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Abstract

This paper uses real-time data for the U.S. to estimate out-of-sample forecast uncertainty about the Federal Funds Rate. By combining a Taylor rule with an unobserved components model of economic fundamentals I separate forecast uncertainty into economically interpretable components that represent uncertainty about future economic conditions and uncertainty about future monetary policy. The estimation results indicate important time variation in uncertainty about the future Federal Funds Rate.

Keywords: monetary policy reaction function, interest rate uncertainty, state-space model

JEL Classification: E52, C32, C53

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1 Introduction

The aim of this paper is to study forecast uncertainty in the U.S. money market by estimating changes in uncertainty about forecasts of the Federal Funds Rate in the U.S. Estimates of interest rate uncertainty are important for a wide range of financial market applications such as portfolio allocation, derivative pricing, risk management etc. Furthermore, as the Federal Funds Rate is the indicator of monetary policy in the U.S., it is also important for evaluating monetary policy. For example, uncertainty about future money market rates is an indicator of the credibility and predictability of the central bank's monetary policy. To keep this uncertainty low is an important goal of central banks' communication policy which "guides" expectations about future policy decisions (for example, European Central Bank (2008), Reinhart (2003)). Concern about interest rate uncertainty is also due to possible negative effects of increasing uncertainty about future interest rates on economic stability (e.g. Poole (2005)).[1]

The empirical importance of time-variation in uncertainty about short-term interest rates has been documented in many studies. Mostly, measures of interest rate uncertainty are constructed from the time series of historical interest rate changes, either by estimating ARCH/GARCH models (e.g. Chuderewicz (2002) and Lanne and Saikkonen (2003)), stochastic volatility models (e.g. Caporale and Cipollini (2002)) or regime switching models of volatility (e.g. Sun (2005)).[2] An important drawback of these approaches is however, that changes in the extracted measure of uncertainty are difficult to interpret economically.

Since the most important driving force of short-term interest rates is monetary policy much can be gained by basing any interpretation of forecast uncertainty about short-term interest rates on a model that accounts for how financial markets perceive monetary policy to respond to changes in economic conditions. Combining an interest

[1]For example, an increase in the volatility of money market rates can be transmitted through the yield curve (Ayuso et al. (1997)) causing the volatility of longer-term interest rates to rise as well which has negative effects on real growth (e.g. Muellbauer and Nunziata (2004)) and investment (e.g. Byrne and Davis (2005)).

[2]A third approach uses derivative prices to estimate interest rate uncertainty. See, for example Fornari (2005).

rate rule which is widely accepted as an approximation to the behavior of the Federal Reserve (Fed) with a model of the evolution of economic fundamentals I am able to separate various components of forecast uncertainty about the Federal Funds Rate in an economically meaningful way.

The starting point of the analysis is the famous Taylor rule (Taylor (1993)) that is generally accepted as a descriptive model of how the Fed sets the Federal Funds Rate in response to (expected) economic conditions. Even though the Fed certainly does not follow a Taylor rule mechanically, financial market participants often use Taylor-type rules as a forecasting tool.

Forecasting the Federal Funds Rate using a Taylor rule requires predictions of how the economic situation the Fed will have to respond to in the future will look like. Hence, uncertainty concerning the forecasts of the information the central bank is expected to act upon, is one source of uncertainty about future interest rates (uncertainty about economic fundamentals).

The second element of uncertainty is related to imperfect knowledge about the central bank's reaction to given future economic conditions. The reaction coefficients in estimated simple interest rate rules such as the Taylor rule have been shown to change over time (e.g. Mehra (1999), Judd and Rudebusch (1999), Clarida et al. (2000), Tchaidze (2001), Gordon (2005)). One reason for this is that the coefficients in optimally derived monetary policy reaction functions depend on the central bank's preferences about output stabilization, inflation and possibly other goals as well as on structural parameters of the model of the economy. Changes in preferences and changes in the structure of the economy will both affect the coefficients in the monetary policy reaction function. Another reason is that simple interest rate rules generally are only crude approximations to an optimal monetary policy reaction function. Central banks base their policy decisions on a much more comprehensive data set than a simple Taylor-type interest rate rule which only accounts for (forecasts of) the output gap and inflation. Hence, situations with identical (forecast) values of the output gap and inflation can be significantly different economically if judged by the much larger optimal information set. Thus, the central bank does not necessarily have to react to (apparently) identical economic situations in the same way and this will lead to changing reaction coefficients in estimated simple interest rate rules. Finally, changes in the reaction coefficients can also result from fitting a linear reaction function when the true reaction function is in fact non-linear. Time variation in the coefficients in the Fed's reaction function are a second source of uncertainty about the future Federal Funds Rate.

The third element of Federal Funds Rate forecast uncertainty is due to the fact that the estimated reaction function is an approximation. The approximation error of the Taylor rule relative to the actual Federal Funds Rate is represented by the error term in the empirically estimated interest rate rule.

Changes in the Fed's reaction function and financial market participants' learning about these changes are modelled empirically by estimating a time-varying interest rate rule. Empirical studies on monetary policy rules have shown that estimation from ex-post revised data results in distorted estimates of reaction coefficients (e.g. Orphanides (2001), Perez (2001)). The estimation of a monetary policy reaction function using ex-post revised data assumes to much information on part of the monetary policy authority: First it contains observations that actually were not available at the time of the actual monetary policy decision and second, some observations have undergone revisions relative to the information that the central bank had to act upon.[3] Hence, the results presented in this paper are derived from recursive estimates using a real-time data set of macroeconomic variables.

An important contribution of this paper is to offer a new application in the growing empirical literature on time-varying monetary policy rules: the study of uncertainty about future monetary policy. Previous analyses have focused on ex-post descriptions of central bank behavior: For example, Clarida, Galì and Gertler (2000) provide evidence of pronounced changes in Taylor-type interest rate rules for the U.S. using split-sample regressions. They show a strong shift in the Fed's reaction function related to the appointment of Fed Chairman Volcker in 1979. More recently Boivin (2006) and Kim and Nelson (2006) estimate forward-looking Taylor rules with time-varying parameters and report sizeable but more gradual changes in the coefficients. Trecroci and Vassali (2006) show that time-varying monetary policy reaction functions for the U.S., the U.K., Germany, France and Italy perform superior to constant parameter rules in

[3] See also Orphanides (2002, 2003) for a discussion of the importance of using real-time data for the empirical modelling of monetary policy.

accounting for observed changes in interest rates.[4] However, most of these studies on time-varying monetary policy reaction functions use ex-post revised data which might bias the results.[5]

The two-step estimation approach of using model-generated forecasts in the estimation of a Taylor-type interest rate rule is related to the one advocated in Nikolsko-Rzhevsky (2008). Since the Fed's internal forecasts of future economic conditions (Greenbook forecasts) are available only with a lag of five years he looks among different univariate and multivariate forecasting models for one which is able to generate out-of-sample forecasts closely tracking the Greenbook forecasts. Using forecasts generated from this model he then estimates a forward-looking Taylor rule for the Fed. Similarly, McCulloch (2007) estimates a forward-looking Taylor rule using an adaptive least squares technique. The forecasts which enter the monetary policy reaction function are generated from structural vector autoregressions. While the two-step procedures employed in these papers is similar to the one presented here, these papers do not consider forecast uncertainty.

The paper is structured as follows: Section 2 outlines the empirical models for the monetary policy reaction function and for the economic fundamentals that enter into it. Section 3 presents the data set and explains how the real-time data are used in the estimation. The results are discussed in Section 4.

2 A model of policy and economic fundamentals

The empirical model for the Federal Funds Rate is based on the notion that the Fed adjusts the Federal Funds Rate in response to the current or expected state of the economy. Thus, Federal Funds Rate forecasts suffer from two elements of uncertainty: (i) uncertainty about the future state of the economy and (ii) uncertainty about future policy response to a given state of the economy. The first type of uncertainty concerns forecasting future values of the variables in the central bank's reaction function while

[4]Time-varying Taylor rules have also been estimated for the Deutsche Bundesbank by Kuzin (2005) and using a regime-switching model by Assenmacher-Wesche (2008).

[5]An exception is Boivin (2006) who uses the Fed's own forecasts of economic fundamentals.

the second type concerns forecasts of future values of the reaction function's coefficients. The next sections outline the empirical model for the monetary policy reaction function and the model from which the forecasts for economic fundamentals are generated.

2.1 The Taylor rule

I assume that the central bank follows a Taylor-type rule in setting the short-term interest rate[6]

$$i_t = \bar{r}_t + \pi_t + \alpha_{\pi,t}(\pi_t - \bar{\pi}_t) + \alpha_{z,t} z_t \tag{1}$$

where i_t is the Federal Funds Rate, $\bar{r}_t$ is the time-varying equilibrium real interest rate, π_t is the inflation rate, $\bar{\pi}$ is the time-varying inflation target, and z_t is the output gap. Equation (1) allows for time variation in the reaction coefficients $\alpha_{\pi,t}$ and $\alpha_{z,t}$. The interest rate rule can be rewritten as

$$i_t = \alpha_{0,t} + \alpha_{\pi,t}\pi_t + \alpha_{z,t} z_t, \tag{2}$$

where $\alpha_{0,t} = \bar{r}_t + \bar{\pi}_t - \alpha_\pi \bar{\pi}_t$.

In empirical studies of interest rate rules of this type it is standard practice to assume that equation (2) describes the interest rate desired by the central bank while the actual interest rate is adjusted gradually towards this target, i.e.

$$i_t = (1-\rho)(\alpha_{0,t} + \alpha_\pi \pi_t + \alpha_z z_t) + \rho i_{t-1}, \qquad 0 \leq \rho \leq 1. \tag{3}$$

(3) can be rewritten as

$$i_t = \beta_{0,t} + \beta_{\pi,t}\pi_t + \beta_{z,t} z_t + \rho_t i_{t-1} + \epsilon_t, \tag{4}$$

[6] For the following analysis to be valid it is not absolutely necessary that the central bank exactly follows such a rule. The model presented here would be also valid if, for example, participants in financial markets perceived the central bank to do so or if they themselves use a Taylor rule to describe the setting of the short-run interest rate.

with $\beta_{i,t} = (1-\rho_t)\alpha_{i,t}$. Equation (4) includes an error term to capture the non-systematic component of monetary policy or the approximation error of the Taylor rule relative to the actually observed Federal Funds Rate.

Since the economy responds to changes in the monetary policy instrument with a time lag, the central bank generally does not react to the contemporaneous values of inflation and of the output gap but to forecasts of these variables. I assume that the central bank sets its policy rate in response to forecasts of inflation and of the output gap two quarters ahead. Furthermore, contemporaneous observations of inflation and output are not available to policy makers and the central bank has to base these forecasts on information from period $t-1$[7]

$$i_t = \beta_{0,t} + \beta_{\pi,t}\pi_{t+2|t-1} + \beta_{z,t}z_{t+2|t-1} + \rho_t i_{t-1} + \epsilon_t, \tag{5}$$

where $x_{t+2|t-1}$ denotes the conditional expectation of variable x in period $t+2$ based on information available in period $t-1$.

2.2 Output gap and inflation forecasts

The output gap which enters the Taylor rule (5) is an unobservable variable and can only be inferred indirectly from the observed output dynamics. Various empirical decompositions of actual output into a long-run trend component (potential output) and a short-run cyclical component (output gap) have been suggested in the literature.[8] The output gap is related to the inflation rate – the second independent variable in the interest rate rule equation (5) – by a Phillips curve-type relationship. To exploit both sources of information, it is preferable to jointly model the dynamics of inflation and of the output gap using an unobserved components model suggested by Kuttner (1994): The output equation is based on Watson (1986) and decomposes the log of real GDP (y) into a random walk and a stationary AR(2) component

[7] Various assumptions about the length of the forecasting horizon have been used in the literature. Due to the high degree of autocorrelation of the forecasts the choice of the forecast horizon has only modest effects on the results. See also Boivin (2006).

[8] These include the Hodrick-Prescott filter als well as decompositions suggested by Watson (1986) and Clark (1989).

$$
\begin{aligned}
y_t &= n_t + z_t && (6) \\
z_t &= \phi_1 z_{t-1} + \phi_2 z_{t-2} + e_t^z && (7) \\
n_t &= \mu_y + n_{t-1} + e_t^n. && (8)
\end{aligned}
$$

n is the trend component and follows a random walk with drift μ_y while z is the (log) deviation of real GDP from potential output, i.e. the output gap. Note that a negative output gap represents a recession as for $z_t < 0$ it follows from (6) that $y_t < n_t$, i.e. actual output falling short of potential output.

Inflation dynamics are modelled as an ARIMA process in which the change in the rate of inflation depends on the lagged output gap.[9]

$$\alpha(L)\Delta\pi_t = \mu_\pi + \gamma(L)z_{t-1} + \delta(L)\nu_t, \tag{9}$$

where $\alpha(L)$, $\gamma(L)$, and $\delta(L)$ are polynomials in the lag operator of order p, r, and q, μ_π is a constant and ν is a normally i.i.d error term. Preliminary estimations not shown here suggested lag orders of p=0, q=3, and r=1. Furthermore μ_π was restricted to zero.

The model (6 - 9) can be written in state-space form which yields the observation equation - already including the restrictions from the previous paragraph –

$$
\begin{aligned}
Y_t &= \mu + H\tilde{x}_t + e_t, && (10) \\
& Y_t = \begin{bmatrix} \Delta y_t \\ \Delta\pi_t \end{bmatrix}, \mu = \begin{bmatrix} \mu_y \\ 0 \end{bmatrix}, e_t = \begin{bmatrix} e_t^n \\ 0 \end{bmatrix} \\
& H = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & \gamma & 1 & \delta_1 & \delta_2 & \delta_3 \end{bmatrix} \\
& \mathbf{E}e_t e_t' = \Sigma_Y = \begin{bmatrix} \sigma_{e,n}^2 & 0 \\ 0 & 0 \end{bmatrix},
\end{aligned}
$$

[9] Preliminary unit-root tests strongly reject the hypothesis of a stationary inflation rate and suggest a model in first differences.

and the transition equation for the state variables

$$\tilde{x}_{t+1} = F\tilde{x}_t + \zeta_{t+1} \tag{11}$$

$$\tilde{x}_t = \begin{bmatrix} z_t \\ z_{t-1} \\ \nu_t \\ \nu_{t-1} \\ \nu_{t-2} \\ \nu_{t-3} \end{bmatrix}, \zeta_t = \begin{bmatrix} e_t^z \\ 0 \\ e_t^\nu \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$F = \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{E}\zeta_t\zeta_t' = \Sigma_\zeta = \begin{bmatrix} \sigma_{e,z}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{e,\nu}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The shocks e^ν, e^n and e^z are assumed to be serially and mutually uncorrelated. The model parameters can be estimated by maximum likelihood using the Kalman filter.

For each period t forecasts for the output gap and for the inflation rate in $t+2$ based on period $t-1$ information can be obtained from the estimated model. As the estimate of the period-t output gap from data up to and including $t-1$ is $z_{t|t-1}$, which is the first element of $\tilde{x}_{t|t-1}$, the forecast for the output gap in $t+2$ based on period-$t-1$ information is

$$z_{t+2|t-1} = 1_z' FFF\tilde{x}_{t|t-1}, \tag{12}$$

where 1_z is a unit vector for the first element of $\tilde{x}$. Forecasts of inflation in $t+2$ based on data available in $t-1$ can be constructed as

$$\pi_{t+2|t-1} = \pi_{t-1} + 1'_{\pi}\left[3\mu + H(I + F + FF)\tilde{x}_{t|t-1}\right]. \tag{13}$$

These forecasts are used as explanatory variables in the estimation of the Taylor rule (5). Thus, I assume that either the central bank uses this or a related model to estimate the current state of the economy or that financial market participants accept this model as an approximation of how the central bank arrives at its estimates of economic fundamentals.

The monetary policy reaction function (5) can be written in state-space form as

$$\begin{aligned} i_t &= x'_t\beta_t + \epsilon_t, \\ & x'_t = \begin{bmatrix} 1 & \pi_{t+2|t-1} & z_{t+2|t-1} & i_{t-1} \end{bmatrix} \\ & \mathbf{E}\epsilon_t^2 = \sigma^2_{\epsilon,t}. \end{aligned} \tag{14}$$

The time-varying parameters are assumed to follow a random walk (Cooley and Prescott (1976))

$$\begin{aligned} \beta_{t+1} &= \beta_t + w_{t+1} \\ & \beta_t = \begin{bmatrix} \beta_{0,t} \\ \beta_{\pi,t} \\ \beta_{z,t} \\ \rho_t \end{bmatrix}, w_t = \begin{bmatrix} w_t^c \\ w_t^{\pi} \\ w_t^z \\ w_t^i \end{bmatrix} \\ & \mathbf{E}w_t w'_t = \Sigma_w. \end{aligned} \tag{15}$$

The shocks within w and ϵ are serially and mutually uncorrelated, as well as uncorrelated with any shocks in the output gap/inflation model. The parameters of this model again can be estimated by maximum likelihood and application of the Kalman filter. The estimates of the time-varying parameters β will be interpreted as representing market participants' view of the currently relevant central bank reaction function and will be used for forecasting future interest rates.

The version of (13-14) which is actually estimated in this paper contains two modifications: First, the interest-rate smoothing parameter ρ has to be restricted to $0 \leq \rho_t \leq 1$ resulting in a non-linear interest rate rule and a modification of the Kalman filter approach shown in Appendix B. Second, the error in the Taylor rule ϵ_t is modelled as an ARCH(2) process in order to account for temporary deteriorations in the Taylor rule's ability of tracking the Federal Funds Rate[10]

$$\sigma^2_{\epsilon,t} = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + +\alpha_2 \epsilon^2_{t-2}.$$

3 Data and Estimation

Quarterly observations of output and inflation for the U.S. are obtained from the Real-time data set for Macroeconomists (RTDSM) at the Federal Reserve Bank of Philadelphia.[11] Output is real GNP (from 1993 on real GDP) while the inflation rate is 100 times the quarterly log difference of the GNP/GDP deflator. The output and inflation series are grouped into data vintages containing only time series that would have been available at a specific point in time. In the RTDSM the first real-time vintage is available for 1965Q4 and contains time series from 1947Q1 to 1965Q3. For each of the following quarters new vintage series are available with new observations for the most recent quarter and revised data for some of the previous observations. Since both the price level and real output are observed with a one period lag each vintage ends one quarter before the date it applies to. The four vintages from 1993 are missing observations for the time period from 1947Q1 to 1959Q1. The policy indicator i_t is the quarterly average of the Federal Funds Rate. In contrast to the data on output and inflation the Federal Funds Rate is not subject to revisions.

Table 1 is a stylized representation of real-time observations on a variable x. The columns contain the data vintages beginning with $\tau_0 = 1965Q4$ and ending in $T = 2007Q3$. $x_{t|\tau}$ is variable x in period t as observed in period τ. For the RTDSM

[10]The most obvious example is the period between 1979 and 1983 in which the Fed targeted non-borrowed reserves.

[11]A detailed description of the data set can be found in Croushore and Stark (1999, 2001, 2003).

	τ_0	τ_0+1	$\dots$	T-1	T
t_0	$x_{t_0\|\tau_0}$	$x_{t_0\|\tau_0+1}$	$\dots$	$x_{t_0\|T-1}$	$x_{t_0\|T}$
t_0+1	$x_{t_0+1\|\tau_0}$	$x_{t_0+1\|\tau_0+1}$	$\dots$	$x_{t_0+1\|T-1}$	$x_{t_0+1\|T}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
τ_0-1	$x_{\tau_0-1\|\tau_0}$	$x_{\tau_0-1\|\tau_0+1}$	$\dots$	$x_{\tau_0-1\|T-1}$	$x_{\tau_0-1\|T}$
τ_0	-	$x_{\tau_0\|\tau_0+1}$	$\dots$	$x_{\tau_0\|T-1}$	$x_{\tau_0\|T}$
τ_0+1	-	-	$\dots$	$x_{\tau_0+1\|\tau_0}$	$x_{\tau_0+1\|\tau_0+1}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$T-2$	-	-	$\dots$	$x_{T-2\|T-1}$	$x_{T-2\|T}$
$T-1$	-	-	$\dots$	-	$x_{T-1\|T}$
T	-	-	$\dots$	-	-

Table 1: Stylized real-time data set

$t_0 = 1947Q1$ and $t < \tau$ because the variables are observed with a lag of one period.

The empirical model of the output gap and the inflation rate (9-10) is estimated recursively from the real-time data to generate forecasts of these variables for period $t+2$ based on information up to $t-1$. At each of these dates only the time series of the variables that would have actually been available to the central bank are used to estimate the model parameters, the output gap series, and to derive the forecasts. The sample period for each estimation starts in 1959Q4. The first vintage used is 1966Q1 with the last observation for 1965Q4. Hence, the first forecasts for the output gap and for the inflation rate are $z_{1966Q3|1965Q4}$ and $\pi_{1966Q3|1965Q4}$. For 1966Q2 the model is re-estimated from the 1966Q2 vintage and new forecasts $z_{1966Q4|1966Q1}$ and $\pi_{1966Q4|1966Q1}$ are made etc. The coefficients of the time-varying Taylor rule are estimated recursively from these model-generated forecasts starting in 1966Q1 since the Federal Funds Rate cannot be taken to be the principal indicator for the Fed's monetary policy before this date (e.g. Lansing (2003)). For each quarter from 1966Q1 to 2007Q3 the free parameters in (13-14) are re-estimated using the real-time forecasts for the output gap.

Two assumptions are required to actually estimate the monetary policy reaction function from the model-generated forecasts of economic fundamentals: First, the contemporaneous value of $x_t = (1\pi_{t+2|t-1}z_{t+2|t-1}i_{t-1})'$ that underlies the central bank's decision is known to the public. Second, x_t must be exogenous to β_t. For example, the model does not allow for asymmetries in the interest rate response to the output gap or inflation, i.e. for the β parameters to vary systematically with changes in the output gap or inflation forecasts.[12]

4 Estimation results

Figure 1 presents two time series of one-sided Kalman filter estimates of output gaps. The solid line is the output gap estimated from ex-post revised data (1959Q4 - 2007Q3) while the dashed line represents output gap estimates in real-time, i.e. the estimates that would have been obtained at each point in time using the most recent available data *at that specific point in time*. The difference between both time series is the real-time measurement error in the terminology of Orphanides and van Norden (2002). The estimated output gap from ex-post revised data is smoother than the real-time output gap and the real-time estimates for particularly negative values of the output gap are much more pronounced.

« insert Figure 1 »

Figure 2 compares one-sided estimates of output gaps over time for three different vintages. While the dashed line in Figure 1 shows the output gap at each point in time estimated from the latest available vintage of data. Figure 2 traces estimated output gaps obtained from three specific vintages for the time period from 1964Q4 to 1997Q2. In contrast to Figure 1 the data used in the estimation of the output gap does not change along a specific line. The solid line shows output gap estimates from the data set from 2007Q2, the dashed line from 2002Q2, and the dotted line from 1997Q2. The data sets differ in the extent to which the data has been revised and in the number of observations which is higher for later vintages. It can be seen that the low points of the business cycle tend to be more pronounced for shorter data sets with less revisions. As we move to the right and approach the vintage data of each data set the estimates diverge more strongly since data revisions are more drastic closer to the release date

[12]Note that the regressors in the Taylor rule are not endogenous in the sense of Kim (2006) and Kim and Nelson (2005) since e_t and ϵ_t are assumed to be uncorrelated.

of the data.[13]

« insert Figure 2 »

Figure 3 presents the inflation forecast which is used together with the forecast of the output gap in the estimation of the forward-looking Taylor rule. The top panel shows actual inflation together with the inflation forecast. Forecast errors are presented in the bottom panel. The RMSE of the inflation forecast is 0.039.

« insert Figure 3 »

The next two figures contain graphs of the recursive estimates of the model parameters. They show how the estimated parameters of the structural economic model change as more and improved data becomes available and economic agents learn about the structural relations in the economy. The autoregressive coefficients on the output gap ϕ_1 and ϕ_2 are shown in Figure 4 together with the drift of potential output μ_y. The dashed line are bands of two standard deviations around the parameter estimates. All parameter estimates are statistically significant. Both autoregressive parameters are relatively stable over time and are highly correlated. As shown in the bottom right panel their sum is is roughly constant and highly significant.

« insert Figure 4 »

Of special interest is the "Phillips-curve" parameter γ which describes the effect of the output gap on the change in the inflation rate. Figure 5 shows recursive estimates of γ together with error bands of two standard deviations. Except for two short periods of time in the 1970s and in the mid 1980s the Phillips-curve coefficient is significantly different from zero. However, the size of the effect of the output gap on inflation is relatively low with estimates between 0.02 and 0.3 from the mid 1980s up to the present.

« insert Figure 5 »

The estimated time series for the output gap forecast $z_{t+2|t-1}$ and for the inflation forecasts $\pi_{t+2|t-1}$ together with observations on the Federal Funds Rate are used to estimate the parameters of the time-varying Taylor rule. These parameters are estimated

[13] For similar results see Orphanides and van Norden (2002).

recursively implying that at each point in time the parameter estimates are based only on the model generated real-time forecasts for the output gap and inflation running from 1966Q1 up to the date of the estimation. The recursive one-sided estimates of the coefficients in the Fed's reaction function are shown in Figure 6.

The Taylor-rule coefficients exhibit strong variations over time. Often the coefficient on the inflation forecast (upper right panel) is less than one thus violating the Taylor principle (Taylor (1999)). It sometimes even becomes negative, for example in the mid 1970s, the mid 1990s and after the bursting of the new economy bubble in 2001. The coefficient on the output gap (lower left panel) trends upward from the mid 1980s on but exhibits pronounced cyclical swings. Changes in the intercept (upper left panel) can be attributed to both changes in the equilibrium real interest rate and changes in the Fed's inflation target. The intercept is extremely high in the high-inflation era of the 1970s and early 1980s. This is caused by the breakdown in fit of TR which occurs at this time.

« insert Figure 6 »

5 Federal Funds Rate forecast uncertainty

5.1 The one-period ahead interest-rate forecast

Forecast uncertainty about the Federal Funds Rate in the next quarter is defined as

$$\mathbf{E}_t \left[(i_{t+1} - \hat{i}_{t+1|t})^2 | \Omega_t \right], \tag{16}$$

where

$$\hat{i}_{t+1|t} = \mathbf{E}_t \left[i_{t+1} | \Omega_t \right] = \mathbf{E}_t \left[x'_{t+1} \beta_{t+1} | \Omega_t \right]. \tag{17}$$

Ω_t represents the information available to market participants immediately *after* the interest rate is set at time t. This information set consists of the estimated reaction function in (14) and (15), the estimated model in (10) and (11), and the series of current and past interest rates and output gap and inflation forecasts.[14]

Since β and x to be uncorrelated.[15]

$$\hat{i}_{t+1|t} = \mathbf{E}_t\left[x'_{t+1}|\Omega_t\right]\mathbf{E}_t\left[\beta_{t+1}|\Omega_t\right] = \hat{x}'_{t+1|t}\beta_{t+1|t}. \tag{18}$$

Note that since $x_t = (1 \quad \pi_{t+2|t-1} \quad z_{t+2|t-1} \quad i_{t-1})$, the forecast of x_{t+1} based on Ω_t, is $\hat{x}_{t+1|t} = (1 \quad \pi_{t+3|t-1} \quad z_{t+3|t-1} \quad i_t)$. However the forecast of β_{t+1} based on Ω_t is $\beta_{t+1|t}$ as i_t is part of the information set in period t.

The interest rate forecasts from the time-varying Taylor rule using the recursively estimated coefficients from Figure 6 in combination with real-time predictions of next quarter's forecasts of the output gap and the inflation rate are shown in Figure 7. The estimated model provides a reasonable approximation to the observed Federal Funds rate with a RMSE of *1.54*. Some very high forecast errors result for the period from 1979 to 1982 in which the Fed targeted nonborrowed reserves instead of the Federal Funds Rate and for the mid 1970s. Generally, forecast errors for the time period up to the mid 1980s are larger than those for the later time period.

« insert Figure 7 »

Combining (14), (16) and (18) leads to

[14]Instead of assuming that market participants know the model the central bank uses to estimate the output gap and the current inflation rate the results could also be obtained under the assumption that market participants accept the model as a relatively accurate representation of the way the central bank acquires and uses its information.

[15]This assumption is implied by using the Kalman filter to estimate β.

$$
\begin{aligned}
\mathbf{E}_t\left[(i_{t+1}-\hat{i}_{t+1|t})^2|\Omega_t\right] &= \mathbf{E}_t\left[(x'_{t+1}\beta_{t+1}-\hat{x}'_{t+1|t}\beta_{t+1|t})^2|\Omega_t\right] \\
&= \mathbf{E}_t\left[\beta'_{t+1}x_{t+1}x'_{t+1}\beta_{t+1}|\Omega_t\right]-\beta'_{t+1|t}\hat{x}_{t+1|t}\hat{x}'_{t+1|t}\beta_{t+1|t} \\
&\quad +\sigma^2_{\epsilon,t+1|t} \qquad (19) \\
&= \hat{x}'_{t+1|t}\mathbf{E}_t\left[(\beta_{t+1}-\beta_{t+1|t})(\beta_{t+1}-\beta_{t+1|t})'|\Omega_t\right]\hat{x}_{t+1|t} \\
&\quad +\beta'_{t+1|t}\mathbf{E}_t\left[(x_{t+1}-\hat{x}_{t+1|t})(x_{t+1}-\hat{x}_{t+1|t})'|\Omega_t\right]\beta_{t+1|t} \\
&\quad +\sigma^2_{\epsilon,t+1|t} \\
&= \hat{x}'_{t+1|t}P_{\beta,t+1|t}\hat{x}'_{t+1|t}+\beta'_{t+1|t}P_{x,t+1|t}\beta_{t+1|t}+\sigma^2_{\epsilon,t+1|t}. \qquad (20)
\end{aligned}
$$

$\sigma^2_{\epsilon,t+1|t}$ is the forecast of the variance of the approximation error using the estimated ARCH coefficients. $P_{\beta,t+1|t} = \mathbf{E}_t\left[(\beta_{t+1}-\beta_{t+1|t})(\beta_{t+1}-\beta_{t+1|t})'\right]$ is obtained from the Kalman filter. The first term in (20) is the component of the overall interest rate forecast uncertainty due to possible changes in the way the Fed responds to the fundamental variables in its reaction function. This uncertainty rises if there is an increase in absolute value of the variables that enter the policy rule. The reason is that even if uncertainty about the β-parameters remains unchanged, uncertainty about the size of the interest rate response of the central bank increases when the absolute values of the variables the β-coefficients are multiplied with rise.
$P_{x,t+1|t} = \mathbf{E}_t\left[(x_{t+1}-x_{t+1|t})(x_{t+1}-x_{t+1|t})'|\Omega_t\right]$ represents the uncertainty about the forecast of the economic variables the interest rate responds to. A detailed derivation of this expression can be found in Appendix C.

The results for the one-quarter ahead forecast uncertainty from (20) are presented in Figure 8. The solid line indicates aggregate interest rate uncertainty while the other two lines represent uncertainty about the reaction coefficients in the Taylor rule which will prevail in the next quarter

$$\beta_{unc} = \hat{x}'_{t+1|t}P_{\beta,t+1|t}\hat{x}'_{t+1|t},$$

and uncertainty about economic fundamentals in the next quarter

$$x_{unc} = \beta'_{t+1|t}P_{x,t+1|t}\beta'_{t+1|t}.$$

Aggregate interest rate uncertainty is the sum of the two other series plus the time-varying residual uncertainty.

« insert Figure 8 »

Figure 8 indicates considerable changes in uncertainty about one-quarter ahead forecasts of the Federal Funds Rate. Peaks in forecast uncertainty were in the mid 1970s, in the early 1980s, in 1984Q4 and in 2002Q2. The lower panel shows a cropped version of the graph not including the very high estimated uncertainty in 1980Q3. It shows that even when ignoring extreme values uncertainty about the one-quarter ahead Federal Funds Rate was significantly higher in the 1970s and 1980s than in the 1990s and 2000s.

The first strong rise beginning in 1973 is caused by an increase in uncertainty about the the reaction coefficients in the Taylor rule. After a brief decline, uncertainty about future policy coefficients increases once more after 1977 and remains high up to the mid 1980s. The extreme hike in forecast uncertainty in the early 1980s however, can only partially explained by uncertainty about the coefficients in the Fed's reaction function. Its primary cause is a strong increase in residual uncertainty, i.e. a massive deterioration of Taylor rule's ability to track the actual Federal Funds Rate.[16] The same applies to the peak in uncertainty in 1984Q4.

Uncertainty about economic fundamentals, i.e. output gap and inflation forecasts, prevailing in the next quarter increases temporarily in the mid 1970s but remains fairly low throughout the whole sample period. From the late 1980s on uncertainty about future fundamentals and about future policy reactions are very low and close to each other. Up to the late 1980s however, uncertainty about the Taylor rule coefficients dominates uncertainty about future fundamentals.

« insert Figure 9 »

Figure 9 presents the time-varying variance of ϵ which results from the recursive estimation of the Taylor rule and is assumed to follow a ARCH process. The lower panel contains a cropped version of the upper panel without the extreme value estimated for

[16]Residual uncertainty is the difference between aggregate uncertainty and the sum of the other two components

1980Q2. Surprisingly the conditional variance is very low up to 1980Q2.[17] After 1980 however, the conditional variance exhibits pronounced changes over time.

The estimates of interest rate forecast uncertainty shown in Figure 10 were obtained from a Taylor rule specification under the assumption of a constant variance of ϵ. In this case changes in the estimated residual variance result only from the re-estimation of the Taylor rule parameters at each point in time. The omission of ARCH effects lead to almost all uncertainty about the Federal Funds Rate forecast being attributed to uncertainty about the Taylor rule coefficients. Residual uncertainty is extremely low even for the 1979-82 period.[18] This causes the Kalman filter algorithm to attribute forecast errors predominantly to changes in the Taylor rule coefficients and leads to strong revisions in the coefficients of the interest rate rule.

« insert Figure 10 »

Figure 11 compares forecast uncertainties for the ARCH specification and the Taylor rule without ARCH errors. Until the early 1980s both specifications lead to almost identical results. However, the peaks in uncertainty in the early and mid 1980s are less pronounced for the model without ARCH effects. For this model interest rate uncertainty increases less drastically but comes down much slower as well. The hike in 2001/2002 which the ARCH model attributes mostly to an increase in the conditional variance of the error term is not captured by the model without ARCH errors.

« insert Figure 11 »

5.2 The two-period ahead interest-rate forecast

Forecast uncertainty about the Federal Funds Rate two quarters ahead is

$$\mathbf{E}_t\left[(i_{t+2} - \hat{i}_{t+2|t})^2|\Omega_t\right], \tag{21}$$

[17]This might be caused by problems with the estimation of the ARCH parameters using the relatively short sample at the beginning. Up to 1980Q4 the estimates for the sum of α_1 and α_2 is well below 0.1 and not statistically significant. From 1980Q4 on the sum is greater than one and significant.

[18]This is evident from the fact that aggregate forecast uncertainty is tracked almost perfectly by the time series of uncertainty about policy reactions alone.

where

$$
\begin{aligned}
\hat{i}_{t+2|t} &= \mathbf{E}_t\left[(i_{t+2}|\Omega_t\right] = \mathbf{E}_t\left[x'_{t+2}\beta_{t+2}|\Omega_t\right] \qquad (22)\\
&= \mathbf{E}_t\left[x'_{t+2}|\Omega_t\right]\mathbf{E}_t\left[\beta_{t+2}|\Omega_t\right] = \hat{x}'_{t+2|t}\beta_{t+2|t},
\end{aligned}
$$

Expanding (22) gives

$$
\begin{aligned}
\mathbf{E}_t\left[(i_{t+2}-\hat{i}_{t+2|t})^2|\Omega_t\right] &= \mathbf{E}_t\left[(x'_{t+2}\beta_{t+2}-\hat{x}'_{t+2|t}\hat{\beta}_{t+2|t})^2|\Omega_t\right]\\
&= \mathbf{E}_t\left[\beta'_{t+2}x_{t+2}x'_{t+2}\beta_{t+2}|\Omega_t\right] - \beta'_{t+2|t}\hat{x}_{t+2|t}\hat{x}'_{t+2|t}\beta_{t+2|t}\\
&\quad +\sigma^2_{\epsilon,t+2|t} \qquad (23)\\
&= \hat{x}'_{t+2|t}\mathbf{E}_t\left[(\beta_{t+2}-\beta_{t+2|t})(\beta_{t+2}-\beta_{t+2|t})'|\Omega_t\right]\hat{x}_{t+2|t}\\
&\quad +\beta'_{t+2|t}\mathbf{E}_t\left[(x_{t+2}-\hat{x}_{t+2|t})(x_{t+2}-\hat{x}_{t+2|t})'|\Omega_t\right]\beta_{t+2|t}\\
&\quad +\sigma^2_{\epsilon,t+2|t}\\
&= \hat{x}'_{t+2|t}P_{\beta,t+2|t}\hat{x}'_{t+2|t} + \beta'_{t+2|t}P_{x,t+2|t}\beta'_{t+2|t} + \sigma^2_{\epsilon,t+2|t}. \qquad (24)
\end{aligned}
$$

$P_{\beta,t+2|t} = \mathbf{E}_t\left[(\beta_{t+2}-\beta_{t+2|t})(\beta_{t+2}-\beta_{t+2|t})'|\Omega_t\right]$ can be computed using the delta method from $P_{\tilde{\beta},t+2|t} = GP_{\tilde{\beta},t+1|t}G' + \Sigma_{w,t+1|t}$, where $P_{\tilde{\beta},t+1|t}$ follows from (B13). See Appendix B for details.

As expected Figure 12 shows forecast uncertainty over two quarters to be generally higher than that over one quarter. The relative importance of residual uncertainty declines while uncertainty about future economic fundamentals becomes more important in explaining periods of high forecast uncertainty. Uncertainty about the Taylor rule coefficients is still the main reason for the increase in overall forecast uncertainty in the mid 1970s and around 1980. However, as shown in Figure 13 for the longer forecast horizon uncertainty about future output gap and inflation forecasts is quantitatively more important than uncertainty about the future policy reaction function for most of the time after the 1980s.

« insert Figure 12 »

« insert Figure 13 »

6 Conclusion

This paper has presented a simple model of monetary policy in the U.S. that separates the forecast uncertainty about future values of the Federal Funds Rate into uncertainty about the state of the economy in the future and uncertainty about how the central bank will react to it.

The results from real-time U.S. data show considerable time variation in the parameters of the policy rule as well as marked changes in the components of Federal Funds rate forecast uncertainty. In particular, uncertainty about the strength of the Fed's future responses to economic fundamentals changed strongly through time and was most pronounced in mid 1970s and the in the late 1970s through the early 1980s. For a short forecasting horizon of one quarter uncertainty about future economic conditions has a very limited impact. However, increasing the forecast horizon to two quarters the situation is reversed and uncertainty about the future state of the economy becomes relatively more important.

Appendix A: The Kalman filter equations

The estimates of the unobserved component $\tilde{x}_{t|t-1} = E_{t-1}[\tilde{x}_t]$ and of its covariance matrix $P_{\tilde{x},t|t-1} = E_{t-1}[(\tilde{x}_t - \tilde{x}_{t|t-1})(\tilde{x}_t - \tilde{x}_{t|t-1})']$ are formed recursively

$$\begin{aligned} \tilde{x}_{t|t-1} &= F\tilde{x}_{t-1|t-1}, & \text{(A1)} \\ P_{\tilde{x},t|t-1} &= FP_{\tilde{x},t-1|t-1}F' + \Sigma_\zeta, & \text{(A2)} \end{aligned}$$

with $\tilde{x}_{t|t} = E_t[\tilde{x}_t]$ and its covariance matrix $P_{\tilde{x},t|t} = E_t[(\tilde{x}_t - \tilde{x}_{t|t})(\tilde{x}_t - \tilde{x}_{t|t})']$.

After the information on Y_t has become available, the estimates are updated as

$$\begin{aligned} \tilde{x}_{t|t} &= \tilde{x}_{t|t-1} + K_{t|t-1}(Y_t - Y_{t|t-1}) \\ &= \tilde{x}_{t|t-1} + K_{t|t-1}(Y_t - \mu - H\tilde{x}_{t|t-1}) \\ &= \tilde{x}_{t|t-1} + K_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t) & \text{(A3)} \\ P_{\tilde{x},t|t} &= P_{\tilde{x},t|t-1} - K_{t|t-1}HP_{\tilde{x},t|t-1}, & \text{(A4)} \end{aligned}$$

with $K_{t|t-1} = P_{\tilde{x},t|t-1}H'[HP_{\tilde{x},t|t-1}H' + \Sigma_Y]^{-1}$.
The second second row of (A3) is used to generate the estimates $\tilde{x}_{t|t}$ while the third row is used to compute the expressions for interest rate uncertainty (see below).

Appendix B: The linearized state-space model for the Taylor rule

The Taylor rule is rewritten as

$$i_t = \beta_{0,t} + \beta_{\pi,t}\pi_{t+2|t-1} + \beta_{z,t}z_{t+2|t-1} + f(i_{t-1}, \beta_{\rho,t}) + \epsilon_t, \tag{B1}$$

with

$$f(i_{t-1}, \beta_{\rho,t}) = \frac{1}{1 + exp(-\beta_{\rho,t})} i_{t-1} \equiv \rho_t i_{t-1},$$

and

$$\beta_t = \beta_t + w_{t+1}, \tag{B2}$$

where

$$\beta_t = (\beta_{0,t}, \beta_{\pi,t}, \beta_{z,t}, \beta_{\rho,t})'.$$

The Kalman filter is applied to a linearized version of (B1) (see Harvey (1989)): A Taylor approximation to (B1) around $\beta_{\rho,t} = \beta_{\rho,t|t-1}$ results in

$$\begin{aligned} i_t \quad = \quad & \beta_{0,t} + \beta_{\pi,t}\pi_{t+2|t-1} + \beta_{z,t}z_{t+2|t-1} + \frac{1}{1 + exp(-\beta_{\rho,t|t-1})} i_{t-1} \\ & + \frac{exp(-\beta_{\rho,t|t-1}) i_{t-1}}{(1 + exp(-\beta_{\rho,t|t-1}))^2} (\beta_{\rho,t} - \beta_{\rho,t|t-1}) + \epsilon_t. \end{aligned} \tag{B3}$$

This can be written as

$$\tilde{i}_t = \beta_{0,t} + \beta_{\pi,t}\pi_{t+2|t-1} + \beta_{z,t}z_{t+2|t-1} + \frac{exp(-\beta_{\rho,t|t-1}) i_{t-1}}{(1 + exp(-\beta_{\rho,t|t-1}))^2} \beta_{\rho,t} + \epsilon_t, \tag{B4}$$

with

$$\tilde{i}_t = i_t - \frac{i_{t-1}}{1 + exp(-\beta_{\rho,t|t-1})} + \frac{exp(-\beta_{\rho,t|t-1}) i_{t-1}}{(1 + exp(-\beta_{\rho,t|t-1}))^2} \beta_{\rho,t|t-1}.$$

In each iteration of the Kalman filter there is now an additional step to compute $\tilde{i}$ using the estimate from the previous estimation $\tilde{\beta}_{\rho,t|t-1}$.

The modifications that result from the assumption of an ARCH-process for the error term are as shown in Kim und Nelson (2006). The error term is included in the unobserved component. Thus

$$\tilde{i}_t = \begin{bmatrix} 1 & \pi_{t+2|t} & z_{t+2|t} & \frac{exp(-\beta_{\rho,t|t-1})i_{t-1}}{(1+exp(-\beta\rho,t|t-1))^2} & 1 \end{bmatrix} \begin{bmatrix} \beta_t \\ \epsilon_t \end{bmatrix} \tag{B5}$$

$$= \tilde{x}_t' \tilde{\beta}_t, \tag{B6}$$

and

$$\tilde{\beta}_t = G\tilde{\beta}_{t-1} + \tilde{w}_t, \tag{B7}$$

where

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{B8}$$

$$\tilde{w}_t = \begin{bmatrix} w_t & \epsilon_t \end{bmatrix}, \tag{B9}$$

and

$$E\tilde{w}_t\tilde{w}_t' = \Sigma_{\tilde{w},t} = \begin{bmatrix} \Sigma_w & 0 \\ 0 & \sigma^2_{\epsilon,t} \end{bmatrix}, \tag{B10}$$

$$\sigma^2_{\epsilon,t} = \alpha_0 + \alpha_1\epsilon^2_{t-1} + \alpha_2\epsilon^2_{t-2}. \tag{B11}$$

The forecasting equations of the Kalman filter become

$$\tilde{\beta}_{t|t-1} = G\tilde{\beta}_{t-1|t-1}, \tag{B12}$$

$$P_{\tilde{\beta},t|t-1} = GP_{\tilde{\beta},t-1|t-1}G' + \Sigma_{\tilde{w},t}. \tag{B13}$$

After i_t is observed the estimates are updated as

$$\begin{aligned}
\tilde{\beta}_{t|t} &= \tilde{\beta}_{t|t-1} + P_{\tilde{\beta},t|t-1}\tilde{x}_t[\tilde{x}_t' P_{\tilde{\beta},t|t-1}\tilde{x}_t]^{-1}(\tilde{i}_t - \tilde{x}_t'\tilde{\beta}_{t|t-1}), &\text{(B14)}\\
P_{\tilde{\beta},t|t} &= P_{\tilde{\beta},t|t-1} - P_{\tilde{\beta},t|t-1}\tilde{x}_t[\tilde{x}_t' P_{\tilde{\beta},t|t-1}\tilde{x}_t]^{-1}\tilde{x}_t' P_{\tilde{\beta},t|t-1}. &\text{(B15)}
\end{aligned}$$

The covariance matrix of the unobserved states $P_{\tilde{\beta}}$ is based on $\tilde{\beta} = (\beta_{0,t}, \beta_{\pi,t}, \beta_{z,t}, \beta_{\rho,t})'$. It can be transformed to the covariance matrix for $(\beta_{0,t}, \beta_{\pi,t}, \beta_{z,t}, \rho_t)'$ by using the delta method.

Since I require only the one-sided estimates for $\tilde{x}$ and β the equations for the smoothing algorithm are not necessary and thus not reproduced here.[19]

[19]For more details on the Kalman filter see, for example, Hamilton (1996) or Kim and Nelson (1999).

Appendix D: Uncertainty measures

Uncertainty about economic conditions in the one-period case

Derivation of (20): A Taylor-Approximation can be used for

$$
\begin{aligned}
\mathbf{E}\left[\beta'_{t+1}x_{t+1}x'_{t+1}\beta_{t+1}|\Omega_t\right] \;\approx\;& \beta'_{t+1|t}\hat{x}_{t+1|t}\hat{x}'_{t+1|t}\beta_{t+1|t} \\
&+2\mathbf{E}(\beta_{t+1}-\beta_{t+1|t}|\Omega_t)'\hat{x}_{t+1|t}\hat{x}'_{t+1|t}\beta_{t+1|t} \\
&+2\mathbf{E}(x_{t+1}-\hat{x}_{t+1|t}|\Omega_t)'\beta_{t+1|t}\beta'_{t+1|t}\hat{x}_{t+1|t} \\
&+\hat{x}'_{t+1|t}\mathbf{E}\big[(\beta_{t+1}-\beta_{t+1|t})(\beta_{t+1}-\beta_{t+1|t})'|\Omega_t\big]\hat{x}_{t+1|t} \\
&+\beta'_{t+1|t}\mathbf{E}\big[(x_{t+1}-\hat{x}_{t+1|t})(x_{t+1}-\hat{x}_{t+1|t})'|\Omega_t\big]\beta_{t+1|t} \\
&+4\hat{x}'_{t+1|t}\mathbf{E}\big[(\beta_{t+1}-\beta_{t+1|t})(x_{t+1}-\hat{x}_{t+1|t})'|\Omega_t\big]\beta_{t+1|t} \\
\approx\;& \beta'_{t+1|t}\hat{x}_{t+1|t}\hat{x}'_{t+1|t}\beta_{t+1|t} \qquad\qquad \text{(D1)} \\
&+\hat{x}'_{t+1|t}\mathbf{E}\big[(\beta_{t+1}-\beta_{t+1|t})(\beta_{t+1}-\beta_{t+1|t})'|\Omega_t\big]\hat{x}_{t+1|t} \\
&+\beta'_{t+1|t}\mathbf{E}\big[(x_{t+1}-\hat{x}_{t+1|t})(x_{t+1}-\hat{x}_{t+1|t})'|\Omega_t\big]\beta_{t+1|t}.
\end{aligned}
$$

Substituting this expression into (19) yields (20).

Since $x_{t+1} = (1 \quad \pi_{t+3|t} \quad z_{t+3|t} \quad i_t)$ and $\hat{x}_{t+1} = (1 \quad \pi_{t+3|t-1} \quad z_{t+3|t-1} \quad i_t)$ I can write

$$
\begin{aligned}
P_{x,t+1|t} &= \mathbf{E}_t\left[(x_{t+1}-\hat{x}_{t+1|t})(x_{t+1}-\hat{x}_{t+1|t})'|\Omega_t\right] \\
&= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & p_{\pi,\pi,t+1} & p_{\pi,z,t+1} & 0 \\ 0 & p_{\pi,z,t+1} & p_{z,z,t+1} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad\qquad \text{(D2)}
\end{aligned}
$$

where $p_{\pi,\pi,t+1} = \mathbf{E}\left[(\pi_{t+3|t}-\pi_{t+3|t-1})^2|\Omega_t\right]$, $p_{z,z,t+1} = \mathbf{E}\left[(z_{t+3|t}-z_{t+3|t-1})^2|\Omega_t\right]$, and $p_{\pi,z,t+1} = \mathbf{E}\left[(\pi_{t+3|t}-\pi_{t+3|t-1})(z_{t+3|t}-z_{t+3|t-1})|\Omega_t\right]$.

The individual elements can be derived as follows: The inflation forecast the central bank will react to in the next period is $\pi_{t+3|t} = \pi_{t-1} + \Delta\pi_t + \sum_{i=1}^{3}\Delta\pi_{t+i|t}$ while the forecast of this variable based on information dated $t-1$ is $\pi_{t+3|t-1} = \pi_{t-1} + \sum_{i=0}^{3} \Delta\pi_{t+i|t-1}$. Hence,

$$
\begin{aligned}
\pi_{t+3|t} - \pi_{t+2|t-1} &= = (\Delta\pi_t - \Delta\pi_{t|t-1}) + \sum_{i=1}^{3}(\Delta\pi_{t+i|t} - \Delta\pi_{t+i|t-1}) \\
&= \mathbf{1}_2'\big[(Y_t - Y_{t|t-1}) + \sum_{i=1}^{3}(Y_{t+i|t} - Y_{t+i|t-1})\big] \\
&= \mathbf{1}_2'\big[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + \sum_{i=1}^{3} H(\tilde{x}_{t+i|t} - \tilde{x}_{t+i|t-1})\big] \qquad \text{(D3)} \\
&= \mathbf{1}_2'\big[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(F + FF + FFF)(\tilde{x}_{t|t} - \tilde{x}_{t|t-1})\big].
\end{aligned}
$$

At the time the policy rate in period t is announced, uncertainty about $\pi_{t+3|t}$, the estimate of inflation the central bank will react to in the next period stems from two sources: First, $(\Delta\pi_t - \Delta\pi_{t|t-1})$ is the error made in estimating the change in the inflation rate from the previous to the current period. Second, $\sum_{i=1}^{3}(\Delta\pi_{t+i|t} - \Delta\pi_{t+i|t-1})$ is the difference between the changes in inflation from period $t+1$ to $t+3$ forecast by the central bank at the time it has to set i_{t+1} – and thus formed with knowledge of π_t – and the forecast of the changes in inflation made by the public in $t-1$ without knowing π_t.

With $\mathbf{1}_2 = (0 \quad 1)'$. Using (10), (11) and (A3) and (C3) we get

$$
\begin{aligned}
\pi_{t+3|t} - \pi_{t+3|t-1} &= \mathbf{1}_2'\big[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(F + FF + FFF)K_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t)\big] \\
&= \mathbf{1}_2'\big[H(I + (F + FF + FFF)K_{t|t-1}H)(\tilde{x}_t - \tilde{x}_{t|t-1}) \qquad \text{(D4)} \\
&\quad + (I + H(F + FF + FFF)K_{t|t-1})e_t\big],
\end{aligned}
$$

which can be written as

$$
\pi_{t+3|t} - \pi_{t+3|t-1} = \mathbf{1}_2'\Big[A_{1,\tilde{x}}(\tilde{x}_t - \tilde{x}_{t|t-1}) + A_{1,e_t}e_t\Big]. \qquad \text{(D5)}
$$

Using this expression the result is

$$
\begin{aligned}
p_{\pi,\pi,t+1} &= \mathbf{E}\left[(\pi_{t+3|t} - \pi_{t+3|t-1})^2 | \Omega_t\right] \\
&= \mathbf{1}_2' \left[A_{1,\tilde{x}} P_{\tilde{x},t|t-1} A_{1,\tilde{x}}' + A_{1,e_t} \Sigma_Y A_{1,e_t}' \right] \mathbf{1}_2. \qquad \text{(D6)}
\end{aligned}
$$

$z_{t+3|t}$ is the (1,1) element of $\tilde{x}_{t+3|t} = FFF\tilde{x}_{t|t}$, while $z_{t+3|t-1}$ is the (1,1) element of $\tilde{x}_{t+3|t-1} = FFF\tilde{x}_{t|t-1}$. Hence,

$$
\begin{aligned}
z_{t+3|t} - z_{t+3|t-1} &= \mathbf{1}_1' FFF(\tilde{x}_{t|t} - \tilde{x}_{t|t-1}) \\
&= \mathbf{1}_1' FFFK_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t). \qquad \text{(D7)}
\end{aligned}
$$

Defining

$$
z_{t+3|t} - z_{t+3|t-1} = \mathbf{1}_1' \left[B_{1,\tilde{x}}(\tilde{x}_t - \tilde{x}_{t|t-1}) + B_{1,e_t} e_t \right], \qquad \text{(D8)}
$$

with the respective coefficients shown in (C7) leads to

$$
\begin{aligned}
p_{z,z,t+1} &= \mathbf{E}\left[(z_{t+3|t} - z_{t+3|t-1})^2 | \Omega_t\right] \\
&= \mathbf{1}_1' \mathbf{E}\left[(\tilde{x}_{t+1|t} - \tilde{x}_{t+1|t-1})(\tilde{x}_{t+1|t} - \tilde{x}_{t+1|t-1})' | \Omega_t\right] \mathbf{1}_1 \\
&= \mathbf{1}_1' \left[B_{1,\tilde{x}} P_{\tilde{x},t|t-1} B_{1,\tilde{x}}' + B_{1,e_t} \Sigma_Y B_{1,e_t}' \right] \mathbf{1}_1, \qquad \text{(D9)}
\end{aligned}
$$

with $\mathbf{1}_1 = (1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)'$. Uncertainty about the central bank's forecast for the output gap is due to the fact that when policy is set next period additional information in form of observations of π_t and y_t will be available.

Finally, combining (C5) with (C8) yields

$$
\begin{aligned}
p_{\pi,z,t+1} &= \mathbf{E}\left[(\pi_{t+3|t} - \pi_{t+3|t-1})(z_{t+3|t} - z_{t+3|t-1}) | \Omega_t\right] \\
&= \mathbf{1}_2' \left[A_{1,\tilde{x}} P_{\tilde{x},t|t-1} B_{1,\tilde{x}}' + A_{1,e_t} \Sigma_Y B_{1,e_t}' \right] \mathbf{1}_1. \qquad \text{(D10)}
\end{aligned}
$$

All these expressions can be evaluated using the model estimates from section 3 and the results from the Kalman filter.

Uncertainty about economic conditions in the two-period case

$$P_{x,t+2|t} = \mathbf{E}_t\left[(x_{t+2}-\hat{x}_{t+2|t})(x_{t+2}-\hat{x}_{t+2|t})'|\Omega_t\right] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & p_{\pi,\pi,t+2} & p_{\pi,z,t+2} & p_{\pi,i,t+2} \\ 0 & p_{\pi,z,t+2} & p_{z,z,t+2} & p_{i,z,t+2} \\ 0 & p_{\pi,i,t+2} & p_{i,z,t+2} & p_{i,i,t+2} \end{bmatrix}, \quad \text{(D11)}$$

where $p_{\pi,\pi,t+2} = \mathbf{E}\left[(\pi_{t+4|t+1}-\pi_{t+4|t-1})^2|\Omega_t\right]$, $p_{z,z,t+2} = \mathbf{E}\left[(z_{t+4|t+1}-z_{t+4|t})^2|\Omega_t\right]$, $p_{\pi,z,t+2} = \mathbf{E}\left[(\pi_{t+4|t+1}-\pi_{t+4|t-1})(z_{t+4|t+1}-z_{t+4|t})|\Omega_t\right]$, $p_{i,i,t+2} = \mathbf{E}\left[(i_{t+1}-\hat{i}_{t+1|t})^2|\Omega_t\right]$, $p_{\pi,i,t+2} = \mathbf{E}\left[(\pi_{t+4|t+1}-\pi_{t+4|t-1})(i_{t+1}-\hat{i}_{t+1|t})|\Omega_t\right]$, and $p_{i,z,t+2} = \mathbf{E}\left[(i_{t+1}-\hat{i}_{t+1|t})(z_{t+4|t+1}-z_{t+4|t})|\Omega_t\right]$.

The inflation forecast the central bank will react to two periods in the future is $\pi_{t+4|t+1} = \pi_{t-1} + \Delta\pi_t + \Delta\pi_{t+1} + \sum_{i=2}^4 \Delta\pi_{t+i|t+1}$, while $\pi_{t+4|t-1} = \pi_{t-1} + \Delta\pi_{t|t-1} + \Delta\pi_{t+1|t-1} + \sum_{i=2}^4 \Delta\pi_{t+i|t-1}$. Thus,

$$\begin{aligned}
\pi_{t+4|t+1} - \pi_{t+4|t-1} &= (\Delta\pi_t - \Delta\pi_{t|t-1}) + (\Delta\pi_{t+1} - \Delta\pi_{t+1|t-1}) + \sum_{i=2}^{4}(\Delta\pi_{t+i|t+1} - \Delta\pi_{t+i|t-1}) \\
&= \mathbf{1}_2'\big[(Y_t - Y_{t|t-1}) + (Y_{t+1} - Y_{t+1|t-1}) + \sum_{i=2}^{4}(Y_{t+2|t+1} - Y_{t+2|t-1})\big] \\
&= \mathbf{1}_2'\big[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(\tilde{x}_{t+1} - \tilde{x}_{t+1|t-1}) + e_{t+1} \\
&\quad + \sum_{i=2}^{4} H(\tilde{x}_{t+i|t+1} - \tilde{x}_{t+i|t-1})\big] \\
&= \mathbf{1}_2'\big[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(\tilde{x}_{t+1} - \tilde{x}_{t+1|t-1}) + e_{t+1} \\
&\quad + H(F + FF + FFF)(\tilde{x}_{t+1|t+1} - \tilde{x}_{t+1|t-1})\big] \\
&= \mathbf{1}_2'\big[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(\tilde{x}_{t+1} - \tilde{x}_{t+1|t-1}) + e_{t+1} \\
&\quad + H(F + FF + FFF)(\tilde{x}_{t+1|t+1} - \tilde{x}_{t+1|t} + \tilde{x}_{t+1|t} - \tilde{x}_{t+1|t-1})\big] \\
&= \mathbf{1}_2'\big[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(\tilde{x}_{t+1} - \tilde{x}_{t+1|t-1}) + e_{t+1} \\
&\quad + H(F + FF + FFF)(\tilde{x}_{t+1|t+1} - \tilde{x}_{t+1|t} + F(\tilde{x}_{t|t} - \tilde{x}_{t|t-1}))\big], \quad \text{(D12)}
\end{aligned}$$

with $\mathbf{1}_2 = (0 \quad 1)'$. Using (10) and (11) yields

$$
\begin{aligned}
\pi_{t+4|t+1} - \pi_{t+4|t-1} &= \mathbf{1}_2'\big[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(F(\tilde{x}_t - \tilde{x}_{t|t-1}) + \zeta_{t+1}) + e_{t+1} \\
&\quad + H(F + FF + FFF)[K_{t+1|t}(H(\tilde{x}_{t+1} - \tilde{x}_{t+1|t}) + e_{t+1}) \\
&\quad + FK_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t))]\big] \\
&= \mathbf{1}_2'\big[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(F(\tilde{x}_t - \tilde{x}_{t|t-1}) + \zeta_{t+1}) + e_{t+1} \\
&\quad + H(F + FF + FFF)[K_{t+1|t}(H(F(\tilde{x}_t - \tilde{x}_{t|t}) + \zeta_{t+1}) + e_{t+1}) \\
&\quad + FK_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t))]\big] \\
&= \mathbf{1}_2'\big[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(F(\tilde{x}_t - \tilde{x}_{t|t-1}) + \zeta_{t+1}) + e_{t+1} \\
&\quad + H(F + FF + FFF)[K_{t+1|t}(H(F(\tilde{x}_t - \tilde{x}_{t|t-1} - K_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) \\
&\quad + e_t) + \zeta_{t+1}) + e_{t+1}) + FK_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t))]\big] \\
&= \mathbf{1}_2'\Bigg[H(I + F + (F + FF + FFF)[K_{t+1|t}(HF(I - K_{t|t-1}H)) \\
&\quad + FK_{t|t-1}H])(\tilde{x}_t - \tilde{x}_{t|t-1}) \\
&\quad + (I + H(F + FF + FFF)(FK_{t|t-1} - K_{t+1|t}HFK_{t|t-1}))e_t \\
&\quad + (I + H(F + FF + FFF)K_{t+1|t})e_{t+1} \\
&\quad + H(I + (F + FF + FFF)K_{t+1|t}H)\zeta_{t+1}\Bigg],
\end{aligned} \tag{D13}
$$

where in going from the second to the third step we have added and subtracted $x_{t|t-1}$ to and from the expression $(\tilde{x}_t - \tilde{x}_{t|t})$. Define

$$
\begin{aligned}
\pi_{t+4|t+1} - \pi_{t+4|t-1} &= \mathbf{1}_2'\Bigg[A_{2,\tilde{x}}(\tilde{x}_t - \tilde{x}_{t|t-1}) + A_{2,e_t}e_t + A_{2,e_{t+1}}e_{t+1} \\
&\quad + A_{2,\zeta}\zeta_{t+1}\Bigg],
\end{aligned} \tag{D14}
$$

where the respective coefficients are shown in (C13). This leads to

$$
\begin{aligned}
p_{\pi,\pi,t+2} &= \mathbf{E}\left[(\pi_{t+4|t+1} - \pi_{t+4|t-1})^2 | \Omega_t\right] \qquad \text{(D15)} \\
&= \mathbf{1}_2'\Bigg[A_{2,\tilde{x}}P_{\tilde{x},t|t-1}A_{2,\tilde{x}}' + A_{2,e_t}\Sigma_Y A_{2,e_t}' + A_{2,e_{t+1}}\Sigma_Y A_{2,e_{t+1}}' + A_{2,\zeta}\Sigma_\zeta A_{2,\zeta}'\Bigg]\mathbf{1}_2.
\end{aligned}
$$

$z_{t+4|t+1}$ is the (1,1) element of $\tilde{x}_{t+4|t+1} = FFF\tilde{x}_{t+1|t+1}$, while $z_{t+4|t-1}$ is the (1,1) element of $\tilde{x}_{t+4|t-1} = FFF\tilde{x}_{t+1|t-1}$. Hence,

$$
\begin{aligned}
z_{t+4|t+1} - z_{t+4|t-1} &= \mathbf{1}_1' FFF(\tilde{x}_{t+1|t+1} - \tilde{x}_{t+1|t-1}) \\
&= \mathbf{1}_1' FFF(\tilde{x}_{t+1|t+1} - \tilde{x}_{t+1|t} + \tilde{x}_{t+1|t} - \tilde{x}_{t+1|t-1}) \\
&= \mathbf{1}_1' FFF[K_{t+1|t}(H(\tilde{x}_{t+1} - \tilde{x}_{t+1|t}) + e_{t+1}) + FK_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t)] \\
&= \mathbf{1}_1' FFF[K_{t+1|t}(H(F(\tilde{x}_t - \tilde{x}_{t|t}) + \zeta_{t+1}) + e_{t+1}) + FK_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t)] \\
&= \mathbf{1}_1' FFF[K_{t+1|t}(H(F(\tilde{x}_t - \tilde{x}_{t|t-1} - K_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t)) + \zeta_{t+1}) + e_{t+1}) \\
&\quad + FK_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t)] \\
&= \mathbf{1}_1' \Big[FFF[K_{t+1|t}HF(I - K_{t|t-1}H) + FK_{t|t-1}H](\tilde{x}_t - \tilde{x}_{t|t-1}) \\
&\quad + FFF[FK_{t|t-1} - K_{t+1|t}HFK_{t|t-1}]e_t \\
&\quad FFFK_{t+1|t}e_{t+1} + FFFK_{t+1|t}H\zeta_{z+1} \Big], \qquad \text{(D16)}
\end{aligned}
$$

where in going from the fourth to the fifth step we have added and subtracted $x_{t|t-1}$ to and from the expression $(\tilde{x}_t - \tilde{x}_{t|t})$. Define

$$
\begin{aligned}
z_{t+4|t+1} - z_{t+4|t-1} &= \mathbf{1}_1' \Big[B_{2,\tilde{x}}(\tilde{x}_t - \tilde{x}_{t|t-1}) + B_{2,e_t}e_t + B_{2,e_{t+1}}e_{t+1} \qquad \text{(D17)} \\
&\quad + B_{2,\zeta}\zeta_{t+1} \Big],
\end{aligned}
$$

with the respective coefficients shown in (C16). Hence

$$
\begin{aligned}
p_{z,z,t+2} &= \mathbf{E}\left[(z_{t+4|t+1} - z_{t+4|t-1})^2 | \Omega_t\right] \qquad \text{(D18)} \\
&= \mathbf{1}_1' \mathbf{E} \Big[B_{2,\tilde{x}} P_{\tilde{x},t|t-1} B_{2,\tilde{x}}' + B_{2,e_t}\Sigma_Y B_{2,e_t}' + B_{2,e_{t+1}}\Sigma_Y B_{2,e_{t+1}}' + B_{2,\zeta}\Sigma_\zeta B_{2,\zeta}' \Big] \mathbf{1}_1.
\end{aligned}
$$

From (C14) and (C17) it follows that

$$
\begin{aligned}
p_{\pi,z,t+2} &= \mathbf{E}\left[(\pi_{t+4|t+1} - \pi_{t+4|t-1})(z_{t+4|t+1} - z_{t+4|t-1}) | \Omega_t\right] \qquad \text{(D19)} \\
&= \mathbf{1}_2' \Big[A_{2,\tilde{x}} P_{\tilde{x},t|t-1} B_{2,\tilde{x}}' + A_{2,e_t}\Sigma_Y B_{2,e_t}' + A_{2,e_{t+1}}\Sigma_Y B_{2,e_{t+1}}' + A_{2,\zeta}\Sigma_\zeta B_{2,\zeta}' \Big] \mathbf{1}_1.
\end{aligned}
$$

Next are the correlations of the forecast errors for the output gap and inflation with the forecast error for the interest rate. The latter one is

$$
\begin{aligned}
i_{t+1} - \hat{i}_{t+1|t} &= x'_{t+1}\beta_{t+1} - \hat{x}'_{t+1|t}\beta_{t+1|t} + \epsilon_{t+1} \\
&= x'_{t+1}(\beta_t + w_{t+1}) - \hat{x}'_{t+1|t}\beta_{t|t} + \epsilon_{t+1} \\
&= (x_{t+1} - \hat{x}_{t+1|t})'\beta_{t|t} + x'_{t+1}(\beta_t + w_{t+1} - \beta_{t|t}) + \epsilon_{t+1}. \qquad \text{(D20)}
\end{aligned}
$$

Since $x'_{t+1} = (1 \quad \pi_{t+3|t} \quad z_{t+3|t} \quad i_t)$ and $\hat{x}'_{t+3|t} = (1 \quad \pi_{t+3|t-1} \quad z_{t+3|t-1} \quad i_t)$ the above expression can be expanded to

$$
\begin{aligned}
i_{t+1} - \hat{i}_{t+1|t} &= (\pi_{t+3|t} - \pi_{t+3|t-1})\beta_{\pi,t|t} + (z_{t+3|t} - z_{t+3|t-1})\beta_{z,t|t} \\
&\quad + (\beta_{c,t} - \beta_{c,t|t}) + \pi_{t+3|t}(\beta_{\pi,t} - \beta_{\pi,t|t}) \\
&\quad + z_{t+3|t}(\beta_{z,t} - \beta_{z,t|t}) + i_t(\rho_t - \rho_{t|t}) \\
&\quad + x'_{t+1}w_{t+1} + \epsilon_{t+1}. \qquad \text{(D21)}
\end{aligned}
$$

The inflation forecast made in period $t+1$ is

$$
\begin{aligned}
\pi_{t+3|t} &= \pi_{t-1} + \Delta\pi_t + \sum_{i=1}^{3} \Delta\pi_{t+i|t} \\
&= \pi_{t-1} + \mathbf{1}'_2\left[Y_t + \sum_{i=1}^{3} Y_{t+i|t}\right] \\
&= \pi_{t-1} + \mathbf{1}'_2\left[4\mu + H\tilde{x}_t + e_t + H(F + FF + FFF)\tilde{x}_{t|t}\right] \\
&= \pi_{t-1} + \mathbf{1}'_2\Big[4\mu + H((\tilde{x}_t - \tilde{x}_{t|t-1}) - (\tilde{x}_{t|t} - \tilde{x}_{t|t-1})) \\
&\quad + e_t + H(I + F + FF + FFF)\tilde{x}_{t|t}\Big] \\
&= \pi_{t-1} + \mathbf{1}'_2\Big[4\mu + H(\tilde{x}_t - \tilde{x}_{t|t-1}) - HK_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t) \\
&\quad + e_t + H(I + F + FF + FFF)(\tilde{x}_{t|t-1} + K_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t))\Big] \qquad \text{(D22)} \\
&= \pi_{t-1} + \mathbf{1}'_2\Big[4\mu + H(I - K_{t|t-1}H + (I + F + FF + FFF)K_{t|t-1}H)(\tilde{x}_t - \tilde{x}_{t|t-1}) \\
&\quad + (I - HK_{t|t-1} + H(I + F + FF + FFF)K_{t|t-1})e_t + H(I + F + FF + FFF)\tilde{x}_{t|t-1}\Big],
\end{aligned}
$$

and $(\pi_{t+3|t} - \pi_{t+3|t-1})$ is shown in (C5).

$$
\begin{aligned}
z_{t+3|t} &= \mathbf{1}_1'\tilde{x}_{t+3|t} \\
&= \mathbf{1}_1' FFF\tilde{x}_{t|t} \\
&= \mathbf{1}_1' FFF(\tilde{x}_{t|t-1} + K_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t)) && \text{(D23)} \\
&= \mathbf{1}_1' \Big[FFFK_{t|t-1}H(\tilde{x}_t - \tilde{x}_{t|t-1}) + FFFK_{t|t-1}e_t + FFF\tilde{x}_{t|t-1} \Big], && \text{(D24)}
\end{aligned}
$$

and $(z_{t+3|t} - z_{t+3|t-1})$ is shown in (C8).

Hence,

$$
\begin{aligned}
p_{\pi,i,t+2} &= \mathbf{E}\left[(\pi_{t+4|t+1} - \pi_{t+4|t-1})(i_{t+1} - i_{t+1|t})|\Omega_t\right] \\
&= \mathbf{1}_2' \left[A_{2,\tilde{x}} P_{\tilde{x},t|t-1} A_{1,\tilde{x}}' \beta_{\pi_{t|t}} + A_{2,e_t}\Sigma_Y A_{1,e_t}' \beta_{\pi_{t|t}} \right] \mathbf{1}_2, && \text{(D25)} \\
&\quad + \mathbf{1}_2' \left[A_{2,\tilde{x}} P_{\tilde{x},t|t-1} B_{1,\tilde{x}}' \beta_{z_{t|t}} + A_{2,e_t}\Sigma_Y B_{1,e_t}' \beta_{z_{t|t}} \right] \mathbf{1}_1.
\end{aligned}
$$

and

$$
\begin{aligned}
p_{i,z,t+2} &= \mathbf{E}\left[(z_{t+4|t+1} - z_{t+4|t-1})(i_{t+1} - i_{t+1|t})|\Omega_t\right] \\
&= \mathbf{1}_1' \left[B_{2,\tilde{x}} P_{\tilde{x},t|t-1} A_{1,\tilde{x}}' \beta_{\pi_{t|t}} + B_{2,e_t}\Sigma_Y A_{1,e_t}' \beta_{\pi_{t|t}} \right] \mathbf{1}_2, && \text{(D26)} \\
&\quad + \mathbf{1}_1' \left[B_{2,\tilde{x}} P_{\tilde{x},t|t-1} B_{1,\tilde{x}}' \beta_{z_{t|t}} + B_{2,e_t}\Sigma_Y B_{1,e_t}' \beta_{z_{t|t}} \right] \mathbf{1}_1.
\end{aligned}
$$

Finally, $p_{i,i} = \mathbf{E}\left[(i_{t+1|t} - \hat{i}_{t+1|t-1})^2|\Omega_t\right]$ is known from the one-step-ahead forecast uncertainty.

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Figure 1: Output gap estimates from historical and real time data

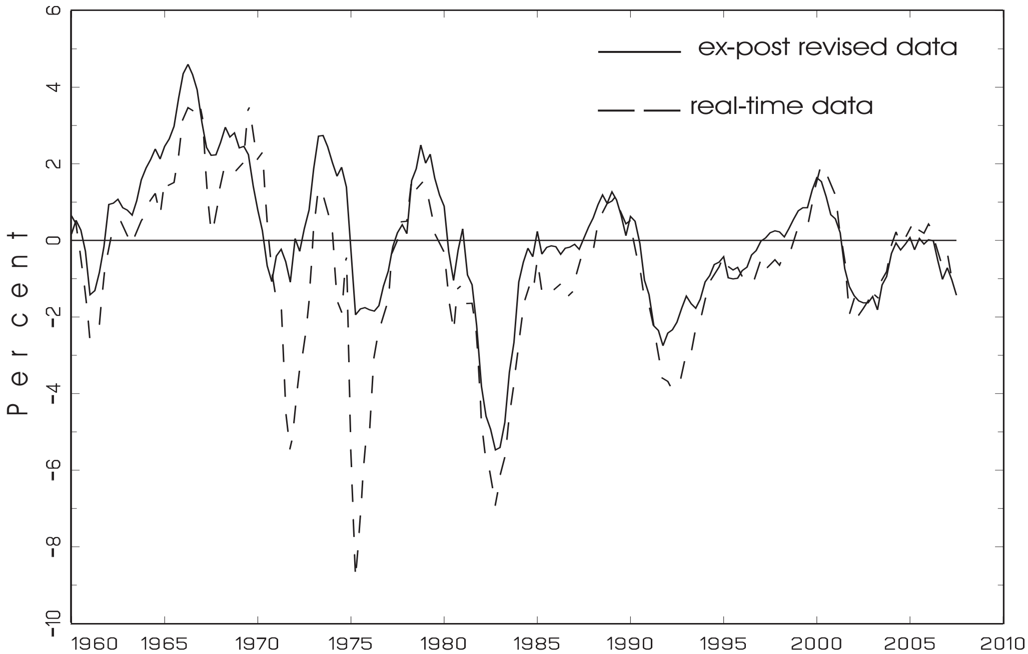

Figure 2: Output gap estimates from different vintages of real time data

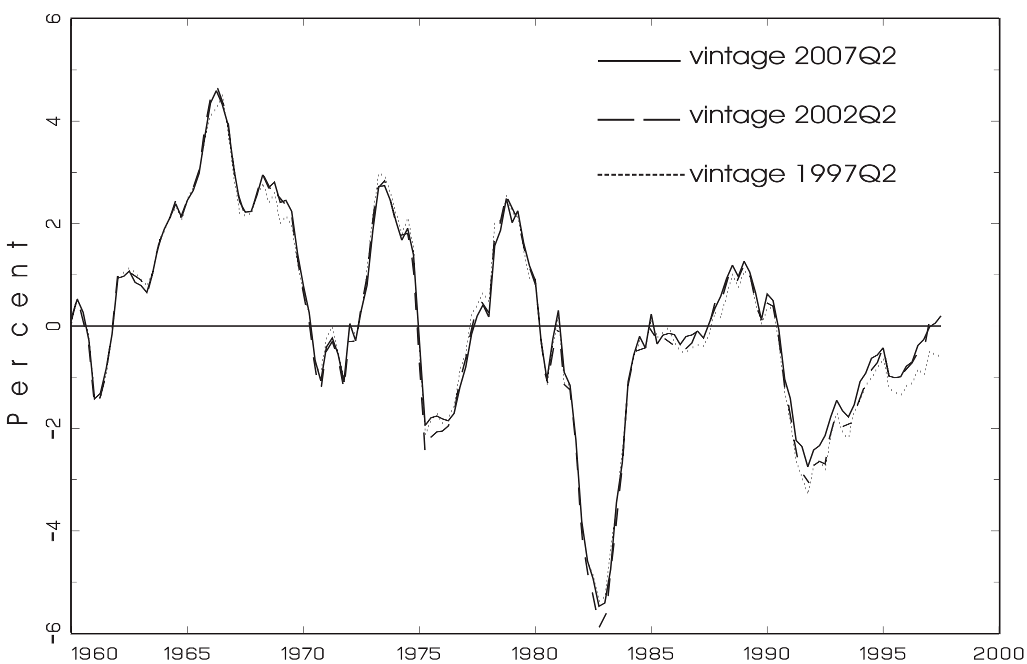

Figure 3: Actual inflation and real-time inflation forecasts

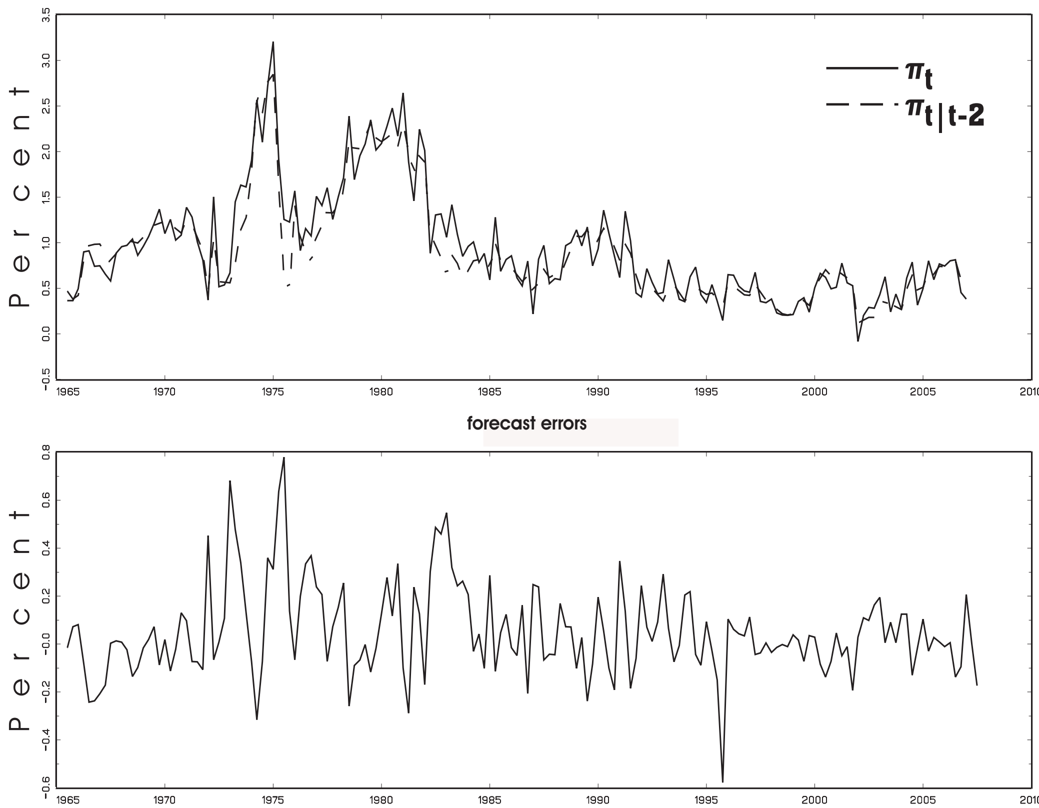

Figure 4: Real-time estimates of output-inflation equation coefficients

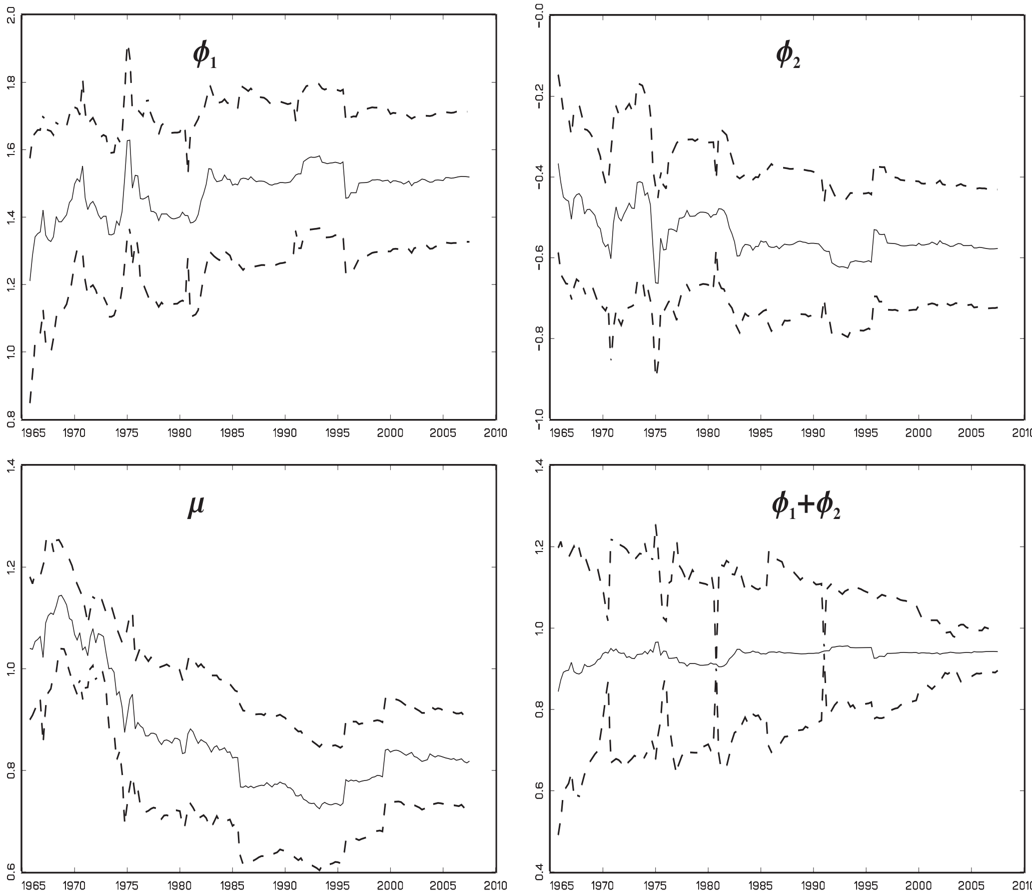

Figure 5: Real-time estimates of γ

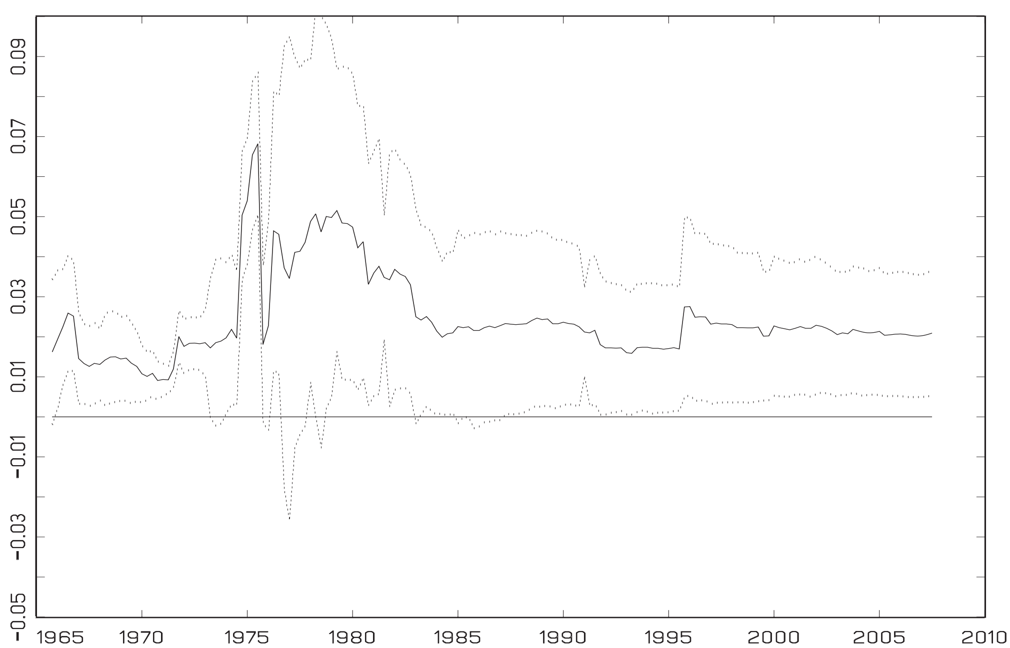

Figure 6: One-sided coefficient estimates

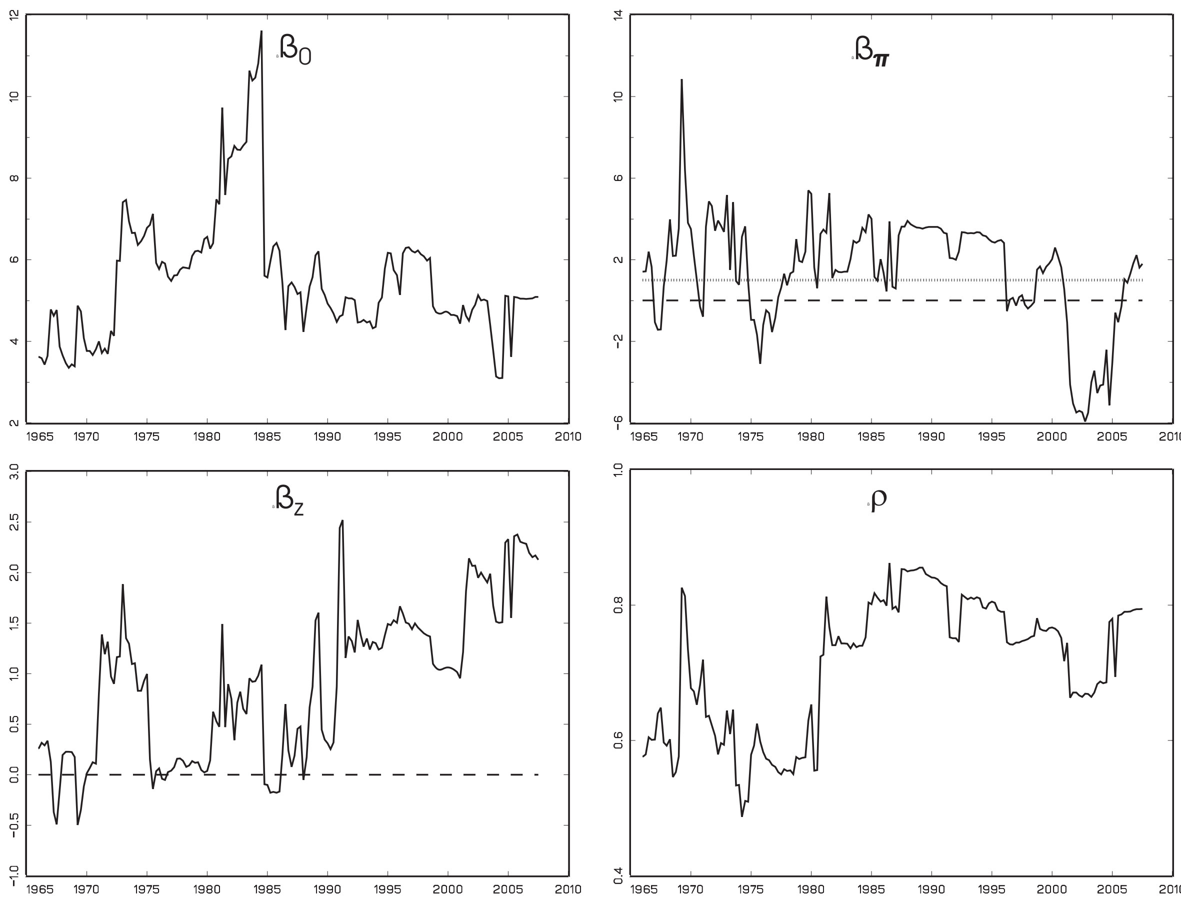

Figure 7: Real-time Federal Funds rate forecasts and forecast errors

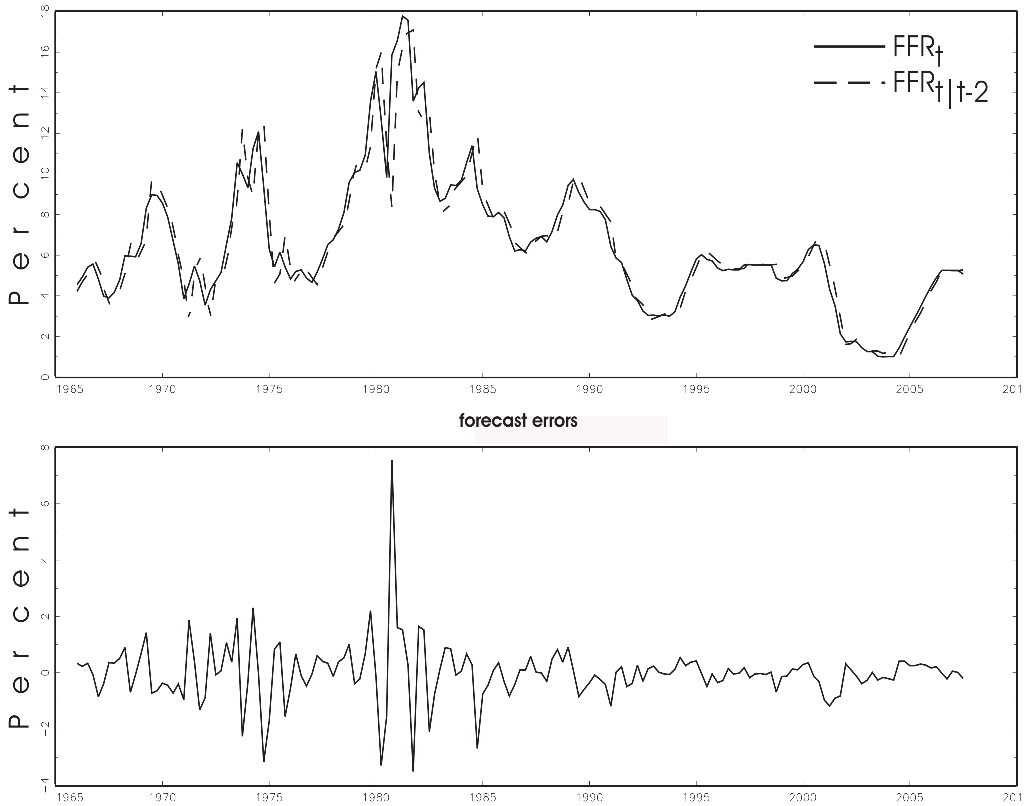

Figure 8: One-quarter ahead forecast uncertainty for Federal Funds Rate

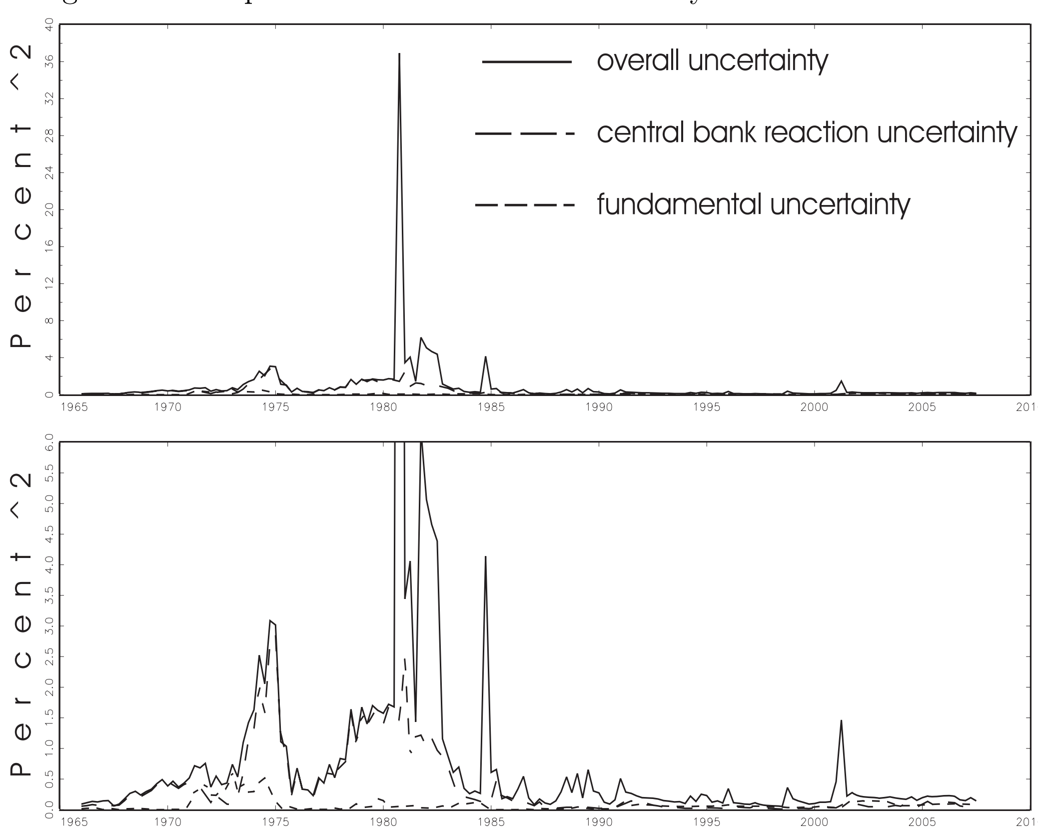

Figure 9: One-quarter ahead residual forecast uncertainty for Federal Funds Rate

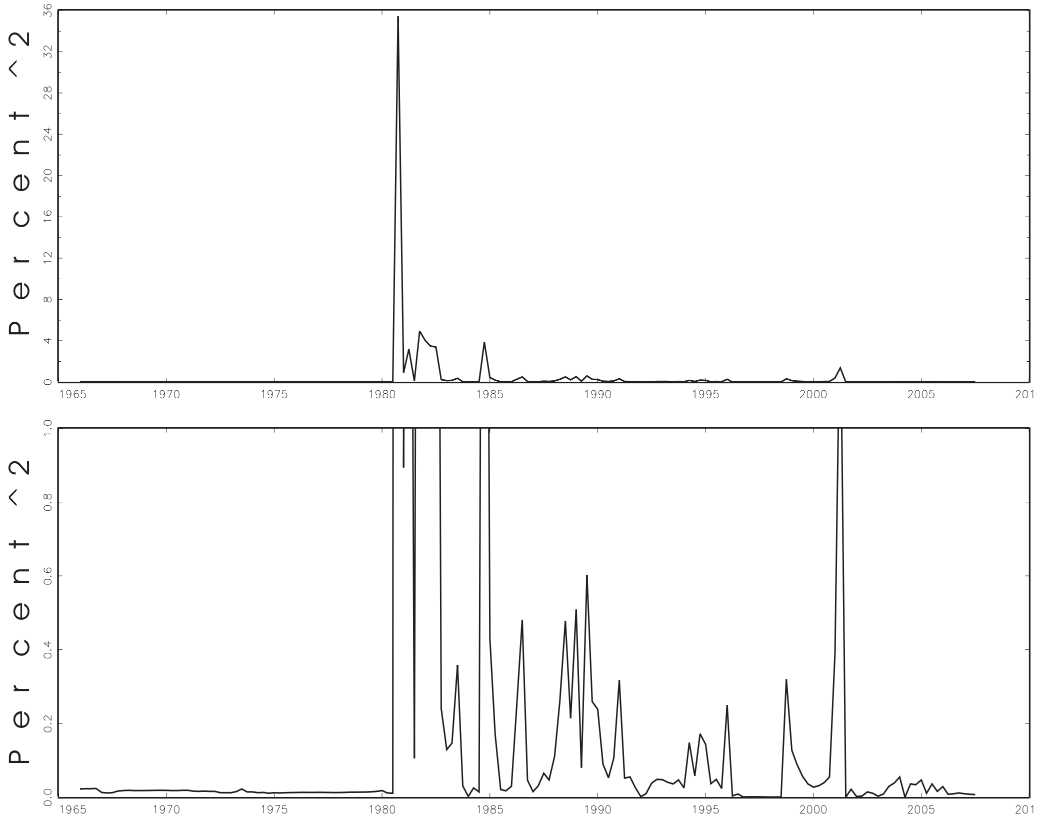

Figure 10: One-quarter ahead forecast uncertainty for Federal Funds Rate (homoskedastic errors)

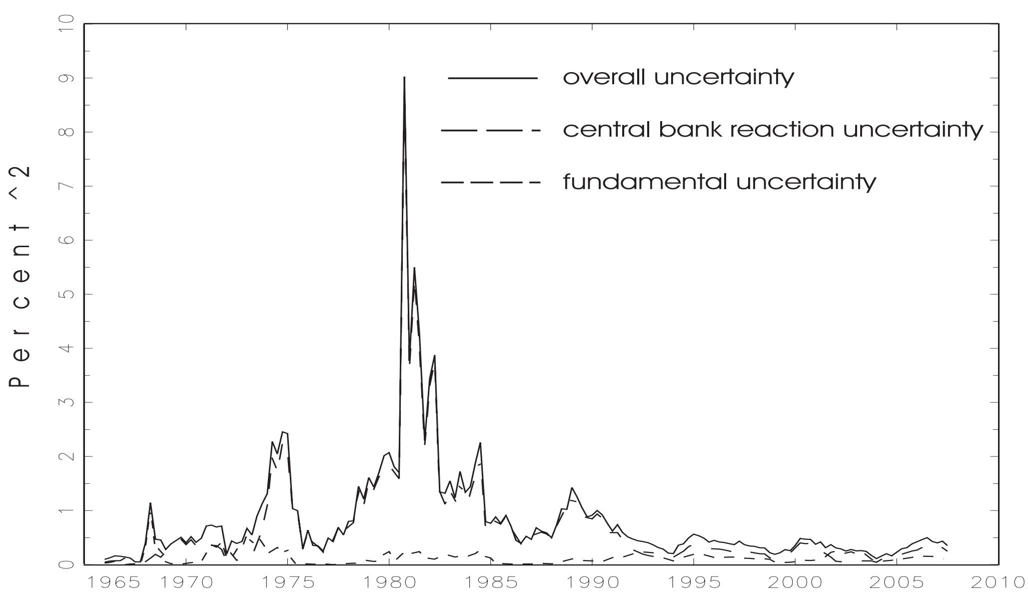

Figure 11: One-quarter ahead forecast uncertainty for Federal Funds Rate (homoskedastic vs. heteroskedastic errors)

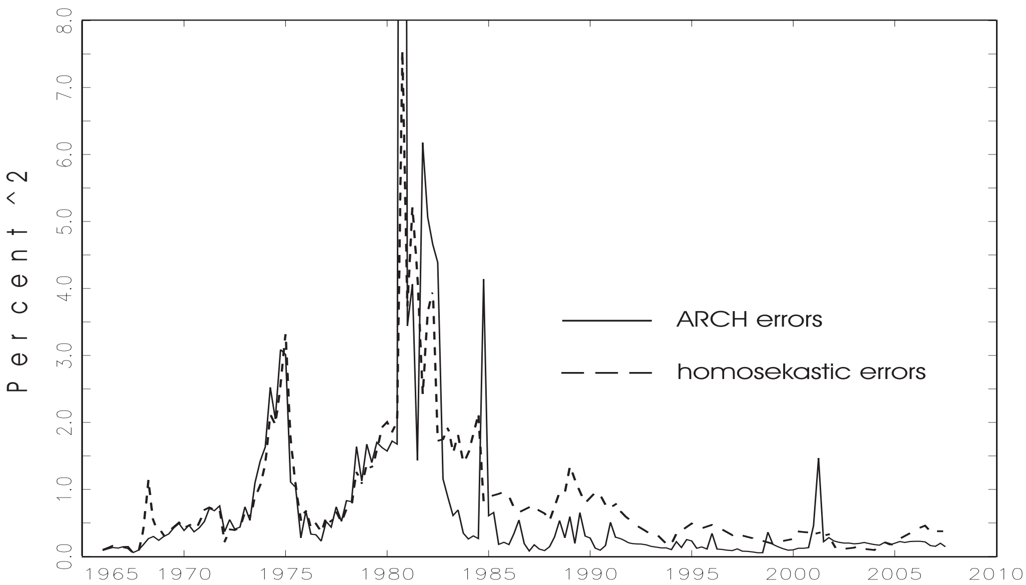

Figure 12: Two-quarter ahead residual forecast uncertainty for Federal Funds Rate

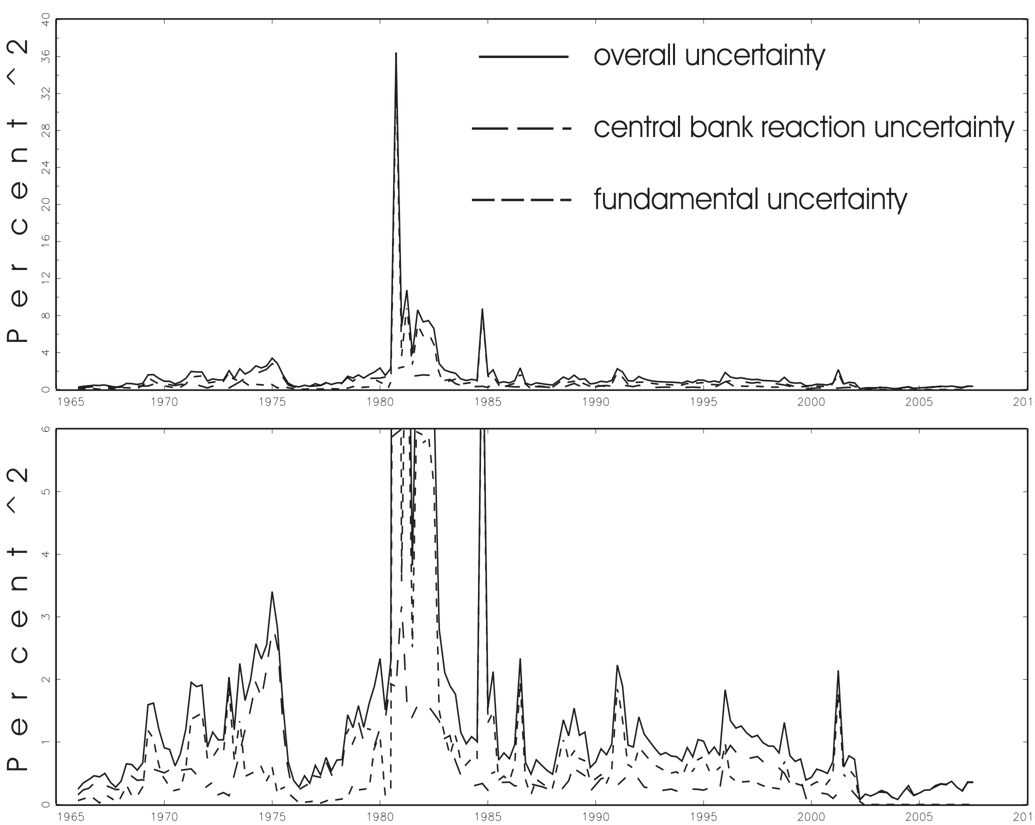

Figure 13: Two-quarter ahead residual forecast uncertainty for Federal Funds Rate