_{t+1} \). If more children receive an education in period \( t \), in period \( t + 1 \) firms observe that a higher fraction of workers in the labor market are educated. Given that this raises the expected revenue from a matched pair, firms create more jobs in the modern sector and this increases the probability of workers finding a job.

4 Equilibrium

4.1 Rational Expectations Equilibria

Given the number of educated parents in the previous generation \( e_t \) and the expected matching probability \( E_t[q_{t+1}] \), equation (1) determines the number of educated workers in period \( t + 1 \), \( e_{t+1} \). Then, as a function of \( e_{t+1} \), the actual matching probability \( q_{t+1} \) is determined according to (6). As long as agents are rational, the expected matching probability \( E_t[q_{t+1}] \) must coincide with the actual probability \( q_{t+1} \). Therefore, under a given value of \( e_t \), the rational expectations equilibrium for this period is given by a pair of \( q_{t+1} (= E_t[q_{t+1}]) \) and \( e_{t+1} \) that simultaneously satisfy (1) and (6).

Figure 1 depicts conditions (1) and (6) in \((e_{t+1}, q_{t+1})\) space, given small and large values of \( e_t \). In the figure, the schooling locus represents the number of agents who choose to become educated given expectation \( E_t[q_{t+1}] \). The job creation locus shows the probability of a worker finding a vacant job as a function of \( e_t \). If these loci intersect, then it is a rational expectation equilibrium.

Observe that \( E_t[q_{t+1}] = 0 \) is a rational expectation for any \( e_t \). If the children in period \( t \) hold a pessimistic expectation that the matching probability in period \( t + 1 \) will be zero, no one attends school \( (e_{t+1} = 0) \). Then in period \( t + 1 \), and knowing that
there is no chance of finding an educated worker, firms do not create any vacant jobs in the modern sector. Thus, the zero matching probability \( q_{t+1} = 0 \) is realized, which verifies that the initial expectation \( E_t[q_{t+1}] = 0 \) is rational. We refer to this rational expectations equilibrium as the low-education equilibrium.

When the number of educated parents is sufficiently large (we shortly derive a precise condition for this), as illustrated in the right-hand panel of Figure 1, another rational expectation exists in which both \( E_t[q_{t+1}] \) and \( e_{t+1} \) are positive. Suppose that in period \( t \) agents in childhood hold an expectation that \( E_t[q_{t+1}] \geq z/\beta y \). Then, children in good environments receive an education and become educated in period \( t+1 \); i.e., \( e_{t+1} = 1 - p + pe_t \). Observing this value of \( e_{t+1} \), firms create vacant jobs, and the actual matching probability for workers becomes \( q_{t+1} = Q(1 - p + pe_t) \), where function \( Q(\cdot) \) is defined by (6). Therefore, there is a rational expectation satisfying \( E_t[q_{t+1}] > z/\beta y \) if \( Q(1 - p + pe_t) \) is actually higher than \( z/\beta y \).

Let us examine when this is actually the case. Note that as \( 0 < z/\beta y < 1,7 \) equation (6) implies that \( Q(e_{t+1}) \geq z/\beta y \) holds if and only if both \( e_{t+1} \geq (k - y_u)/y \) and \( m[e_{t+1}y + y_u]^\alpha \geq z/\beta y \) are satisfied. Substituting \( e_{t+1} = 1 - p + pe_t \) for these

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7Recall that \( z, \beta > 0 \) and \( 0 < y < z/\beta \).
conditions and solving for $e_t$ gives\(^8\)

$$e_t \geq \frac{k - ye + py}{py} \equiv \Phi, \quad \text{and}$$

$$e_t \geq \frac{1}{py} \left[ \left( \frac{z}{\beta y} \right)^{1/\alpha} - ye + py \right] \equiv \Psi(z).$$

Condition (7) is necessary for the expected profit from a matched pair to be positive. Given this, condition (8) is necessary for firms to create enough jobs such that children are willing to receive an education.

Figure 2 depicts conditions (7) and (8) against the opportunity cost of education $z$. If $e_t$ is above the $\max\{\Phi, \Psi(z)\}$ curve, expectation $E_t[q_{t+1}] > z/\beta y$ is self-fulfilling: when agents have a high prospect of finding jobs, many receive an education, which stimulates firms to create jobs, and the matching probability rises. We refer to this as the high-education equilibrium. The following proposition summarizes the result.

**Proposition 1** The set of (stage) equilibria in period $t+1$ is determined by the number of educated parents $e_t$ in period $t$ as follows.

(i) When $e_t < \max\{\Phi, \Psi(z)\}$, the only rational expectations equilibrium is $e_{t+1} = q_{t+1} = 0$.

(ii) When $e_t \geq \max\{\Phi, \Psi(z)\}$, there are multiple rational expectations equilibria. One is $e_{t+1} = q_{t+1} = 0$, and the other is $e_{t+1} = 1 - p + pe_t$ and $q_{t+1} = Q(1 - p + pe_t)$.

\(^8\)In deriving (7) and (8), we used $y_u + (1 - p)y = y_u + y - py = y_u + (ye - y_u) - py = ye - py$. 
4.2 Dynamics and the Poverty Trap

In what follows, we consider the long-term dynamics of the economy. From Proposition 1, the number of educated adult agents $e_t$ evolves over generations according to

$$e_{t+1} = \begin{cases} 0 & \text{if } e < \max\{\Phi, \Psi(z)\}, \\ \text{either 0 or } 1-p+pe_t & \text{if } e \geq \max\{\Phi, \Psi(z)\}. \end{cases}$$

Figure 3 illustrates the pattern of dynamics. We find that two steady-state values of $e_t$ exist: the first is a good steady state where all agents receive an education ($e_t = 1$), whereas the second is a poverty trap where no agent receives an education ($e_t = 0$).

The economy converges to the good steady state only if (i) the number of educated agents in the initial adult generation, $e_0$, is larger than the threshold level of $\max\{\Phi, \Psi(z)\}$, and (ii) for every generation, agents in childhood hold the optimistic expectation that the high-education equilibrium will be realized when they grow up. If either of the above conditions is violated, the economy falls into the poverty trap. Formally, the following proposition and corollary follow from equation (9).

**Proposition 2** Suppose that $\max\{\Phi, \Psi(z)\}$ is positive. If the number of educated

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$^9$From (7) and (8), we find that $\Phi > 0$ holds when $k > ye - py$ and that $\Psi(z) > 0$ holds when $z > \beta y m(y_e - py)$. Therefore, $\max\{\Phi, \Psi(z)\} > 0$ holds that either the cost of creating the vacancy $k$ is above $y_e - py$, or the opportunity cost of receiving an education $z$ is above $\beta y m(y_e - py)$. 

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agents in some generation is below $\max\{\Phi, \Psi(z)\}$, then no agent after this generation will receive an education.

Corollary 1 Suppose that $\max\{\Phi, \Psi(z)\}$ is positive. Even when the number of educated agents in some generation is above $\max\{\Phi, \Psi(z)\}$, if agents in this generation hold a pessimistic expectation $E_t[q_{t+1}] = 0$, then no agent after this generation will receive an education.

Let us explain the mechanisms that jointly create the poverty trap. The first mechanism is intrageneration coordination failure. Suppose that agents in some period have a pessimistic expectation $E_t[q_{t+1}] = 0$, which is always one of the rational expectations. Then they do not receive an education, and the low-education equilibrium is realized (i.e., $e_{t+1} = q_{t+1} = 0$). Even though this is a worse outcome than the high-education equilibrium, each agent cannot rationally change their expectation given the expectations of other agents: if other agents have a pessimistic expectation, one can rationally expect that other agents will not receive an education and that in the next period firms observing this will not create jobs ($q_{t+1} = 0$). Therefore, we cannot escape this bad outcome unless all children within the same generation coordinate their expectations to the high-education equilibrium. The other mechanism is the intergenerational linkage. When the number of educated parents $e_t$ is low, many children ($p(1 - e_t)$ of population 1) are in difficult environments and therefore do not receive an education. Specifically, given the number of educated parents in period $t$ ($e_t$), the number of educated parents in period $t + 1$ ($e_{t+1}$) is bounded above by $1 - p - pe_t$, which is smaller when $e_t$ is smaller. Thus, the low educational attainment of a generation is partly inherited by the next generation.

Although the above mechanisms are important, each is not significant enough to individually explain the persistent poverty and low education found among low-income countries. The intragenerational coordination problem does not mean that agents necessarily fail to coordinate on the high-education problem; the coordination failure is merely one possibility among multiple equilibria. Similarly, the problem of an intergenerational linkage does not by itself mean that low educational attainment is persistent; even among children born to uneducated parents, a fraction $1 - p$ are in (relatively)

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10Earlier studies focused on the strategic complementarity between workers and firms because workers and firms determine their investment decisions at the same time (see Laing, Parivos, and Wang 1995; Takii 1997; Acemoglu 1997; and Burdett and Smith 2002). However, firm investment, and therefore job creation, does not usually take as long as the time required by children to obtain an education. Therefore, we assume that firms choose to invest after they confirm the fraction of educated workers in the economy. In this case, a strategic complementarity emerges, not between firms and workers, but rather among children.
good environments. If they receive an education, the problem of low educational attainment will then be gradually resolved over succeeding generations.

However, when the intra- and intergenerational problems combine, they create a far more serious situation than can be created by each of the problems alone. Suppose that there are only a few educated parents (specifically, $e_t$ is below $\max\{\Phi, \Psi(z)\}$). This means that a significant fraction of children is in a difficult environment and therefore does not receive an education. Given this, agents can rationally expect that in the next period, the fraction of educated workers will be low. Firms will then not create sufficient jobs, and as a result the matching probability will be low. Therefore, even children in good environments do not receive an education. As a result, and as shown in Proposition 1, coordination on the high-education equilibrium is impossible when $e_t < \max\{\Phi, \Psi(z)\}$. In this way, the intergenerational linkage of low education necessarily causes the intragenerational coordination failure.

Once intragenerational coordination failure occurs, there are no educated parents in period $t + 1$ ($e_{t+1} = 0$). That is, the intragenerational coordination failure fortifies the intergenerational linkage of low education ($e_t = 0$ means not only $e_{t+1} \leq 1 - p$ but $e_{t+1} = 0$). This induces the majority of children in period $t + 1$ to choose not to receive an education, and again causes the intragenerational coordination failure in generation $t + 1$, which is inherited by period $t + 2$, and so on. In this way, the intra- and intergenerational mechanisms interact with each other, and perpetuate the poverty and low education found in Proposition 2. We describe this situation as a dual poverty trap.

Finally, observe that the interaction not only makes the poverty trap more persistent, but also raises the possibility that an economy falls into the poverty trap. Corollary 1 shows that, even when the initial $e_t$ is above the threshold, there remains a possibility of falling into the persistent poverty trap once a generation fails to coordinate on the high-education equilibrium.

5 Economic Policies

The previous section has shown that if $e_t < \max\{\Phi, \Psi(z)\}$, the economy is dually trapped in the sense that the intra- and intergenerational problems interact with each other to perpetuate poverty and low educational attainment. This section considers the policy prescriptions for escaping from this dual poverty trap.
5.1 Necessity of Combined Implementation

In a dually trapped economy, the majority of children are born to uneducated parents. A significant fraction of children are in difficult environments and do not receive an education. This in turn lowers the number of jobs created by firms in the next period, which lowers the matching probability for educated workers. Given the low matching probability, even children in good environments choose not to receive an education. To resolve this situation, one can envisage several types of policy. One is to induce children in good environments to receive an education, even when they expect the probability of finding a job is not very high. This can be through a schooling subsidy and/or the provision of free education. Another type of policy is to support some children in difficult environments to remove the barriers to education. This increases the mass of children who can potentially receive an education and helps children coordinate on the high-education equilibrium.\(^1\)

Note that even when these kinds of programs make the high-education equilibrium possible, the low-education equilibrium \((E_t[q_{t+1}] = e_{t+1} = 0)\) is still a rational expectations equilibrium. In a dually trapped economy, children in period \(t\) know that the probability of finding a job in the modern sector has been low for a number of generations, including the current period \(t\) \((q_t = 0)\). Given this, they are likely to believe that other children in the same generation will hold a pessimistic rational expectation that a similar equilibrium will be realized in the next period \((q_{t+1} = 0)\).\(^2\) Therefore, in a dually trapped economy, the above-mentioned policies are likely to be effective only when combined with appropriate policies that convince children that their lives can be different from their parents.

An example of this type of policy is the Female Secondary School Assistance Project in Bangladesh, jointly initiated by the Government of Bangladesh and the World Bank in 1993. In the project, stipends for female students attending school are combined with a female education awareness campaign, where videos and print materials are distributed nationally aiming “...to promote a supportive community environment for

\(^1\)Yet another type of policy is to influence the labor market, for example, by subsidizing job creation. However, in the present setting, where it takes time for children to be educated, this type of policy suffers from a time-inconsistency problem. That is, to induce children in period \(t\) to receive an education, the authority (either a government or an aid organization) must commit to subsidizing job creation in period \(t+1\), when these children search for jobs. However, once the agent becomes educated in period \(t+1\), the authority’s objective has been achieved and it has no incentive to actually subsidize job creation.

\(^2\)Rostow (1990) refers to this phenomenon as “long run fatalism,” whereas Hoff and Pandey (2004) call it “historically created social identities.” Chamley (2002) shows that a similar equilibrium will be chosen repeatedly, even if there is only a small uncertainty about the structure of the future economy.
girls’ education through widespread awareness about the merits of female educational, economic, and social development.\footnote{Each year the awareness campaign distributes about 340,800 brochures, 841,500 calendars, 300,000 stickers, and 3,250 diaries. Awards are given and documentaries are produced, with a total cost of US$1.7 million dollars. See the World Bank’s web site and search for project ID P009555.} Similarly, the World Bank’s Africa division placed special emphasis on awareness campaigns in enhancing the education of girls in Mauritania and Guinea (see the World Bank 2000), successfully raising the gross school enrollment rates of girls in Mauritania to 83.2% in 1997–98 from 39.3% in 1989–90 and in Guinea to 36.9% in 1997–98 from 21.7% in 1989–90. In the following subsections, we consider subsidies, free education and support programs in more detail when they are combined with public awareness programs.

5.2 Subsidy for Schooling: The Case of $\Phi \leq 0$

Let us consider a subsidy program in which all children that finish schooling in (generic) period $t$ receive subsidy $s_t$, where $0 < s_t < z$. With the subsidy program, the opportunity cost of education falls from $z$ to $z - s_t$.\footnote{In fact, the subsidy on schooling works in a similar way to a penalty on child labor. Recall that $z$ is the income of agents when they are children. If there is a penalty on child labor, and the expected amount of the fine is $s_t$, the expected net income from working during childhood falls to $z - s_t$. This means that the opportunity cost of education falls by the amount of $s_t$, similar to a subsidy on schooling. However, the enforcement of a ban on child labor is generally difficult in low-income countries and may cost more than the subsidy program. Basu (1999) questioned whether a child labor ban was effective in eliminating the poverty trap. He also examined the connection between child labor and adult minimum wages (Basu 2000) and product boycotts (Basu and Zarghamee 2008). Doepke and Zilibotti (2005) showed that multiple political equilibria arise through the introduction of voting on a ban on child labor.} For simplicity, we assume that the expenditure required for this program is covered by foreign aid and/or a nondistortionary tax on agents.

Suppose that the economy is initially in the dual poverty trap: i.e., $\max\{\Phi, \Psi(z)\} > 0$ and $e_t = 0$. Can the subsidy on schooling then help the economy escape from the trap? The answer depends on whether $\Phi$ is positive or negative. The following considers an economy with $\Phi \leq 0$, which holds when parameters satisfy $p < (y_e - k)/y$ (corresponding to the left-hand panel of Figure 2).

Recall from Proposition 1 that the high-education equilibrium exists only when the number of educated parents $e_t$ is at least as large as $\max\{\Phi, \Psi(z)\}$. Note that, when $\Phi \leq 0$, the threshold is simply given by $\Psi(z)$. With subsidy $s_t$, $\Psi(z)$ falls to $\Psi(z - s_t)$ and the locus of the high-education equilibrium extends to $\Psi(z - s_t)$. In particular, when $z - s_t \leq \beta y m(y_e - py)^{\alpha}$, the locus of the high-education equilibrium extends to...
$e_t = 0$, as illustrated in Figure 4. This means that even when the number of educated parents is zero, the high-education equilibrium becomes the rational expectations equilibrium. Therefore, by combining subsidy $z - \beta y m (y_e - py)^{\alpha} \equiv S(0)$ and an awareness program to coordinate expectations, the number of agents receiving an education can be increased to $e_{t+1} = 1 - p > e_t$.

Note that the subsidy program must continue for some number of periods until $e_t$ reaches the initial threshold level at $\Psi(z)$. From (8), it turns out that the high-education equilibrium exists if and only if

$$e_t \geq \Psi(z - s_t) \iff s_t \geq z - \beta y m [(1 - p + pe_t) y + y_u]^{\alpha} \equiv S(e_t),$$

where $S(e_t)$ represents the required amount of subsidy. If the subsidy falls short of $S(e_t)$, the high-education equilibrium disappears and the economy reverts to the dual poverty trap in one period. Observe that $S(e_t)$ is decreasing in $e_t$. This means that when there are more educated parents, a low subsidy is enough to induce children to receive an education. Because $e_t$ grows over generations during the program, equation (10) implies that the amount of subsidy must be largest in the initial period and then can be reduced over the generations as $e_t$ grows. Once $e_t$ reaches or exceeds $\Psi(z)$, then no subsidy is thereafter necessary. Even after this takeoff period, children must coordinate upon the high-education equilibrium for all periods. This is typically not difficult given that they observe that firms are creating enough jobs for their parents when they form their own expectations. Over generations, the economy approaches the good steady state at $e_t = 1$. 

Figure 4: Dynamic effects of a subsidy ($\Phi \leq 0$)
5.3 A Two-step Approach with Free Education: $\Phi > 0$

The previous subsection has shown that a subsidy program is effective when $\Phi \leq 0$. In an economy with $\Phi > 0$, however, the subsidy program alone cannot save the dually trapped economy because the threshold is determined by $\max\{\Phi, \Psi(z - s_t)\} \geq \Phi > 0$. The locus of the high-education equilibrium cannot extend below $\Phi > 0$ and the trapped economy at $e_t = 0$ cannot switch to the high-education equilibrium.

To see why a subsidy is not helpful, note that from the definition in (7), $\Phi > 0$ means that parameters satisfy

\[ p > \frac{(y_e - k)}{y}, \]

where $p$ is the fraction of children in difficult environments among those born to uneducated parents. Intuitively, if $p$ is high, a trapped economy where all parents are uneducated has only a few children in good environments. Even when all of these children receive an education, the expected profit of a firm from a matched job will be negative, and no jobs will be created. In this particular situation, any partial subsidy for schooling cannot induce agents to receive an education as there is no private benefit from becoming educated.

In this case, children receive an education only when the opportunity cost of schooling (i.e., the foregone earnings while attending school) is fully compensated. Specifically, suppose that the authority provides $f_t \leq 1 - p + pe_t$ of children raised in good educational environments with “free education” in period $t$. This means that these $f_t$ children receive $z$ in return for attending school.\(^{15}\) The dynamics of the economy now change from (9) to

\[
e_{t+1} = \begin{cases} f_t & \text{if } e_t < \max\{\Phi, \Psi(z - s_t)\}, \\ \text{either } f_t \text{ or } 1 - p + pe_t & \text{if } e_t \geq \max\{\Phi, \Psi(z - s_t)\}, \end{cases}
\]

where $0 \leq s_t < z$ represents the amount of schooling subsidy for children not receiving free education but completing school by themselves. As shown in Figure 5, the locus of low-education equilibrium shifts up by $f_t$. Recall also that the locus of the high-education equilibrium can be extended to the point of $\Phi$ by a subsidy program. In the following, we show that the dually trapped economy can be saved in two steps through the combined policies of providing both free education and subsidies.

**Step 1:** Suppose that in period 1 the economy is in the dual poverty trap with positive $\Phi$, i.e., $e_1 = 0 < \Phi$. The first step of the program is to induce $\Phi$ children to receive an education through the provision of free education. If $1 - p \geq \Phi$, which is the case when $p < \sqrt{(y_e - k)/y}$, this can be done in one period by providing $f_1 = \Phi$ children in good environments with free education.\(^{16}\)

\(^{15}\)Note that $f_t$ cannot exceed the number of children in good environments $1 - p + pe_t$ because children in difficult environments will never attend school, even when they receive $z$.

\(^{16}\)If $1 - p < \Phi$, it is impossible to choose $f_1 = \Phi$ when $e_1 = 0$ as there are not enough children.
Step 2: Once the number of educated agents reaches $\Phi$, the provision of free education is no longer necessary ($f_t$ can be set to zero). Instead, the authority needs to subsidize education and coordinate expectations so that agents choose the high-education equilibrium every period, as in the policy package discussed in the previous subsection. Figure 5 shows that by providing subsidy $s_2 = S(e_1),^{17}$ $\Psi(z - s_2)$ falls to $\Phi$. This means that the locus of the high-education equilibrium extends to $\Phi$, as $\max\{\Phi, \Psi(z - s_t)\} = \Phi = \Psi(z - s_2)$. Then, through an appropriate awareness program, the agents choose the high-education equilibrium. The subsidy program must be continued until $e_t$ reaches the original threshold level at $\max\{\Phi, \Psi(z)\}$.

Note that while this two-step program is effective, even when $\Phi > 0$, it requires provision of free education for $f_t = \Phi$ children in the first step, which necessitates a lump-sum expenditure of $z\Phi$. Given that $z$ is the entire income foregone while attending school, $z\Phi$ can be a considerable financial burden. Moreover, the recipients of free education in the first step will not be employed in the modern sector because $e_1 = \Phi$ is not enough to induce firms to create jobs, and therefore the provision of free education has no immediate effect on output. Given this, it may be politically difficult to persuade the constituency or donor countries to bear a large burden of expenditure.

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17Recall that function $S(\cdot)$ is defined by (10).
In the next subsection, we show that the expenditure on the free education program can be reduced when it is possible to support some of the children in difficult environments and remove the barriers to education.

### 5.4 Providing Education Support

We have so far focused on policies to induce children in good environments to receive an education, as it is assumed that children in difficult environments cannot receive an education. In low-income countries, children face various barriers to education depending on their family and social circumstances, and removing these barriers will require individual treatments. While it is difficult for the authorities to systematically solve the problem, there are many nongovernment organizations (NGOs) around the world that seek to support these disadvantaged children. This subsection investigates how such efforts contribute to saving the economy from the poverty trap.

Suppose that there are NGOs that support children in difficult environments, and in (generic) period \( t \), they successfully provide educational opportunities with \( a_t \) among \( p(1 - e_t) \) children in difficult environments.\(^ {18} \) These \( a_t \) children act the same way as the children in good environments, i.e., they receive an education if so doing results in a higher lifetime income. Thus, in total, there are \( 1 - p + pe_t + a_t \) children who are willing to receive an education if the expected matching probability \( E_t[q_{t+1}] \) is sufficiently high. Note that this is the same situation as the case where there are \( e_t + a_t/p \) educated parents in an economy without NGO support. (In that case there are \( 1 - p + p(e_t + a_t/p) \) children in good environments). Therefore, by replacing \( e_t \) in (11) by \( e_t + a_t/p \), we obtain the dynamics of the economy in the presence of education support:

\[
e_{t+1} = \begin{cases} 
  f_t & \text{if } e_t + a_t/p < \max\{\Phi, \Psi(z - s_t)\}, \\
  \text{either } f_t \text{ or } 1 - p + pe_t + a_t & \text{if } e_t + a_t/p \geq \max\{\Phi, \Psi(z - s_t)\}.
\end{cases}
\]  

As illustrated in Figure 6, the introduction of education support shifts the locus of the high-education equilibrium to the left by the amount of \( a_t/p \). Accordingly, the threshold level of \( e_t \) falls from \( \max\{\Phi, \Psi(z - s_t)\} \) to \( \max\{\Phi, \Psi(z - s_t)\} - a_t/p \). This implies that the provision of education support for disadvantaged children can benefit not only these children, but also children in good environments: that is, if the supported children receive an education, the number of educated increases in the next period, this induces firms to create more jobs, and thereby raises the expected matching probability.

\(^{18}\)We assume that \( a_t \) is exogenous to the government because there is no systematic way to solve the problems of troubled families and therefore to increase \( a_t \) easily.
In this way, support for children in difficult environments can even benefit children already in good environments by raising the possibility of coordinating expectations on the high-education equilibrium.

In particular, we can show that the introduction of education support can significantly reduce the financial burden required in the first step of the two-step policy package discussed in the previous subsection, and through this the policy increases the financial feasibility of the package. Suppose that $\Phi > 0$ and the economy is in the dual poverty trap in period 1 ($e_1 = 0$), and it is expected that NGOs will provide support for $a_2$ children in difficult environments in the next period. Then, in period 1 it suffices to provide free education opportunities with $\Phi - \frac{a_2}{p}$ children in good environments. This requires an initial lump-sum expenditure of $z\Phi - z(a_2/p)$ instead of $z\Phi$. Then, in period 2 there are $e_2 = \Phi - (a_2/p)$ educated parents. As illustrated in Figure 6, the locus of the high-education equilibrium can be extended down to $\Phi - (a_2/p)$ by the combined policy of providing education support for $a_2$ disadvantaged children and providing an education subsidy $s_2 = S(\Phi)$ for all children who attend school. (This can be confirmed by substituting these values in equation (12). The combined policy should be further combined with an appropriate policy of coordinating agents’ expectation (e.g., awareness programs) so that they choose the high-education equilibrium at $e_3 = 1 - p + p\Phi$. From the next period on the economy can converge to the good steady state, even without the provision of education support, as $e_t$ is already higher.

\[ f_1 = \Phi - \frac{a_2}{p} \]

\[ 0 \leq \Phi - \frac{a_2}{p} \leq 1 \]

\[ 1 - p + p\Phi \]

\[ e_t+1 \]

\[ \Phi \]

\[ \Phi \]

\[ \Psi(z) \]

\[ 1 \]

Figure 6: Dynamic effects of providing education support

\[ e_t = 1 - p + p\Phi + a_t \]

\[ \text{High edu. eq. with Subsidy and Supports} \]

\[ \text{Low edu. eq. with Free Education} \]

\[ \text{Good S.S.} \]

\[ 19\]

\[ 20\]
than Φ. \( e_3 = 1 - p + p\Phi = \Phi + (1 - p)(1 - \Phi) > \Phi \).

6 Conclusion

Using a simple overlapping generations model with a frictional labor market, this paper develops a theory of persistent poverty and low education by focusing on the interaction between the intergenerational transmission of educational attainment and the intragenerational coordination failure problem.

We show that the economy may have high- and low-education equilibria because of strategic complementarity between children of the same generation. When many children receive an education, firms can easily find educated workers and thus will create many jobs, and this justifies children’s choice to receive an education. On the contrary, if no children receive an education, it becomes difficult for firms to find educated workers, not many jobs are created, and the children’s decision of not receiving an education becomes rational. In principle, which of the two equilibria is realized depends on the expectations among children. However, the previous generation’s educational attainment may limit the type of expectation that can be held by the following generation. When the majority of parents in the economy are uneducated, even children in good environments cannot hold an optimistic expectation that firms will create many jobs, because they know that there are many disadvantaged children who will not receive an education. This may explain the stagnant situation of low-income countries, where even nondisadvantaged children are reluctant to receive an education: as a result, low educational attainment is inherited from one generation by the next.

To save the economy from this poverty trap, it is necessary both to make the high-education equilibrium possible and to make children believe that the high-education equilibrium will actually occur. When the fraction of children in difficult environments is not very high, an appropriate policy package is to combine a schooling subsidy and an awareness program to convince children that their employment prospects are better than their parents’. However, any partial subsidy for education does not work when a considerable fraction of children are in difficult environments. In this case, the authority initially needs to provide complete compensation for the opportunity cost of education (i.e., the foregone income in childhood) for a certain number of children before switching to subsidy programs. We also consider the effectiveness of an education support program for children in difficult environments and find that it is beneficial, even for those who do not receive the support, because it can raise the possibility of coordinating their expectations on the high-education equilibrium.

This paper analyzed the situation where two particular poverty trap mechanisms
previously separately analyzed may reinforce each other. Obviously, there are many other reasons for persistent poverty (e.g., credit constraints, high fertility, insufficient capital accumulation, and so on), and low-income countries usual suffer from more than one of these problems, possibly in addition to the problems analyzed here. Our results suggest the necessity of a comprehensive policy package that should not be simply a collection of separate actions for each issue, but must be derived from a model that takes into account the interactions among the underlying issues.

**Appendix A: Job Creation when Firms Can Identify Educated Agents**

In Section 3, we derived the job creation curve (6) by assuming that firms cannot distinguish between educated and uneducated workers until their vacancies match workers. Here, we show that a job creation curve similar to (6) can be obtained in the setting where firms can search only for educated workers and it is more costly for them when the fraction of educated workers among all adult workers is smaller. (e.g., in this case, the required advertisement cost will be higher).

Recall that among adult workers of population 1, $e_{t+1}$ are educated in period $t+1$. The cost of creating a vacancy and searching for an educated worker for one period is assumed to depend on the fraction of educated workers $e_{t+1}/1 = e_{t+1}$ and denoted by $K(e_{t+1})$, where $K'(\cdot) < 0$, $K(0) > y_e$ and $K(1) < y_e$. Similar to equation (2), the number of matches between the educated workers of size $e_{t+1}$ and the vacancies of size $v_{t+1}$ in period $t+1$ is given by

$$M(e_{t+1},v_{t+1}) = \min\{Ae_{t+1}^{1-\eta}v_{t+1}^{\eta}, e_{t+1}, v_{t+1}\}. \tag{13}$$

If a vacancy in a firm successfully matches an educated worker, the profit from the match is $y_e$. Obviously, if $K(e_{t+1})$ is larger than $y_e$, firms never create jobs. That is, if $e_{t+1} < K^{-1}(y_e)$, then $v_{t+1} = 0$. When $e_{t+1} \geq K^{-1}(y_e)$, firms create jobs until the free entry condition, $y_e M(e_{t+1},v_{t+1})/v_{t+1} = K(e_{t+1})$, is satisfied.\textsuperscript{20} This yields

$$v_{t+1} = \min \left\{ \left( \frac{Ay_e}{K(e_{t+1})} \right)^{1/(1-\eta)} e_{t+1}, \frac{y_e}{K(e_{t+1})} e_{t+1} \right\} \text{ for } e_{t+1} \geq K^{-1}(y_e).$$

Substituting the obtained value of $v_{t+1}$ back into the matching function (13) gives the number of matches in equilibrium. The matching probability of an educated worker

\textsuperscript{20}When $e_{t+1}$ happens to exactly coincide with $K^{-1}(y_e)$, it turns out that the free entry condition holds within a range of $v_{t+1}$. In this case, we assume that the firms create as many jobs as possible under the free entry condition.
is given by dividing it by \( e_{t+1} \), yielding

\[
q_{t+1} = \begin{cases} 
0 & \text{for } e_{t+1} < K^{-1}(y_e), \\
\min \left\{ A^{1/(1-\eta)} \left( \frac{y_e}{K(e_{t+1})} \right)^{\eta/(1-\eta)}, 1 \right\} & \text{for } e_{t+1} \geq K^{-1}(y_e). 
\end{cases}
\]  

(14)

Note that because \( K'(\cdot) < 0 \), the expression \( A^{1/(1-\eta)} \left( \frac{y_e}{K(e_{t+1})} \right)^{\eta/(1-\eta)} \) is increasing in \( e_{t+1} \). Similar to the job creation curve (6) derived in section 3, equation (14) implies that the matching probability \( q_{t+1} \) is zero for small values of \( e_{t+1} \), jumps when \( e_{t+1} \) reaches a threshold at \( K^{-1}(y_e) \), and gradually increases with \( e_{t+1} \) until it reaches the upper bound of 1. When the schooling curve (1) is combined with the job creation curve (14), instead of (6), we obtain essentially the same results as derived in Sections 4 and 5.

References


