Ambiguity, Infra-Marginal Investors, and Market Prices

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Abstract

It is difficult to explain the price insensitive or infra-marginal behavior, an example of which is the behavior of credit markets during the recent financial crisis, by risk aversion alone. It is known that infra-marginal behavior may arise with ambiguity aversion. Furthermore, there appears to be fairly strong evidence of a close connection between ambiguity and conformity. Here we propose an extension of the standard ambiguity framework to incorporate conformity. We find that there are open sets of state-price ratios over which the entire market is price insensitive or infra-marginal. This result has important implications for market equilibrium and volatility.

JEL Classification: G1, D8

Key Words: *Ambiguity, Infra-Marginal Behavior, Arrow Securities*

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A salient feature of the 2008 global financial crisis is freezing up of the credit markets. The problem is not the price of credit; rather, it's the lack of availability of credit that caused the crisis implying that at least a large segment of the credit market became infra-marginal or price insensitive. That is, changes in the price of credit did not induce participation. It is well known that with expected utility maximization, infra-marginality is a knife-edge condition and there are no open sets of state price ratios in which agents remain infra-marginal or price insensitive. It seems that expected utility maximization may not be sufficient to understand infra-marginal behavior.

As argued by Knight (1921), Keynes (1936) and others, and as motivated by the famous Ellsberg paradox in Ellsberg (1961), agents appear to distinguish between risk (outcomes with known probabilities) and ambiguity (outcomes with unknown probabilities). In particular, Ellsberg (1961) and later experimental replications clearly demonstrate that agents display aversion to ambiguity.

Due to strong experimental and anecdotal evidence of ambiguity aversion, economics literature has started to explore its implications recently. Notable contributions are Epstein and Wang (1994), Maenhout (2000), Cagetti, Hansen, Sargent and Williams (2002), Uppal and Wang (2003), Skiadas (2005), Trojani, Leippold, and Vanini (2005), and Bossaerts, Ghirardato, Guarneschelli, and Zame (2007). In particular, Bossaerts et al (2007) use the "$\alpha - \max\min$" model of Ghirardato, Maccheroni, and Marinacci (2004) to show that infra-marginal responses with interesting implications arise with ambiguity aversion.

In contrast, psychology literature offers undeniable evidence of a strong link between ambiguity and conformity. There is clear evidence that when individuals do not know how to respond, they look to other people, observe how they behave, and mimic their behavior. Notable examples from this literature are Sherif (1936), Deutsch and Gerard (1955), Asch (1956), Allen and Wilder (1977), Bandura (1986), Turner (1991), and Cialdini and Trost (1998).

How do we reconcile ambiguity with conformity? More importantly, how does the interaction between ambiguity and conformity affect infra-marginality? What are the implications for market equilibrium? In this paper, we make an initial tentative attempt at providing answers to these questions. We accomplish this by extending the $\alpha - \max\min$ framework of Ghirardato et al (2004) to incorporate conformity. Our main finding is that in a market comprising of ambiguity averse agents of all types and of ambiguity neutral agents, there are open sets of state price ratios in which the entire market is infra-marginal or price insensitive. This is in contrast with Bossaerts et al (2007) where, for state-price ratios different than one, there are always some ambiguity averse agents who remain marginal.

The above finding has stark implications. If aggregate wealth differs across states then there are open sets of state price ratios that cannot support any market equilibrium with ambiguity averse agents. In order to find equilibrium, prices must jump over such intervals. Implications for volatility are immediately seen.

The Model

Consider a market over one time period marked by two points in time; time 0 and time1. All trade takes place at time 0 and all consumption takes place at time 1. There is only one consumption good. There are three states of nature; state 1, state 2, and state 3. Any one of these states may be realized at time 1 so uncertainty is only about time 1. At time 0, each agent is endowed with a risk-less security and Arrow securities. The Arrow securities are of three types corresponding to the three states; type 1, type 2, and type 3. They pay exactly one unit of consumption good in corresponding states at time 1 and nothing otherwise. That is, type 1 pays 1 unit if state 1 is realized and nothing otherwise. Similarly, types 2 and 3 pay 1 unit if states 2 and 3 are realized respectively and nothing otherwise.

Denote the price vector of Arrow securities by $p = (p_1, p_2, p_3)$. Normalize so that the price of the risk-less security is 1. It follows from the no arbitrage condition that

$p_1 + p_2 + p_3 = 1$. Since it's a three-state, three security model, we can conveniently take the market to be complete (an Arrow-Debreu market which is complete), so the risk-less asset is redundant.

Here we consider three different frameworks for individual choice in the setting described above; the expected utility framework, the ambiguity attitudes framework of Ghirardato et al (2004), and an extension of Ghirardato et al (2004) framework to incorporate conformity.

The most familiar individual choice framework is expected utility maximization. In that framework, given the state probabilities $\pi_1, \pi_2, 1-\pi_1-\pi_2$, and the budget constraint, each individual makes demand choices so as to maximize the expected utility of wealth given by

$$U(w) = \pi_1 U(w_1) + \pi_2 U(w_2) + (1-\pi_1-\pi_2)U(w_3) \tag{1}$$

where $U(.)$ is assumed to be twice differentiable, strictly increasing, and strictly concave.

In contrast, Ghirardato et al (2004) propose a framework for ambiguity attitudes. Suppose the probability of state 1 is known but the probabilities of states 2 and 3 are unknown. That is, agents face ambiguity regarding states 2 and 3. To deal with such a situation, Ghirardato et al (2004) propose the following framework. With $U(.)$ being twice differentiable, strictly increasing, and strictly concave, an agent maximizes,

$$\begin{aligned} U(w) = \pi_1 U(w_1) + \alpha \min_{\beta \in [0, (1-\pi_1)]} [\beta U(w_2) + (1-\pi_1-\beta)U(w_3)] \\ (1-\alpha) \max_{\gamma \in [0, (1-\pi_1)]} [\gamma U(w_2) + (1-\pi_1-\gamma)U(w_3)] \end{aligned} \tag{2}$$

An agent's pessimistic scenario is captured by the value of parameter β that minimizes expected utility over ambiguous states and his optimistic scenario is determined by the value of γ that maximizes his expected utility over the ambiguous states. The parameter $\alpha \in [0,1]$ captures ambiguity attitudes. Extreme ambiguity aversion is captured by $\alpha = 1$, extreme ambiguity loving is captured by $\alpha = 0$, and ambiguity neutrality is captured by $\alpha = 0.5$. Bossaerts et al (2007) study the implications of this framework for infra-marginality. They show that there is an open set of state price ratios over which an agent is infra-marginal and prefers to hold an unambiguous portfolio.

As mentioned in the introduction, there appears to be a strong link between ambiguity and conformity. A large body of psychology literature confirms this link (see introduction for references). To our knowledge, this apparent link between ambiguity and conformity has not been modeled in economic literature. Here we take an initial tentative step in this direction and explore the implications of this link for economic behavior. We accomplish this by extending the Ghirardato et al (2004) framework as follows,

$$\begin{aligned} U(w) = \pi_1 U(w_1) + c\{\alpha \min_{\beta \in [0,(1-\pi_1)]} [\beta U(w_2) + (1-\pi_1-\beta)U(w_3)] + \\ (1-\alpha)\max_{\gamma \in [0,(1-\pi_1)]}[\gamma U(w_2) + (1-\pi_1-\gamma)U(w_3)]\} + \\ (1-c)\{\eta \min_{\beta \in [0,1-\pi_1]}[\beta U(w_2) + (1-\pi_1-\beta)U(w_3)] + \\ (1-\eta)\max_{\gamma \in [0,1-\pi_1]}[\gamma U(w_2) + (1-\pi_1-\gamma)U(w_3)]\} \end{aligned} \tag{3}$$

$U(.)$ is assumed to be twice differentiable, strictly increasing, and strictly concave. (3) has two additional parameters when compared with (2); $c \in [0,1]$ and $\eta \in [0,1]$. They capture an agent's confidence in his own judgment and average ambiguity attitude in the environment respectively. The idea is that an agent is a social being and is connected to other agents through the media or otherwise and gets an idea about their attitudes regarding ambiguity and consequently gets influenced. An agent may not even be consciously aware of these influences since such influences may operate subconsciously or implicitly. Different types of implicit environmental influences have been extensively documented in the psychology literature. See Forgas and Moylan (1987), Wells and Petty (1980), and Steele and Aronson (1995).

When $c = 1$, the framework in (3) reduces to the Ghiraradato et al (2004) framework and agent displays full confidence in his own judgment and does not conform. An agent is maximally conforming when $c = 0$. When $c = 0.50$, an agent gives equal weights to his own ambiguity attitude and the average ambiguity attitude in his environment. This framework is general enough to potentially capture a rich class of market behaviors, involving interplay between ambiguity and conformity. In this framework, verifiable implications of the psychology assertion that higher the perception of ambiguity-stronger is the need to conform, can be derived and experimentally tested as negative correlations between α *and* c .

The extended framework leads to a number of interesting results, which are discussed next.

Proposition 1 ***In an ambiguity averse environment, there is a non-empty set of state price ratios over which an agent, who is less ambiguity averse than the environment, prefers not to be exposed to ambiguity, provided his confidence falls below a certain threshold.***

Proof. If $w_2 > w_3$, the minimum and maximum conditions in (3) are satisfied when $\beta = 0$ and $\gamma = 1 - \pi_1$ respectively. So (3) becomes,

$$U(w) = \{c\alpha + (1-c)\eta\}(1-\pi_1)U(w_3) + \{c(1-\alpha) + (1-c)(1-\eta)\}(1-\pi_1)U(w_2) \quad (4)$$

The first order condition of optimality leads to,

$$\frac{\{c(1-\alpha) + (1-c)(1-\eta)\}U'(w_2)}{p_2} = \frac{\{c\alpha + (1-c)\eta\}U'(w_3)}{p_3} \quad (5)$$

Since, $U(.)$ is strictly concave,

$$\frac{p_2}{p_3} < \frac{c(1-\alpha) + (1-c)(1-\eta)}{c\alpha + (1-c)\eta} \qquad \text{if } w_2 > w_3 \quad (6)$$

Similarly, if $w_3 > w_2$, we obtain,

$$\frac{p_2}{p_3} > \frac{c\alpha + (1-c)\eta}{c(1-\alpha) + (1-c)(1-\eta)} \quad (7)$$

It follows,

$$w_2 = w_3 \Leftrightarrow \frac{c(1-\alpha)+(1-c)(1-\eta)}{c\alpha+(1-c)\eta} \le \frac{p_2}{p_3} \le \frac{c\alpha+(1-c)\eta}{c(1-\alpha)+(1-c)(1-\eta)} \qquad (8)$$

That is, if the price ratio is in the range defined by (8), then the agent prefers not be exposed to ambiguity (holds an unambiguous portfolio). The agent is infra-marginal or price insensitive over that range.

Given that the environment is ambiguity averse ($\eta > 0.50$), and the agent is less ambiguity averse than the environment ($\eta - \alpha > 0$), (8) is a non-empty set if

$$c < \frac{\eta - 0.50}{\eta - \alpha}$$

■

<u>Corollary 1</u> *If an agent is ambiguity neutral but the environment is ambiguity averse then there is always a non-empty set of state price ratios over which he prefers no to be exposed to ambiguity.*

<u>Corollary 2</u> *Even if an agent is extremely ambiguity loving, there is a threshold confidence level below which he prefers not to be exposed to ambiguity.*

Proposition 2 ***In a market where all types of ambiguity averse attitudes (including ambiguity neutrality) are uniformly present, there is a non-empty set of state price ratios over which the entire market is infra-marginal.***

Proof. Here $\alpha \in [0.5,1]$ and $\eta = 0.75$. Let L denote the lower limit of the set of state price ratios over which an agent of ambiguity attitude α is infra-marginal and U denote the upper limit of the set.

It is easy to see that $\frac{\partial L}{\partial \alpha} < 0$ & $\frac{\partial U}{\partial \alpha} > 0$, it follows that the highest value of L and the lowest value of U occur at $\alpha = 0.50$.

It follows that the entire market is infra-marginal over the following price range,

$$\frac{c \times 0.50 + (1-c) \times 0.25}{c \times 0.50 + (1-c) \times 0.75} \leq \frac{p_2}{p_3} \leq \frac{c \times 0.50 + (1-c) \times 0.75}{c \times 0.50 + (1-c) \times 0.25} \tag{9}$$

■

Corollary 1 Even if the market includes a significant fraction of ambiguity loving agents, apart from ambiguity averse and ambiguity neutral agents, there exists a non-empty set of state price ratios over which the entire market is infra-marginal.

Remark If there is no conformity, that is, only ambiguity is present, as in Ghirardato et al (2004) framework, marginal agents always exist for any state price ratio different than 1. Hence, the possibility of entire market being infra-marginal arises only due to the interaction between ambiguity and conformity.

Proposition 3 ***If aggregate wealths over ambiguous states are unequal, and all types of ambiguity averse attitudes (including ambiguity neutrality) are uniformly present in the market, there exists a non-empty set of state price ratios that cannot support any market equilibrium.***

Proof. It follows from proposition 2 that the entire market is infra-marginal for the set of prices defined by (9). Market clearing requires that aggregate demands of state securities equal their respective aggregate supplies. However, infra-marginal investors demand equal wealths in ambiguous states. Consequently, markets cannot clear for the set of state price ratios defined by (9) if aggregate wealths are different across ambiguous states.

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Conclusion

We propose an extension of the framework in Ghirardato et al (2004) to incorporate conformity. The implications are explored for a three state – three security complete market world. We find that there is a non-empty set of state price ratios for which the entire market, composed of ambiguity averse and ambiguity neutral agents, becomes infra-marginal or price insensitive. Hence, if aggregate wealths differ across states, then such a set cannot support market equilibrium.

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