Disinflation and the NAIRU in a New-Keynesian New-Growth Model (Extended Version)

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Abstract

Unemployment in the big continental European economies like France and Germany has been substantially increasing since the mid 1970s. So far it has been difficult to empirically explain the increase in unemployment in these countries via changes in supposedly employment unfriendly institutions like the generosity and duration of unemployment benefits. At the same time, there is some evidence produced by Ball (1996, 1999) saying that tight monetary policy during the disinflations of the 1980s caused a subsequent increase in the NAIRU, and that there is a relationship between the increase in the NAIRU and the size of the disinflation during that period across advanced OECD economies. There is also mounting evidence suggesting a role of the slowdown in productivity growth, e.g. Nickell et al. (2005), IMF (2003), Blanchard and Wolfers (2000).

This paper introduces endogenous growth via a capital stock externality into an otherwise standard New Keynesian model with capital accumulation and unemployment. We subject the model to a cost push shock lasting for 1 quarter, in order to mimic a scenario akin to the one faced by central banks at the end of the 1970s. Monetary policy implements a disinflation by following a standard interest feedback rule calibrated to an estimate of a Bundesbank reaction function. About 40 quarters after the shock has vanished, unemployment is still about 1.7 percentage points above its steady state, while annual productivity growth has decreased. Over the same horizon, a higher weight on the output gap increases employment (i.e. reduces the fall in employment below its steady state). Thus the model generates an increase in unemployment following a disinflation without relying on a change to labour market structure.

We are also able to coarsely reproduce cross country differences in unemployment. A higher disinflation generated by a larger cost push shock causes a stronger persistent increase in unemployment, the correlation noted by Ball. For a given cost push shock, a policy rule estimated by Clarida, Gali and Gertler (1998) for the Bundesbank and the Federal Reserve Bank produces a stronger persistent increase in the case of the Bundesbank than of the Federal Reserve. Testable differences in real wage rigidity between continental Europe and the United States, namely, as pointed out by Blanchard and Katz (1999), the presence of the labour share in the wage setting function for Europe with a negative coefficient but it’s absence in the U.S. also imply different unemployment outcomes following a cost push shock. If real wage growth does not depend on the labour share, the persistent increase in unemployment is about one percentage point smaller than when it does. To the extent that the wage setting structure is determined by labour market rigidities, "Shocks and Institutions" jointly determine the unemployment outcome, as suggested by Blanchard and Wolfers (2000).
The calibration of unobservable model parameters is guided by a comparison of second moments of key variables of the model with Western German data. The endogenous growth model matches the moments better than a model without endogenous growth but otherwise identical features. This is particularly true for the persistence in employment as measured by first and higher order autocorrelation coefficients.

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1 Introduction

The persistent increase in continental European unemployment since the 1970s is often blamed on labour markets having become more rigid. There is however growing evidence that labour market institutions, while powerful at explaining cross country differences in unemployment at a given point in time, are less so at explaining the evolution of unemployment across time, or at least leave a lot to be explained. Findings along these lines include the IMF (2003), Nickell (2002), Blanchard and Wolfers (2000), Fitoussi et al. (2000) and Elmeskov (1998).

This paper contributes to the explanation of the rise in European unemployment by introducing endogenous growth into a New Keynesian model featuring unemployment. We implement this in a simple fashion by assuming that technological progress is realised through investment and thus linking total factor productivity to the capital stock. We subject the economy to a 1 quarter non-serially correlated cost-push shock and let the central bank disinflate the economy - as happened in many industrialised economies at the beginning of the 1980s. This temporary shock can cause a persistent and substantial increase in unemployment, lasting over 10 to 20 years in an order of magnitude of 1 percentage points or more. The model also sheds light on some cross-country differences in the unemployment experience.

More precisely, we aim to shed light on the following set of stylised facts and empirical findings:

- Unemployment has increased substantially in many big European OECD economies since the 1970s. Figure 1 displays quarterly unemployment rates from 1975 to 2000 for six selected OECD European Economies and the United States. Note that unemployment is very persistent: It increases relatively quickly, as for instance during the recessions at the beginning of the 80s, but reverts only relatively slowly, incompletely, or not at all. By contrast, unemployment in the United States shows less persistence. It also does not show much of a trend.

- There has been a decline in the growth rate of labour productivity (measured as output per hour worked) across OECD countries in the 1980s. This decline has been substantially larger in Western European Economies than in the United States. Average annual productivity growth in Western European economies was 1.5% lower in the period from 1981 to 1990 than in the previous decade, while it declined by merely 0.2% in the United States. Skoczylas and Tissot (2005) estimate changes in trend productivity growth for OECD economies from 1960 to 2004. They locate declines between one and 3.9% between 1976 and 1985 in 9 Western European Economies but none in the United States.

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1 The number is based on cross country averages for 1971-1980 and 1981-1990 of the productivity growth rates of Belgium, Denmark, Western Germany, Ireland, Spain, France, Italy, the Netherlands, Finland, Sweden, the United Kingdom and Norway. These rates are based on the series on GDP at constant prices and total hours worked from AMECO (2008).
• It is a consistent finding that a decline in productivity growth increases unemployment. Examples include Bassanini and Duval (2006), Pissarides and Vallanti (2005), Nickel (2002, 2005), Ball and Moffitt (2001) and Fitoussi et al. (2000). Three of these studies (Bassanini and Duval, Blanchard and Wolfers, Fitoussi et al.) explicitly model interactions between productivity growth declines and labour market institutions. They find that macroeconomic shocks help to explain the evolution of unemployment across time while cross country-differences in institutions help to explain why in some countries unemployment responds more strongly to macroeconomic shocks than in others.

• Based on a study of 17 OECD countries, Ball (1999) argues that those central banks willing to aggressively lower real interest rates during the recessions of the early 1980s reduced the subsequent increase in the NAIRU in their countries.

• There seems to be a negative medium run relationship between the change in inflation and the change in the NAIRU. This is illustrated in Figure 2, which plots the change in the NAIRU against the change in CPI Inflation for 21 OECD countries from 1980 to 1990 and from 1990 to 2000. The negative correlation is not perfect but still obvious: Countries with a larger decrease in inflation suffered on average a larger increase in their NAIRU.² Ball (1996) was the first to draw attention to this link and also investigated it more formally.

The paper is structured as follows: Section 2 develops a model which coarsely encompasses the mainstream consensus on the long and short run dynamics of unemployment. In this model, a temporary cost push shock only has a short lived effect on unemployment and so has the monetary policy response to the shock. We coin this model "Jackman, Layard, Nickell", or JLN economy. We then add the New Growth extension. Section 3 discusses the calibration, which is informed by empirical evidence on some of the model parameters and by the comparison of the second moments of a couple of model variables with their empirical counterparts, which is conducted in section 4. Section 5 then discusses the response of the economy to a one quarter cost push shock calibrated to induce a disinflation of about 4 percentage points and focuses on the induced evolution of unemployment across time. It also looks at the tradeoffs of policymakers face between stabilising inflation and stabilising unemployment. Section 6 adds a cross-country dimension to our analysis. First, we vary the size of the cost push shock and record the resulting changes in Inflation and the NAIRU over a 10 year horizon. We then compare the differences in the unemployment response generated by a Bundesbank and a Federal Reserve Policy rule as estimated by Clarida, Gali and Gertler (1998), and finally we investigate the effects

²The data is taken from the OECD Economic Outlook. The countries are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, U.S.A.
differences in real wage rigidity between Europe and the United States. Section 7 concludes.
2 The Model

In this section we will develop a New Keynesian model with unemployment and endogenous growth which contributes to explaining the above findings. To stress the fact our results stem from the introduction of endogenous growth, we also present an otherwise identical model without endogenous growth which we take as the starting point of our analysis. This is a model to approximate the ruling consensus on the relationship between unemployment and the NAIRU. We will refer to this model as the JLN economy as the model that develops the current consensus on the short and long run causes of unemployment. This consensus says that while unemployment both in the short and in the long run is determined by aggregate demand, only the NAIRU is consistent with stable inflation. Inflation targeting central banks will push unemployment towards this level. The NAIRU itself will be affected by any variable which directly increases wages in spite of excess supply in the labour market, increases the pricing power of firms or reduces the efficiency of the labour market to match jobs to workers (Nickell et al. 2002: 2-3, Jackman et al.1991).

Sections 2.1 to 2.4 develops the JLN economy, while section 2.5 shows which modifications are necessary once we introduce endogenous growth.

2.1 Households

Danthine and Kurmann (2004) introduce unemployment in a general equilibrium model without moving away from the representative agent framework. In the Danthine-Kurmann setup individuals are organized in families in a zero-one continuum of families which are infinitely lived. All decisions regarding the intertemporal allocation of consumption and the accumulation of capital are made at the family level. Each family member supplies one unit of labour in-elastically but derives disutility from the effort $G(e_l)$ he or she supplies in their job. The share of unemployed members is the same for each family. The large family assumption means that although there are unemployed individuals in the economy, it is not necessary to track the distribution of wealth.

In addition, some workers supply overhead labour, whose nature will be described in more detail below. They can be thought of as the owners of the monopolistically competitive firms. Overhead workers never become unemployed because no firm can produce without managerial staff. A share $n^s$ of the workforce is employed by the government who is assumed to pay the same wage as the private sector. They are funded by lump sum taxes. All families have the same share of managers and

---

3 The reason for introducing both state employees and overhead workers $n^s$ is to achieve a reasonable calibration of steady state values. In the Romer endogenous growth model, the level of employment affects the growth rate. This is due to the fact that the marginal product of capital is an increasing function of employment. The marginal product of capital governs the growth rate by determining the willingness of households to save. To achieve a reasonable steady state growth rate, we remove part of the labour force from "productive" sector and thus to reduce the impact...
government employees.

Families solve the following constrained maximisation problem by choosing consumption \( C_t \) (which is a CES consumption basket \( C_t = \left[ \int_0^1 (c_t(i))^{\frac{\sigma-1}{\sigma}} \, di \right]^{\frac{\sigma}{\sigma-1}} \)), bond-holding \( B_t \), investment expenditure \( I_t \), next periods capital stock \( K_{t+1} \) and the effort level \( e_t \) of the typical working family member:

\[
U = E_t \left\{ \sum_{i=0}^{\infty} \beta^i [u(C_{t+i} - \text{hab}_{t+i-1}) - (n_{t+i} - \bar{n}) G(e_{t+i})] \right\}, \, u > 0, \, w^u < 0 \tag{1}
\]

s.t. \( (n_t - \bar{n}) w_t + \frac{B_{t-1}}{P_t} (1 + i_{t-1}) + F_t + r^K_t K_t \geq C_t + \frac{B_t}{P_t} + T_t + I_t \) and \( K_{t+1} = (1 - \delta) K_t + I_t \left( 1 - S \left( \frac{I_t}{I_{t-1}} - (1 + g) \right) \right) \), \( S(0) = 0, \, S(0)^I = 0, \, S(0)^{II} > 0 \) \tag{2}

where \( P_t \) denotes the price index of the consumption basket. A family's period income consists wages \( w_t \), interest income \( i_{t-1} \) on risk less bonds they bought in the previous period \( B_{t-1} \), the profits of the monopolistically competitive firms in the economy \( \Gamma_t \), and dividends \( r^K_t \) from renting out their accumulated capital \( K_t \). They have to pay lump sum taxes \( T_t \). We assume adjustment costs in investment: Only a fraction of one unit of investment expenditure is actually turned into additional capital. This fraction decreases in the investment growth rate. The assumptions on the first derivative of the \( S(.) \) function imply that adjustment costs vanish when the economy is growing at its steady state growth rate \( g \).\(^4\) This implies that the steady state growth rate does not depend on the parameters of the adjustment cost function \( S \). Setting up the lagrangian and denoting the lagrange multipliers of the budget constraint and the capital accumulation constraint as \( \lambda_t \) and \( \lambda_q t \) yields the following first order conditions with respect to consumption, capital and investment:

\(^4\)There are two advantages of assuming investment adjustment costs and external habit formation. Firstly, it facilitates matching the second moments of investment and consumption, and secondly, it makes the on impact response of unemployment to the cost push shock in the simulation we are going to perform later more reasonable. By contrast, the impact on the longer run response of unemployment is rather small.
\[ u'(C_t - hab_{t-1}) = \beta E_t \left[ u'(C_{t+1} - hab_t) \frac{1}{1 + \pi_{t+1}} \right] [1 + i_t] \]  
(4)

\[ \lambda_t = u'(C_t - hab_{t-1}) \]  
(5)

\[ \beta E_t (\lambda_{t+1} r_{t+1}^p + \lambda_{t+1} q_{t+1} (1 - \delta)) = \lambda_t q_t \]  
(6)

\[ \lambda_t q_t \left[ \left( 1 - S \left( \frac{I_t}{I_{t-1}} - (1 + g) \right) \right) - \frac{I_t}{I_{t-1}} S' \left( \frac{I_t}{I_{t-1}} - (1 + g) \right) \right] \]  
(7)

\[ + \beta E_t \left[ \lambda_{t+1} q_{t+1} \left( \frac{I_{t+1}}{I_t} \right)^2 S' \left( \frac{I_{t+1}}{I_t} - (1 + g) \right) \right] = \lambda_t \]  
(8)

Note that with this notation, \( q_t \) denotes the present discounted value of the future profits associated with buying an additional unit of capital today, also known as Tobin’s q. We assume logarithmic instantaneous utility. Following Schmitt-Grohe and Uribe (2005), we assume

\[ S \left( \frac{I_t}{I_{t-1}} - (1 + g) \right) = \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - (1 + g) \right)^2. \]  
(10)

Applying these functional forms yields the expressions discussed in section 2 of the paper.

The cost of effort function of individual j \( G(e_{t+i}(j)) \) is of the form

\[ G(e_{t+j}(j)) = \left( e_{t+j} - \left( \phi_0 + \phi_1 \log w_{t+j} + \phi_2 (n_t - \bar{n}) \right) \right)^2 \]  
(9)

\[ \log b_t = \phi_6 \log (Y_{t-1} / (n_{t-1} - \bar{n} - n^s)) + (1 - \phi_6) \log w_{t-1} + \phi_7 \]  
(10)

\[ \phi_1, \phi_7 > 0, 1 \geq \phi_6 \geq 0, \phi_2, \phi_3, \phi_4 < 0, \phi_1 > -\phi_3 \]  
(11)

where \( Y_t \) is private sector output. Note that the effort function enters the families’ utility separately which implies that it is independent of the budget constraint. Furthermore, state employees are assumed not to perform any effort while at work. The first order condition with respect to effort is

\[ e_{t+i}(j) = \phi_0 + \phi_1 \log w_{t+j} + \phi_2 (n_t - \bar{n}) + \phi_3 \log w_t \]  
(12)

\[ + \phi_4 \log w_{t-1} - \phi_5 \log b_t - \phi_8 (Y_{t-1} / (n_{t-1} - \bar{n} - n^s)) \]

The structure of the cost of effort function is motivated by the idea of "gift exchange" between the firm and the worker. The worker’s gift to the employer is effort. The employer has to show his appreciation for the employees’ contribution by paying an appropriate wage \( w_t(j) \). A higher contemporary average wage \( w_t \) reduces effort because it represents a "reference level" to which the current employers’ wage offer is compared. Put differently, it requires the firm to pay a higher wage if it wants to extract the same amount of effort. A higher average past real wage \( w_{t-1} \) boosts
the workers’ aspirations as well.\(^5\) The aggregate employment level of non-overhead workers \(n_t - \bar{n}\) summarizes labour market tightness. It is thus positively related to the workers’ outside working opportunities, and thus also tends to reduce effort.

The view that wages have a big effect on workers morale and thus productivity because they signal to the worker how his contribution to the organizational goals is valued is supported by an extensive microeconomic survey conducted by Bewley (1989). Bewley found that wage changes (in particular wage cuts) seem to be especially important. Bewley interviewed over 300 business people, labour leaders and business consultants in search for an explanation why wages are rarely cut in recessions.\(^6\)

The terms \(b_t\) and \((Y_{t-1}/ (n_{t-1} - \bar{n} - n^s))\) represent and a modification to the Danthine and Kurman (2004) cost of effort function. \(b_t\) denotes unemployment income. This will be chiefly unemployment benefits and black market income. It tends to lower the level of effort.\(^7\) Workers want to be valued more than someone who receives benefits or does not have a legal job. \(b_t\) is linked both to past real wages and past productivity in the private sector, where \(Y_t\) denotes private sector output. This may reflect both the structure of benefits and the manner in which the black market is linked to the official economy. Productivity also has a direct effect on morale and effort as employees desire their due share of the companies’ success. Unions might play a role in this to the extent that they instil a sense of entitlement among employees.

The employer takes this relationship into account when setting the wage, as will be discussed further below.

### 2.2 Cost Minimisation and Efficiency Wages

The production technology is a Cobb Douglas production function,

\[
Y_t(i) = AK_t(i)\alpha (TFP_t \epsilon_t(i) (n_t(i) - \bar{n}))^{1-\alpha}
\]

where the output of firm \(i\) \(Y_t(i)\) depends on the capital stock of firm \(i\) \(K_t(i)\), the efficiency of its workers \(\epsilon_t(i)\) and the number of non-overhead workers \(n_t(i) - \bar{n}\). In the Danthine and Kurman model (2004), in a first stage the firm minimises its cost of producing a given amount of output. Capital and labour are hired in economy wide

\(^5\)See Danthine and Kurmann (2004), pp. 111-113. It would be desirable to have the individual workers past real wage \(w_t(j)\) in the equation but that would considerably complicate the maximisation problem of the representative firm dealt with later, so we follow Danthine and Kurman in assuming a dependence of effort on the average wage. For the same reason we include average productivity rather than the respective firm’s productivity.

\(^6\)See Bewley (1998), pp. 459-490. A discussion of further evidence is Bewley (2004). Bewley also argues that his findings contradicts essentially all theoretical justifications of real wage rigidity not based on gift exchange considerations, like implicit contracts, insider outsider models or the efficiency wage models based on no-shirking conditions.

\(^7\)Danthine and Kurman (2007) introduce the benefit level as a factor which, ceteris paribus, reduces effort.
factor market. However, the firm does not take the real wage as given but sets it taking into account the relationship between effort and wages given by (12).\(^8\) Hence the firm’s problem is:

\[
\begin{align*}
\min_{K_t(i), n_t(i), w_t(i), e_t(i)} & \quad r_t^k K_t(i) + w_t(i)(n_t(i) - \bar{n}) \quad \text{s.t.} \quad Y_t(i) = AK_t(i)^\alpha (TFP_t e_t(i) (n_t(i) - \bar{n}))^{1-\alpha} \\
\end{align*}
\]

and \(e_t(i) = \phi_0 + \phi_1 \log w_t(i) + \phi_2 (n_t - \bar{n}) + \phi_3 \log w_t + \phi_4 \log w_{t-1} - \phi_5 \log b_t - \phi_8 (Y_{t-1} / (n_{t-1} - \bar{n} - n^s))\)

The first order conditions for capital and labour are

\[
\begin{align*}
r_t^k &= \alpha mc_t(i) \frac{Y_t(i)}{K_t(i)} \\
w_t(i) &= (1 - \alpha)mc_t(i) \frac{Y_t(i)}{n_t(i) - \bar{n}}
\end{align*}
\]

where \(mc_t(i)\) and \(r_t^k\) refer to real marginal costs of firm \(i\) and the capital rental rate. It will be shown below that even though all firms set the wage individually, firms will find it optimal to set the same wage and the same efficiency level. Dividing the two first order conditions gives \(\frac{K_t(i)}{n_t(i) - \bar{n}} = \frac{\alpha w_t}{1 - \alpha r_t^k}\). Thus the capital labour ratio is the same across firms. It is then easily shown using the production function that the same holds for the output-capital ratio and the output-labour ratio. Hence we have

\[
\begin{align*}
r_t^k &= \alpha mc_t \frac{Y_t}{K_t} \\
w_t &= (1 - \alpha)mc_t \frac{Y_t}{n_t - n^s - \bar{n}}
\end{align*}
\]

This then means that the capital to (productive) labour ratio, the output per unit of productive labour ratio and marginal costs are the same in all firms, as can be easily verified by dividing the two first order conditions. This gives the capital to productive labour ratio as \(\frac{K_t(i)}{n_t(i) - \bar{n}} = \frac{\alpha w_t}{1 - \alpha r_t^k}\). Substituting this back into equation (15) yields an equation for \(mc_t(i)\) containing only labour augmenting technological progress and the factor price, implying that marginal costs are the same across all firms:

\[
mc_t = \frac{(r_t^k)^\alpha w_t^{1-\alpha}}{A \alpha^\alpha (1 - \alpha)^{1-\alpha}(\phi_1 TFP_t)^{1-\alpha}}
\]

We now turn to wage setting. The first order conditions with respect to effort and the real wage are

---

\[ n_t(i) - \pi = \frac{\zeta_t \phi_1}{w_t(i)} \]  

\[ \zeta_t = (1 - \alpha)mc_t \frac{Y_t(i)}{e_t(i)} \]  

Combining those with the first order condition with respect to labour yields an optimal effort level of \( \phi_1 \). Substituting this back into the effort function (12), we note that, as the firm’s wage depends only on aggregate variables which are the same for all firms, it must indeed hold that \( w_t(i) = w_t \). Substituting for \( \log b_t \) and rearranging then yields

\[
(\phi_1 + \phi_3) \log w_t = (\phi_5 (1 - \phi_6) - \phi_4) \log w_{t-1} + \phi_1 - \phi_0 + \phi_5 \phi_7 - \phi_2 (n_t - \pi) \\
- (\phi_5 \phi_6 + \phi_8) \log \left( \frac{1}{\left(n_{t-1} - \pi - n^*\right)} \right)
\]

Subtracting \((\phi_5 \phi_6 + (1 - \phi_3))\log w_{t-1}\) on both sides and dividing by \((\phi_1 + \phi_3)\) then yields

\[
\log w_t = \frac{\phi_1 - \phi_0 + \phi_5 \phi_7}{\phi_1 + \phi_3} - \frac{\phi_2}{\phi_1 + \phi_3} (n_t - \pi) + \frac{\phi_5 + \phi_8 - \phi_4}{\phi_1 + \phi_3} \log w_{t-1} \\
- \frac{(\phi_5 \phi_6 + \phi_8)}{\phi_1 + \phi_3} \log \left( \frac{w_{t-1} (n_{t-1} - \pi - n^*)}{Y_{t-1}} \right)
\]  

(19)

Hence with the coefficient restrictions imposed above, the wage depends positively on the past real wage and non managerial employment. It will be above its market clearing level and thus there is unemployment in the economy.

Note that the last term in brackets is in fact the private sector labour share. If this were constant in the steady state, as it would be at a constant employment level, equation (19) could be solved for a long run real wage if \( \frac{\phi_5 + \phi_8 - \phi_4}{\phi_1 + \phi_3} < 1 \). As mentioned above however, in our model, Danthine and Kurmann’s, is a growth model. Therefore the real wage must be growing in the steady state. Thus a wage setting function simply relating the wage level to employment would not be consistent with a stable employment level. The easiest way to deal with the issue therefore is to set \( \frac{\phi_5 + \phi_8 - \phi_4}{\phi_1 + \phi_3} = 1 \). This does not seem too restrictive: It simply says that an increase in the log of the time t real wage in the economy (including firm i) has in absolute value the same net effect on effort (remember we have \( \phi_1 + \phi_3 > 0 \)) as an increase in the exogenous reference as represented by \( \log w_{t-1}, \log b_t \) and \( \log (Y_{t-1} / (n_{t-1} - \pi - n^*)) \).
Thus we arrive at a real wage Phillips Curve with a labour share term:

\[ \log w_t - \log w_{t-1} = a + b (n_t - \bar{n}) + c \log \left( \frac{w_{t-1} (n_{t-1} - \bar{n} - n^s)}{Y_{t-1}} \right), \]

with

\[ a = \frac{\phi_2 - \phi_1 + \phi_5 \phi_7}{\phi_1 + \phi_3}, \quad b = -\frac{\phi_2}{\phi_1 + \phi_3} > 0 \quad \text{and} \quad c = -\frac{(\phi_5 \phi_6 + \phi_8)}{\phi_1 + \phi_3} < 0 \] \tag{20}

Equation (20) is a real wage Phillips Curve plus an "error correction term" represented by the log of the labour share. Note that if there is no effect of productivity on effort and \((\phi_8 = 0)\) and no effect of productivity on benefits \((\phi_6 = 0)\) we have \(c = 0\).

It remains to determine the size of the overhead labour force. Following Rotemberg and Woodford (1999), we assume that in the steady state, all economic profit generated by employing productive labour and capital goes to the overhead staff. Hence the firm ends up with zero profits.\(^9\) This is justified because setting up production is impossible without overhead labour and the firms profit is thus essentially equal to the collective marginal product of its overhead staff. We assume that the overhead staff splits this profit equally. We assume the amount of overhead workers required to enable production is such that the real wage for overhead and non-overhead workers will be exactly the same in the steady state. These assumptions allow for a straightforward way to determine the amount of overhead and non-overhead workers as a function of total employment. Zero profit requires

\[ \frac{\mu - 1}{\mu} Y_t - w_t \bar{n} = 0 \]

where \(\frac{\mu - 1}{\mu}\) is the share of firms profits in output. Substituting \(w_t = (1 - \alpha) \frac{Y_t}{n - n^s - \bar{n}}\) gives, after some manipulation

\[ \frac{\mu - 1}{1 - \alpha} = \frac{\bar{n}}{n - n^s - \bar{n}} \equiv \bar{s} \]

This is the ratio of overhead labour to productive labour, which we call \(\bar{s}\). Solving for \(\bar{n}\) then gives

\[ \bar{n} = \frac{\bar{s}}{1 + \bar{s}} (n - n^s) \]

2.3 Price Setting and Nominal Rigidities

Each firm produces one of the variants of the output good in the CES basket. Households spread their expenditures across the different varieties in the basket in a cost minimising fashion. Assuming that investment expenditure stretches over these variants in precisely the same way as consumption demand, we can write the demand

---

for variant $j$ as $y_{t+i}(j) = Y_{t+i} \left( \frac{p_{t+i}(j)}{P_{t+i}} \right)^{-\theta}$. Following Rotemberg (1983) we assume that the representative firm faces quadratic costs if it alters its individual price inflation from a reference level $\Pi - 1$. This is the steady state level of inflation in the economy. These cost arise because frequent price changes are bad for the reputation of the company. Convincing customers to remain with the company nevertheless is costly. Additional costs arise because deviating from the "standard" level of inflation requires the firm to engage in a costly re-optimisation process. This has to be carried out by high paid marketing professionals, while price changes close to average inflation can be decided by lower paid "frontline" staff. Both kinds of costs are likely to increase in the firms output as well. We assume the following functional form:

$$AC_{t+i}(j) = \frac{\varphi}{2} \left( \frac{p_{t+i}(j)}{P_{t+i}(j)} - \Pi \right)^2 y_{t+i}(j)$$  \hspace{1cm} (21)

The firm $j$ chooses its price $p_{t+i}(j)$ in order to maximise

$$\sum_{i=0}^{\infty} E_t \left[ \rho_{t,t+i} \left( \frac{p_{t+i}(j)}{P_{t+i}} y_{t+i}(j) - mc_{t+i} y_{t+i}(j) - AC_{t+i}(j) \right) \right]$$  \hspace{1cm} (22)

where $\rho_{t,t+i}$ denotes the discount factor used to discount real profits earned in period $t+i$ back to period $t$. Note that because households own the firms, we have $\rho_{t,t+i} = \beta^i \frac{u'(C_{t+i})}{u'(C_t)}$. Differentiating with respect to $p_t(j)$ and noting that, as all firms are the same, $p_t(j) = P_t$ holds ex post, yields

$$(1 - \theta) + \theta mc_t - \varphi \left( \frac{P_t}{P_{t-1}} - \Pi \right) \frac{P_t}{P_{t-1}} + \frac{\varphi}{2} \left( \frac{P_t}{P_{t-1}} - \Pi \right)^2$$

$$+ E_t \left[ \rho_{t,t+1} \varphi \frac{Y_{t+1}}{Y_t} \left( \frac{P_{t+1}}{P_t} - \Pi \right) \frac{P_{t+1}}{P_t} \right] = 0$$  \hspace{1cm} (23)

which is a nonlinear version of the standard New Keynesian Phillips curve. It is, however, a consistent feature of empirical estimations of Phillips curves that specifications which include lagged inflation as well ("hybrid" Phillips curves) perform better than purely forward looking Phillips Curves. This is because inflation has inertia.\(^{10}\) Backward looking elements are easily introduced into the price setting considerations of the firm by assuming that the reference level of inflation does not remain constant over time. Instead, we assume that it equals last periods inflation, i.e. $\Pi_t = \frac{P_{t-1}}{P_{t-2}}$. If the inflation rate becomes higher for several periods, firms will mandate frontline staff to handle price increases of that size in order to keep costs low. Customers will get used to the different pace of price changes as well, making a higher rate of price

\(^{10}\)See for instance Gali and Gertler (2000).
change less costly for the individual firm. Hence we have

\[(1 - \theta) + \theta mc_t - \varphi \left( \frac{P_t}{P_{t-1}} - \frac{P_{t-1}}{P_{t-2}} \right) \frac{P_t}{P_{t-1}} + \theta \frac{\varphi}{2} \left( \frac{P_t}{P_{t-1}} - \frac{P_{t-1}}{P_{t-2}} \right)^2 \]

\[+ E_t \left[ \rho_{t+1} \varphi \frac{Y_{t+1}}{Y_t} \left( \frac{P_{t+1}}{P_t} - \frac{P_t}{P_{t+1}} \right) \frac{P_{t+1}}{P_t} \right] = 0 \quad (24)\]

The experiment we want to conduct later is a disinflation. Inflation is brought into the economy by a so called "cost-push shock" $u_t$ widely used in the New Keynesian literature.\(^{11}\) This shock increases current inflation, holding the values of past inflation and marginal costs constant, and is added directly to the Phillip's curve equation:

\[(1 - \theta) + \theta mc_t - \varphi \left( \frac{P_t}{P_{t-1}} - u_t \right) \frac{P_t}{P_{t-1}} + \theta \frac{\varphi}{2} \left( \frac{P_t}{P_{t-1}} - u_t \right)^2 \]

\[+ E_t \left[ \rho_{t+1} \varphi \frac{Y_{t+1}}{Y_t} \left( \frac{P_{t+1}}{P_t} - \frac{P_t}{P_{t+1}} - u_t \right) \frac{P_{t+1}}{P_t} \right] = 0 \quad (25)\]

It is easily shown that up to first order, this Phillips Curve resembles very closely specifications which are obtained by Woodford (2003) under the assumption of Calvo contracts and full indexation of the prices of those firms which can not re-optimize prices to past inflation.\(^{12}\) It is a forward looking accelerationist Phillips Curve. If present and future marginal costs are at their steady state level and present and future values of cost push shock are zero, inflation will remain constant. It will accelerate or decelerate otherwise. Hence the model has a well defined NAIRU.

### 2.4 Monetary Policy

Monetary Policy is assumed to follow a simple Taylor type nominal interest rate rule. The exact specification will vary across simulations. However, all specifications will include a lagged dependent variable in order to account for the interest rate inertia observed in the data. In the baseline, the interest rate reacts to current inflation and the lagged output gap:

\[i_t = (1 - \rho) \left( \bar{\iota} + \psi_\pi \pi_t + \psi_Y \frac{g_{\pi_{t-1}}}{4} \right) + \rho i_{t-1} \quad (26)\]

$\bar{\iota}$, $\rho$ and $g_{\pi_t}$ denote the long-run real interest rate (recall that inflation is zero in the steady state), the degree of interest rate smoothing and the output gap, respectively. $\psi_\pi$ and $\psi_Y$ denote the long run coefficients on inflation and the output gap. The central bank responds to the lagged value of the output gap but current values of

\(^{11}\)See for instance Clarida et al (1999), pages 1665 and 1667.

\(^{12}\)See Woodford (2003), p. 215. In fact, the coefficients on expected future inflation and the coefficient on lagged inflation exactly match Woodfords' results.
inflation. We choose this form because it matches the specification estimated by Clausen and Meier (2003) on a real time output gap. We will discuss later why this is more consistent with our model than the alternatives. However, our basic results are not sensitive to changing the form of the policy rule.

The output gap is the percentage deviation of total output, i.e. private sector plus the output of government employees, from its natural level. We calculate the output of government employees by simply adding up their wages, following the convention of national accounts. We assume that government employees earn the same wage as in the private sector. For total output, we then have $Y_t + w_t n^s$, while total natural output is given by $Y^n_t + w^n_t n^s$. $w_t$ and $Y^n_t$ denote the wage rate and the private sector output level consistent with natural employment, or the NAIRU. Thus we have

$$gp_t = \frac{Y_t + w_t n^s - Y^n_t - w^n_t n^s}{Y^n_t + w^n_t n^s}$$

(27)

$Y^n_t$ denotes the private sector output level which would set marginal costs equal to its long run level $\mu^{-1}$, given the capital stock and the previous period’s real wage. As can be obtained from equation (25), this would ensure that in the absence of cost push shocks, inflation is neither rising nor falling. The employment level corresponding to this output level will be referred to as "natural employment" $n^n_t$. The natural levels of output and employment are derived by substituting the equation for the rental on capital (15) and the wage setting equation (20) into (17) and setting $mc_t = \mu^{-1}$. The natural levels of output, employment and the real wage are then given by the values of $Y^n_t$, $n^n_t$ and $w^n_t$ solving

$$\mu^{-1} = \frac{(n^n_t - n^s - \pi)^\alpha w^n_t}{A(1-\alpha)(\phi_1 TFP_t)^{1-\alpha}K^n_t}$$

$$\log w^n_t - \log w_{t-1} = a + b * (n^n_t - \pi) + c \log \left(\frac{w_{t-1} (n_{t-1} - \pi - n^s)}{Y_{t-1}}\right)$$

$$Y^n_t = AK^n_t(\phi_1 (n^n_t - n^s - \pi))^{1-\alpha}$$

(28)

Note that given the past real wage, the capital stock has a positive effect on natural employment given the past real wage. This effect works through the negative effect of a higher capital stock on the capital rental. This tends to lower marginal costs and thus accommodates a higher real wage given the mark-up. This allows the employer to meet the demands of wage setters associated with higher employment.

### 2.5 Introducing Endogenous Growth

The basic idea in the knowledge spill-over model of Romer (1986) is to start off with a standard neoclassical production function with labour augmenting technical progress as above.\footnote{The exposition here follows Barro and Sala-i-Martin (2004), pp.212-222.} An additional feature is that labour augmenting technological progress
might be firm specific. Thus the output of firm $i$ is given by

$$Y_t(i) = F(K_t(i), TFP_t(i) n_t(i)) \quad (29)$$

Romer then makes two crucial assumptions:

- Increasing it’s physical capital simultaneously teaches the firm how to produce more efficiently. This idea was first suggested by Arrow (1962). For simplicity, in the Romer setup, $TFP_t(i)$ is simply proportional to the firm’s capital stock.

- Knowledge is a public good. Hence each firm’s knowledge is in fact proportional to the aggregate capital stock rather than to its own.\(^{14}\) However, the impact of the firm’s capital stock on the aggregate capital stock is so small that they can be neglected. Thus the production function of firm $i$ becomes

$$Y_t(i) = F(K_t(i), K_t n_t(i)) \quad (30)$$

This implies that there are now constant returns to capital at the economy wide level, allowing per capita output to grow. However, there are still decreasing returns to capital at the firm level. In the Romer model, this leads to a growth rate which is inefficiently low. This is because saving is so low as the individual return on capital falls short of the social return on capital.

Thus we set $TFP_t = K_t$ in the equations derived in the previous section. The marginal cost equation (17) and the aggregate production function become

$$mc_t = \frac{(r_t^L)^\alpha w_t^{1-\alpha}}{A\alpha^\alpha(1 - \alpha)^{1-\alpha}(\phi_1 K_t)^{1-\alpha}} \quad (31)$$

$$Y_t = AK_t(\phi_1 (n_t - n^s - \pi))^{1-\alpha} \quad (32)$$

To arrive at from the production function (29), after setting $TFP_t = K_t$, we divide by $n_t(i) - \pi$. As the capital labour ratio and the output per unit of productive labour ratio are the same across all firms, we arrive at (32).

The capital stock now has a stronger effect on both marginal costs and output than in the JLN economy. An increase in the capital stock by 1% for a given employment level (implying that output expands at the same rate) reduces marginal costs by 1%. In the absence of endogenous growth the effect is only $\alpha\%$. This can be seen by substituting the capital rental out of equations (17) and (31) and then substituting $\frac{Y_t}{K_t}$ using the respective production functions.

\(^{14}\)See Barro/ Sala-i-Martin (2004), pp.21-22.
Accordingly, the capital stock also has a greater effect on natural employment and the NAIRU. The equivalents of equations (28) are

\[ \mu^{-1} = \left( n^n_t - n^s - \bar{n} \right) w^n_t \frac{A}{1 - \alpha} \left( \phi_1 \right)^{1-\alpha} K_t \]

\[ \log w^n_t - \log w_{t-1} = a + b \left( n^n_t - \bar{n} \right) + c \log \left( \frac{w_{t-1} \left( n_{t-1} - \bar{n} - n^s \right)}{Y_{t-1}} \right) \]

\[ Y^n_t = A K_t \left( n^n_t - n^s - \bar{n} \right)^{1-\alpha} \] (33)

Clearly, an increase in the capital stock now accommodates a larger increase in natural employment than in (28).

### 2.6 The Aggregate Equations

This section summarises the models aggregate equations developed above for convenience of the reader and introduces explicit functional forms where that has not yet been done above. As many of the economy’s variables are growing in the steady state \((Y_t, C_t, I_t, w_t, K_t)\), simulation of the model requires normalising those variables with a cointegrated variable. It is very convenient from a technical point of view to normalise with respect to the capital stock. How that is done is shown in the appendix, as well as the computation of steady state values of the stationarised variables.

Aggregate demand is the sum of consumption, investment, the amount of price adjustment costs and government expenditure:

\[ AD_t = C_t + I_t + \frac{\varphi}{2} (\pi_t - \pi_{t-1})^2 Y_t + w_t n^s \] (34)

We will assume logarithmic utility so that the consumption Euler equation becomes

\[ 1/ \left( C_t - hab_{t-1} \right) = \beta (1 + i_t) E_t \left[ \frac{1}{(C_{t+1} - hab_t) (1 + \pi_{t+1})} \right] \] (35)

The level of habit is given by

\[ hab_{t-1} = j C_{t-1} \]
Investment expenditures is governed by the following equations:

\[ \lambda_t = \frac{1}{C_t - hab_{t-1}} \]  
(36)

\[ \beta E_t \left( \lambda_{t+1}r_t^k + \lambda_{t+1}q_{t+1} (1 - \delta) \right) = \lambda_t q_t \]  
(37)

\[ \lambda_t q_t \left[ \left( 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - (1 + g) \right)^2 \right) - \frac{I_t}{I_{t-1}} \kappa \left( \frac{I_t}{I_{t-1}} - (1 + g) \right) \right] \]  
(38)

\[ + \beta E_t \left[ \lambda_{t+1}q_{t+1} \left( \frac{I_{t+1}}{I_t} \right)^2 \kappa \left( \frac{I_{t+1}}{I_t} - (1 + g) \right) \right] = \lambda_t \]

while capital accumulation is given by

\[ K_{t+1} = (1 - \delta) K_t + I_t \left( 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - (1 + g) \right)^2 \right) \]  
(39)

The capital rental is given in both models by

\[ r_t^k = \alpha mc_t \frac{Y_t}{K_t} \]  
(40)

However, with endogenous growth, we can write \( r_t^k \) as a function of employment and marginal costs alone, namely as

\[ r_t^k = \alpha mc_t A(\phi_1 (n_t - n^s - \bar{\pi}))^{1-\alpha} \]  
(41)

Marginal cost in the JLN economy becomes

\[ mc_t = \frac{(r_t^k)^{\alpha} w_t^{1-\alpha}}{A\alpha^{\alpha}(1 - \alpha)^{1-\alpha}(\phi_1 TFP_t)^{1-\alpha}} \]  
(42)

while in the presence of endogenous growth, we have

\[ mc_t = \frac{(r_t^k)^{\alpha} w_t^{1-\alpha}}{A\alpha^{\alpha}(1 - \alpha)^{1-\alpha}(\phi_1 K_t)^{1-\alpha}} \]  
(43)

Wages are set according to equation (20):

\[ \log w_t - \log w_{t-1} = a + b * (n_t - \bar{\pi}) + c \log \left( \frac{w_{t-1} (n_{t-1} - \bar{\pi} - n^s)}{Y_{t-1}} \right) \]  
(44)

Total output in the absence of endogenous growth is given by private sector output \( Y_t \) plus the output of state employees:

\[ Output_t = AK_t^\alpha (TFP_t \phi_1 (n_t - \bar{\pi} - n^s))^{1-\alpha} + w_t n_t^s \]  
(45)
while in the presence of endogenous growth, we have

$$Output_t = AK_t((n_t - \pi - n^s) \phi_1)^{1-\alpha} + w_t n^s$$

Markets clear:

$$AD_t = Output_t$$

The evolution of prices is determined by the Phillips Curve, where we replace the stochastic discount factor by its definition

$$\rho_{t,t+1} = \beta u'(C_{t+1} - hab_t) = \beta \frac{C_t - hab_{t-1}}{C_{t+1} - hab_t}$$

$$(1 - \theta) + \theta mc_t - \varphi \left( \left( \frac{P_t}{P_{t-1}} - u_t \right) - \frac{P_{t-1}}{P_{t-2}} \right) \left( \frac{P_t}{P_{t-1}} - u_t \right) + \theta \frac{\varphi}{2} \left( \frac{P_t}{P_{t-1}} - u_t \right) - \frac{P_{t-1}}{P_{t-2}} \right)^2$$

$$+ \beta E_t \left[ \frac{C_t - hab_{t-1}}{C_{t+1} - hab_t} \varphi Y_{t+1} \left( \frac{P_{t+1}}{P_t} - \left( \frac{P_t}{P_{t-1}} - u_t \right) \right) \frac{P_{t+1}}{P_t} \right] = 0 \quad (46)$$

Finally, monetary policy is specified by equation (26)

$$i_t = (1 - \rho) \left( \bar{i} + \psi_{\pi} \pi_t + \frac{\psi_Y}{4} g p_{t-1} \right) + \rho i_{t-1} \quad (47)$$

with $gp_t$ as defined in (27) with natural output as determined in (28) for the JLN economy and as determined in (33) for the New Growth economy.

### 3 Simulation Setup and Calibration

We aim to create a scenario akin to the one faced by central banks in Western Europe at the end of the seventies and the beginning of the 1980s. That means we would like to create a situation where annual inflation increases several percentage points above its target level for some time and is then subsequently reduced. Therefore $u_t$ is set equal to 0.03 for the first quarter and the model is simulated under perfect foresight. To put it differently, for given values of marginal cost, past and expected inflation, inflation in that quarter is increased by three percentage points. In the baseline simulation, this will give rise to a disinflation of a bit more than 4.6 percentage points over 5 years, if we compare annual rates in the first and the sixth year. This is at the lower end of disinflations actually experienced during that period. For instance, in Germany, annual inflation was at 6.3% in 1981, which was then reduced to -0.1% in 1986, which is a rather small disinflation compared to the UK, France or Italy were inflation declined by 8.6, 10.8 and 13.7 percentage points over the same period, respectively. Note that there is no endogenous persistence in the shock itself beyond the first quarter, implying that any persistence in the path of the variables and in particular unemployment beyond that point is endogenous. The models are solved employing a second order approximation to the policy function using the approach
of Schmitt-Grohe and Uribe (2004). We use the software Dynare to implement the solution.\footnote{To be able to solve our two growth models, we normalise with respect to the capital stock and total factor productivity (see Appendix).}

The calibration of the non-monetary policy model parameters for the experiment described above is presented in table 1. It was arrived at as follows. We distinguish between four different types of parameters. The first set is calibrated according to standard values in the literature. This set contains the utility discount factor $\beta$, the private output elasticity of capital $\alpha$, the elasticity of substitution between varieties of goods $\theta$, the depreciation rate $\delta$, and the price adjustment cost parameter $\varphi$. $\varphi$ is calibrated as to generate marginal cost coefficient in the linearised version of equation 12 which would also be generated in a Calvo Phillips Curve with full backward indexing of unchanged prices and a probability of no re-optimisation is $2/3$.

The second set, consisting of $n^s$, $a$, $b$ and $c$, is based on empirical evidence. $n^s$ is calculated from data of the German statistical office on the number of full time equivalent employees in the public sector and on total hours worked in the economy in 2006. $b$ and $c$ are calibrated to be consistent with an estimate of that function. We estimate (44) on German data on hourly labour costs, unemployment (instead of employment, as is done in the empirical literature) and the labour share in GDP ranging from 1970 to 2000. We then calibrate the intercept $a$ to achieve a steady state unemployment rate of 4%.

The third set consists of the three "free" parameters $A$, $κ$ and $j$ the production function multiple, the parameter indexing adjustment costs and the degree of habit formation. They were calibrated to match second moments of a couple of important variables in German data. The results are discussed in an extended version of this paper.

Table 1: Calibration of non-policy Parameters

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$j$</th>
<th>$A$</th>
<th>$\theta$</th>
<th>$\delta$</th>
<th>$\varphi_1$</th>
<th>$\varphi$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$κ$</th>
<th>$u_1$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.33</td>
<td>0.99</td>
<td>0.4</td>
<td>0.38</td>
<td>6</td>
<td>0.025</td>
<td>0.452</td>
<td>-0.1123</td>
<td>0.08</td>
<td>-0.1</td>
<td>0.65</td>
<td>0.03</td>
<td>0.1793</td>
<td></td>
</tr>
</tbody>
</table>

$n^s$  | $i$   | $g_{TFP}$ | $\sigma_u$ |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.18</td>
<td>0.0181</td>
<td>0.0079</td>
<td>0.003</td>
</tr>
</tbody>
</table>

The baseline calibration of the monetary policy reaction function is taken from Clausen and Meier (2003), who estimate a Bundesbank policy rule over the period from 1973 to 1998 for quarterly data. Clausen and Meiers best performing model yields the statistically significant coefficients on output, inflation and the lagged interest rate reported in table 2 which in fact correspond to the original coefficients proposed by Taylor (1993) to characterise the policy of the Federal Reserve. Their estimate of the output gap coefficient is of particular interest because the Bundesbank was often perceived as paying less attention to output than the Fed. This is also borne out by other Taylor-rule estimates, one of which we discuss below. For the purpose at hand, we consider using the least hawkish baseline coefficients for the
policy rule in the literature of Bundesbank Taylor rule estimates. It will become clear why this is the case when we discuss the simulation results.

**Table 2: Baseline Calibration of the Policy Rule: Clausen and Meier (2003)**

<table>
<thead>
<tr>
<th>$\psi_\pi$</th>
<th>$\psi_Y$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.52</td>
<td>0.75</td>
</tr>
</tbody>
</table>

However, we are also interested in comparing the effects of different policies estimated for the Bundesbank and the Federal Reserve. Therefore we would like to draw on a study using the same methodology to estimate policy rules for different countries, Clarida et al. (1998). Their rule is estimated using monthly data. A quarterly data version of their specification would be

$$i_t = (1 - \rho) \left( \bar{i} + \psi_\pi E_t \left( \frac{\pi_{t+1} + \pi_{t+2} + \pi_{t+3} + \pi_{t+4}}{4} \right) + \frac{\psi_Y}{4} \hat{g}_t \right) + \rho i_{t-1} \quad (48)$$

Hence the central bank responds to a one year forecast of inflation, the current output gap and the lagged interest rate. They measure potential output using a quadratic trend of a West German industrial production index and their data set stretches from 1979 to 1993 and estimate the policy rule using the general method of moments. The point estimates are replicated in table 3. Clearly, the small coefficient on the output gap corresponds more to the conventional wisdom on how the Bundesbank was conducting policy.

**Table 3: Forward looking interest rate Rule: Clarida, Gali and Gertler (1998)**

<table>
<thead>
<tr>
<th>$\psi_\pi$</th>
<th>$\psi_Y$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.31</td>
<td>0.25</td>
<td>0.91</td>
</tr>
</tbody>
</table>

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4 Some Moment Comparison

We now report the results of comparing the second moments generated by stochastic simulations of the model economy to the corresponding empirical moments for German data. The moment comparison forms an important part in the calibration of the model. The three free parameters $\kappa$, $A$ and $j$ where calibrated with an eye on the empirical standard deviation of the investment/capital ratio to the output capital ratio and the persistence of employment and consumption, both as measured by the first to fifth order autocorrelation. We report some selected second moments of other important variables to give an idea how the model in the chosen calibration matches the data. We carry out the same comparison for the JLN economy, and for both the baseline policy reaction function and the Clarida, Gali Gertler (1998) estimate.

We consider the following variables: The ratios of (total) output, consumption, investment and real wages to capital, denoted as $F_t$, $D_t$, $R_t$ and $H_t$ respectively (recall that we have to normalise all the trended variables with the capital stock to render them stationary) and employment $n_t$ (measured as linearly detrended log hours), the nominal interest rate $i_t$, inflation $\pi_t$ (measured as the change in the consumer price index (CPI)), productivity growth $p_t$ (measured as change in real GDP per hour worked), capital stock growth $g_t$, and the investment/savings rate $I/Y$. From those, we compute the following moments: The coefficient of variation for output, the relative standard deviations of $D_t$, $R_t$ and $\pi_t$ to $F_t$, the standard deviations of employment, the savings rate and capital stock growth, the cross-correlation of all variables with $F_t$ and the autocorrelation of each variable up to the fifth order. We conduct the moment comparison for both the baseline case and the reaction function estimated by Clarida, Gali and Gertler.

The construction of the data for $F_t$, $D_t$, $R_t$ and $H_t$ are discussed in the Appendix. The raw data was obtained from the Statistische Bundesamt, except for the nominal interest rate and the inflation data which was obtained from the "International Financial Statistics" CD-ROM. The data set ranges 1970:Q1 to only 1990:Q4 because reunification is associated with a big drop in $F_t$, $D_t$ and $R_t$, which would distort the moments. Furthermore, there are strong theoretical reasons to believe that all variables other than employment, inflation and the nominal interest rate are stationary. This is why we do not detrend or filter them. However we adjust the sample to induce stationarity if stationarity is not confirmed for the full sample by either an ADF test (by rejecting the null of a unit root) or a KPSS test (by not rejecting the null of stationarity). Where we have to detrend, we use a linear time trend. The details are given in the appendix.

Table 4 reports the various standard deviations, relative standard deviations and cross-correlations with the output capital ratio $F_t$ listed above. Column 1 contains the data, while column 2 and 3 refer to the baseline policy reaction function. The standard deviation of employment for the New Growth economy is on the mark because we have calibrated the standard deviation of the cost push shock accordingly. The resulting
The coefficient of variation of $F_t$ for the New Growth Model (NGM) is smaller than in the data. It is in fact almost equal to the standard deviation of employment, which is in fact also true for the JLN economy. The relative standard deviation of $D_t$ in the New Growth model is very close to the data, while in the JLN economy, it is far too low. The relative standard deviation of $R_t$ with respect to $F_t$ is close to the data in both models but closer in the New Growth economy. The standard deviations of capital stock growth is very close to the data in the New Growth economy. The same holds for the standard deviation of capital stock growth relative to the standard deviation of employment (0.0766 as opposed to 0.0714 in the data). This is important because movements capital stock growth rates drive the results (and in particular employment) in the New Growth economy discussed in the next section. Therefore we would like the standard deviation of capital stock growth relative to employment to be close to the data. In the JLN economy, this relative standard deviation is too high.

Turning to the cross-correlations, what is most striking is that for the JLN economy, $corr(i_t, F_t)$, $corr(\pi_t, F_t)$, $corr(p_t, F_t)$ are wrongly signed. They are negative where the data is positive. The New Growth model produces wrong signs for $corr(\pi_t, F_t)$, though the absolute value is much smaller than for the JLN Economy, and $corr(H_t, F_t)$. The magnitudes of $corr(D_t, F_t)$ and $corr(R_t, F_t)$ are not too far away from the data for both models, while for $corr(n_t, F_t)$, both models produce considerably too high values. It is particularly interesting that the New Growth model produces a positive correlation between the output capital ratio and the nominal interest rate. Correctly matching the correlation of output with inflation and the nominal interest rate is generally perceived as a difficulty in New Keynesian models if demand shocks are absent.20

<table>
<thead>
<tr>
<th>Moments</th>
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<th>NGM</th>
<th>CGG: JLN</th>
<th>CGG: NGM</th>
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</thead>
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<tr>
<td>$sd: F_t/\text{mean} F_t$</td>
<td>0.0272</td>
<td>0.0115</td>
<td>0.0192</td>
<td>0.0077</td>
<td>0.0215</td>
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<tr>
<td>$sd: D_t/sd: F_t$</td>
<td>0.6179</td>
<td>0.4447</td>
<td>0.5936</td>
<td>0.4619</td>
<td>0.5910</td>
</tr>
<tr>
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<td>0.4888</td>
<td>0.5783</td>
<td>0.4540</td>
<td>0.6072</td>
<td>0.4812</td>
</tr>
<tr>
<td>$sd: n_t$</td>
<td>0.0196</td>
<td>0.0112</td>
<td>0.0209</td>
<td>0.0074</td>
<td>0.0235</td>
</tr>
<tr>
<td>$sd: (I_t/Y_t)$</td>
<td>0.0092</td>
<td>0.0048</td>
<td>0.0053</td>
<td>0.0035</td>
<td>0.0061</td>
</tr>
<tr>
<td>$sd: \pi_t/(sd: F_t/\text{mean} F_t)$</td>
<td>0.208</td>
<td>0.3645</td>
<td>0.2001</td>
<td>0.8801</td>
<td>0.1982</td>
</tr>
<tr>
<td>$sd: g_t$</td>
<td>0.0014</td>
<td>0.0012</td>
<td>0.0016</td>
<td>0.0009</td>
<td>0.0018</td>
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<tr>
<td>$corr(D_t, F_t)$</td>
<td>0.8658</td>
<td>0.95</td>
<td>0.9923</td>
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<tr>
<td>$corr(R_t, F_t)$</td>
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<tr>
<td>$corr(n_t, F_t)$</td>
<td>0.5921</td>
<td>0.95</td>
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<td>0.1557</td>
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<td>0.2263</td>
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<tr>
<td>$corr(p_t, F_t)$</td>
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<td>0.7587</td>
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<tr>
<td>$corr(H_t, F_t)$</td>
<td>0.4924</td>
<td>0.4476</td>
<td>-0.6729</td>
<td>0.4468</td>
<td>-0.7258</td>
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</tbody>
</table>

Table 5 reports the autocorrelation up to the fifth order for the data and the baseline case. For those variables which we do not reject the null of stationarity over the full sample we use the dataset starting in 1970 rather than the reduced dataset starting in 1974 in order not to unnecessarily sacrifice information. When the i-th order autocorrelation of a variable is within $\pm 0.1$ of the corresponding autocorrelation in the sample, it is printed in bold. A number in italics means that the value is closer to the data than the i-th order autocorrelation of the same variable in the competing model. Concerning the variables $F_t$, $D_t$, and $n_t$, we observe that the New Growth economy is matching the persistence the data quite closely. By contrast, $R_t$, $g_t$, $i_t$ and $I_t/Y_t$ are considerably less persistent in the New Growth model than in the data, although they are still considerably closer to the data than in the JLN economy. Conversely, all these variables show far too little persistence in the JLN economy (and for all variables less than in the New Growth economy): The autocorrelations are dying off too quickly.

For $\pi_t$, both models produce very similar autocorrelations. They match the first order autocorrelation but all the remaining ones are incorrectly signed. For $p_t$, both models produce incorrectly signed first and second order autocorrelations. The JLN economy then does match the sign of the third order autocorrelation but produces wrong signs for the remainder. The New Growth economy produces a wrong sign for the third order autocorrelation but almost matches the fourth and matches the sign of the fifth. For the real wage to capital ratio $H_t$, both models match the first to fourth order autocorrelation, though the JLN economy comes closer to the data. The New Growth economy fails match the fifth order autocorrelation, while the JLN economy does.

Thus the New Growth model does mostly better than the neoclassical at matching the data’s second moments for the baseline central bank reaction function, with very few exceptions.

**Table 5: Autocorrelations, Baseline**
We now turn to the reaction function estimated by Clarida, Gali and Gertler (1998). The relative standard deviations and cross correlations can be obtained from columns 4 and 5 of table 4. Again the standard deviations of $F_t$ and $i_t$ are quite close to each other for both models, unlike in the data. The New Growth economy still closely matches the relative standard deviation of $D_t$ and $R_t$ (the later even better than before). In the JLN economy the relative standard deviation of $D_t$ is still a good deal too low and the relative standard deviation of $R_t$ is even further from the data than before. $corr(D_t, F_t)$ and $corr(R_t, F_t)$ are almost equal while $corr(n_t, F_t)$ is considerably reduced (and thus brought closer to the data) for the JLN economy.
$corr(H_t, F_t)$ and $corr(p_t, F_t)$ also show some change in magnitude but not in signs. By contrast, $corr(\pi_t, F_t)$ becomes positive in both models, with the New Growth model coming very close to the data. Concerning the autocorrelations, which are reported in Table 6, note that they generally increase somewhat in the New Growth model, much so in case of $i_t$, but decrease in the neoclassical model, with the exception of $i_t$ and $\pi_t$. Thus we conclude that the New Growth model is still better at matching the second moments discussed here, in particularly the persistence in the data, than the JLN economy.
Table 6: Autocorrelations, Clarida, Gali Gertler Reaction Function

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<tr>
<th>Order of Autocorrelation</th>
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<th>NGM</th>
<th>Data</th>
<th>JLN</th>
<th>NGM</th>
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<td>0.34</td>
<td>0.91</td>
</tr>
<tr>
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<td>$D_t$</td>
<td>$\pi_t$</td>
<td>$\pi_t$</td>
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<td>0.58</td>
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</table>

5 Simulation Results

Figure 3 plots the response of actual unemployment for the JLN and the New Growth economy to the one quarter cost push shock. In all figures, the initial value is the steady state value of the respective variable. Furthermore, when we refer to “Baseline” in figures or in the text, we always mean the New Growth economy in its baseline calibration.
In the JLN economy, unemployment increases by about 3 percentage points on impact but starts recovering after reaching a maximum of 10.4%. It then quickly recovers and in quarter 8 practically returns to its steady state value and then slightly overshoots for some time. Employment would be expected to decrease because the cost push shock will increase inflation which will ultimately lead to an increase in ex ante real interest rates via equation 13. As consumers and investors are forward looking, this causes a contraction of aggregate demand on impact. Figure 4 plots the inflation rate, which peaks in quarter 1 at a value of about 3.8% and then quickly declines back to zero.
By contrast, in the New Growth economy, unemployment increases by more on impact than in the JLN economy. Even more important, the increase is far more persistent. After about 11 quarters (10 quarters after the end of the shock), when employment is already overshooting in the JLN economy, only a bit more than half of the on-impact loss in employment has vanished and employment is still about 3.2 percentage points below its steady state value. What is more, employment growth then comes to a halt: quarterly increases are around 0.06 percentage points per
quarter or less. As can be seen in table 4, in the New Growth economy, after 10 years unemployment is still about 1.8 percentage points above its steady state value and after 15 years the difference is still about 1.2 percentage points. Thus as often observed in the Europe, unemployment increases quickly but falls only very slowly. Furthermore, Figure 5 reveals that the persistent increase in actual unemployment is matched by an increase in the NAIRU, as after six quarters, actual unemployment falls below the NAIRU, which gradually increases during and after the recession. A glance at Figure 4 shows that inflation (after peaking in quarter 1 at a quarterly rate of about 3.3 percentage point) indeed stops declining at about the same time actual unemployment falls below the NAIRU, as we would expect from the definition of the NAIRU.

**Table 4: Unemployment deviation from the Steady State in the New Growth Economy, Baseline and \( \psi_y = 5 \)**

<table>
<thead>
<tr>
<th>Quarters</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
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</thead>
<tbody>
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<td>Baseline</td>
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<td>2.3</td>
<td>1.8</td>
<td>1.5</td>
<td>1.2</td>
<td>1.0</td>
<td>0.8</td>
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<tr>
<td>( \psi_y = 5 )</td>
<td>1.9</td>
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<td>1.2</td>
<td>1.0</td>
<td>0.8</td>
<td>0.6</td>
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We know from (44) that an increase in unemployment will reduce real wage growth which would tend to lower marginal costs, so there must be a strong countervailing force pushing marginal costs up in order to explain why inflation stops falling. Figure 6 shows that while real wage growth drops sharply, in quarter 2 the growth rate of the capital stock falls by even more and remains considerably below real wage growth for about 9 quarters. After that they are about equal. Slower capital stock growth entails slower technological progress and thus slower growth of labour productivity, which will tend to generate a higher trajectory of marginal cost for a given level of real wage growth. In the New Growth model, the movement of real wages relative to labour productivity for a given employment level is thus captured by the evolution of the wage capital ratio. This variable matters a lot for marginal cost, as shown by (43). Figure 7, which plots the deviations of marginal cost and the wage capital ratio from their steady state values confirms that it is the movement of the real wage capital ratio which drives marginal cost back up, as both move broadly in parallel.
By contrast, in the JLN economy, the effect of the capital stock on marginal costs is much weaker. The major determinant of marginal costs apart from real wages is total factor productivity $T F P_t$. This grows exogenously no matter whether
output and investment are contracting or growing. Thus marginal costs or, to put it differently, the permissible, non-inflationary rate of real wage growth are much less affected by changes to the capital stock.

Turning back to the New Growth economy, the recovery of actual employment has to slow down after about 6 quarters because unemployment arrives at a level beyond which any reduction would cause inflation to accelerate as it pushes real wage growth above the growth rate of the capital stock and thus pushes up marginal cost. This would trigger interest rate increases via the policy rule. In fact this is already happening as actual unemployment is falling below the NAIRU and inflation starts to pick up. To put it differently, the central bank does not have a reason to boost employment by aggressively lowering the interest rate because although inflation is somewhat below target, the output gap is closed as marginal cost equals its steady state value. Figure 8 shows that the central bank stops lowering the real interest rate \( i_t - E_t \pi_t \) after 8 quarters, when it is 0.45 percentage points (about 1.81 percentage points at an annualised rate) below the steady state value, and begins to tighten again.

This level of the real interest rate is not very expansionary, while below its steady state value, is not very expansionary. Figure 9 summarises the benefits from investing by plotting the present discounted value of an additional unit of capital, \( q_t \). \( q_t \) recovers quickly after the shock has passed and reaches its steady state value of one after five quarters. It then slightly exceeds it’s steady state level for six quarters. However, this is not sufficiently high to move the capital stock growth rate up quickly because of the investment adjustment costs: The first order condition \((??)\) determines the investment growth rate, which due to fast recovery of \( q_t \), moves much closer to it’s steady state value as well. However, the capital stock growth rate depends on the investment capital ratio, as can be seen from equation (39), which has declined during the recession and subsequent period of slow growth. Thus a faster recovery of capital stock growth would require an investment growth rate exceeding the steady state, which would have to be induced by an above steady state \( q_t \) which in turn would require a lower real interest rate.
The speed of recovery is then governed by the relative growth rates of real wages and the capital stock. From quarter 9 onwards, the capital stock grows slightly faster than real wages. This causes a slow decline in the wage-capital ratio, as can be obtained from figure 8, and allows for a slow reduction in unemployment because
higher productivity growth implies firms can accommodate the increased real wage growth associated with a tighter labour market without facing an increase in marginal costs. This, in turn, again increases capital stock growth by increasing the marginal product of capital.

Thus the disinflation engineered by the central bank, while clearly successful, has come at a cost beyond a temporary reduction in employment: The unemployment level consistent with constant inflation, has increased. Just as found by Ball, a successful disinflation during which the economy goes into a recession is followed by an increase in the NAIRU.

Associated with the increase in unemployment is a persistent slowdown in labour productivity growth. This is in line with the evidence cited above. It is easily shown that labour productivity growth across all employees in the New Growth model can be written as $p_t = \frac{\text{Output}_t}{\text{Output}_{t-1}} - 1$. Thus if quarterly employment changes are negligible, productivity growth equals output growth. Furthermore, with employment approximately constant, output growth is essentially equal to capital stock growth. Therefore, from quarter 9 onwards, $p_t$ approximately equals capital stock growth. At this point it falls short of its steady state value by about 0.23% per quarter or 0.92% at an annualised rate, while 40 quarters after the shock it is still about 0.11% lower than in the steady state, or 0.44% at an annualised rate. Average annualised productivity growth over the first 10 years after the shock equals 2.46%. Assuming that average productivity growth before the shock hit equalled its steady state rate of 3.42%, this implies a decline of average productivity growth from one decade to the next of 0.96%. Interestingly, average German productivity growth did decline by 1.44% from the 1970s to the 1980s.21

These results provoke the question how changes to the central bank's reaction function would affect the long-run paths of employment and inflation. Intuition would expect that a stronger weight on the output gap in the reaction function would lead to a smaller decrease in employment not just in the short but also in the long run. As investment would be squeezed less, there would be a smaller decline in capital stock growth which could accommodate higher of non-inflationary employment after the recovery from the recession. To show this we increase the coefficient on the output gap, $\psi_y$, to 5, leaving all other parameters the same. The corresponding evolution of unemployment can be obtained from figure 10. Indeed unemployment not only increases considerably less in the short run (in fact it decreases on impact), and after 40 quarters, it is still about 0.8 percentage point lower than in the Baseline case, as can be obtained from the second line of table 7. Hence a less hawkish monetary policy has indeed very long-lasting benign effects on unemployment.

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21Productivity is measured as real GDP per hour worked. The data was taken from Statistisches Bundesamt Wiesbaden (2007b). A sophisticated analyses of changes in trend productivity growth by Skoczylas and Tissot (2005) finds a negative break for Germany in 1979 of -2.75%
The lower increase in unemployment comes at the cost of a considerably stronger short run inflation surge. While in the baseline simulation, inflation peaks a (quarterly) rate of 3.3%, it now increases as high as 4.9% in the first quarter, as can be obtained from figure 11, while the annual inflation rate over the first year amounts to 15%. Note however that the increase in inflation is only temporary. After 10 quarters,
it has already decreased to 0.42%. Thus the stronger acceleration in inflation is a short run phenomenon. The gain in employment is of more persistent nature.

As mentioned above, Ball (1999) finds that during the recessions of the early 1980s, countries whose central banks aggressively lowered interest rates experienced smaller increases in the NAIRU than those which did not. Ball calculates the difference between the NAIRU in the year before the recession and five years after. He defines a recession as one year with GDP growth below 1%. He regresses this on the maximum reduction of the ex-post real interest rate during any time of the recessions first year, which he refers to as maximum easing. The coefficient on maximum easing is -0.42 and is significant at the 10% level. We try to replicate this relationship with our New Growth model by varying the output gap coefficient between 0 and 4, leaving everything else the same, thus obtaining data on maximum easing and the five year change in the NAIRU. Our resulting coefficient on maximum easing is negative as well and varies between -0.24 and -1.16. This is for the most part consistent with Balls estimate.

6 Cross Country Aspects

The previous section shows that our New Keynesian model with endogenous growth is able to produce a persistent increase in unemployment as a consequence of a disinflation. This is an important result because economists have been struggling to explain the evolution of unemployment in continental Europe over time. This begs the question whether we can also use the model to replicate differences in unemployment evolutions across countries. We address this issue in three different ways in this section. For that purpose, we will draw on the differences in the size of the disinflation across the OECD, in (estimates of) the policy reaction function coefficients between the Bundesbank and the Federal Reserve and in real wage rigidity.

We noted earlier that there is a negative correlation between the change in inflation and the change in the NAIRU. Ball (1996) investigated this for the 1980s and we plotted it over two decades and across 21 OECD countries in figure 2. There are various possible reasons why countries might have differently sized disinflations. Economies might differ in the way they respond to global supply shocks, perhaps due to differences in energy intensity of production. Their past record of monetary policy might differ as well, (in the sense that some central banks have let inflation spiral more out of bounds than others, leading to larger deviations of inflation from target), as might choices of how much to disinflate (a central bank might just be willing to accept a higher inflation rate following a supply shock). Finally, exchange rate volatility might differ as well. Incorporating these various sources of inflation volatility into our model would be far beyond the scope of this paper. However, we

\footnote{Ball controls for the duration of unemployment benefits.}

\footnote{See Ball (1999), p. 207.}
do try to mimic their inflationary impact by varying the size of the cost push shock. We vary the size of the cost push shock from 0.01 to 0.05, leaving all other parameters unchanged. Then we calculate the change from year 1 to year 10 of the inflation rate during those years and the NAIRU in the first quarter of those years, and plot the later against the former in figure 12. There is a clear negative correlation. The slope of the line varies between -0.41 and -0.56, which not too far away from the simple regression coefficient of -0.33 (or -0.36 if, like Ball (1996) we exclude Greece) resulting from a regression of the change in the NAIRU on the change in inflation using the OECD data presented earlier.

Let us now take a look at the effect of observable differences in the monetary policy rule. To get a proper idea of the effects of these it is obviously important to have comparable estimates. Therefore we make use of the fact that Clarida et al. (1998) estimated the same policy rule using the same methodology for several countries, including Germany and the United States. We would have liked to draw on real time estimates as in the previous section but to our knowledge, internationally comparable estimates of this kind do not exist. The coefficient estimates of Clarida et al. of (48) for the Federal Reserve are reproduced in table 3.

We now repeat the same experiment we conducted in the last section for both the estimates for the Bundesbank reaction function and the coefficients of the Federal Reserve.

---

24We take the difference of the first quarter of both years since the NAIRU moves up very fast during the first four quarters. Differencing the annual averages of the two years would create a misleading impression of the correlation between the medium run change in the NAIRU (by unduly reducing this change) and the change in inflation. The quarterly movements of the NAIRU in the OECD data are very slow and redoing figure one with the difference in the NAIRU between 1980 quarter 1 1990 quarter 1 rather than with the differences in the annual averages as is the case now would not change the result.
Reserve. The first two lines of Table 8 show the deviation of unemployment from
is steady state for both set of coefficients. Note first that the persistent increase in
unemployment with the policy rule as specified and estimated by Clarida Gali, and
Gertler for the Bundesbank is substantially higher than the increase we saw with the
policy rule used in the Baseline. This illustrates that, in terms of the unemployment
effects which are the subject of this paper, we were quite conservative in specifying and
calibrating our Baseline policy rule. Apart from that, unemployment is persistently
higher under the Bundesbank rule than under the Federal Reserve one, though the
difference is for the most part less than one percentage point. For instance after 10
years, unemployment and the NAIRU are about 0.5 percentage points higher under
the Bundesbank Rule than under the Federal Reserve rule.

It is, however, informative to take a look at the standard errors associated with
Clarida, Gali and Gertlers (CGGs) estimate. For instance, the standard error asso-
ciated with the coefficient on the lagged interest rate \( \rho \) has as standard error of 0.03.
Thus a value for \( \rho \) of 0.06 is still consistent (at a 5% level of confidence) with the CGG
estimate. The third row of table 9 shows the implied evolution of unemployment if
we set \( \rho = 0.91 \). The resulting unemployment trajectory is substantially lower than
before. After 40 quarters, the unemployment and the NAIRU are now 1.1 percentage
points lower than under the Bundesbank rule, while after 50 quarters, the difference
is still 1 percentage point. In the same manner, we can also make use of the standard
error of the estimate of \( \psi_Y \), which equals 0.16. Increasing \( \psi_Y \) to 0.88 yields the em-
ployment trajectory shown in the final row of table 8, which is again lower than with
the point estimate. After 40 quarters, unemployment is and the NAIRU are about
1.1 percentage points lower than under the Bundesbank policy rule. Thus in the New
Growth model, differences in policy function parameters consistent with the CGG
evidence can contribute to explaining the different evolutions of the unemployment
rate in Germany as compared to the United States.

Accordingly, differences in monetary policy also explain differences in the change
in the productivity growth rate between Germany and United States from the 1970s
to the 1980s. As noted above, average US productivity growth declined by only about
0.18% from the 1970s to the 1980s, whereas the decline in Germany was about 1.4%.
Table 9 displays the difference between average annualised productivity growth during
the first decade after the shock and the decade before the shock.\(^{25}\) Thus within the
New Growth model, differences in monetary policy would account for between 0.24
and 0.6 percentage points of the difference in productivity growth.

\(^{25}\) As above we assume that during the decade before the shock hits, the average productivity
growth rate equalled its steady state.
### Table 8: Results for CGG Policy Rules

<table>
<thead>
<tr>
<th>Quarter</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bundesbank</td>
<td>3.8</td>
<td>3.8</td>
<td>3.1</td>
<td>2.5</td>
<td>2.1</td>
<td>1.7</td>
<td>1.4</td>
<td>1.1</td>
</tr>
<tr>
<td>Federal Reserve</td>
<td>2.9</td>
<td>3.1</td>
<td>2.5</td>
<td>2</td>
<td>1.7</td>
<td>1.3</td>
<td>1.1</td>
<td>0.9</td>
</tr>
<tr>
<td>Federal Reserve, $\rho = 0.91$</td>
<td>2.1</td>
<td>2.1</td>
<td>1.7</td>
<td>1.4</td>
<td>1.1</td>
<td>0.9</td>
<td>0.7</td>
<td>0.6</td>
</tr>
<tr>
<td>Federal Reserve, $\psi_Y = 0.88$</td>
<td>2.4</td>
<td>2.5</td>
<td>2</td>
<td>1.6</td>
<td>1.3</td>
<td>1.1</td>
<td>0.9</td>
<td>0.7</td>
</tr>
</tbody>
</table>

### Table 9: Results for CGG Policy Rules

<table>
<thead>
<tr>
<th>Change in ten Year Average Productivity Growth, percentage Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bundesbank</td>
</tr>
<tr>
<td>Federal Reserve</td>
</tr>
<tr>
<td>Federal Reserve, $\rho = 0.91$</td>
</tr>
<tr>
<td>Federal Reserve, $\psi_Y = 0.88$</td>
</tr>
</tbody>
</table>

Finally, we explore the effects of the observed cross continental differences in the nature of real wage rigidity. Estimating (44) on U.S. data confirms the finding of other researchers that $c = 0$, while the U.S. estimate of $b$ is not significantly different from the value we employed so far (0.08). Therefore, in our final experiment aimed at highlighting cross country dimensions, we set $c = 0$ in the Baseline calibration, leaving everything else as in the Baseline. The resulting deviation of unemployment from its steady state can be obtained from table 11. Clearly, the increase in unemployment is persistently lower. After 40 quarters, unemployment is only 0.6 percentage points higher than in the steady state, compared to 1.7 percentage points in the Baseline. Average annualised productivity growth is only 0.36% lower than in the previous decade as opposed to 0.96% in the baseline calibration.

Within our model, $c=0$ would arise if there is no direct effect of productivity on effort and if benefits are not linked productivity. We suggested above that these results might be rooted in stronger unions and perhaps a stronger link between unemployment benefits and productivity in Europe. Blanchard and Wolfers (2000) find that the impact of macroeconomic shocks on unemployment is affected by the labour market structure. They find that both unobservable macroeconomic shocks (captured by a time effect) as well as a one percentage point reduction in total factor productivity growth increase unemployment by more the higher is union density. This result is confirmed by Fitoussi et al. (2000). In that sense, our model provides some theoretical to the notion that both "shocks and institutions" (Blanchard and Wolfers) are crucial to explaining the cross country evidence on the evolution of unemployment.

### Table 10: $c = 0$ - Percentage point Deviation of Unemployment from its Steady State for selected Quarters

<table>
<thead>
<tr>
<th>Quarter</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>1.1</td>
<td>0.9</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>
7 Conclusion

This paper applies a New Keynesian with unemployment and endogenous growth to explain the persistent increase in continental European unemployment and the lack thereof in the United States. We calibrate key parameters like the coefficients in the wage setting equation and the interest feedback rule of the central bank to Western German data. The model economy is hit with a one quarter cost-push shock calibrated to induce a disinflation of an order of magnitude seen at the beginning of the 1980s in many industrialised OECD economies. We perform the same experiment on a model without endogenous growth: The JLN economy.

Under the baseline calibration, unemployment will still be about 1.8 percentage points above its pre-shock value after about 10 years. As can be observed with continental European unemployment rates, unemployment increases quickly but reverts only very slowly. At the same time, inflation stops declining soon after the cost push shock has vanished, implying that the NAIRU has increased. Unsurprisingly, no such effect is seen in the JLN economy, where unemployment is back to its steady state after about two years.

The increase in the NAIRU in the New Growth economy is brought about by the decline in investment during the recession required to disinflate the economy. In the New Growth economy, for a given employment level, capital stock growth determines labour productivity growth. Hence the real wage-capital ratio is the main driver of marginal costs. Thus, although wage growth declines as employment contracts, marginal cost returns back to its steady state level soon after the shock has vanished because capital stock growth for while declines even more. This stops the disinflation. The subsequent recovery is very slow because the central bank has no reason to lower interest rates. Its reaction function dictates that it reacts solely to inflation, being close to target, and the output gap, defined as the deviation of output from the level consistent with constant inflation, which is zero.

The model thus also contributes, to explaining the productivity slowdown observed across advanced OECD economies, and why negative shocks to productivity growth are frequently significant variables in regression of unemployment on this variable and others.

Finally, apart from generating a persistent increase in unemployment, the model also contributes to explaining cross country differences in unemployment. Varying the size of the cost push shock generates a relationship between the change in the inflation rate and the change in the NAIRU over a ten year horizon similar to a relationship in the data first observed by Ball (1996). Using comparable policy rule estimates of Clarida, Gali and Gertler (1998) for the Bundesbank and the Federal Reserves while holding the cost-push shock constant creates higher persistent unemployment with the later than with the former. Finally, taking account of a well established cross-continental difference in the structure of the wage setting function, namely the absence of a labour share term, also proves informative. In the absence of the labour
share term, we see a lower increase in the NAIRU. The size of the labour share term in wage setting can be linked, if coarsely so, to features of the labour market like union density or the benefit system. Thus the paper lends support to the view that, as suggested by Blanchard and Wolfers (2000), it is both "shocks and institutions" which are at the heart of explaining the evolution of unemployment across time and the differences across countries.

8 Appendix A: Normalised Version of the New Growth Model

As we are dealing with two growth models, we have to stationarise all variables which would otherwise be trended in order to be able to solve the model. This appendix applies this normalisation to the New Growth model. The resulting equations are those which have been solved and simulated. We define $C_t, K_t, \theta_{t-1}, L_t, R_t$, and $w_t$ as $D_t, H_{ab_{t-1}}, F_t$, $R_t$ and $H_t$, while the gross capital stock growth rate $\frac{K_{t+1}}{K_t} - 1$ is defined as $g_{t+1}^k$.

We directly apply the normalisation to the equations of the aggregate demand block:

$$F_t = D_t + R_t + \frac{\varphi}{2} (\pi_t - \pi_{t-1})^2 (F_t - H_t n^s) + H_t n^s$$ (49)

Consumption (remember $hab_{t-1} = jC_{t-1}$, thus $H_{ab_{t-1}} = \frac{D_{t-1}}{1 + g_{t-1}^k}$)

$$1/(D_t - H_{ab_{t-1}}) = \beta E_t \left[ (1 + i_t)/((1 + \pi_{t+1}) (D_{t+1} - H_{ab_t}) (1 + g_{t+1}^k)) \right]$$ (50)

$$H_{ab_t} = \frac{D_t}{1 + g_{t+1}^k}$$ (51)

Investment:

$$\beta E_t \left( \frac{1}{(D_{t+1} - H_{ab_t}) (1 + g_{t+1}^k)} \left( r_{t+1}^k + q_{t+1} (1 - \delta) \right) \right) = \frac{1}{D_t - H_{ab_{t-1}}} q_t$$ (52)

$$\frac{1}{D_t - H_{ab_{t-1}}} q_t \left[ \left( 1 - \frac{\kappa}{2} \left( \frac{R_t}{R_{t-1}} (1 + g_t^k) - (1 + g) \right)^2 \right) - \frac{R_t}{R_{t-1}} \kappa \left( \frac{R_t}{R_{t-1}} (1 + g_t^k) - (1 + g) \right) \right]$$ (53)

$$+ \beta E_t \left[ \frac{1}{D_{t+1} - H_{ab_{t+1}}} \left( \frac{R_{t+1}}{R_t} (1 + g_{k+1}^k) \right)^2 \kappa \left( \frac{R_{t+1}}{R_t} (1 + g_{k+1}^k) - (1 + g) \right) \right] = \frac{1}{D_t - H_{ab_{t-1}}}$$

From (39) we have

$$g_{t+1}^k = -\delta + R_t \left( 1 - \frac{\kappa}{2} \left( \frac{R_t}{R_{t-1}} (1 + g_t^k) - (1 + g) \right)^2 \right)$$ (54)
The rental on capital becomes:

\[ r^k_t = \alpha mc_t (F_t - H_t n^s) \]  \hspace{1cm} (55)

Substituting (55) into (17) and multiplying by \( \frac{K_t^{1-\alpha}}{K_t^{1-\alpha}} \) yields

\[ mc_t = \frac{(F_t - H_t n^s)^{1-\alpha}}{X} H_t \]  \hspace{1cm} (56)

where \( X = A^{\frac{1}{1-\alpha}} (1-\alpha) \phi_1 \).

The wage setting function \( \ln w_t = \ln w_{t-1} + a + b(n_t - \bar{n}) + c \log \left( \frac{w_{t-1} (n_{t-1} - \bar{n} - n^s)}{u_{t-1}} \right) \)

can be rewritten as (using equation (16)) \( \ln H_t = a + b(n_t - \bar{n}) + \ln \left( \frac{H_{t-1}}{(1 + g_t)} \right) + c \log ((1-\alpha) mc_{t-1}) \)

\[ H_t = \exp(a + b(n_t - \bar{n})) \frac{H_{t-1}}{(1 + g_t^c)} ((1-\alpha) mc_{t-1}) \]  \hspace{1cm} (57)

Employment: from \( Output_t = AK_t((n_t - \bar{n} - n^s) \phi_1)^{1-\alpha} + w_t n^s \), we have

\[ F_t = A((n_t - \bar{n} - n^s) \phi_1)^{1-\alpha} + H_t n^s \]  \hspace{1cm} (58)

The Phillips Curve and the Policy rule do not contain any trended variables and therefore does not need to be normalised. However, we will substitute the real profits stochastic discount factor by its definition, i.e. \( \rho_{t,t+1} = \beta \frac{u(C_{t+1} - Hab)}{u(Hab_{t-1})} = \beta \frac{C_{t+1} - Hab_{t-1}}{C_{t} - Hab_t} \),

which gives

\[ (1-\theta) + \theta mc_t - \varphi \left( \left( \frac{P_t}{P_{t-1}} - u_t \right) - \frac{P_{t-1}}{P_{t-2}} \right) \left( \frac{P_t}{P_{t-1}} - u_t \right) + \theta \frac{\varphi}{2} \left( \left( \frac{P_t}{P_{t-1}} - u_t \right) - \frac{P_{t-1}}{P_{t-2}} \right)^2 \]

\[ + \beta \frac{D_t - Hab_{t-1}}{F_t} \varphi E_t \left[ \frac{F_{t+1}}{D_{t+1} - Hab_t} \left( \frac{P_{t+1}}{P_t} - \left( \frac{P_t}{P_{t-1}} - u_t \right) \right) \left( \frac{P_{t+1}}{P_{t+1}} - u_t \right) \right] = 0 \]  \hspace{1cm} (59)

Replacing \( \frac{P_{t+1}}{P_{t+1}} = 1 + \pi_{t+1} \) gives

\[ (1-\theta) + \theta mc_t - \varphi \left( (\pi_t - u_t) - \pi_{t-1} \right) (1 + \pi_t - u_t) + \theta \frac{\varphi}{2} ((\pi_t - u_t) - \pi_{t-1})^2 \]

\[ + \beta \frac{D_t - Hab_{t-1}}{F_t} \varphi E_t \left[ \frac{F_{t+1}}{D_{t+1} - Hab_t} (\pi_{t+1} - (\pi_t - u_t)) (1 + \pi_{t+1}) \right] = 0 \]  \hspace{1cm} (62)
For natural output, natural employment and the natural real wage from equation

\[ \mu^{-1} = \frac{(F^n_1)^{\alpha}{H^n_t}}{X} \quad (63) \]

\[ F^n_t = A_t((n^n_t - \bar{n} - n^s) \phi_1)^{1-\alpha} + n^s H^n_t \]

\[ H^n_t = \exp(a + b(n^n_t - \bar{n})) \frac{H^n_{t-1}}{(1 + g^K_t)} ((1 - \alpha) mc_{t-1})^c \quad (64) \]

given last periods wage/capital ratio \( H_{t-1} \) and this periods capital stock growth rate \( g^K_t \) (which was also determined in the t-1 by the then investment decision). The output gap \( gp_t \) is then calculated as

\[ gp_t = \frac{Output_t - Output^n_t}{Output^n_t} \left( \frac{K_t}{K_t} \right) = \frac{F_t - F^n_t}{F^n_t} \quad (65) \]

9 Appendix B: Steady State Relations

This Appendix shows how to calculate the steady state values for the system developed in Appendix B. We will first derive a steady state relation between the level of employment and the steady state growth rate for the New Growth Economy.

First apply the fact that in the steady state, \( g^K_t = g \) to (53) which yields \( q = 1 \). We then apply this to (52) which yields

\[ \beta \left( r^{k}_{t+1} + (1 - \delta) \right) = (1 + g) \quad (66) \]

In the New Growth economy, we now replace the capital rental with equation (55) and, after using (58) and noting that in the steady state we have \( mc = \mu^{-1} \), arrive at

\[ g = \left[ \beta \left( (1 - \delta) + \alpha \mu^{-1} A((n - \bar{n} - n^s) \phi_1)^{1-\alpha} \right) \right] - 1 \quad (67) \]

This is the steady state growth rate which is borne out by the marginal product of capital in the endogenous growth economy. It is easily verified that it is increasing and concave in employment. It is straightforward to show that in the steady state, the real wage implied by the desired mark-up grows at the same rate as output and the capital stock by using \( mc_t = \mu^{-1} \) and \( r^K_t = r^k \) on (31). This yields

\[ w_t = K_t \phi_1 \left( \frac{\mu^{-1} A\alpha^\alpha (1 - \alpha)^{1-\alpha}}{(r^k)^{\alpha}} \right)^{1/(1-\alpha)} \quad (68) \]

Hence in the steady state, the real wage has to grow at the same rate as the capital stock. This means that equation (68) actually the dynamic, endogenous growth version of the familiar macroeconomic textbook price setting function: It gives the real wage growth rate compatible with marginal costs remaining constant and at it’s
long run level. Unlike the textbook price setting function, this real wage growth rate is not constant but increases in employment: A higher steady state employment level implies a higher marginal product of capital, which triggers higher investment and thus faster capital stock- and thus productivity growth. Accordingly, the steady state levels of employment and the growth rate are determined by the intersection of (67) with the wage setting function (20), (making again use of the fact that \( mc = \mu^{-1} \) in the steady state).

In practice, we choose a desired steady state employment rate (here 0.96) and then compute the wage setting function intercept \( a \) to support this value, given \( g \), \( b \) and \( \alpha \) and \( \pi \).

Having determined \( g \) and \( n \), the determination of the steady state values of \( F_t, D_t, R_t, H_t, r^k_t \) and \( i_t \) is now straightforward. For \( F \) we have

\[
F = A(((n - \pi - n^s) \phi_1)^{1-\alpha} 
\]  \quad (69)

from the production function. For \( R_t \), we have from the capital accumulation equation in (54)

\[
R = g + \delta 
\]  \quad (70)

\( D \) can then be determined as a residual via

\[
D = F - R 
\]  \quad (71)

\( H \) is computed using the cost-minimisation first order condition for labour (16)

\[
H = (1 - \alpha)\mu^{-1} \frac{F}{n - \pi - n^s} 
\]  \quad (72)

\( r^k \) is computed via

\[
r^k = \alpha \mu^{-1} A((n - \pi - n^s) \phi_1)^{1-\alpha} 
\]  \quad (73)

The steady state value of \( i_t \) is computed from (50)

\[
i = \frac{1 + g}{\beta} - 1 
\]  \quad (74)

Note that this is also the intercept of the interest rate rule \( \tilde{i} \) of the central bank.

10 Appendix C: Normalised Version of the JLN Economy

Most of the equations from Appendix B just carry over to the JLN economy. However, there are a few changes related to the production function and the marginal cost equation. The aggregate production function is now \( Output_t = AK^\alpha_t (TFF_t\phi_1 (n_t - \pi - n^s))^{1-\alpha} + w_t n^s \). Dividing both sides by \( K_t \) gives

\[
F_t = (l_t \phi_1 (n_t - \pi - n^s))^{1-\alpha} + H_t n^s 
\]  \quad (75)
where \( l_t \) is defined as \( \frac{T_{FP} t}{K_t} \). This variable evolves according to

\[
l_t = \frac{1 + g_{TFP} t}{1 + g^K_t} l_{t-1}
\]

(76)

In the JLN model, it convenient to normalise the real wage with respect to \( T_{FP} t \) rather than with respect to \( K_t \), while all the remaining normalisations carry over to the JLN model. Denoting \( \frac{w_t}{T_{FP} t} \) as \( H^\text{nc}_t \), we have from (42), after making use of (40)

\[
m_c t = \frac{F_t^{(\alpha/(1-\alpha))} H^\text{nc}_t}{A^{1/(1-\alpha)}} (1 - \alpha) \phi_1
\]

(77)

Concerning the capital rental, we employ the JLN expression for \( F_t \) to have

\[
r^k_t = \alpha m_c t A_t^{1-\alpha} \left( (n_t - \bar{n} - n^s) \phi_1 \right)^{1-\alpha}
\]

(78)

The normalised wage setting equation becomes

\[
H^\text{nc}_t = \exp(a + b (n_t - \bar{n})) \frac{H^\text{nc}_{t-1}}{(1 + g_{TFP})} \left( (1 - \alpha) m_c_{t-1} \right)^c
\]

(79)

All the remaining equations are just the same as in the New Growth version. The computation of the steady state values in the neoclassical model is slightly different. The steady state growth rate (of output, consumption, the capital stock, the real wage) is now given by the parameter \( g_{TFP} \) rather than being endogenously determined, which means we have \( g = g_{TFP} \). Hence we can compute the steady state real interest rate from 74, while we compute \( r^k \) from (66). From (79), we have the steady state employment rate. Setting \( m_c t = \mu^{-1} \) in (78) then gives the steady state value for \( l_t \) as

\[
l = \frac{1}{(n_t - \bar{n} - n^s) \phi_1} \left( \frac{r^k \mu}{\alpha A} \right)^{1/(1-\alpha)}
\]

(80)

which allows us to compute \( F \) from (75). Rearranging (77) then yields \( H^\text{nc} \).

11 Appendix E: Estimation of the Wage Setting Function

We estimate the real wage growth function using German data ranging from 1970Q1 to 2000Q4. Our dataset includes Western German data up to 1991Q4 and following that data for the unified country. All data is taken from a publication of the German "Statistisches Bundesamt", all of which has been seasonally adjusted.\(^{26}\) When estimating the function, we replace the employment rate with one

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minus the unemployment rate. As a measure for labour costs, we use the "Arbeitnehmerentgeld" per hour worked, which is employee compensation including the full tax wedge. This is deflated using the GDP price index. The labour share is given by total nominal compensation (i.e. total "Arbeitnehmerentgeld") divided by total nominal GDP. Denoting the unemployment rate as $U$, we then estimate 
\[
\Delta \log w_t = a + b \times U_t + c \log (LS_{t-1}) + d_{1992Q1}, \]
where $LS_{t-1}$ denotes the previous periods labour share in GDP, $d_{1992Q1}$ denotes an intercept dummy equaling one in 1992Q1 and zero everywhere else. The later is to account for reunification. We use two stage least squares to account for the possible endogeneity of employment. As instruments, we choose $\Delta \log$ real wage$_{t-1}$, unemploymentrate$_{t-1}$ (following Danthine and Kurman (2004)), $c$ and $d_{1992Q1}$.$^{27}$

Note that we use Newey-West Standard Errors serial correlation consistent standard errors because the Breusch-Godfrey LM test for serial correlation rejects the hypotheses of no serial correlation at the 5% level. The result is reported in table E1, where WG denotes the change in log real wages and $U$ denotes the unemployment rate.

---

$^{27}$See Danthine/ Kurman (2004), p. 121.
Table E1
Dependent Variable: WG
Method: Two-Stage Least Squares
Date: 06/03/08 Time: 12:27
Sample (adjusted): 1970Q3 2000Q4
Included observations: 122 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=4)
Instrument list: WG(-1) C U(-1) LOG(LS(-2)) D92Q1

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.042273</td>
<td>0.018893</td>
<td>-2.237531</td>
<td>0.0271</td>
</tr>
<tr>
<td>U</td>
<td>-0.120587</td>
<td>0.046582</td>
<td>-2.588689</td>
<td>0.0108</td>
</tr>
<tr>
<td>LOG(LS(-1))</td>
<td>-0.089599</td>
<td>0.032530</td>
<td>-2.754342</td>
<td>0.0068</td>
</tr>
<tr>
<td>D92Q1</td>
<td>-0.112300</td>
<td>0.002188</td>
<td>-51.33630</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared 0.574362 Mean dependent var 0.005890
Adjusted R-squared 0.563541 S.D. dependent var 0.014077
S.E. of regression 0.009300 Sum squared resid 0.010205
F-statistic 52.46032 Durbin-Watson stat 2.535359
Prob(F-statistic) 0.000000

Note that our calibrated value of \( \beta \) is lower than the point estimate of 0.12. However, it is not statistically different from 0.12 with any reasonable level of confidence, in fact it is less than one standard deviation away from the point estimate. The reason for this choice is that while it is possible to preserve the results of this paper in face of higher wage flexibility, this calibration has certain undesirable features. If we aim to achieve a steady growth rate of GDP in the order of magnitude of a reasonable order of magnitude (and one that makes lifetime utility converge), we would have to choose either relatively high depreciation rates or a lower individual discount factor \( \beta \), the latter implying a very high steady state risk less rate. We think that these considerations justify the choice of a value smaller than the point estimate.

The reason why the coefficient of the labour share \( c \) also falls short of the calibrated coefficient, though the distance is again less than one standard deviation. This is due to the fact that we have experimented with different computations of the labour share in GDP. The alternative computation was based on real values of GDP and employee compensation. The later computations methods generated a value of -0.12 than the -0.1 we use in the simulations. It is not a priori clear which measure is more appropriate. In fact to is common to interpret the labour share term as real wages divided by productivity (i.e. real GDP/hour or real GDP/ employee) and enter these variables separately.\(^{28}\) Furthermore, a reduction of \( c \) by 0.01 has only small effects on


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our simulation results.

For the United States, we estimate the wage setting equation using the BLS series on real hourly compensation, BLS series PRS85006153, to calculate $\Delta \log w_t$, the seasonally adjusted unemployment rate, series LNS14000000Q, nominal GDP from the BEA NIPA table 1.1.5 and total nominal employee compensation from the BEA NIPA table 2.1. In order to get a significant coefficient on the unemployment rate, we were forced to include five years more than in our estimate for Germany. The result can be obtained from Table E2. As expected, the LS is not significant. Re-estimating the equation after dropping $\log (LS_{t-1})$ leads to an almost unchanged estimate of the coefficient on the unemployment rate. Note that the coefficient on the unemployment rate in Germany is not statistically different from the coefficient estimated for the U.S. at any reasonable level of confidence.
Table E2
Dependent Variable: WG
Method: Least Squares
Date: 06/03/08 Time: 16:03
Sample: 1965Q1 2000Q4
Included observations: 144
Newey-West HAC Standard Errors & Covariance (lag truncation=4)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.005364</td>
<td>0.033562</td>
<td>0.159838</td>
<td>0.8732</td>
</tr>
<tr>
<td>U</td>
<td>-0.066995</td>
<td>0.033021</td>
<td>-2.028888</td>
<td>0.0444</td>
</tr>
<tr>
<td>LS</td>
<td>0.003206</td>
<td>0.059322</td>
<td>0.054043</td>
<td>0.9570</td>
</tr>
</tbody>
</table>

R-squared 0.030102  Mean dependent var 0.003202
Adjusted R-squared 0.016345  S.D. dependent var 0.006138
S.E. of regression 0.006088  Akaike info criterion -7.344412
Sum squared resid 0.005226  Schwarz criterion -7.282541
Log likelihood 531.7977  F-statistic 2.188064
Durbin-Watson stat 1.642216  Prob(F-statistic) 0.115927

Table 3
Dependent Variable: WG
Method: Two-Stage Least Squares
Date: 06/03/08 Time: 13:15
Sample: 1965Q1 2000Q4
Included observations: 144
Instrument list: WG(-1) C U(-1)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>-0.067605</td>
<td>0.032425</td>
<td>-2.084951</td>
<td>0.0389</td>
</tr>
<tr>
<td>C</td>
<td>0.007253</td>
<td>0.002008</td>
<td>3.612856</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

R-squared 0.030069  Mean dependent var 0.003202
Adjusted R-squared 0.023238  S.D. dependent var 0.006138
S.E. of regression 0.006077  Sum squared resid 0.005226
F-statistic 4.347020  Durbin-Watson stat 1.643279
Prob(F-statistic) 0.038864
12 Appendix F: Construction of the Dataset used in the Moment Comparison

This appendix explains the construction of the dataset for $F_t, D_t, R_t$ and $H_t$. The German federal statistical office ("Statistisches Bundesamt") supplies annual data for the capital stock in constant prices of the year 2000.\(^{29}\) Thus we had to construct quarterly observations for the capital stock. We decided on the following method. We first calculated the annual change. Then we allocated the total changed to the four quarters according to the share these quarters had in real gross fixed investment. This gave as a beginning of quarter value for the capital stock.

Our data on real output, consumption and investment expenditure was preferably also to be in prices of 2000. However, the Statistisches Bundesamt only supplies chained indices for these variables.\(^{30}\) We therefore used nominal GDP, consumption and investment 2000 to recursively calculate our series in absolute numbers. As the indices for post and pre reunification years have different bases, we used the ratio of unified Germany to Western Germany from 1991 to downscale the index for each variable.

Furthermore, as the total labour force in our model is normalised to one, Output, consumption and investment are essentially expressed in per capita terms in our model, as is the capital stock. Hence case of $F_t, D_t$ and $R_t$, the number of inhabitants cancels out and we can divide real GDP by our capital stock measure, and accordingly for $D_t$ and $R_t$. By contrast, $H_t$ is computed by multiplying the real wage as measured in the previous section times the average number of hours worked across the sample. We tried a linear trend for hours worked instead but this would have turned our measure of $H_t$ non-stationary.

The null of stationarity is rejected at the 5% level for $D_t$ and $F_t$ using the KPSS test. After removing the years 70 to 73, we are not rejecting the null of stationarity anymore at the 10% level for these variables. For $H_t$, the null of stationarity is not rejected at the 5% level for the full sample. For $R_t$, the unit root can be rejected over the entire sample at the 5% level using an ADF test, as is the case for $g_t$ and the savings rate. The same holds for the nominal interest rate, and so we do not detrend this variable either. We do detrend the inflation rate, because the null of stationarity is rejected for this variable using a KPSS test.

References


\(^{29}\)See statistisches Bundesamt (2006b), table 3.2.19.1.


[27] Juillard, M. (1996), Dynare: A program for the resolution of and simulation of dynamic models with forward variables through the use of relaxation algorithm, CEPREMAP.


