Acceptable Risk in a Portfolio Analysis

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ACCEPTABLE RISK IN A PORTFOLIO ANALYSIS

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ABSTRACT
A social network has been used to simulate how agents of different levels of risk aversion under different circumstances behave in financial markets when deciding between risk-free and a risky asset. This is done by a discrete time version evolutionary game of risk-loving and risk-averse agents. The evolutionary process takes place on a social network through which investors acquire information they need to choose the strategy. A significant feature of the paper is that first-order stochastic dominance is a key determinant of the decision-making, while second-order stochastic dominance is not, with the level of omniscience and preferences of agents also having a significant role. Under most of the circumstances, pure risk-aversion turns out to be dominated strategy, while pure risk-taking "almost" dominant.

JEL Classification: C73, D85, G11
Keywords: social networks, stochastic dominance, evolutionary finance, portfolio analysis.

1 Introduction

Uncertainty determines the central problem in a portfolio analysis and force agents to decide. Such decision-making process can generally be defined as a two-stage process. It starts first with "*observation and experience, and ends with beliefs about the future performance of available securities. The second stage starts with the relevant beliefs about the future performances and ends with the choice of portfolio*" as set by Markowitz (1952, 77). The first stage can be explained through very complex processes that determine bids and asks. They both are done by many heterogeneous interacting agents with different preferences, knowledge (which does not exist in a concentrated form but is dispersed (Hayek 1945)), experiences, endowment, social networks, perceptions, reasoning abilities and abilities to learn, preferences towards risk and time valuations, expectations (Kahneman and Tversky 1979) and other agent-specific characteristics.[1] Agents on markets "*continually adjust their market moves, buying decisions, prices, and forecasts to the situation these moves or decisions or prices or forecasts together create…*" as Arthur (2006, 1554) argues. we do not tackle the very important first stage as it goes far beyond the feasible in the current science, but similar to Tversky and Kahneman (1974) assume that agents use heuristics, and focus on the second stage only.

The most important element of the decision-making is action. Action is a purposeful behavior and a will put into operation, by which agents satisfy their goals by employing their means (Mises 1949). In the paper, agents make actions using a social network (Watts and Strogatz 1998; Wasserman and Faust 1994). In a social network, agents make ties with their relatives, friends and colleagues, and use those ties to acquire information and experience they need to guide their decisions. In such framework, agents do not only learn of their past behavior, but also from the past behavior of others. Because of this feature, social networks usually imply herd behavior, with strategies that bring higher payoffs being dominant (Banerjee 1992; 1993; Lux 1995). We try to overcome this by introducing two kinds of agents differing to their levels of omniscience. Omniscient agents always opt for that strategy which is expected to bring them higher payoff. Non-omniscient agents follow Rubinstein's tradeoff between the optimality and complexity of the strategy (Rubinstein 1998) and respond to the circumstances by using their intuition (Kahneman 2003). A paper is related to agent-based behavioral finance where agents trust more and compare their wealth to other agents they know (Granovetter 2005), thus sharing information about the efficiency of different investment activities. For an individual, it is important how close to the agents with "the right" information he is, determining the time the one is able to get information. Closer to important agents one is, faster can such individual adapt to changing circumstances.

In the paper, agents decide between two securities of different riskiness and can opt either to make the portfolio of one of the two available, or a portfolio of both of them. By introducing expected return and the variance into the simulations, we tackle the decision-making under the risk and test for the effects of small perturbations.

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[1] Hirshleifer (2002) offers a brief discussion on psychology and decision-making.

The rest of the paper is organized as follows. Section 2 gives a short introduction of the social network used in simulations and develops simulation games. Section 3 gives results and Section 4 concludes.

2 The model

2.1 Social network

The network $G = \{g_{i,j} \mid i, j \in N\}$ is populated with a finite set of $N = \{1,2,...,n\}$ agents, denoted with nodes, connected with each other with a finite set of $E = \{1,2,3,\ldots,l\}$ links. Let us denote $g_{i,j} = 1$ if agents i and j are directly connected and $g_{i,j} = 0$ if they are not. The network is undirected if it satisfies the following condition: $g_{i,j} = 1 \Leftrightarrow g_{j,i} = 1$ and is directed otherwise. I assume undirected networks where two agents are either connected or not. No loops are possible in the networks, thus $g_{i,i} \neq 1$ for all $i \in N$. A network is directed if there is a path between all pairs of nodes in the network, while it is complete if all nodes are connected. The diameter of the network denoted the maximum length of shortest paths over all pairs of nodes in the network. The degree of a node is the number of nodes to which a node is directly connected.

In the game, we have a discrete time interval $t = 1,2,3,\ldots,T$. There are $n = 1.000$ agents in the game, distributed on the ring lattice and connected according to the small world principle. u represents a share of agents that are risk-averse and the rest $(1-u)$ are risk-loving. Let us denote the first A_A and the later A_D. The average degree of agents in the network equals $k_i(g) = 6$. The probability that a connection from an agent A_i is rewired to the randomly chosen agent A_j in the network, equals $p = 0.01$. Thus, agents have local connections to their closest neighbors, while they also have some long-range connections to distant agents, representing their global acquaintances. Once connections between agents are rewired and agents populated on the lattice, the network remains unchanged.

2.2 Assets

There are two different kinds of securities in the game, while we assume such small agents who are not able to affect the market. A return of a risk-free security is denoted Br, while Sr denotes the return of a risky security. $Br = 0.0002$ in every time period.

The return of a risky asset is defined as a stable Levy process (Mandelbrot 1963a, b; Mantenga and Stanley 1995). Stable Levy distribution requires four parameters, an index of stability $\alpha \in (0,2]$, a skewness parameter $\beta \in [-1,1]$, a scale parameter $\sigma > 0$ and a location parameter $\mu \in \Re$. Characteristic function $\varphi_X(t)$ of such distribution is stable iff (Lukacs 1970):

$$\log \varphi_X(t) = \begin{cases} i\mu t - \sigma^\alpha |t|^\alpha \left\{1 - i\beta sign(t) \tan \frac{\pi\alpha}{2}\right\}, & \alpha \neq 1 \\ i\mu t - \sigma |t| \left\{1 + i\beta sign(t) \frac{2}{\pi} \log|t|\right\}, & \alpha = 1 \end{cases} \text{ and } sign(t) = \begin{cases} -1, & t < 0 \\ 0, & t = 0 \\ 1, & t > 0 \end{cases}.$$

Chambers et al. (1976) and Weron (2001) propose an algorithm for generating stochastic variable, which proceeds as follows.

For $\alpha \neq 1$, we compute

$$X = S_{\alpha,\beta} \left[\frac{\sin\left(\alpha\left(V + B_{\alpha,\beta}\right)\right)}{\left(\cos(V)\right)^{1/\alpha}} \right] \cdot \left[\frac{\cos\left(V - \alpha\left(V + B_{\alpha,\beta}\right)\right)}{W} \right]^{(1-\alpha)/\alpha}, \quad \text{where} \quad B_{\alpha,\beta} = \frac{\arctan\left(\beta \tan \frac{\pi\alpha}{2}\right)}{\alpha}$$

and $S_{\alpha,\beta} = \left[1 + \beta^2 \tan^2 \frac{\pi\alpha}{2}\right]^{1/(2\alpha)}$. Thus, $Sr = \sigma X + \mu$.

For $\alpha = 1$, we compute

$$X = \frac{2}{\pi} \left[\left(\frac{\pi}{2} + \beta V\right) \tan V - \beta \log\left(\frac{W \cos V}{\frac{\pi}{2} + \beta V} \right) \right] \text{ and } Sr = \sigma X + \frac{2}{\pi} \beta\sigma \log \sigma + \mu.$$

In both cases, V is a random variable that is uniformly distributed on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and W is an independent exponential random variable with mean 1.

According to their initial preferences, agents can choose between pure strategy of having only risk-free or risky assets or make a portfolio of the two. Agents who decide for pure risky strategies are denoted A_S, while risk-dominant agents who opt for a portfolio are denoted A_{Sp}. Contrary, risk-averse agents who prefer risk-free securities are denoted A_B and A_{Bp} those risk-averse agents who opt for a portfolio. In either case, portfolio is selected from the part of stocks one prefers, $0 \leq pi \leq 1$, while $(1 - pi)$ represents securities of the type one does not prefer. Simulations start with $A_B = A_{Bp}$ and $A_S = A_{Sp}$, while $A_B + A_{Bp} + A_S + A_{Sp} = 1$. Agents accumulate their wealth in time according to the strategy they choose and are free to change between the strategies in every period during the game.

$$W_{t+1}\left(A_S\right)=W_t\left(A_S\right)\cdot\left[1+Sr\right]$$
$$W_{t+1}\left(A_{Sp}\right)=W_t\left(A_{Sp}\right)\cdot\left[1+Sr\cdot pi+Br\cdot\left(1-pi\right)\right]$$
$$W_{t+1}\left(A_{Bp}\right)=W_t\left(A_{Bp}\right)\cdot\left[1+Br\cdot pi+Sr\cdot\left(1-pi\right)\right] \qquad (1).$$
$$W_{t+1}\left(A_B\right)=W_t\left(A_B\right)\cdot\left[1+Br\right]$$

$W_{t+1}(\bullet)$ and $W_t(\bullet)$ represent wealth of an agent in time t and $t+1$, while $(\bullet)$ denotes the strategy played by an agent in time. Returns of securities are exogenous to agents. Agents are not able to foresee them, neither do they know the system how prices change in time. Each agent's objective is to maximize his own wealth. When sharing information of their wealth, they might help others in maximizing their wealth.

2.3 Omniscience

The important element in the game is the acquisition of knowledge, which agents adopt by information they get from others. Simulations are done using two sorts of agents, omniscient and non-omniscient, where in particular simulations all agents are of equal level of (non)omniscience. Omniscience of agents is modeled through logistic probability function (Szabo and Toke 1998), defined as

$$\wp=\left[1+\exp[(W_i-W_j)/\kappa]\right]^{-1} \qquad (2).$$

W_i and W_j represent the outcomes of agents i and j, while the coefficient $\kappa=\left[0,1\right]$ denotes a level of omniscience of agents. This means that in every period an agent i does not choose the strategy that had in the past award him with the highest return and is expected to do it in the future as well. For $\kappa=0$, an agent i chooses the strategy of an agent j if the later one brought him higher returns, i.e. $W_j>W_i$, otherwise he remains at his strategy set. This means that an agent takes every strategy from his neighbor so long as they award him with higher returns. The bigger the difference in outcomes of the two agents and the smaller the value of coefficient κ, the higher the probability that such strategy is adopted, an vice versa. There is no common knowledge among agents in the game.

If not indicated differently $\alpha=1.996$, $\beta=0.2$, $\sigma=0.42$, $\mu=0.1$ and $T=10.000$. Games are iterated forward in time, using a synchronous update scheme, which means that agents in the game simultaneously update their strategies by considering the level of their omniscience.

3 Simulation analysis

3.1 Basic framework

Omniscient agents

We first analyze how initial preferences of agents affect their decision-making when confronted to the share of risky (risk-free) securities risk-loving (risk-averse) agents can add to a portfolio using omniscient agents with $\kappa = 0.001$. Figures 1 depict the averages of 20 independent realizations of the game on the entire definition-spaces $0 \le u \le 1$, $0 \le pi \le 1$ by using a step of 0.1 units to variables u and pi. Thus, $u\_step = 0.1$ and $pi\_step = 0.1$, with $u = u + u\_step$ and $pi = pi + pi\_step$.

Color-palettes in figures through the paper depict the shares of agents who opt for a particular strategy in $T = 10.000$. Throughout the paper S represents the share of agents who opt for risky strategies, Sp represents the share of agents who prefer risky stocks but opt for a portfolio, Bp represents the share of agents who prefer risk-free securities and opt for portfolio, and B represents the share of agents who prefer risk-free securities.

Figure 1a: Omniscient agents opting for *S*

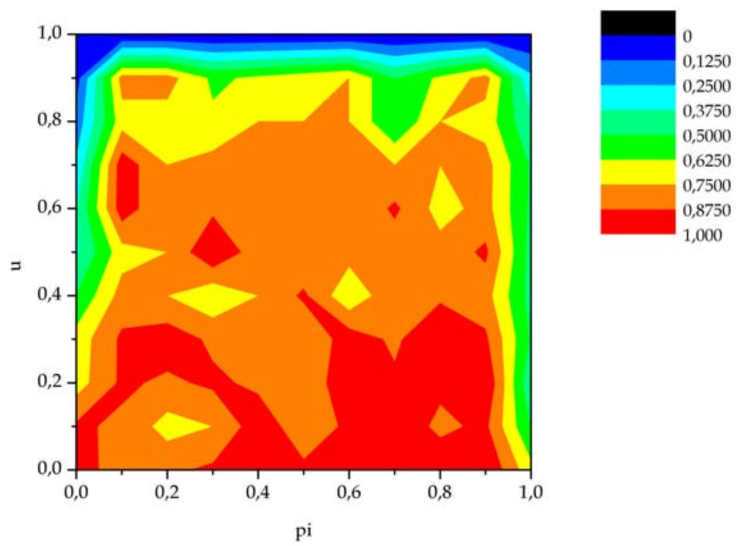


Figure 1b: Omniscient agents opting for *Sp*

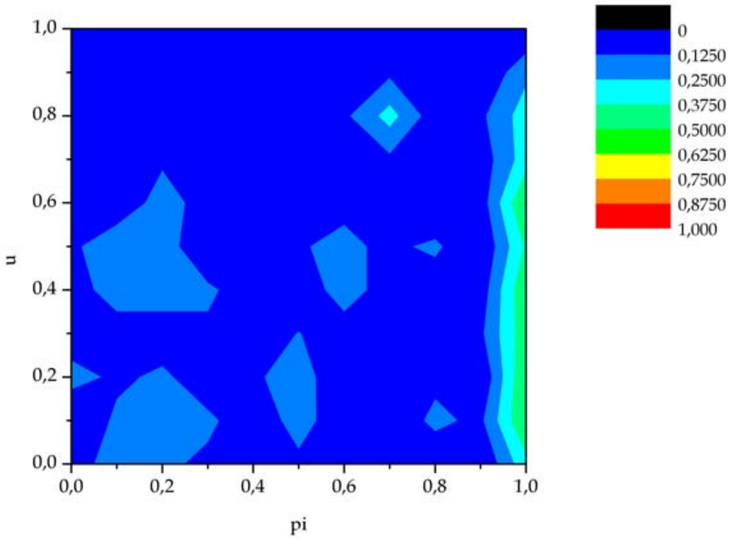


Figure 1c: Omniscient agents opting for *Bp*

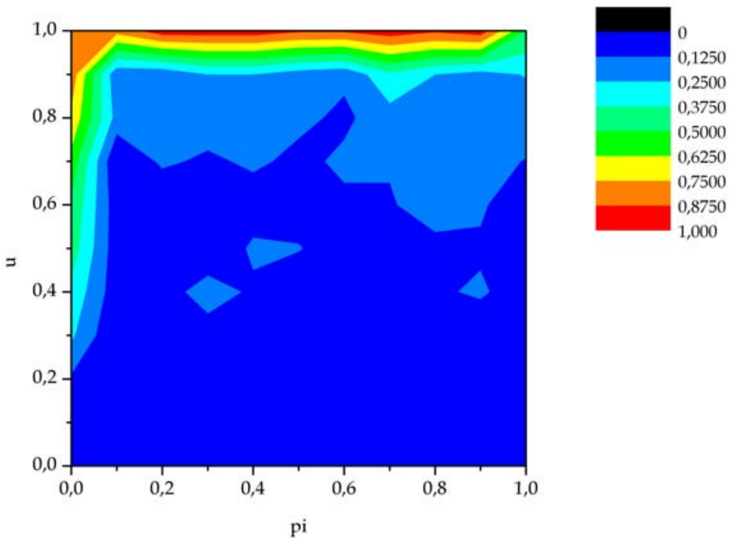
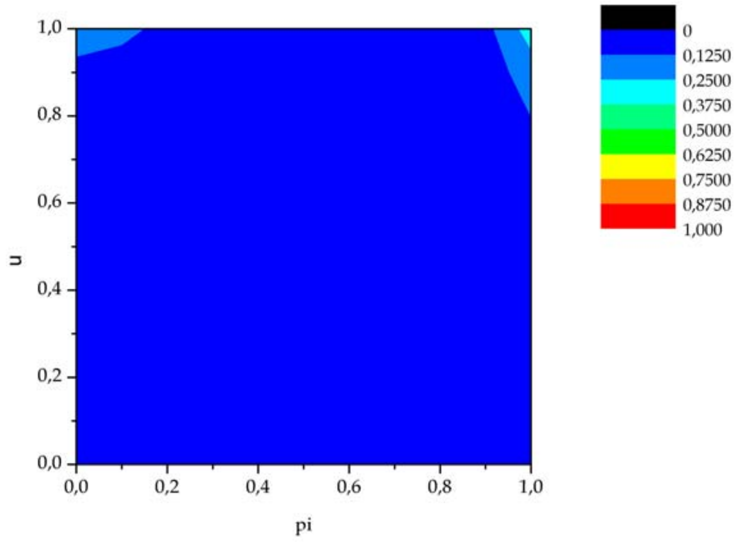


Figure 1d: Omniscient agents opting for *B*



Comparing the figures, we see that pure risk-taking is a dominant strategy used by omniscient agents on a great part of the definition space (Figure 1a), while pure risk-aversion is a dominated strategy on the "almost" entire definition space (Figure 1d).[2] Even though we start the game with risk-averse agents only, they opt for a portfolio with as much risky assets as allowed (Figure 1c).

Non-omniscient agents

We now substitute omniscient agents with non-omniscient. This is done by setting $\kappa = 1.0$. Color-palettes in Figures 2 again depict averages of 20 independent games.

Figure 2a: Non-omniscient agents opting for *S*

[2] For definitions of dominant and dominated strategies, see Fudenberg and Tirole (1994).

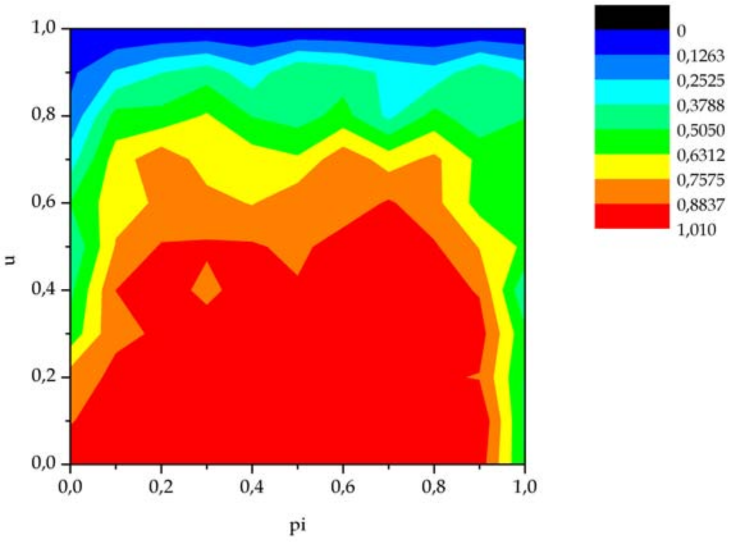


Figure 2b: Non-omniscient agents opting for *Sp*

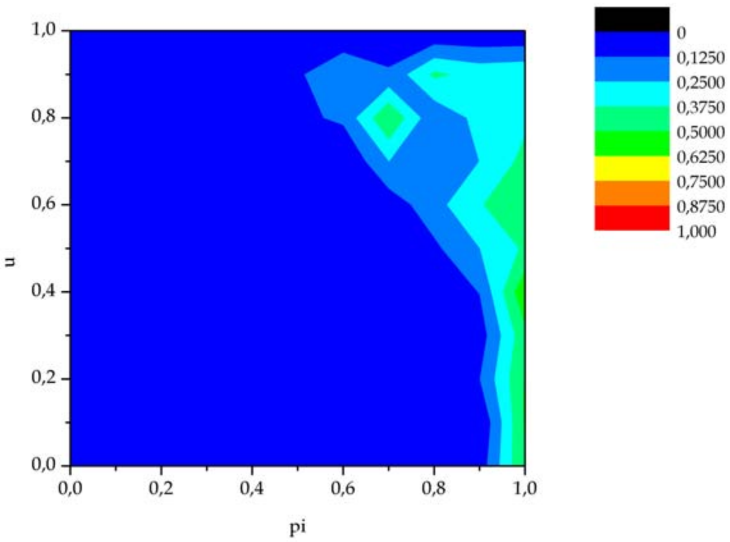


Figure 2c: Non-omniscient agents opting for *Bp*

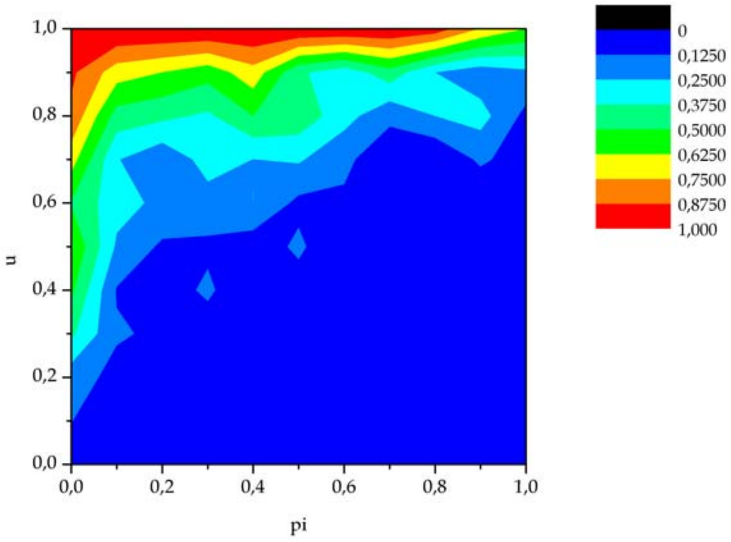
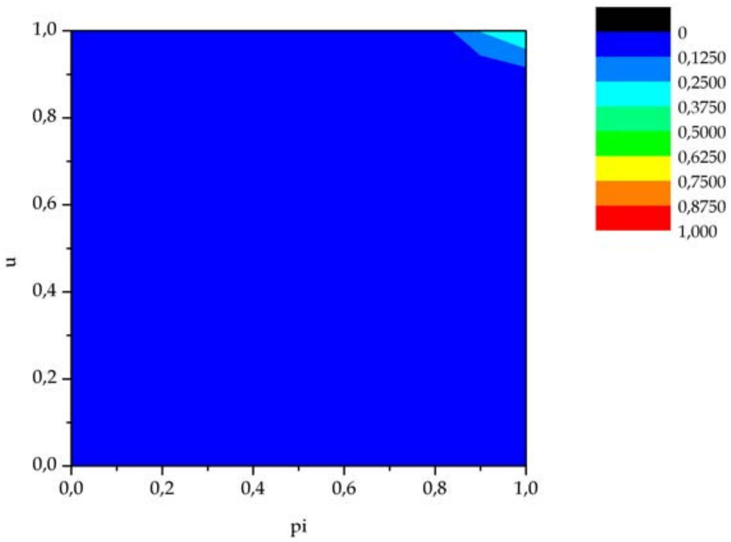


Figure 2d: Non-omniscient agents opting for *B*



Following the figures, non-omniscience does have a slight effect on strategies played by agents in the game. Despite pure risk-aversion still is a dominated strategy (Figure 2d), non-omniscient agents are much more prone to playing portfolio strategies than omniscient agents are. Pure risk-taking is no such strict dominant strategy for $u > 0.5$ (Figure 2a). However, for $0 \le u < 0.5$, it is even "more dominant" as in the case of omniscient agents. Finally, the more risk-averse agents there are in the beginning of the game, the more likely it is that all of them end the game with a portfolio (Figure 2c). This effect is especially significant for $u > 0.7$.

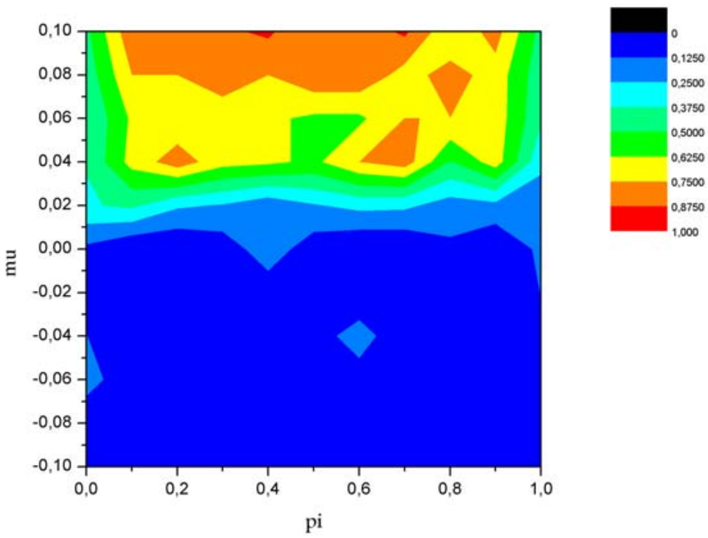
3.2 Changing expected return

In this part, we test for the validity of a first-order stochastic dominance in the model. If $F(\bullet)$ and $G(\bullet)$ represent any two distributions, $F(\bullet)$ first-order stochastically dominates $G(\bullet)$ if for every concave function, we have $E\left(u\left[F(\bullet)\right]\right) \geq E\left(u\left[G(\bullet)\right]\right)$ (Rothschild and Stiglitz 1970). We do this by analyzing the effects of small perturbations of the expected return of risky security μ (denoted *mu* in figures) and *pi* on the decision-making. Figures below depict the averages of 20 independent realizations of the game on the entire definition-space $0 \leq pi \leq 1$ by using $pi\_step = 0.1$, and on $-0.1 \leq \mu \leq 0.1$ by using $\mu\_step = 0.01$ with $\mu = \mu + \mu\_step$ and $pi = pi + pi\_step$. Simulations start with $u = 0.5$, which means $A_S = A_{Sp} = A_B = A_{Bp} = 0.25$.

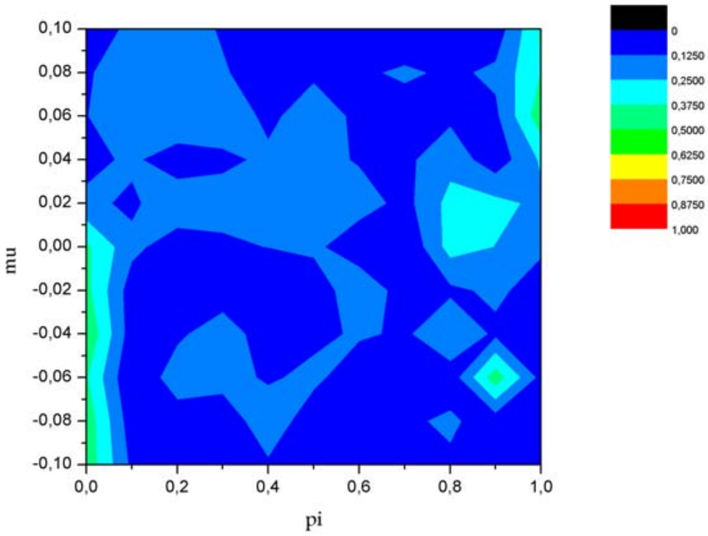
Omniscient agents

Figures 3 depict the averages of omniscient agents with $\kappa = 0.001$.

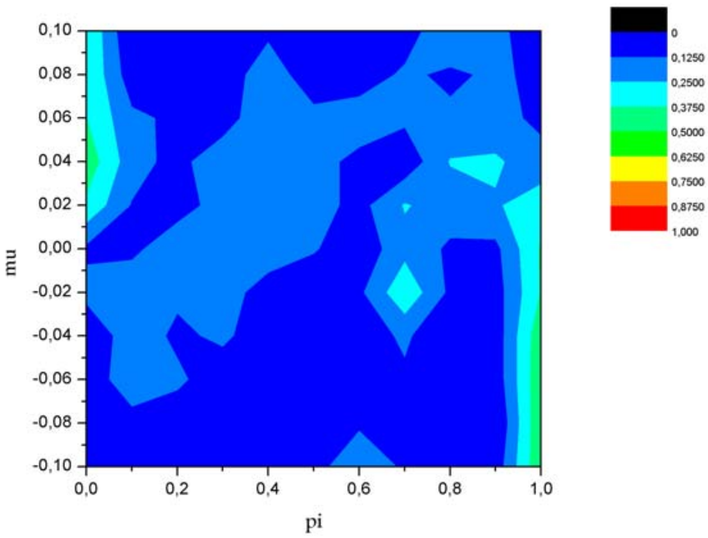
3a: Omniscient agents opting for *S*



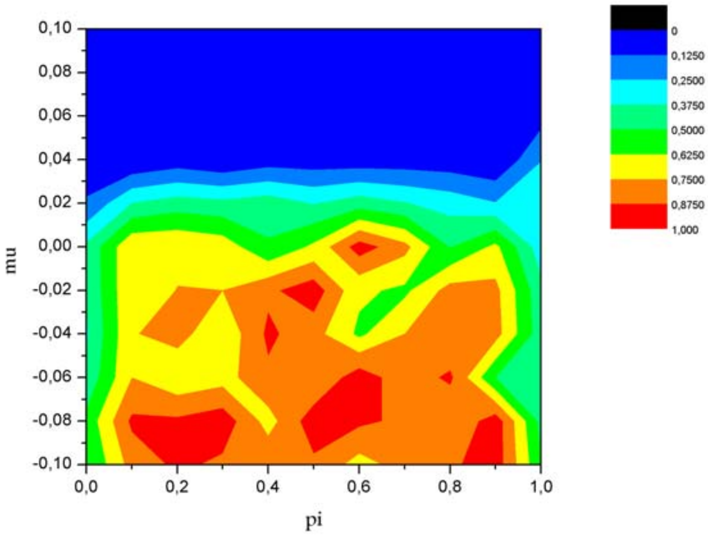
3b: Omniscient agents opting for *Sp*



3c: Omniscient agents opting for *Bp*



3d: Omniscient agents opting for *B*

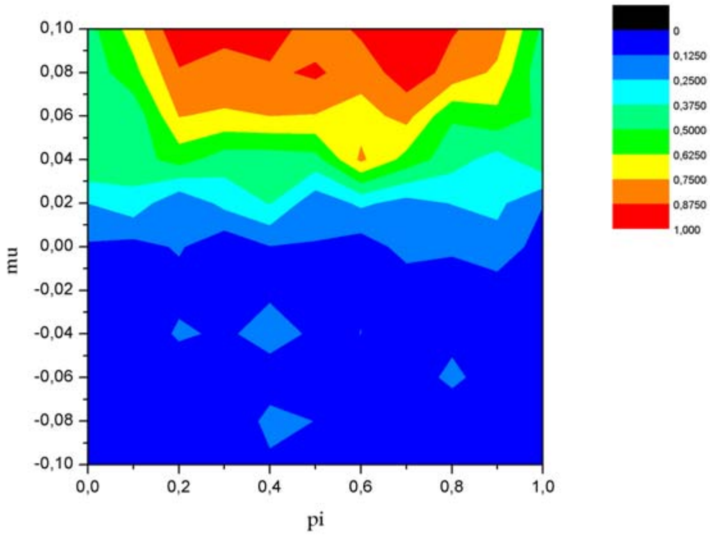


Figures reveal that expected return of securities is a deciding factor in the decision-making in the game. Considering Figure 3a and Figure 3d, the bifurcation starts within the range of $0.02 < \mu < 0.03$, which determines whether agents opt for pure risk-taking strategies, or through portfolio for pure risk-averse strategies. For $\mu < 0.00$, pure risk-taking strategies are "almost dominated" strategies, while there is no dominant strategy. On the other hand, for $\mu > 0.03$ pure risk-averse strategies become dominated strategies with no strict dominant strategy. However, bigger (lower) the expected return, more prone agents are to adopt pure risk-taking (risk-averse) strategies.

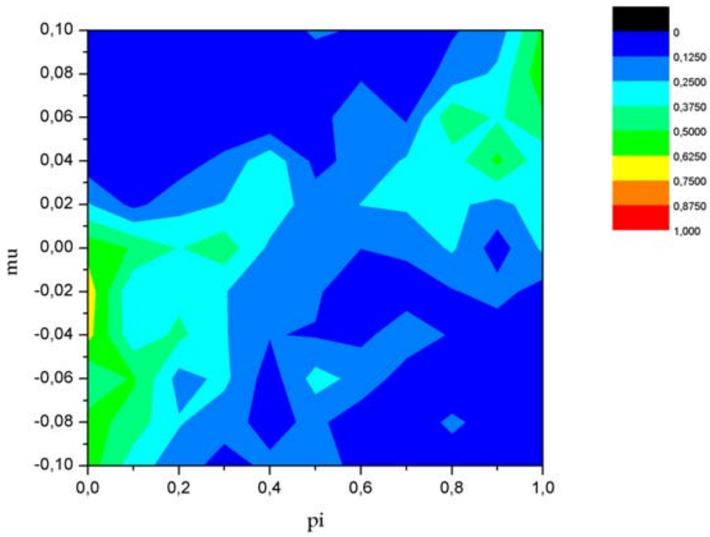
Non-omniscient agents

Non-omniscience of agents is introduced through the coefficient $\kappa = 1.0$. Averages of 20 independent simulations are depicted in Figures 4.

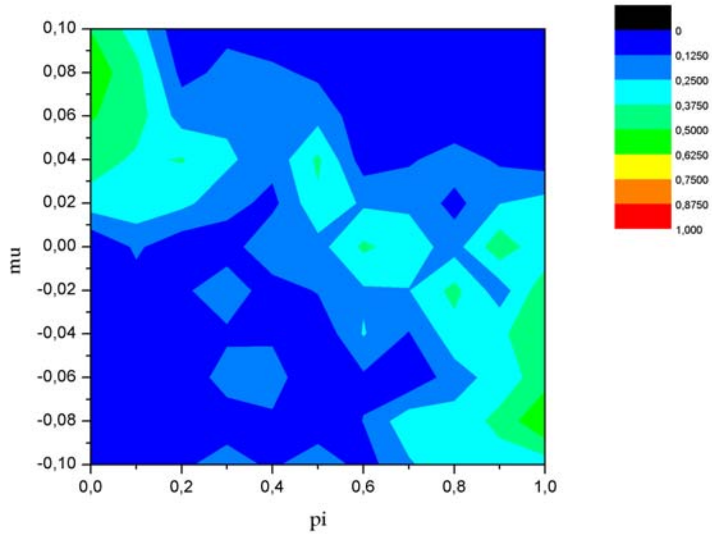
4a: Non-omniscient agents opting for *S*



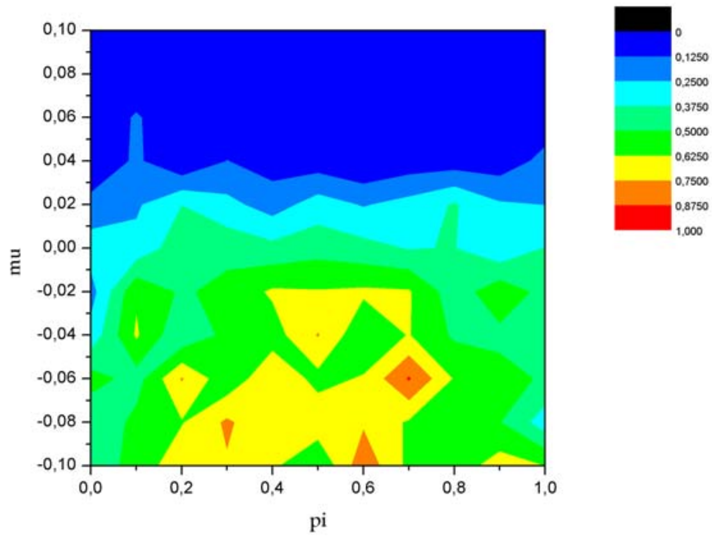
4b: Non-omniscient agents opting for *Sp*



4c: Non-omniscient agents opting for *Bp*



4d: Non-omniscient agents opting for *B*



We see again that first-order stochastic dominance is the most significant element of the decision-making of agents. Comparing Figures 4 with Figures 3 reveals of the importance of non-omniscience to the decision-making of agents. Non-omniscient agents are little more prone to the portfolio, which offsets the islands in Figures 4b and 4c. Comparing the figures to the figures of omniscient agents, a bifurcation is not so sharp, despite still present.

3.3 Changing variance

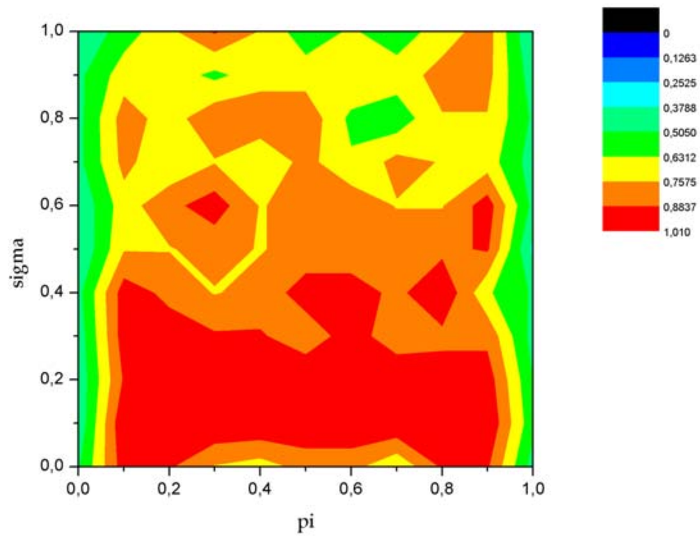
Now we test for the second-order stochastic dominance in the model as we simulate how agents manage their portfolios with changing variance of a risky security. If $F(\bullet)$ and $G(\bullet)$

represent any two distributions with the same mean, $F(\bullet)$ second-order stochastically dominates $G(\bullet)$ if for every concave function, we have $E\left(u\left[F(\bullet)\right]\right) \geq E\left(u\left[G(\bullet)\right]\right)$ (Rothschild and Stiglitz 1970). We do this by analyzing perturbations of variance σ (denoted *sigma* in figures) and *pi* on the decision-making. Steps in figures below equal $pi\_step = 0.1$ on $0 \leq pi \leq 1$ with $pi = pi + pi\_step$, and $\sigma\_step = 0.1$ on $0 \leq \sigma \leq 1$ with $\sigma = \sigma + \sigma\_step$. We set the value $\mu = 0.2$ to correspond the expected return of a risk-free security. Simulations start with $u = 0.5$, which means $A_S = A_{Sp} = A_B = A_{Bp} = 0.25$.

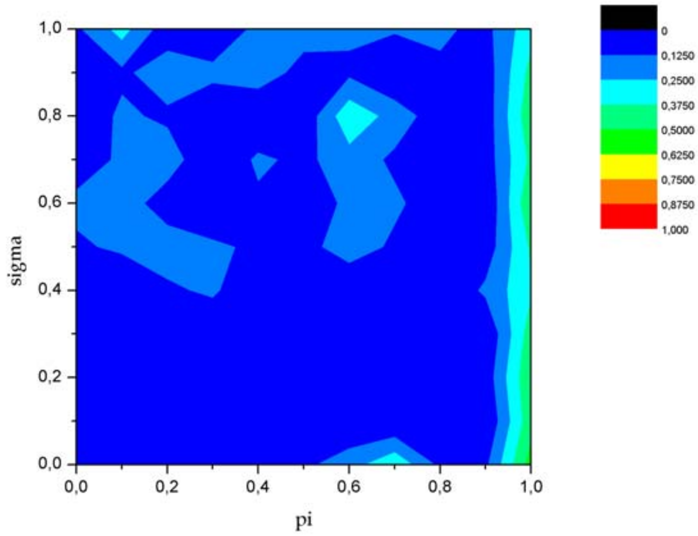
Omniscient agents

We first do the simulations with omniscient agents and plot the averages of 20 independent realizations in Figures 5.

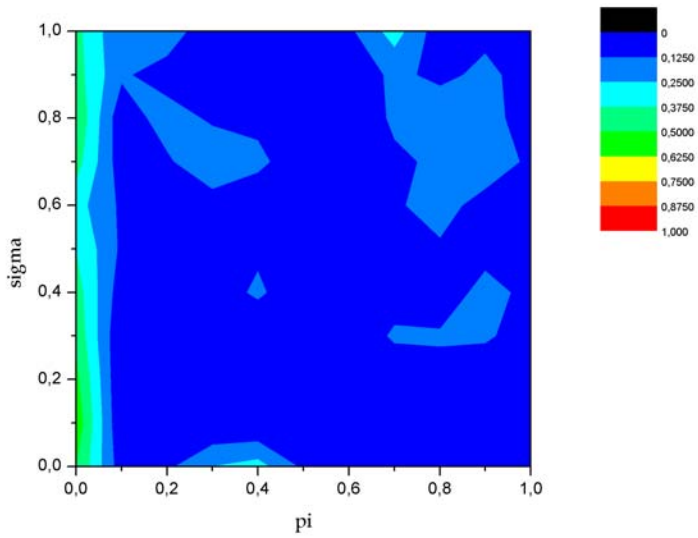
5a: Omniscient agents opting for *S*



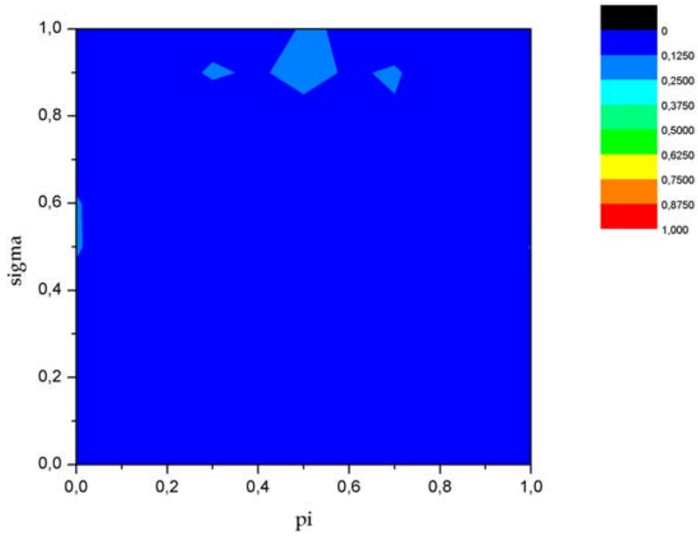
5b: Omniscient agents opting for *Sp*



5c: Omniscient agents opting for *Bp*



5d: Omniscient agents opting for *B*

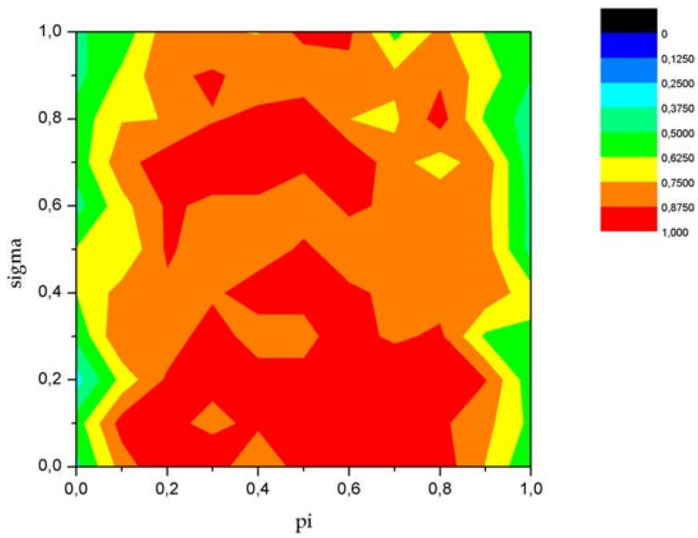


We see that under the modeled circumstances second-order stochastic dominance does not affect the decision-making of agents. Pure risk-averse strategies are dominated strategies on the entire space area, which could be seen from Figure 5d. There is no such strictly dominant strategy on the entire definition-space, despite pure risk-taking turns out to be "almost" dominant under a great part of the alternatives, as can be seen in Figure 5a.

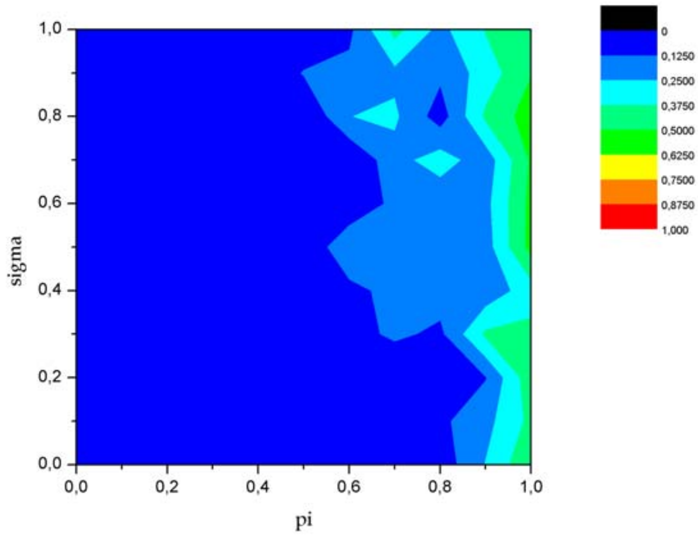
Non-omniscient agents

Finally, we do the simulations with non-omniscient agents with $\kappa = 1.0$ and plot the results in Figures 6.

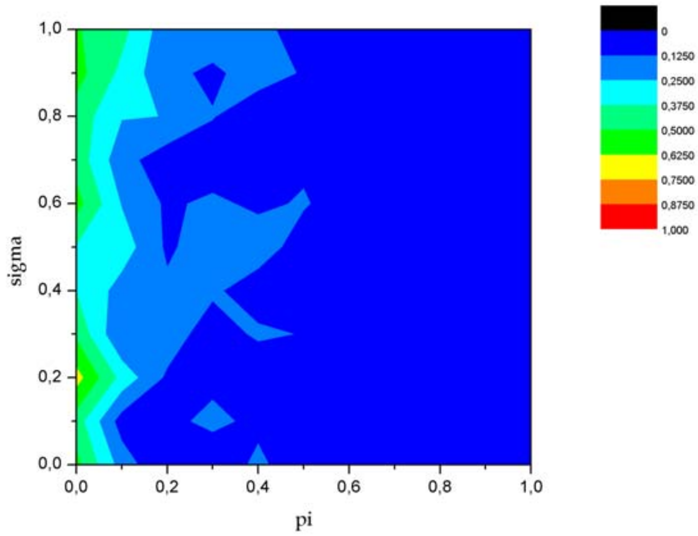
6a: Non-omniscient agents opting for *S*



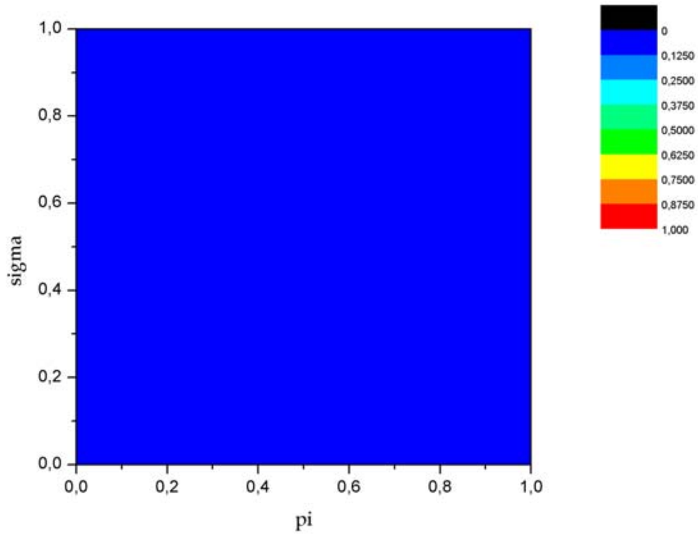
6b: Non-omniscient agents opting for *Sp*



6c: Non-omniscient agents opting for *Bp*



6d: Non-omniscient agents opting for *B*



We can see from the figures that also in the environment of non-omniscient agents, pure risk-aversion strategies are strictly dominated (Figure 6d), while risk-taking preserves the status of "almost" strict dominant strategy (Figure 6a). Once again, the introduction of non-omniscience slightly changed the behavior of agents, who become more prone to opt for portfolio (Figure 6b and Figure 6c).

4 Concluding remarks

In the paper, social networks are used for simulating agents' decision-making in financial markets under different circumstances when choosing among risk-free and risky securities or a portfolio of the two. It has been demonstrated that the realized return of securities is a key determinant of the decision-making and not its variance. The second feature of the paper is that the level of omniscience slightly affects the decision-making of agents who depart from pure risk-taking or risk aversion and more often take portfolio. Under most of the circumstances, pure risk-aversion turns out to be dominated strategy, while pure risk-taking "almost" dominant.

A side effect of non-omniscience is also that it slows down the process of getting towards the synchronous behavior of agents, for which much more simulations do not end with the unanimous behavior in the prescribed time.

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